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Does social capital explain the Solow residual? A DSGE approach

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Abstract

Social capital has been credited with playing a role in many desirable economic outcomes. We analyze how these potentially beneficial effects translate into the macro-performance of economies by developing a dynamic stochastic general equilibrium (DSGE) model featuring the role of social capital in the explanation of the Solow residual. We then simulate and estimate the model with Bayesian techniques using Italian data. Our framework fits actual data better than a standard DSGE model, suggesting that social capital may improve the economic performance via its impact on total factor productivity.

Keywords: social capital; total factor productivity; Solow residual; DSGE models.

JEL Classification: E12, E22, O11, Z1, Z13.

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1 Introduction

The literature has credited social capital with playing a role in many desirable outcomes such as loan repayment and access to credit (Karlan et al., 2009), investments in education (Coleman, 1988) and innovation (Knack and Keefer, 1997), the efficiency of job matching processes (Granovetter, 1973), the mitigation of agency problems in organizations (Costa and Kahn, 2003), political accountability and the performance of institutions (Putnam et al., 1993; Nannicini et al., 2013), just to name a few.

How do these effects translate into the macro-performance of economies? Based on increasingly refined identification strategies, empirical studies found evidence of a relationship between aspects of social capital - such as trust, networks and forms of prosocial behavior - and growth across countries (e.g. Knack and Keefer, 1997; Algan and Cahuc, 2010) or regions (e.g. Tabellini, 2010; Guiso et al., 2016). However, our knowledge of the mechanisms allowing the effects of social capital to result in a better economic performance and, in the long run, sustained growth is still limited.

In this paper we argue that social capital is a public good whose accumulation creates economy-wide externalities. This shared stock improves the economic and institutional environment within which individuals acquire their skills and firms accumulate means of production and produce output, resulting in a more efficient allocation of resources and a higher total factor productivity. Since trust enhances access to credit (Karlan, 2005; Feigenberg et al., 2013; Karlan et al., 2009), enrollment in higher education may be easier; at the firm level, higher credit opportunities might simplify the financing of innovative projects. The better performance and accountability of public institutions generally linked to social capital raises the enforceability of contracts (Rota et al., 2017) and the quality of public services, including publicly provided education, thereby creating a less uncertain environment for investment decisions (Knack and Keefer, 1997; Hall and Jones, 1999). The mitigation of agency problems typical of a more trusting society also plays a role in improving the management of human resources and lowers monitoring costs both in the workplace and in inter-firm relationships (La Porta et al., 1997; Costa and Kahn, 2003). Job placement mechanisms are further refined by the fact that, in high trust societies, hiring decisions are more likely to be influenced by talent and effort instead of the personal attributes of applicants, such as blood ties and personal knowledge - which are common surrogates of trustworthiness in low-trusting societies (Knack and Keefer, 1997; Alesina and La Ferrara, 2005; Zenou, 2015). As a result, social capital also increases the return to specialized and vocational educa-

tion, resulting in stronger incentives to invest in human capital (Knack and Keefer, 1997; Guiso et al., 2010; Coppier et al., 2018).

Overall, these mechanisms create a social infrastructure more favorable to high levels of output per worker, which, in the words of Hall and Jones (1999): “Gets the prices right so that individuals capture the social returns to their actions as private returns”.

We study the contribution of social capital to the economic performance by developing a dynamic stochastic general equilibrium (DSGE) model featuring the role of social capital in a constant returns to scale production function. We simulate and estimate the model using Bayesian techniques to match Italian data for total factor productivity over the period 1950-2014. Italy is a remarkable case study in the social capital literature for its well known regional differences in terms of cultural traits, trust, and civic engagement and because seminal studies have found evidence of a relationship between aspects of social capital and the economic development of the country (e.g. Putnam et al., 1993; Guiso et al., 2004; Felice, 2012; Guiso et al., 2016). We then compare the actual pattern of total factor productivity with the same time series simulated through the DSGE model in a benchmark case, i.e. not including social capital, and in our framework explicitly modeling the role of social capital.

The empirical analysis shows that accounting for the role of social capital allows an otherwise standard DSGE model to better fit actual data in the long run. This result suggests that the Solow residual may be credibly explained by the macroeconomic effects of social capital.

Our paper bridges two strands of literature. The first broadly studies the aggregate returns to social capital by empirically analyzing its correlation with growth rates across countries (Knack and Keefer, 1997; Isiche and Sawada, 2009; Algan and Cahuc, 2010; Anchorena and Anjos, 2015) or regions (Akçomak and ter Weel, 2009; Tabellini, 2010; Guiso et al., 2016), or with other macro outcomes, such as tax compliance (Andriani, 2016), the performance of institutions (Nannicini et al., 2013), and the size of the welfare state (Bjørnskov and Svendsen, 2013; Cerqueti et al., 2016).

The second strand encompasses macroeconomic studies investigating the possible determinants, in addition to technology, of the Solow residual, such as knowledge spillovers (Chang et al., 2016), entrepreneurial innovation (Grossman, 2009), sectoral composition (Casler and Gallatin, 1997), public health (Kelly, 2017), and the depletion of natural resources (Antoci et al., 2009; Antoci et al., 2011a).

We contribute to these strands of literature by developing, and empirically testing, the first DSGE model aimed at explaining the Solow residual as

a result of social capital's macroeconomic effects. Our findings suggest that the relationship between social capital and growth highlighted in empirical studies may be the result of a higher level of total factor productivity.

The rest of the paper is organized as follows. Section 2 describes the theoretical model. In Section 3 we describe our data, the methodology employed to estimate the parameters, the model's dynamics through the impulse response functions, and the related results. In section 4, we compare the performance of the different model's specifications through the measures of entropy. Section 5 concludes.

2 The model

The economy is populated by infinitely living households, who maximize the expected discounted value of an inter-temporal utility function, i.e.:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t(C_t, N_t) \quad (1)$$

with β^t corresponding to the subjective discount factor, C_t is private consumption and N_t are the hours worked, under the following inter-temporal budget constraint:

$$P_t C_t + K_t^p + K_t^s \leq R_t^p K_{t-1}^p + R_t^s K_{t-1}^s + W_t N_t \quad (2)$$

where K_{t-1}^p and K_{t-1}^s are the endowments of physical and social capital respectively at the beginning of time t , P_t is the consumer price index, W_t are nominal wages, and R_t^i ($i = s, p$) are the gross rates of return

$$R_t^i = r_t^i + 1 - \delta^i \quad (3)$$

with r_t^i ($i = s, p$) representing the net capital rentals and δ^i ($i = s, p$) the capital depreciation rates.

According to the seminal work of Becker (1974) and Bourdieu (1986), social capital is in part accumulated through rational investment decisions. Agents, in fact, also invest in the creation of connections and social obligations, and in building a reputation of trustworthiness, to the purpose of pursuing particular or general goals that could not be achieved without coordination and cooperation. The stock resulting from these decisions is a shared resource having the nature of a public good (Bourdieu (1986); Putnam et al., 1993; Dasgupta, 2001). In line with the extensive literature on

social capital and growth (see for example Knack and Keefer, 1997, and Antoci et al., 2011b), we assume this resource being an input of production. As suggested by Bourdieu (1986), agents can appropriate the outcomes of production to the extent of their personal or corporate wealth of social capital, which therefore determines the rental rate of this factor of production. In the words of Bourdieu, “The profits which derive from membership in a group are the basis of the solidarity which makes them possible” (Bourdieu, 1986, 250). Social capital, however, requires an endless and costly effort to produce and reproduce lasting relationships (Bourdieu, 1982; 1986). Like the other forms of capital, it is therefore subject to depreciation, as relationships, networks and trust can slacken over time as a result of the decline in social participation and of negative shocks (see for example Antoci et al., 2011b; Bilancini and D’Alessandro, 2012; Guriev and Melnikov, 2016).

Both physical and social capital, K_t^i ($i = s, p$), evolve according to the standard law of motion, i.e.:

$$K_t^i - K_{t-1}^i = I_t^i - \delta^i K_{t-1}^i \quad (4)$$

where I_t^i ($i = s, p$) are investments in social capital, achieved through social participation. The more actors engage socially and civically, the stronger, wider and more trust-intensive are the networks they create or belong to (Bourdieu, 1986).

Following Antoci et al. (2011b), we can think of I_t^s as the average level social participation. A higher social participation, in fact, provides individual and corporate actors with higher opportunities of creating new, trust-intensive, ties, and makes it easier to preserve the existing ones. If, by contrast, the social environment is poor of participation opportunities, the creation or new ties and the activation of existing, latent, relationships, require more time and effort, to the point that agents may have the incentive to replace social participation (e.g. civic engagement and the consumption of relational goods) with physical activities (Antoci et al., 2011b; 2015).

Private consumption C_t is defined as follows

$$C_t = \left[\int_0^1 C_t(j)^{1-\frac{1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (5)$$

with j representing the variety of goods produced by each firm acting as a monopolistic competitor, $C_t(j)$ is the consumption of the good $j \in [0; 1]$ and $\varepsilon > 1$ indicating the elasticity of substitution between differentiated goods.

The optimal allocation of expenditures across the households reads as

$$C_t(i) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t \quad (6)$$

with $P_t(j)$ representing the price of the good j implying that

$$\int_0^1 P_t(j) C_t(j) dj = P_t C_t \quad (7)$$

and

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (8)$$

We assume that the period utility function follows a semi-logarithmic form:

$$U(C_t, N_t) = \log(C_t) - \frac{N_t^{1+\gamma}}{1+\gamma} \quad (9)$$

where γ is the inverse of the Frish elasticity of labor supply.

The supply side of the economy is composed of a representative perfectly competitive final goods firm whose intermediate inputs are aggregated according to the following CES technology

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (10)$$

where $j \in [0; 1]$ is a continuum of firms, each one producing a different variety of intermediate good with the same constant returns to scale technology:

$$Y_t(j) = A_t [N_t(j)]^\zeta [K_{t-1}^p(j)]^\nu [K_{t-1}^s(j)]^{1-\zeta-\nu} \quad (11)$$

where $Y_t(j)$ is the production function of good j , $N_t(j)$, $K_{t-1}^p(j)$ and $K_{t-1}^s(j)$ are labor, physical and social capital employed in the productive process of good j , whereas A_t is a productivity shifter whose law of motion in logs reads as

$$a_t = \rho a_{t-1} + \epsilon_t^a \quad (12)$$

where $a_t = \log A_t$, $\rho \in [0, 1]$ is a persistence coefficient and ϵ_t^a is a white noise.

Moreover, each firm has a probability of resetting prices in any given period, $1-\theta$, independent across firms (staggered price setting, Calvo, 1983), with $\theta \in [0; 1]$, indicating an index of price stickiness.

The aggregate price dynamics reads as

$$P_t = \left[\theta (P_{t-1})^{1-\varepsilon} + (1-\theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (13)$$

with P_t^* indicating the prices resetted in period t ; inflation is positively related to the discounted expected value of the inflation of one period ahead

and to the log-deviation of real marginal cost according to the degree of price stickiness captured by the parameter¹ θ :

$$\pi_t = \beta\pi_{t+1}^e + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{mc}_t \quad (14)$$

The level of output associated to a fully-flexible price scenario is \bar{y}_t with the following corresponding log-linear definition of output gap

$$\tilde{y}_t = y_t - \bar{y}_t \quad (15)$$

The market clearing conditions for the good j can be expressed as follows:

$$Y_t(j) = C_t(j) \quad (16)$$

The previous relationship states that production of good j is allocated to private consumption.

Then, using the definitions of $C_t(j)$, equation (16) can be rewritten as follows:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t \quad (17)$$

Finally, by plugging (17) into the definition of the aggregate output (10) the aggregate market clearing condition is obtained:

$$Y_t = C_t \quad (18)$$

In the case of absence of social capital (i.e. the benchmark case compared to a scenario where social capital is present, in order to evaluate which of the two models better matches the time series of Solow residual), the previous

¹This expression is analytically derived in Appendix.

model collapses to the following equations:

$$\begin{aligned}
U(C_t, N_t) &= \log(C_t) - \frac{N_t^{1+\gamma}}{1+\gamma} & (19) \\
P_t C_t + K_t^p &\leq R_t^p K_{t-1}^p + W_t N_t \\
\int_0^1 P_t(j) C_t(j) dj &= P_t C_t \\
Y_t(j) &= A_t [N_t(j)]^\zeta [K_{t-1}^p(j)]^{1-\zeta} \\
a_t &= \rho a_{t-1} + \epsilon_t^a \\
Y_t &= \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \\
\pi_t &= \beta \pi_{t+1}^e + \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{m}c_t \\
Y_t &= C_t
\end{aligned}$$

3 Methodology, data and model dynamics

We empirically test the model using Italian annual data. Since the seminal study of Putnam et al. (1993), Italy is a typical case study in the social capital literature, due to its remarkable regional differences. Several studies have provided evidence of the role of social capital in the economic development of Italy (e.g. Felice, 2012; Guiso et al., 2004; Guiso et al., 2016).

In order to estimate the parameters, simulate the time series and evaluate their dynamic responses in the presence of the total factor productivity shock, we adopt the inferential procedure based on the Monte Carlo Markov Chains (MCMC) methods and, in particular, on the Metropolis-Hastings algorithm, which belongs to the family of Bayesian estimation methods (see among others Canova, 2007 and Smets and Wouters, 2007). In particular, we have built a multi-chain MCMC procedure based on four chains of size 100,000. The algorithm converges within 45,000 iterations to its expected value. Therefore, to remove any dependence from the initial conditions, we remove the first 45,000 observations from each chain. This high number of iterations, together with the 90% highest posterior density (HPD) credible interval for the estimates, ensures the robustness of our results². All the calculations have been performed through the software DYNARE.

²In detail, the estimation procedure is based on two steps. In the first, we have estimated the mode of the posterior distribution by maximizing the log posterior density function, which is a combination of the prior information on the structural parameters

Below, we summarize the measurement equation considered, i.e. the relationship between the data (left side) and the model variables (right side):

$$[\Delta \ln Y_t] = [\Upsilon] + 100 * [y_t - y_{t-1}] \quad (20)$$

where $\Delta \ln Y_t$ is the real GDP annual growth rate for Italy expressed in percentage terms from 1950 to 2014 drawn from Fred Economic Data, and $\Upsilon = 100 * \ln(v)$ is the annual real GDP trend growth rate, expressed in percentage terms.

We choose the real GDP growth rate as the observable variable due to its important informative role: in fact, real GDP growth encompasses both Solow residual and the contribution to growth linked to the productive factors.

The parameters and their definitions are shown in Table 1.

Table 1: Definitions of the parameters

Parameters	Definitions
ζ	output elasticity of labor
ν	output elasticity of private capital
δ^p	depreciation rate of private capital
δ^s	depreciation rate of social capital
γ	Frisch elasticity of labor supply
β	Inter-temporal discount factor
θ	Price stickiness
Υ	Annual real GDP growth rate
ρ	Persistence of total factor productivity
σ	Standard deviation of total factor productivity shock

The prior densities are consistent with the domain of the parameters. Following Del Negro and Schorfheide (2008), in the prior elicitation process we divided the parameters into three groups, on the basis of the information used to calibrate the priors.

The first group of parameters consists of those that determine the steady state $[\zeta, \nu, \delta^p, \delta^s]$ and whose calibration derives from macroeconomic ‘great

with the likelihood of the data. In the second, we have used the Metropolis-Hastings algorithm in order to draw a complete picture of the posterior distribution and compute the log marginal likelihood of the model. Moreover, following Brooks and Gelman (1998), we carried out the univariate convergence diagnostic based on a comparison between pooled and within MCMC moments, whose results are available upon request.

ratios' mainly referred to the sample information. In the second group there are parameters that are related to policy, households, production $[\gamma, \beta, \theta, \Upsilon]$, taken either from micro-level data or from the literature or from out-of-the-sample information. In the third group there are parameters describing the propagation mechanism of the stochastic shocks, such as standard deviations of them and autocorrelations $[\rho, \sigma]$. These last parameters are calibrated on the basis of the second moments of the observable variables, which are also consistent with the results found by the literature.

The calibrated values compared with the posterior ones are shown in Table 2.

The posterior values of the parameters are estimated using the observable variable (the real GDP annual growth rate) conditionally to the model. The posterior estimates of the parameters are composed of the posterior means together with the 90% HPD (Highest Posterior Density) credible interval for the estimated parameters obtained by the Metropolis-Hastings algorithm³.

Table 2: Prior and posterior distributions of the parameters

Parameters	Prior distribution			Posterior distribution		
	Distribution	Mean	St. Dev.	Mean	90% HPD interval	
ζ	beta	0.40	0.10	0.60	0.45	0.73
ν	beta	0.30	0.10	0.32	0.15	0.50
δ^p	beta	0.10	0.10	0.16	0.00	0.38
δ^s	beta	0.09	0.10	0.14	0.00	0.38
γ	gamma	3.00	0.75	2.55	1.32	3.67
β	beta	0.80	0.1	0.78	0.63	0.93
θ	beta	0.75	0.1	0.95	0.92	0.98
Υ	normal	1.55	0.1	1.53	1.36	1.68
ρ	beta	0.90	0.10	0.98	0.95	1.00
σ	inv. gamma	0.10	2.00	0.04	0.02	0.06

The elasticities of the production function (ζ, ν) have been calibrated considering the average share of wages and capital rentals on GDP for Italy from 1980 to 2011 (provided by the Italian National Institute of Statistics, Istat) with a small standard deviation; the posterior value of labor share is higher than the prior one, showing the relative importance of labor input

³We have increased the standard deviations of the prior distributions of the parameters by 50 percent in order to evaluate the sensitivity of the estimation results with the assumptions on prior estimates (Smets and Wouters, 2007). Overall, the estimation results are quite the same (results are available upon request).

in the Italian production function, whereas the posterior estimate of physical capital is almost the same than the calibrated value. The prior value of the depreciation rates for physical and social capital has been measured through the steady state ratio $\left(\delta = \frac{\bar{I}^i}{K^i}\right)$ ($i = s, p$): for physical capital we have used Italian data on investments and capital stocks from 1980 to 2011. To measure social capital we followed an established approach (see for example Guiso et al., 2004; Nannicini et al., 2013; Guriev and Melnikov, 2016) and relied on an indicator of the volume of blood donations as given by the number of 16-ounce blood bags collected per inhabitant provided by the Italian Association of Voluntary Blood Donors (AVIS), which collects 90 percent of the whole blood donations and 100 percent of anonymous blood donations. In particular, the parameter δ^s has been calibrated by considering the yearly blood donations from 1980 to 2011 as a measure of I_t^s , i.e. the average level social participation, and the accumulation over time of them as an indicator of social capital stock. The posterior values of δ^i are both higher than prior ones.

The prior value for the inverse of Frish elasticity of labor supply (γ) is able to match four empirical moments for the Italian data from 1980 to 2011 in accordance with Cho and Cooley (1994) and Argentiero and Bollino (2015): the ratio of standard deviation of total output to the standard deviation of total consumption, the correlation between total output and total consumption, the correlation between underground production and total consumption and the correlation between regular production and total consumption. The posterior value for γ is slightly lower than the prior one. The annual real GDP trend growth rate (Γ) is normally distributed, has been calibrated on Italian data with a prior mean of 1.55 that is almost the same of the posterior estimated value.

The price stickiness coefficient, i.e. the fraction of firms that does not reset its price in a period, is calibrated to a value of 0.75, following Galí and Monacelli (2008). The posterior value of this parameter is higher than the prior one, thus showing a higher degree of price stickiness for the Italian economy.

Following the real business cycle literature (see for example King and Rebelo, 1999) and the second moments of Italian total factor productivity data (provided by FRED Economic Data), we set a high value for the persistence coefficient of total factor productivity, which has also been confirmed by the estimation procedure, and a loose prior value for the standard deviation of the productivity shifter (σ).

The dynamic response of the main variables, in log-deviations from their

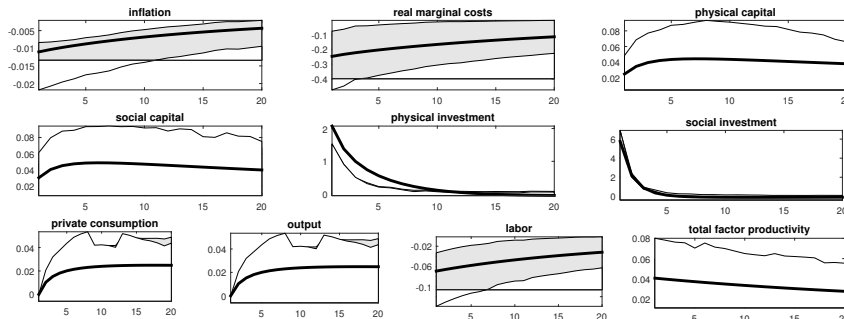


Figure 1: Impulse response functions for a positive productivity shock

steady state values, to stochastic shocks to total factor productivity is represented by impulse response functions (IRFs) in figure 1. Note that for all of the IRFs, the size of the standard deviations of the stochastic shocks and the variables' responses relate to the posterior average of the IRFs for each draw of the MCMC algorithm, together with 90% credible intervals.

In the aftermath of a positive technology shock, output increases, but less than the positive growth of total factor productivity. This stylized fact is consistent with the empirical findings of Galì (1999), Smets and Wouters (2003) and Galì and Monacelli (2008) according to which price stickiness determines an increase of aggregate demand (increase in private consumption) lower than the rise in supply. Hence, firms, due to the increased productivity, are able to produce the same quantity of goods with less hours worked, although capital stocks and investments increase due to the rise of capital rentals. Real marginal costs (mc) fall as well as inflation, but this last variable decrease less than in a fully flexible price scenario.

4 Analysis of the performance

In this section, we compare the empirical annual time series of Italian total factor productivity from 1950 to 2014 with the series obtained by implementing the DSGE model in the benchmark case (19) of absence of social capital (*benchmark series*, BS hereafter) and in presence of social capital (*social capital series*, SC hereafter). In doing so, we want to understand if adding social capital to the productivity function allows the model to better fit actual data.

A MCMC method has been used to generate the simulated time series

for the BS and SC models. The simulated series span the same period of the original sample with the same periodicity, to allow the comparison experiments. Thus, we have 65 years for the period 1950-2014. We consider 100,000 realizations of the random shocks described in the considered DSGE models (see section 2). Next, the expected value of all the simulations at each time has been taken, and this will be the corresponding values at each year. Therefore, the length of the original sample and of the two simulated series will be $n = 65$.

We denote by $x = (x_i)_{i=1,\dots,n}$, $b = (b_i)_{i=1,\dots,n}$ and $s = (s_i)_{i=1,\dots,n}$ the original sample, the series of type BS and SC, respectively.

To discuss the models, three strategies have been adopted. First, the distance between x and b has been compared with the one between x and s . The times of the realizations will be included in this part of the analysis, so that the concept of distance between two series will involve the contemporaneous realizations of the series. As we will see, several concepts of distance have been used, in order to obtain a satisfactory level of information from this procedure. Second, we adopt a data science perspective and discuss a rank-size analysis of the three series. In so doing, we are able to understand the possible presence of common regularities of the realizations of the three series when they are ranked in descending order. As a side analysis of data science type, the linear trends of the series have been also compared. Third, the empirical distributions of the three series have been considered and compared under the point of view of the descriptive statistics. In this framework, an entropy between the series distributions has been also taken into account.

4.1 Time series distance approach

The distances employed in the first approach are the Euclidean one, the maximum, the minimum and the Euclidean one. They are defined, respectively, as follows

$$d_M(x, y) = \max_{i=1,\dots,n} |x_i - y_i|, \quad (21)$$

$$d_m(x, y) = \min_{i=1,\dots,n} |x_i - y_i|, \quad (22)$$

$$d_E(x, y) = \frac{1}{n} \sum_{i=1}^n (x_i - y_i)^2, \quad (23)$$

where $y \in \{b, s\}$. The three concepts of distance are quite natural and jointly offer a panoramic view on how the original sample is close to the benchmark simulations or to the ones with social capital in a time-wise form.

Distance d	$d(x, s)$	$d(x, b)$
$d = d_M$	0,524	1,814
$d = d_m$	0	0
$d = d_E$	0,038	0,445

Table 3: Distances between the original sample x and the two competing simulated series b and s , according to formulas in (21), (22) and (23).

Results can be found in Table 3.

By looking at Table 3, it is clear that the model with social capital has a remarkably smaller distance to the empirical sample than the model without social capital. The average (Euclidean) distance $d_E(x, b)$ is more than eleven times greater than $d_E(x, s)$, while the maximum distance is more than three times bigger.

4.2 Data science approach

Time series are here viewed as collections of numbers. We aim at understanding whenever b and s share some regularity properties with x , and which one is closer to x in this respect.

The first step of this analysis is the assessment and the discussion of the linear trend of the three series. Time plays a relevant role, in that trend is intended on a temporal basis and allows to observe the overall behavior of the time series. To achieve our scopes, a simple linear regression has been implemented over the three series, according to equation

$$y = \alpha t + \beta, \tag{24}$$

with $y \in \{x, b, s\}$ and $t > 0$ represents time. α and β are the parameters to be calibrated, and represent the slope and the intercept, respectively.

Results can be find in Figures 2, 3, 4 and Table 4.

Some insights can be derived from the linear trend exploration. First of all, it is rather evident that one can hardly observe a reliable linear trend for b , while x and s exhibit a better looking linear regression. This is confirmed also by the values of R^2 , which are reported in Table 4. Notice also that the R^2 for the empirical case is around 60% and similar to that of SC, hence suggesting an analogous explanation power of the linear regression of the scatter plot. Moreover, both linear trends for x and s show an increasing behavior.

In the second step, a rank-size analysis approach has been adopted. The elements of the series have been ranked in decreasing order, so that $rank = 1$

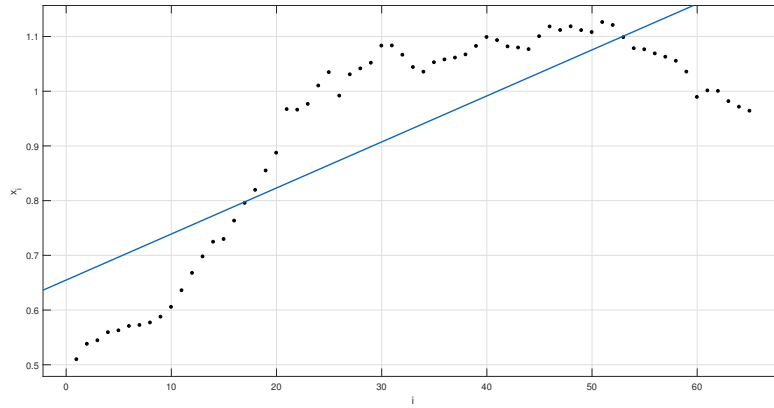


Figure 2: Linear trend for x . For a better visualization, the scatter plot is also presented.

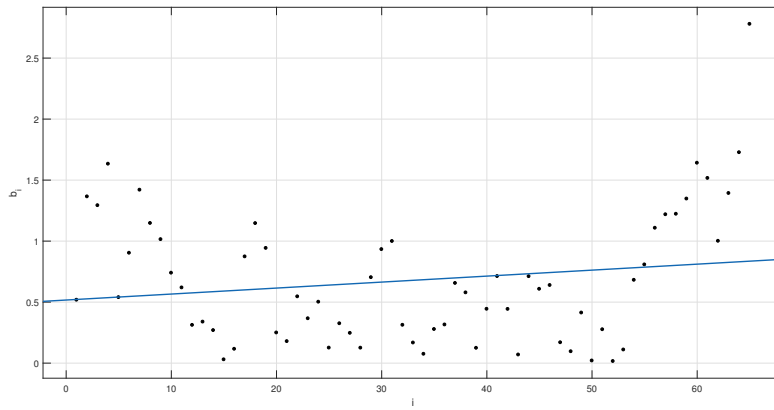


Figure 3: Linear trend for the benchmark series b . The scatter plot is juxtaposed to the best fit straight line.

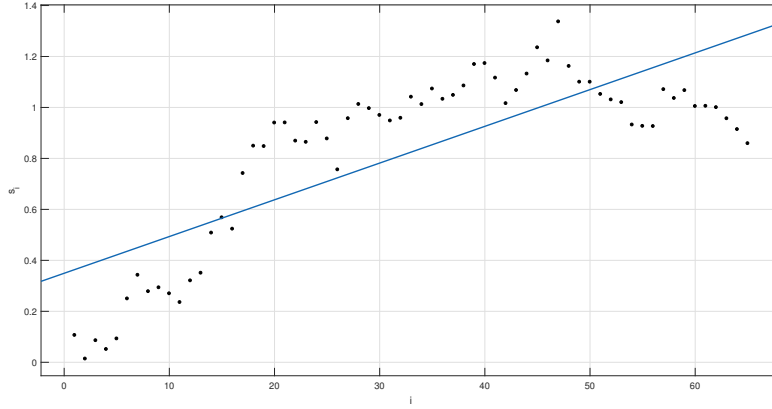


Figure 4: Linear trend for s . Also in this case, the scatter plot and the calibrated linear function are jointly shown.

Series y	$\hat{\alpha}$	$\hat{\beta}$	R^2
$y = x$	0.008415 (0.006849, 0.00998)	0.6549 (0.5954, 0.7143)	0.6468
$y = b$	0.0049 (-0.002205, 0.01201)	0.5171 (0.2474, 0.7869)	0.0293
$y = s$	0.01441 (0.01157, 0.01726)	0.3493 (0.2412, 0.4575)	0.6188

Table 4: The calibrated parameters $\hat{\alpha}$ and $\hat{\beta}$ of the linear regression exercise, according to formula (24), for the three cases of original sample, the benchmark series and the one with social capital. In brackets, the confidence interval at a 95% confidence level.

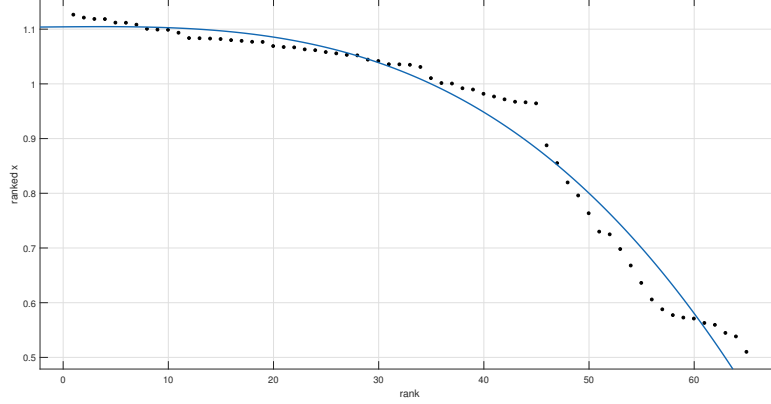


Figure 5: Rank-size best fit for x , according to formula (25), along with the scatter plot of the real data.

is associated to the largest value of the series while $rank = n$ is the smallest one. In so doing, the temporal dimension of the considered series is lost. The scatter plot of the series realizations with respect to $rank$ is then fitted with a decreasing curve $y = f(rank)$ belonging to a preselected parametric family of functions. The comparison of the calibrated parameters obtained for x , b and s say much about the similarities of BS and SC with the empirical sample.

By a preliminary visual inspection of the rank-size scatter plot, we here consider a third degree polynomial of the type

$$f(rank) = \gamma_3 \cdot rank^3 + \gamma_2 \cdot rank^2 + \gamma_1 \cdot rank + \gamma_0 \quad (25)$$

where $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are real parameters to be calibrated.

Figures 5, 6 and 7 allows a visual inspection of the best fit, which is rather satisfactory for the three cases. Such an idea is confirmed by looking at the goodness of fit R^2 , which is reported for completeness along with the calibrated parameters in Table ??.

Rank-size analysis provides some information about the closeness of b and s to x . Figures 5, 6 and 7 highlight that x and s show a similar shape in terms of concavity of the best fitted curve, hence suggesting a common behavior of the elements of the original series and the SC one when they are ranked in descending order. Differently with such series, the curve associated to BS is convex at high rank and exhibits an inflection point at a middle rank.

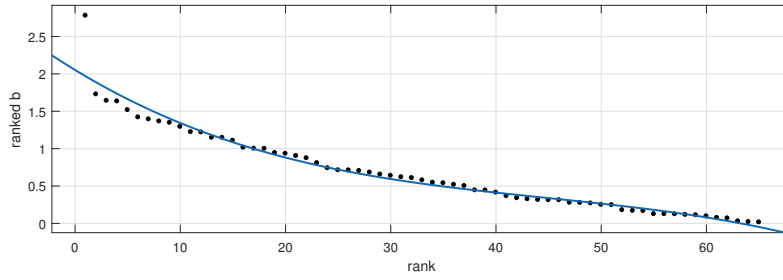


Figure 6: Rank-size best fit for b through function in (25) and related scatter plot.

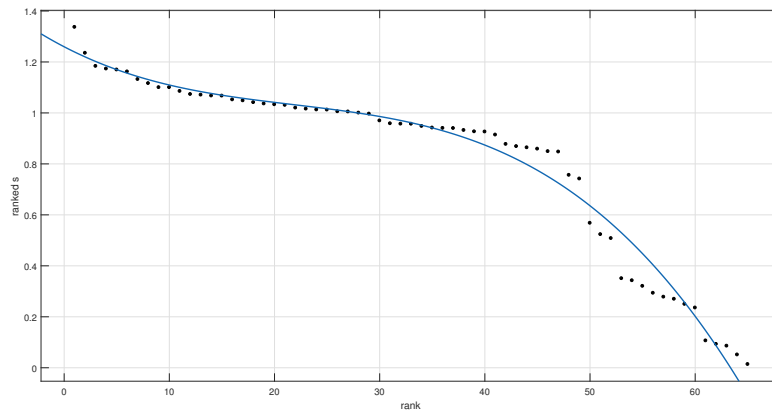


Figure 7: Rank-size best fit for s , obtained by using formula (25). The scatter plot is also shown for comparison purposes.

Table 5: The calibrated parameters $\hat{\gamma}$ and $\hat{\delta}$ of formula (25), for the three series. In brackets, the confidence bounds at 95%. The R^2 is also shown (last column).

Series y	$\hat{\gamma}_3$	$\hat{\gamma}_2$	$\hat{\gamma}_1$	$\hat{\gamma}_0$	R^2
$y = x$	-2.334e-06 (-3.948e-06, -7.198e-07)	-8.21e-06 (-0.0001702, 0.0001538)	0.0001615 (-0.004458, 0.004781)	1.104 (1.069, 1.14)	0.9723
$y = b$	-1.18e-05 (-1.754e-05, -6.069e-06)	0.001587 (0.001011, 0.002163)	-0.08558 (-0.102, -0.06917)	2.052 (1.926, 2.178)	0.9534
$y = s$	-1.156e-05 (-1.435e-05, -8.779e-06)	0.0007569 (0.0004776, 0.001036)	-0.02142 (-0.02939, -0.01345)	1.26 (1.198, 1.321)	0.9732

Table 6: Main statistical indicators associated to the three series x , b and s .

Statistical indicator	x	s	b
Mean μ	0,93	0,83	0,68
Variance σ^2	0,04	0,12	0,29
Standard deviation σ	0,20	0,35	0,54
Skewness	-1,01	-1,09	1,18
Kurtosis	-0,55	-0,09	2,07
Median	1,03	0,96	0,58
Max	1,13	1,34	2,78
Min	0,51	0,01	0,01

4.3 Empirical distribution approach

Time series are here discussed on the basis of their empirical distributions. As in the case of rank-size analysis, the time dimension is lost but a meaningful analysis of the macroscopic properties of the realizations can be carried out.

The main descriptive statistics are presented in Table 6.

By looking at Table 6, one can immediately argue that the series with social capital is much closer to the empirical sample than the series without social capital. Remarkably, skewness is negative and with very similar values for x and s while it is positive with a large value for b , hence leading to an evident violation of the symmetry property of the distributions when social capital does not intervene in the DSGE model. Analogously, kurtosis is negative for x and s and it is positive with a value close to three for b . This means that we are in presence of an original sample of platykurtic type – confirmed also for the series with the addition of social capital – while the case without social capital leads to a leptokurtic distribution. Values of x and s are much closer to the mean than the one of b (smaller standard deviations) and the distance between the means is lower for x and s than for x and b . Moreover, x and s seem to span analogous intervals (quite the same maxima and minima) while the maximum between x and b are noticeably different.

The distance between the distribution of x and those of b and s has been also measured by using entropy. Such a measure is suitable for our scopes, because it is able to capture the overall features of the distribution of the data under investigation. In this respect, entropy summarize in a unified setting the position and variability indicators given by the descriptive statistics.

Table 7: Computation of the entropy for the three series x , b and s , according to formula (26).

	x	s	b
Entropy	4,15	4,06	3,87

The considered entropy is given by:

$$\mathcal{E}(y) = - \sum_{i=1}^n \frac{|y_i|}{\sum_{k=1}^n |y_k|} \cdot \log \left(\frac{|y_i|}{\sum_{k=1}^n |y_k|} \right), \quad (26)$$

where $y = [y_i]_{i=1, \dots, n}$. We use formula (26) for the original sample $y = x$ and when $y = b$ and $y = s$. The reference entropy is the one associated to the original sample x . The model – BS or SC – which fits the empirical data in a more convincing way is the one whose entropy is closer to the one of x . The reasoning behind this evidence lies in the thermodynamic definition of entropy, which is nothing but the disorder associated to the series. Basically, the value of the entropy can be associated to the distance of the distribution from the uniform case. This suggests that similar entropies are associated to analogous macroscopic properties of the probabilistic structure of the data, hence leading to similar series.

Results are reported in Table 7.

The comparison between the entropies gives that both the simulated series underestimate the reference entropy of the original sample x . However, the entropy of s is much closer to the reference one than the entropy of b . This outcome goes in the same direction of what said by the analysis of the descriptive statistics, hence stating the supremacy of the DSGE model with social capital in capturing the real data with respect to the benchmark model without social capital.

5 Conclusions

In this paper, we developed a DSGE model to analyze the possible contribution of social capital in explaining the Solow residual. Our model fits the actual pattern of total factor productivity in Italy from 1950 to 2014 better than a standard DSGE model not featuring the role of social capital, suggesting that the latter can actually explain total factor productivity. This result supports the empirical approaches finding a positive relationship between social capital and the economic performance across countries.

However, our estimated model remains stylized and cannot shed light on the mechanisms through which social capital may increase productivity. Based on previous literature, we suggest the improvement in the efficiency of the credit and the labor market, the higher incentives to invest in human capital and innovation, and the mitigation of agency problems as possible channels of transmission. However, further research is needed for a deeper understanding of the action of social capital and for disentangling the effect of its several dimensions (e.g. networks and trust). In addition, a refinement in measurement methods and an extension of the analysis to other case studies is certainly desirable.

Having said that, our work provides the first attempt to explain productivity dynamics through a DSGE framework featuring the role of social capital, thus providing a new possible interpretation of the Solow residual that contributes to a vast literature at the intersection between social capital and productivity studies.

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6 Appendix

6.1 Equilibrium Characterization

6.1.1 Households

The (9) is maximized under (2) by using the method of Lagrange multipliers⁴, i.e.:

$$L = \max_{[C_t, N_t, K_t^p, K_t^s]_{t=0}^{\infty}} E \left[\sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{N_t^{1+\gamma}}{1+\gamma} \right) + \chi_t (R_t^p K_{t-1}^p + R_t^s K_{t-1}^s + W_t N_t - P_t C_t - K_t^p - K_t^s) \right] \quad (27)$$

where χ_t is the dynamic Lagrange multiplier, with the following three necessary conditions:

$$\frac{\partial L}{\partial C_t} : \frac{1}{(C_t) P_t} = \chi_t \quad (28)$$

$$\frac{\partial L}{\partial N_t} : \frac{N_t^\gamma}{W_t} = \chi_t \quad (29)$$

$$\frac{\partial L}{\partial K_t^p} : -\chi_t + \beta E_t [\chi_{t+1} R_t^p] = 0 \quad (30)$$

$$\frac{\partial L}{\partial K_t^s} : -\chi_t + \beta E_t [\chi_{t+1} R_t^s] = 0 \quad (31)$$

where (30) and (31) state that in equilibrium the value of marginal utility of consumption at time t is equal to the discounted expected value of marginal utility of consumption at time $t + 1$.

The following equation is a result of the combination of (28) and (29), i.e.:

$$N_t^\gamma (C_t) = \frac{W_t}{P_t} \quad (32)$$

The combination of (28) with (30) and (31) reads as:

$$\beta R_t^p E_t \left[\left(\frac{C_t}{C_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \quad (33)$$

$$\beta R_t^s E_t \left[\left(\frac{C_t}{C_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \quad (34)$$

The previous equations imply the following non arbitrage condition between the gross rates of return

$$R_t^p = R_t^s \quad (35)$$

⁴The use of dynamic programming technique would produce the same results.

that in steady state reads as

$$\overline{R}_t^p = \overline{R}_t^s = \frac{1}{\beta} \quad (36)$$

6.1.2 Firms

The profit maximization problem of the final good firm is:

$$\max_{Y_t(j)} P_t \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(j) Y_t(j) dj \quad (37)$$

with the following FOC for the variety of intermediate good j :

$$P_t \frac{\varepsilon}{\varepsilon-1} \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon-1}{\varepsilon} Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}-1} = P_t(i) \quad (38)$$

$$\left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{-\frac{\varepsilon}{\varepsilon-1}} Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} \quad (39)$$

from which is derived the corresponding relative demand for the good j

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \quad (40)$$

Given $(W_t, R_t^p, R_t^s)_{t=0}^\infty$, since the representative intermediate producers face a common price for the productive factors, each firm faces the following problem:

$$\min_{[N_t(j), K_{t-1}^p(j), K_{t-1}^s(j)]_{t=0}^\infty} - (W_t N_t(j) + R_t^p K_{t-1}^p(j) + R_t^s K_{t-1}^s(j)) + \quad (41)$$

$$+ \varphi(j) \left[\begin{aligned} & A_t [N_t(j)]^\zeta [K_{t-1}^p(j)]^\nu [K_{t-1}^s(j)]^{1-\zeta-\nu} + \\ & + (1-\delta^p) K_{t-1}^p(j) + \\ & + (1-\delta^s) K_{t-1}^s(j) - \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t \end{aligned} \right] \quad (42)$$

where the Lagrange multiplier $\varphi(j)$ is associated to the marginal costs.

The problem (41) yields to the following FOCs:

$$W_t = \varphi(j) \left[\zeta A_t [N_t(j)]^{\zeta-1} [K_{t-1}^p(j)]^\nu [K_{t-1}^s(j)]^{1-\zeta-\nu} \right] \quad (43)$$

$$R_t^p = \varphi(j) \left[\nu A_t [N_t(j)]^{\zeta-1} [K_{t-1}^p(j)]^{\nu-1} [K_{t-1}^s(j)]^{1-\zeta-\nu} + (1-\delta^p) \right] \quad (44)$$

$$R_t^s = \varphi(j) \left[\begin{aligned} & (1-\zeta-\nu) A_t [N_t(j)]^{\zeta-1} [K_{t-1}^p(j)]^{\nu-1} * \\ & * [K_{t-1}^s(j)]^{-\zeta-\nu} + (1-\delta^s) \end{aligned} \right] \quad (45)$$

from which an expression for the marginal costs MC_t can be derived

$$MC_t = \zeta^{-\zeta} \nu^{-\nu} (1 - \zeta - \nu)^{-(1-\zeta-\nu)} (W_t)^\zeta (R_t^p)^\nu (R_t^s)^{1-\zeta-\nu} \frac{1}{A_t} \quad (46)$$

From the expression of the aggregate price dynamics (13)

$$P_t = \left[\theta (P_{t-1})^{1-\varepsilon} + (1 - \theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (47)$$

the division of each member of (47) by P_{t-1} reads as

$$\Pi_t^{1-\varepsilon} = \theta + (1 - \theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\varepsilon} \quad (48)$$

The log-linearization of (48) around zero inflation steady state produces the following equivalent results

$$\pi_t = (1 - \theta) (p_t^* - p_{t-1}) \quad (49)$$

$$p_t = \theta p_{t-1} + (1 - \theta) p_t^* \quad (50)$$

A firm in period t chooses a price P_t^* that maximizes the current market value of the profits Υ_t , i.e.

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} (P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})) \} \quad (51)$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} \quad (52)$$

for $k = 0, 1, 2, \dots$ and where $Q_{t,t+k} = \beta^k (C_{t+k}/C_t) (P_t/P_{t+k})$ is the discount factor, $\Psi_t(\cdot)$ is the cost function of the firm, whereas $Y_{t+k|t}$ represents output in period $t+k$ for a firm resetting its price in period t . Next, the first order condition associated with the problem (51) is given by:

$$\sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} Y_{t+k|t} (P_t^* - M \psi_{t+k|t}) \} = 0 \quad (53)$$

where $\psi_{t+k|t} = \Psi'_{t+k}(Y_{t+k|t})$ indicates the nominal marginal cost in period $t+k$ for a firm resetting its price in period t and $M = \frac{\varepsilon}{\varepsilon-1}$ that is the desired markup in the absence of constraints on the frequency of price

adjustment. Note that in the absence of price rigidities ($\theta = 0$) the previous condition collapses to the optimal price setting condition under flexible prices:

$$P_t^* = M\psi_{t|t} \quad (54)$$

Then, the division of both the members of (53) by P_{t-1}^i reads as:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - M * MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0 \quad (55)$$

where $MC_{t+k|t} = \frac{\psi_{t+k|t}}{P_{t+k}}$ is the real marginal cost in period $t+k$ for firms whose last price set is in period t .

Finally, the log-linearization of (55) around the zero inflation steady state with a first-order Taylor expansion reads as

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})] \quad (56)$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - \overline{mc}$ is the log-deviation of marginal cost from its steady state value.

The optimal price setting strategy for the typical firm resetting its price in period t can be derived from (56), after some algebra:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [mc_{t+k|t} + p_{t+k}] \quad (57)$$

with $\mu = \log \frac{\varepsilon}{\varepsilon-1}$ representing the optimal markup in the absence of constraints on the frequency of price adjustment ($\theta = 0$).

Hence, the price setting rule for the firms resetting their prices is represented by a charge over the optimal markup in the presence of fully flexible prices, given by a weighted average of their current and expected nominal marginal costs, with the weights being proportional to the probability of the price remaining effective $(\theta)^k$.

Note that, under the hypothesis of constant returns to scale, implicit in the production function of our model, the marginal cost is independent from the level of production, i.e. $mc_{t+k|t} = mc_{t+k}$ and, hence, common across firms; so, the expression (57) can be rewritten in the following way:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [mc_{t+k}] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t [p_{t+k}] \quad (58)$$

Moreover, the equation (58) can be expressed in the following recursive form:

$$p_t^* - p_{t-1} = \beta\theta E_t [p_{t+1}^*] - (1 - \beta\theta) p_t + (1 - \beta\theta) \widehat{m}c_t \quad (59)$$

and combined with (49) in a log-linear form in order to obtain the domestic inflation equation:

$$\pi_t = \beta\pi_{t+1}^e + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \widehat{m}c_t \quad (60)$$

with $\pi_{t+1}^e = E_t [\pi_{t+1}]$.