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Abstract

This study builds a dynamic balance-of-payments-constrained (BOPC) model that incorporates the endogenous determination of the economic growth rate, conflictive wage/price distribution, and employment rate. Following the Kaleckian–Marxian literature, wages and commodity prices are determined by the reserve army effect and employment is determined by the reserve army creation effect. The relative strength of these two effects generates different outcomes for the transitional dynamics and comparative statics analysis. In particular, the model shows stability, instability, and a cyclical nature, the latter of concurs with the evidence reported by previous empirical studies.

Keywords: Balance-of-payments-constrained model, Conflictive income distribution, Cyclical growth

JEL Classification: E12, F43, E24, E32

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1 Introduction

This study presents a dynamic balance-of-payments-constrained (BOPC) growth model that has three differences from the conventional BOPC growth model, namely the dynamic determination of the (i) BOPC growth rate, (ii) conflictive wage/price distribution, and (iii) employment rate. The BOPC growth model is a growth theory in post-Keynesian economics. The model postulates that the balance-of-payments position of a country limits effective demand, to which supply can usually adapt. That is, BOPC growth serves as an upper bound on the actual growth rate. Numerous contributions have been made since the seminal work of Thirlwall (1979). Soukiazis and Cerqueira (2012) comprehensively summarize recent contributions on the history, theory, and empirical evidence of BOPC growth. Theoretical research covers many topics such as incorporating capital flows and interest payments (Thirlwall and Hussain (1982); Moreno-Brid (2003); Barbosa-Filho (2001); Alleyne and Francis (2008)), a multi-sectoral model with many tradable goods (Araujo and Lima (2007); Araujo (2013); Araujo et al. (2013)), and an integral approach to internal and external imbalances (Soukiazis et al. (2013-2014, 2014)).

However, as these studies have focused on the equilibrium path of the economy, the nature of transitional dynamics has thus far remained ambiguous. In addition, most models have formalized the BOPC growth model in terms of international trade. Consequently, they have overlooked the effects of the income distribution and employment on trade competitiveness. Indeed, although the causes and consequences of employment and income distribution dynamics are relevant research topics in post-Keynesian economics, their impacts on BOPC growth have not yet been examined.

In this vein, the present study addresses the following three points. First, most BOPC growth models analyse economic growth on the equilibrium path. Their analysis is thus limited to the steady-state path, and the transitional dynamics towards the long-run path are not investigated. In other words, existing models by nature exclude the situation when a gap between the growth rates of exports and imports exists. Given the trade imbalance in the real world, it is empirically unrealistic to suppose that this is the case. The BOPC growth model should theoretically investigate not only the on-path process but also the off-path process caused by balance-of-payments disequilibria, which motivates the current study to consider the transitional dynamics in the BOPC growth model. Such an extension reveals the potential instability involved in the standard BOPC growth model.

Second, the BOPC growth model generally assumes away the effects of relative prices or the real exchange rate in deriving the long-run growth rate, and then derives Thirlwall's law. These effects are supposed to be neutral on economic growth, either because the price elasticities of exports and imports are assumed to be low or because the evolution of the exchange rate tends to follow purchasing power parity (PPP) in the long run. Consequently, the interrelationship between the determinants of relative prices and BOPC growth remains unexplored in the literature. The evolutions of nominal exchange and inflation follow different mechanisms. For example, inflation is, according to post-Keynesian economics, an outcome of the class conflict over the income distribution in the economy (Rowthorn (1977)). The change in the labour productivity growth rate also affects inflation. On the contrary, the evolution of the nominal exchange rate may be affected by other factors such as the interest rate difference and political interventions (Carlin and Soskice (2015)). As a result, the nominal exchange rate may not absorb countries' inflation at each point in time. Taking these factors into consideration, the current model analyses a case in which a price change has a strong effect on relative prices, which induces the dynamics of BOPC growth.[1]

Lastly, I introduce the endogenous determination of the labour productivity growth rate and employment rate. If labour productivity is constant over time, then labour demand growth is determined by the BOPC growth rate, which most BOPC growth models suppose implicitly. These dynamics run from the goods market to the labour market and there is no feedback from the latter to the former. This is an analytically insufficient setting in two senses. Firstly, if labour demand growth is insufficient to cover labour supply growth, the employment rate would be zero over time, which is unrealistic. By contrast, if labour supply growth is insufficient to cover labour demand growth, then the economic growth rate is eventually determined by the natural growth rate. Since the natural growth rate does not necessarily guarantee a balanced growth between exports and imports, an economy will perpetually have an imbalance of payments, which is un-

[1]In fact, a body of the empirical literature advocates the importance of the effect of relative prices on BOPC growth. On the basis of their empirical study, Bagnai et al. (2012) insist that the assumption of a constant real exchange rate, routinely made in most studies, is inappropriate. Regarding the effect of relative prices, Soukiazis et al. (2014) show that relative prices affect Italian growth, revealing that models without such prices under-predict the actual growth rate. Soukiazis et al. (2013-2014) find that models with non-neutral relative prices predict the growth rate better than does Thirlwall's law, and thus relative prices are relevant in BOPC growth. Bahmani-Oskoee (1995) reveals that PPP fails to hold for developing countries.

sustainable. Secondly, labour productivity growth is a key determinant of price competitiveness in international trade as well as labour demand. Therefore, the effect of productivity growth dynamics should not be assumed away, especially when one supposes that the commodity produced in each country is a substitute and that the home country is in price competition with the foreign country.[2] For these reasons, by introducing the dynamic determination of labour productivity growth, the current study analyses its impact on the BOPC growth rate and employment rate.

Taking these three analytical pillars into consideration, the current study contributes to the body of knowledge by revealing the mechanisms of stable, unstable, and cyclical BOPC growth. In particular, it adds new insights to previous findings on cyclical BOPC growth. Garcimartin et al. (2016) capture the business cycle as the difference in the short-run growth rates and the growth rate determined by Thirlwall's law (i.e. business cycles are merely the gap between these two growth rates). However, the model used in this study is more dynamic than theirs in the sense that it illustrates how the BOPC growth rate perpetually and endogenously deviates from Thirlwall's law. The current study shares inspirations from Goodwin (1967), Kalecki (1971), Thirlwall (1979), and Dávila-Fernández and Libaino (2016). However, the BOPC growth rate in their models is always stable and constant at Thirlwall's law. The current study, by contrast, reveals that the BOPC growth rate may be cyclical or unstable. Moreover, I theoretically prove the existence of cyclical BOPC growth and numerically depict the cyclical configuration. Similar to Barbosa-Filho (2001) and Pugno (1998), the current study considers the dynamic adjustment of the BOPC growth rate according to the trade imbalance over time. It further introduces the conflictive wage/price distribution and dynamic adjustment of the employment rate, which differentiates my model from that of Barbosa-Filho (2001) and Pugno (1998). Finally, the current study sheds light on the potential instability hidden in the basic BOPC growth model when combined with distributional and labour market dynamics.[3]

[2]This study exclusively focuses on the productivity effect on price competitiveness. The empirical evidence suggests that a change in labour productivity also has a favourable impact on non-price competition (Romero and MacCombie, 2017). However, this is beyond the scope of the current study.

[3]Since the current model is similar to that of Pugno (1998) in that both construct a dynamic model including the income distribution and employment rate and generate cyclical behaviour in the BOPC growth model, it is better to elaborate on the difference in detail. His model contains autonomous demand, the export-import ratio, the real exchange rate, as well as labour demand and supply. Although he simulates his model and shows cyclical behaviour, the driving force behind these dynamics is unclear because of the inherent complexity. In addition, his proof of local stability is insufficient because it only explains a part (i.e. trace and determinant) of the Routh–Hurwitz stability

The remainder of the paper is organized as follows. Section 2 sets up the BOPC growth model in which the BOPC growth rate, conflictive wage/price distribution, and employment rate are all endogenously determined. Section 3 investigates the dynamic properties of the model. Section 4 numerically confirms the cyclical nature of the model analysed in the previous section. Section 5 is a comparative statics analysis of some of the parameters. Finally, Section 6 concludes.

2 Model

The main notations for the home country used in this study are as follows: Y_{Dt}: total output (total income), L_t: labour demand, K_t: capital stock, N_t: labour supply, q_t: labour productivity, p_t: commodity price, w_t: nominal wage rate, e_t: nominal exchange rate between two countries, X_t: export demand, M_t: import demand, σ_t: wage share, z_t: employment rate. Variables with t refer to time changes over time, but I omit the subscript t for parsimony. These notations are employed to express the home country's variables unless stated otherwise. The same variables in the foreign country are expressed by adding the subscript F to the variable (e.g. the commodity price in the foreign country is p_F).

This study considers the international trade of commodities between the home country and foreign country, focusing on the determinants of economic growth in the former. The firms in both countries produce the same commodity, but at different prices. The commodity produced in each country is a substitute and the home country is in price competition with the foreign country. When the commodity price of the home country is higher than that of the foreign country, its exports (imports) decrease (increase), and vice versa.

The firms in the home country operate with the following fixed coefficient production function using labour and capital:

$$Y_D = \min(qL, u_n K),$$

meaning they are producing at a normal rate of capacity utilization u_n. I assume that the normal rate of capacity utilization is constant over time. If labour and capital are efficiently used in the

condition for higher-dimensional differential equations. If one defines a five-dimensional dynamic model, it is necessary to check the principle minor of the Jacobian matrix as well as the trace and determinant, which is not clear in his study. Instead of his model, I straightforwardly focus on the dynamics of the BOPC growth rate, the income distribution, and labour demand. As the model consists of three dimensions, it thus proves the local stability and these dynamics are explained based on Marxian and Kaleckian concepts.

home country, the output is produced by the following condition $Y_D = u_n K = qL$, of which the dynamic expression is

$$\hat{Y}_D = \frac{\dot{u_n}}{u_n} + \frac{\dot{K}}{K} = \frac{\dot{q}}{q} + \frac{\dot{L}}{L}, \tag{1}$$

where the dot symbol means the derivative of the variable with regard to time (e.g. $\dot{x} = dx/dt$) and the hat symbol means the growth rate of the variable (e.g. $\hat{x} = \dot{x}/x$). Since the normal rate of capacity utilization is constant, $\dot{u_n}/u_n = 0$, and the growth rates of capital stock and actual output are the same. Capital accumulation is supposed to be accompanied by BOPC growth.

The export and import demand functions are formalized by using the following conventional BOPC growth model. The demand functions for each commodity are given by the Cobb–Douglas functional form. First, the export demand function for the commodity is given by

$$X = \bar{X}\left(\frac{ep_F}{p}\right)^{\varepsilon_1} Y_F^{\eta_1}, \tag{2}$$

where $\bar{X}$ is a constant term, $\varepsilon_1 > 0$ is the elasticity of relative prices, and $\eta_1 > 0$ is the income elasticity of demand for exports. This formalization means that if the real exchange rate depreciates (i.e. a rise in ep_F/p), the export demand of the home country's commodity increases. Equation (2) also means that booms in the foreign country (i.e. a rise in Y_F) induce higher export demand for the home country's commodity.

By taking the logarithms of equation (2) and differentiating with respect to time, the growth rate of exports is obtained as follows:

$$\hat{X} = \varepsilon_1(\hat{e} + \hat{p}_F - \hat{p}) + \eta_1 \hat{Y}_F. \tag{3}$$

This is the dynamic form of the export demand function.

Second, the import demand function for the foreign country's commodity is given by

$$M = \bar{M}\left(\frac{ep_F}{p}\right)^{-\varepsilon_2} Y_D^{\eta_2}, \tag{4}$$

where $\bar{M}$ is a constant term, $\varepsilon_2 > 0$ is the elasticity of relative prices, and $\eta_2 > 0$ is the income elasticity of demand for imports of the foreign commodity. When the real exchange rate depreciates, the import demand of the foreign country's commodity decreases. Equation (4) also means that an increase in the home country's income (i.e. a rise in Y_D) induces higher import demand for the foreign country's commodity.

The dynamic form of the import demand function is derived by following the same procedure as above. That yields

$$\hat{M} = -\varepsilon_2(\hat{e} + \hat{p}_F - \hat{p}) + \eta_2 \hat{Y}_D. \tag{5}$$

Jointly with $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, I assume that the Marshall–Lerner condition with respect to trade holds. That is, $\varepsilon_1 + \varepsilon_2 > 1$, and exchange rate depreciation (appreciation) improves (deteriorates) the trade balance.

The BOPC condition is measured in nominal terms at a macroeconomic level. If there is a trade balance at the aggregate level at a point in time, it is given by

$$pX = p_F eM,$$

where the left-hand side represents the total value of exports in the home country and the right-hand side represents the total value of imports in the home country. As mentioned above, most BOPC growth models assume that trade is initially balanced, and this balance is also kept over time. This case is given as follows:

$$\hat{p} + \hat{X} = \hat{p}_F + \hat{e} + \hat{M}.$$

I now relax this assumption and introduce the case in which there are disequilibria of the balance of payments. These disequilibria are then dynamically adjusted by the change in the economic growth rate of the home country. That is, when nominal export demand grows faster (slower) than nominal import demand, the growth rate of the home country increases (decreases) to recover the balance of payments. Thus, the Keynesian quantitative adjustment rather than the neoclassical price adjustment is supposed to restore the balance of payments (Blecker (1998)). By using equations (3) and (5), an equation of motion for the growth rate of the home country is defined by

$$\begin{aligned} \dot{\hat{Y}}_D =& \phi(\hat{p} + \hat{X} - \hat{p}_F - \hat{e} - \hat{M}) \\ =& \phi(\hat{p} + \varepsilon_F(\hat{e} + \hat{p}_F - \hat{p}) + \eta_1 \hat{Y}_F - \hat{p}_F - \hat{e} + \varepsilon_2(\hat{e} + \hat{p}_F - \hat{p}_i) - \eta_2 \hat{Y}_D), \end{aligned} \tag{6}$$

where ϕ denotes the speed of adjustment of the growth rate of the home country regarding the disequilibria in the balance of payments.[4]

[4] This study does not explicitly incorporate the role of capital flows. Some BOPC growth models introduce

When the trade balance is realized, the time rate of the change in exports and imports must be equal. Therefore, $\dot{\hat{Y}}_D = 0$ is satisfied and I get

$$\hat{Y}_D = \frac{\eta_1}{\eta_2}\hat{Y}_F - \frac{\epsilon}{\eta_2}(\hat{e} + \hat{p}_F - \hat{p}), \tag{7}$$

where $\epsilon \equiv 1 - \varepsilon_1 - \varepsilon_2$. The sign of ϵ is negative because the Marshall–Lerner condition $\varepsilon_1 + \varepsilon_2 > 1$ has been imposed. The current study does not assume away the role of a change in relative prices and considers its direct and indirect impacts on the BOPC growth rate.

By introducing the rate of change in the commodity price, the current study emphasizes the dynamic chain of the conflictive wage/price distribution in the post-Keynesian literature (e.g. Rowthorn (1977); Sasaki (2013); Lavoie (2014)). When introducing these dynamics into the BOPC growth model, it is important that the real exchange rate does not evolve so that the balance of payments can be realized at an ideal level; however, this change is affected by the domestic distributive conflict. Thus, domestic conflicts (aspiration for target shares of the income distribution) may cause international conflicts (disequilibrium in the balance of payments) and vice versa.

The definition of the wage share is $\sigma = w/(pq)$, from which I obtain the dynamic expression of the wage share as follows:

$$\dot{\sigma} = \sigma(\hat{w} - \hat{p} - \hat{q}). \tag{8}$$

The conflictive wage/price distribution describes the dynamics of the money wage and price in the following manner. Firms set their price to close the gap between their target profit share and the actual profit share. If the actual profit share is lower than their target profit share, then firms attempt to raise the growth rate of prices to realize the target share. By using the wage share target, the dynamics of the price can be described as follows:

$$\hat{p} = \beta(\sigma - \sigma_C), \tag{9}$$

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capital flows according to trade imbalances and relax the external constraint for an economy during a certain period (Thirlwall and Hussain (1982); Moreno-Brid (1998-99, 2003); Barbosa-Filho (2001)). However, the role of capital flows is still controversial since their incorporation into the model may make little difference to the predicted growth rates and export growth is important (Thirlwall (2012)). In addition, capital inflows can be only a temporary way of relaxing the BOPC model because they do not stimulate the export-led cumulative growth rate in the long run. After all, economies cannot depend on capital flows to finance trade imbalances permanently, and the adjustment in aggregate income eventually accommodates such imbalances. Hence, I analyse the basic BOPC growth model of export and import dynamics with an aggregate income adjustment.

where $\sigma_C > 0$ is the target wage share set by firms, which I assume to be an exogenous variable for simplicity.

The growth rate of the money wage that workers negotiate depends on the gap between their target wage share and the actual wage share. If the actual wage share is lower than the target wage share, workers attempt to raise the growth rate of wages to meet the target share. That is,

$$\hat{w} = (1-\beta)(\sigma_W(z) - \sigma), \quad \sigma_W(z) > 0, \quad \sigma_w'(z) > 0, \tag{10}$$

where $\sigma_W(z)$ is the target wage share set by workers. I assume that $\sigma_W(z^*) > \sigma_C$ for their target wage share because firms and workers normally demand a higher profit share and a higher wage, respectively. This is endogenously determined by the "reserve army effect." This effect reflects workers' demands in the bargaining process, which change depending on their position in the labour market. A high employment rate strengthens the position of workers to negotiate a higher target wage share. In equations (9) and (10), β and $1-\beta$ represent the relative bargaining power of workers and firms across the income distribution, respectively, which also concern the speed of the wage and price adjustment.[5]

The third state variable is the employment rate z. From the production function, I obtain $z = Y_D/qN$, where N denotes the exogenous labour supply growing at the exogenously given growth rate n. Hence, the change in the employment rate follows

$$\dot{z} = z(\hat{Y}_D - g_q(z) - n), \quad g_q(z) > 0, \quad g_q'(z) > 0, \tag{11}$$

where the reserve army creation effect $g_q'(z) > 0$ is introduced into the labour productivity growth dynamics. This effect stipulates that the growth rate of labour productivity depends positively on the employment rate. Bhaduri (2006), Dutt (2006), and Sasaki (2013) introduce this effect into their domestic models of growth and distribution, but I apply this effect to the BOPC growth model. It is reasonable to simultaneously introduce this effect with the reserve army effect into the current model for the following reason. The model focuses on the effect of relative price changes on international trade. The upward pressure on the target wage share is one of the determinants of the rise in the commodity price. As the labour market tightens via economic

[5] Sasaki et al. (2013)'s Kaleckian model considers the degree of price competition in wage bargaining and price setting. Such a case may be appropriate to describe the corporatist economy in which labour and capital coorporate to some extent. The background of the current study differs from theirs by illustrating the case in which internal and external conflicts collude independently.

growth, the bargaining power of workers increases because of the reserve army effect, which exerts upward pressure on the target wage share. Without productivity growth, this aspiration leads to a rise in the commodity price. The deterioration of price competitiveness consequently leads to a decrease in net export growth under the Marshall–Lerner condition. To avoid the deterioration of price competitiveness, firms try to introduce labour-saving technological progress to maintain competitiveness. Hence, simultaneously introducing both effects is legitimate in the current BOPC growth model with non-neutral price effects.

3 Analysis

The dynamic system of the BOPC growth model consists of three differential equations. By substituting the price equation (9) into equation (6), I obtain the dynamics of the output growth rate $\dot{\hat{Y}}_D$. By substituting the price equation (9), wage dynamics (10), and productivity growth equation (11) into equation (8), I obtain the dynamics of the wage share $\dot{\sigma}$. Finally, equation (11) defines the dynamics of the employment rate $\dot{z}$. Then, the system is summarized as follows:

$$\dot{\hat{Y}}_D = \phi(\epsilon\beta(\sigma - \sigma_C) - \epsilon(\hat{e} + \hat{p}_F) + \eta_1 \hat{Y}_F - \eta_2 \hat{Y}_D), \tag{12}$$

$$\dot{\sigma} = \sigma((1 - \beta)\sigma_W(z) + \beta\sigma_C - g_q(z) - \sigma), \tag{13}$$

$$\dot{z} = z(\hat{Y}_D - g_q(z) - n). \tag{14}$$

The steady state of the system occurs when $\dot{\hat{Y}}_D = \dot{\sigma} = \dot{z} = 0$ is reached. In the following analysis, it is assumed that there exist steady-state values such that $\sigma^* \in (0, 1)$, $z^* \in (0, 1)$, and $\hat{Y}_D^* > 0$, where the asterisk represents the steady-state value. These values satisfy the following equations:

$$\hat{Y}_D^* = \frac{\eta_1}{\eta_2}\hat{Y}_F + \frac{\epsilon}{\eta_2}\beta(\sigma^* - \sigma_C) - \frac{\epsilon}{\eta_2}(\hat{e} + \hat{p}_F), \tag{15}$$

$$\sigma^* = (1 - \beta)\sigma_W(z^*) + \beta\sigma_C - g_q(z^*), \tag{16}$$

$$\hat{Y}_D^* = g_q(z^*) + n. \tag{17}$$

The standard BOPC growth model assumes PPP and a trade balance in the initial period as well as over time. If I also assume $\hat{e} = \hat{p} - \hat{p}_F$, then I obtain $\hat{Y}_D^* = \frac{\eta_1}{\eta_2}\hat{Y}_F$. Hence, the above dynamic system is reduced to a two-dimensional system composed of the wage share and employment rate. In this case, the BOPC growth rate follows Thirlwall's law, and the steady state of the two-dimensional subsystem is locally stable according to the Jacobian matrix defined below.

In the current model, however, when the Marshall–Lerner condition holds, the conflictive wage/price distribution effectively intervenes in the dynamics of the BOPC growth rate. Then, the growth rate feeds back to the dynamics of the employment rate (equation 14), which further causes a change in the productivity growth rate and target wage share from the reserve army effect and reserve army creation effect, respectively (equation 13). Accordingly, both the commodity price and the nominal wage dynamically evolve further, which again gives feedback to the BOPC growth rate through the wage share (equation 12). If the steady state is locally stable, the BOPC growth rate is equal to the natural growth rate (equation 17). Because the growth rate of labour productivity is endogenously determined in this three-dimensional model, the natural rate of growth is also endogenous, which Leon-Ledesma and Thirlwall (2002) empirically confirm.

To investigate the local asymptotic stability of the steady state, the system of differential equations (12), (13), and (14) is linearized around the steady state. The linearized system is given by

$$\begin{pmatrix} \dot{\hat{Y}}_D \\ \dot{\sigma} \\ \dot{z} \end{pmatrix} = \underbrace{\begin{pmatrix} j_{11} & j_{12} & 0 \\ 0 & j_{22} & j_{23} \\ j_{31} & 0 & j_{33} \end{pmatrix}}_{\boldsymbol{J}} \begin{pmatrix} \hat{Y}_D - \hat{Y}_D^* \\ \sigma - \sigma^* \\ z - z^* \end{pmatrix},$$

where $\boldsymbol{J}$ is the Jacobian matrix. The non-zero elements of the Jacobian matrix and their signs are given as follows:

$$j_{11} \equiv \frac{\partial \dot{\hat{Y}}_D}{\partial \hat{Y}_D} = -\phi\eta_2 < 0,$$

$$j_{12} \equiv \frac{\partial \dot{\hat{Y}}_D}{\partial \sigma} = \phi\epsilon\beta < 0,$$

$$j_{22} \equiv \frac{\partial \dot{\sigma}}{\partial \sigma} = -\sigma^* < 0,$$

$$j_{23} \equiv \frac{\partial \dot{\sigma}}{\partial z} = \sigma^*\Theta \gtrless 0,$$

$$j_{31} \equiv \frac{\partial \dot{z}}{\partial \hat{Y}_D} = z^* > 0,$$

$$j_{33} \equiv \frac{\partial \dot{z}}{\partial z} = -z^* g_q'(z^*) < 0,$$

where

$$\Theta \equiv [(1-\beta)\sigma_w'(z^*) - g_q'(z^*)] \gtrless 0.$$

All the elements are evaluated at the steady-state values. Except for element j_{23}, the signs of the elements are unambiguous. Regarding the non-zero elements, the BOPC growth rate (j_{11}), wage share (j_{22}), and employment rate (j_{33}) all have self-stable dynamics. Further, a rise in the wage share lowers the BOPC growth rate (j_{12}) by deteriorating price competitiveness, and the rise in the BOPC growth rate raises the employment rate (j_{31}).

The sign of j_{23} depends on two parts: the reserve army effect augmented by workers' bargaining power $(1-\beta)\sigma_w'(z^*)$ and the reserve army creation effect $g_q'(z^*)$. Depending on the relative strength of these two effects, the sign may be either positive or negative. The sign of j_{23} plays an important role for both the stability of the equilibrium and the comparative statics analysis below. I consider the dynamic properties of the model by examining the following two cases.

3.1 A stronger reserve army creation effect (Case A)

Let us consider the case in which the reserve army creation effect $g_q'(z)$ works stronger than the reserve army effect augmented by workers' bargaining power $(1-\beta)\sigma_w'(z^*)$. In this case, I have $\Theta < 0$, and the sign of j_{23} is negative, meaning that a rise in the employment rate leads to an increase in the profit share owing to the strong impact of the reserve army creation effect on the labour productivity growth rate. Then, the following proposition is obtained.

Proposition 1. *Suppose that the reserve army creation effect works stronger than the reserve army effect augmented by workers' bargaining power. If the Marshall–Lerner condition is satisfied and $\epsilon < 0$, the steady state of the economy is locally stable when the income elasticity of import demand η_2 is sufficiently large; however, it is locally unstable when the income elasticity of import demand η_2 is sufficiently small.*

Proof. See Appendix A. □

In the BOPC growth literature, η_2 represents the income elasticity of import demand, which is considered to reflect the non-price competitiveness of the foreign country's commodity. A rise in this value resulting from increasing the attractiveness of the imported goods, *ceteris paribus* lowers the BOPC growth rate of the home country. This is also true for the current model (see also the comparative statics analysis). However, Proposition 1 indicates that a higher income elasticity of import demand is required to establish stable BOPC growth when the wage share negatively changes to a rise in the employment rate. This result implies that regarding the income elasticity of import demand, a high BOPC growth rate and its stability may involve a trade-off.

3.2 A stronger reserve army effect (Case B)

Case B is when the reserve army effect augmented by workers' bargaining power works stronger than the reserve army creation effect. Formally, $\Theta > 0$ and the sign of j_{23} is positive. This means that a rise in the employment rate leads to a rise in the wage share. Hence, a rise in the employment rate leads to a rise in the wage share because of the strong impact of the reserve army effect on workers' target wage share. Then, I obtain the following proposition.

Proposition 2. *Suppose that the reserve army effect augmented by workers' bargaining power works stronger than the reserve army creation effect. If the Marshall–Lerner condition is satisfied and suppose that the income elasticity of import demand is small. Then, a limit cycle occurs when the speed of the adjustment of the goods market lies within a certain range.*

Proof. See Appendix A. □

The next section confirms the existence of cyclical BOPC growth by using a numerical study. For instance, a rise in the BOPC growth rate leads to a rise in the employment rate. When the reserve army effect works strongly, it induces a rise in the wage share and simultaneously deteriorates price competitiveness. Consequently, the BOPC growth rate decreases, also leading to a fall in employment. A fall in employment in turn induces a decline in the wage share because of the strong reserve army effect and weak reserve army creation effect, improving price competitiveness. Then, the BOPC growth rate increases again, leading to a rise in employment. Thus, a new cycle begins, and the economy converges to a cyclical path.

However, once the feedback effect in this process is cut off, the long-run equilibrium becomes stable. In this case, the following proposition is obtained.

Proposition 3. *If the Marshall–Lerner condition is not satisfied in the sense of* $\epsilon = 0$ *or if the conflictive wage/price distribution has a neutral effect on the commodity price from* $\Theta = 0$*, the steady state of the economy is locally stable.*

Proof. See Appendix A. □

Comparing Propositions 1–3, both a strong reserve army creation effect (Case A) and a strong reserve army effect (Case B) may have a destabilizing effect on the economy subject to balance-of-payments constraints. Proposition 1 implies that a strong reserve army creation effect is a cause of unstable dynamics when the income elasticity of import demand is low. Proposition 2 implies that a strong reserve army effect is a cause of perpetual fluctuation, combined with a certain adjustment speed of the growth rate. Only when both effects are offset or the effect of the exchange rate is neutral is the steady state necessarily stable (Proposition 3).

4 Numerical Study

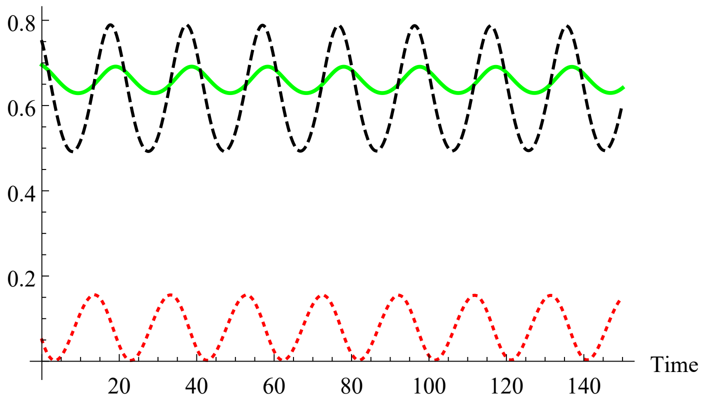
This section employs numerical simulations and shows that the Hopf bifurcation exists when a strong reserve army effect is working (Case B). The qualitative approach used here aims to show how the BOPC growth model behaves cyclically, which Proposition 2 declares. The basic parameters are set as follows:

$$\eta_1 = 0.775,\ \eta_2 = 0.018,\ \epsilon_1 = 0.775,\ \epsilon_2 = 0.700,\ \hat{p}_f = 0.0015,\ \hat{e} = 0.0015,\ \hat{Y}_F = 0.035,$$
$$\beta = 0.425,\quad \sigma_C = 0.500,\quad \delta = 0.575,\quad \gamma = 0.950,\quad q_0 = 0.1000,\quad \theta = 1.000,\quad n = 0.015.$$

By using these parameters, the function of workers' target profit share is defined as $\sigma_W(z) = \delta z^{\theta}$ and that of the productivity growth rate is $g_q(z) = q_0 z^{\gamma}$. In this numerical example, the parameters are set so that the reserve army effect augmented by workers' bargaining power works stronger than the reserve army creation effect. In addition, the Marshall–Lerner condition is satisfied, and the income elasticity of import demand is small. These ensure the preconditions for Proposition 2. To solve the differential equation systems, the initial conditions for the BOPC growth rate, wage share, and employment rate are $\hat{Y}_D(0) = 0.05$, $\sigma(0) = 0.695$, and $z(0) = 0.75$, respectively.

By using these parameters, the steady-state values of the endogenous variables are $\hat{Y}_D^* = 0.079475$, $\sigma^* = 0.658203$, and $z^* = 0.630027$. In addition, there exist two Hopf bifurcation points regarding the speed of the adjustment of the BOPC growth rate, which are $\phi_1 = 4.82401$ and $\phi_2 = 25.7941$. By using ϕ_1, the following figures present the dynamic behaviour of the main variables.

Figure 1 presents the oscillation of the BOPC growth rate, wage share, and employment rate. It shows that when an economy is trapped in this cycle, it never attains the economic growth rate



Note: The red dotted line represents the BOPC growth rate, green solid line represents the wage share, and black dashed line represents the employment rate.

Figure 1: Cyclical behaviour of the BOPC growth rate, wage share, and employment rate

that realizes the balance of payments. The BOPC growth rate reaches its peak (bottom) earlier than the employment rate in this cycle, meaning employment is led by effective demand. On the contrary, the BOPC growth rate moves in an opposite manner to the wage share, meaning that an increase in the wage share deteriorates the price competitiveness of a country and consequently restrains the BOPC growth rate.

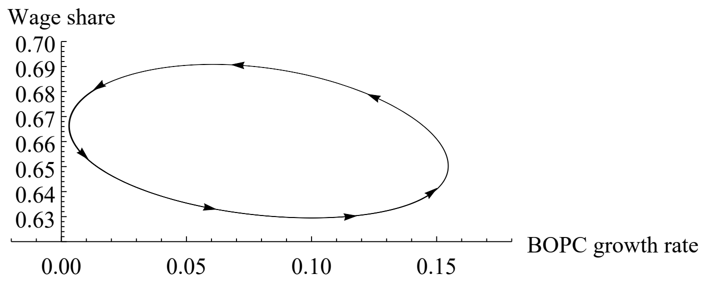


Figure 2: Anticlockwise cycles in the $(\hat{Y}_D, \sigma)$-plane in Case B

Figures 2, 3, and 4 decompose the time series of the main variables into two dimensions. Figure 2 projects the dynamics of the BOPC growth rate and wage share on the $(\hat{Y}_D, \sigma)$-plane, showing that the behaviour of the wage share and BOPC growth rate moves anticlockwise. Figure 3 presents the dynamics of the BOPC growth rate and employment rate on the $(e, \hat{Y}_D)$-plane. The behaviour of the employment rate and BOPC growth rate moves clockwise. This figure also confirms that the variation in the BOPC growth rate plays a leading role in determining the employment rate in the Keynesian fashion. Finally, Figure 4 projects the anticlockwise dynamics of

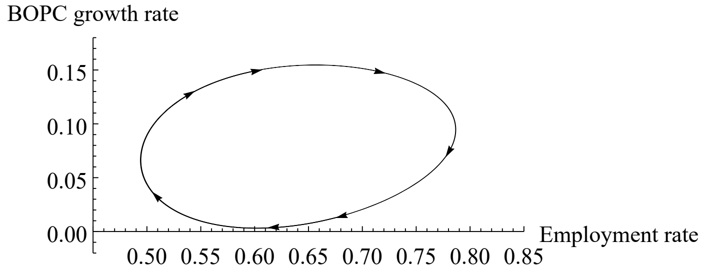


Figure 3: Clockwise cycles in the (e, $\hat{Y}_D$)-plane in Case B

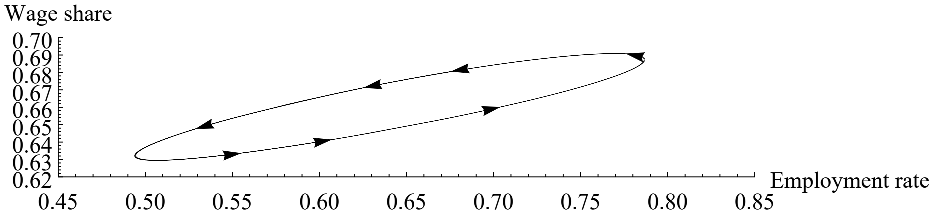


Figure 4: Anticlockwise cycles in the (e, σ)-plane in Case B

the employment rate and wage share, which are similar to those of Goodwin (1967). Zipperer and Skott (2011) and Barbosa-Filho and Taylor (2006) report regular anticlockwise cycles involving output (i.e. the capacity utilization rate) and the wage share in the US economy based on the Goodwin-based model of the domestic economy. However, as Figures 2 and 4 show, if the strong reserve army effect is combined with the BOPC model, anticlockwise cycles between the growth rate and wage share can be reproduced as well. Therefore, the observed cycles can be a form of what Stockhammer and Michell (2016) call the pseudo-Goodwin cycle, namely anticlockwise movements in the output–wage share space that are not completely due to the Goodwin mechanism. The BOPC growth mechanism also generates similar cyclical growth.

5 Comparative Statics Analysis

This section investigates the effects of a shift in the parameters on the BOPC growth rate, wage share, and employment rate for the stable case. Table 1 summarizes the results of the comparative statics analysis in Cases A (stronger reserve army creation effect) and B (stronger reserve army effect). The + sign indicates that the corresponding variable increases with a rise in the parameter, while the − sign indicates that the corresponding variable decreases with it. Appendix B provides the mathematical explanations.

Table 1: Results of comparative statics analysis

	Case A						Case B					
	$\hat{Y}_F$	η_1	η_2	$\hat{e}$	n	σ_W	$\hat{Y}_F$	η_1	η_2	$\hat{e}$	n	σ_W
$\hat{Y}_D^*$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$+$	$-$
σ^*	$-$	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$-$	$+$	$-$	$+$
z^*	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$

As in Thirlwall's law, the BOPC growth rate $\hat{Y}_D^*$ is positively led by a rise in the growth rate of the foreign country $\hat{Y}_F$ and a rise in the income elasticity of export demand η_1, whereas it is restrained by a rise in the income elasticity of import demand η_2. An acceleration in the depreciation rate of currency $\hat{e}$ also has a positive impact on the BOPC growth rate. The impact of these parameters on the employment rate is the same as the one for the BOPC growth rate. This result explicitly shows that the employment rate is demand-led in the BOPC growth model. These results hold in both Cases A and B.

The steady-state value of the wage share σ^* is negatively related to $\hat{Y}_F$, η_1, and $\hat{e}$, but positively related to η_2 in Case A. By contrast, it is positively related to $\hat{Y}_F$ and η_1, but $\hat{e}$ is negatively related to η_2 in Case B. When the BOPC growth rate positively diverges from the initial steady state, it increases the employment rate. This rise in the employment rate then stimulates the target wage share of workers through the reserve army effect as well as stimulates the labour productivity growth rate through the reserve army creation effect. If the latter is stronger than the former (Case A), the wage share decreases; by contrast, if the former is stronger than the latter (Case B), the wage share increases.

By introducing labour supply growth and the target wage share of workers into the current BOPC growth model, the current model can evaluate these impacts. In Case A, when the employment rate falls short of the initial steady-state value by a rise in the labour supply growth rate n, it strongly limits the productivity growth rate. Then, the actual wage share begins to rise and the commodity price also rises, which deteriorates the price competitiveness of the home country. Consequently, the BOPC growth rate and employment rate fall. In Case B, the same initial shock reduces the employment rate, which in turn strongly restrains the target wage share of workers. Accordingly, the actual wage share begins to fall, while the commodity price also falls. As a result, price competitiveness improves, which increases the BOPC growth rate and employment rate.

The impact of an autonomous rise in the target wage shares of workers σ_W decreases the BOPC growth rate and employment rate, while it raises the equilibrium wage share in both cases. A rise in workers' target share provides strong momentum to the dynamics of the wage share, which increases this value at the new steady state. At the same time, a rise in this share directly deteriorates price competitiveness and restrains net export growth. Because of the decrease in net export growth, the BOPC growth rate is lower than before. Consequently, the employment rate also decreases.

6 Conclusion

This paper presented a dynamic BOPC growth model incorporating a conflictive wage/price distribution and the endogenous determination of the labour productivity growth and employment rates. Most BOPC growth models have exclusively focused on the stable equilibrium path, assuming away the causes and consequences of relative prices. However, the commodity price in the home country may change through the bargaining process for the income distribution, change in labour productivity growth, and accordingly the employment rate. In addition, the adjustment process of the trade disequilibria in reality is not as immediate as the conventional model supposes. Therefore, interactions among the BOPC growth rate, income distribution, and employment rate may have a lasting effect on each other. By incorporating these ideas into the BOPC growth model, the current study revealed the potentially unstable properties of this model.

If one ignores the role of relative prices by assuming PPP or assuming away the Marshall–Lerner condition, the BOPC growth rate follows Thirlwall's law in a stable manner. By contrast, when a change in relative prices has a substantial impact on the trade balance, the steady state may become unstable in certain conditions. First, if the reserve army creation effect works stronger than the reserve army effect augmented by workers' bargaining power, when the income elasticity of import demand is low, the economy undergoes unstable dynamics. Although the lower income elasticity of imports has been considered to be one of the sources of higher BOPC growth, it is also a source of instability. Second, when the reserve army effect works stronger than the reserve army creation effect, if the income elasticity of import demand is low and the speed of the growth rate adjustment for the disequilibria in the balance of payments lies within a certain range, then limit cycles may occur. Thus, the lower income elasticity of imports is a source of cyclical BOPC growth as well. Section 4 confirms this phenomenon by using a numerical simulation. The anticlockwise cycles of the growth rate and employment rate for the wage share, which some empirical research has also shown for the US economy, can be observed in the current BOPC growth model.

This study also conducted a comparative statics analysis. A rise in the foreign growth rate and income elasticity of exports as well as the depreciation of the exchange rate or fall in that of imports raise the BOPC growth rate and employment rate. However, these impacts on the wage share differ according to the relative strength of the reserve army effect and reserve army creation effect. By introducing the labour supply growth rate and endogenizing the wage share, it was also revealed that rises in the labour supply growth rate and workers' target wage share also decrease the BOPC growth rate and employment rate in both cases. As for their impacts on the wage share, the impact of a rise in the labour supply growth rate differs from case to case, whereas an autonomous rise in workers' target wage share necessarily increases the wage share in both cases.

The conventional BOPC growth model arrives in most cases at Thirlwall's law, in which the BOPC growth rate remains constant. This law is independent of the income distribution and employment. By contrast, the extension in this study reveals the causes and consequences of the employment and income distribution dynamics within the BOPC growth framework. Once we introduce into the model the core of post-Keynesian economics, namely that the income distribution matters and the employment rate is led by effective demand, substantial implications ensue.

Appendix A: Local stability of the steady state

To prove Propositions 1, 2, and 3, let us state a preliminary argument. The characteristic equation that corresponds to the Jacobian matrix $\boldsymbol{J}$ is given as

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0,$$

where λ denotes a characteristic root. Coefficients a_1, a_2, and a_3 are given by

$$
\begin{aligned}
a_1 &= -\mathrm{tr}\boldsymbol{J} = -(j_{11} + j_{22} + j_{33}), \\
a_2 &= \begin{vmatrix} j_{11} & j_{12} \\ 0 & j_{22} \end{vmatrix} + \begin{vmatrix} j_{22} & j_{23} \\ 0 & j_{33} \end{vmatrix} + \begin{vmatrix} j_{11} & 0 \\ j_{31} & j_{33} \end{vmatrix} = j_{11}j_{22} + j_{22}j_{33} + j_{11}j_{33}, \\
a_3 &= -\det \boldsymbol{J} = -(j_{11}j_{22}j_{33} + j_{12}j_{23}j_{31}),
\end{aligned}
$$

where tr $\boldsymbol{J}$ denotes the trace of $\boldsymbol{J}$, a_2 is the sum of the principal minors' determinants, and $a_3 = \det \boldsymbol{J}$ is the determinant of $\boldsymbol{J}$.

The necessary and sufficient condition for local stability is that all the characteristic roots of the Jacobian matrix have negative real parts, which, from the Routh–Hurwitz condition, is equivalent to

$$
a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1a_2 - a_3 > 0.
$$

I investigate whether these conditions are satisfied. The current model has the property that the coefficients of the characteristic equation are linear functions of the adjustment speed ϕ for the disequilibrium in the balance of payments. These coefficients are

$$
\begin{aligned}
&a_1 \equiv \Delta_1\phi + \Delta_2, \\
&a_2 \equiv \Delta_3\phi + \Delta_4, \\
&a_3 \equiv \Delta_5\phi, \\
&a_1a_2 - a_3 \equiv f(\phi) = (\Delta_1\Delta_3)\phi^2 + (\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5)\phi + \Delta_2\Delta_4,
\end{aligned}
$$

where Δ_1 to Δ_5 and their signs are as follows:

$$
\begin{aligned}
&\Delta_1 \equiv \eta_2 > 0, \\
&\Delta_2 \equiv \sigma^* + z^*g_q'(z^*) > 0, \\
&\Delta_3 \equiv \eta_2(\sigma^* + z^*g_q'(z^*)) > 0, \\
&\Delta_4 \equiv \sigma^*z^*g_q'(z^*) > 0, \\
&\Delta_5 \equiv \sigma^*z^*(\eta_2g_q'(z^*) - \epsilon\beta\Theta) \gtrless 0.
\end{aligned}
$$

Since the signs of $\Delta_1, \Delta_2, \Delta_3$, and Δ_4 are uniquely positive, $a_1 > 0$ and $a_2 > 0$ are satisfied for positive values of ϕ. In addition, $f(\phi)$ is a convex downwards and its intercept is positive. The axis of this is given by

$$\bar{\phi} = \frac{-(\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5)}{2\Delta_1\Delta_3},$$

where the denominator is positive. When the interception of $f(\phi)$ is positive, as long as its axis is located in $\phi < 0$, $f(\phi) > 0$ is necessarily satisfied for $\phi > 0$, and the sign of $a_1a_2 - a_3$ is also positive. However, when the value of Δ_5 is positive and large, the axis of the function on the graph may be located in $\phi > 0$, and the graph of the function may cross the horizontal axis in the positive domain.

Thus, the conditions $a_3 > 0$ and $a_1a_2 - a_3 > 0$ are concerned with the sign of Δ_5. Therefore, it is necessary to check the stability conditions with these two terms altogether.

Proof of Proposition 1. As proven above, $a_1 > 0$ and $a_2 > 0$ are always satisfied. When the reserve army creation effect is stronger (Case A), $\Theta < 0$. Then, let us focus on a_3 and the coefficient of $\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5$ that determines the axis of parabola $f(\phi)$. By expanding this coefficient, I have

$$\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5 = \eta_2(\sigma^* + z^* g_q'(z^*))^2 + z^*\sigma^*\beta\epsilon\Theta,$$

where Θ is negative and ϵ is also negative from the Marshall–Lerner condition. Hence, $f(\phi) > 0$ is always satisfied for $\phi > 0$, meaning $a_1a_2 - a_3 > 0$.

On the contrary, a_3 is

$$a_3 = \sigma^* z^*(\eta_2 g_q'(z^*) - \epsilon\beta\Theta)\phi,$$

in which both Θ and ϵ are negative from the strong reserve army creation effect and Marshall–Lerner condition, respectively. For a_3 to be positive, I need

$$\eta_2 > \frac{\epsilon\beta\Theta}{g_q'(z^*)},$$

meaning that if η_2 is sufficiently large, it ensures $a_3 > 0$, and that if η_2 is sufficiently small, it does not ensure $a_3 > 0$. Therefore, as long as η_2 is sufficiently large (small), the necessary and sufficient conditions for local stability are all satisfied (not satisfied). □

Proof of Proposition 2. In the case of a stronger reserve army effect augmented by workers' bargaining power, I have $\Theta > 0$. In addition to $a_1 > 0$ and $a_2 > 0$, it is always satisfied that

$$a_3 = \sigma^* z^*(\eta_2 g_q'(z^*) - \epsilon\beta\Theta)\phi > 0,$$

where Θ is negative and ϵ is negative from the Marshall–Lerner condition.

On the contrary, a coefficient of $f(\phi)$ that determines the position of the axis of the parabola is

$$\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5 = \eta_2(\sigma^* + z^* g_q'(z^*))^2 + z^*\sigma^*\beta\epsilon\Theta.$$

If the following inequality is satisfied

$$\eta_2(\sigma^* + z^* g_q'(z^*))^2 < -z^*\sigma^*\beta\epsilon\Theta,$$

then the axis of parabola $\bar{\phi}$ comes to $\phi > 0$. This is true when the income elasticity of import demand η_2 is small, while the Marshall–Lerner condition works strongly (i.e. a large absolute value of ϵ). In this case, the discriminant of $f(\phi) = 0$ is positive, and the equation $f(\phi)$ has two positive real roots, on which the sign of $f(\phi)$ alternates. That is, for $\phi \in (0, \phi_1)$, $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ are all satisfied; for $\phi \in (\phi_1, \phi_2)$, $a_1 > 0$, $a_2 > 0$, $a_3 > 0$ are satisfied, but $a_1a_2 - a_3 < 0$; for $\phi > \phi_2$, $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ are all satisfied again.

Therefore, the limit cycle occurs by the Hopf bifurcation at $\phi = \phi_1$ and $\phi = \phi_2$. Indeed, at $\phi = \phi_1$ and $\phi = \phi_2$, I get

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad \left.\frac{\partial(a_1a_2 - a_3)}{\partial\phi}\right|_{\phi=\phi_1 \text{or } \phi_2} \neq 0,$$

which means that all the conditions for the Hopf bifurcation are satisfied. □

Proof of Proposition 3. Again, $a_1 > 0$ and $a_2 > 0$ are always satisfied. Let us first impose $\epsilon = 0$ in the dynamic system. Then, I obtain $a_3 \equiv \sigma^* z^* \eta_2 g_q'(z^*)\phi > 0$. In addition, $\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5$ determining the position of parabola $f(\phi)$ is

$$\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5 = \eta_2(\sigma^* + z^* g_q'(z^*))^2 > 0.$$

Therefore, the axis of this parabola is always located in $\phi < 0$ in this case. Hence, $a_1a_2 - a_3 > 0$ is necessarily satisfied for $\phi > 0$.

Second, let us consider the case in which the reserve army effect and reserve army creation effect offset each other and $\Theta = 0$. Since the rate of change in the commodity price at the steady state is $\hat{p}^* = \beta[(1-\beta)\sigma_W(z^*) - g_q(z^*)]$, if these effects offset each other, the impact on the rate of change in the commodity price is neutral. In this case, it is satisfied that $a_3 \equiv \sigma^* z^* \eta_2 g_q'(z^*)\phi > 0$. In addition, $\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5$ that determines the position of parabola $f(\phi)$ is

$$\Delta_1\Delta_4 + \Delta_2\Delta_3 - \Delta_5 = \eta_2(\sigma^* + z^* g_q'(z^*))^2 > 0.$$

Therefore, the axis of this parabola is always located in $\phi < 0$ in this case, too. Similar to the first case, $a_1 a_2 - a_3 > 0$ is necessarily satisfied for $\phi > 0$. □

Appendix B: Comparative Statics Analysis

The steady-state values of the BOPC growth rate, wage share, and employment rate satisfy equations (15), (16), and (17). The comparative statics analysis is conducted in the stable case. When the equilibrium of the system is locally stable, the determinant of the Jacobian matrix is negative. Therefore, the sign of the following value

$$\Lambda \equiv \eta_2 g_q'(z^*) - \epsilon\beta\Theta$$

used in the comparative statics analysis is positive. Then, the impact of a change in the following parameters on the steady-state values is as follows:

- The impact of a change in the growth rate of the foreign country

$$\frac{d\hat{Y}_D^*}{d\hat{Y}_F} = \Lambda\eta_1 g_q'(z^*) > 0,$$
$$\frac{d\sigma^*}{d\hat{Y}_F} = \Lambda\eta_1\Theta \gtrless 0,$$
$$\frac{dz^*}{d\hat{Y}_F} = \Lambda\eta_1 > 0.$$

  In Case A where $\Theta < 0$, I obtain $d\sigma^*/d\hat{Y}_F < 0$, while in Case B where $\Theta > 0$, I obtain $d\sigma^*/d\hat{Y}_F > 0$.

- The impact of a change in the income elasticity of exports (η_1)

$$\frac{d\hat{Y}_D^*}{d\eta_1} = \Lambda\hat{Y}_F g_q'(z^*) > 0,$$
$$\frac{d\sigma^*}{d\eta_1} = \Lambda\hat{Y}_F\Theta \gtrless 0,$$
$$\frac{dz^*}{d\eta_1} = \Lambda\hat{Y}_F > 0.$$

  In Case A where $\Theta < 0$, I obtain $d\sigma^*/d\eta_1 < 0$, while in Case B where $\Theta > 0$, I obtain $d\sigma^*/d\eta_1 > 0$.

- The impact of a change in the income elasticity of imports (η_2)

$$\frac{d\hat{Y}_D^*}{d\eta_2} = -\Lambda \hat{Y}_D^* g_q'(z^*) < 0,$$
$$\frac{d\sigma^*}{d\eta_2} = -\Lambda \hat{Y}_D^* \Theta \gtrless 0,$$
$$\frac{dz^*}{d\eta_2} = -\Lambda \hat{Y}_D^* < 0.$$

In Case A where $\Theta < 0$, I obtain $d\sigma^*/d\eta_2 > 0$, while in Case B where $\Theta > 0$, I obtain $d\sigma^*/d\eta_2 < 0$.

- The impact of a change in the nominal exchange rate ($\hat{e}$)

$$\frac{d\hat{Y}_D^*}{d\hat{e}} = -\Lambda \epsilon g_q'(z^*) > 0,$$
$$\frac{d\sigma^*}{d\hat{e}} = -\Lambda \epsilon \Theta \gtrless 0,$$
$$\frac{dz^*}{d\hat{e}} = -\Lambda \epsilon > 0.$$

In Case A where $\Theta < 0$, I obtain $d\sigma^*/d\hat{e} < 0$, while in Case B where $\Theta > 0$, I obtain $d\sigma^*/d\hat{e} > 0$.

- The impact of a change in the labour supply growth rate (n)

$$\frac{d\hat{Y}_D^*}{dn} = -\Lambda \epsilon \beta \Theta \gtrless 0,$$
$$\frac{d\sigma^*}{dn} = -\Lambda \eta_2 \Theta \gtrless 0,$$
$$\frac{dz^*}{dn} = -\Lambda \eta_2 < 0.$$

In Case A where $\Theta < 0$, I obtain $d\hat{Y}_D^*/dn < 0$ and $d\sigma^*/dn > 0$, while in Case B where $\Theta > 0$, I obtain $d\hat{Y}_D^*/dn > 0$ and $d\sigma^*/dn < 0$.

- The impact of an autonomous change in workers' target wage share (σ_W)

$$\frac{d\hat{Y}_D^*}{d\sigma_W} = \Lambda \epsilon g_q'(z^*) \beta (1-\beta) \sigma_w'(z^*) < 0,$$
$$\frac{d\sigma^*}{d\sigma_W} = \Lambda \eta_2 (1-\beta) \sigma_w'(z^*) g_q'(z^*) > 0,$$
$$\frac{dz^*}{d\sigma_W} = \Lambda \epsilon \beta (1-\beta) \sigma_w'(z^*) < 0.$$

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