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Inflation, Unemployment and Economic Growth in a Schumpeterian Economy

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Abstract

This study explores the long-run relationship between inflation and unemployment in a monetary Schumpeterian growth model with matching frictions in the labor market and cash-in-advance (CIA) constraints on consumption and R&D investment. Under the CIA constraint on R&D, higher inflation that raises the opportunity cost of cash holdings leads to a decrease in innovation and economic growth, which in turn decreases labor-market tightness and increases unemployment. Under the CIA constraint on consumption, higher inflation instead decreases unemployment in addition to stifling innovation and economic growth. Therefore, the two CIA constraints have drastically different implications on the long-run relationship between inflation and unemployment. This theoretical result is consistent with our empirical finding and provides a plausible explanation (via the relative magnitude of the two CIA constraints) for the mixed empirical results on the relationship between inflation and unemployment in the literature. Finally, we also calibrate our model to aggregate data in the US and Eurozone to explore quantitative implications on the relationship between inflation and unemployment.

JEL classification: E24, E41, O30, O40
Keywords: inflation, unemployment, innovation, economic growth

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1 Introduction

The relationship between inflation, unemployment and economic growth has long been a fundamental question in economics. This study provides a growth-theoretic analysis on this important relationship in a Schumpeterian model with equilibrium unemployment. Creative destruction refers to the process through which new technologies destroy existing firms. On the one hand, the destructive part of this process leads to job losses. On the other hand, new technologies also create new firms with new employment opportunities. In a frictionless labor market, these job destructions and job creations could offset each other leaving the labor market with full employment. However, given the presence of matching frictions between firms and workers, this continuous turnover in the labor market as a result of creative destruction leads to what Joseph Schumpeter (1939) referred to as "technological unemployment". At the first glance, it may seem that technological unemployment is a very specific kind of unemployment; however, as Schumpeter [1911] (2003, p.89) wrote, "[i]t doubtlessly explains a good deal of the phenomenon of unemployment, in my opinion its better half."

To explore the effects of inflation on unemployment and economic growth, we introduce money demand via cash-in-advance (CIA) constraints on consumption and R&D investment into a scale-invariant Schumpeterian growth model with equilibrium unemployment. Early empirical studies such as Hall (1992) and Opler *et al.* (1999) find a positive and significant relationship between R&D and cash flows in US firms. Bates *et al.* (2009) document that the average cash-to-assets ratio in US firms increased substantially from 1980 to 2006 and argue that this is partly driven by their rising R&D expenditures. Brown and Petersen (2011) provide evidence that firms smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Berentsen *et al.* (2012) argue that information frictions and limited collateral value of intangible R&D capital prevent firms from financing R&D investment through debt or equity forcing them to fund R&D projects with cash reserves. A recent study by Falato and Sim (2014) provides causal evidence that R&D is indeed an important determinant of firms' cash holdings. They use firm-level data in the US to show that firms' cash holdings increase (decrease) significantly in response to a rise (cut) in R&D tax credits, which vary across states and time. Furthermore, these effects are stronger for firms that have less access to debt/equity financing. These results suggest that due to the presence of financing frictions, firms need to hold cash to finance their R&D investment. We capture these cash requirements on R&D using a CIA constraint as in Chu and Cozzi (2014).

Under the CIA constraint on R&D, an increase in inflation that determines the opportunity cost of cash holdings raises the cost of R&D investment. Consequently, higher inflation decreases R&D. Given that we remove scale effects[1] by considering a semi-endogenous-growth version of the Schumpeterian model in which the long-run rate of creative destruction is determined by exogenous parameters, a decrease in R&D leads to a decrease in the growth rate of technology only in the short run but decreases the level of technology in the long run. Although the rate of creative destruction decreases temporarily, the decrease in innovation in the long run increases the number of unemployed workers relative to labor-market vacancies causing a negative effect on labor market tightness and a positive effect on unemployment. In other words, via the CIA constraint on R&D, higher inflation increases unemployment in the long run by decreasing the demand for workers.

We also consider a conventional CIA constraint on consumption. Interestingly, under the CIA constraint on consumption, higher inflation instead decreases unemployment in addition to

[1] See Jones (1999) for a discussion of scale effects in the R&D-based growth model.

stifling innovation and economic growth. Intuitively, higher inflation leads to an increase in the opportunity cost of cash holdings, which in turn increases the cost of consumption relative to leisure. As a result, the household consumes more leisure and reduces labor supply. The decrease in labor supply reduces R&D labor and also the number of workers searching for employment. The resulting increase in labor-market tightness decreases unemployment. In other words, via the CIA constraint on consumption, higher inflation decreases unemployment by decreasing the supply of labor.

Therefore, the two CIA constraints have drastically different implications on the long-run relationship between inflation and unemployment. This theoretical result is consistent with our empirical finding. We use data in the US and consider two variables that capture financial frictions faced by firms and consumers, respectively. We find that inflation has a positive (negative) effect on unemployment via the financial friction faced by firms (consumers), which is a proxy for the CIA constraint on R&D (consumption). Our analysis provides a plausible explanation (via the relative magnitude of the two CIA constraints) for the mixed empirical results on the relationship between inflation and unemployment in the literature.[2] Finally, we calibrate our model to aggregate data in the US to explore quantitative implications and find that the model delivers a positive (negative) relationship between inflation and unemployment when we use data on M0 (M1) as the measure of money. Interestingly, when we calibrate the model to data in the Eurozone, we find that the model delivers a negative relationship between inflation and unemployment under both measures of money. We discuss intuition behind these results in the main text.

This study relates to the literature on Schumpeterian growth; see Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) for seminal studies and Aghion, Akcigit and Howitt (2014) for a recent survey. However, these seminal studies and many subsequent studies in this literature feature full employment rendering them unsuitable for the purpose of analyzing unemployment. Early contributions in the Schumpeterian theory of unemployment are Aghion and Howitt (1994, 1998), Cerisier and Postel-Vinay (1998), Mortensen and Pissarides (1998), Pissarides (2000), Şener (2000, 2001) and Postel-Vinay (2002).[3] A recent study by Aghion, Akcigit, Deaton and Roulet (2016) explores the effects of job creation and job destruction on the welfare of workers in a Schumpeterian model with unemployment and uses data on subjective wellbeing to verify their theoretical results. The present study complements these interesting studies by introducing money demand into the Schumpeterian model with unemployment and analyzing the effects of inflation on unemployment and economic growth. To our knowledge, this combination of Schumpeterian growth, money demand and equilibrium unemployment is novel to the literature.

This study also relates to the literature on inflation and economic growth. In this literature, Stockman (1981) and Abel (1985) analyze the effects of inflation via a CIA constraint on capital investment in a monetary version of the Neoclassical growth model. Subsequent studies in this literature explore the effects of inflation in variants of the capital-based growth model. This study instead relates more closely to the literature on inflation and *innovation-driven* growth. In this literature, an early study by Marquis and Reffett (1994) analyzes the effects of inflation via a CIA constraint on consumption in a variety-expanding growth model based on Romer (1990).[4]

[2] For example, Ireland (1999), Beyer and Farmer (2007), Russell and Banerjee (2008) and Berentsen *et al.* (2011) document a positive relationship between inflation and unemployment in the US, whereas Dolado *et al.* (2000) and Karanassou *et al.* (2005, 2008) find a negative relationship between inflation and unemployment in the US and some European countries.

[3] See also Parello (2010) who considers a Schumpeterian model with unemployment due to efficiency wage.

[4] See also Arawatari *et al.* (2018) who consider monetary policy in the Romer variety-expanding model with

In contrast, we explore the effects of inflation in a Schumpeterian quality-ladder model. Chu and Lai (2013), Chu and Cozzi (2014), Chu, Cozzi, Lai and Liao (2015), He and Zou (2016), Chu, Cozzi, Furukawa and Liao (2017, 2018), Hori (2017), Huang *et al.* (2017) and He (2018) also analyze the relationship between inflation and economic growth in the Schumpeterian model. However, all these studies exhibit full employment due to the absence of matching frictions in the labor market. The present study provides a novel contribution to the literature by introducing equilibrium unemployment driven by matching frictions to the monetary Schumpeterian growth model. Recent studies by Wang and Xie (2013) and Chen *et al.* (2018) also analyze the effects of inflation on economic growth and unemployment driven by matching frictions in the labor market. Their models generate money demand via CIA constraints on consumption and wage payment to production workers. In contrast, we model money demand via a CIA constraint on R&D. More importantly, they consider capital accumulation as the engine of economic growth whereas our analysis complements these interesting studies by exploring a different growth engine that is R&D and innovation.

The rest of this study is organized as follows. Section 2 documents some stylized facts. Section 3 describes the Schumpeterian model. Section 4 provides a qualitative analysis on the effects of inflation on unemployment and economic growth. Section 5 presents our quantitative results. The final section concludes.

2 Stylized facts

In this section, we document some stylized facts on the relationship between inflation and unemployment in the US. We consider the following regression specification:

$$\begin{aligned} u_{it} &= \psi_1\pi_t + \psi_2\pi_t \times BC_t + \psi_3\pi_t \times BF_t + \psi_4\pi_t \times BF_t \times RD_i + \psi_5\pi_t \times RD_i \\ &\quad + \psi_6 BF_t \times RD_i + \psi_7 BC_t + \psi_8 BF_t + \Psi X_{i,t} + \varphi_i + \varepsilon_{it}, \end{aligned}$$

where u_{it} is the unemployment rate in industry i at time t,[5] π_t is the inflation rate computed from the consumer price index in the US at time t, BF_t is bank lending policy to firms at time t, BC_t is bank lending policy to individual consumers at time t, and RD_i is the average level of $R\&D$ intensity in industry i.[6] $X_{i,t}$ is a vector of other explanatory variables, which include GDP per capita in the US, the log of trade volume (i.e., import plus export) in the US and industry-specific time trends. φ_i is the industry fixed effect.

Bernanke and Blinder (1988), Bernanke and Gertler (1995), Stein (1998), Van den Heuvel (2002) and Bolton and Freixas (2006) show that bank lending affects the transmission of monetary policy to the real economy. The tightness of bank lending determines the financing abilities of firms (BF_t) and individual consumers (BC_t). Therefore, BF_t is a reasonable proxy for the CIA constraint faced by firms, whereas BC_t is a reasonable proxy for the CIA constraint faced by consumers. US Federal Reserve Board conducts a quarterly survey of eighty domestic banks

heterogeneity in the productivity of R&D entrepreneurs.

[5] Data on the unemployment rate is collected from the Current Population Survey of the Bureau of Labor Statistics (BLS). The BLS releases yearly reports on the unemployment rate by industry. Because data is not available for 2000−2001, we use data from 1998−1999 and 2002−2017.

[6] Data on industry-level R&D intensity is from Kugler and Verhoogen (2012); see also Sutton (1991, 1998).

and twenty-four foreign banks with US branches/agencies on their lending policies to firms and consumers from 1998 to 2017.[7] The survey classifies the tightness reported by banks on lending to firms and individuals into five levels: tightened considerably, tightened somewhat, unchanged, eased somewhat and eased considerably.[8] Here BF_t (BC_t) is computed as the percentage of banks that report lending policies on firms (consumers) being tightened "considerably" or "somewhat".

Table 1: Correlation between unemployment and inflation

	"considerably" or "somewhat"		only "considerably"	
	All banks	Large banks	All banks	Large banks
$BF_t > BC_t$	0.3671	0.3675	0.3548	0.3548
$BC_t > BF_t$	-0.4306	-0.4417	-0.4906	-0.4882

Table 1 reports the correlations between unemployment and inflation under different states of bank lending policies reported by banks. Table 1 shows that unemployment and inflation are positively correlated if $BF_t > BC_t$ (i.e., firms face a stronger degree of financial frictions than consumers). Table 1 also shows that unemployment and inflation are negatively correlated if $BC_t > BF_t$ (i.e., consumers face a stronger degree of financial frictions than firms).

Table 2: Effects of inflation on unemployment

	All banks			Large banks		
	(1)	(2)	(3)	(4)	(5)	(6)
BF × Inflation	0.389***	0.320***	0.192	0.367***	0.296***	0.191
	(0.096)	(0.096)	(0.140)	(0.083)	(0.086)	(0.119)
BC × Inflation	-0.504***	-0.504***	-0.398***	-0.368***	-0.368***	-0.304***
	(0.104)	(0.101)	(0.144)	(0.070)	(0.068)	(0.091)
RD × BF × Inflation		2.231***	2.547***		2.323**	2.723***
		(0.805)	(0.887)		(0.916)	(1.037)
RD × BF		-7.526***	-8.641***		-7.634***	-9.002***
		(2.353)	(2.670)		(2.700)	(3.145)
RD × Inflation		-12.459	-24.992		-11.163	-24.964
		(24.413)	(28.143)		(25.320)	(30.000)
Inflation, BF and BC	Yes	Yes	Yes	Yes	Yes	Yes
Other controls	No	No	Yes	No	No	Yes
Industry fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Observations	252	252	252	252	252	252
R-squared	0.341	0.385	0.399	0.346	0.384	0.393

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The first three columns correspond to lending policy reported by all banks, whereas the last three columns correspond to lending policy reported by large banks. In all columns, we control inflation, bank policy for firms (BF) and bank policy for consumers (BC). In columns (3) and (6), we further control GDP per capita, log trade volume and industry-specifc time trends.

Table 2 reports our main empirical results. The first three columns correspond to lending policies reported by all banks, whereas the last three columns correspond to policies reported by

[7] We take an average of the quarterly data to construct yearly data for the regression.

[8] Bank lending data is from the CEIC database.

only large banks. Column (1) shows that the coefficient on the interaction term between inflation and BF_t is positive and significant whereas it is negative and significant for the interaction between inflation and BC_t. This pattern is consistent with Table 1. This means that higher inflation increases the unemployment rate if bank lending policy is tightened for firms, but decreases the unemployment rate if lending policy is tightened for consumers instead. In column (2), we add a triple interaction term of inflation, R&D intensity and BF_t in the regression. Given that a larger R&D intensity may indicate that firms need to borrow more for R&D activities and face a tighter constraint as a result, we expect a positive coefficient on this triple interaction term. As expected, the coefficient on the triple interaction term in column (2) is positive and significant, which means that higher inflation increases the unemployment rate more in the industries that are more exposed to R&D financing constraints. Furthermore, it should be noted that the coefficient on the interaction term between inflation and BC_t is still negative and significant. In column (3), we add other control variables, including GDP per capita, the log of trade volume and industry-specific time trends. In this case, the coefficient on the triple interaction term continues to be positive and significant whereas the coefficient on the interaction between inflation and BC_t remains negative and significant. When we use data on large banks only in columns (4)-(6), the above results still hold. We also consider focusing on the case of bank lending policies being tightened "considerably" as a robustness check for which the results are similar and reported in Appendix A.

In this section, we have documented the following stylized facts. First, under the financial friction faced by firms (i.e., a proxy for the CIA constraint on R&D), higher inflation increases unemployment. Second, under the financial friction faced by consumers (i.e., a proxy for the CIA constraint on consumption), higher inflation instead decreases unemployment. Finally, when higher inflation increases unemployment, this effect is larger in the industries that have higher R&D intensity.

3 A monetary Schumpeterian model with unemployment

In the Schumpeterian model developed by Aghion and Howitt (1992), economic growth is driven by quality improvement. R&D entrepreneurs invent higher-quality products in order to dominate the market and earn monopolistic profits. After R&D entrepreneurs create new inventions, they open up vacancies to search for workers in the labor market,[9] in which the number of job separations is determined by creative destruction and the number of job matches is determined by an aggregate matching function and labor market tightness. Due to matching frictions between workers and firms with new technologies, the economy features equilibrium unemployment in the long run. Unlike the important precedent of Mortensen (2005), we (a) remove the scale effect via increasing R&D difficulty as in Segerstrom (1998), (b) introduce money demand via CIA constraints on R&D investment and consumption as in Chu and Cozzi (2014), and (c) consider elastic labor supply.

[9] Michelacci (2003) develops a search-theoretic R&D-based growth model in which after R&D scientists create new inventions, they have to search for entrepreneurs to implement these inventions.

3.1 Household

The representative household has L members. The household's utility function is given by

$$U = \int_0^\infty e^{-\rho t} \left[\ln c_t + \gamma \ln(L - l_t) \right] dt, \tag{1}$$

where c_t denotes the household's total consumption of final good (numeraire) at time t. Each member of the household supplies one unit of labor, and l_t is the household's total supply of labor at time t. The parameter $\rho > 0$ determines subjective discounting, and $\gamma \geq 0$ determines leisure preference.

The asset-accumulation equation expressed in real terms is given by

$$\dot{a}_t + \dot{m}_t = r_t a_t - \pi_t m_t + i_t d_t + I_t - \tau_t - c_t. \tag{2}$$

a_t is the real value of financial assets (in the form of equity shares in monopolistic intermediate goods firms) owned by the household. r_t is the real interest rate. π_t is the inflation rate. m_t is the real money balance accumulated by the household. d_t is the amount of money lent to R&D entrepreneurs subject to the following constraint: $d_t + \xi c_t \leq m_t$, where $\xi \in [0, 1]$ parameterizes the strength of the CIA constraint on consumption. The interest rate on money lending d_t to R&D firms is the nominal interest rate,[10] which is equal to $i_t = r_t + \pi_t$ from the Fisher identity. τ_t is a lump-sum tax levied on the household. I_t is the total amount of labor income given by $I_t \equiv w_t x_t + \omega_t R_t + b_t u_t$,[11] where w_t is the wage rate of production workers x_t, ω_t is the wage rate of R&D workers R_t, and b_t is unemployment benefits provided to unemployed workers u_t who are searching for jobs in the labor market.[12] To ensure balanced growth, we assume that $b_t = \bar{b} y_t / L$ is proportional to output per capita, where $\bar{b} \in (0, 1)$ is an unemployment-benefit parameter. Given the labor force l_t, the resource constraint on labor at time t is

$$x_t + R_t + u_t = l_t. \tag{3}$$

The household chooses consumption c_t and labor supply l_t and accumulates assets a_t and money m_t to maximize (1) subject to (2), (3) and the CIA constraint $d_t + \xi c_t \leq m_t$. The resulting optimality condition for labor supply is

$$l_t = L - \frac{\gamma(1 + \xi i_t) c_t}{\omega_t}, \tag{4}$$

where the opportunity cost of leisure is the R&D wage rate ω_t because individuals can freely choose between employment in the R&D sector and job search.[13] The intertemporal optimality condition is given by

$$-\frac{\dot{\zeta}_t}{\zeta_t} = r_t - \rho, \tag{5}$$

[10] It can be easily shown as a no-arbitrage condition that the interest rate on d_t must be equal to i_t.

[11] The household pools the different sources of labor income among members, who share all the risks. See Michelacci and Schivardi (2013) for an interesting analysis on how idiosyncratic risks affect entrepreneurial activity and growth.

[12] Here we assume a uniform level of unemployment benefits for all unemployed workers. Michelacci and Ruffo (2015) explore the welfare implications of age-dependent unemployment benefits and find that an optimal unemployment replacement rate should decline with age.

[13] Given that R&D is essentially the search for a higher-quality product, there is no need to have it preceded by another search activity.

where ζ_t is the Hamiltonian co-state variable on (2) and determined by $\zeta_t = [(1+\xi i_t)c_t]^{-1}$. In the case of a constant nominal interest rate i, (5) becomes the familiar Euler equation $\dot{c}_t/c_t = r_t - \rho$.

3.2 Final good

Final good y_t are produced by perfectly competitive firms that aggregate a unit continuum of intermediate goods using the following Cobb-Douglas aggregator:

$$y_t = \exp\left\{\int_0^1 \ln\left[A_t(j)x_t(j)\right] dj\right\}, \tag{6}$$

where $A_t(j) \equiv q^{n_t(j)}$ is the productivity or quality level of intermediate good $x_t(j)$.[14] The parameter $q > 1$ is the exogenous step size of each quality improvement, and $n_t(j)$ is the number of innovations that have been invented *and implemented* in industry j as of time t. From profit maximization, the conditional demand function for $x_t(j)$ is

$$x_t(j) = y_t/p_t(j), \tag{7}$$

where $p_t(j)$ is the price of $x_t(j)$ for $j \in [0,1]$. All prices are denominated in units of final good, chosen as the numeraire.

3.3 Intermediate goods

The unit continuum of differentiated intermediate goods are produced in a unit continuum of industries. Each industry is temporarily dominated by a quality leader until the arrival and implementation of the next higher-quality product. The owner of the new innovation becomes the next quality leader.[15] The current quality leader in industry j uses one unit of labor to produce one unit of intermediate good $x_t(j)$. We assume - as in Mortensen (2005) - that the employer has no outside option and the workers' outside option is unemployment benefit b_t. In this case, the generalized Nash bargaining game is[16]

$$\{x_t(j), w_t(j)\} = \arg\max\{[w_t(j) - b_t]x_t(j)\}^{\beta}\{[p_t(j) - w_t(j)]x_t(j)\}^{1-\beta}, \tag{8}$$

where the parameter $\beta \in (0,1)$ measures the bargaining power of workers. The bargaining outcome on wage is[17]

$$w_t(j) = \beta p_t(j) + (1-\beta)b_t, \tag{9}$$

which is an average between the marginal revenue product $p_t(j)$ of each worker and the value of unemployment benefit b_t weighted by the bargaining power of workers. The employer and workers

[14] Given we will assume that one unit of labor produces one unit of intermediate goods, we use x_t to denote both the quantity of intermediate goods and the quantity of production workers, for notational convenience.

[15] This is known as the Arrow replacement effect; see Cozzi (2007a) for a discussion of the Arrow effect.

[16] Using a more general bargaining condition with the value functions of employment and unemployment would complicate the model without providing new insight; see for example footnote 3 in Mortensen (2005).

[17] This bargaining outcome can also be obtained from $w_t(j) = \arg\max\{[w_t(j) - b_t]^{\beta}[p_t(j) - w_t(j)]^{1-\beta}\}$ (i.e., individual wage bargaining).

commit to this wage schedule over the lifetime of the firm. Substituting (9) into (8) shows that the $x_t(j)$ that maximizes (8) is the same as the $x_t(j)$ that maximizes the following profit function:

$$\Pi_t(j) = [p_t(j) - w_t(j)]x_t(j) = (1-\beta)[p_t(j) - b_t]x_t(j) = (1-\beta)[y_t - b_t x_t(j)], \tag{10}$$

where the second equality uses (9) and the third equality uses (7).

In Grossman and Helpman (1991) and Aghion and Howitt (1992), the markup is assumed to be given by the quality step size q, due to limit pricing between the current and previous quality leaders. Here we follow Howitt (1999) and Dinopoulos and Segerstrom (2010) to consider a more realistic scenario in which new quality leaders do not engage in limit pricing with previous quality leaders because after the implementation of the newest innovations, previous quality leaders exit the market and need to search for workers before reentering. Given the Cobb-Douglas aggregator in (6), the unconstrained monopolistic price would be infinity (i.e., $x_t(j) \to 0$). We follow Evans *et al.* (2003) to consider price regulation under which the regulated markup ratio cannot be greater than $z > 1$.[18] The equilibrium price is

$$p_t(j) = z w_t(j) = z\frac{1-\beta}{1-\beta z}b_t, \tag{11}$$

where the second equality uses (9). We impose an additional parameter restriction given by $\beta z < 1$. Substituting (11) into (7) yields

$$x_t(j) = \frac{1-\beta z}{(1-\beta)z}\frac{y_t}{b_t} = \frac{1-\beta z}{(1-\beta)z\bar{b}}L \equiv x, \tag{12}$$

where the last equality uses $b_t = \bar{b}y_t/L$. Finally, the amount of monopolistic profit is

$$\Pi_t(j) = \Pi_t = (p_t - w_t)x = \frac{z-1}{z}y_t. \tag{13}$$

Given that the amount of monopolistic profit is the same across industries, we will follow the standard treatment in the literature to focus on the symmetric equilibrium, in which the arrival rate of innovations is equal across industries.[19]

3.4 R&D

R&D is performed by a continuum of competitive entrepreneurs. If an R&D entrepreneur sinks $\widetilde{R}_t$ units of labor to engage in innovation in an industry, then she is successful in inventing the next higher-quality product in the industry with an instantaneous probability given by

$$\widetilde{\delta}_t = \frac{h_t \widetilde{R}_t}{A_t}, \tag{14}$$

[18] This formulation enables us to separate the markup and the quality step size, allowing for a more realistic calibration exercise.

[19] See Cozzi (2007b) for a discussion of multiple equilibria in the Schumpeterian model. Cozzi *et al.* (2007) provide a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian model.

where h_t is an exogenous process of innovation productivity that grows at a constant rate g.[20] We assume that innovation productivity h_t/A_t decreases in aggregate quality $A_t \equiv \exp\left(\int_0^1 \ln A_t(j)dj\right)$ in order to capture increasing difficulty of R&D in the economy,[21] and this specification removes the scale effect in the innovation process of the quality-ladder model as in Segerstrom (1998).[22] The expected benefit from investing in R&D is $V_t\widetilde{\delta}_t dt$, where V_t is the value of the expected discounted profits generated by a new innovation and $\widetilde{\delta}_t dt$ is the entrepreneur's probability of having a successful innovation during the infinitesimal time interval dt. To facilitate the payment of R&D wages, the entrepreneur borrows money from the household, and the cost of borrowing is determined by the nominal interest rate i_t. To parameterize the strength of this CIA constraint on R&D, we assume that a fraction $\sigma \in [0,1]$ of R&D expenditure requires the borrowing of money from households. Therefore, the total cost of R&D is $(1+\sigma i_t)\,\omega_t \widetilde{R}_t dt$. Free entry implies

$$V_t\widetilde{\delta}_t dt = (1+\sigma i_t)\,\omega_t \widetilde{R}_t dt \Leftrightarrow V_t = (1+\sigma i_t)\,\omega_t A_t/h_t, \tag{15}$$

where the second equality uses (14).

3.5 Matching and unemployment

When an R&D entrepreneur has a new innovation, she is not able to immediately launch the new product to the market due to matching frictions in the recruitment of manufacturing workers.[23] Instead, she has to open up x vacancies to recruit x workers for producing and launching her products to the market. We follow the standard treatment in the search-and-matching literature to consider an aggregate matching function $F(v_t, u_t)$, where v_t is the number of vacancies in the labor market and u_t is the number of unemployed workers. $F(v_t, u_t)$ has the usual properties of being increasing, concave and homogeneous of degree one in v_t and u_t. In the economy, the number of successful matches at time t is given by $F(v_t, u_t)$; in other words, the number of workers who find jobs is $F(v_t, u_t)$. Therefore, the job-finding rate is

$$\lambda_t = F(v_t, u_t)/u_t = F(v_t/u_t, 1) \equiv M(\theta_t), \tag{16}$$

where $\theta_t \equiv v_t/u_t$ denotes labor market tightness, and $\lambda_t = M(\theta_t)$ is increasing in θ_t. Similarly, the number of vacancies filled is also $F(v_t, u_t)$, so the vacancy-filling rate is

$$\eta_t = F(v_t, u_t)/v_t = M(\theta_t)/\theta_t, \tag{17}$$

where $\eta_t = M(\theta_t)/\theta_t$ is decreasing in θ_t. Following the usual treatment in the literature,[24] we assume that when matching occurs to a firm at time t, the firm matches with x workers simultaneously. In other words, the number of successful matches at time t is first determined by the

[20] Here we introduce an exogenous growth component into our model for two reasons. Theoretically, without population growth, the semi-endogenous growth model requires an exogenous growth component to sustain long-run growth. Empirically, previous studies find a residual exogenous growth component when using data to calibrate or estimate R&D-based growth models; see for example Comin (2004) and Cozzi *et al.* (2017).

[21] See Venturini (2012) for empirical evidence based on US manufacturing data that supports the semi-endogenous growth model with increasing difficulty of R&D.

[22] Segerstrom (1998) considers an industry-specific index of R&D difficulty. Here we consider an aggregate index of R&D difficulty to simplify notation without altering the aggregate results of our analysis.

[23] Dinopoulos *et al.* (2013) consider an interesting setting, aimed at studying the importance of rent-seeking activities on unemployment, in which new firms are able to immediately recruit a fraction $\phi \in (0,1)$ of the desired number of workers x.

[24] See for example Mortensen (2005) and Dinopoulos *et al.* (2013).

matching function $F(v_t, u_t)$, and then, these matches are randomly assigned to $F(v_t, u_t)/x$ firms. Therefore, the probability for a firm with opened vacancies to match with x workers at time t is also η_t.

3.6 Asset values

Each unemployed worker faces the probability λ_t of being employed at any point in time. Once a worker is hired by a firm, he/she begins employment and faces the probability $\widetilde{\delta}_t$ of the next innovation being invented in his/her industry. After the innovation is invented, the worker faces the probability η_t of the next innovation being implemented and his/her firm being forced out of the market due to creative destruction. Let U_t denote the value of being unemployed. The familiar asset-pricing equation of U_t is

$$r_t = \frac{b_t + \dot{U}_t + \lambda_t(W_t - U_t)}{U_t}, \tag{18}$$

where b_t is unemployment benefit, λ_t is the rate at which an unemployed worker becomes employed and W_t denotes the value of being employed in an industry in which the subsequent innovation has not been invented. The asset-pricing equation of W_t is

$$r_t = \frac{w_t + \dot{W}_t + \widetilde{\delta}_t(S_t - W_t)}{W_t}, \tag{19a}$$

where w_t is the wage of a production worker, $\widetilde{\delta}_t$ is the rate at which the subsequent innovation is invented and S_t denotes the value of being employed in an industry in which the subsequent innovation has been invented but not yet been launched to the market.[25] The asset-pricing equation of S_t is

$$r_t = \frac{w_t + \dot{S}_t + \eta_t(U_t - S_t)}{S_t}, \tag{19b}$$

where η_t is the rate at which the subsequent innovation is launched to the market and the worker becomes unemployed. Given that a worker must be indifferent between being employed by an R&D entrepreneur and engaging in job search, the wage of R&D workers is equal to

$$\omega_t = r_t U_t - \dot{U}_t. \tag{20}$$

The life cycle of an innovation can be described as follows. When an innovation is invented, its owner creates vacancies in the labor market to recruit workers, and the probability of successfully recruiting workers and beginning production at any point in time is η_t. Once an innovation is launched to the market, it faces the probability $\widetilde{\delta}_t$ of the next innovation being invented. The subsequent innovation cannot be invented until the current innovation has been launched to the market and directly observed.[26] After the next innovation is invented, the probability of it being

[25] Unlike Mortensen (2005) who assumes that the current quality leader must stop its operation as soon as the next innovation is *invented*, we allow the current quality leader to continue its operation until the next innovation is *implemented*. This generalization is rational for the current quality leader, who continues to earn profits, and also for the workers because $S_t > U_t$.

[26] This assumption, shared by Mortensen (2005), captures the realistic feature of the intertemporal spillovers, of equally benefiting from patent description and actual use of the good. This aspect is often remarked in the microeconomic literature on innovation.

launched to the market is η_t. Once the next innovation is launched to the market, the value of the current innovation becomes zero. Let V_t be the value of a new innovation for which its vacancies have not been filled. Its asset-pricing equation is given by

$$r_t = \frac{\dot{V}_t + \eta_t (Z_t - V_t)}{V_t}, \tag{21a}$$

where η_t is the rate at which the product is launched to the market. The asset-pricing equation of Z_t, which is the value of the innovation when its vacancies have been filled, is given by

$$r_t = \frac{\Pi_t + \dot{Z}_t + \widetilde{\delta}_t (X_t - Z_t)}{Z_t}, \tag{21b}$$

where Π_t is the monopolistic profit earned by a launched product and $\widetilde{\delta}_t$ is the rate at which the subsequent innovation is invented. The asset-pricing equation of X_t, which is the value of the current innovation when the subsequent innovation has been invented but not yet been launched to the market, is given by

$$r_t = \frac{\Pi_t + \dot{X}_t - \eta_t X_t}{X_t}, \tag{21c}$$

where η_t is the rate at which the subsequent innovation is launched to the market and the current monopolist loses the market.

3.7 Government

The monetary policy instrument that we consider is the inflation rate π_t, which is exogenously set by the monetary authority. Given π_t, the nominal interest rate is endogenously determined according to the Fisher identity such that $i_t = \pi_t + r_t$, where r_t is the real interest rate. The growth rate of the nominal money supply is $\mu_t = \pi_t + \dot{m}_t/m_t$.[27] Finally, the government balances the fiscal budget subject to the following balanced-budget condition: $\tau_t + \mu_t m_t = b_t u_t$.

3.8 Steady-state equilibrium

We will define the *aggregate* innovation-arrival rate as $\delta_t \equiv (1 - f_t)\widetilde{\delta}_t$, where f_t is the measure of industries with unlaunched innovations. The outflow from the pool of firms searching for workers is given by $\eta_t f_t$, and the inflow into this pool is given by $(1 - f_t)\widetilde{\delta}_t = \delta_t$. Therefore, in the steady state, we must have $\eta_t f_t = \delta_t$. The aggregate production function of final good is given by $y_t = A_t x$, where (the log of) aggregate technology A_t is defined as

$$\ln A_t \equiv \int_0^1 \ln A_t(j) dj = \int_0^1 n_t(j) dj \ln q = \int_0^t \eta_v f_v dv \ln q, \tag{22}$$

[27] It is useful to note that in this model, it is the *growth rate* of the money supply that affects the real economy in the long run, and a one-time change in the *level* of money supply has no long-run effect on the real economy. This is the well-known distinction between the neutrality and superneutrality of money. Empirical evidence generally favors neutrality and rejects superneutrality, consistent with our model; see Fisher and Seater (1993) for a discussion on the neutrality and superneutrality of money.

where we have normalized $A_0 = 1$ (i.e., $\ln A_0 = 0$). Differentiating (22) with respect to t yields $\dot{A}_t / A_t = \eta_t f_t \ln q$, where $\eta_t f_t$ is the measure of industries with newly launched innovations at time t. The steady-state growth rate of A_t is

$$\frac{\dot{A}_t}{A_t} = \eta f \ln q = \delta \ln q = g, \tag{23}$$

where the third equality holds because R and $\delta = h_t R / A_t$ are constant on the balanced growth path implying that h_t and A_t both grow at the exogenous rate g in the long run.[28] From the last equality of (23), the steady-state rate of creative destruction is determined by exogenous parameters such that $\delta = g / \ln q$.

On the balanced growth path,[29] (20) becomes

$$\omega_t = \rho U_t. \tag{24}$$

Solving (18) and (19) yields the balanced-growth value of ρU_t given by

$$\rho U_t = \rho \frac{(\rho + \eta)\,(\rho + \widetilde{\delta}) b_t + (\rho + \eta + \widetilde{\delta}) \lambda w_t}{(\rho + \eta)\,(\rho + \widetilde{\delta})\,(\rho + \lambda) - \widetilde{\delta} \lambda \eta}, \tag{25}$$

where $\widetilde{\delta} = \delta / (1 - f) = \delta / (1 - \delta / \eta)$. From (21), the balanced-growth value of V_t is

$$V_t = \frac{\eta (\Pi_t + \widetilde{\delta} X_t)}{(\rho + \eta)(\rho + \widetilde{\delta})} = \frac{\rho + \eta + \widetilde{\delta}}{(\rho + \eta)^2 (\rho + \widetilde{\delta})} \eta \Pi_t. \tag{26}$$

Substituting (24)-(26) into the R&D free-entry condition in (15) yields

$$\frac{\rho + \eta + \widetilde{\delta}}{(\rho + \eta)^2 (\rho + \widetilde{\delta})} \eta \Pi_t = \frac{A_t}{h_t} \rho \, (1 + \sigma i) \, \frac{(\rho + \eta)\,(\rho + \widetilde{\delta}) b_t + (\rho + \eta + \widetilde{\delta}) \lambda w_t}{(\rho + \eta)\,(\rho + \widetilde{\delta})\,(\rho + \lambda) - \widetilde{\delta} \lambda \eta}. \tag{27}$$

For convenience, we define a transformed variable $\alpha_t \equiv A_t / h_t$, which is the level of R&D difficulty (measured by aggregate technology) relative to innovation productivity. Substituting (11), (13) and $b_t = \bar{b} y_t / L$ into (27) and then rearranging terms yield

$$\alpha = \frac{z - 1}{(1 + \sigma i)\, z \bar{b}} \frac{L}{\rho (\rho + \widetilde{\delta})} \frac{(\rho + \eta)\,(\rho + \widetilde{\delta})\,(\rho + \lambda) - \widetilde{\delta} \lambda \eta}{(\rho + \eta)\,(\rho + \widetilde{\delta}) + \lambda (\rho + \eta + \widetilde{\delta})(1 - \beta)/(1 - \beta z)} \Theta(\theta), \tag{28}$$

where $\widetilde{\delta} = \delta / (1 - \delta / \eta)$, $\lambda = M(\theta)$, $\eta = M(\theta)/\theta$ and

$$\Theta(\theta) \equiv \frac{\eta}{\rho + \eta} \left(1 + \frac{\widetilde{\delta}}{\rho + \eta} \right).$$

[28] The semi-endogenous growth model does not require the growth rate of technology to be equal to the rate g. If we consider a more general specification $\delta = h_t R / A_t^{\chi}$, then $\dot{A}_t / A_t = g / \chi$ in the long run. We consider a special case $\chi = 1$ for simplicity.

[29] It is useful to note that on the balanced growth path, the following set of variables $\{l, x, R, u, v\}$ is constant, whereas the remaining variables $\{A_t, w_t, \omega_t, y_t, c_t, b_t, \Pi_t, U_t, W_t, S_t, V_t, X_t, Z_t\}$ grow at the rate g.

We refer to (28) as the R&D free-entry (FE) condition, which contains two endogenous variables $\{\alpha, \theta\}$.[30] It is useful to note that the FE condition depends on the nominal interest rate i via the CIA constraint on R&D (i.e., $\sigma > 0$). From the R&D free-entry condition in (15), we have $\alpha = [\rho(1+\sigma i)]^{-1}V/U$, where we have also used (24). Whenever an increase in labor market tightness θ reduces the vacancy-filling rate η and increases the job-finding rate λ, it decreases the value V of an invention relative to the value U of unemployment, which in turn requires α to fall in the long run in order for the R&D free-entry condition to hold. We summarize this result in Lemma 1.

Lemma 1 *The FE curve shows a negative relationship between α and θ if ρ is sufficiently large.*

Proof. See Appendix B. ■

To close the model, we use the following steady-state condition that equates the inflow δ into the pool of firms searching for workers to its outflow ηf:

$$\delta = \eta f = \eta v/x = M(\theta)u/x. \tag{29}$$

The second equality follows from $fx = v$, where f is the number of firms with opened vacancies and x is the number of vacancies per firm. The third equality in (29) follows from (17) and uses the definition of $\theta \equiv v/u$. Furthermore, we need to derive the equilibrium supply of labor l. Substituting (11), (24), (25), $b_t = \bar{b}y_t/L$ and $c_t = y_t$ into labor supply in (4) yields

$$l(\underset{-}{i}, \underset{+}{\theta})/L = 1 - \frac{\gamma(1+\xi i)}{\rho\bar{b}} \frac{(\rho+\eta)(\rho+\widetilde{\delta})(\rho+\lambda) - \widetilde{\delta}\lambda\eta}{(\rho+\eta)(\rho+\widetilde{\delta}) + (\rho+\eta+\widetilde{\delta})\lambda(1-\beta)/(1-\beta z)}, \tag{30}$$

which is increasing in θ if ρ is sufficiently large as we will show in the proof of Lemma 2. Substituting (3), (12), (14) and (30) into (29) and applying the definition of $\alpha \equiv A/h$ yield

$$\alpha = \frac{L}{\delta}\left\{ l(\underset{-}{i}, \underset{+}{\theta})/L - \left[1 + \frac{\delta}{M(\theta)}\right] \frac{1-\beta z}{(1-\beta)z\bar{b}} \right\}. \tag{31}$$

We refer to (31) as the labor-market (LM) condition, which also contains two endogenous variables $\{\alpha, \theta\}$. It is useful to note that the LM condition depends on the nominal interest rate i via the CIA constraint on consumption (i.e., $\xi > 0$). From (14), we have $\alpha = R/\delta$. An increase in labor-market tightness θ reduces unemployment u, which in turn increases labor for R&D R. As a result of increased R&D, innovation becomes more difficult (i.e., α increases) in the long run, and this effect is present regardless of whether labor supply is elastic or inelastic. We summarize this result in Lemma 2. Finally, (28) and (31) can be used to solve for the steady-state equilibrium values of $\{\theta, \alpha\}$; see Figure 1 for an illustration.

Lemma 2 *The LM curve shows a positive relationship between α and θ if ρ is sufficiently large.*

Proof. See Appendix B. ■

[30] Recall that $\delta = g/\ln q$ is determined by exogenous parameters in the steady state.

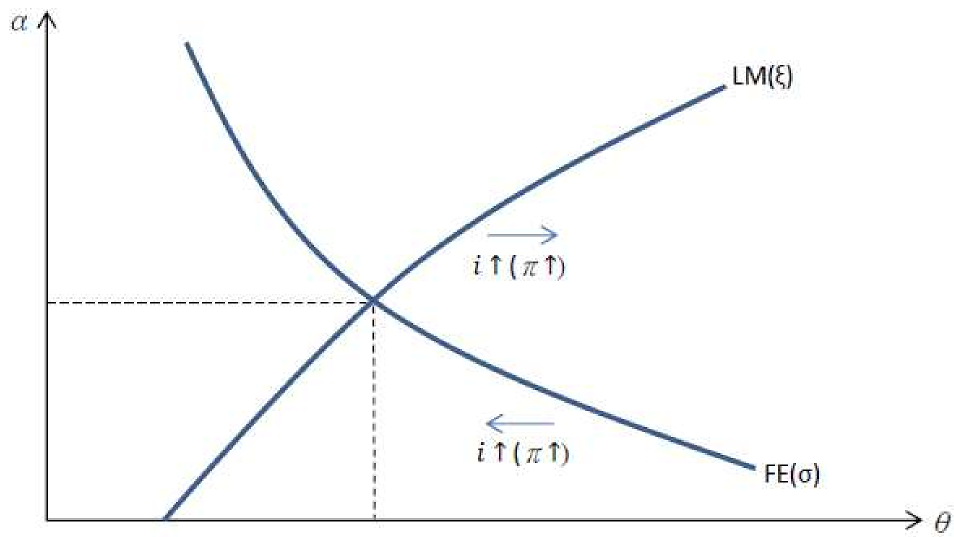

Figure 1: Steady-state equilibrium

4 Inflation, unemployment and economic growth

In this section, we explore the relationship between inflation, unemployment and economic growth. Section 3.1 considers the effects of inflation via the CIA constraint on R&D (i.e., $\sigma > 0$ and $\xi = 0$). Section 3.2 considers the effects of inflation via the CIA constraint on consumption (i.e., $\sigma = 0$ and $\xi > 0$).

4.1 Inflation via the CIA constraint on R&D

In this subsection, we explore the effects of inflation on unemployment and economic growth under the CIA constraint on R&D. From the Fisher identity, we have $i = \pi + r = \pi + \rho + g$, where the second equality uses the Euler equation and $\dot{c}_t/c_t = \dot{A}_t/A_t = g$. Therefore, a one-unit increase in the inflation rate leads to a one-unit increase in the nominal interest rate in the long run.[31] In Figure 1, we see that an increase in the nominal interest rate i (caused by an increase in inflation π) shifts the FE curve to the left reducing labor market tightness θ and the level of technology α. As for the resulting effect on unemployment u, we see from (29) that unemployment $u = \delta x/M(\theta)$ (where δ and x are determined by exogenous parameters and independent of i) is decreasing in the job-finding rate $M(\theta)$. Therefore, the increase in inflation π raises unemployment u by reducing labor market tightness θ and the job-finding rate $M(\theta)$. From (14), aggregate R&D is given by $R = \alpha\delta$; therefore, higher inflation π (that decreases the level of technology α) also reduces R&D. Now we consider the effect of inflation on economic growth. The dynamics of technology $\alpha_t \equiv A_t/h_t$ is given by $\dot{\alpha}_t/\alpha_t = \dot{A}_t/A_t - g$. Therefore, given that higher inflation π decreases

[31] For example, Mishkin (1992) and Booth and Ciner (2001) provide empirical evidence for a positive relationship between inflation and the nominal interest rate in the long run.

the steady-state value of α, it must also decrease the growth rate of A_t temporarily such that $\dot{A}_t/A_t < g$ before α_t reaches the new steady state. We summarize all these results in the following proposition.

Proposition 1 *Under the CIA constraint on R&D, higher inflation has (a) a positive effect on unemployment, (b) a negative effect on R&D, (c) a negative effect on the growth rate of technology in the short run, and (d) a negative effect on the level of technology in the long run.*

Proof. Proven in text. ■

The intuition of Proposition 1 can be explained as follows. Higher inflation leads to an increase in the opportunity cost of cash holdings, which in turn increases the cost of R&D investment via the CIA constraint on R&D. As a result, R&D decreases resulting into a lower growth rate of technology in the short run and a lower level of technology in the long run. The negative relationship between inflation and R&D is consistent with the empirical evidence based on cross-sectional regressions in Chu and Lai (2013) and panel regressions in Chu, Cozzi, Lai and Liao (2014). The negative relationship between inflation and economic growth is also supported by the cross-country evidence in Fischer (1993), Guerrero (2006) and Vaona (2012). Although the rate of creative destruction decreases temporarily, the decrease in innovation in the long run increases the number of unemployed workers relative to labor-market vacancies, causing a negative effect on labor-market tightness and a positive effect on long-run unemployment. In other words, via the CIA constraint on R&D, inflation increases long-run unemployment by decreasing the demand for labor. Therefore, under the CIA constraint on R&D, inflation and unemployment have a positive relationship in the long run, and this theoretical result is consistent with empirical studies, such as Ireland (1999), Beyer and Farmer (2007), Russell and Banerjee (2008) and Berentsen *et al.* (2011) who consider data in the US. In Section 2, we also find a positive effect of inflation on unemployment via the financial friction faced by firms, which is a proxy for the CIA constraint on R&D.

Finally, it is easy to see from the FE condition in (28) and Proposition 1 that relaxing the liquidity constraint on R&D (i.e., a decrease in σ) would reduce unemployment and increase R&D and innovation.

4.2 Inflation via the CIA constraint on consumption

In this subsection, we explore the effects of inflation on unemployment and economic growth under the CIA constraint on consumption. In this case, Figure 1 shows that an increase in inflation π shifts the LM curve to the right increasing labor market tightness θ and decreasing the level of technology α. As for the resulting effect on unemployment u, we see from (29) that unemployment $u = \delta x/M(\theta)$ is decreasing in the job-finding rate $M(\theta)$. Therefore, the increase in inflation π reduces unemployment u. From (14), aggregate R&D is given by $R = \alpha\delta$; therefore, the higher inflation π also reduces R&D. As for the effect of inflation on economic growth, given that inflation π decreases the steady-state value of α, it must decrease the growth rate of A_t temporarily before α_t reaches the new steady state. We summarize these results in Proposition 2.

Proposition 2 *Under the CIA constraint on consumption, higher inflation has (a) a negative effect on unemployment, (b) a negative effect on R&D, (c) a negative effect on the growth rate of technology in the short run, and (d) a negative effect on the level of technology in the long run.*

Proof. Proven in text. ■

The intuition behind the negative relationship between inflation and unemployment can be explained as follows. Higher inflation leads to an increase in the opportunity cost of cash holdings, which in turn increases the cost of consumption relative to leisure. As a result, the household consumes more leisure and reduces labor supply. The decrease in labor supply reduces R&D labor and also the number of workers searching for employment. The resulting increase in labor-market tightness decreases unemployment. In other words, via the CIA constraint on consumption, inflation decreases unemployment by decreasing the supply of labor. Therefore, under the CIA constraint on consumption, inflation and unemployment have a negative relationship, and this theoretical result is consistent with empirical studies, such as Dolado *et al.* (2000) and Karanassou *et al.* (2005, 2008) who consider data in Europe and the US. In Section 2, we also find a negative effect of inflation on unemployment via the financial friction faced by consumers, which is a proxy for the CIA constraint on consumption.

Finally, it is easy to see from (30) and Proposition 2 that relaxing the liquidity constraint on consumption (i.e., a decrease in ξ) would increase unemployment, R&D and innovation.

5 Quantitative analysis

In this section, we first calibrate the model to US data to explore its quantitative implications. To facilitate this analysis, we follow the standard approach in the literature to specify a Cobb-Douglas matching function $F(v_t, u_t) = \varphi v_t^{\varepsilon} u_t^{1-\varepsilon}$, where the parameter $\varphi > 0$ captures matching efficiency and the parameter $\varepsilon \in (0, 1)$ is the elasticity of matches with respect to vacancies. Given this matching function, the job-finding rate is $\lambda_t = \varphi \theta_t^{\varepsilon}$, and the vacancy-filling rate is $\eta_t = \varphi \theta_t^{\varepsilon-1}$.

The model features the following structural parameters $\{\rho, \varepsilon, \beta, g, \bar{b}, z, q, \gamma, \varphi, \sigma, \xi\}$ and a policy instrument π. We follow Acemoglu and Akcigit (2012) to set the discount rate ρ to 0.05. We follow Berentsen *et al.* (2011) to set $\varepsilon = 1 - \beta = 0.28$, so that the elasticity of matches with respect to vacancies is equal to the bargaining power of firms satisfying the Hosios (1990) rule. We consider a long-run technology growth rate g of 1% and a long-run inflation rate π of 3%. Then, we calibrate the remaining parameters $\{\bar{b}, z, q, \gamma, \varphi, \sigma, \xi\}$ by matching theoretical moments to data. We calibrate $\bar{b}$ to match data on unemployment benefits as a ratio of per capita income, which is about one quarter in the US. We calibrate the markup ratio z to match data on R&D as a share of GDP, which is about 3% in the US. We calibrate the quality step size q, which determines $\delta = g/\ln q$, to match a long-run unemployment rate u/l of 6% in the US. We calibrate the leisure parameter γ to match the ratio of labor force to the working-age population (aged 16 to 64), which is about three quarters in the US. We calibrate matching efficiency φ to match a long-run job-finding rate λ of 0.3, as estimated in Hall (2005). We calibrate the CIA-R&D parameter σ to match the semi-elasticity of R&D/GDP with respect to inflation $\partial \ln(R\&D/GDP)/\partial\pi = 0.4$ estimated in Chu, Cozzi, Lai and Liao (2015). We calibrate the CIA-consumption parameter ξ to match the money-output ratio m/y. We consider two conventional measures of money: M0 and M1. In the US, the average M0-output ratio is about 0.06, whereas the average M1-output ratio is about 0.12. The calibrated parameter values are summarized in Table 3.

Table 3: Calibrated parameter values for the US											
	ρ	ε	β	g	$\overline{b}$	z	q	γ	φ	σ	ξ
M0	0.05	0.28	0.72	0.01	0.25	1.30	1.65	0.25	0.17	0.43	0.05
M1	0.05	0.28	0.72	0.01	0.25	1.30	1.65	0.25	0.16	0.04	0.12

Given the above parameter values, we proceed to simulate the long-run Phillips curve (with inflation on the horizontal axis) in this calibrated US economy. Figure 2a shows an upward-sloping Phillips curve under the M0 specification, whereas Figure 2b shows a downward-sloping Phillips curve under the M1 specification. The intuition behind these contrasting results can be explained as follows. Under the M0 specification, the relatively low money-output ratio implies a small degree of CIA on consumption (i.e., a small ξ). As a result, in order to match the empirical semi-elasticity of R&D with respect to inflation, the degree of CIA on R&D must be relatively large (i.e., a large σ). In this case, the effect of inflation on unemployment works through mainly the R&D channel giving rise to a positive relationship between the two variables. Under the M1 specification, the relatively high money-output ratio implies a larger degree of CIA on consumption (i.e., a larger ξ), which in turn is almost sufficient to deliver the empirical semi-elasticity of R&D with respect to inflation. As a result, the implied degree of CIA on R&D becomes much smaller (i.e., a much smaller σ). In this case, the effect of inflation on unemployment works through mainly the consumption-leisure tradeoff giving rise to a negative relationship between the two variables. This ambiguous relationship between inflation and unemployment in the US is consistent with the contrasting empirical results in the literature.

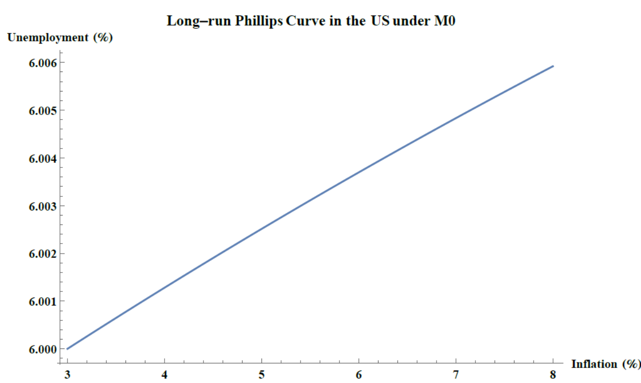

Figure 2a

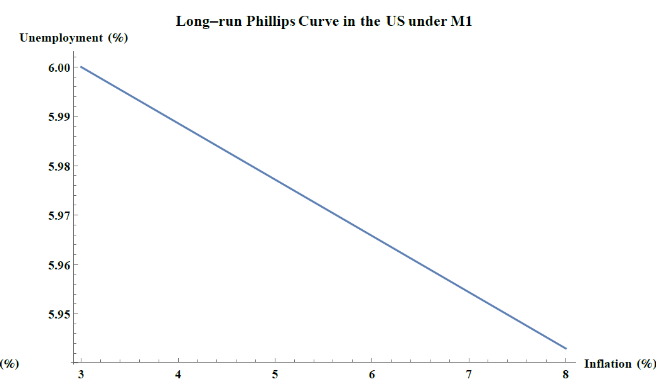

Figure 2b

In the rest of this section, we recalibrate the model to the Eurozone, which features lower R&D, higher unemployment, lower job-finding rate, higher unemployment benefits and higher money-output ratio than the US. Specifically, we consider an R&D-output ratio of 2%, a long-run unemployment rate of 9%, an average job-finding rate λ of 0.07,[32] and unemployment benefits as a ratio of per capita income of 0.4.[33] Finally, we consider the two measures of money as before: an average M0-output ratio of 0.08, and an average M1-output ratio of 0.4. Table 4 summarizes the calibrated parameter values.

[32] See Hobijn and Sahin (2009) for estimates of the job-finding rates in a number of European countries.

[33] See Esser *et al.* (2013) for data on unemployment benefits in European countries.

Table 4: Calibrated parameter values for the Eurozone											
	ρ	ε	β	g	$\bar{b}$	z	q	γ	φ	σ	ξ
M0	0.05	0.28	0.72	0.01	0.40	1.26	4.03	0.21	0.09	0.38	0.07
M1	0.05	0.28	0.72	0.01	0.40	1.26	4.03	0.20	0.09	0.02	0.40

Given the above parameter values, we proceed to simulate the long-run Phillips curve in this calibrated European economy. Figures 3a shows a downward-sloping Phillips curve under the M0 specification, whereas Figure 3b also shows a downward-sloping Phillips curve under the M1 specification. The reason why the Phillips curve is always downward sloping in this case is the stronger CIA friction on consumption, which in turn is implied by the higher money-output ratio in the Eurozone. Under the M0 specification, the calibrated value for the CIA-consumption parameter ξ is 0.07, compared to $\xi = 0.05$ in the US. The stronger CIA friction on consumption in Europe implies that the effect of inflation on unemployment works through the consumption-leisure tradeoff giving rise to a negative relationship between inflation and unemployment even under the M0 specification. This finding of a downward-sloping Phillips curve in Europe is consistent with the empirical evidence in Dolado *et al.* (2000) and Karanassou *et al.* (2008).

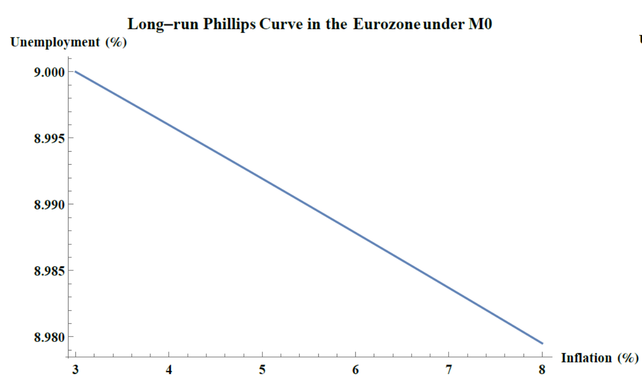

Figure 3a

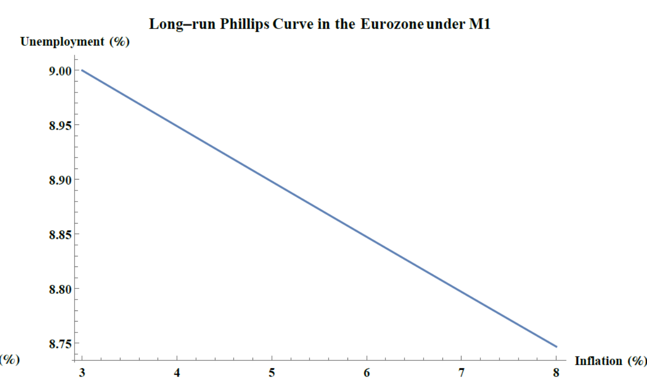

Figure 3b

6 Conclusion

In this study, we have explored a fundamental question in economics that is the long-run relationship between inflation, unemployment and economic growth. We consider a standard Schumpeterian growth model with the additions of money demand via CIA constraints and equilibrium unemployment driven by matching frictions in the labor market. In this monetary growth-theoretic framework with search frictions, we discover a positive (negative) relationship between inflation and unemployment under the CIA constraint on R&D (consumption), a negative relationship between inflation and R&D, and a negative relationship between inflation and economic growth. These theoretical predictions are largely consistent with empirical evidence.

An important policy implication from our analysis is that monetary expansion could be useful to reduce the rather high unemployment rate in the Eurozone, but that it would come at the expense of innovation and long-run technological competitiveness. A better policy prescription for the European banking authorities would be to manage to ease the liquidity problems that plague R&D activities. According to our results, this policy, unlike monetary expansion, would at the same time decrease unemployment and increase growth and technological competitiveness.

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Appendix A

Table A1: Effects of inflation on unemployment (tightened "considerably")

	All banks			Large banks		
	(1)	(2)	(3)	(4)	(5)	(6)
BF × Inflation	1.805	1.205	1.593	1.686*	1.204	1.864
	(1.390)	(1.365)	(2.291)	(0.972)	(0.958)	(1.551)
BC × Inflation	-2.541*	-2.541*	-3.183	-2.460**	-2.460**	-3.454**
	(1.408)	(1.365)	(2.324)	(1.101)	(1.068)	(1.729)
RD × BF × Inflation		19.531***	19.685***		15.682***	15.718***
		(6.953)	(7.117)		(5.517)	(5.648)
RD × BF		-69.315***	-69.696***		-55.774***	-55.863***
		(20.061)	(20.423)		(16.298)	(16.589)
RD × Inflation		-3.964	-4.821		-2.695	-2.963
		(22.425)	(23.867)		(22.299)	(23.727)
Inflation, BF and BC	Yes	Yes	Yes	Yes	Yes	Yes
Other controls	No	No	Yes	No	No	Yes
Industry fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Observations	252	252	252	252	252	252
R-squared	0.315	0.365	0.406	0.320	0.369	0.406

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The first three columns correspond to lending policy reported by all banks, whereas the last three columns correspond to lending policy reported by large banks. In all columns, we control inflation, bank policy for firms (BF) and bank policy for consumers (BC). In columns (3) and (6), we further control GDP per capita, log trade volume and industry-specifc time trends.

Appendix B

Proof of Lemma 1. First, we restrict the range of values for θ to ensure that (a) $\lambda = M(\theta) < 1$ (i.e., the number of workers who find jobs at a given point in time must be less than the number of workers searching for jobs at that time), (b) $\eta = M(\theta)/\theta < 1$ (i.e., the number of vacancies filled at a given point in time must be less than the number of vacancies on the market at that time), and (c) $\eta > \delta$ so that $f = \delta/\eta < 1$ (i.e., the number of industries with unlaunched innovations must be less than the total number of industries, which is normalized to unity). Then, we examine each term on the right-hand side of (28) separately. The first term in (28) is independent of θ, whereas the second term in (28) is decreasing in θ given that $\tilde{\delta}$ increases with θ. The third term in (28) can be reexpressed as

$$\vartheta(\theta) \equiv \rho \frac{\rho + \tilde{\delta} + \lambda(1 + \frac{\tilde{\delta}}{\rho+\eta})}{\rho + \tilde{\delta} + \lambda(1 + \frac{\tilde{\delta}}{\rho+\eta})(1-\beta)/(1-\beta z)}. \tag{B1}$$

Given $(1-\beta)/(1-\beta z) > 1$, we can show that $\vartheta'(\theta) < 0$ holds if[34]

$$\{[1 - \delta\theta/M(\theta)]^2 \rho/\delta + 1\}/\delta > 1/M'(\theta), \tag{B2}$$

which holds if ρ is sufficiently large because $1 - \delta\theta/M(\theta) = 1 - \delta/\eta > 0$. As for the fourth term in (28), noting $\eta = M(\theta)/\theta$ and $\tilde{\delta} = \delta/[1 - \delta\theta/M(\theta)]$, we can show that $\Theta'(\theta) < 0$ holds if and only if[35]

$$\underbrace{\frac{\rho}{\delta}\left([1 - \delta\theta/M(\theta)]^2 + \frac{2\delta\,[1 - \delta\theta/M(\theta)]\,\theta/M(\theta)}{\rho\theta/M(\theta) + 1} \right)}_{\equiv \chi(\theta)} > 1. \tag{B3}$$

Note that $\chi(\theta) > 0$ because $\rho > 0$ and $1 - \delta\theta/M(\theta) = 1 - \delta/\eta > 0$. Therefore, we can conclude this proof by saying that a large value of ρ is a sufficient (but not necessary) condition for the FE curve in (28) to be downward sloping in θ. ■

Proof of Lemma 2. By (30) and (B1), $l(i,\theta)/L = 1 - \vartheta(\theta)\gamma(1+\xi i)/(\rho\bar{b})$. From the proof of Lemma 1, $\vartheta(\theta)$ is decreasing in θ if ρ is sufficiently large. Together with (31), one can easily show that the LM curve is upward sloping in θ. ■

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[34] Note that $\vartheta'(\theta) < 0$ holds if and only if

$$(\rho + \tilde{\delta}) \left[\lambda \left(1 + \frac{\tilde{\delta}}{\rho+\eta} \right) \right]' > \tilde{\delta}' \lambda \left(1 + \frac{\tilde{\delta}}{\rho+\eta} \right),$$

which can be expressed as

$$\lambda(\rho + \tilde{\delta}) \left[\frac{\tilde{\delta}'(\rho+\eta) - \eta'\tilde{\delta}}{(\rho+\eta)^2} \right] > [\tilde{\delta}'\lambda - (\rho+\tilde{\delta})\lambda'] \left(1 + \frac{\tilde{\delta}}{\rho+\eta} \right).$$

Given $\lambda' > 0$, $\eta' < 0$, and $\tilde{\delta}' > 0$, this holds if $\tilde{\delta}'\lambda - (\rho+\tilde{\delta})\lambda' < 0$, which is equivalent to (B2) by $\lambda = M(\theta)$, $\tilde{\delta} = \delta/\left[1 - \delta\theta/M(\theta)\right]$, and $\tilde{\delta}' = \tilde{\delta}^2 \left[M(\theta) - \theta M'(\theta)\right]/M(\theta)^2$. Note that $M(\theta) > \theta M'(\theta)$ by the properties of $M(\theta)$.

[35] Note that

$$\Theta'(\theta) = \frac{\varkappa'(\theta)}{[1 - \delta\varkappa(\theta)]^2 \left[\rho\varkappa(\theta) + 1\right]^2} \left[\delta - \rho\left([1 - \delta\varkappa(\theta)]^2 + \frac{2\delta\varkappa(\theta)\,[1 - \delta\varkappa(\theta)]}{\rho\varkappa(\theta) + 1} \right) \right],$$

where $\varkappa(\theta) \equiv \theta/M(\theta)$ and thus $\varkappa'(\theta) > 0$.