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Abstract. R&D investment are an important engine of growth and development. Yet economists have often claimed under-investment, also due to the asymmetric information between inside investors and outside investors and financiers, and the consequent capital and financial market imperfections. Some recent empirical evidence robustly supports these claims. Motivated by this evidence, we study the effects of asymmetric information and financial frictions on R&D investment within a dynamic GE economy of Shumpeterian tradition. The model and equilibrium concept we propose is rich enough to represent investment and innovation decisions, financial decisions and decisions regarding technology adoption/diffusion through patent licensing. Qualitative predictions indicate that the financial policy of the firm matters in explaining both entrepreneurial production and innovation decisions. Young R&D-intensive firms might rely more heavily on internal sources and equity than on debt financing, relatively to what would otherwise be observed in absence of frictions. These findings contribute to explain the type of financial hierarchy recently highlighted in the empirical studies.

Keywords. Innovation; R&D; creative destruction; Shumpeterian growth; intertemporal firm choice; firm financial structure; Modigliani-Miller; asymmetric information; financial markets; venture capital; general equilibrium.

JEL Classification Numbers. O310, O330, O340, O400, D530, D920.

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1. Introduction

It is a widely accepted view that research and development (R&D) is an important determinant of competitiveness, economic growth and development. Despite this central role, economists have often stressed the difficulty that firms might encounter in financing this type of activities. This, first of all, depends on the fact that the final outcome of an R&D project is perceived as riskier, relatively to more traditional investment in physical capital, both in terms of maturity and probability of success. A second reason is that the assessment of risk is complicated by the intrinsic characteristic of R&D activities which exacerbate the information asymmetries between inside investors (the innovator entrepreneurs) and the outside investors and financiers.

R&D activities are, typically, skilled-labor intensive. During the development of a project, a large fraction of R&D expenditure goes into the formation of the firm's 'soft' capital, a peculiar intangible asset which constitutes the firm's knowledge base. 'Soft' capital includes, for example, the organizational competence of the entrepreneur/managers and the technical experience of the employees. More generally, it can be thought to consist of any type of un-codified, or tacit, firm-specific input which improves the company expected profitability; the likelihood that a certain expenditure on physical capital and labor will result in 'high' profit realizations.[1] An immediate consequence of these characteristics for the financial policy of the firm is that knowledge capital represents a poor source of collateral: because of its idiosyncratic nature and serendipity, it is clearly hard to measure; and, being firm-specific, it has a low salvage value. This is true even in absence of asymmetric information (or conditional to the available information as in *e.g.* see Berger and Udell, 1990). A second relevant consequence is that these characteristics of R&D activities are a natural and powerful source of information asymmetries.

Even when R&D investment project leads to a verifiable final outcome, a certifiable product or process innovation, asymmetric information makes harder for outside investors and financiers to assess its risk both at an ex-ante and at an interim stage. At the *ex-ante stage*, at least two main difficulties arise. First, the likelihood of success and realization of a R&D project strongly depends on the quality of the project and/or on entrepreneurial abilities; characteristics on which the innovator firm is better informed. Thus, analogously to Akerlof (1970), outside investors might ask the entrepreneur to pay a lemon-premium to compensate for the possibility that the project is of a lower quality than claimed; something that is reinforced for longer term projects, in Leland and Pyle (1977). Alternatively to prevent *adverse selection*, debt contracts might entail moderate risk premia but face severer credit restrictions, or rationing, as in Stiglitz and Weiss (1981). A second argument for information asymmetries at the ex-ante stage is that the outcome of a R&D project strongly depends on entrepreneurial effort, and thus it is subject to *moral hazard*. In fact, for any given project and entrepreneur qualities, the probability of innovating depends on the entrepreneur's commitment to the project, also in terms of actual expenditure, and might induce lenders to raise credit margins or collateral requirements (*e.g.* see Chan and Thakor, 1987).

All these characteristics, the quality of a project as well as the ability and effort/commitment of its entrepreneur proponent, are typically neither directly observable by outside investors, nor fully revealed by data on competitors and industry performance. Indeed, R&D projects in the technology

[1] See, for example, the discussion in Hubbard (1988), page 198.

or science base sector are normally unique to the developer firm, hence not much can be learned by observing the return from other firms operating in the industry. This is not so in more traditional capital intensive sectors, where the expected returns from investing in a firm tend to comove more closely with the sectorial average; something that makes applicable standard models of risk.

Research projects are usually more difficult to evaluate also at an *interim-stage*. In fact, during a project implementation, inside investors, fearing imitation, are reluctant to disclose information, either directly or through delegated monitoring (*e.g.*, see Bhattacharya and Ritter, 1983; Anton and Yao, 1998). Information disclosure is also discouraged by the accounting measurement and reporting rules, which tend to treat R&D differently from other investments: in the US, R&D expenditure is immediately inscribed in financial statements, so that no information on the value and productivity changes of research projects is reported to investors. In contrast, such information for investments in tangible capital are available on a quarterly base.

The empirical literature has recently provided some strong evidence in support of the negative effect of asymmetric information and financial frictions on R&D investments, even in economies with thick financial markets, a well developed system of specialized intermediaries and *venture capitalists*. The capital structure of R&D-intensive firms tends to exhibit considerably less leverage than that of other firms (see Friend and Lang, 1988; Hall, 1992; and Bhagat and Welch, 1995). Hall and Lerner (2010) surveying the up-to-date empirical studies, essentially, conclude that,

- large established firms do appear to resort more to internal funds for financing R&D investments and are especially active in managing their cash flow to ensure this;
- small and young innovative firms experience higher costs of capital that are only partly mitigated by the presence of venture capital;
- there is a clear and diffused evidence that debt (mostly bank loans) is the least used source of finance.

In other words, R&D intensive firms seem to display a *financial hierarchy*, privileging internal sources to equity and equity to debt financing. Of these empirical findings, the greater responsiveness of R&D investments to cash flow in economies with thicker financial markets is the most difficult to explain. One may argue that this responsiveness signals that firms rely more on outside financing and are borrowing constrained; but may as well say that it is simply the result of a greater sensitivity of firms to the general economic conditions, such as the demand level. The hypothesis of a *demand-linked* effect has been rejected by Brown, Fazzari and Petersen (2012), who show that financial constraints matter for R&D investments of publicly traded European firms. Borisova and Brown (2013) reinforce this result documenting that innovative firms tend to liquidate tangible assets to finance R&D in economic downturns, when the market demand drops.

The fact that traditional bank loans seem to be the least preferred source of outside funding could either be interpreted as the consequence of asymmetric information and of the implied credit restrictions (rationing and collateral requirements), or as the consequence of a firm's rational choice. The first explanation seems convincing especially in the absence of a developed market for venture capital, if one considers that standard bank loans have some very appealing characteristics for a firm with intense R&D activities. In fact, its payment schedule prescribes that, if the firm is successful, it is granted all net profits (*i.e.* effort and abilities are rewarded with the investment surplus they generate); if instead it fails and the firm goes bankrupt, usual limited-liability clauses leaves an high knowledge-base firm exposed to mild losses from asset confiscation (*e.g.* see Gale and Hellwig, 1985; Innes, 1990). Instead, in an economy with a highly developed system of venture capitalists, low-leverage financing can be preferable especially by start-up and innovative firms in their early stage. Specialized venture capital funds are technically more experienced and have a better monitoring technology, which they use both to discriminate projects and to extract information at their interim stage. Thus, venture-capital contracts may simply be more attractive to some firms because they mitigate information asymmetries and do not expose entrepreneurs to rationing or expensive collateral requirements.

In spite of the recognized relevance of asymmetric information and financial frictions, these have been largely ignored in modern innovation-based endogenous growth theory, started by Romer (1990), Aghion and Howitt (1992), Grossman and Helpman (1991). In the present paper we try to fill this gap proposing a simple dynamic GE model in which entrepreneurs are simultaneously engaged in technological innovation and financial decisions. In short, our economy is a Shumpeterian version of a growth model. Innovation, when it occurs, results in an increase of factor productivity and in a patent award to the innovator. Patents can be licensed by the innovator entrepreneur before they expire. Innovation depends on entrepreneurs ability and commitment (R&D expenditure), and the individual likelihood to innovate is assumed to be an increasing function of both such ingredients. Because ability is private information, adverse selection and moral hazard might emerge. This motivates the introduction of financial frictions in a particular form of 'markets segmentation'. Precisely, we assume that firms' equity are not traded directly, but they are intermediated by private equity-funds, who 'pool' them into a composite security. Because of limited commitment (*i.e.*, possibility of default) also loans financing is subject to debt limits and the severity of such limits reflects asymmetric information. Except for the market of patent licenses, which operates as a monopoly, the economy is perfectly competitive.

The equilibrium concept proposed is rich enough to represent investment and innovation decisions, firms' financial policy decisions and decisions regarding technology adoption/diffusion through patent licensing. We argue, and illustrate in a simple example, that the qualitative predictions of the model are in line with those emerging in the empirical evidence. Indeed, asymmetric information and financial imperfections explain why entrepreneurs might find individually optimal to adopt the financial hierarchy found in the data. Potentially successful innovators might find more convenient (or simply be forced to rely more) on cash-flow and equity financing than on debt financing; this seems to be especially true for young start-up firms. Firms' financial policy matters for production and innovation decisions. This explains also why financial frictions might reduce the capacity of individual decisions, such as R&D expenditure, to signal entrepreneurial abilities, making the asymmetric information a severer, persistent problem.

Our paper has some features in common with the Shumpeterian growth literature. The economy proposed has an OLG structure and entrepreneurs with different skills or abilities face credit constraints, similarly to Acemoglu, Aghion and Zilibotti (2006). However, differently from them, and similarly to Aghion and Howitt (1992) and to all the related literature (*e.g.* see Aghion and Howitt, 2005), we assume that the probability to achieve a successful innovation increases in the entrepreneur R&D expenditure/effort and ability. The closest, in spirit, to our entrepreneurial model is the one sketched by Greenwald and Stiglitz's (1990) and, more generally, individual models of moral hazard. Finally, differently from all the earlier growth literature, we propose a richer financial structure: besides loans and internal finance, which are considered in Acemoglu et al. (2006), we assume that firms can also enter in equity contracts, which we represent as competitive 'venture capital' funds or 'pools'. The use of 'pool' securities in competitive economies links our model to the literature on general equilibrium economies with asymmetric information (*e.g.* see Dubey Geanakoplos and Shubik, 2005, and, for production economies, Drèze, Minelli and Tirelli, 2008).

The paper is organized as follows. **Section 2** describes the economy and defines an equilibrium concept. **Section 3** presents a simple example that is used to illustrate how asymmetric information and financial frictions can distort firms' financial and investment decisions in a way that is in line with the empirical evidence mentioned above. **Section 4** considers large economies (*i.e.* economies with a large number of entrepreneurs of each type) and establishes equilibrium existence.

2. The economy and equilibrium

2.1. **Time, individual characteristics and technology.** Let us start with a general description of the economy. The economy is one with: a single commodity, also serving as production capital; infinite time periods $\mathcal{T} = (0, 1, ..., t, ..)$; and overlapping generations of individuals and R&D projects. Every generation t is formed by a finite number of types.[2] Individuals are either entrepreneurs or households. For the entrepreneurs, the OLG structure aims at capturing the two stages of a R&D project; in the first period of life, *young-age*, an entrepreneur engages into a research activity and in the next period, *old age*, the outcome of this activity realizes. Entrepreneurs are indexed by i in $\mathcal{E} = (1, 2, .., I)$. Every entrepreneur i, born in date t, is characterized by her ability to innovate ρ_t^i, we shall define later, an initial endowment of the commodity (e_t^i, e_{t+1}^i) and identical risk-neutral preferences on her old-age consumption. Households are indexed by h in $\mathcal{H} = (1, 2, ..., H)$. Every household h is endowed with (e_t^h, e_{t+1}^h) units of the commodity and has identical preferences, possibly, exhibiting risk aversion.

Each entrepreneur manages a single firm whose technology is represented by a neoclassical production function. At all dates t, given a capital input k_t and *factor productivity* A_t, production output is,[3]

$$y_t = f(A_t, k_t).$$

Technologies evolve only due to changes of total factor productivity $(A_{t+s})_{s\in\mathcal{T}}$, where we assume that A_t is non-decreasing over time and its growth rate is either zero or $\eta - 1 > 0$. In other words, our economy is one in which technological innovation follows a *quality ladder* growth model with A_t representing the *frontier* technology at t, and a technology increment, an innovation, is a step of height η,

$$A_{t+1} = \begin{cases} \eta A_t, & \textit{when an innovation occurs} \\ A_t, & \textit{otherwise} \end{cases}$$

We further assume that an innovation, when it occurs, is granted of a perfectly enforceable *patent*, which has a finite life, we assume 1 period. Patents can be licensed. We assume that each

[2]A similar overlapping-generations framework is used, for example, by Acemoglu, Aghion and Zilibotti (2006), and by Howitt and Mayer-Foulkes (2004). There, entrepreneurs types are interpreted as dynasties.

[3]Without loss of generality, we assume that the commodity (hence capital) depreciates completely in every period. Thus, the amount of individual capital k_t equals individual investment.

entrepreneur decides on the technological adoption when young, either purchasing a license to use the patented, frontier technology or freely accessing to the pre-existing one. We label as *mature* (and index it by $\iota = 1$) the firms that hold a license and as *start-up* those that start behind the frontier and try to innovate. Thus, although the total number I of entrepreneur types is assumed to be constant over time, its distribution between firms (or technologies) is endogenous. For later use, we denote by n_t the number of patent licenses (*i.e.* mature firms) at date t; accordingly, we index the set of entrepreneurs running the two firm-types in t, respectively, by $\iota = 1$ and $\iota = 0$, so that $\mathcal{E}$ splits into two subset $\mathcal{E}^\iota$, for $\iota = 0, 1$. The time subscript t used for the type-set, $\mathcal{E}_t$ and $\mathcal{H}_t$ serves to indicate that agents refer to a particular generation t.

A third type of agents in the economy is 'financial intermediaries'. These individuals are profit maximizing, risk neutral agents who intermediate funds between consumers and firms. Their role will be better understood once we describe the uncertainty and information characterizing the economy.

2.2. **Uncertainty.** For simplicity, the only source of uncertainty is technological innovation.[4] Thus, we respectively denote by S and U the *aggregate states* in which an innovation either does occur or it does not and describe a history of technological change in t by $A^t = (A_0, .., A_t)$.

Technological innovation, when it occurs, is the outcome of a sigle entrepreneur's research and development (R&D) activity. Each entrepreneur, who invests in a R&D activity when young, might either be successful s or unsuccessful u in her old-age. These events define two possible *idiosyncratic states*. Accordingly, at every date t, economic uncertainty can be described by $I + 1$ mutually exclusive states of the world,

$$\xi^u = (u, u, ..., u)\,, \quad \xi^i = (u, .., \underbrace{s}_{i^{th}}, u.., u), \quad i \in \mathcal{E}$$

where ξ^u corresponds to the event in which all entrepreneurs fail to innovate, and so does the economy as a whole; while, ξ^i, in $\mathcal{E}$, corresponds to the event that entrepreneur i innovates, and so does the economy as a whole. The resulting state-space is denoted by Ξ; $U \equiv \{\xi^u\}$ and $S \equiv \xi^s \in \Xi \setminus \{\xi^u\}$.

2.3. **Information.** All individuals know the objective probability ρ_t that a technological innovation occurs at time $t + 1$. Hence, at all t, they compute the economy expected growth of technological progress as,

$$\mathbb{E}_t \frac{A_{t+1}}{A_t} = \rho_t \eta + (1 - \rho_t) = \rho_t(\eta - 1) + 1$$

This is exactly equal to η if an innovation occurs almost surely and is equal to one (*i.e.* no growth) if the probability ρ_t equals zero. Individuals observe the outcome of innovation.

Our economy is one with *private information* too: *i)* the entrepreneur type i, which is characterized by a probability to innovate ρ^i, is private information, and *ii)* investment in innovation x_t^i (the firm knowledge base capital), is carried out by a young entrepreneur i after she has traded in all markets open at t. Instead, all the other characteristics of the economy (including the type of technology used by each firm) are common knowledge.

[4]As in a stochastic quality ladder growth model, in every period t, given a frontier technology A_t, the next period frontier A_{t+1} is regarded as a binary random variable taking values, ηA_t if an innovation occurs and A_t otherwise.

In our private-information economy, each entrepreneur i, when young, faces a partition of Ξ,

$$\Omega^i = \big\{\{\xi^u\}, \{\xi^i\}, \{\xi^{-i}\}\big\}$$

where, ξ^{-i} is any state in which an innovation occurred thanks to an entrepreneur different from i. In words, Ω^i contains the no-innovation state ξ^u, the personal success state ξ^i, the innovation of some competitor firm.

Instead, each household h observes the realizations of the aggregate states only. Accordingly, her information partition is,

$$\Phi = \{\{U\}, \{S\}\} \equiv \{\{\xi^u\}, \{(\xi^i)_{i\in\mathcal{E}}\}\}$$

of typical element ϕ.

Finally, for every entrepreneur i, joint probabilities are defined by $P_t^i : \Omega^i \to [0,1]$, and for every consumer h, by $P_t : \Phi \to [0,1]$ as follows,[5]

$$P_t^i(\xi^i) = \rho_t^i, \quad P_t^i(\xi^s) = \sum_{i\in\mathcal{E}} \rho_t^i \equiv \rho_t = P_t(S), \quad P_t^i(\xi^u) = 1 - \rho_t = P_t(U).$$

To simplify notation, we shall later use $\mathbb{E}_t^i X$ to denote the expected value of X (a random variable with domain Ξ) computed by entrepreneur i according to P_t^i; and simply $\mathbb{E}_t X$ when X is measured with respect to P_t. Analogously, we define conditional moments.[6]

As for intermediaries, they are essentially monitoring entities. We shall distinguish between two types of intermediaries, equity-funds and banks, characterized by different information and monitoring technologies. In particular, similarly to what happens in reality, we assume that equity-funds can verify, at no cost, profit realizations on the individual firms with whom they contract. While banks can only exercise a costly monitoring on profits.

2.4. **Markets.** Since individuals do only trade contracts whose payout is contingent on states that they can verify, the economy has four markets in every period: a spot commodity market, an equity market, a loan/debt market and a market for patent licenses. Except for the patent market, which operates as a monopoly, all the others are perfectly competitive.

Patent licenses. A patent is awarded to the entrepreneur who innovates first: individual i in state ξ^i. The patent holder in t becomes a monopolists and can eventually license the patent by offering licenses to other entrepreneurs at some price p_t. Licenses are standardized and allow the buyers to exploit the technology in full. In $t+1$ the patent expires and the associated technology becomes

[5]The time subscript t attached to P_t^i aims to capture the fact that entrepreneur i is young in t and operates with a success probability ρ_t^i.

[6] In particular, consider a r.v. X^i with domain Ξ, measurable with respect to $(P_t^i)_i$. Keep in mind that $(P_t^i)_i$ are joint probability distributions of all states ξ (*i.e.*, in this setting, of both idiosyncratic and aggregate states). Then, the following definitions are standard.

$$\mathbb{E}_t^i[X^i] \equiv \sum_j P_t^i(\xi^j)X^i(\xi^j), \quad \mathbb{E}_t^i[X^i|S] \equiv \mathbb{E}_t^i[X^i|\xi^s] = \frac{1}{P_t(S)} \sum_{j\neq 0} P_t^i(\xi^j)X^i(\xi^j)$$

Similarly, $\mathbb{E}_t^i[X^i|U] = P_t^i(\xi^u)X^i(\xi^u)/P_t(U) = X^i(\xi^u)$. It will be useful to remember that if X^i has domain Φ (*i.e.* it varies across aggregate states only), $\mathbb{E}_t^i[X^i|\phi] = X^i(\phi)$.

freely accessible to all current producers. This essentially makes the innovation a perfectly non-rival, club good supplied by a monopolist in a market where the demand side is perfectly competitive.

Equities. The informational structure of the economy and, in particular, the fact that households can only verify aggregate states, motivates our assumption that individual firms' equities are not directly traded, but can only be sold as a 'pool', by private equity funds.[7]

Equity funds have an OLG structure too. Each one is linked to a generation of entrepreneurs' investment projects and lasts for two periods. Therefore, at every date, a new generation of funds appears in the economy, to serve currently young entrepreneurs, and an old one liquidates.

An entrepreneur i, who is young in t, might enter on an equity-contract by selling at a competitive market price q_t any share $0 \leq \zeta_t^i \leq \widehat{\zeta}$ of her firm's operating profits (π_t^i, π_{t+1}^i) to an intermediary. The intermediary participation to young-age profits is granted by the fact that he observes π_t^i and entails a contribution to firm i costs in the same proportion. In the old age, when uncertainty realizes, the intermediary monitors and, eventually, seize profit realizations π_{t+1}^i. This occurs for all entrepreneurs i who contract with an intermediary and for all intermediaries.

On the other side of the equity market, intermediaries sell to the households claims on the return to the fund at the market price q_t. The return is a 'pool' of profits from equity financing and consists of a current payoff Π_t^b and a future one, Π_{t+1}^a, which is contingent to aggregate states in Φ.[8] The exact definition of Π_t^b and Π_{t+1}^a is postponed to section 2.6.3.[9]

Loans. At all t, a loan l is an exclusive, linear contract characterized by a sure gross rate of return and a quantitative debt limit, a positive pair (R_{t+1}^l, B_t^l). Loan contracts are offered by competing, risk neutral intermediaries (*e.g.* banks) who contextually collect deposits. Every young entrepreneur i in t can subscribe a (unique) loan contract l, issuing debt $b_t^i < 0$ up to the limit $-B_t^l$, for the promise to pay a sure, gross rate of return R_{t+1}^l. Whenever $b_t^i > 0$ we say that i is subscribing a bank deposit. In the following period, in case the entrepreneur defaults on her loan obligations, she is monitored by the intermediary and her assets/income are sized. This intends to capture the idea that bank monitoring is costly and is only carried out when an entrepreneur fails to honor her contract.

2.5. **Assumptions.** The technology is represented by a standard production function.

Assumption 1 (Production technology)**.**
The function $f : \mathbb{R}_+^2 \to \mathbb{R}_+$, such that $y = f(A, k)$ is twice continuously differentiable, increasing and concave in k and increasing in A; moreover, it satisfies, $f(A, 0) = f(0, k) = 0$, $\lim_{k\to 0} \partial f/\partial k > 1$, $\lim_{k\to +\infty} \partial f/\partial k = 0$.

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[7]See, for example, Dubey, Geanakoplos and Shubik (2005), Bisin and Gottardi (1999) and Lisboa (2001). For production economies and other references, see Drèze, Minelli and Tirelli (2008).

[8]The superscript b and a, respectively, stand for *before* and *after*.

[9]An alternative specification is to assume that funds are indexed by the firms' technology $\iota \in \{0, 1\}$. However, this does not add any particular insight, inasmuch firms profits and fund returns are collinear; in other words, inasmuch the adding this distinction does not complete financial markets, allowing consumers to efficiently diversify aggregate risk.

Recall that the innovation ability of an entrepreneur i is defined as the probability she will successfully innovate ρ_t^i. We assume that $\rho_t^i = \rho^i(\cdot, x_t^i)$ is an increasing function of both type i's, intrinsic, innovation ability i, and her R&D investment/effort x_t^i: for any level of R&D expenditure $x \geq x'$ and entrepreneur type $i \geq j$, $\rho^i(\cdot, x) \geq \rho^j(\cdot, x)$ and $\rho^i(\cdot, x) \geq \rho^i(\cdot, x')$; where these hold with strict inequality, respectively, if $i > j$ and $x > x'$. Moreover, we assume that ρ_t^i is a decreasing function of the aggregate level of R&D expenditure and the technological frontier, $X_t = g(x_t, A_t)$, where $x_t := (x^j)_{j \in \mathcal{E}_t}$ and g is an aggregator function of x_t. This aims to capture a *'fishing out'* effect; namely, the fact that economies with higher research activity and/or more advanced technology require an higher individual R&D effort by the individual firm in order to achieve an innovation (*i.e.* ρ^i is increasing in x^i/X). Finally, for consistency, at all t,

$$(\rho) \qquad \rho_t = \sum_{i \in \mathcal{E}_t} \rho_t^i \leq 1$$

Therefore, the probability that the economy experiences a technological innovation $P_t(S)$ equals the mass of entrepreneurs who innovate, which must not exceed one. Again, the economy as a whole might either be one in which an innovation occurs almost surely or not.

These properties are formally stated next, while the illustration of a parametrization of ρ_t^i is presented in an example, in section 3 below (see also Howitt and Mayer-Foulkes, 2005).

Assumption 2 (Innovation probabilities)**.**
At all dates t in $\mathcal{T}$ and for every entrepreneur i in $\mathcal{E}_t$, $P_t^i(\xi^i) = \rho_t^i$, and ρ_t^i denotes the value of $\rho^i(x_t^i, X_t)$, attained when x_t^i is entrepreneur i R&D expenditure and $X_t \geq x_t^i > 0$ is the indicator of aggregate R&D in t. $\rho^i : [0,1]^2 \to [0,1]$ is a function that, with respect to the first argument, is twice continuously differentiable, strictly increasing, strictly concave and satisfies $\rho_t^i(0, \cdot) = 0$; it also satisfies, $\lim_{x_t^i/X_t \to 0} \partial\rho_t^i/\partial x_t^i = +\infty$, $\lim_{x_t^i/X_t \to +\infty} \partial\rho_t^i/\partial x_t^i = 0$. Finally, $(\rho_t^i)_{i \in \mathcal{E}_t}$ is such that condition (ρ) holds.

Assumption 3 (R&D aggregator function)**.**
For any given A_t, the aggregator $g : [0, I] \to [0,1]$, such that $X_t = g(x_t, A_t)$ and $x_t := (x^j)_{j \in \mathcal{E}_t}$, is a function which is once continuously differentiable, non-decreasing, concave.

Each household's preferences are defined on her old-age consumption, represented by a standard expected utility function,

Assumption 4 (Households utility function)**.**

$$U(c_{t+1}) = \mathbb{E}_t u(c_{t+1})$$

where the function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, strictly increasing and strictly quasi-concave.

Finally, to ensure that individuals are ex-ant identical, we assume the following.

Assumption 5 (Endowments).
Individuals have identical, positive endowments: $e^i = e^J \geq 0$ *and* $e^h = e^H \geq 0$*. Moreover, for all* t *in* $\mathcal{T}$*,* $(e^J_t, e^J_{t+1}) \neq 0$*,* $(e^H_t, e^H_{t+1}) \neq 0$*.*

2.6. **Decisions.**

2.6.1. *Entrepreneurs.* An entrepreneur i who starts a production activity in t, essentially, faces four decisions:

- technology adoption or decision about the firm type (A_t, A_{t-1}),
- R&D investment x^i_t,
- production input in young- and old-age, $(k^i_{t+s})_{s=0,1}$,
- financial policy, (ζ^i_t, b^i_t).

In addition, a successful innovator is involved in patent license sales.

Conditional on the event that a technological innovation has occurred, a young entrepreneur at t, starts by deciding on technology adoption, between A_{t-1} and $A_t = \eta A_{t-1}$. The first technology is patent-free, of type $\iota = 0$; the second, $\iota = 1$, requires to purchase a license at the market price p_t. Contextually, given a technology $A_{t+\iota-1}$, the entrepreneur chooses a R&D expenditure x^i_t, production input (k^i_t) and decides on external financing, a equity-loan pair (ζ^i_t, b^i_t). Internal financing, takes the form of firm *cash flow*,

$$y^i_t = f(A_{t+\iota-1}, k^i_t) - k^i_t - x^i_t \tag{y_y}$$

Hence, a young firm i has current *operating profits*,

$$\pi^i_t = y^i_t - p_t\iota \tag{π_y}$$

In her old-age, at $t+1$, entrepreneur i observes the outcome of her R&D activity, as well as the realization of aggregate risk. Then, conditionally on the state ξ, she makes her production decisions, collects and make financial payments and (if $\xi = \xi^i$ as occurred) sells patent licenses. In the i's old-age, cash flow and operating profits, respectively, are,

$$y^i_{t+1}(\xi) = \left(f(A_{t+1}, k^i_{t+1}) - k^i_{t+1}\right)(\xi), \quad \xi \in \Xi \tag{y_o}$$

$$\pi^i_{t+1}(\xi) = y^i_{t+1}(\xi) + p_{t+1}n_{t+1}(\xi), \quad \xi \in \Xi \tag{π_o}$$

where $p_{t+1}n_{t+1}(\xi)$ equals $p_{t+1}n_{t+1}$ if $\xi = \xi^i$ and is zero otherwise.

We can now define entrepreneur i budget set corresponding to any technology ι. At market prices $\mathfrak{p} = (\mathfrak{p}_t)_t = (q_t, p_t, R^{-1}_{t+1})_t$, borrowing limit B^ι_t, technology history A^t, the *budget set* of an entrepreneur i in $\mathcal{E}^\iota_t$,

$$\mathbb{B}^{i,\iota}_t = \mathbb{B}^{i,\iota}\left(\mathfrak{p}, B_t, A^t, (\pi^i_{t+k})_{k=0,1}\right)$$

is the set of all possible (contingent) consumption levels c^i_{t+1} such that, at all ξ in Ξ,

$$c^i_{t+1}(\xi) \leq e^i_{t+1}(\xi) + \left(1 - \zeta^i_t\right)\pi^i_{t+1}(\xi) + b^i_t R_{t+1}$$

at some financial policy (b^i_t, ζ^i_t) and operating profits (π^i_t, π^i_{t+1}) satisfying,

$$-B^\iota_t \leq b^i_t = e^i_t + \left(1 - \zeta^i_t\right)\pi^i_t + q^\iota_t \zeta^i_t, \quad 0 \leq \zeta^i_t \leq \widehat{\zeta}$$

and $(\pi_y), (\pi_o)$, for some choice, $0 \leq x_t^i, (k_{t+j}^i)_{j \in \{0,1\}}$.[10]

The corresponding life-time expected, indirect-utility of a young entrepreneur i at t is,

$$V_t^{i,\iota} = \max \left\{ \mathbb{E}_t^i c_{t+1}^i : (c_{t+1}^i, x_t^i, b_t^i, \zeta_t^i) \in \mathbb{B}_t^{i,\iota} \right\}$$

and the entrepreneur (contingent) problem can be solved backwards, from old-age to young.

A successful entrepreneur, when old, sells patent licenses at the price p_{t+1} that maximizes sales profits (*i.e.* revenue, sales costs being zero), for a given patent demand conjecture.[11] The old entrepreneur also decides on current production. Going backwards to young age, the entrepreneur decides on current production and financing, so as to maximize expected future consumption on her budget set, for given future realizations of firm profits and returns to innovation and patent sales. This optimization is solved in t for all the available technologies ι. Finally, at t, a young age entrepreneur i purchases a patent license and adopt a frontier technology, if and only if $V_t^{i,1} \geq V_t^{i,0}$.

2.6.2. *Households.* Households manage their finances so has to maximize their expected utility of old-age consumption. Having assumed that utility is monotonic, we let,

$$b_t^h = e_t^h - \left(q_t - \Pi_t^b\right) \theta_t^h, \quad \theta_t^h \in \mathbb{R}_+^2$$

and define h's budget set $\mathbb{B}_t^h(\mathfrak{p}, \Pi_t^b, \Pi_{t+1}^a)$ as the set of contingent consumption levels satisfying,

$$c_{t+1}^h(\phi) \leq e_{t+1}^h(\phi) + \left[e_t^h - \left(q_t - \Pi_t^b\right) \theta_t^h\right] R_{t+1} + \theta_t^h \Pi_{t+1}^a(\phi), \quad \phi \in \Phi$$

at some fund-portfolio θ_t^h in $\mathbb{R}_+^2$. The household's indirect utility is

$$V_t^h = \max \left\{ \mathbb{E}_t u\left(c_{t+1}^h\right) : c_{t+1}^h \in \mathbb{B}_t^h \right\}$$

The next subsection clarifies why, in our economy, households' old-age consumption will only vary across aggregate states, which are the only ones they can effectively verify.

2.6.3. *Equity funds.* For simplicity, we have assumed that funds are essentially monitoring entities, with an identical technology, facing no operating costs. Free accessibility to the monitoring technology and free entry of intermediaries imply zero profits: each fund 'passively' balances state-contingent revenues with costs, period by period. This implies that, without loss of generality, we can assume that, in each period, a single equity fund is available for trade. More precisely, for a representative fund, budget balance at the 'before' and 'after' investment stage is specified as follows. When the fund (and firms) are young,

$$q_t \sum_{i \in \mathcal{E}_t} \zeta_t^i + \Pi_t^b \sum_{h \in \mathcal{H}} \theta_t^h = q_t \sum_{h \in \mathcal{H}} \theta_t^h + \sum_{i \in \mathcal{E}_t} \pi_t^i \zeta_t^i$$

At old age, the fund correctly assesses firms' profits and form their conjecture on entrepreneur types and actions based on the information collected at the contracting stage. Assuming that such

[10] Observe that outside investors, holding a fraction ζ_t^i of the firm are entitled to receive a proportional share of operating profits and loan financing is represented by a negative value of b_t^i.

[11] In the simplest case in which there is only one entrepreneur type, the monopolist will set the license price such as to make young entrepreneurs indifferent between the two technologies. See section 3 below. The whole decision process will have a precise definition in section 4.

conjectures are correct (see remark 2.1 below), we have,

$$\Pi^a_{t+1}(\phi)\sum_{h\in\mathcal{H}}\theta^h_t = \bar{\zeta}_t\sum_{i\in\mathcal{E}_t}\left(\frac{\zeta^i_t}{\bar{\zeta}}\mathbb{E}^i_t\left[\pi^i_{t+1}(\xi^i)|\phi\right]\right)$$

where, for a precise definition of conditional probabilities we refer to footnote 6 above.

Next, assuming that the total number of claims sold by the firms are (strictly) positive and that they balance with those subscribed by the households, $\bar{\zeta}_t = \sum_{i\in\mathcal{E}_t}\zeta^i_t > 0$ and $\bar{\theta}_t = \sum_{h\in\mathcal{H}}\theta^h_t$, we obtain,

$$(\Pi^b_t) \qquad \Pi^b_t \equiv \frac{1}{\bar{\zeta}_t}\sum_{i\in\mathcal{E}_t}\zeta^i_t\pi^i_t,$$

$$(\Pi^a_{t+1}) \qquad \Pi^a_{t+1}(\phi) \equiv \frac{1}{\bar{\zeta}_t}\sum_{i\in\mathcal{E}_t}\left(\zeta^i_t\mathbb{E}^i_t\left[\pi^i_{t+1}(\xi^i)|\phi\right]\right), \quad \phi\in\Phi_t$$

which we assume to be sero if $\bar{\zeta}_t = 0$.

We now argue that, in our simple economy, definition (Π^a_{t+1}) simplifies considerably. First, observe that firms' profits are made of two components, the revenues eventually obtained by the innovator firm from license sales $p_{t+1}n_{t+1}$, and the profits from the production activity y_{t+1}. The first component turns out to be independent of types and idiosyncratic states. In fact, *i)* the license demand at $t+1$ is independent of the realized idiosyncratic state and, *ii)* the patent winner at $t+1$ set p_{t+1} so as to extract the maximal revenue-surplus from current young firms, and this– conditional on the aggregate state S– is independent of her type. The production component y_{t+1} is also independent of i, essentially, as a consequence of the assumption that patent expire after one period; this implies that, a part from the innovator who jumps to A_{t+1}, all the other old-age produce with an A_t-technology. As we shall illustrate in greater detail later, once a state realizes, the only type-specific elements that enter Π^a_{t+1} are the individual participation/shares, $(\zeta^i_t/\bar{\zeta}_t)$. Accordingly, the fund payoff can be written in a more intuitive and simple notation as follows. Let $\widehat{\rho}^i \equiv (\zeta^i/\bar{\zeta})\rho^i$, and denote by $\pi(S)$ and $\pi(U)$, respectively, the level of profit achieved by an innovator firm and by one who has failed to innovate. Then, equation (Π^a_{t+1}) reads,

$$\Pi^a_{t+1}(\phi) = \begin{cases} \frac{\widehat{\rho}_t}{\rho_t}\left[\pi_{t+1}(S) - \pi_{t+1}(U)\right] + \pi_{t+1}(U), & \text{if } \phi = S; \\ \pi_{t+1}(U), & \text{if } \phi = U. \end{cases}$$

In words, if ϕ is a state of the economy S, in which some firm innovates, the fund payoff equals a weighted sum of the the no innovation profit $\pi_{t+1}(U)$, plus the incremental profit due to the innovation. This last innovation component is also weighted by the participation weights in the fund (*i.e.* the terms $\zeta^i_t/\bar{\zeta}_t$ in $\widehat{\rho}_t$). Indeed, if all equity shares were the same, $\widehat{\rho}_t = \rho_t$, and $\Pi^a_{t+1}(S) = \pi_{t+1}(S)$. Otherwise, the return to the fund, when an innovation occurs, is increasing in the 'quality' of firms' ventures, measured by $\Sigma_i(\zeta^i/\bar{\zeta})\rho^i$, which depends on entrepreneurs' ability (ρ^i), on their effort (x^i) and on the participation weights in the fund.

Remark 2.1. *(Incentive compatibility) We point out that the intermediary knows that equity contracts are ex-ante incentive compatible and ex-post enforceable. In fact, consider an entrepreneur i in $\mathcal{E}_t$, who enters a contract $(q_t, \pi^i_t, \pi^i_{t+1})$, by selling some share ζ^i_t to the fund. Ex-ante, i set $\zeta^i_t > 0$ only if this contract is individually optimal at q_t. Ex-post, once uncertainty realizes, i has* *no other choice than delivering the realization of* π^i_{t+1}*, as the fund always detects miss-reported profits. Notice that, even if the fund observed R&D investment* x^i_t*, this would not be enough to correctly identify i's intrinsic ability (*i.e. *to recover her probability distribution* P^i*). However, due to the ex-post monitoring on profits, entrepreneurs have no incentive to misreport their true type, provided the equity price* q_t *is anonymous. Indeed, suppose for a moment that, based on the observation of R&D expenditure and the supply of equity, a fund decides to offer different equity prices* $(.., q^i, ..)$*, conditional upon the type reported by the entrepreneur at the contracting stage. Then, every entrepreneur would have an incentive to claim to be the type for whom intermediaries require the lowest interest rate; that is, for given profits, the highest price,* $\bar{q} \equiv \max_i(.., q^i, ..)$*. This is particularly true in our context, in which (old-age) profits are independent of entrepreneur types. Therefore, the only incentive-compatible contract (of the linear type considered here) is one with a uniform price (*i.e. *type-independent).*

2.6.4. *Banks.* We argue that, in this context, it is reasonable to assume that banks compete to offer *standard debt contracts*, in sense of Gale and Hellwig (1985), which are also characterized by debt limits. We assume that banks monitor entrepreneurs who default and size their income. Such punishment implies that no entrepreneur finds optimal to implement strategic default; since this, yielding zero old-age wealth and consumption, is dominated by the decision not to set up any firm to begin with.

Even ruling out strategic default, entrepreneurs might fail to repay due to insufficient funds, in unsuccessful states.[12] This implies that default by i is observed in state (ξ^{-i}) if and only if, $(1-\zeta^i_t)\pi^i_{t+1}(\xi^{-i}) < -b^i_t R_{t+1}$, suggesting, for every i in $\mathcal{E}_t$ and any ι in $\{0,1\}$, a debt limit $B^i_t = (1-\zeta^i_t)\pi_{t+1}(U)/R_{t+1}$, where π_{t+1} is independent of i for reasons we have explained above.

Are loans contracts (R_{t+1}, B^i_t) default-proof for the lenders? To answer this, recall that types are private information at the time of loan contracting. A lender knows the type distribution, but cannot effectively detect types. Ex-post verification, based on output observability, does not help the implementation of such contracts: at the contracting stage, an entrepreneur j might have an incentive to misreport her type, claiming to be of type i, if she sees she would be borrowing constrained at $B^j_t < B^i_t$; at the repayment stage, as we have seen discussing equity-funds, profits are anonymous and do only allow to separate a successful innovator from all its unsuccessful competitors. Therefore, the only linear debt-contracts which guarantee full repayment are those with an anonymous debt limit, with ζ^i_t replaced by $\max_i \zeta^i_t$. Yet, for simplicity, we assume the form,

$$B_t = (1-\widehat{\zeta})\frac{\pi_{t+1}(U)}{R_{t+1}} \qquad (B_t)$$

Observe that, as debt repayment coincides with the firm liquidation, other mechanisms based on reputation, such as market exclusion, are unfeasible.

Suppose that, at all t, loan contracts $(R_{t+1}, B_t,)$ are supplied competitively by intermediaries who finance themselves by collecting funds from savers at the interest rate R_{t+1}. Then, it is straightforward to verify that market clearing on loans yields zero profits to the intermediaries.

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[12]Indeed, to have a strictly positive, expected return from entrepreneurship, it must be that in the successful state every firm has no advantage to default: for any entrepreneur i of generation t, $(1-\zeta^i_t)\pi^i_{t+1}(\xi^{-i}) \geq -b^i_t R_{t+1}$.

2.7. **Analyzing individual decisions.** A complete characterization of individual decisions is deferred to section 4. Here we only focus on those optimality conditions which have a clear economic interpretation and will help understanding some salient, qualitative predictions that our economy may deliver.

Entrepreneurs' decisions. For any entrepreneur i in $\mathcal{E}_t$,

$$((c^i_{t+1}, k^i_{t+j})_{j\in\{0,1\}}, x^i_t)$$

is an interior individual optimum, with respect to capital inputs, at financial decisions $0 \leq \zeta^i_t \leq \widehat{\zeta}$, $b^i_t \geq -B_t$, if and only if it satisfy, at all ξ in Ξ,

$$(*) \qquad [\textit{young-age}] \quad \begin{aligned} &\frac{\partial \rho^i_t}{\partial x^i_t}\left[\pi^i_{t+1}(\xi^i) - \pi^i_{t+1}(\xi^u)\right] \geq R_{t+1}, \ \textit{with equality if} \ \ b^i_t > -B_t \\ &A_{t+\iota-1} f_{k,t} = 1 \end{aligned}$$

$$(**) \qquad [\textit{old-age}] \quad \begin{aligned} &c^i_{t+1}(\xi) = e^i_{t+1}(\xi) + \left(1-\zeta^i_t\right)\pi^i_{t+1}(\xi) + b^i_t R_{t+1}, \\ &A^\iota_{t+1}(\xi) f_{k,t+1}(\xi) = 1 \end{aligned}$$

The timing of actions explains production decisions. Working backwards from $t+1$, after the state ξ has realized, entrepreneur i in $\mathcal{E}_t$ decides production inputs and, if $\xi = \xi^i$, she also decides patent-license sales. 'Production' decisions are equivalent to those of a (static) standard neoclassical firm (see the second line of $(*)$ and of $(**)$). Under our assumptions, the solution to production decisions is unique and yields continuous functions of input-demand and profits. Moreover, for all $0 \leq \zeta^i_t \leq \widehat{\zeta}$, debt repayment is feasible and individually rational, according to (B_t) above: productivity inaction is always feasible, $\pi^i_{t+1} \geq 0$, by individual rationality ($c^i_{t+1} \geq 0$), $(1-\zeta^i_t)\pi^i_{t+1}(\xi^u) + e^i_{t+1}(\xi^u) \geq R_{t+1}B_t$.

At young age t, given prices and future profits, all other decisions can be determined. First, considering the first line of $(*)$, R&D expenditure is chosen so that marginal benefits are above marginal costs (equal if the entrepreneur is financially unconstrained); where marginal benefits equal the expected increase in profits due to an improvement of the innovation probability,[13]

$$\rho^i_{x^i_t}\left[\pi^i_{t+1}(\xi^u) - \pi^i_{t+1}(\xi^{-i})\right]$$

while marginal costs are the actualized value of investing into the safe asset R_{t+1}. Second, firm's financial policy is also driven by very simple considerations. The budget constraint is,

$$-B_t \leq b^i_t = e^i_t + \left(1-\zeta^i_t\right)\pi^i_t + q_t\zeta^i_t - x^i_t$$

Thus, if at t firm's cash flow π^i_t and personal income e^i_t are insufficient to cover expenditure, $k^i_t + x^i_t$, the entrepreneur has to demand outside finance. The optimal loan-equity mixed depends on the

[13] Here one should bear in mind that an entrepreneur who does not innovate attains a profit $\pi(U)$, no matter what the others do. In fact $\pi^i_{t+1}(\xi^{-i}) = \pi^i_{t+1}(\xi^u) = f(A_t, k^*) - k^*$; which is due to the fact that any patent on A_t (if it existed!) expires in $t+1$, making A_t the only rational technological choice of old-age entrepreneurs.

relative cost of the two instruments. Equity financing is cheaper than loan if it promises a lower expected return,

$$\frac{\mathbb{E}_t^i \pi_{t+1}^i}{q_t + \pi_t^i} < R_{t+1}$$

in which case firm i sells the maximum feasible amount of equities, $\widehat{\zeta}$. Observe that this tends to happen for agents who have a relatively low expected return from entrepreneurship, either because they are of low-ability or because they have chosen to invest relatively little in R&D. Though, things are not so trivially predictable; it is also possible that

$$\frac{\mathbb{E}_t^i \pi_{t+1}^i}{q_t + \pi_t^i} > R_{t+1}$$

and still the entrepreneur sells equities, even up to $\widehat{\zeta}$, because she is borrowing constrained (γ_t^i sufficiently high in (ζ)). Hence, one might still observe a promising young entrepreneur entering equity financing, just because banks are not offering her enough credit. This, obviously, makes financial decisions a weak signal of entrepreneurs' ability and R&D effort.

Summarizing, ζ_t^i is the value of,

$$(\zeta) \qquad \zeta_t^i(\mathfrak{p}, n_{t+1}, B_t, A^t) = \begin{cases} \widehat{\zeta}, & \text{if } \frac{\mathbb{E}_t^i \pi_{t+1}^i}{q_t+\pi_t^i} \leq R_{t+1} + \gamma_t^i; \\ \left[0, \widehat{\zeta}\right], & \text{if } \frac{\mathbb{E}_t^i \pi_{t+1}^i}{q_t+\pi_t^i} = R_{t+1} + \gamma_t^i; \\ 0, & \text{otherwise} \end{cases}$$

where $\gamma_t^i \geq 0$ is the multiplier of the borrowing constraint, which is zero if $b_t^i > -B_t$

Households' decisions. For a typical household h, let the individual stochastic discount factor be ∇_{t+1}^h. Then, $(c_t^h, b_t^h, \theta_t^h)$ is individually optimal if only if,

$$R_{t+1} \geq \frac{\mathbb{E}_t \nabla_{t+1}^h \Pi_{t+1}^a}{q_t - \Pi_t^b},$$

with equality if $\theta_t^h > 0$,

$$b_t^h = e_t^h - \left(q_t - \Pi_t^b\right) \theta_t^h$$

and

$$c_{t+1}^h = e_{t+1}^h + b_t^h R_{t+1} + \theta_t^h \Pi_{t+1}^a$$

is the state-contingent, consumption profile. Clearly, ∇_{t+1}^h equals the gradient of the expected utility at c_{t+1}^h.

2.8. **Equilibrium.** We identify an economy by,

$$\mathfrak{E} = (\mathcal{E}, \mathcal{H}, A_0, \eta, f, \widehat{\zeta}, (\rho^i, e^i)_{i\in\mathcal{E}}, g, (u^h, e^h)_{h\in\mathcal{H}})$$

where, it is understood that, f, ρ^i, g, u^h are functions.

Definition 1 (Equilibrium)**.** *An equilibrium of an economy $\mathfrak{E}$ is a sequence of measurable functions of allocations and prices*

$$\left((c_{t+1}^i, (y_{t+k}^i)_{k=0,1}, x_t^i, \zeta_t^i, b_t^i)_{i\in\mathcal{E}_t}, ((c_{t+k}^h)_{k=0,1}, b_t^h, \theta_t^h)_{h\in\mathcal{H}}\right)_{t\in\mathcal{T}}, \quad \mathfrak{p} = (q_t, p_t, R_{t+1}^{-1})_{t\in\mathcal{T}},$$

returns from equity-funds $(\Pi^a_t, \Pi^a_{t+1})_{t\in\mathcal{T}}$*, number of licenses sold* $(n_t)_{t\in\mathcal{T}}$*, debt limits* $((B_t)_{t\in\mathcal{T}})_{\iota\in\{0,1\}}$ *and technological progress* $(A_t)_{t\in\mathcal{T}}$*, such that, at all dates* t *and state* ϕ *in* Φ*,*

(1) *all entrepreneurs* i *in* $\mathcal{I}$ *and households* h *in* $\mathcal{H}$ *solve their optimal decision problems at* $\big(\mathfrak{p}, A^t, B_t, (e^i_{t+k})_{k=0,1}\big)$*,* X_t *and* (Π^b_t, Π^a_{t+1})*;*
(2) *equity market clears,* $\sum_{h\in\mathcal{H}} \theta^h_t - \sum_{i\in\mathcal{E}} \zeta^i_t = 0$*;*
(3) *loan market clears,* $\sum_{h\in\mathcal{H}} b^h_t + \sum_{i\in\mathcal{E}_t} b^i_t = 0$*;*
(4) *returns from equity-funds are defined according to* $(\Pi^b_t), (\Pi^a_{t+1})$*;*
(5) p_t *solves the monopoly problem and* n_t *is the number of license sales at* p_t*;*
(6) *debt limits are determined according to* (B_t)*;*
(7) $X_t = g(x_t, A_t)$*.*

It can be easily verified that, for all generation t, an equilibrium of definition 1 satisfies a commodity market clearing in expectation. This suggests we could have presented an equilibrium definition in which item (4) is substituted by a requirement on the commodity market clearing. This is common in the general-equilibrium literature with asymmetric information, at least since Helpman and Laffont (1973) and has the 'advantage' to avoid the need to explicitly define intermediaries and their actions (*e.g.*, see Bisin et al. (2011), Lisboa (2001), Bisin and Gottardi (1999) and the references therein). However, this simplification tends to 'hide' the type of information requirements and institutions that would be actually needed to implement the corresponding equilibrium outcomes. To clarify this issue we add the following remark.

Remark 2.2 (The exact form of commodity market clearing)**.** *The type of market clearing condition implied by definition 1 is one in which the commodity market clears in expectations, computed with respect to the finest information partition (*i.e. *the join of* $(\Omega^i)_i$*). This is equivalent to require that the commodity-market-auctioneer has a (point-) expectation on types and, precisely, on* $(\rho^i_t)_i$*, which turns out to be exact, at equilibrium. Hence, for a type* i *in* $\mathcal{E}_t$*, the auctioneer takes the expectation of each variable with domain* Ξ *using the information* $(\Omega^i)_i$ *at that point in time, as in footnote 6. In fact, by doing this along* i*'s old-age budget constraint one finds, for each aggregate state* ϕ*,* $\mathbb{E}^i_t[c^i_{t+1}|\phi] = \mathbb{E}^i_t[e^i_{t+1}|\phi] + (1-\zeta^i_t)\mathbb{E}^i_t[\pi^i_{t+1}|\phi] + b^i_t\mathbb{R}_{t+1}$*. Hence, aggregating across old-age entrepreneurs* i *of generation* t *and using (4), one obtains,*

$$\bar{c}^t_{t+1} - \bar{e}^t_{t+1} - \bar{y}^t_{t+1} - p_{t+1}n_{t+1} = 0$$

where the time-superscript indexes the $t-$*generation,* $\bar{y}^t_{t+1} = y^t_{t+1}(S) + (J-1)y^t_{t+1}(U)$ *is the aggregate (expected) net-output produced by generation* t *in* $t+1$*, and* $p_{t+1}n_{t+1}$ *is the revenue from license sales that the innovator monopolist of generation* t *collects from the* $(t+1)-$*young (*i.e. *the entrepreneurs* i *in* $\mathcal{E}_{t+1}$ *who establish a mature firm). Finally, at equilibrium, the term* $p_{t+1}n_{t+1}$ *can be computed by a simple aggregation of the budget constraints of all agents, households and entrepreneurs, who are young in* $t+1$*. This, by (4), yields,*

$$p_{t+1}n_{t+1} = \bar{x}_{t+1} - \bar{e}^{t+1}_{t+1}$$

Therefore, upon substitution,

$$\bar{c}_{t+1} - \bar{e}_{t+1} - \bar{y}_{t+1} = 0$$

where $\bar{e}_{t+1} = \bar{e}^t_{t+1} + \bar{e}^{t+1}_{t+1}$ is the aggregate endowment at $t+1$ and $\bar{y}_{t+1}$, similarly defined, is the net-output produced in $t+1$.[14]

To summarize, we have shown that an equilibrium in definition 1 is also an equilibrium in which (4) is substituted by the assumption that the commodity market clears in expectations. The reverse is straightforward.

3. A simple example

In this section, we present a simplified version of our economy and discuss some of its possible balanced-growth equilibrium outcomes, also by mean of a numerical example.

The economy is one with only two entrepreneurs $\mathcal{E} = \{1, 2\}$, and a single household $\mathcal{H} = \{h\}$. All individuals are endowed with a certain amount of commodity when young, defined by $e = ((e^1_t, e^2_t), e^h_t)_t$ such that, at all t, $e_t = \eta^t e_0 > 0$.

We also start by focusing on the case of no-aggregate uncertainty, $\Xi = \{\xi^1, \xi^2\}$. Precisely, we parameterize the aggregator function g such that $X_t = g(x_t, 1) \equiv \lambda^1(x^1_t)^\beta + \lambda^2(x^2_t)^\beta$, and assume that the probability distributions are as follows: for every entrepreneur i, who conjectures $X_t > 0$, the success probability is,

$$(\rho^i) \qquad \rho^i(x^i_t, X_t) = \frac{\lambda^i(x^i_t)^\beta}{X_t}, \quad 0 < \beta < 1;$$

where the parameters $0 < \lambda^1 < \lambda^2 < 1$ capture entrepreneurial abilities. The production technology is Cobb-Douglas; at all t, given input k_t and frontier technology $A_t = \eta^t A_0$, commodity output is,

$$f(A_t, k_t) = A_t^{1-\alpha} k_t^\alpha, \quad 0 < \alpha < 1.$$

Since we focus on balanced growth equilibria, for simplicity and whenever it causes no confusion, we drop the time-subscript, indicate with A the frontier technology of the current young, and with ηA the next period frontier. We also indicate with subscripts y and o, respectively, young and old agents.

Information and markets are as in the general economy. Old-age individual profits are, $\pi_o = (\pi_o(S), \pi_o(U))$. There is a single equity-fund, which is traded at price q and whose payoffs are,[15]

$$\Pi^b = \frac{1}{\bar{\zeta}_t} \sum_i \pi^i_y \zeta^i, \quad \Pi^a = \widehat{\rho}_t \Delta\pi_o + \pi_o(U)$$

where $\Delta\pi_o = \pi_o(S) - \pi_o(U)$ and $\widehat{\rho}_t = \sum_i \rho^i(\zeta^i/\bar{\zeta})$.

3.1. Individual decisions.

Entrepreneurs decision process can be solved backwards, from old- to young-age, as we now illustrate.

[14] To be precise, notice that, $\bar{y}_{t+1} = \bar{y}^t_{t+1} + \bar{y}^{t+1}_{t+1}$, where $\bar{y}^{t+1}_{t+1}$ is the simple aggregation of $y^i_{t+1} = f_{t+1} - k^i_{t+1} - x^i_{t+1}$ over $\mathcal{E}_{t+1}$ (see equation (y_y) above).

[15] Recall that, in this example economy, an innovation occurs with probability $\rho_t = 1$, at all dates t.

For any technological adoption ι and current licence price p, entrepreneur i optimal *cash flow* (y^i_y, y^i_o) are defined as follows,

$$y_{y,\iota} = \max_{k_y \geq 0}[(\eta^\iota A)^{1-\alpha} k_y^\alpha - k_y - x] = \frac{1-\alpha}{\alpha} k_{y,\iota} - x, \quad k_{y,\iota} = \eta^\iota A \alpha^{\frac{1}{1-\alpha}}$$
$$y_o(U) = \max_{k_o(U) \geq 0}[(\eta A)^{1-\alpha}(k_o(U))^\alpha - k_o(U)] = \frac{1-\alpha}{\alpha} k_o(U), \quad k_o(U) = \eta k_{y,0}$$
$$y_o(S) = \max_{k_o(S) \geq 0}[(\eta^2 A)^{1-\alpha}(k_o(S))^\alpha - k_o(S)] = \frac{1-\alpha}{\alpha} \eta k_o(U), \quad k_o(S) = \eta^2 k_{y,0}$$

Accordingly, operating profits π can also be specified so as to account for differences across the firm's technology ι managed by young entrepreneurs (*i.e.* those for whom a patent protection is effective). Old-age, profits π_o are independent of types and technologies, but are state-contingent, also due to the uncertain reward from innovation $(p'(S), p'(U)) = (p', 0)$:

$$\pi^i_y = y_{y,\iota} - p\iota, \quad \pi^i_o(\phi) = y_o(\phi) + p'(\phi), \ \phi \in \{S, U\}$$

Next, we observe that, conditional of having innovated, each old entrepreneur acts as a monopolist and fix the patent price p' so as to fully extracts the cash-flow-rent of the current young. Here, this occurs by setting $p' = \eta(\eta - 1)y^0_y = \eta p$; implying that the licence price steadily grows at the rate of technological progress η. To see why this is so, first, notice that, for a symmetric R&D expenditure, $x = x^i = x^{-i}$, the high-ability entrepreneur 2 has the highest success probability (ρ^i), implying the highest expected reward from innovation,

$$\frac{\lambda^2}{\lambda^1 + \lambda^2} \times \frac{\Delta \pi_o}{x}$$

This also reveals that the only, exogenous, source of firms' heterogeneity is the ability λ^i, while technological adoption does not affect innovation decisions. Indeed, the choice of purchasing a patent license and initiate a mature firm gives to the entrepreneur a productivity advantage in her young-age only (*i.e.*, as long as the patent lasts); that is, gross of patent cost, a mature firm has higher cash-flow only at the time in which the investment decision takes place. However, knowing this, the current monopolist sets the license price p so as to extract all the technological rent; thus, at equilibrium, entrepreneurs are indifferent between setting up a start-up or a mature firm (*i.e.* p implies that $\pi_y = \pi_o$).

This all description yields the following young-age budget constraint,

$$(b.c._y) \qquad -B^i \leq b^i = e^i + (q - \pi_y)\zeta^i$$

and contingent, consumption profile,

$$(b.c._o) \qquad c^i(\xi) = (1 - \zeta^i)\pi_o(\xi) + Rb^i, \quad \xi \in \Xi$$

Therefore, going backwards, a young entrepreneur i solves,

$$V^i_\iota = \max_{(x^i, b^i, \zeta^i)} (1 - \zeta^i) \left[\rho^i(x^i)\pi_o(S) + (1 - \rho^i(x^i))\pi_o(U)\right] + Rb^i_\iota$$

subject to $(b.c._y)$ and a limit on equity issue $0 \leq \zeta^i \leq \widehat{\zeta} \leq 1$.

The households problem can be solved in a similar way. It is simpler, in this economy, as households bear no risk.

3.2. **A *balanced growth* equilibrium.** We conjecture that the economy has a *balanced growth* equilibrium, with: physical capital, R&D expenditure, output, consumption, loan/deposit positions and debt limits growing at the gross-rate of technological progress η; p and q growing at η; a constant interest rate on bank loans R, constant equity positions ζ^i and stationary innovation probabilities ρ^i. Our conjecture can be easily verified, by tedious computations, which we omit. Instead, what we think it is interesting to highlight here is the sort of financial structure that might emerge at equilibrium. Thus, in the rest of this section, we are going to illustrate, also through a parametric example, that a 'balanced growth' equilibrium can have the following features:

(1) The high-ability entrepreneur 2 chooses to initiate a start-up firm and the low-ability 1 purchases a patent license;
(2) the start-up finances R&D using all the available cash-flow and loanable funds; however, being short of cash and borrowing-constrained, entrepreneur 2 opts for an 'inefficiently high' share of equity financing;
(3) the low-ability entrepreneur 1, running the mature firm, saves almost all her young-age cash-flow in the bank deposit and invests only a limited amount in R&D;
(4) in the aggregate, credit frictions imply an under-investment in R&D by the high-ability entrepreneur, who is running a startup;
(5) both firms issue equities to the fund, whose shares are purchased by the consumer.

Moreover, at equilibrium, expected returns faced by a young entrepreneur have the following structure,

$$\texttt{Exp.return to invest in firm 1} < \mathbf{R} \leq \frac{\mathbf{\Pi^a}}{\mathbf{q-\Pi^b}} < \texttt{Exp.return to invest in firm 2}$$

This clearly entails an inefficient allocation of capital across ventures. In an economy with no capital-markets segmentation and financial restrictions, this return structure would create arbitrage opportunities; as a result, capital would flow across firms (from the low-ability 1 to the high-ability 2) and across markets, so as to equalize their returns with R. Thus, with respect to this efficient benchmark, there is an under-investment of R&D in the start-up firm, run by the high-ability entrepreneur and an over-investment in the firm run by the low-ability entrepreneur.

More formally, this equilibrium outcome simply follows by the property that the success probability (ρ^i) is strictly concave in the relative, innovation expenditure x^i/X. An individual innovation decision x^i is optimal if and only if,

$$\rho^i_{x^i} \Delta\pi_o \geq R$$

holding with equality if $i's$ borrowing constraint is non-binding. Solving, under our simple parameterization, one finds,

$$x^i = \left(\frac{\lambda^i \beta}{X} \times \frac{\Delta\pi_o}{R + \gamma^i} \right)^{\frac{1}{1-\beta}}, \quad X := \sum_j \lambda^j (x^j)^\beta$$

with γ^i denoting the Lagrange multiplier associated to the borrowing constraint of i. Clearly, although higher ability entrepreneurs are more prone to innovate, their debt limits might increase investment cost (measured by γ^2), resulting in an under-investment in R&D. Precisely, R&D expenditure satisfies $x^2 \geq x^1$ if and only if,

$$\frac{\lambda^2}{\lambda^1} \geq 1 + \frac{\gamma^2}{R};$$

in words, the high-ability entrepreneur invests more than the low-ability if and only if the cost of the financial friction she faces is 'small' relative to her ability advantage.

As for financial decisions, the return structure is one indicating that the high-ability, potentially more innovative entrepreneur, faces higher expected returns. This implies that, in absence of financial constraints, she would rather use the loan instead of equities. However, as financial constraints start binding, entrepreneur 2 opts to increase equity financing, selling shares of her firm to the fund. In contrast, the low-ability entrepreneur 1 is one that prefers to sell equities and save into a bank deposit (*i.e.* withholds funds from real investments and allocate them to purely financial ones). This behavior is still desirable, at equilibrium, since it allows to make funds flow to the high-ability, helping to keep low the interest rate and to relax the borrowing limits.

In the following numerical exercise we derive an equilibrium with the property we have just described, and compare its figures with those achieved at an equilibrium of an economy with 'weaker' financial frictions. This *benchmark* economy is such as firms' equities are still pooled into a fund but their financial decisions are left unrestricted (no limits on borrowing or on equity issues are imposed). Hence, in the benchmark, asymmetric information between consumers and entrepreneurs remains, although intermediaries are fully informed on entrepreneurs types and can purchase equity contracts at firm-specific prices.

For an economy with the following fundamentals,

$$\Big(A, (\lambda^1, \lambda^2), \alpha, \widehat{\zeta}, \beta, \eta, (e_0^1, e_0^2, e_0^c)\Big) = (1, (.25, .75), .35, .29, .85, 2.5, (.25, .25, .4))$$

the main equilibrium values are reported in Table 1.[16] In both cases, license prices are determined

Table 1. Results

Variables	x^1	ζ^1	b^1	x^2	ζ^2	b^2	θ^h	b^h	R
Equilibrium	.008	.19	.87	1.63	.04	- .87	.23	0	1.06
Benchmark	.001	0	.62	1.64	0	- 1.01	0	.39	1.44

so as to make entrepreneurs indifferent with respect to the technology to adopt, and grow at the rate of technological progress η (*i.e.* $p' = \eta p$).

The benchmark equilibrium is 'efficient' in the sense that it maximizes aggregate expected utilities, given the information available to each individual and market structure. At this equilibrium, returns to alternative investments and financial sources are equalized:[17]

$$\texttt{Exp.return to invest in firm 1} = \mathbf{R} = \texttt{Exp.return to invest in firm 2}$$

[16]The equilibrium algorithm is written in *Mathematica* and it is available on request.

[17]Here the fund price q is computed as $q = \frac{1}{\zeta^1+\zeta^2}(q^1\zeta^1 + q^2\zeta^2)$.

with the fund that is (voluntarily) not traded. This implies efficiency of both production and R&D decisions: at equilibrium, capital freely flows across firms, so as to equate their expected returns and the returns to investing in innovation.

Our numerical results illustrate features (1)-(4) at the beginning of this subsection. By going from the equilibrium figures to the 'efficient' benchmark one immediately observes that the R&D expenditure of the high-ability increases and that of the low-ability decreases. Financial decisions are also revised as expected, with the high-ability that decreases the proportion of equity financing over both the total outside financing and its own R&D expenditure, $q\zeta/(q\zeta+|b|)$ and $q\zeta/x_0$, falling to zero, respectively, from 9.6 percent and 5.7 percent. Also the low-ability entrepreneur, who is exploiting her informational advantage to profit from selling equities, essentially, finds optimal not to invest in the firm and to act as a saver rather than as an entrepreneur.

Again, weakening asymmetric information allows to implement an allocation in which savings are allocated more efficiently because transferred, through the bond, to the high-ability entrepreneur. This is possible essentially because *i)* it allows intermediaries to achieve a precise evaluation of equity prices, which damages the low-ability, and *ii)* it increases the possibility of borrowing, which lowers the cost of outside funds faced by the high type. As for the later, *Figure* 1 plots the supply (marginal cost) schedule associated to debt financing. It closely resembles to the standard representation used in the literature to explain financing hierarchy of entrepreneurs with market imperfections.[18] Here, the innovation expenditure, x, is measured on the horizontal axis and the marginal cost of debt is reported on the vertical axis. The marginal cost perceived by an high-type entrepreneur 2 is equal to the market interest rate R up to the expenditure level that makes her debt constraint binding, then it increases according to the shadow marginal cost associated to her debt limit γ. The marginal cost of cash-flow financing is the cheapest, being equal to one, at equilibrium. The marginal cost of funding is measured by the marginal cost of using an extra unit of capital to produce output divided by its marginal productivity: for a firm with technology $\iota \in \{0,1\}$,

$$\frac{1}{MPK_\iota} = \frac{1}{\alpha} \times \left(\frac{k_{y,\iota}}{\eta^\iota A}\right)^{1-\alpha}$$

It is important to remark that, while R is taken as given by each entrepreneur, economies in which financial constraints are introduced, or simply become more severe, tend to be associated to a fall in the equilibrium value of R. This equilibrium effect is also well understood and, in the essence, it is the consequence of the fact that a fall in the loan demand has to be balanced by a proportional fall of the (unconstrained) loan supply. In our economy a drop of R not only reduces the entrepreneur's cost of outside financing, but also relaxes her debt-limit; yet, this beneficial effect does not eliminate the overall distortion produced on R&D investment and financing.

A final comment concerns the benchmark equilibrium. The fact that the high type prefers to subscribe a debt contract and issues equities only when this is insufficient to fund investments, is in line with the corporate finance literature. Debt contracts have very good incentive properties under moral hazard, as their payment schedule is flat for every level of profit exceeding the fixed

[18] See Hubbard (1998) for the original discussion (in particular, see his figure 1 on p.196) and, for example, Carpenter and Petersen (2002) for one focussing on R&D financing.

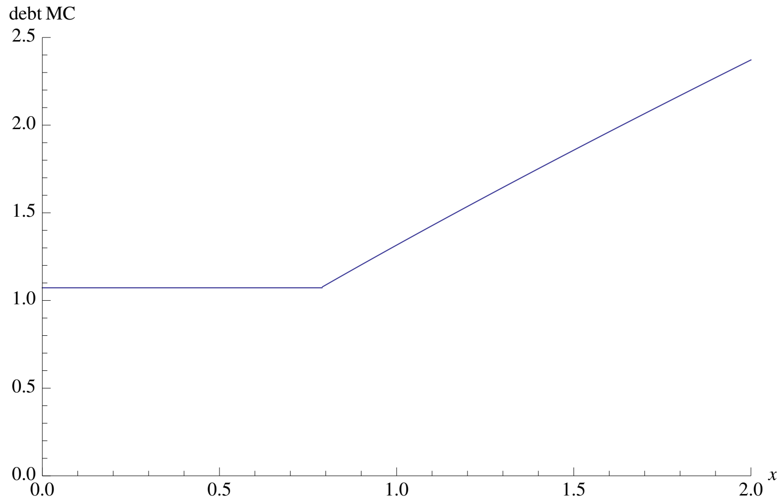

FIGURE 1. Entrepreneur 2, debt supply schedule

debt cost (*e.g.*, see Innes, 1990). This clearly creates an incentive for the entrepreneur who borrows to maximize the probability of achieving high profit realizations (ρ^i in our setting).

3.3. **An example with aggregate risk.** Our previous simple economy can be slightly changed to allow for aggregate risk, by adopting a different functional form for the innovation probability ρ^i. For example, it suffices to consider,

$$\rho^i = \lambda^i \left(\frac{x^i}{X}\right)^\beta, \quad 0 < \beta < 1, \quad X = x^1 + x^2$$

for $\sum \lambda^i = 1$ and $0 < \lambda^i < 1$, for all i.

We can explore how financial frictions and innovation decisions affect expected growth at the whole economy, without going through the whole exercise of computing equilibria. Recalling that individual optimality implies,

$$\rho^i_{x^i} = \frac{R + \gamma^i}{\Delta \pi_o}$$

under our parameterization, we have that an equilibrium must satisfy,

$$\frac{x^i}{X} = \left(\beta \lambda^i \frac{\Delta \pi_o}{R + \gamma^i}\right)^{\frac{1}{1-\beta}}$$

This yields an economy innovation probability,

$$\rho = \sum \rho^i = \sum \lambda^i \left(\frac{x^i}{X}\right)^\beta = \sum (\lambda^i)^{\frac{1}{1-\beta}} \left(\beta \frac{\Delta \pi_o}{R + \gamma^i}\right)^{\frac{\beta}{1-\beta}}$$

Financial frictions (binding borrowing limits) tend to lower the expect growth rate, for given interest rate R. As in our previous example, R adjusts downwards with credit restrictions, therefore the equilibrium effect on ρ depends on the overall effects on the terms $R + \gamma^i$.

4. Equilibrium existence in large economies

We shall consider a version of our economy in which there still is a finite number of entrepreneur types I, but there are a continuum of identical individuals of each type; without loss of generality, we assume the space of each type of entrepreneurs to be the unit interval. This essentially allows us to overcome a technical problem related to the non-convexity generated by the binary choice of technological adoption. Hence it would not be needed if we had to simplify our setting to one in which technologies are somehow assigned to each young entrepreneur. As we prove later, passing to large economies makes each type aggregate demand for licenses a convex correspondence. The economy becomes observationally equivalent to one with I entrepreneurs in which every i who is young at t, decides the fraction of activity $\nu_t^{i,\iota}$ to carry out using a technology ι in $\{0,1\}$. Yet, as she does so, she will not be constrained to define an R&D activity and financial policy which splits with the same proportions.

Accordingly, in this new economy aggregation has to be redefined and so has to be the equilibrium. For the latter it suffices to notice the following. Market clearing are now,

$$\sum_{i\in\mathcal{E}_t}\nu_t^i\zeta_t^i-\sum_{h\in\mathcal{H}_t}\theta_t^h=0,\quad \sum_{i\in\mathcal{E}_t}\nu_t^i b_t^i+\sum_{h\in\mathcal{H}_t}b_t^h=0$$

Moreover, letting, $\bar{\zeta}_t=\sum_{i\in\mathcal{E}_t}\nu_t^i\zeta_t^i$, the fund-payoffs are,

$$\Pi_t^b\equiv\frac{1}{\bar{\zeta}_t^\iota}\sum_{i\in\mathcal{E}_t}\nu_t^i\zeta_t^i\pi_t^i, \tag{Π_t^b}$$

$$\Pi_{t+1}^a(\phi)\equiv\frac{1}{\bar{\zeta}_t^\iota}\sum_{i\in\mathcal{E}_t}\nu_t^i\left(\zeta_t^i\mathbb{E}_t^i\left[\pi_{t+1}^i(\xi^i)|\phi\right]\right),\quad \phi\in\Phi_t \tag{Π_{t+1}^a}$$

4.1. Equilibrium existence.

Theorem 1. *For every large economy $\mathfrak{E}$ satisfying our assumptions an equilibrium exists.*

Without loss of generality, we restrict to economies where at $t=0$ there is no innovation, coming form the unrepresented past, and with the available technology being accessible at no cost, $p_0=0$.

The proof of this theorem is divided in two parts. In the first part, we prove the existence of an equilibrium in a *$T-$finite OLG economy*, with a finite number of generations T. As an immediate corollary, it follows that an equilibrium exists also in a *$T-$truncated OLG economy*; where a truncated economy is one in which assets and licenses can only be traded up to date $T-1$ (not in T) and spot markets are open up to T, but sequences of prices and allocations are defined over the infinite horizon (see, for example, Levine, 1989). Finally, because the equilibrium set of a $T-$truncated economy is also compact, the limit of such truncated equilibria exists and it is an equilibrium of our original overlapping generation economy.

4.1.1. *Equilibrium existence in a finite horizon OLG economy.* It is worth spending a few more words on the proof logic. Consider a $T-$finite economy; an OLG economy, as the one describes, which ends with the T^{th} generation. Time and uncertainty can have a tree representation, whose nodes are defined with respect to aggregate states $\phi\in\{S,U\}$ occurring at each date. Denote by $\mathbb{P}_s(\phi_t)$ the unique predecessor node of ϕ_t at time $s<t$. Then, for any $t\geq 0$ and node $\phi_{t+1}^s=S$

we can construct a 2-period, 2-state economy populated by a single generation, with the same characteristics as those assumed for the general economy, and aggregate uncertainty in the second period characterized by a node ϕ^u_{t+1} with the predecessor node $\phi_t = \mathbb{P}_t(\phi^s_{t+1}) = \mathbb{P}_t(\phi^u_{t+1})$. This one-generation economy is also characterized by a the fact that each entrepreneur, when old, has a license demand-conjecture $n(\phi_{t+1})$ that she can use to compute the monopoly profit from patent sales, in the event she had successfully innovated.

Definition 2 (A one-generation economy $\mathfrak{E}(\phi^s_{t+1})$)**.** *Any node ϕ^s_{t+1} and demand conjecture $n(\phi^s_{t+1})$ identifies a one-generation economy,*

$$\mathfrak{E}(\phi^s_{t+1}) = (A(\phi^s_{t+1})/\eta, \eta, f, u, \widehat{\zeta}, (\rho^i)_{i\in\mathcal{E}}, g, (e_t, e_{t+1}), n(\phi^s_{t+1}))$$

where t is used to index the date-event $\phi_t = \mathbb{P}(\phi^s_{t+1}) = \mathbb{P}(\phi^u_{t+1})$ and the variables in $t+1$ are contingent to the states $(\phi^s_{t+1}, \phi^u_{t+1}) = (S, U)$.

We now claim that we can obtain $\mathfrak{E}(T)$ as the union of $2\left(2^{T-1}\right) - 1$ distinct, one-generation economies,

$$(+) \qquad \mathfrak{E}(T) = \bigcup_{k=1}^{K} \left\{\mathfrak{E}(\phi^s_{t+1,k}) : t = 0, .., T-1\right\}, \quad K \equiv 2 \times 2^{T-1} - 1$$

Indeed, there are 2^{T-1} terminal nodes which correspond to innovation states, S. For each one of these 2^{T-1} nodes, $\phi^s_{T,j}$ in $\{\phi^s_{T,1}, ...\phi^s_{T,2^{T-1}}\}$, take the history terminating with $\phi^s_{T,j}$. Along each of these 2^{T-1} histories there are K distinct, one-generation economies. By patching these economies one can give a tree-representation to $\mathfrak{E}(T)$. We do it for $T = 2$ in figure 4.1.1 below.

Definition 3 (Equilibrium of a one-generation economy $\mathfrak{E}(\phi^s_{t+1})$)**.** *An equilibrium of a one-generation economy $\mathfrak{E}(\phi^s_{t+1})$ is a profile of measurable allocations and prices,*

$$\left(((y^i_{t+k})_{k=0,1}, x^i_t, \zeta^i_t, b^i_t, \gamma^i_t, n^i_t)_{i\in\mathcal{E}}, (\Pi^b_t, \Pi^a_{t+1}), (c^h_{t+1}, b^h_t, \theta^h_t)_{h\in\mathcal{H}}, \mathbf{p}_t = (q_t, p_t, p_{t+1}, R^{-1}_{t+1}, B_t), X_t\right)$$

such that,

1. *every individual i in $\mathcal{I}$ adopts her decisions optimally, at $(\mathbf{p}_t, (\Pi_t, \Pi_{t+1}), A_{t+1}, B_t, e_t)$, X_t and at the conjecture of license demand $n(\phi^s_{t+1})$;*
2. *the loan market clears, $\sum_{i\in\mathcal{E}_t} \nu^i_t b^i_t + \sum_{h\in\mathcal{H}_t} b^h_t = 0$;*
3. *the equity market clears: $\sum_{i\in\mathcal{E}_t} \nu^i_t \zeta^i_t - \sum_{h\in\mathcal{H}_t} \theta^h_t = 0$;*
4. *the return to equity funds satisfies $(\Pi^b_t), (\Pi^a_{t+1})$;*
5. *p_{t+1} solves the monopoly problem at the conjecture license-demand $n(\phi^s_{t+1})$ and license sales n_{t+1} are determined. p_t solves the monopoly problem at the actual license-demand $n(\phi_t)$ and license sales n_t are determined.*
6. *debt limit is B_t;*
7. *$X_t = g(x_t, A_{t+1}/\eta)$.*

Here, the only thing to notice is that an equilibrium of a one-generation economy $\mathfrak{E}(\phi^s_{t+1})$ defines the license demand correspondence of the generation when young, at t. Since the trade of patent-licenses is the only form of trade occurring across generations, the later piece of information is the only one we need to establishing equilibrium in a $T-$finite economy. To illustrate how this can be done, consider the following procedure.

Now, having in mind the representation in (+) and its corresponding tree (see figure 2), start at the terminal node of the terminal history in which there has been an innovation in every predecessor node (an history of full success at the aggregate level); call this node $\phi^s_{T,1}$. This identifies a one-generation economy $\mathfrak{E}(\phi^s_{T,1})$ with a license demand conjecture of $n_T(\phi^s_{T,1}) = 0$. At the terminal nodes $(\phi^s_{T,1}, \phi^u_{T,1})$ (where ϕ^u_T is s.t. $\phi^s_{T-1,1} = \mathbb{P}_{T-1}(\phi^s_{T,1}) = \mathbb{P}_{T-1}(\phi^u_{T,1})$), technologies are $A(\phi^s_{T,1}) = \eta^T A_0$ and $A(\phi^u_{T,1}) = \eta^{T-1} A_0$, respectively. Thus, profit functions can be computed. We point out that the identity (or type) of the innovator entrepreneur in $\phi^s_{T,1} = S$ does not affect market variables and, consequently, the other agents' behaviors. Working backwards, at $\phi^s_{T-1,1}$ we can define the individuals' optimal decisions (including R&D and financial ones) as functions of prices and fund payoffs only. Moreover, for the entrepreneurs, we can also compute the license demand (the technology adoption correspondence) $n(\phi^s_{T-1})$. This is obtained observing that the decision is between a costly technology $A(\phi^s_{T-1}) \equiv A(\phi^s_T)/\eta$ and a freely available one, equal to $A(\phi^s_{T-1}) \equiv A(\phi^s_T)/\eta^2$ (the most productive among all the free technologies available). Finally, at every node $(\phi^s_{T-1}, \phi^u_{T-1})$ financial market prices can be derived so as to satisfy market clearing, which only involve currently young individuals. Thus, an equilibrium for this one-generation economy can be derived. Notice that this procedure is a backward induction one which is used to derive demand and supply functions of prices and competitive prices that have to be determined by the 'auctioneer'; thus, finding an equilibrium of a one-generation economy involves a fixed-point argument. Next, if, for example, $T = 2$ (as in figure 2), working backwards, we can consider the economy $\mathfrak{E}(\phi^s_{T-1})$ equipped with a license-demand conjecture $n(\phi^s_{T-1})$, whose equilibrium we can compute as above. We are now at the origin of the tree, ϕ_0 in which the technology is free. It remains to reiterate the two-step procedure just illustrated, by starting from the only other terminal node of type S, $\phi^s_{T,2} = S$. This defines an economy $\mathfrak{E}(\phi^s_{T,2})$ of which we can compute the equilibrium. Clearly, this reasoning extends to any finite $T > 2$.

We now provide a simpler (and yet more precise and general) definition of the equilibrium algorithm for a $T-$finite economy.

Definition 4 (Iteration $I(\phi)$)**.** *Given a $T-$finite economy $\mathfrak{E}(T)$, for every $0 \leq t \leq T-1$, an iteration $I(\phi^s_{t+1})$ starts at a node $\phi^s_{t+1} = S$, chosen such that there exists another node ϕ^u_{t+1} with common predecessor $\phi_t = \mathbb{P}_t(\phi^s_{t+1}) = \mathbb{P}_t(\phi^s_{t+1})$. For all $0 \leq t \leq T-1$, iteration $I(\phi^s_{t+1})$ identifies a one-generation economy $\mathfrak{E}(\phi^s_{t+1})$. The outcome of an iteration is an equilibrium of $\mathfrak{E}(\phi^s_{t+1})$ and a conjecture $n(\phi_t)$ corresponding to the license demand correspondence of its currently young entrepreneurs; where, $n(\phi_0) = 0$.*

Definition 5 (Equilibrium algorithm)**.** *The equilibrium algorithm over a $T-$finite economy $\mathfrak{E}(T)$ consists in performing the iterations $(I(\phi^s_{t+1}))_{0\leq t\leq T-1}$ with the following rules:*

- *For all $0 \leq t \leq T-1$, $\phi^s_{t+1} = S$ and $\phi^s_t = \mathbb{P}(\phi^s_{t+1})$, the conjecture $n(\phi^s_t)$ in $\mathfrak{E}(\phi^s_t)$ is the outcome of $I(\phi^s_{t+1})$;*
- *For all terminal nodes $\phi^s_T = S$, a conjecture $n(\phi^s_T) = 0$ characterizes $\mathfrak{E}(\phi^s_T)$;*
- *Iterations are performed backwards, starting at all the terminal nodes of type S.*

The outcome of an application of the algorithm is an equilibrium of $\mathfrak{E}(T)$.

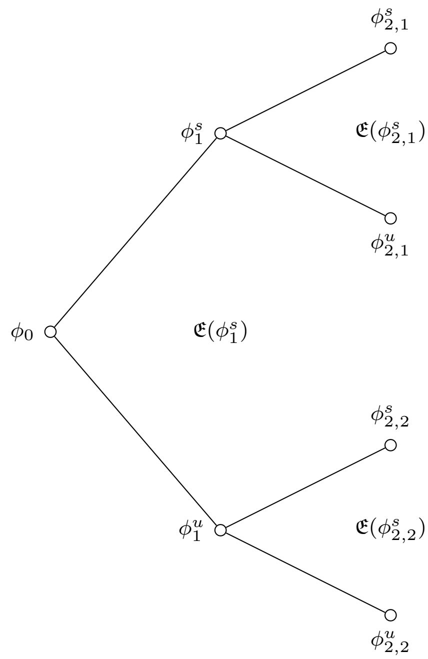

FIGURE 2. $\mathfrak{E}(2)$

Having observed all this, the existence of an equilibrium of a $T-$finite economy $\mathfrak{E}(T)$ follows from the existence of an equilibrium of $\mathfrak{E}(\phi^s_{t+1})$, something we establish in the following proposition.

Proposition 2. *An equilibrium of the economy $\mathfrak{E}(\phi^s_{t+1})$ exists. Moreover, the equilibrium set is compact.*

Equilibrium existence of $\mathfrak{E}(\phi^s_{t+1})$, essentially, follows from Debreu (1958), Radner (1972) and (for an economy with equity funds) Drèze, Minelli and Tirelli (2008), opportunely extended to our context. Thus, first, we artificially bound the economy, restricting real resources and asset positions in a box of edge κ. Individuals' real actions (c, y, k, x) are measurable, positive sequences, which are uniformly bounded: at every date t, the $t-$element of the allocation sequence is assumed to be contained in a cube of uniform edge $\kappa > 0$. An analogous assumption is introduced for households' financial actions (b, θ). S^κ denotes the intersection of a set S with a $\kappa-$cube of equal dimension. Prices $\mathbf{p}_t = (q_t, p_t, p_{t+1}, R^{-1}_{t+1})$ are restricted to be measurable, positive sequences which are uniformly bounded, below by $\epsilon > 0$ and above by $1/\epsilon$; so that, at every date t, the $t-$element of the each price sequence is contained in $G^\epsilon := [0, 1/\epsilon]$. Similarly, for any entrepreneur i and every date t, γ^i_t is restricted on G^ϵ. At $t = 0$, $\mathbb{P}^\epsilon, \mathbb{G}^\epsilon$, respectively, denote the set of al possible, bounded, measurable sequences of prices and individual multipliers γ. Second, we prove that an equilibrium of this box-T-economy exists. Finally, we show that, by taking κ and $1/\epsilon$ sufficiently large, the latest is also an equilibrium of the original, $T-$ finite economy.

Production and innovation choices. Consider any entrepreneur i in $\mathcal{E}_t$ and, for notational simplicity, drop the superscript i, unless necessary.[19] Using her young-age budget constraint to substitute for b_t, we can write the entrepreneur maximum problem as,

$$(fP) \qquad \max_{\left(x_t,(k_{t+j})_{j\in\{0,1\}},\zeta_t,\gamma_t\right)} J^\iota\left(x_t,k_{t+1},\zeta_t,b_t,\cdot\right) = (1-\zeta_t)\,\mathbb{E}_t^i\pi_{t+1}+$$
$$+\left[\left(f(A_{t+\iota-1},k_t)-k_t-x_t-p_t\iota+e_t^i\right)(1-\zeta_t)+q_t\zeta_t\right](R_{t+1}+\gamma_t)+\gamma_t B_t;$$
$$\textit{such that } \left(x_t,(k_{t+j})_{j\in\{0,1\}}\right)\in[0,\kappa]\times\mathbb{R}_+^4,\; 0\le\zeta_t\le\widehat{\zeta},\quad \gamma_t\in G^\epsilon$$

$$\begin{aligned}\mathbb{E}_t^i\pi_{t+1} &:= (1-\rho_t)\,\pi_{t+1}(\xi^u)+\rho_t^i\pi_{t+1}(\xi^i)+\left(\rho_t-\rho_t^i\right)\pi_{t+1}(\xi^{-i})\\ &= \left(1-\rho_t^i\right)\pi_{t+1}(\xi^u)+\rho_t^i\pi_{t+1}(\xi^i)\end{aligned}$$

and, according to (y_o) and (π_o), $\pi_{t+1}(\xi)=\left[f(A_{t+1}^\iota,k_{t+1})-k_{t+1}+p_{t+1}n_{t+1}\right](\xi)$ is independent of i; also, $\pi_{t+1}(U)\equiv\pi_{t+1}(\xi^u)\equiv\pi_{t+1}(\xi^{-i})$ and $\pi_{t+1}(S)\equiv\pi(\xi^i)_{t+1}$, for all i.

First, we observe that an optimal production choice corresponds to the solution of a finite number of static problems determining the net-supply or -output function. For a young-age,

$$s_t(\mathbf{p}_t)=\max_{k_t\ge0}\left[f(A_{t+\iota-1},k_t)-k_t\right]$$

For an old-age, $I+1$ state-contingent ex-post problems define a net-output profile: for all $\xi\in\Xi$,

$$s_{t+1}(\mathbf{p}_t)(\xi)=\max_{k_{t+1}(\xi)\ge0}\left[f(A_{t+1},k_{t+1})-k_{t+1}\right](\xi)$$

These problems are (strictly) concave programs, delivering a unique solution, a maximizer input and net-output profile,

$$\left(k_t^*,(k_{t+1}^*(\phi))_{\phi\in\Phi}\right),\left(y_t^*,(y_{t+1}^*(\phi))_{\phi\in\Phi}\right)\gg0$$

which also identifies optimal profits $\pi_{t+1}^*(\phi)$ for all $\phi\in\{S,U\}$.

Second, default-proof debt limits are defined according to (B_t) as the function, $B_t^{\iota,\kappa}:\mathbb{P}^\epsilon\to[0,\kappa]$,

$$B_t^{\iota,\kappa}(\mathbf{p}_t)=\frac{(1-\widehat{\zeta})\pi_{t+1}^*(\mathbf{p}_t)(\xi^u)}{R_{t+1}}$$

This function is continuous and bounded.

Third, the firm maximum problem is also concave in x_t, therefore the individually optimal R&D investment is characterized by its first order condition,

$$\frac{\partial\rho_t^i}{\partial x_t^i}\left[\pi_{t+1}^*(S)-\pi_{t+1}^*(U)\right]-(R_{t+1}+\gamma_t)=0$$

The left-hand-side of this condition is a continuous function of $(\mathbf{p}_t,\gamma_t,X_t)$, we call it $\varrho_t(\mathbf{p}_t,\gamma_t,X_t)$, where X_t is the aggregate R&D expenditure at t. For any $(\mathbf{p}_t,\gamma_t,X_t)$, ϱ_t has a zero x_t^{i*}, by the Intermediate Value Theorem. Indeed, under our assumptions, ϱ_t is a strictly decreasing function of x_t^i; strictly positive as $x_t^i\to0$ (bc. $\lim_{\widehat{x}^i\to0}\rho_x^i(\widehat{x}^i,\cdot)=+\infty$); negative (or zero) for large enough x_t^i (bc. $\lim_{\widehat{x}^i\to+\infty}\rho_x^i(\widehat{x}^i,\cdot)=0$). The zero is unique, by strict monotonicity of ϱ_t. By the IFT, the solution,

$$\chi_t^{i*}(\mathbf{p}_t,\gamma_t,X_t)$$

[19] In particular, we shall use the superscript to distinguish between ρ^i and ρ.

is a continuous function on $\mathbb{P}^{\epsilon} \times G^{\epsilon} \times [0, I\kappa]$ with positive values x_t^{i*} in $[0, \kappa]$.

Firm financial decisions. Fourth, optimal equity financing is characterized by a correspondence $\zeta_t^{\kappa,\epsilon*} : \mathbb{P}^{\epsilon} \times G^{\epsilon} \times [0, I] \times [0, I\kappa] \to [0, \widehat{\zeta}]$ defined as in (ζ) above. This correspondence is clearly nonempty, upper hemi-continuous (uhc) and convex valued. These properties are inherited by the bond-holding correspondence,

$$b_t^{\kappa,\epsilon*}(\mathbf{p}_t, \gamma_t, n_{t+1}, X_t) = \left(1 - \zeta_t^{\kappa,\epsilon*}(\mathbf{p}_t, \gamma_t, n_{t+1})\right)\left(\pi_t^* - \chi_t^*(\mathbf{p}_t, \gamma_t, X_t) + e_t^i\right) + q_t \zeta_t^{\kappa,\epsilon*}(\mathbf{p}_t, \gamma_t, n_{t+1})$$

We are now going to define a multiplier-correspondence. This is, $\Gamma_t^{\kappa,\epsilon*} : \mathbb{P}^{\epsilon} \times G^{\epsilon} \times [0, I] \times [0, 1] \to G^{\epsilon}$,

$$\Gamma_t^{\kappa,\epsilon*}(\mathbf{p}_t, \gamma_t, n_{t+1}, X_t) = arg \min_{\gamma_t' \in G^{\epsilon}} \gamma_t' \left[b_t^{\kappa,\epsilon*}(\mathbf{p}_t, \gamma_t, n_{t+1}, X_t) + B_t(\mathbf{p}_t)\right]$$

Notice that $\Gamma_t^{\kappa,\epsilon*} = 0$ when the borrowing limit is non-binding, $b_t^{\kappa,\epsilon*}(\cdot, \gamma_t) > -B_t$ and $\Gamma_t^{\kappa,\epsilon*} = G^{\epsilon}$ otherwise. Hence, such correspondence is clearly closed with non-empty and convex values; and, because G^{ϵ} is compact, it is also uhc. Notice that a fixed point of this correspondence identifies a multiplier that makes the complementary slackness condition of problem (fP) hold.

Firm choice correspondence. Concluding, we can define the firm choice correspondence, $S_t^{\iota,\epsilon,\kappa} : \mathbb{P}^{\epsilon} \times [0, I] \times [0, I\kappa] \to \mathbb{F}_t^{\kappa} \times [0, \widehat{\zeta}] \times [-B_t, \kappa] \times G^{\epsilon}$ such that,

$$S_t^{\iota,\epsilon,\kappa}(\mathbf{p}_t, n_{t+1}, X_t) = (\times_{k \in \{0,1\}} s_{t+k}^{\iota} \times x_t^{\iota*}) \times \zeta_t^{\kappa,\epsilon*} \times b_t^{\kappa,\epsilon*} \times \Gamma_t^{\kappa,\epsilon*}$$

where $\mathbb{F}_t^{\kappa} := [0, \kappa]^3 \times [0, \kappa]$. By the properties of each of one of its components, we conclude that,

Lemma 3. *$S_t^{\iota,\epsilon,\kappa}$ is nonempty, compact and convex valued, upper hemi-continuous.*

License demand. Consider a single entrepreneur of type i, born in t. $S_t^{i,\iota,\epsilon,\kappa}$ represents the choice of i corresponding to her decision to initiate either a mature $(\iota = 1)$ or a start-up $(\iota = 0)$. Accordingly, for any tuple $(\mathbf{p}_t, n_{t+1}, X_t)$ in $\mathbb{P}^{\epsilon} \times [0, I] \times [0, I\kappa]$, $(V_t^{i,1}, V_t^{i,0})$ represent the expected (indirect) utilities of an entrepreneur i, born in t, resulting from the two alternatives. These are continuous functions of $(\mathbf{p}_t, n_{t+1}, X_t)$ by Berge's maximum theorem. The technology adoption decision, to buy a patent license, at t is represented by the correspondence $\iota_t^{i,\epsilon} : \mathbb{P}^{\epsilon} \times [0, I] \times [0, I\kappa] \to \{0, 1\}$ defined such that,

$$\iota_t^{i,\epsilon}(\mathbf{p}_t, n_{t+1}, X_t) = \begin{cases} 1 & \text{if } V_t^{i,1}(p_t, \cdot) \geq V_t^{i,0} \\ 0 & \text{otherwise.} \end{cases}$$

Next, assume that there is a continuum of entrepreneurs of every type i, measurable in the unit interval. The measure of type i entrepreneurs demanding a licence is, $\mu^{i,\epsilon} : \mathbb{P}^{\epsilon} \times [0, I] \times [0, I\kappa] \to [0, 1]$ such that,

$$\mu_t^{i,\epsilon}(\mathbf{p}_t, n_{t+1}, X_t) = \begin{cases} 1 & \text{if } V_t^{i,1}(p_t, \cdot) > V_t^{i,0} \\ [0, 1] & \text{if } V_t^{i,1}(p_t, \cdot) = V_t^{i,0} \\ 0 & \text{otherwise.} \end{cases}$$

The aggregate demand of licenses at t is $\mu_t^{\epsilon} = \sum_i \mu_t^{i,\epsilon} : \mathbb{P}^{\epsilon} \times [0, I] \times [0, I\kappa] \to [0, I]$.

Lemma 4. *$\mu_t^{i,\epsilon}$ is nonempty, compact and convex valued, upper hemi-continuous. These properties aggregate to μ_t^{ϵ}.*

Proof. For all i, t, $\mu_t^{i,\epsilon}$ is clearly nonempty, convex valued, with a closed graph. The graph is one of a step function, taking value 1 up to some $\bar{p}_t \geq 0$, being the whole interval $[0,1]$ at $\bar{p}_t$, and taking value zero for any $p_t > \bar{p}_t$. As the range $[0,1]$ is compact, by the closed graph theorem, the correspondence is uhc. Finally, these properties aggregate to μ_t^ϵ. $\square$

Entrepreneur choice correspondence. We now define a type i entrepreneur, choice correspondence as,

$$\left(S_t^i \times \mu_t^i\right)^{\epsilon,\kappa}$$

Lemma 3 and 4 imply,

Lemma 5. $\left(S_t^i \times \mu_t^i\right)^{\epsilon,\kappa}$ *is nonempty, compact and convex valued, uhc.*

License supply and equilibrium. Suppose we keep considering time t. License are supplied by a single, successful entrepreneur, who is currently old. This entrepreneur acts as a monopolist, fixing the price p_t so as to maximize revenue from licence sales, given the market demand correspondence μ_t^ϵ of currently young entrepreneurs.

The monopolist solves the maximum problem,

$$(mP) \qquad \psi_t^\epsilon(\mathbf{p}_t, n_{t+1}, X_t) = \arg \max_{p'_t \in [0,1/\epsilon]} p'_t \cdot \mu_t^\epsilon(p'_t, n_{t+1}, X_t)$$

where $\psi_t^\epsilon : \mathbb{P}^\epsilon \times [0, I] \times [0, I\kappa] \to [0, 1/\epsilon]$. This–if it is nonempty– determines a monopoly equilibrium price in the license market open at time t.

We point out that the monopoly problem is independent of i. It is obvious that whoever is the patent holder acts as a monopolist facing a market demand correspondence that is type-independent. This is used both to determine the equilibrium prices p_t^* and p_{t+1}^*; although, for p_t^*, the monopolist is an agent who is a member of another economy (i.e. a member of the generation preceding the one in $\mathfrak{E}(\phi_{t+1}^a)$–*i.e.* of the generation in $\mathfrak{E}(\phi_t)$- with $\phi_t = \mathbb{P}_t(\phi_{t+1}^a)$).

Lemma 6. *The monopolist problem* (mP) *has a solution.*

Proof. Define a licence price correspondence to be $\varsigma_t^\epsilon : [0, I] \to \mathbb{P}^\epsilon$ such that,

$$(+) \qquad \varsigma_t^\epsilon(n_t) = \left\{ arg \max_{0 \leq p'_t \leq 1/\epsilon} p'_t \cdot n_t \right\}$$

where n_t is the overall measure of entrepreneurs demanding a licence at t. Then, define the licence-market correspondence $\xi_t^\epsilon : \mathbb{P}^\epsilon \times [0, I] \times [0, I\kappa] \to \mathbb{P}^\epsilon \times [0, I]$ such that

$$\xi_t^\epsilon(p_t, n_t, X_t) = \varsigma_t^\epsilon(n_t) \times \mu_t^\epsilon(p_t, n_t, X_t).$$

The licence-market correspondence computes all the alternative pairs (p_t, n_t) which the patent holder can possibly face, when it comes to sell licences. This computation has a solution if this correspondence has a fixed point, in which case the monopoly problem is well defined. More precisely, we are going to establish that ς_t^ϵ and ξ_t^ϵ, are nonempty, compact and convex valued, uhc; hence, by Kakutani's fixed point theorem, ξ_t^ϵ has a fixed point (p_t^*, n_t^*). Among such fixed points, which lie in a compact set, some will yield maximum revenue to the patent holder, *i.e.* yield licence prices in ψ_t^ϵ.

The correspondence ς_t. As all variables considered are contemporaneous, let us drop time subscript t. ς^ϵ is nonempty by Weierstrass' theorem, because for every n_t, $\varsigma^\epsilon \cdot n_t$ is a continuous (linear) function defined on a compact set. If we view the constrained set of the maximization problem in (+) as a constant correspondence, $\jmath : \mathbb{P}^\epsilon \to [0, I]$ such that $\jmath(p_t) = [0, I]$ for all p_t in $\mathbb{P}^\epsilon$; then the constrain set is obviously a nonempty, compact valued and continuous correspondence. Therefore, by Berge's Maximum theorem, we conclude that, for all $n_t \in \jmath(\mathbb{P}^\epsilon)$, $\varsigma^\epsilon(n_t)$ defined in (+) is an uhc and compact-valued correspondence. Finally, the linearity of $\varsigma^\epsilon(n_t)$ implies that it is also convex valued.

We have shown that ξ^ϵ has a fixed point (p_t^*, n_t^*), which says that $p_t^* \in \varsigma^\epsilon(n_t^*)$ and $n_t^* \in \mu^\epsilon(p_t^*, \cdot)$. Let P_t^* be the set of fixed points licence-prices. Observe that for all $p_t^* \in P_t^*$, $0 \le p_t^* \cdot \mu^\epsilon(p_t^*, \cdot) \le (1/\epsilon)I$ identifies monopoly revenue values on the real line. As such points can be ordered, a maximum solution to (mP) exists, although its argument p_t^* may not be unique.

The same proof applies to derive the equilibrium price p_{t+1}^* for the given conjecture of the the license-demand correspondence in $t+1$. □

Fund payoff correspondence. The fund payoff (perturbed) correspondence is defined as, $\beta^{\epsilon,\kappa} : [0,\kappa]^{4J} \to [0,\kappa]^3$, $\beta^{\epsilon,\kappa}(\pi_t, \pi_{t+1}, \zeta) = (\Pi_t^{b,\epsilon,\kappa}, \Pi_{t+1}^{a,\epsilon,\kappa})$, where,

$$\Pi_t^{b,\epsilon,\kappa} = \frac{1}{\sum_{i\in\mathcal{E}_t} n_t^i \zeta_t^i + J\epsilon} \left[\sum_{i\in\mathcal{E}_t} n_t^i (\zeta_t^i + \epsilon) \pi_t^i \right],$$

$$\Pi_{t+1}^{a,\kappa}(\phi) = \frac{1}{\sum_{i\in\mathcal{E}_t} n_t^i \zeta_t^i} \sum_{i\in\mathcal{E}_t} n_t^i \zeta_t^i \mathbb{E}_t^i[\pi_{t+1}^i | \phi], \quad \pi \in \Phi$$

Household decisions. Let the budget set of h be, $\boldsymbol{B}_t^{h,\epsilon,\kappa} \equiv \boldsymbol{B}_t^{h,\epsilon,\kappa}\left(\mathbf{p}_t, \Pi_{t+1}^{\epsilon,\kappa}\right)$

$$\boldsymbol{B}_t^{h,\epsilon,\kappa} = \left\{ (c_{t+1}, \theta_t^h, b_t^h) \in [0,\kappa]^2 \times [0,\kappa] \times [-\kappa,\kappa] : \begin{array}{l} b_t^h \le e_t^h - \left(q_t - \Pi_t^b\right)\theta_t^h \\ c_{t+1}^h(\phi) \le e_{t+1}^h + b_t^h R_{t+1} + \theta_t^h \Pi_{t+1}^a(\phi), \ \phi \in \Phi \end{array} \right\}$$

The household h demand correspondence is, $D_t^{h,\epsilon,\kappa} : \mathbb{P}^\epsilon \times [0,\kappa] \to [0,\kappa]^2 \times [0,\kappa] \times [-\kappa,\kappa]$ such that,

$$D_t^{h,\epsilon,\kappa}\left(\mathbf{p}_t, \Pi_{t+1}^{\epsilon,\kappa}\right) = \arg\max \left\{ u^h\left(c_{t+1}\right) : (c_{t+1}, \theta_t^h, b_t^h) \in \boldsymbol{B}_t^{h,\epsilon,\kappa}\left(\mathbf{p}_t, \Pi_{t+1}^{\epsilon,\kappa}\right) \right\}$$

Omitting, for simplicity, the superscript h, we can establish the following. Since $\boldsymbol{B}_t^{\epsilon,\kappa}$ is compact and u is continuous, $D^{\epsilon,\kappa}$ is nonempty. Since $\boldsymbol{B}_t^{\epsilon,\kappa}$ is convex and u is quasi-concave, $D^{\epsilon,\kappa}$ is convex valued. By the continuity of u and Berge's maximum theorem, $D_t^{\epsilon,\kappa}$ is uhc if the budget correspondence is continuous (lhc and uhc). The graph of $\boldsymbol{B}_t^{\epsilon,\kappa}$ is closed in $\mathbb{P}^\epsilon \times [0,\kappa] \times [0,\kappa]^2$ and $[0,\kappa]$ is compact, so $\boldsymbol{B}_t^{\epsilon,\kappa}$ is uhc. lhc follows by standard arguments (*e.g.* Dréze et al., 2008), analogous to those used above for the firm's constrained correspondence $\chi_t^{\iota,\epsilon,\kappa}$.

Competitive markets auctioneer. The idea is to determine the equilibrium price of competitive markets, (q_t^*, R_{t+1}^*), defining an auctioneer demand correspondence, in the spirit of Debreu (1958), as $\Phi_t^{\epsilon,\kappa} : \left(\mathbb{F}_t^\kappa \times [0, \widehat{\zeta}]\right)^J \times ([-\kappa,\kappa] \times [0,\kappa])^H \to [0, 1/\epsilon]^2$ such that,

$$\Phi_t^{\epsilon,\kappa} = \arg\max_{\widehat{q}_t, \widehat{R}_t} \left\{ \widehat{q}_t\left(\bar{\theta}_t - \bar{\zeta}_t\right) + \widehat{R}_{t+1}^{-1}\bar{b}_t : \ \left(\widehat{q}_t, \widehat{R}_{t+1}^{-1}\right) \in [0, 1/\epsilon]^2 \right\}$$

This correspondence has all the desired properties: it is nonempty, compact and convex valued, uhc.

R&D aggregator. Using the short hand notation $g_t = g(x_t, A_t)$, for any given A_t, we define the following composition,

$$\alpha_t := g_t \circ \left(\times_{i\in\mathcal{E}_t}\chi_t^{i*}\right) : \mathbb{P}_t^\epsilon \times G_t^\epsilon \times [0, I\kappa] \to [0, I\kappa]$$

such that $X_t^* = g_t\left(\times_i \chi_t^{i*}, A_t\right)$. Hence, α_t is a continuous function.

Equilibrium existence.

Lemma 7. *For all $(\epsilon,\kappa)-$box one-generation economy, $\mathfrak{E}^{\epsilon,\kappa}(\phi_{t+1}^a)$, an equilibrium exists.*

Proof. For all $t \in \{0, 1, .., T\}$, the equilibrium is described by a tuple,

$$\boldsymbol{e}_t^{\epsilon,\kappa} = \Big(((y_{t+k}^i)_{k=0,1}, x_t^i, \zeta_t^i, b_t^i, \gamma_t^i, n_t^i)_{i\in\mathcal{E}}, (\Pi_t^{b,\epsilon,\kappa}, \Pi_{t+1}^{a,\epsilon,\kappa}), (c_{t+1}^h, b_t^h, \theta_t^h)_{h\in\mathcal{H}}, (q_t, p_t, p_{t+1}, R_{t+1}^{-1}, B_t), X_t\Big),$$

an element of the set,

$$E_t^{\epsilon,\kappa} = \Big(\mathbb{F}_t^\kappa \times [0, \widehat{\zeta}] \times [-\kappa, \kappa] \times G_t^\epsilon \times [1, I]\Big)^I \times [0,\kappa]^3 \times \big([0,\kappa]^2 \times [-\kappa,\kappa] \times [0,\kappa]\big)^H \times \mathbb{P}_t^\epsilon \times [-B_t, \kappa] \times [0, I\kappa]$$

The equilibrium correspondence is $\Sigma_t^{\epsilon,\kappa} : E_t^{\epsilon,\kappa} \to E_t^{\epsilon,\kappa}$ be defined as,

$$\Sigma_t^{\epsilon,\kappa} = \times_{i\in\mathcal{E}}\left(S_t^i \times \mu_t^i \times \Gamma_t^{i*}\right)^{\epsilon,\kappa} \times \beta_t^{\epsilon,\kappa} \times_{h\in\mathcal{H}} \left(D_t^h\right)^{\epsilon,\kappa} \times \Phi_t^{\epsilon,\kappa} \times \psi_{t+1}^\epsilon \times \alpha_t$$

Using the above lemmata, we conclude that the equilibrium correspondence has a fixed point $e_t^{\epsilon,\kappa}$, by Kakutani's fixed point theorem. □

We are now ready to prove proposition 2, by showing that an equilibrium $\boldsymbol{e}_t^{\epsilon,\kappa}$ of a box-economy converges to an equilibrium of the original economy, $\mathfrak{E}(\phi_{t+1}^a)$. To this end, first, we prove that $\boldsymbol{e}_t^{\epsilon,\kappa}$ converges to $\boldsymbol{e}_t^{\kappa}$, as ϵ goes to zero, and then that it remains in the interior of the $\kappa-$box as κ goes to infinity. Observe that, as the equilibrium set is interior to a box, it is also compact.

Proof of proposition 2. Consider an equilibrium $e^{\epsilon,\kappa}$ and the associated aggregate variables, as above, denoted with an upper-bar. Using the auctioneer correspondence, we now show that –as ϵ goes to zero (and for given κ)– $e_t^{\epsilon,\kappa}$ remains bounded and markets clear.

By monotonicity of preferences, at equilibrium, all budget constraints hold with equality. The typical young entrepreneur i and household h, respectively, have the following constraints, which only differ from the original because we let R_{t+1}^{-1} denote the price of a zero-coupon bond,

$$e_t^i + \left(1 - \zeta_t^i\right)\pi_t^i + q_t\zeta_t^i - R_{t+1}^{-1}b_t^i - x_t^i = 0, \quad e_t^h - R_{t+1}^{-1}b_t^h - \left(q_t - \Pi_t^{b,\epsilon,\kappa}\right)\theta_t^h = 0;$$

Aggregating the budget constraint of young agents in t, at a patent-market equilibrium, we find,

$$\begin{aligned} 0 =& \bar{e}_t + \bar{y}_t - \bar{x}_t - \sum_{i\in\mathcal{E}_t}\nu_t^i\zeta_t^i\pi_t^i + q_t\bar{\zeta}_t - R_{t+1}^{-1}\sum_{i\in\mathcal{E}_t}\nu_t^i b_t^i + \\ & - R_{t+1}^{-1}\sum_h b_t^h - q_t\bar{\theta}_t + \Pi_t^{b,\epsilon,\kappa}\bar{\theta}_t \end{aligned}$$

Then, using the definition of the equity-fund return, substitute in the latest for,

$$\sum_{i\in\mathcal{E}_t}\nu_t^i\zeta_t^i\pi_t^i=\bar{\zeta}_t\Pi_t^{b,\epsilon,\kappa}+\epsilon\left[J\Pi_t^{b,\epsilon,\kappa}-\sum_{i\in\mathcal{E}_t}\nu_t^{i,\iota}\pi_t^i\right]$$

This, changing signs, yields,

$$0=\bar{x}_t-\bar{e}_t-\bar{y}_t-\Pi_t^{b,\epsilon,\kappa}\left(\bar{\theta}_t-\bar{\zeta}_t\right)-\mathcal{L}_t^{b,\epsilon}+q_t\left(\bar{\theta}_t-\bar{\zeta}_t\right)+R_{t+1}^{-1}\bar{b}_t \tag{f1}$$

where,

$$\mathcal{L}_t^{b,\epsilon}\equiv-\epsilon\left[J\Pi_t^{b,\epsilon,\kappa}-\bar{\pi}_t\right]$$

Next, observe that $\bar{x}_t-\bar{e}_t-\bar{y}_t-\Pi_t^{b,\epsilon,\kappa}\left(\bar{\theta}_t-\bar{\zeta}_t\right)\le\mathcal{L}_t^{b,\epsilon}$; otherwise, if this were holding with $>$, $q_t\left(\bar{\theta}_t-\bar{\zeta}_t\right)+R_{t+1}^{-1}\bar{b}_t<0$, which cannot be, at equilibrium, without violating the auctioneer optimality (there would be a gain in setting the prices to zero). More precisely, we notice that $(\bar{\theta}_t-\bar{\zeta}_t)>0$ implies an equilibrium price $q_t=1/\epsilon$ and $(\bar{\theta}_t-\bar{\zeta}_t)=0$ any price $q_t\in[0,1/\epsilon]$. Therefore, using also the fact that, at equilibrium, $p_tn_t\ge0$,

$$\begin{aligned}\frac{1}{\epsilon}(\bar{\theta}_t-\bar{\zeta}_t)&=\bar{e}+\bar{y}_t-p_tn_t-x_t+\Pi_t^{b,\epsilon,\kappa}\left(\bar{\theta}_t-\bar{\zeta}_t\right)+\mathcal{L}_t^{b,\epsilon}\\&\le\bar{e}_t+\bar{y}_t+\Pi_t^{b,\epsilon,\kappa}J\widehat{\zeta}+\mathcal{L}_t^{b,\epsilon}\end{aligned}$$

All this implies,

$$0\le\bar{\theta}_t-\bar{\zeta}_t\le\epsilon\left|\bar{e}_t+\bar{y}_t+\Pi_t^{b,\epsilon,\kappa}J\widehat{\zeta}+\mathcal{L}_t^{b,\epsilon}\right|$$

and, similarly,

$$0\le\bar{b}_t\le\epsilon\left|\bar{e}_t+\bar{y}_t+\Pi_t^{b,\epsilon,\kappa}J\widehat{\zeta}+\mathcal{L}_t^{b,\epsilon}\right|$$

Finally, we can use the latest in (f1) to bound the net-expenditure,

$$\begin{aligned}\bar{x}_t+p_tn_t-(\bar{e}_t+\bar{y}_t)&=\Pi_t^{b,\epsilon,\kappa}\left(\bar{\theta}_t-\bar{\zeta}_t\right)+\mathcal{L}_t^{b,\epsilon}-\left[q_t\left(\bar{\theta}_t-\bar{\zeta}_t\right)+R_{t+1}^{-1}\bar{b}_t\right]\\&\le\Pi_t^{b,\epsilon,\kappa}\left(\bar{\theta}_t-\bar{\zeta}_t\right)+\mathcal{L}_t^{b,\epsilon}\\&\le\epsilon\Pi_t^{b,\epsilon,\kappa}\left|\Pi_t^{b,\epsilon,\kappa}J\widehat{\zeta}+\mathcal{L}_t^{b,\epsilon}\right|+\mathcal{L}_t^{b,\epsilon}\end{aligned}$$

Now, consider the budget constraints of the old individuals, at $t+1$,

$$c_{t+1}^i=e_{t+1}^i+(1-\zeta_t^i)\pi_{t+1}^i+b_t^i,\quad c_{t+1}^h=e_{t+1}^h+b_t^h+\Pi_{t+1}^{a,\epsilon,\kappa}\theta_t^h,$$

By taking expectations over Ξ with respect to $(P_t^i)_i$ for all entrepreneurs $i\in\mathcal{E}_t$, conditional to each aggregate state ϕ, and aggregating over the individuals,

$$(\bar{c}_{t+1}-\bar{e}_{t+1}-\bar{y}_{t+1})(\phi)+\left[\sum_{i\in\mathcal{E}_t}\nu_t^i\zeta_t^i\mathbb{E}^i[\pi_{t+1}^i|\phi]\right]-\Pi_{t+1}^{a,\kappa}(\phi)\bar{\theta}_t-\bar{b}_t=0$$

Reiterating the above transformation, we find,

$$(\bar{c}_{t+1}-\bar{e}_{t+1}-\bar{y}_{t+1})(\phi)-p_{t+1}n_{t+1}(\phi)-\Pi_{t+1}^{a,\kappa}(\phi)\left(\bar{\theta}_t-\bar{\zeta}_t\right)-\bar{b}_t-\mathcal{L}_{t+1}^{a,\epsilon}(\phi)=0$$

where, $\mathcal{L}_{t+1}^{a,\epsilon}(\phi) \equiv \epsilon \left[J\Pi_{t+1}^{a,\epsilon,\kappa} - \bar{\pi}_{t+1}\right](\phi)$

$$\begin{aligned}(\bar{c}_{t+1} - \bar{e}_{t+1} - \bar{y}_{t+1})(\phi) &= p_{t+1}n_{t+1}(\phi) + \Pi_{t+1}^{a,\kappa}(\phi)\left(\bar{\theta}_t - \bar{\zeta}_t\right) + \bar{b}_t - \mathcal{L}_{t+1}^{a,\epsilon}(\phi) \\ &\leq p_{t+1}n_{t+1}(\phi) + \Pi_{t+1}^{a,\kappa}(\phi)\epsilon\left[\bar{e} + \bar{y}_t + \Pi_t^{b,\epsilon,\kappa}J\widehat{\zeta} + \mathcal{L}_t^{b,\epsilon}\right] + \bar{b}_t - \mathcal{L}_{t+1}^{a,\epsilon}(\phi) \\ &\leq p_{t+1}n_{t+1}(\phi) + \epsilon\left(\Pi_{t+1}^{a,\kappa}(\phi) + 1\right)\left[\bar{e} + \bar{y}_t + \Pi_t^{b,\epsilon,\kappa}J\widehat{\zeta} + \mathcal{L}_t^{b,\epsilon}\right] + |\mathcal{L}_{t+1}^{a,\epsilon}(\phi)|\end{aligned}$$

As $\epsilon \to 0$, the equilibrium sequence $\boldsymbol{e}_t^{\epsilon,\kappa}$ remains finite if prices stay finite. First, consider the monopoly price p_{t+1}. As p_{t+1} increases without bound, the current license demand goes to zero, hence $p_{t+1}n_{t+1} \leq \epsilon^{-1}n_{t+1}$ would converge to zero. Since the latest is not optimal for the monopolist, we achieve a contradiction.

For completeness, suppose q_t goes to infinity as ϵ goes to 0. Then, currently young entrepreneurs will want to issue equities and reduce debt and households would like to adjust theirs portfolios in the opposite direction. If the box defined by κ is large enough to contain the loan supply, a contradiction to households' optimality is reached. An analogous argument applies to R_{t+1}^{-1}. Hence, we conclude that the whole equilibrium sequence converges as ϵ goes to zero.

Finally, to complete the the proof, we now show that, by choosing κ large enough, the equilibrium variables remain in the interior of the cube. We know that for all κ an equilibrium of the box-economy exists. Considering an increasing sequence (κ_n), by compactness of the set of attainable allocations, the corresponding sequences of attainable consumption, capital, and equity-fund payoffs converge. Furthermore, by old-age individuals' budget constraints, also financial positions in loans and equities converge. Hence, for sufficiently large n, the cube is non-binding and all the allocations are interior.

□

Proposition 8. *For all $T \geq 1$ and all $T-$finite economy $\mathfrak{E}(T)$ an equilibrium exists. Moreover, the equilibrium set is compact.*

Proof. An equilibrium exists for every $T-$ finite economy $\mathfrak{E}(T)$, by proposition 2 and the application of the equilibrium algorithm in definition 5. This allows to determine the equilibrium set as a subset of the product $\times_{t=0}^{T-1}\Sigma_t$. Moreover, the equilibrium set of finite economies is compact as $\times_{t=0}^{T-1}\Sigma_t$ is compact. □

4.1.2. *Equilibrium existence in a OLG truncated economy.* Recall that a $T-$ truncated economy is a $T-$finite economy $\mathfrak{E}(T)$ in which assets and patent licenses can be traded at all dates $t < T$ and spot markets are open at all dates $t \leq T$. We think at this economy as one in which debt limits are zero for all individuals and licenses $n_T = 0$, at all truncation dates T. Equilibrium existence for such an economy trivially follows from proposition 8 and is therefore stated without proof.

Corollary 9. *For all $T \geq 1$ and all $T-$truncated economy, an equilibrium exists. Moreover, the equilibrium set is compact.*

4.1.3. *Equilibrium existence - proof of theorem 1.* Consider equilibria of T-truncated economies. Passing to equilibrium subsequences, as T goes to infinity, such subsequences converge, in the product topology, to an equilibrium; this is so since, for every T, the equilibrium set of a $T-$truncated economy is (nonempty and) compact by corollary 9. This completes the proof of theorem 1.

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