The factors inefficient allocation of investment between economies

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Abstract

The article deals with the problem of possibility of allocation of investment capital by economies. The situation, when the less effective investors find themselves in more favorable investment climate despite of rationality principle is considered, i.e. there is a situation adverse selection.

The model describing investors’ behavior in the economy, characterized by an investment climate of some favorability, is developed.

There was considered a behaviour of investors–maximizers and investors–satisficers.

2 cases are modeled: independence of the economic climate from the volume of available investments and limitedness of maximal volume of investments, and dependence of the economic climate from the volume of available investments and unlimitedness of maximal volume of investments.

The profitableness of investments on the economic climate considered for exponential and logistic function.

The analysis shows reasons for adverse selection have a fundamental behavioral basis. The given study allows concluding the investment market is not fully self-controlled and sometimes needs government intervention.

Keywords: capital allocation, behavior of agents, opportuneness economic climate, maximizer, satisficer
Introduction

The article is devoted to the problem of the efficiency of the distribution of investors by economy. The problem is as follows.

The functioning of group of the related economies can consider from the viewpoint of each their association and of some single economy inside this group. The efficiency of the economic activity in above 2 cases bases on different criteria: while we consider the efficiency as the welfare maximization, but in the first case it is aggregate welfare of all economies of the group, and in the latter one it is welfare of a particular economy. Also, roles of the generally accepted regulatory instrument - of taxes will be different. Then the issue of optimization of capital allocation is to find the optimal allocation of economic agents in economies, i.e. to find allocation that maximizes welfare (of all economies or a particular economy).

In opinion of the rationality of agents' behaviour, the best allocation of investors in economies is such that the most productive agents act in the economy with the most favorable climate, leaving the least favorable economy to the least productive investors. Indeed all agents try to get into the most favorable economy, but the most powerful ones have a better chance of success.

At the same time in practice, even in open economies there function agents of varying efficiency, in a more favorable climate may be relatively weak investors but in less favorable climate – more effective ones.

The question arises, is such situation explained only isolated factors, such as incomplete openness of economies (that causes the emergence of administrative and economic barriers), random influences, temporal events, institutional distortions, or also by rather general regularities that there are regardless of the presence or absence of the above isolated parameters?
Literature review

One of the most well-known mechanisms that explains the formation of above situations, is the mechanism of so-called “adverse selection”, was first described by G. Akerlof (1970). The issue adverse selection as a consequence of incompleteness of information was discussed by Besley (1994) and by Tirole (2006). In particular Besley (1994) studied market failure and underlined the need to consider the full array of factors that lead to the imperfection of a market. He concluded there are weighty arguments for government intervention to the function of the market. Tirole (2006) analyze information asymmetry between insiders and outsiders at the funding stage by adapting adverse selection's model and moral hazard model to capital allocation. Stiglitz & Weiss (1981) showed that in an equilibrium credit market interest rates may influence on the selection and behaviour of potential borrowers, causing sometimes the adverse selection of creditors by borrowers because of the different probability repayment of them debts.

D’Aspremont & Gérard-Varet (1979) investigated optimal public design mechanisms. They have used for the first time linear inequality methods to study adverse selection. Boyd & Smith (1993) considered credit rationing in a case of adverse selection. They studied the equilibrium allocation of investment capital in an environment with certain specificity of the investment activity. Philippon & Skreta (2012) studied a design of interventions to stabilize financial markets faced with adverse selection. They have shown the efficiency of an intervention can be assessed by its impact on the market interest rate.

There is also number of researches about modeling of adverse selection in entrepreneurship (Huberman & Kandel (1993), Heinkel & Stoughton (1994), Martin (2009)).

Tirole (2012) provided analysis of market jumpstarting and its two-way interaction between mechanism design and participation constraints. He fined that the government can reduce adverse selection enough to let the market rebound, but not too much, so as to limit the cost of intervention.
Optimization of investment allocation (and in general of capital allocation) was studied by B. Becker, D. Reinhardt, S. Alexiadis, C. A. Ladias, E. Cavallo, A. Galindo, T. Bandyopadhyay, T. Biswas etc. In particular, Becker et al. consider the influence of corporate payout is taxed on investments. They note a large impact of corporate payout on the dynamics of investments and on the capital allocation. Reinhardt (2010) verifies the trend of investment allocation. The author accents the main factors influencing the volume FDI, in addition a productivity, and compares the power of this influence to different sectors of the economy. Alexiadis & Ladias (2011) provide a model of optimal allocation of investment however not for countries as a whole, but for regions in a particular country.

A number of studies focus on emerging economies.

Cavallo et al. (2013) note the distribution of investment is not always effective, especially in emerging economies. They focus on the dependence of this fact on of price volatility. Galindo et al. (2001) introduces the index developed by them for the distribution of investments. Their research studies, first of all, the impact of financial liberalization on the efficiency of investment. Similarly, emerging economies (similar to those created in Eastern European countries) were the subject of study of Bandyopadhyay & Biswas (1997). As the main factor in improving the distribution of investments, the authors identified a temporary price intervention.

Model

To analyze the mentioned problem was developed a formal model of the behavior of economic agents – investors, act in a certain economic system that is characterized by an investment climate of certain favorability. In turn, economic agents have qualifical parameters: productivity of investment activity, income per unit of invested funds.

Let’s introduce the following formal model parameters:

\[ I_i(t)_{t=1,...,m} \] is the set of investors in the country at time \( t \);
\( Y_j \mid_{j=1\ldots n} \) is the set of economic systems (countries), \( i=1,2,\ldots n; \)

\( x_{ij}(t) \) is the investment of the \( i \)-th investor in the \( j \)-th economy at time \( t \);

\( X_j \) is sizes of assets of agents in the \( j \)-th economy, \( j=1,2,\ldots n; \)

\( P_j \) is the capacity of the \( j \)-th economy, \( j=1,2,\ldots n; \)

\[ x_j(t) := \sum_{i=1}^{m} x_{ij}(t) \] is total investments of all investors in the \( j \)-th economy at time \( t \). For the discrete time \((t_0, t_1, \ldots, t_n)\):

\[ x_{nj}(t) = x_j(t_n) = \sum_{i=1}^{m} x_{ij}(t_n); \]

\[ r_{ij}(t) = r(x_{ij}, t) \] is income of the \( i \)-th investor on his investments in the \( j \)-th economy at time \( t \).

Below, a group of economies is considered as a whole. There are 2 variants of initial filling with investments of economies:

1) under condition of independence of the climate of the economy from the volume of investments available in it and of limitedness them volume, and

2) under condition of dependence of the climate from the volume of available investments and of unlimitedness them volume.

**The independence of income of the economy from the volume of investments in it**

It can be show that in this case, the optimal allocation of agents’ assets to economies is such when the most productive agents act in the most favorable economy’s climate; the most productive agents from the rest act in the economy with climate, which is next by the favorability etc.

Really, assume that exactly one investor can act in a single economy. Then for any 2 investors \((i_1, i_2)\) and any 2 economies \((j_1, j_2)\), for which the following conditions

\[ r(x_{i_1}, t; a_{i_1}, \alpha_{i_1}, b_{j_1}, \beta_{j_1}) > r(x_{i_2}, t; a_{i_2}, \alpha_{i_2}, b_{j_1}, \beta_{j_1}); \]

\[ r(x_{i_1}, t; a_{i_1}, \alpha_{i_1}, b_{j_2}, \beta_{j_2}) > r(x_{i_2}, t; a_{i_2}, \alpha_{i_2}, b_{j_2}, \beta_{j_2}); \]
are true, the predicate
\[
 r \left( x_i, t; a_i, \alpha_i, b_j, \beta_j \right) + r \left( x_i, t; a_j, \alpha_j, b_j, \beta_j \right) > \\
 r \left( x_i, t; a_i, \alpha_i, b_j, \beta_j \right) + r \left( x_i, t; a_j, \alpha_j, b_j, \beta_j \right)
\] (1)
also is true.

Obviously, it is equally true for any number of economies and of investors. I.e., the optimal allocation (in terms of efficiency) will be the following: in the most favorable economic environment the most individual efficient agents act and their number is necessary and sufficient for the investment demand saturation inside of this economy. The most effective agents among the remainder act in the less favorable environment etc. The least favorable environment remains for least effective investors (Fig. 1).

Formally it can be written by considering the decision-making (behavior) of investors depending on the parameters of economic environment, in which the investors act. Let for \( j=1,2,...,n \):

\( \psi(j) \) is the function of the allocation of investments of agents acting in the economy \( i \):
\[
\forall i : i = 1,2,\ldots,M : \psi(j) \in \{1,2,\ldots,n\};
\]

\( \varphi(j) \) is the function of agents ordering, that has follows property:
let \( r \left( x,t,a_{j_1},\alpha_{j_1},b_{k},\beta_{k} \right) > r \left( x,t,a_{j_2},\alpha_{j_2},b_{k},\beta_{k} \right) \),
then \( \varphi_{j_1} < \varphi_{j_2} \);
\[
\forall j : j = 1,2,\ldots,M : \varphi(j) \in \{1,2,\ldots,M\}.
\]

It is obviously that
\[
\forall k : k = 1,2,\ldots,n : \exists j_k : j_k \in \{1,2,\ldots,M\}:
\]
\[
\left( \sum_{\varphi^{-1}(j) \leq j_k} x_{\varphi^{-1}(j)} \leq \sum_{kk=1}^{k} Y_{kk} \right) \wedge \left( \sum_{\varphi^{-1}(j) \leq j_k + 1} x_{\varphi^{-1}(j)} > \sum_{kk=1}^{k} Y_{kk} \right).
\] (2)

Then the following allocation gives the most productivity:
\[
\psi \left( j : \varphi^{-1}(j) \leq j_1 \right) = 1; \forall k = 2,\ldots,M : \varphi \left( j : j_{k-1} < \varphi^{-1}(j) \leq j_k \right) = k.
\] (3)
In addition agents differ in their cognitive abilities. In the context of this study, this means that the agents belong either to maximizers aiming to maximize their profit, or to satisficers, for who it is quite enough to just be profitable (see e.g. the definition of a satisficer in (Simon, 1955; 1956). Also, in each economy, there is a government that sets rates of taxes, collects them, and eventual commit funds to improving the investment climate.

The dependence of investment income for \(i\)-th agent on his qualification and on investment climate of \(j\)-th economy may looks differently, but it must correspond to logic of interdependence of above parameters. In particular, the behaviour of sigmoid function (e.g., the logistic function, the Gompertz function) and of exponential function corresponds to this logic. We get for exponential function:
\forall i: i = (1, N), \forall j: j = (1, N): r(x_{ij}) = a_i b_j e^{-(\alpha_i + \beta_j)x_{ij}};

for sigmoid ("S"-shaped) function:

\forall i: i = (1, N), \forall j: j = (1, N): r(x_{ij}) = \frac{a_i b_j}{1 + e^{(\alpha_i + \beta_j) - x_{ij}}};

where parameters $a_i$ and $\alpha_i$ represent the investor’s efficiency, and parameters $b_i$ and $\beta_i$ – the investment climate of economy, in addition

\forall i: i = (1, N), \forall j: j = (1, N): a_i > 0, b_j > 0, \alpha_i > 0, \beta_j > 0.

**The dependence of economy’s income on the volume of investments in it**

For an unlimited economy, the climate of which depends on the investment market saturation, the arrangement of agents in economies in accordance with the effectiveness of their investment activity is violated.

Investment effectiveness depends on market saturation; market saturation is directly related to demand and market capacity.

The financial market cannot be completely saturated, i.e. it is not able of accepting funds to $+\infty$. With the increase of money supply in the market, their value, expressed in the bank interest rate for a loan, will be steadily decreasing and will tend to zero as $t$ tends to $+\infty$.

In the absence of funds on the market, the price of the initial offer, probably, will not be equal to $+\infty$, but will be finite. Under such a definition, the logic of initial fills with investments of economic systems changes somewhat Filling of the economy with the most favorable climate continue, as long as the money supply does not cause the investments (money) to fall below their current value in the economy that has next in order of favorableness of climate.

At the same time, financial filling of this second economy will eventually reduce the cost of funds in it less than not only their cost in the economy that is next on the efficiency (into which funds was not filled yet), but also in the first one. As a result, there will be an incentive for new investments in the economy with a priori the most favorably climate. In consequence of such alternating filling, more efficient
investors may find themselves in economies with the less-favored climate and vice versa.

We start to consider the logic of filling the economy with investments with an ideal starting situation, when none of the $n$ economies has any investment yet. Obviously, in such a situation and without constraints, the economy with the best climate will be filled with investment first. At a time when such a filling will lead to a reduction in the current profitability of the 1st economy to the profitability of 2st one, these two economies will be filling parallel.

In the same way, there will be the third, fourth etc. economies sequential connected (table 1), in addition,

$$r_1(t_1) > r_2(t_1) = r_3(t_1) = \ldots = r_n(t_1) \equiv 0;$$

$$r_1(t_2) = r_2(t_2) > r_3(t_2) = \ldots = r_n(t_2) \equiv 0;$$

$$r_1(t_3) = r_2(t_3) = r_3(t_3) > r_4(t_3) > r_5(t_3) = \ldots = r_n(t_3) \equiv 0;$$

$$r_1(t_{n-1}) = r_2(t_{n-1}) = \ldots = r_n(t_{n-1}) = r_n(t_n) \equiv 0;$$

$$\forall i \geq n \ \forall j_1, j_2 = 1, 2, \ldots, n: \quad r_{j_1}(t_i) = r_{j_2}(t_i). \quad (4)$$

Table 1. The allocation of investments and incomes for economic systems

<table>
<thead>
<tr>
<th>Economy number</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_1$</td>
</tr>
<tr>
<td>1</td>
<td>$x_{11}; r_{11}$</td>
</tr>
<tr>
<td>2</td>
<td>$0; 0$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$n$</td>
<td>$0; 0$</td>
</tr>
</tbody>
</table>

Conditions of the ratio of investment volumes in economies $j$ and $k$ at time $t_n$ for the exponential function are follows:

$$r(x_{ij}) = r(x_{ik}) \iff a_i b_j e^{-(\alpha_i + \beta_j)x_{ij}} = a_i b_k e^{-(\alpha_i + \beta_k)x_{ik}};$$

$$\left(\alpha_i + \beta_j\right) x_{ij} - \left(\alpha_i + \beta_k\right) x_{ik} = \ln \left(\frac{b_j}{b_k}\right);$$
\[ x_{ij} = \frac{\ln b_j - \ln b_k}{\alpha_i + \beta_j} + \frac{\alpha_i + \beta_k}{\alpha_i + \beta_j} x_{ik}. \]  
\(5\)

In particular for \(k=n\):

\[ r(x_{ij}) = r(x_{in}) \iff x_{ij} = \frac{\ln b_j - \ln b_n}{\alpha_i + \beta_j}. \]  
\(6\)

The same conditions for the Gompertz function are follows:

\[ r(x_{ij}) = r(x_{ik}) \iff \frac{a_i b_j}{1 + e^{(\alpha_i + \beta_j)-x_{ij}}} = \frac{a_i b_k}{1 + e^{(\alpha_i + \beta_k)-x_{ik}}}; \]

\[ b_j \left(1 + e^{(\alpha_i + \beta_k)-x_{ik}}\right) = b_k \left(1 + e^{(\alpha_i + \beta_j)-x_{ij}}\right); \]

\[ x_{ij} = \left(\alpha_i + \beta_j\right) - \ln \left(\frac{b_j}{b_k} e^{(\alpha_i + \beta_k)-x_{ik}} - 1\right). \]  
\(7\)

For \(k=n\):

\[ r(x_{ij}) = r(x_{in}) \iff x_{ij} = \left(\alpha_i + \beta_j\right) - \ln \left(\frac{b_j}{b_n} e^{(\alpha_i + \beta_n)} - 1\right). \]  
\(8\)

Thus for every time, may calculate rational allocation of investments under economies in the absence of government intervention into economies' climate and/or tax policy.

In the future, in the absence of interference in the climate of the economy, the total investment volume is increasing, and the profitability of each economy is decreasing, in addition, levels of profitability of all economies remains equal to each other.

An ideal, continuous case of filling the economy with investments is described above. In reality, this process is discrete, so the general situation is somewhat different (though not fundamentally). Fig. 2 shows the allocation of investors in economies for the discrete nature of their entry.
Figure 2. The optimal allocation of investors in economies, if economy’s profitability depends on the volume of investment

a) exponential dependence

b) sigmoid dependence
From Fig. 2 shows, the general view of the agents’ allocation for exponential and sigmoidal dependencies of economy’s income from its capital filling are similar to each other: in both cases, investors with approximately the same productivity do not concentrate in one economy with a certain climate’s favorability, but are allocated to all (at least several) economies.

**Discussion**

We analyzed the possible behavioral incentives for the secondary reallocation of agents between economies by the above model. It could help answer following questions:

1) under what conditions and which agents will aim to switch to the economy with a more favorable climate, and who ones to stay in that economy where they are act currently?
2) what will be the agents’ attitude towards emergence in the economy of another investors?
3) will agents aim to invest into improvement of economic climate in which they act? And what are exactly agents?
4) will the government aim to invest in improving the national economy?

There were identified a set of basic incentives of the presence or absence of agents’ desire to change the location both of own and other investments:

1) an effective investor can get the incentive to change the economy with a better investment climate to the economy with worse one if the second economy contains substantially less funds than the first one, that increases its profitability. This behavior is typical for all agents: both for maximizers and for satisfies;
2) though all investor–maximizers aim to the economy with the most favorable climate, the presence of competition in the “world without taxes” leads to the fact that more productive investors crowd out less productive to worse conditions of activity. As a result, after some time it will be possible to observe the above described picture of effective allocation. That is, according to (2), in a particular economy there will be agents-maximizers, that in their qualification not able to
stand in the more efficient economy. Obviously, from this point of view, they can be consider their current location optimal, therefore they will not aim neither to move to a more efficient economy nor to improve the climate of the economy in which they are currently acting, fearing that in such case more productive investors may push out them of a better climate;

3) the fact that the investor-satisficer's qualification is sufficient for a profitable activity in a certain economy can well serve as an incentive for his refusal to pass to the economy with a better climate because the aim of the satisficer is not maximum profit, but any profit from him activity. Simultaneously they will do nothing to improve the favorability of investment climate, since they are satisfied by their current position;

4) at the same time, at the same time, the unprofitability of the satisficer's activity cause a search for economy with a more favorable investment climate for his founds that would allow him to make a profit. Of course in completely static conditions of a perfect market these investors would have to go bankrupt. However in reality there are new markets emerge, barriers to entry can hinder some effective investors, therefore inefficient investors get his chance also.

Moreover entering of additional investors into economy can lead to deterioration of investment climate (due to glut of economy by equity) and also to strengthening of competition between new agents and existing ones. Thus it is obvious, the resources of rational agents will be directed to resisting the arrival of new actors, in addition, they will resist the arrival more powerful investors more intensely, because more powerful investors create more competition.

The directions of potential transformation of the investors’ location on the basis of the above incentives are shown on Fig. 3, from which follows that the average productivity of economy decrease with time rather than increase.

Consequently agents’ migration can lead to situation opposite to one shown on Fig. 1: the least efficient investors find themselves in the economy with the most favorable investment climate, and the best efficient investors replace them in the most
unfavorable economy, that corresponding to classic case “adverse selection” (Akerlof, 1970).

**Figure 3. The trends of agents’ migration**

![Graph showing the trends of agents' migration](image)

The realized logical analysis was verified and confirmed by computer simulation of the agents' behavior. The pattern of optimal allocation of investors to economies with a fixed income of economies is shown on Fig. 4; the pattern of allocation in accordance with concept of adverse selection is shown on Fig. 5.

At the same time, inefficient agents may aim to invest in improving the climate of the economy, in which they are currently working, more than efficient ones. At the same time obviously, low-yielding investors will be able to spend comparatively small free funds to improve the economic climate (if at all, they will have free funds). Therefore eventually they should hope only that the government will improve the economic climate by means of collected taxes.
Figure 4. The optimal allocation of investors in economies for the fixed profitability of economies (the computer simulation)

Figure 5. The optimal allocation of investors in economies according to the logic of adverse selection (computer modeling)
The analysis shows that in such case, the average productivity of the economy decrease over time, finally it reasonable to consider that it can lead to “closure” of economy and to the appearance of incomplete market as a form of market failure (Bator, 1958, Stiglitz, 1989).

Conclusion

Analysis of the model of investor behavior in economies allows to assert that the reasons for their “motley” distribution in economies, when less effective investors can get into a more favorable climate, and more effective - into less favorable, are not only separate institutional and random factors, they have a deeper behavioral ground. In turn, the fact that, under minimal constraints, rather rational agents' behavior leads to non-optimal decisions, even by in terms of profitability, shows that of the self-regulation of market of investment is incomplete in the outlined situation, and hence he need the additional regulation.
**Reference**


