# Research on Optimization Strategy of CPPI

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# Abstract

Economic globalization and financial market integration have increased fluctuation in financial markets. Investors expect an investment and wealth management product that can maintain the minimum protection when the price decreases, while it brings profits when the market grows. So the portfolio insurance strategy just meets the investors' requirements. The research analyzes the operation mechanism of CPPI, and then empirically tests the performance and risk of CPPI strategy based on the back-testing method.



**Keywords:** CPPI; operating mechanism; investment and wealth management


## Introduction

Investors encounter a variety of risks when investing in the financial markets, the key of risk control is that seeks ways to quantify risks. In other words, the meaning is how to quantify the risk, so that venture capital investors can more clearly understand the market they face. The rapid development of new financial derivatives makes the financial market more difficult to control, resulting in a sharp improvement in risk. It is important to investors and market regulators that grow the supervision of market risks and ensure the stable development of the market.

## CPPI's operating mechanism

The core idea of CPPI is that investors set a minimum acceptable level of portfolio value, to calculate the safety margin that goes beyond the bottom line and invest in risky assets with some amplification multiplier (Dichtl and Drobetz, 2010). In order to implement the CPPI strategy of portfolio insurance, investors must confirm the capital preservation bottom line $F$ firstly at any moment during the capital preservation cycle. This bottom line adds value to the risk-free return on assets and guarantees capital preservation after maturity (Dichtl and Drobetz, 2010). CPPI strategy portfolio consists of risk-free capital preservation assets $C$ such as government bonds and risky assets $R$ like equities. The margin of safety $N$ is between the portfolio value $V$ and the bottom line $F$ . CPPI strategy decision rule is that in the portfolio, the risk assets $R$ and safety margin $N$ maintain a constant ratio $m$ greater than 1, also known as amplification multiplier. Then constitute the following model:

$$V = R + C = N + F \tag{1}$$

$$R = \mathrm{m} \times (V \sim F) \tag{2}$$

The current capital preservation bottom line, which is based on the final capital reserve target for the portfolio (which is a percentage of the principal level, such as 90%), and a reasonable discount rate (according to the bond yield to maturity Kobe) can also be referred to as the value reference line(Louis and Xavier, 2010). For ensuring that the portfolio value is not lower than the capital preservation target due to frequent fluctuations, a risk mat is further designed on the basis of the capital

preservation. The risk mat is a certain proportion of the investment principal, which is generally within 5%. The minimum target value line for the portfolio is the sum of the capital preservation bottom line and the risk cushion, and the safety margin is the current portfolio value minus the portfolio minimum target value line, which represents the asset portfolio's ability to withstand impairment when the market fluctuates (Chen and Liao, 2007).

Moreover, the Portfolio Insurance CPPI strategy requires ensuring that the amplification multiplier is within the target's monitoring range. However, too frequent adjustments also imply huge transaction costs, for which reason dynamic adjustment rules of risk assets must be determined on a regular basis. For convenience of explanation, the following symbols are defined: $T$ is the CPPI strategy to protect the deadline; $R_t$ is the value of a risky asset at time $t$ ($t \in [0, T]$); $N_t$ is the safety offset of $t$; $m_t$ is the calculated amplification multiplier for $t$, $m_d$ is the lower bound for the calculation of the amplification multiplier, $m_u$ is the upper bound for the calculation of the amplification multiplier, $m$ is the standard amplification multiplier, and $C_t$ is the capital preservation value of $t$ (If $C_t$ is a negative cash position when the CPPI strategy allows borrowing, further assume the largest cash lending position is $C_d$).

During CPPI strategy $[0, T]$, on a certain dynamic day, the dynamic adjustment rules of risky assets ratio can be divided into the following four cases:

1. When $N_t > 0$, and $m_d \le m_t \le m_u$, it indicates that the proportion of risk assets within the scope of monitoring will not be adjusted. The value of the risky asset configured at this time is $R_t = N_t \times m_t$.

2. When $N_t > 0$, and $m_t > m_u$, it indicates that the proportion of risky assets is too large. At this point, the ratio of risky assets must be adjusted downwards to the standard amplification multiplier $m$ on the period of regular adjustment, and the adjusted risk asset value is $R_t = N_t \times m$.

3. When $N_t > 0$, and $m_t < m_d$, At this point, if $C_t > C_d$, should further reduce the capital preservation of assets and increase the risk-holding assets; if $C_t = C_d$, that has reached the maximum amount of borrowing, maintaining the number of original risk assets, will not be adjusted.

4. When $N_t \le 0$, the safety offset is 0 or negative, which is called a cash incident.

The occurrence of cash events is generally due to the sudden and drastic decline of the market, which cannot result in the timely liquidation of risky assets. For ensuring that the portfolio is above the capital preservation level at the end of the capital preservation period, when the safety margin is reduced to $N_t=0$, the risk assets in the investment portfolio must be cleared and allocated to the capital-guaranteed assets, with $R_t=0$.

In summary, the following proposition can be drawn.

Proposition 1: When the decrease of market causes the calculation of amplification multiplier to exceed the upper limit of the monitoring ratio, the CPPI strategy requires the reduction of holdings of risky assets and the preservation of assets; When calculated multiplier fall to the lower limit of the monitoring ratio because of the improvement of market, the CPPI strategy requires the holdings of risky assets to be capital-protected up to the maximum borrowing limit. Therefore, in essence, the portfolio insurance CPPI is a" more and more to buy, more or sell more" investment strategy.

## CPPI's payment function

Assume that a CPPI portfolio can invest in two types of assets: one is a money market account, denoted by $B$; the other is a tradable portfolio of risk assets, denoted by $S$, such as a composite stock index. The investment period is [0, $T$] and each investment strategy is self-financing. The value of $B$ risk-free asset obeys $dB_t=B_t rdt$.

Among them, r is the risk-free interest rate of continuous compound interest. The market value of a risk portfolio obeys the following classic stochastic diffusion process(Joossens and Schoutens, 2015):

$$dS_t=（udt+\alpha dW_t） \tag{3}$$

At any moment t, the CPPI strategy must ensure that the portfolio value is above the bottom line value $F_t$, $F_t$ provides the dynamically variable maturities, apparently $F_t$ obeys the following:

$$dF_t=F_t rdt \tag{4}$$

According to the operating mechanism of the portfolio CPPI strategy, $V_t=C_t+R_t$; where $C_t$ is capital preservation, $R_t$ is risky asset, and $R_t=mN_t$. Due to the

Taylor formula, the CPPI portfolio value at time t can be expressed as:

$$dV_t = (V_t - R_t)\frac{dF_t}{F_t} + R_t\frac{dS_t}{S_t} \tag{5}$$

And because of $V_t = F_t + N_t$, according to type (3), type (4) and type (5), the safety offset meets the following random process:

$$\begin{aligned} dN_t &= d(V_t - F_t) \\ &= (V_t - R_t)\frac{dF_t}{F_t} + R_t\frac{dS_t}{S_t} - dF_t \\ &= (N_t + F_t - mN_t)\frac{dF_t}{F_t} + mN_t - dF_t \\ &= N_t[(m(u-r)+r)dt + m\sigma dW_t] \end{aligned} \tag{6}$$

Obviously, $F_0$ must be less than $V_0$, and the difference between the two $V_0 - F_0$ is denoted by $N_0$. Based on formula (6):

$$N_t = N_0\exp[(m(u-r)+r-\frac{m^2\sigma^2}{2})t + m\sigma W_t] \tag{7}$$

From (3), the stochastic differential equation $S_t = S_0\exp[\sigma W_t + (u-\frac{\sigma^2}{2})t]$ of risk asset $S$ can be drawn as:

$$W_t = \frac{1}{\sigma}\ln(\frac{S_t}{S_0}) - (u-\frac{1}{2}\sigma^2)t \tag{8}$$

Substituting formula (8) into formula (7):

$$N_t = N_0(\frac{S_t}{S_0})^m\exp\{[r - m(r-\frac{\sigma 2}{2}) - m^2\frac{\sigma^2}{2}]t\} \tag{9}$$

Letting $\alpha_t = (\frac{N_0}{S_0{}^m})\exp[\beta t]$, $\beta = [r - m(r-\frac{\sigma^2}{2}) - m^2\frac{\sigma^2}{2}]$, according to $V_t = F_t + N_t$, from (4) and (9), the value of CPPI strategy portfolio at any time t in the investment interval [0, $T$] is a function of the amplification multipliers m and $S$ as follows:

$$V_t(m, S_t) = F_0e^{rt} + \alpha_t S_t^{\ m} \tag{10}$$

Generally, the portfolio insurance CPPI strategy has a capital preservation cycle of 1 year, a risk asset initial price of 100, a volatility of 20%, a capital preservation target of 100% of the principal, a risk-free interest rate of 5%, and the range of change in

maturity asset prices is 0 ~ 200. And CPPI standard amplification multipliers m respectively is 2, 4, 6 and 8(Joossens and Schoutens, 2010).

Proposition 2: Portfolio Insurance CPPI strategy effectively controls the downside risk in the market while also giving investors an opportunity to participate in an upside market.

Proposition 3: The payment curves of any two CPPI strategies intersect, and the payment curve of one CPPI strategy will not be completely above another curve. Therefore, there is no dominant CPPI strategy.

## CPPI strategy and its optimization of empirical research

The following report empirically tests CPPI strategy performance and its risk based on back-testing. At present, 55 surviving capital preservation funds have relatively short term of protection, usually 3 years. In order to examining the impact of the capital preservation term on the CPPI strategy, this paper selects two capital preservation periods: the first, the long-term portfolio, April 23, 2009 ~ December 24, 2014; the second, the short-term portfolio, 2014 July 1 ~ June 28, 2017 This paper selects the SSE 180 Index and the CSI 300 Index as risk assets for long-term and short-term portfolios respectively.

The main parameters of the retrospective test are as follows: (1) Guaranteed level, is generally ≤100%; (2) Risk mat, is generally ≤5%;(3) target multiplication factor, with normally ≤5 times, in order to reducing the cost of frequent adjustments to the asset portfolio, set the lower limit of amplification multiplier and calculate the upper limit of amplification multiplier respectively. There is no adjustment when the calculation of amplification multiplier is within this range, if not, adjust to the target amplification multiplier; (4) According to the relevant policies of margin financing and lending, the maximum loan ratio is set to 50%; (5) The dynamic adjustment interval is two weeks, which is every Friday of the second week. Dynamic monitoring and calculation of daily amplification multiplier, if the 5 consecutive working days to adjust the market to achieve a substantial trigger conditions for cash events, dynamic

adjustment mechanism can be activated in advance; (6) The CPPI strategy costs are as follows: the management fee is 1% and the hosting fee is 0.5%; (7) The main interest rates are as follows: the interest rate of short-term funds is 2.5%, and the term structure of capital preservation assets refers to Interest rate level of the inter-bank bond market; (8) Mandatory dividend or split clause. If the asset portfolio share is more than 1.5 yuan for two consecutive dynamic adjustment days, the manager can choose to dividend or divide to return the share face value back to $ 1 and start a new CPPI simulation; (9) After the cash incident, investors further have the following options: Fixed-income securities held with the remaining duration of the guaranteed period achieves the capital preservation target, Or the combined share of assets after the cash incident causes the share of the value of the restoration of 1 yuan and a new round of CPPI strategy simulation.

Table 1 shows the results of the short-cycle CPPI strategy back-testing. Among them, the target amplification multipliers are set to 3, 4, and 5 respectively. The risk mats for strategy 1 to strategy 3 are 3.9% and the level of principal protection is set to 100%; while the risk mat of Strategy 4 ~ Strategy 6 is 0, and the guaranteed level is set as 90%. During the sample period, the CSI 300 Index fluctuated from 2526 points to 2201 points with a gain of -12.59%. However, strategies 1 ~ 3 with relatively low risk levels all achieved the capital preservation successfully, while strategies 4 ~ 6 with relatively high risk levels also achieved 90% of the capital preservation targets.

Table 1 Short-cycle CPPI Strategy Back-testing Results (2014-07-01 ~ 2017-06-28)

| | Strategy 1 | Strategy 2 | Strategy 3 | Strategy 4 | Strategy 5 | Strategy 6 |
|---|---|---|---|---|---|---|
| Risk assets | CSI 300 Index | CSI 300 Index | CSI 300 Index | CSI 300 Index | CSI 300 Index0 | CSI 300 Index |
| Risk mats | 3.90% | 3.90% | 3.90% | 0 | 0 | 0 |
| Target magnification | 3.0 | 4.0 | 5.0 | 3.0 | 4.0 | 5.0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| multiplier | | | | | | |
| Amplify the lower limit of the multiplier | 2.5 | 3.5 | 4.5 | 2.5 | 3.5 | 4.5 |
| Enlarge the multiplier limit | 3.5 | 4.5 | 5.5 | 3.5 | 4.5 | 5.5 |
| Guaranteed level | 100% | 100% | 100% | 91% | 91% | 91% |
| CPPI revenue | 4.22% | 0 | 0 | -3.34% | -5.01% | -7.52% |
| CPPI Volatility | 6.99% | 8.91% | 10.95% | 12.90% | 15.17% | 17.11% |
| CSI 300 Index earnings | -12.59% | -12.59% | -12.59% | -12.59% | -12.59% | -12.59% |
| CSI 300 Index volatility | 20.61% | 20.61% | 20.61% | 20.61% | 20.61% | 20.61% |

As the stock market continued to fall, Strategy 2, Strategy 3, had a cash incident on January 2, 2017. Compared with the 20.61% volatility of the CSI 300 Index, the volatility of all CPPI strategies is significantly reduced. In particular, the volatility of Strategy 1 is only 6.99%(Schöttle and Werner, 2015).

The long-period CPPI strategy returns and volatility characteristics are shown in Table 2, in which the target amplification multiplier is set as 4, the upper and lower limit of amplification multipliers are calculated as 4.5 and 3.5, the risk cushion is 4% and the

guaranteed level is 100%. During the sample period, the SSE 180 index rose 41.17%, but it experienced a surge and then a sharp decline. This paper introduces the cash lending mechanism, mandatory dividend or disagreement clause and investors' cash event option mechanism while examine the impact of CPPI strategy optimization on performance and risk characteristics. Among them, Strategy 1 is the standard CPPI strategy; Strategies 2 and 5 introduce investors' cash event options; Based on this, strategies 3 and 6 also introduce a mandatory dividend or split clause strategy; the maximum loan ratio of strategy 4 to strategy 6 is 50% (Schöttle and Werner, 2015).

Table 2: Long-cycle CPPI Strategy Back-testing Results (2009-04-23 ~ 2014-12-24)

| | Strategy 1 | Strategy 2 | Strategy 3 | Strategy 4 | Strategy 5 | Strategy 6 |
|---|---|---|---|---|---|---|
| Risk assets | SSE 180 index | SSE 180 index | SSE 180 index | SSE 180 index | SSE 180 index | SSE 180 index |
| Risk mats | 4.00% | 4.00% | 4.00% | 0 | 0 | 0 |
| Target magnification multiplier | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 |
| Amplify the lower limit of the multiplier | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 |
| Enlarge the multiplier limit | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 | 4.5 |
| Guarantee | 100% | 100% | 100% | 100% | 100% | 100% |

| | | | | | | |
|---|---|---|---|---|---|---|
| d level | | | | | | |
| The maximum loan ratio | 0 | 0 | 0 | 50% | 50% | 50% |
| Mandatory dividend or split | No | No | Yes | No | No | Yes |
| Cash incident option | No | Yes | Yes | No | Yes | Yes |
| CPPI revenue | 0 | 42.05% | 169.85% | 0 | 46.24% | 188.25% |
| CPPI Volatility | 6.54% | 27.12% | 15.74% | 6.54% | 31.14% | 17.23% |
| SSE 180 Index earnings | 41.17% | 41.17% | 41.17% | 41.17% | 41.17% | 41.17% |
| SSE 180 Index volatility | 29.41% | 29.41% | 29.41% | 29.41% | 29.41% | 29.41% |

Table 2 shows that Strategy 1 had a cash event on January 18, 2012 with a simulated return of zero. Strategy 2 sets the investor option after a cash incident but does not set a mandatory dividend or split clause. For example, a cash event occurred on January 18, 2012 with simulated earnings of 42.05%. Strategy 3 not only has set a mandatory dividend or split clause, but also set the investor cash event option mechanism. There were four mandatory dividends or split events, including: October 20, 2010, July 5, 2012, January 31, 2013, October 17, 2013, and a cash incident occurred on February 13, 2014. The simulated revenue increased significantly to 169.85%. Compared with

CPPI strategies 1 ~ 3, after the introduction of the lending mechanism, the simulation benefits of strategies 4 ~ 6 have been improved, at the same time the volatility has also increased accordingly.

Back-testing results show that CPPI strategy volatility is significantly lower relative to the SSE 180 Index and the CSI 300 Index, except for the long-term CPPI strategy 5. In addition, optimizing the CPPI strategy through innovative mechanisms, such as the introduction of cash event option mechanism, can make full use of possible reversal opportunities after the market falls sharply (Moresi, et al. 2011); In a significantly rising market, the introduction of a mandatory dividend or split mechanism can lock in huge income earlier; The introduction of a cash lending mechanism will help to capture the higher returns from the market as it suffers from a higher level of risk.

## Conclusion

Portfolio Insights CPPI strategies not only enjoy high returns in the bull market cycle, but also achieve capital preservation goals in the bear market cycle where the market is either sharply or declining. CPPI strategy is a "more and more to buy, more or sell more" strategy in the magnification of the monitoring range relative to the risk of assets. Relative to the major investment benchmark of market, the improved CPPI strategy outperformed the passive index investment strategy during the simulation period, with less volatility.

The corrective measures adopted in this paper include: 1.compulsory dividend or split clause. In the increasing bull market, the proportion of investment in risky assets should be reduced; 2. Design investors' options after the cash-in event. It is the chance to capture a rebound after the market has dropped sharply; 3. Establish a cash lending mechanism. The reason is that improve the effectiveness of CPPI's strategy under greater risk.

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