The Costly Quest for a Better Price - A Model of Search and Information Costs

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30 June 2017

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Abstract

In many markets, consumers obtain price quotes before making purchases. This paper considers a fixed-sample size model of consumer search for price quotes when sellers must spend resources to learn the costs of providing goods/services. It is found that (1) even with ex ante identical consumers and sellers, there is price dispersion in the equilibrium; (2) despite price dispersion and zero search costs, it may be optimal to search just two sellers; (3) the optimal number of searches can increase with sellers' information costs; (4) the expected equilibrium price can decrease with consumers' search costs. (JEL D40, L00)

Keywords: Price dispersion, Precontract cost, Search cost, Sealed-bid Auction.

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In order to make informed purchase decisions, consumers search. In order to earn their business, sellers provide relevant information such as prices. The standard economic models of consumer search assume that price search is costly, but price setting is costless.[1] In many markets, however, even a simple price quote may involve nontrivial costs for the seller. For example, a repair shop must diagnose the problem before giving a cost estimate; a mortgage lender must evaluate a borrower's creditworthiness before issuing a rate quote; an insurance agent must assess an applicant's risk characteristics before determining the premium; a travel agent has to compare different itineraries and consult with multiple airlines before offering an airfare deal. In these markets, production costs depend on consumers' individual needs. Sellers set prices after consumer search takes place. The search process works much like a first-price sealed-bid procurement auction: each seller submits a bid without knowing his competitors' bids and the consumer pays a price equal to the lowest bid. A consumer can canvass a number of sellers, but cannot contract on any seller's effort in preparing the price quote.

This paper incorporates the above features into a model of consumer search to study the market impact of precontract costs, including consumer search cost and price setting cost. The latter cost is due to uncertainty in the production cost. As such, we call it information cost or diagnosis cost. For concreteness, consider a homeowner who asks contractors for price quotes on a repair job.[2] A contractor may send an unskilled worker or a skilled worker. Sending the skilled worker gives an accurate diagnosis, but is costly and potentially duplicative. The homeowner cannot distinguish the skilled from the unskilled worker. If the contractor expects the homeowner to do comparison shopping and canvass a large number of contractors, then he will be less inclined to send the skilled worker. In the equilibrium, the expected profits net of diagnosis costs from sending the skilled worker must be the same as the profits from sending the unskilled worker. This means that diagnosis costs will be built into the final price quotes. While this observation suggests an equivalence between consumer search and contractor diagnosis — both are paid by the consumer — our analysis of the contractor's problem reveals an important difference: the only way to save on total search costs is to search less, but paradoxically a consumer can save on total diagnosis

[1] It is a long tradition that began with Stigler's seminal paper (1961). More recently, a class of search models with an "information clearinghouse" assume nearly the opposite, that is, zero (marginal) search cost but positive (fixed) advertising cost (Baye and Morgan 2001).

[2] This example is adapted from Pesendorfer and Wolinsky (2003). Their original example assumes that the costs of performing any potential services are identical, but the outcomes can be different. Our example assumes the opposite.

costs by searching more. The latter is true if the contractors' willingness to incur diagnosis costs drops sharply when they face more competitors. Consequently, information costs and search costs can have different, even opposite, impacts on consumer search behavior. For example, the optimal number of searches is two when information costs are zero, but can be infinity when search costs are zero. More generally, while the optimal number of searches always *decreases* with search costs, it can sometimes *increase* with information costs. At the same time, price dispersion persists even with ex ante identical consumers and sellers, and even if search is costless.

The introduction of price setting costs also affects the timing of the game, which raises a commitment issue that has not been previously investigated. When prices are set upon consumer request, the game between the consumer and sellers can either be sequential in nature, or simultaneous, depending on whether the consumer can commit. If a consumer can precommit to the number of searches before sellers submit their bids, then the number of searches will be optimal, but this is not always the case. Holding constant the sellers' pricing strategies, a searcher always benefits from sampling a larger size, and therefore she cannot make a credible commitment when the number of price quotes is private information. In this setting, we find that the expected price can decrease with search costs. Moreover, consumers may engage in excessive search that is detrimental to their own welfare.

This paper contributes to the understanding of transaction costs. Dahlman (1979) classifies transaction costs into three categories based on the stages of a contract: search and information costs (precontract), bargaining and decision costs (contract), policing and enforcement costs (postcontract). While the impact of consumer search costs on market outcome has been extensively studied,[3] its interaction with sellers' information costs has so far received scant attention. A notable exception is French and McKormick (1984), whose informal analysis of the service market anticipates many of the themes explored in this paper. After showing that the winner's expected profit equals the sum of his competitors' sunk costs of bid preparation under a free-entry condition, they argue that consumers indirectly pay for sellers' information costs. The focus of their paper,

[3] See Anderson and Renault (2017) for a recent survey of consumer search theories and Baye, Morgan and Scholten (2006) for an extensive review of the search literature that focuses on price dispersion.

however, is on sellers' marketing strategies, such as how likely sellers charge for their estimates or advertise, whereas the focus of this paper is on the problem faced by the consumer side.[4]

To the best of our knowledge, Pesendorfer and Wolinsky (2003) and Wolinsky (2005) are the only other papers that consider consumer search in the presence of information costs. By assuming that repair outcomes are not contractible (but price searches are costless), Pesendorfer and Wolinsky (2003) examine market inefficiencies when a consumer must rely on second opinions to pick the right contractor. Under the assumption that sellers can provide better matching via costly investments, Wolinsky (2005) shows that consumers' inability to internalize sellers' costs leads to excessive search. Despite the similarity, these two papers have a different focus than ours: they are concerned with prior information on product characteristics (so there is no price dispersion), whereas our paper is concerned with prior information on prices. Because of this difference, most of our results on prices do not exist in their models.[5]

The remainder of this paper is organized as follows. Section 1 introduces the model and discusses assumptions. Section 2 presents some preliminary results. Section 3 analyzes equilibrium properties and shows our main results. Section 4 concludes. Any formal proofs omitted from the main text are contained in the appendix.

1 The Model

A consumer is willing to pay v for a good or service (henceforth, the product), which can be provided by any one of the N sellers. In order to find the best deal, the consumer visits sellers to collect price quotes. The cost of effort for each visit is s, i.e., the search cost. Sellers face the same production cost, but it can take two values: a low cost of c_l with a probability of q or a high cost of c_h with a probability of $1 - q$. A seller can acquire information to learn the actual production cost at a cost of t, i.e., the information cost. Sellers are risk neutral. The values of all above variables are assumed to be common knowledge.

The game is played in the following order:

[4] Due to the lack of formal game-theoretic analysis, the connection between assumptions and results is somewhat opaque in their paper. For example, it is not clear whether the predicted pattern is the result of collective behavior among sellers or the noncooperative outcome.

[5] For tractability, these two models make two assumptions that are somewhat unrealistic: (1) prices are set before diagnoses; (2) search costs are not paid until a contract offer is accepted.

1. The consumer requests prices quotes from n sellers;

2. Each seller, upon request, chooses whether to incur t to acquire information about the production cost: if a seller acquires information, it quotes a price from the distribution of either $F_l(p)$ or $F_h(p)$, depending on whether the cost is c_l or c_h; if a seller does not acquire information, it quotes a price from the distribution of $F_b(p)$.

3. After receiving all price quotes, the consumer buys from the seller that offers the lowest price.

A complete specification of the game also requires us to specify whether n belongs to the sellers' information set when they compete at stage 2. If n is publicly observable, then the consumer can act as a Stackelberg leader and commit to the optimal number of searches; but if n is non-contractible private information, then the consumer and sellers will be playing a simultaneous move game. Both possibilities will be considered.

We focus on symmetric equilibria, in which sellers choose the same bidding strategy, and restrict our attention to equilibria in which the consumer uses a pure strategy, i.e., the consumer does not randomize over her number of searches. It is also not difficult to see that $F_h(p)$ is a degenerate distribution, where $F_h(p) = 0$ for $p < c_h$ and $F_h(p) = 1$ for $p \geq c_h$. Therefore, the equilibrium will be characterized by the consumer's choice of n and sellers' strategies in a triplet $\{\alpha, F_l(p), F_b(p)\}$, where α is the probability of a seller choosing to acquire information about the production cost, $F_l(p)$ is the cumulative distribution function of price quote when a seller learns that the production cost is low, and $F_b(p)$ is the distribution function when a seller submits a "blind" quote. Sellers use pure strategies in pricing if and only if price distributions are degenerate. The solution concept is a Bayesian Nash equilibrium. The consumer and sellers form beliefs about each other's strategies. In the equilibrium, their beliefs are correct.

For ease of exposition, we impose the following restrictions on parameter values. First, we assume that $s << v$ so that it is never optimal for the consumer to search just one seller, in which case she minimizes the total search costs but has to pay a monopoly price for the product. Second, we assume that the pool of sellers is so large that the consumer's number of searches is never constrained by the number of sellers. These restrictions cut down the number of cases we have to consider and allow us to focus on only the nontrivial cases, but they do not change any of the qualitative results.

1.1 Discussion of Assumptions

In order to keep the model tractable and have a stark contrast with the existing literature, this paper has introduced a very simple model that focuses on only the most important aspects of the markets and assumes away some of the admittedly more realistic features. Here we discuss the key assumptions and justify the use of some simplifying assumptions.

Homogenous Sellers At the last stage of the game, the consumer choice is based on price only. This is a common assumption in the consumer search literature. At the same time, it is a natural assumption given the paper's focus on the transaction costs at the precontract stage. After all, without having to incur additional transaction costs at later stages, sellers can offer complete contracts that cover all aspects of the product, including design, quality, warranty, price, etc. This means that we can collapse all these variables into a single one — "price" — and assume all other aspects of the product to be uniform across sellers. Essentially, sellers can be viewed as competing in utility space.

At the bidding stage, the game is modeled as a common value procurement auction since sellers are assumed to be ex ante identical. This may not be a realistic assumption: sellers' costs are likely correlated, but not necessarily the same. Nonetheless, we make this assumption to ensure that equilibrium price dispersion cannot be attributed to different cost realizations across sellers.

The assumption of homogenous sellers is also consistent with empirical research in related markets. For example, in her study of the auto insurance market, Honka (2014) assumes that the only search dimension is the premium charged by each provider. Moreover, she reports that over 93% of consumers kept their coverage choice the same during the last shopping occasion and were searching only for the lowest premium. Similarly, in their study of the Canadian mortgage market, Allen, Clark and Houde (2014) find that contracts are homogeneous, and for a given consumer costs are mostly common across lenders due to loan securitization and a government insurance program.

Uncertainty in Production Cost The production cost is assumed to be a binary random variable for tractability. Admittedly, this is a crude assumption, but it is in line with the home repair example. For other markets, it is also relevant: a mortgage broker or an insurance agent's precontract effort involves identifying the variety of discounts available to an applicant based on her risk characteristics; a travel agent or a car dealer's effort involves finding out the existence of airline promotions or manufacturer incentives. The same assumption is also used by Pesendorfer and Wolinsky (2003). As for the source of the cost uncertainty, it can be internal, (i.e., consumer specific), or external, (e.g., due to input cost variations).

Fixed-sample Size Search There are two basic types of models used in the search literature: fixed-sample size search models assume that consumers sample a fixed number of sellers and choose to buy the lowest priced alternative,[6] whereas sequential search models assume that consumers visit sellers one-by-one and do not stop searching until their reservation prices are met.

There are several reasons why we assume fixed-sample size search. First, existing empirical evidence suggests that fixed-sample size search provides a more accurate description of observed consumer search behavior (De los Santos, Hortacsu, and Wildenbeest 2012, Honka and Chintagunta 2017); second, fixed-sample size search strategies can be optimal when there is a fixed-cost component to search (Hong and Shum 2006). Recall the example of a homeowner in need of repair cost estimates: if she has to take a day off from work for the visits of the contractors, then it will be more efficient to schedule the visits such that they all take place on the same day than to follow a prolonged process from sequential searches. Third, but particularly relevant to this model, costly diagnosis can lead to delay and delay is a more significant problem for sequential search than for fixed-sample size search.[7]

It should also be noted that there is a subtle difference between the commitment problem behind the common criticism against the fixed-sample size approach and the commitment problem highlighted in this paper. The former is about a consumer's inability to commit to the fixed-sample size *strategy* itself (Rothschild 1974), but the latter is about the consumer's inability to commit to the optimal *number* of searches. In economic environments where fixed-sample size search is more

[6] MacMinn (1980) is the first to show that equilibrium price dispersion can arise when consumers engage in fixed-sample size search, but his result relies on cost heterogeneity among sellers. Burdett and Judd (1983) provides a model of equilibrium price dispersion with ex ante identical consumers and sellers. In the equilibrium price distribution, all sellers charge positive markups. A fraction of consumers visit one store and purchase, while the remaining fraction of consumers search two stores and buy from whichever offers the lower price. Janssen and Moraga-Gonzalez (2004) develop an oligopolistic version of Burdett and Judd (1983) where some consumers search costlessly. They show that the equilibrium expected price may be constant, increasing or non-monotonic in the number of sellers, depending on the equilibrium consumers' search intensity and the existing number of sellers. In particular, they find that duopoly yields identical expected price and price dispersion but higher welfare than an infinite number of sellers.

[7] In the same vein, Morgan and Manning (1985) and Janssen and Moraga-Gonzalez (2004) argue that fixed-sample size search is more appealing when a consumer needs to gather price information quickly.

advantageous than sequential search, the issue of commitment to strategy does not bite, but the issue of commitment to the number of searches remains.

Estimation Fee A problem highlighted in this paper is that sellers cannot be compensated directly for their precontract efforts. One may wonder whether charging consumers an estimation fee will solve the problem. In fact, a large part of French and McKormick's discussion centers around the use of an estimation fee to recoup the costs of sellers' efforts. There are two reasons why the current model does not include an estimation fee. First, as emphasized by Pesendorfer and Wolinsky (2003), the diagnosis and estimation of the repair cost is a type of "credence" service — the sellers' diagnosis effort and outcome are unobservable and thus incontractible. Second, French and McKormick's discussion is based on the assumption that a repair shop has some captive consumers — due to prohibitively high search costs — who will choose to pay the estimation fee even if other repair shops can provide costless estimates.[8] In the current model, as long as estimation fees are public, all sellers will end up charging zero estimation fees (or a fee equal to the diagnosis cost of an unskilled worker). The same result, based on the standard Bertrand style argument, is obtained in Pesendorfer and Wolinsky (2003) and Wolinsky (2005).

2 Preliminary Results

2.1 A Benchmark Result

A useful point of departure is to consider what happens if $t = 0$, i.e., sellers can costlessly learn their production costs. This is not only the assumption of a frictionless market, but also the working assumption of almost all consumer search models. In this case, based on the standard Bertrand style argument, we can see that the prices will be set equal to the (realized) production cost as long as there are at least two sellers. A consumer cannot do better by visiting more sellers because she cannot get a better offer, nor can she do better by visiting just one seller, who will charge a monopoly price. Therefore, it is sufficient for the consumer to visit just two sellers to obtain competitive price quotes while economizing on the search costs. The consumer earns a surplus of

[8] One of their remarks - shops in low-income neighborhoods are more likely to charge for estimates because of potential customers' lower search costs - appears to contradict this assumption.

$v - c_E - 2s$, where $c_E = qc_l + (1-q)\,c_h$ is the expected production cost. This serves as a natural benchmark for the current analysis.

Zero information cost, however, is not a necessary condition for the above benchmark outcome. The same outcome can be obtained even if the information cost is positive. It is not difficult to see why: acquiring information about the production cost allows sellers to earn information rents, but it is wasteful from the consumer's point of view. If she and sellers can contract on the latter's information acquisition efforts, then the consumer can prevent sellers from earning information rents by simply requiring sellers not to acquire product cost information.[9] The sellers will again compete *a la* Bertrand, with each of them quoting a price of c_E and earning zero profits. The consumer earns the same amount of expected surplus as in the benchmark. Albeit straightforward, this result demonstrates that the information cost, in itself, does not necessarily cause welfare loss for the consumer. Rather, any loss of efficiency is due to the inability to contract on sellers' information acquisition efforts. Of course, if the cost of effort is so high that it exceeds the private value of information, which equals $q\,(1-q)\,(c_h - c_l)$, then neither seller will make an effort even if they are not contractually precluded from doing so.

In all above cases, consumer surplus is maximized when the consumer obtains two competing price quotes. There is no further gain from requesting additional price quotes.[10] Therefore, it does not matter whether the consumer can commit. Accordingly, we summarize the results in the following proposition:

Proposition 1 *Regardless of whether the number of price quotes is publicly observable or non-contractible private information, consumer surplus is maximized in the unique equilibrium if either (i) $t = 0$; or (ii) $t \geq q\,(1-q)\,(c_h - c_l)$, or (iii) the consumer and sellers can contract on the information acquisition effort. In all three cases, the optimal number of searches is two.*

[9] This requirement may not work if the consumer has private information, which can potentially lead to an adverse selection problem. Here we assume away consumer private information on the ground that sellers have more expertise than consumers in the relevant markets.

[10] In the case of private information, there will be savings from requesting one fewer price quote so the consumer might randomize between searching once and twice, giving rise to the same result as in Burdett and Judd (1983), but this possibility is ruled out here by the assumption that the consumer uses a pure strategy (in the number of searches).

Interestingly, Proposition 1 also shows that the optimal number of searches is the same for extreme values of t. This suggests that the optimal number of searches may not be monotonic in t, an observation that will be confirmed later in the analysis.

2.2 The Subgame Equilibrium at the "Bidding" Stage

For the remainder of the paper, we focus on the more interesting case, in which the cost of information acquisition is small but positive, i.e., $t \in (0, q(1-q)(c_h - c_l))$. In the (stage 2) subgame, n sellers receive requests for price quotes. When deciding what price to quote, these sellers face a problem similar to a sealed-bid common value auction: each seller chooses whether or not to acquire production cost information before submitting a bid without knowing his competitors' bids. This is not a trivial problem, because a pure strategy symmetric equilibrium does not exist.

Lemma 1 *If $t \in (0, q(1-q)(c_h - c_l))$, no pure strategy equilibrium exists at the "bidding" stage in which* all *sellers make an effort to learn the true cost.*

Lemma 2 *If $t \in (0, q(1-q)(c_h - c_l))$, no pure strategy equilibrium exists at the "bidding" stage in which* no *seller makes an effort to learn the true cost.*

Lemma 1 and 2 imply that a symmetric equilibrium in the subgame at the bidding stage must be in mixed strategies. It involves two randomizations for sellers: first, sellers randomize between submitting an informed quote and submitting a blind quote; second, sellers randomize their price quotes. The solution is given by Lemma 3.

Lemma 3 *If $t \in (0, q(1-q)(c_h - c_l))$, each seller chooses to acquire information about the production cost with a probability of $\alpha = 1 - \left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)^{1/(n-1)}$. If a seller learns that the production cost is c_h, he quotes a price of c_h; otherwise he quotes a price according to the distribution of $F_l(p) = \frac{1-\left(\frac{t}{q(p-c_l)}\right)^{1/(n-1)}}{\alpha}$, with $\underline{p_l} = c_l + t/q$ and $\bar{p}_l = \frac{(1-q)c_h + q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. If a seller does not acquire information, he quotes a price according to the price distribution of $F_b(p) = 1 - \frac{\alpha}{1-\alpha}\left(\left(\frac{q(p-c_l)}{(1-q)(c_h-p)}\right)^{\frac{1}{n-1}} - 1\right)^{-1}$, with $\underline{p_b} = \bar{p}_l$ and $\bar{p}_b = c_h$. The expected price is $c_E + n\alpha t$.*

Figure 1 illustrates how the information cost, t, affects price distributions. The red solid curve depicts the price distribution when $t = 0.2(c_h - c_l)$ and the black dashed curve when

$t = 0.1\,(c_h - c_l)$. For each level of information cost, there are two segments of price distributions, corresponding to informed bids and blind bids.

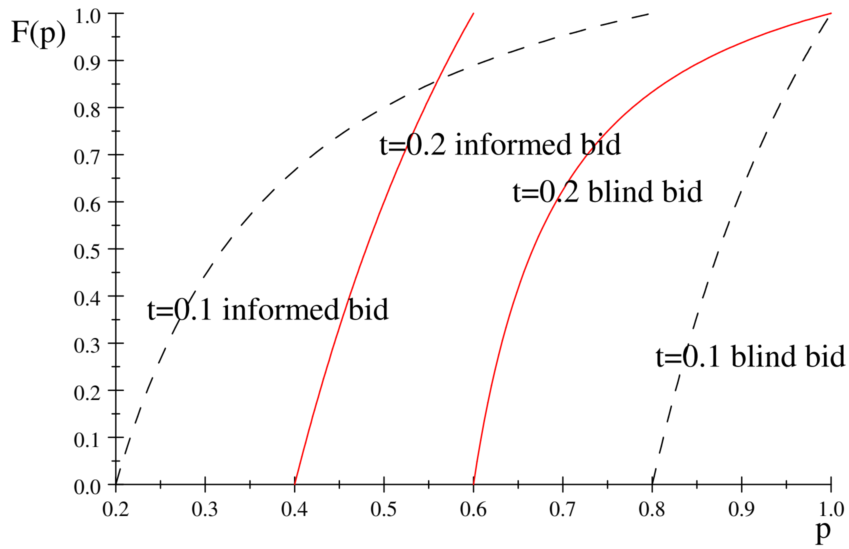

Figure 1: The red solid curve depicts the price distribution when $t = 0.2\,(c_h - c_l)$ and the black dashed curve when $t = 0.1\,(c_h - c_l)$. For both, $q = 1/2$.

From the graph, we can see that sellers are more likely to set low blind bids when t is large. Intuitively, a larger t makes sellers less likely to acquire costly information and this gives uninformed sellers a greater chance to win. In response to that, uninformed sellers bid more aggressively. The effects of a larger t on the bidding behavior of a seller informed of a low cost, however, are more complicated: on the one hand, having fewer competing bids raises the lower bound of informed bids; on the other hand, more aggressive bidding by uninformed sellers lowers the upper bound of informed bids. Therefore, a larger t decreases the degree of dispersion in informed bids.

Lemma 3 also shows that the consumer pays for information costs indirectly in the form of a higher expected price. This gives us the basic intuition why prices can increase with the number of searches and why the consumer may want to limit her number of searches. However, this does not mean that the consumer should always search as little as possible, since a smaller number of searches will give each seller a greater incentive to earn information rents. In other words, neither total information costs nor the expected price is monotonic in n. Basically, as n increases, there are two competing effects on prices: a competitive effect that pushes down prices and a compensation effect that drives up prices. The relative importance of these two effects determines the optimal number of searches.

3 Equilibrium Properties

Consumer surplus is maximized in the benchmark case, but it requires the consumer and sellers to be able to contract on the latter's information acquisition effort. In reality, sellers' information acquisition effort is not verifiable so the benchmark outcome cannot be obtained. The second best outcome is for the consumer to commit to the number of searches before sellers offer competing bids. This is possible only if the number of searches is publicly observable (Bagwell 1995). Accordingly, we analyze two cases, depending on whether n is publicly observable or private information.

3.1 The Number of Price Quotes is Publicly Observable

The consumer surplus can be written as $v - c_E - \min_{n\geq 2}(\alpha t + s)\, n$. Relative to the benchmark case, it is lower by $\min_{n\geq 2}(\alpha t + s)\, n - 2s$. The term $(\alpha t + s)\, n$ captures the overall impact of precontract costs, including consumer search cost and seller information cost, on consumer welfare. It does not contribute to sellers' profit margin and is merely a waste caused by market frictions, but for the lack of better names we shall call it the expected markup and denote it by $\psi(n, s, t)$. Lemma 4 summarizes how the expected markup (essentially the negative of consumer surplus) varies with the number of searches for different combinations of parameter values. For ease of exposition, we ignore the integer constraint on n in this section.

Lemma 4 *For given values of s and t, let $\psi(n) = \psi(n, s, t) = (\alpha t + s)\, n$ and $\gamma = -\ln \frac{t(1-q)}{q((c_h - c_l)(1-q) - t)}$,*

(i) $\psi(n)$ monotonically increases with n on $(1, \infty)$ if $1 + s/t \geq e^{\gamma - 2}(4/\gamma - 1)$; otherwise,

(ii.a) if $s = 0$, then $\psi(n)$ is unimodal on $(1, \infty)$;

(ii.b) If $s > 0$, then $\psi(n)$ has two critical points on $(1, \infty)$. Denote them by $\{n_1, n_2\}$, where $n_1 < n_2$, $\psi(n)$ is maximized at n_1 and minimized at n_2. If $1 + s/t < e^{-\gamma}(2\gamma + 1)$, then $n_1 < 2 < n_2$; if $1 + s/t \in \left(e^{-\gamma}(2\gamma + 1), e^{\gamma - 2}(4/\gamma - 1)\right)$ and $\gamma < 1$, then $n_1 < n_2 \leq 2$; if $1 + s/t \in \left(e^{-\gamma}(2\gamma + 1), e^{\gamma - 2}(4/\gamma - 1)\right)$ and $\gamma > 1$, then $2 < n_1 < n_2$.

(iii) if $n \to \infty$, then $\psi(n) \to ns + \gamma t$.

According to Lemma 4, there are three possible patterns of how the expected markup varies with the number of searches. Figure 2 illustrates these possibilities. When the information cost t is small (the blue dashed curve at the bottom), the expected markup monotonically increases with n; when t is large but s is zero (the black dotted curve in the middle), the expected markup first increases with n, then decreases; when t is large and s is positive (the red solid curve at the top), the expected markup first increases, then decreases between n_1 and n_2, and then increases again for $n > n_2$, where n_1 is a local maxima and n_2 is a local minima as defined in Lemma 4.

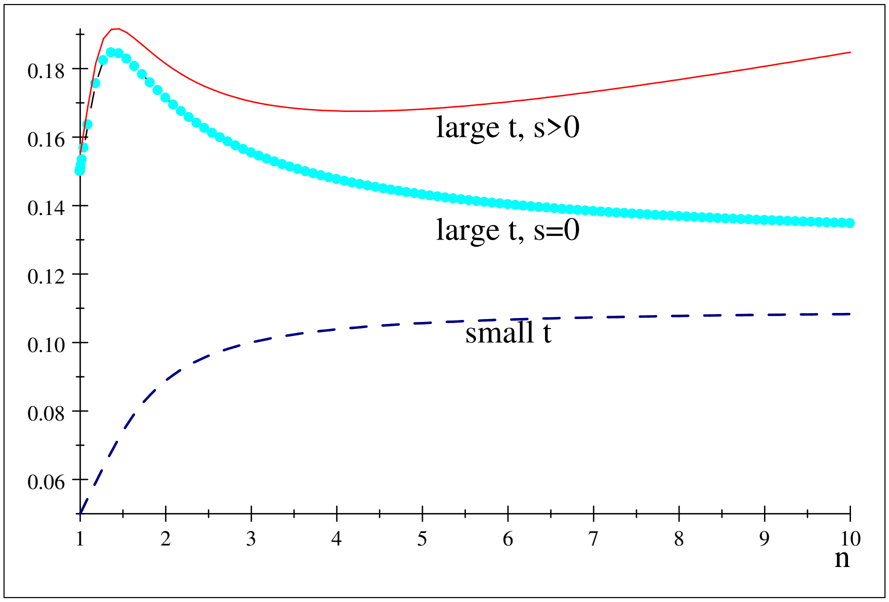

Figure 2 ($q = 1/2$): The expected markup as a function of n. Blue dashed $(t = 0.05\,(c_h - c_l)\,, s = 0)\,,$ Black dotted $(t = 0.15\,(c_h - c_l)\,, s = 0)\,,$ Red solid $(t = 0.15\,(c_h - c_l)\,, s = 0.005\,(c_h - c_l))\,.$

Hence, there are at most three candidates for the optimal number of price quotes: 2, n_2, or infinity, with the last candidate, infinity, being optimal only if $s = 0$. Therefore, the optimal number of price quotes can simply be determined by comparing three numbers, $\psi(2)\,,$ $\psi(n_2)$ and $\psi(\infty)\,.$ The remaining difficulty is to determine $\psi(n_2)\,,$ as n_2 does not have an analytical solution, but we rely on studying the monotonicity and oscillation of the expected markup as a function of n to accomplish the task.

3.1.1 The Optimal Number of Searches

Suppose that n^o is the optimal number of price quotes, then we must have $n^o = \arg\min_{n\geq 2} \psi(n, s, t)$. Proposition 2 summarizes the choices of n^o for different parameter values of s and t.

Proposition 2 *Suppose that the number of price quotes is publicly observable. If* $s = 0$, *then* $n^o > 2$ *if and only if* $\frac{t}{c_h - c_l} > \frac{q(1-q)}{4.92(1-q)+q}$; *if* $s > 0$, *then* $n^o > 2$ *if and only if* $\frac{t}{c_h - c_l} > \frac{q(1-q)}{(1-q)e+q}$ *and* $s/t < e^{-\gamma}(2\gamma + 1) - 1$, *or* $\frac{t}{c_h - c_l} < \frac{q(1-q)}{(1-q)e+q}$ *and* $s/t < e^{\gamma-2}(4/\gamma - 1) - 1$ *and* $2(1 + s/t - e^{-\gamma}) > \frac{(z^*+\gamma)^2}{\gamma} \exp(-z^*)$, *where* $\gamma = -\ln \frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}$ *and* z^* *is the larger root for* $\gamma \frac{(1+s/t)\exp z - 1}{z(z+\gamma)} = 1$.

Proposition 2 shows that, frequently, the optimal number of searches is two.[11] We can see this result more clearly from Figure 3, where $q = 1/2$. The optimal number of searches is two in all regions but the lower-right corner. It is true even when the search cost goes to zero. At first glance, the result that a consumer only needs to search twice in a market of homogeneous sellers may not be surprising. For example, the same result holds in Burdett and Judd (1983) based on the standard Bertrand style argument. However, there is a crucial difference. In Burdett and Judd (1983), if a consumer searches twice, then there will be no price dispersion, eliminating the need for further search. In the present model, a consumer searches twice despite price dispersion, because additional searches would change sellers' bidding strategies, potentially raising prices. It follows that, if search costs are zero, the "law of one price" will hold in Burdett and Judd (1983), but not in the present model.

[11] Honka (2014) finds that consumers get on average 2.96 quotes with the majority of consumers collecting two or three quotes when purchasing auto insurance policies.

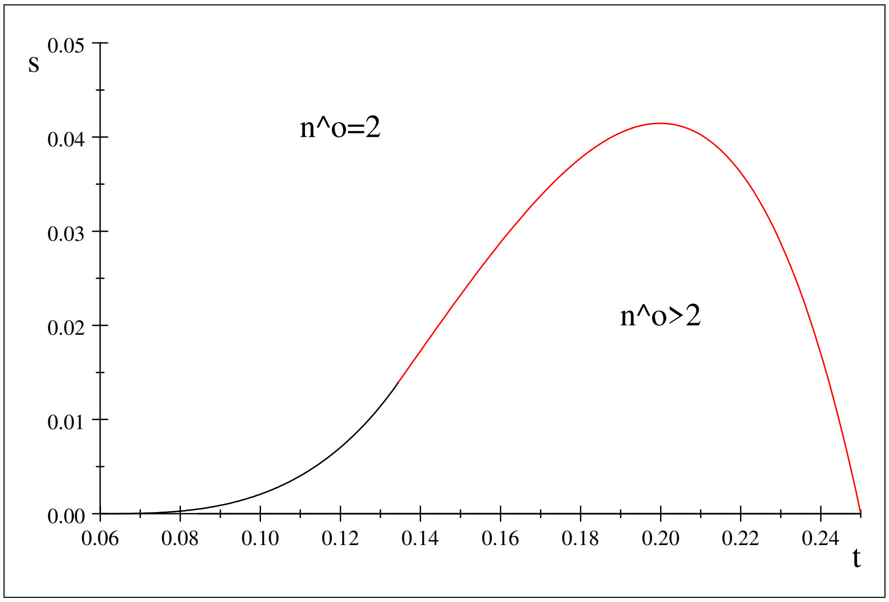

Figure 3: The Optimal Number of Searches.

Our result is more similar to that of Che and Gale (2003), who find it optimal to include only two contestants in a research contest.[12] In their model, restricting entry to two competitors decreases the coordination problem of competing contestants and minimizes the duplication of fixed costs. Similarly, in the present model, limiting the number of bidders creates better incentives at the information acquisition stage and reduces duplication in efforts. However, unlike other papers with a similar result, the present paper also shows that the consumer can sometimes benefit from expanding her search effort, especially when the search cost is small and the information cost is relatively large. The first effect is quite obvious, but the second one is not. When the information cost rises, one might expect the consumer to search less since she has to indirectly pay for sellers' information costs, but this intuition is incomplete because it ignores the effect of an increase in n on sellers' incentive to acquire information. To see this effect clearly, we plot $\alpha(n)/\alpha(2)$ in Figure 4. The graph illustrates three results: first, the propensity to acquire information, α, decreases with n across all ranges of t; second, sellers are less likely to acquire information when t is large; these two results are obvious. Less obvious is the third result, namely, α decreases at a faster rate when t is large. This means that increasing the number of searches can potentially reduce the wasteful and duplicative information acquisition efforts for a large t.

[12] A similar result is also found in auctions with entry (e.g., Harstad 1990, Levin and Smith 1994), and for research tournaments (Taylor 1995, Fullerton and McAfee 1999).

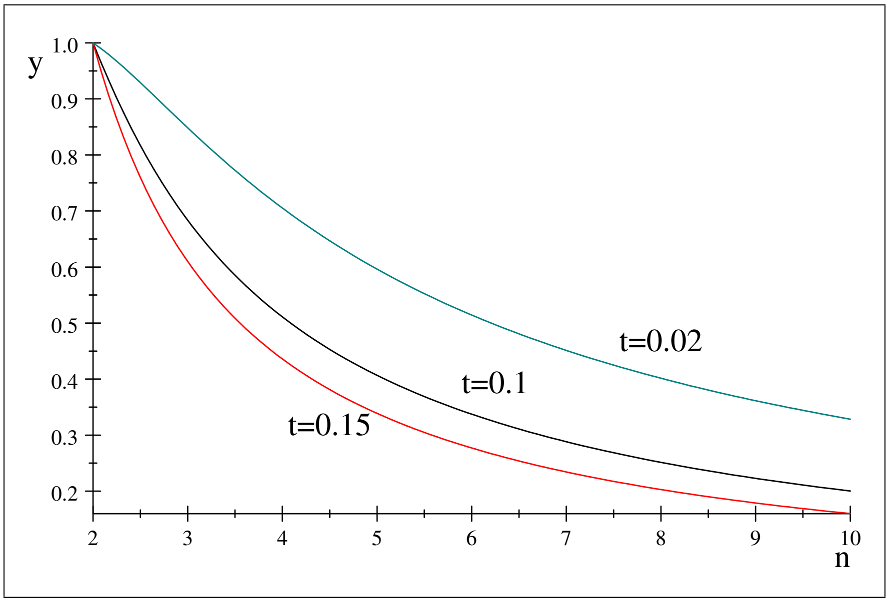

Figure 4: $y = \alpha(n)/\alpha(2)$, when $t = 0.02, 0.1, 0.15$

3.1.2 Comparative Statics

In this section, we examine the impact of search cost and information cost on the equilibrium outcome. The first result is immediate from the preceding discussion.

Proposition 3 *Suppose that the number of price quotes is publicly observable, the optimal number of searches* n^o

(i) *decreases with* s*;*

(ii) *is non-monotonic in* t *if* $s > 0$, *but increases with* t *if* $s = 0$.

According to Proposition 3, search costs and information costs have different impacts on consumers' search behavior, even though they both contribute to the overall precontract costs. This means that it is not only the total costs, but also the composition of the costs, that matter to consumer search. A similar result exists for two-sided markets, but the underlying mechanism is much different. In two-sided markets the composition of costs matters because the costs imposed on one side cannot be fully internalized by the other side of the market, whereas in this model it matters despite consumers' full internalization of sellers' costs.

Let $\varphi(s,t) = \min_{n\geq 2}(\alpha t + s)\,n$ denote the minimized expected markup as a function of s and t. Proposition 4 examines the welfare impact of the two costs.

Proposition 4 *Suppose that the number of price quotes is publicly observable, $\varphi(s,t)$*

(i) *monotonically increases with s;*

(ii) *is unimodal in t.*

Perhaps not surprisingly, the consumer benefits from a lower search cost. More interestingly, the consumer can benefit from a further increase in the information cost when it is already large. This directly follows from our earlier observation that a high information cost can discourage sellers from engaging in wasteful and duplicative information acquisition efforts, which are indirectly paid by the consumer. Another way to understand why the expected total expense is not monotonic in the information cost is to recall Proposition 1: when $t = 0$, by definition the cost of information acquisition effort will be zero; when $t \geq q(1-q)(c_h - c_l)$, there will be no wasteful expenditure on information acquisition because the cost exceeds the value of the information. Between the two extremes, however, there will be positive amounts of (wasteful) information acquisition efforts.

Last, we consider the price effects.

Proposition 5 *Suppose that the number of price quotes n is publicly observable, the expected price*

(i) *increases with n if $\frac{t}{c_h-c_l} < \frac{q(1-q)}{(1-q)e^2+q}$;*

$(ii.a)$ *decreases with n if $\frac{t}{c_h-c_l} \geq \frac{q(1-q)}{(1-q)e^2+q}$ and $n_2 \leq 2$; $(ii.b)$ increases with n on the interval of $[2, n_2]$ and then decreases with n on (n_2, ∞) if $\frac{t}{c_h-c_l} \geq \frac{q(1-q)}{(1-q)e^2+q}$ and $n_2 > 2$;*

(iii) *increases with s.*

In classic models of consumer search, prices are either set before consumer search or at the same time. This implies that consumers' search intensity cannot alter price distributions, so the expected price has to decrease with the number of searches. However, if prices are set after consumer search, then the effect of more searches on the final price is not obvious: on the one hand, competition lowers price; on the other hand, each seller has a smaller chance of being the winning bidder and so will in equilibrium seek a higher expected profit in the event of winning, which translates into a higher price. If the second effect is stronger, more searches will result in a high price. This counterintuitive result has also been noted in a few search models (e.g., Rosenthal 1980, Stahl 1989), but for different reasons.

The result that the expected price increases with the search cost is more conventional. It implies that the standard competitive effect dominates the compensation effect if the consumer can commit to the optimal number of searches. However, if we relax the assumption of commitment, the opposite result can sometimes be obtained, as shown in Section 3.2.

3.1.3 Economic Significance of Information Costs

While the above model generates some interesting results, they will not matter much if the information cost only has a small impact on consumer welfare. In order to evaluate its economic significance, we compare the expected total expense under costly effort to the benchmark case, in which the total expense is just the expected production cost plus the search costs, $c_E + 2s$. To focus on the impact of information cost, we further assume $s = 0$, in which case n^o is either 2 or ∞. Another measure that can be used is the expected price plus the information costs, but it generates the same qualitative result.[13]

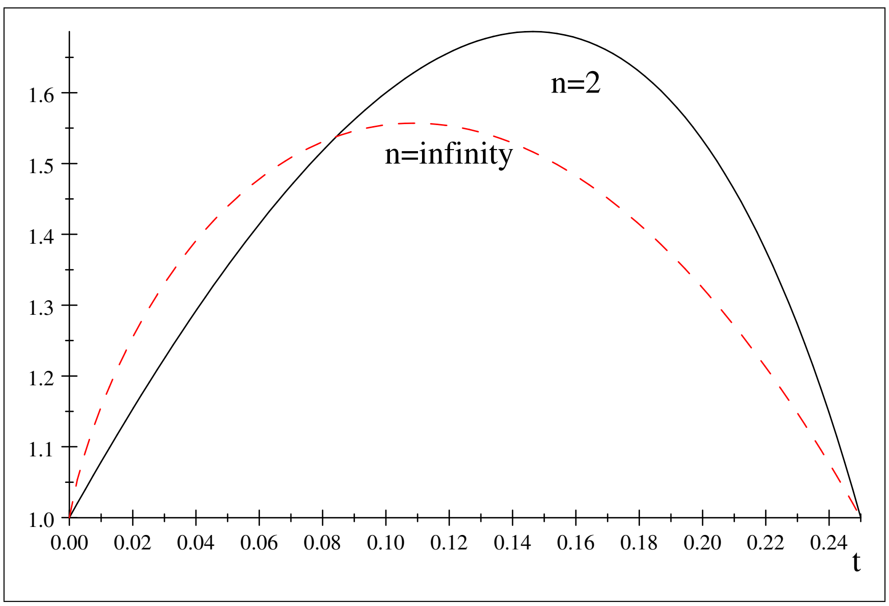

Figure 5: $\varphi(s,t)/c_E$ as a function of $t/(c_h - c_l)$ when $q = 1/2$. The solid curve has $n = 2$ and the dashed $n \to \infty$.

Figure 5 plots $\varphi(s,t)/c_E$ when $q = 1/2$, with the the solid curve corresponding to the case of $n = 2$ and the dashed $n \to \infty$, so the lower envelope is the expected cost when n is optimally chosen. As we can see from the graph, the existence of an incontractible information cost can potentially increase the consumer's total expense by more than one half. This clearly demonstrates that it is not a negligible cost and should be taken seriously not only for its theoretical interests,

[13] Consumer surplus is a less appropriate measure, because it depends on v, an arbitrary parameter in the model.

but also for its practical importance. Interestingly, it offers an alternative explanation for why car buyers obtain significantly more of the surplus available under customer rebate than under dealer discount, a finding that is counter to the simple invariance of incidence analogy (Busse, Silva-Risso and Zettelmeyer 2006). Busse et al. test several hypotheses and find evidence consistent with the asymmetric information hypothesis, that is, car buyers are disadvantaged in negotiations because they are less informed than dealers about the availability of dealer discounts. In contrast, the parties are symmetrically informed about the availability of customer rebates, which are always publicized to potential customers, often in prime-time television advertisements. Note that their explanation is based on the assumption that the information about dealer discounts is readily accessible to dealers. However, these discounts are often in the form of conditional discounts, depending on the geographical location and/or the specific equipment package, or “trim level”. This means that there may be higher information costs for dealer discounts than for customer rebates.[14] Thus, the result that dealer discounts have a smaller pass-through can also be predicted by our model.

3.2 The Number of Price Quotes is Private Information

As mentioned earlier, if n is non-contractible private information, then sellers' bidding strategy depends on their conjecture of the number of competitors; but holding their belief and the corresponding bidding strategy constant, the consumer always benefits from having a larger sample size. To see this, consider the lowest price from obtaining n price quotes, taking the price distribution as given. Denote it by $p_{\min}$. Since the overall CDF for a seller's price distribution is $F(p) = (1-\alpha) F_b(p) + \alpha (qF_l(p) + (1-q)\mathbf{1}(p \geq c_h))$, where $\mathbf{1}(\cdot)$ is the indicator function, $F_l(p)$ and $F_b(p)$ are given in Lemma 3, the CDF of $p_{\min}$ is $\Pr(p_{\min} < p) = 1 - \Pi_{i=1}^{n} \Pr(p_i > p) = 1 - (1-F(p))^n$. Assuming that these expectations are finite, we have $E(p_{\min}) = \int_0^\infty (1-(1-F(p))^n)\, dF(p) = \int_0^\infty (1-F(p))^n\, dn$.

$$\frac{d}{dn} E(p_{\min}) = \int_0^\infty (1-F(p))^n \ln(1-F(p))\, dp < 0. \tag{3.1}$$

[14] According to a web site specialized on automobile markets: "Even if you are the only customer in the dealership, there is still no guarantee you'll be able to get a deal offer in a flash. If you're taking out a loan, the sales manager might have to run your credit to get your credit score. He'll call the finance department to get your interest rate, and then look up specials and incentives on your car to make sure you're getting the right program offer for the right car. Sometimes it just takes a while to get all the information together." Matt Jones, "Behind the Scenes at A Car Dealership", April 29th, 2016, https://www.edmunds.com/car-buying/behind-the-scenes-at-a-car-dealership.html.

$$\frac{d^2}{dn^2}E(p_{\min}) = \int_0^\infty (1-F(p))^n (\ln(1-F(p)))^2 \, dp > 0. \tag{3.2}$$

Equation (3.1) and (3.2) show that, given the sellers' bidding strategies, a consumer obtains lower prices as she searches more sellers, but the incremental gain from further price reductions becomes smaller and smaller. This is essentially the original insight of Stigler (1961). Under the assumption that the price distributions are completely exogenous, he argues that increased search will yield positive but diminishing returns as measured by the expected reduction in the minimum asking price whatever the precise distribution of prices. Figure 6 illustrates (3.1) and (3.2) for the case where sellers believe they only face one competitor, but the consumer engages in $n = 1$, 2 or 3 searches (The mathematical expressions are derived in Appendix B.1).

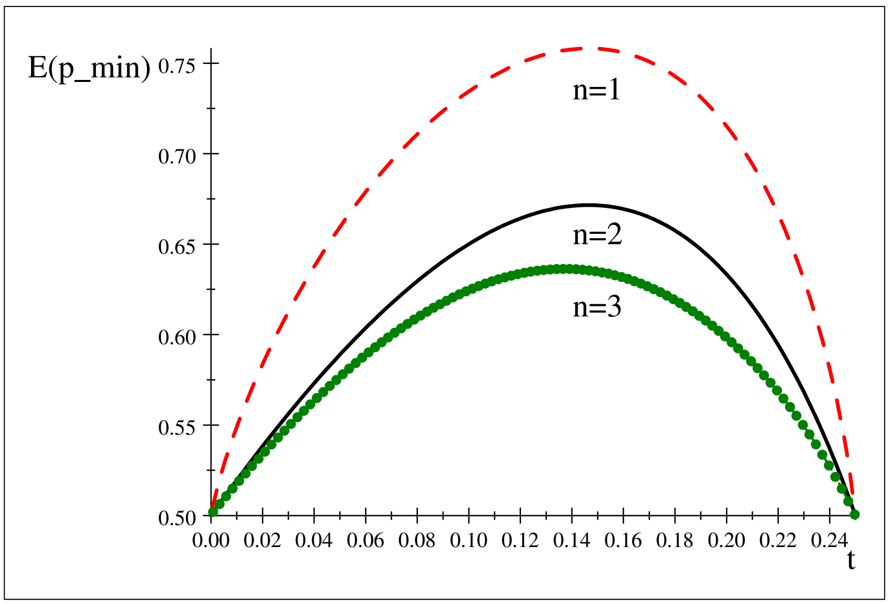

Figure 6: $E(p_{\min})$ as a function of $t/(c_h - c_l)$. The dashed line on the top represents $E(p_{\min})$ when $n = 1$, the solid one in the middle $E(p_{\min})$ when $n = 2$, the dotted one at the bottom $E(p_{\min})$ when $n = 3$. In all case, $q = 1/2$.

In this section, we model the consumer's problem by studying a simultaneous move version of the game. More specifically, we modify the game by assuming that n is not observed by the sellers when they choose their bids and the consumer cannot precommit to the optimal number of searches. Instead, sellers must form beliefs about the number of other sellers visited by the consumer. In the Nash equilibrium, their beliefs must be correct. At the same time, given sellers' beliefs and pricing strategies, the consumer has no incentive to search more or fewer sellers. Formally, let

$\Psi(n,m)$ denote the expected markup, where n represents the actual number of price quotes and m a seller's belief about the number of price quotes including his own. Let the corresponding lowest price quote $p_{\min}$ be denoted by $E(p_{\min}|n,m)$. The equilibrium condition is $m = \arg\min_n \Psi(n,m)$, where $\Psi(n,m) = E(p_{\min}|n,m) + ns - c_E$.

Suppose that n^{PI} is the equilibrium number of price quotes when n is private information, then we must have $\Psi\left(n^{PI}, n^{PI}\right) \leq \Psi\left(n, n^{PI}\right)$ for any n, i.e.,

$$E\left(p_{\min}|n^{PI}, n^{PI}\right) + n^{PI}s \leq E\left(p_{\min}|n, n^{PI}\right) + ns. \tag{3.3}$$

By (3.1) and (3.2), we know that $E(p_{\min}|n,m) - E(p_{\min}|n+1,m)$ is positive and strictly decreases with n. Condition (3.3) can thus be rewritten as

$$E\left(p_{\min}|n^{PI}, n^{PI}\right) - E\left(p_{\min}|n^{PI}+1, n^{PI}\right) \leq s < E\left(p_{\min}|n^{PI}-1, n^{PI}\right) - E\left(p_{\min}|n^{PI}, n^{PI}\right), \tag{3.4}$$

which is the necessary and sufficient condition for the existence of a private information equilibrium. From (3.4), we can easily see that $n^{PI} \to \infty$ when $s \to 0$, regardless of the optimal number of price quotes. This observation gives us the basic intuition why the equilibrium number of price quotes may deviate from the optimum.

Due to the lack of a general analytical solution to expression $(B.1)$, the full characterization of the private information equilibrium is challenging. Therefore, we use examples to show the qualitative results. Proposition 6 considers the price impact of search costs.

Proposition 6 *Suppose that the number of price quotes n is private information, the expected equilibrium price may decrease with s if $\frac{t}{c_h - c_l} < \frac{q(1-q)}{4.92(1-q)+q}$.*

In contrast to Proposition 5 (iii), the expected price is no longer monotonically increasing in the search cost if the consumer is unable to commit. This is because, relative to the commitment case, the search cost has a more direct impact on the number of searches. While a consumer with commitment must take into account the impact of additional searches on the price distribution, a consumer without commitment takes the price distribution as given. Consequently, a lower search cost is more likely to tempt a consumer without the commitment power to engage in additional searches, but this can cause sellers to quote higher prices in order to compensate for their smaller chances of winning the bidding war.

For the same reason, the consumer's lack of commitment can also lead to excessive search that is detrimental to her own welfare. A numerical example is provided in Appendix B.2 to illustrate this possibility.

4 Conclusion

When consumers search, they incur costs. In order to provide consumers the information they search for, sellers may also incur costs. This paper departs from the extant literature by assuming that sellers must make an effort to learn the cost of providing the goods/services before they bid against other sellers, but the consumer is unable to verify or contract on sellers' efforts. Despite its simplicity, the current model is a faithful snapshot of procurement markets. It allows us to derive a number of results that do not exist in the search models of posted-price markets.

These results suggest that the choice of a small sample size when consumers search is not necessarily due to high search costs. Recent empirical studies have documented surprisingly few searches conducted by consumers when shopping for financial products.[15] The lack of consumer search has been attributed to high search costs and non-price preferences, but it is also consistent with the existence of information costs. Indeed, as shown in this paper, the optimal number of searches is often small. Empirical studies that do not take into account sellers' information costs may overestimate consumer search costs or the impact of other factors.

This paper is only one step in trying to understand the impact of sellers' precontract cost on consumer search behavior. For further exploration, it can be extended in a number of directions. First, if the source of uncertainty in product costs is consumer specific, then a consumer may have some private information. This is especially true for insurance markets. How consumer private information affects their search behavior remains an open question.[16] Second, the current model does not consider the possibility that custom solutions exist for different realizations of the

[15] Honka (2014) documents evidence from the US auto insurance market, Allen, Clark and Houde (2014) and Alexandrov and Koulayev (2017) from the Canadian and US mortgage markets, respectively, and Stango and Zinman (2015) from the credit card market.

[16] In a model with a similar setup, Lauermann and Wolinsky (2017) assume that an auctioneer has private information, which affects the number of bidders she solicits. Their paper, however, does not consider bidders' precontract costs.

production costs. This effectively rules out second-degree price discrimination by a single seller or vertical differentiation among multiple sellers. Third, the production cost is assumed to be a binary variable that can be learned with perfect precision. Last, sellers are assumed to compete in a common value auction.[17] Extending the model by relaxing some of these assumptions can be the basis of fruitful future work.

[17] In MacMinn (1980) and Spulber (1995), sellers' price setting is equivalent to bidding in a private value auction. Price dispersion arises from cost heterogeneity of sellers.

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Appendix

A Proof

Proof of Lemma 1. If all sellers learn the true cost, since this is a common value auction, they will all quote the same price. As a result, net of the information cost t, their profits are negative. If a seller deviates by not incurring the information cost and quoting c_h, his profit will be zero, so the deviation is profitable. $\blacksquare$

Proof of Lemma 2. If no sellers learns the true cost, then the price must be c_E and sellers' profits will be zero. If a seller learns the true cost, he can charge $c_E - \varepsilon$ if the production cost is revealed to be c_l. This happens with a probability of q, so his expected profit is $q\left(c_E - c_l\right) = q\left(1-q\right)\left(c_h - c_l\right).$ It is a profitable deviation if $t < q\left(1-q\right)\left(c_h - c_l\right)$. $\blacksquare$

Proof of Lemma 3. By contradiction, it can be shown that the supports of the two price distributions $F_l\left(p\right)$ and $F_b\left(p\right)$ do not overlap in a symmetric equilibrium. Hence, if seller i chooses not to acquire information, then his expected profit is

$$\pi_i = \int_{\underline{p}_b}^{\bar{p}_b} dF_b\left(p\right)\left(\begin{array}{c} q\left(p-c_l\right)\left(1-\alpha\right)^{n-1}\left(1-\tilde{F}_l\left(p\right)\right)^{n-1} \\ +\left(1-q\right)\left(p-c_h\right)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k\left(1-\alpha\right)^{n-1-k}\left(1-\tilde{F}_b\left(p\right)\right)^{n-1-k}\end{array}\right),$$

where $\tilde{F}_l\left(p\right)$ and $\tilde{F}_b\left(p\right)$ are the corresponding price distributions for the $n-1$ other sellers. Because of symmetry, $\tilde{F}_l\left(p\right) \equiv F_l\left(p\right)$ and $\tilde{F}_b\left(p\right) \equiv F_b\left(p\right)$. In a mixed strategy equilibrium, seller i must be indifferent among all choices of p on the support of $\left[\underline{p}_b, \bar{p}_b\right]$, i.e.,

(A.1)

$$q\left(p-c_l\right)\left(1-\alpha\right)^{n-1}\left(1-F_b\left(p\right)\right)^{n-1}+\left(1-q\right)\left(p-c_h\right)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k\left(1-\alpha\right)^{n-1-k}\left(1-F_b\left(p\right)\right)^{n-1-k} = \Delta_1,$$

where Δ_1 is a constant. If seller i chooses to learn the cost, his expected profit can be written as $\pi_i = q\int_{\underline{p}_l}^{\bar{p}_l}\left(p-c_l\right)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k\left(1-\alpha\right)^{n-1-k}\left(1-\tilde{F}_l\left(p\right)\right)^k dF_l\left(p\right) - t$. Applying again the indifference principle, we have

$$(A.2) \qquad (p-c_l)\sum_{k=0}^{n-1}\binom{n-1}{k}\alpha^k(1-\alpha)^{n-1-k}(1-F_l(p))^k=\Delta_2.$$

Since competing sellers are identical, they must all earn zero expected profits in the equilibrium. Hence, we must have $\Delta_1=0$ and $\Delta_2=t/q$. Thus, from $(A.1)$, we can get $q(p-c_l)(1-\alpha)^{n-1}(1-F_b(p))^{n-1}+(1-q)(p-c_h)(\alpha+(1-\alpha)(1-F_b(p)))^{n-1}=0$, i.e., $F_b(p)=1-\frac{\alpha}{1-\alpha}\left(\left(\frac{q(p-c_l)}{(1-q)(c_h-p)}\right)^{\frac{1}{n-1}}-1\right)^{-1}$; from $(A.2)$, we can get $(\alpha-\alpha F_l(p)+(1-\alpha))^{n-1}=\frac{t}{q(p-c_l)}$, i.e., $F_l(p)=\left(1-\left(\frac{t}{q(p-c_l)}\right)^{1/(n-1)}\right)/\alpha$. Since $F_b(\underline{p_b})=0$, we must have $q(\underline{p_b}-c_l)(1-\alpha)^{n-1}+(1-q)(\underline{p_b}-c_h)=0$. Solving, we obtain $\underline{p_b}=\frac{(1-q)c_h+q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. Similarly, we can obtain $\bar{p}_b=c_h$, $\underline{p_l}=c_l+t/q$ and $\bar{p}_l=c_l+\frac{t}{q(1-\alpha)^{n-1}}$. In addition, we have $\bar{p}_l=\underline{p_b}$, so $c_l+\frac{t}{q(1-\alpha)^{n-1}}=\frac{(1-q)c_h+q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. From this, we can solve for the information acquisition probability: $\alpha=1-\left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)^{1/(n-1)}$. The expected price is

$$\begin{aligned}
E(p)&=\alpha^n(1-q)c_h+\alpha^n q\int pd\left(1-(1-F_l(p))^n\right)+(1-\alpha)^n\int pd\left(1-(1-F_b(p))^n\right)\\
&\quad+\sum_{k=1}^{n-1}\binom{n}{k}\alpha^k(1-\alpha)^{n-k}\left((1-q)\int pd\left(1-(1-F_b(p))^{n-k}\right)+q\int pd\left(1-(1-F_l(p))^k\right)\right)\\
&=\alpha^n(1-q)c_h+n\int_0^1\left(\begin{array}{c}(1-\alpha)^n(1-F)^{n-1}\left(q+(1-q)\sum\limits_{k=0}^{n-1}\binom{n-1}{k}\left(\frac{\alpha}{(1-\alpha)(1-F)}\right)^k\right)F_b^{-1}(F)\\ +q\sum\limits_{k=1}^{n}\binom{n-1}{k-1}\alpha^k(1-\alpha)^{n-k}(1-F)^{k-1}F_l^{-1}(F)\end{array}\right)dF\\
&=\alpha^n(1-q)c_h+n\int_0^1\left(\begin{array}{c}(1-\alpha)^n(1-F)^{n-1}\left(q+(1-q)\left(1+\frac{\alpha}{(1-\alpha)(1-F)}\right)^{n-1}\right)F_b^{-1}(F)\\ +q\alpha(1-\alpha F)^{n-1}F_l^{-1}(F)\end{array}\right)dF\\
&=\alpha^n(1-q)c_h+n\int_0^1\left(\begin{array}{c}(1-\alpha)^n(1-F)^{n-1}\left(qc_l+(1-q)c_h\left(1+\frac{\alpha}{(1-\alpha)(1-F)}\right)^{n-1}\right)\\ +\alpha t+q\alpha(1-\alpha F)^{n-1}c_l\end{array}\right)dF\\
&=qc_l+(1-q)c_h+n\alpha t=c_E+n\alpha t.
\end{aligned} \qquad (A.3)$$

■

Proof of Lemma 4. Let $z=\gamma/(n-1)$, we have $n=\gamma/z+1$, $\alpha=1-\exp(-z)$, $\frac{\partial\gamma}{\partial t}<0$, and $\frac{\partial}{\partial n}z=-\frac{\gamma}{(n-1)^2}=-\frac{z^2}{\gamma}$. Thus, $\psi(n)$ can be rewritten as $((1-\exp(-\gamma/(n-1)))t+s)n=$

$((1-\exp(-z(n)))t+s)n = t\left(\frac{(1+s/t)\exp z-1}{z}\right)(\gamma+z)\exp(-z)$. Hence, $\psi'(n) = tze^{-z}\left(\frac{(1+s/t)\exp z-1}{z} - \frac{\gamma+z}{\gamma}\right)$. Let $\phi(z,\gamma) = \gamma\frac{(1+s/t)\exp z-1}{z(z+\gamma)}$. Thus $\psi'(n) \lesseqgtr 0$ as long as $\phi(z,\gamma) \lesseqgtr 1$. Using L'Hopital's rule (Estrada and Pavlovic 2017), we can verify that $\phi(z,\gamma)$ is convex in z, increases with γ, and $\lim_{z\to 0}\phi(z,\gamma) = \gamma\frac{(1+s/t)\exp z}{2z+\gamma}|_{z=0} = 1+s/t \geq 1$.

(i) Since $\phi_z(z,\gamma) = \gamma\frac{(\gamma+2z)+e^z\left(-2z-\gamma+z\gamma+z^2\right)(1+s/t)}{z^2(z+\gamma)^2}$, $\phi(z,\gamma)$ is minimized when $\phi_z(z,\gamma) = 0$, i.e.,

(A.4) $$(1+s/t)\exp z = \frac{\gamma+2z}{\gamma+2z-z\gamma-z^2}.$$

Also note that $\lim_{z\to 0}\phi_z(z,\gamma) = \frac{\gamma-2}{\gamma}(1+s/t) \leq 0$ if $\gamma \leq 2$. Since $\phi(z,\gamma)$ is convex in z, $(A.4)$ has a unique solution. Denote it by z^*. Hence, $\min_z \phi(z,\gamma) = \gamma\frac{(1+s/t)\exp z^*-1}{z^*(z^*+\gamma)} = \frac{\gamma}{\gamma+2z^*-z^*\gamma-z^{*2}} = 1$ if $z^* = 2-\gamma$. Since z^* solves $(A.4)$, $\min_z \phi(z,\gamma) = 1$ if and only if $(A.4)$ holds when $z = 2-\gamma$, i.e., $(1+s/t)\exp(2-\gamma) = 4/\gamma - 1$. Denote its solution by γ^*. Thus, $\min_z \phi(z,\gamma^*) = 1$. Since $\phi_\gamma(z,\gamma) > 0$, we must have $\min_z \phi(z,\gamma) \geq 1$ for all $\gamma \geq \gamma^*$ by the Envelope Theorem. Therefore, $\psi'(n) \geq 0$ if $\gamma \geq \gamma^*$, i.e., $1+s/t \geq e^{\gamma-2}(4/\gamma-1)$.

(ii) Now suppose $\gamma < \gamma^*$. If n is a critical point, then we must have $\phi(z,\gamma) = 1$. There are two critical points if and only if $\min_z \phi(z,\gamma) < 1$.

$(ii.a)$ If $s = 0$, then $\gamma^* = 2$. Since $\phi(z,\gamma)$ is convex in z, $\lim_{z\to 0}\phi(z,\gamma) = 1$ and $\lim_{z\to 0}\phi_z(z,\gamma) < 0$. There is a unique solution of z for $\phi(z,\gamma) = 1$ on $(0,\infty)$. Denote it by $\hat{z}$. $\psi'(n) > 0$ if and only if $z > \hat{z}$. Thus $\hat{n} = \gamma/\hat{z}+1$ must be the unique critical point on $(1,\infty)$ and $\psi'(n) \gtreqless 0$ if $n \lesseqgtr \hat{n}$.

$(ii.b)$ If $s > 0$, then $\gamma^* < 2$, since $e^{\gamma-2}(4/\gamma-1)$ decreases with γ. For $\gamma < \gamma^*$, $e^{\gamma-2}(4/\gamma-1) > 1+s/t$, there are two solutions of z for $\phi(z,\gamma) = 1$ on $(0,\infty)$. Denote them by z_1 and z_2, where $z_1 > z_2$. Thus, $\psi'(n) < 0$ if and only if $z \in (z_2, z_1)$. Hence, $n_i = \gamma/z_i + 1$, $i = 1,2$, must be the only two critical points on $(1,\infty)$, with $\psi'(n) < 0$ if and only if $n \in (n_1, n_2)$. As a result, n_1 maximizes $\psi(n)$ and n_2 minimizes $\psi(n)$. If $s/t < e^{-\gamma}(2\gamma+1)-1$, then $\phi(z,\gamma)|_{z=\gamma} < 1$. Since $\phi(z,\gamma)$ is convex in z and $\phi(z_1,\gamma) = \phi(z_2,\gamma) = 1$, we must have $z_2 < \gamma < z_1$, i.e., $n_1 < 2 < n_2$. If, instead, $s/t > e^{-\gamma}(2\gamma+1)-1$, then $\phi(z,\gamma)|_{z=\gamma} > 1$. It is easy to verify that $e^{-\gamma}(2\gamma+1) < e^{\gamma-2}(4/\gamma-1)$ if $\gamma \neq 1$ and $e^{-\gamma}(2\gamma+1) = e^{\gamma-2}(4/\gamma-1) = 3/e$ if $\gamma = 1$. Since $\phi_z(z,\gamma)|_{z=\gamma} = 0$ when $\gamma = 1$ and $\phi_{z\gamma}(z,\gamma) > 0$, we must have $\phi_z(z,\gamma)|_{z=\gamma} \gtreqless 0$ when $\gamma \gtreqless 1$. Thus, if $\gamma < 1$, we have $\phi_z(z,\gamma)|_{z=\gamma} < 0$, so $\gamma < z_2 < z_1$, i.e., $n_1 < n_2 < 2$; otherwise, $z_2 < z_1 < \gamma$, i.e., $2 < n_1 < n_2$. Last, if $\gamma = 1$, then we cannot have $e^{-\gamma}(2\gamma+1) < 1 + s/t < e^{\gamma-2}(4/\gamma - 1)$.

(iii) $\lim_{n\to\infty} \alpha n = \lim_{z\to 0}(1 - e^{-z})\frac{\gamma+z}{z} = \lim_{z\to 0} e^{-z}\frac{(z+\gamma)^2}{\gamma} = \gamma$, so $\lim_{n\to\infty}(\alpha t + s)n = \gamma t + sn$. ■

Proof of Proposition 2. If $s = 0$, by Lemma 4 $(ii.a)$, we only need to compare $\psi(2)$ and $\psi(\infty)$. By Lemma 4 (iii), $\lim_{n\to\infty}\psi(n) = t\gamma$, whereas $\psi(2) = 2t\left(1 - \left(\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)}\right)\right)$. Since $1 - \exp(-\gamma) < \gamma/2$ for $\gamma > 1.594$, we have $\psi(2) < \lim_{n\to\infty}\psi(n)$ if and only if $\frac{t(1-q)}{q((c_h-c_l)(1-q)-t)} < \exp(-1.594)$, i.e., $t < \frac{q(1-q)}{4.92(1-q)+q}(c_h - c_l)$. The reason why $s = 0$ has to be considered as a special case is due its different asymptotic behavior: $\lim_{n\to\infty}\psi'(n) = s$ if $s > 0$, but $\psi'(n) < 0$ and $\lim_{n\to\infty}\psi'(n) = 0$ if $s = 0$.

If $s > 0$, by Lemma 4 $(ii.b)$, there are four possibilities:

(i) $\psi(n)$ is increasing on $(1,\infty)$ when $\gamma > \gamma^*$, i.e., $\frac{t}{c_h-c_l} \leq \frac{q(1-q)}{(1-q)e^{\gamma^*}+q}$. Hence, $n^o = 2$.

(ii) $n_1 < 2 < n_2$ when $1 + s/t < e^{-\gamma}(2\gamma+1)$. Since $\psi(n)$ is decreasing on the interval of $[2, n_2]$, $\psi(n_2) < \psi(2)$. Therefore, $n^o = n_2 > 2$.

(iii) $n_1 < n_2 < 2$ when $1 + s/t \in \left(e^{-\gamma}(2\gamma+1), e^{\gamma-2}(4/\gamma-1)\right)$ and $\gamma < 1$. Since $\psi(n)$ is increasing on the interval of (n_2, ∞), $\psi(n_2) < \psi(2) < \lim_{n\to\infty}\psi(n)$. In addition, $n^o \geq 2$, so we must have $n^o = 2$.

(iv) $2 < n_1 < n_2$ when $1 + s/t \in \left(e^{-\gamma}(2\gamma+1), e^{\gamma-2}(4/\gamma-1)\right)$ and $\gamma > 1$. Since $\psi(n)$ increases on $[2, n_1]$ and then decreases on $[n_1, n_2]$, $n^o = \arg\min_{n\in\{2,n_2\}}\psi(n)$. Since $\psi(2) = \left(\frac{(1+s/t)\exp z - 1}{z}\right)(\gamma+z)\exp(-z)\,|_{z=\gamma} = 2(1 + s/t - e^{-\gamma})$ and $\psi(n_2) = \left(\frac{(1+s/t)\exp z-1}{z}\right)(\gamma+z)\exp(-z)$ where $\gamma\frac{(1+s/t)\exp z-1}{z(z+\gamma)} = 1$, $n^o > 2$ if $2(1 + s/t - e^{-\gamma}) > \frac{(z+\gamma)^2}{\gamma}\exp(-z)$.

Therefore, in order for $n^o > 2$, if $\gamma < 1$, i.e., $\frac{t}{c_h-c_l} > \frac{q(1-q)}{(1-q)e+q}$, then we must have $s/t < e^{-\gamma}(2\gamma+1) - 1$; if $\gamma > 1$, i.e., $\frac{t}{c_h-c_l} < \frac{q(1-q)}{(1-q)e+q}$, then we must have $s/t < e^{\gamma-2}(4/\gamma-1) - 1$ and $2(1 + s/t - e^{-\gamma}) > \frac{(z^*+\gamma)^2}{\gamma}\exp(-z^*)$, where $z^* is$ the solution to $(A.4)$. ■

Proof of Proposition 3. (i) Suppose that n^o changes to $n^{o\prime}$ and n_2 changes to n_2' when s increases to $s' > s$. To prove that n^o is (weakly) decreasing in s, we need to show $n^{o\prime} \leq n^o$.

If $s = 0$, by Lemma 4 $(ii.a)$, $n^o = \arg\min_{n\in\{2,\infty\}}\psi(n, 0, t)$ and $n^{o\prime} = \arg\min_{n\in\{2,n_2'\}}\psi(n, s', t)$. We only need to prove that $n^{o\prime}$ cannot be $n_2' > 2$ when $n^o = 2$, i.e., $\psi(n_2', s', t) > \psi(2, s', t)$ when

$\psi(n_2', 0, t) > \psi(2, 0, t)$, but this is true because $\psi(n_2', s', t) = \psi(n_2', 0, t) + n_2' s' > \psi(2, 0, t) + 2s' = \psi(2, s', t)$.

If $s > 0$, by Lemma 4 $(ii.b)$, $n^o = \arg\min_{n\in\{2,n_2\}} \psi(n, s, t)$ and $n^{o\prime} = \arg\min_{n\in\{2,n_2'\}} \psi(n, s', t)$. By the Envelope theorem,

$$\text{sign}\frac{dn_2}{ds} = \text{sign} - \frac{\partial^2}{\partial n \partial s}\psi(n, s, t) = (-), \tag{A.5}$$

since $\frac{\partial^2}{\partial n\partial s}\psi(n, s, t) = 1$. Hence, $n_2' < n_2$. Suppose that $n^o = n_2$, then $n^{o\prime}$ will be greater than n^o only if $n_2' > n^o$, but $n_2' < n_2 = n^o$ by $(A.5)$. Suppose that $n^o = 2$, then $n^{o\prime}$ will be greater than n^o only if $n_2' > 2$, i.e., $\psi(2, s, t) < \psi(n_2, s, t)$ and $\psi(2, s', t) > \psi(n_2', s', t)$, but $\psi(2, s', t) = \psi(2, s, t) + 2(s' - s) < \psi(n_2, s, t) + 2(s' - s) \leq \psi(n_2, s, t) + n_2'(s' - s) < \psi(n_2', s, t) + n_2(s' - s) = \psi(n_2', s', t)$, contradiction.

(ii) is immediate from Proposition 2. ■

Proof of Proposition 4. (i) By Lemma 4, $\varphi(s, t) = \min\{\psi(2), \psi(n_2), \psi(\infty)\}$. By the Envelope Theorem, $\frac{d}{ds}\psi(n_2) = n_2 > 0$. At the same time, $\frac{d}{ds}\psi(2) = 2 > 0$ and $\frac{d}{ds}\psi(\infty) = n > 0$. Therefore, $\frac{\partial}{\partial s}\varphi(s, t) > 0$.

(ii) First, $\frac{d}{dt}\psi(2) = \frac{\partial}{\partial t}\left(1 - \frac{t(1-q)}{q((1-q)-t)}\right)t = \frac{t^2-2t(1-q)+q(1-q)^2}{q(1-q-t)^2}$, $\frac{d^2}{dt^2}\psi(2) = \frac{\partial^2}{\partial t^2}\left(1 - \frac{t(1-q)}{q((1-q)-t)}\right)t = -\frac{2}{q}\frac{(1-q)^3}{(1-q-t)^3} < 0$, so $\psi(2)$ increases with t if and only if $t < 1-q-\sqrt{(1-q)^3}$. Second, by the Envelope Theorem, $\frac{d}{dt}\psi(n_2) = \frac{\partial}{\partial t}(1 - \exp(-\gamma/(n_2 - 1)))\,tn_2 = -n_2\left(e^{-\frac{\gamma}{n_2-1}} - 1\right)\frac{\partial\gamma}{\partial t} < 0$, i.e., $\psi(n_2)$ decreases with t. Third, by Lemma 4 (iii), $\frac{d^2}{dt^2}\psi(\infty) = \frac{\partial^2}{\partial t^2}\left(-t\ln\frac{t(1-q)}{q((1-q)-t)}\right) = -\frac{1}{t}\frac{(1-q)^2}{(1-q-t)^2} < 0$. If $s > 0$, by Lemma 4 $(ii.b)$, $\varphi(s, t) = \min\{\psi(2), \psi(n_2)\}$, Since $\psi(2)$ and $\psi(n_2)$ are both quasi-concave, $\varphi(s, t)$ must be quasi-concave in t. If $s = 0$, by Lemma 4 $(ii.a)$, $\varphi(s, t) = \min\{\psi(2), \psi(\infty)\}$. Since $\psi(2)$ and $\psi(\infty)$ are both concave in t, $\varphi(s, t)$ must be concave in t. Last, since $\varphi(s, 0) = \varphi(s, \bar{t})$, where $\bar{t} = q(1 - q)(c_h - c_l)$, $\varphi(s, t)$ must be unimodal in t. ■

Proof of Proposition 5. (i) and (ii) are immediate from the proof of Lemma 4 (i) and $(ii.a)$.

(iii) By the Envelope theorem, $\frac{d}{ds}E(p|n = n_2) = \frac{d}{ds}\psi(n_2) - \frac{d}{ds}n_2 s = n_2 - n_2 - s\frac{dn_2}{ds} \geq 0$. At the same time, $E(p|n = 2)$ is a constant with respect to s. By Proposition 3, n^o decreases with s. This means that, when s increases to s', we cannot have $n^o = 2$ and $n^{o\prime} = n_2 > 2$. Hence, the only possibility for a price decreases after an increase in s is to have $\psi(n_2) < \psi(2)$

but $E(p|n=n_2) > E(p|n=2)$. If this is true, then we must have $\psi(n_2) - 2s < \psi(2) - 2s = E(p|n=2) < E(p|n=n_2) = \psi(n_2) - n_2 s$, contradiction. ■

Proof of Proposition 6. Suppose that initially $E(p_{\min}|2,2) - E(p_{\min}|3,2) < s < E(p_{\min}|1,2) - E(p_{\min}|2,2)$, then we must have $n^{PI} = 2$ according to (3.4) and $E(p_{\min}|2,2) = c_E + \psi(2) - 2s$. If s decreases to 0, then $n^{PI} > 2$, but $E\left(p_{\min}|n^{PI}, n^{PI}\right) = c_E + \Psi\left(n^{PI}, n^{PI}\right) = c_E + \psi\left(n^{PI}\right) > c_E + \psi(2)$ for $n > 2$ if $\frac{t}{c_h - c_l} < \frac{q(1-q)}{4.92(1-q)+q}$ by Proposition 2. ■

B Additional Results When the Number of Price Quotes is Private Information

B.1 The Expected Price

Let $E(p_{\min}|n,m)$ denote the lowest price quoted by n sellers when each of them believes that the consumer obtains m price quotes. Also let $F_{l,m}(p)$ denote the corresponding price distribution of informed bids, $F_{b,m}(p)$ that of blind bids, and α the probability of a seller choosing to acquire information about the production cost. From $(A.3)$, we obtain that

$$
\begin{aligned}
& E(p_{\min}|n,m) \\
&= \alpha^n (1-q) c_h + \alpha^n q \int p d\left(1 - (1 - F_{l,m}(p))^n\right) + (1-\alpha)^n \int p d\left(1 - (1 - F_{b,m}(p))^n\right) \\
&+ \sum_{k=1}^{n-1} \binom{n}{k} \alpha^k (1-\alpha)^{n-k} \left((1-q) \int p d\left(1 - (1 - F_{b,m}(p))^{n-k}\right) + q \int p d\left(1 - (1 - F_{l,m}(p))^k\right)\right) \\
&= \alpha^n (1-q) c_h + n \int_0^1 \begin{pmatrix} q\alpha (1-\alpha F)^{n-1} F_{l,m}^{-1}(F) \\ + (1-\alpha)^n (1-F)^{n-1} \left(q + (1-q)\left(\frac{\alpha}{(1-\alpha)(1-F)} + 1\right)^{n-1}\right) F_{b,m}^{-1}(F) \end{pmatrix} dF \\
&= \alpha^n (1-q) c_h + n \int_0^1 \begin{pmatrix} q\alpha (1-\alpha F)^{n-1} \left(c_l + \frac{t/q}{(1-\alpha F)^{m-1}}\right) + (1-\alpha)^n (1-F)^{n-1} \\ * \left(q + (1-q)\left(\frac{\alpha}{(1-\alpha)(1-F)} + 1\right)^{n-1}\right) \frac{qc_l + (1-q)c_h \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1\right)^{m-1}}{q + (1-q)\left(\frac{\alpha}{(1-\alpha)(1-F)} + 1\right)^{m-1}} \end{pmatrix} dF
\end{aligned}
$$

(B.1)

$$
\begin{aligned}
&= \alpha^n (1-q) c_h + qc_l \left(1 - (1-\alpha)^n\right) + n \int_0^1 \alpha t (1-\alpha F)^{n-m} dF \\
&+ n \int_0^{1-\alpha} G^{n-1} \left(q + (1-q)\left(\frac{\alpha}{G} + 1\right)^{n-1}\right) \left(\frac{qc_l + (1-q) c_h \left(\frac{\alpha}{G} + 1\right)^{m-1}}{q + (1-q)\left(\frac{\alpha}{G} + 1\right)^{m-1}}\right) dG.
\end{aligned}
$$

From $(B.1)$, we can get $E\left(p_{\min}|1,2\right)=c_E+\alpha q\left(1-q\right)\left(c_h-c_l\right)\ln\frac{1-\alpha q}{\alpha(1-q)}-t\ln\left(1-\alpha\right)$, $E\left(p_{\min}|2,2\right)=c_E+2\alpha t$ and $E\left(p_{\min}|3,2\right)=c_E+3\alpha t\left(1-\alpha/2\right)+3\alpha q\left(1-q\right)\left(c_h-c_l\right)\cdot\left(\alpha^2 q\left(1-q\right)\ln\frac{1-\alpha q}{\alpha(1-q)}-\frac{1}{2}\left(1-\alpha\right)\left(1-\alpha+2q\alpha\right)\right)$. It is not difficult to verify that (3.1) and (3.2) hold.

B.2 Excessive Search

Example 1 *Consider the following parameter values:* $q=1/2$, $c_l=0$, $c_h=1$ *and* $t=0.15$. *It is not difficult to verify by Proposition 2 that* $n^o=2$ *as long as* $s>0.023$.

However, from $(B.1)$, *we can get* $E\left(p_{\min}|2,3\right)=\alpha^2/2-2t\ln\left(1-\alpha\right)+\frac{1}{2}\alpha-\alpha^2+\frac{1}{2}+\frac{1}{2}\alpha^2\arctan\frac{-2+\alpha}{\alpha}+\frac{1}{8}\alpha^2\pi=0.706$, $E\left(p_{\min}|3,3\right)=0.655$, *and* $E\left(p_{\min}|3,4\right)=-\frac{2}{3}\alpha^4-2\alpha^2 t+4\alpha t+\frac{1}{2}\alpha^3+\frac{1}{2}\alpha^2-\frac{1}{3}\alpha+\frac{1}{2}+\frac{1}{2}\alpha^4\arctan\frac{-2+\alpha}{\alpha}+\frac{1}{8}\alpha^4\pi=0.623$. *Thus, if* $s\in\left(0.032\,,0.051\right)$, *then* $n=3$ *is an equilibrium that involves excessive search.*