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Chen, Zhijun

School of Economics, Zhejiang University

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# Fighting Collusion in Tournaments<sup>1</sup>

Zhijun Chen<sup>2</sup>

School of Economics, Zhejiang University, China

E-mail: chenzj1219@gmail.com

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This paper proposes a new approach of fighting collusion in tournaments which sheds light on the principle of divide and conquer: the principal can benefit from manipulating information revelation, by which he brings asymmetric information between the agents and thus creates a distortion of efficiency in the coalition. We employ a simple tournament setting where, due to perfect collusion, the efficient effort levels are impossible to be implemented through simple mechanisms. We propose a sophisticated mechanism with a biased promotion rule that allows the principal to manipulate the revelation of information and make asymmetric information between the agents, which brings trade-offs between rent-extraction and distortion of efficiency into the coalition. We show that, it is possible to implement efficient effort levels under the sophisticated mechanism.

JEL Classification: C72, D82

*Key Words:* Collusion, Tournaments

## 1. INTRODUCTION

Rank-order tournaments prevail in organizations when there is difficulty in verifying agents' performance; and they are proved to be effective incentive schemes in many institutions such as bureaucracies and armies, as well as corporations. The fixed-prize tournament contracts serve as commitment devices for contract designers as well as competitive mechanisms for agents whose effort levels are unobservable by others, and these properties are analyzed in the literature of contract theory and labor economics.<sup>3</sup>

There is, however, a great challenge to the efficacy of tournaments in practice: the fixed-prize incentive schemes based on relative performance are vulnerable to collusion. As relative performance is determined by relative rather than absolute effort levels, this allows agents to reduce their absolute effort levels while still keep their relative effort levels unchanged. In the case of homogenous agents, the rank orders when all agents work are the same as when all shirk, which means that the agents can benefit from saving their effort costs. This nature of tournament mechanism will eventually lead to agents' cooperative sabotages, which aims to reduce their efforts collectively. When the agents can form a coalition enforced

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<sup>3</sup>For instance, see Green and Stokey (1983), Lazear and Rosen (1981), Nalebuff and Stiglitz (1983).

by a side contract, it is impossible to induce efficient effort levels under simple tournament mechanisms, as argued by Ishiguro (2004).

The phenomenon of collusion under tournaments is commonly observed in bureaucratic institutes as well as firms, which results to the loss of efficiency in these organizations. For example, as Miller (1992) describes a so-called "binging" game played between works when discussing the compensation scheme of the banking-wiring room in the Hawthorne plant of Western Electric; this game is employed to punish the workers who produce too much, which is indeed a collusion-enforcing device to prevent the workers from exerting high effort levels. While the reported cases of collusion in tournaments can be found in almost every kind of organizations, they are still a tip of the iceberg: due to their illegality, the collusive coalition hide themselves, we observe only a part of discovered coalition.

Since the tournaments were created as a promotion device in bureaucratic systems from ancient dynasties, the phenomenon of collusion has been an austere threat to these organizations. Fighting collusion thus has become a permanent challenge to the politicians and scholars in political science and sociology. However, collusive phenomena have been by and large ignored by the economists, with a few exceptions of Chichago School's approach to regulatory capture in 1970s. Since 1980s, there has been a growing interest in studying collusive behavior, in various environments such as industrial organization, regulation and political economy.

This paper is motivated to bring some new ideas for fighting collusion in tournaments, which highlights the spirit of "divide and conquer": in order to prevent collusion, the principal should create conflicts among coalition members rather than give up stakes to agents. One of the effective way of creating conflicts is to introduce asymmetric information between agents, as recorded by the following famous story in Chinese history.

About 1800 years ago in Chinese Dynasty of Three Kingdoms, a war was broken out between the primer minister of Han Dynasty Cao Cao and a coalition formed by two local governors in Northwestern China, Chao Ma and Sui Han. In the coalition, Chao Ma, whose father was murdered by Cao Cao, is an aggressive young general, and he launched the war to revenge for his father. Sui Han who was a good friend of Ma's father, however, was once to be a colleague of Cao Cao in bureaucracy, and he was involved in the war to help Ma. The war was in stalemate after several rounds of severe battles, which made both sides fretted. At one night, Cao Cao was inspired by a good trick. He wrote a letter to Sui Han, which was nothing but a trivial letter of greeting, and then blacked some sentences deliberately. While Sui Han was wondering at this strange letter, Chao Ma rushed into his room full of anger. A dispute between them was inevitable as Ma insisted that it must be Han who blacked these sentences in order to conceal some important information. The irreconcilable conflict led to a failure of the coalition, as a result, both armies were defeated by Cao Cao finally.<sup>4</sup>

In this famous story, the strange letter with blacked sentences brings asymmetric information between two allies, which creates conflict inside the coalition. This conflict cannot be reconciled since the coalition is sustained by non-binding agreement, and the coalition collapses as a result.

When a coalition is enforced by a binding agreement or side contract, such as in organized crime and political cliques, introducing asymmetric information

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<sup>4</sup>This story is recorded in "A Romance of Three Kingdoms", a famous Chinese historical romance.

between agents can bring trade-offs between rent-extraction and distortion of efficiency, which undermines the coalition and benefits the principal as a result. Based on this idea, we develop a sophisticated mechanism in a simple tournament setting, which allows the principal to manipulate information revelation and thus introduce asymmetric information between agents. In this mechanism, the principal designs a biased promotion rule that favors a lucky agent who is chosen at random. After selecting the lucky agent, the principal hides the information about who is lucky or not and then reveals it only to one agent. This creates asymmetric information between agents and thus brings trade-offs between rent-extraction and efficiency in the coalition. Moreover, the effect of countervailing incentives arises due to the difference of participation costs between lucky and unlucky type, which brings an incentive for the unlucky type to mimic the lucky type. To extract information rents of the unlucky type, the coalition has to distort its efficiency by inducing high effort levels when the informed agent is lucky type. Hence, it is possible to implement efficient effort levels under the sophisticated mechanism.

In this paper, we are not motivated to seek for an optimal collusion-proof mechanism which implements the efficient effort pair at least costs. The main contribution of this paper is of two folds: first, the principle of divide and conquer is highlighted at the first time, which brings a new and active concept to the theory of collusion-proof mechanism design. Second, while the Revelation Principle fails under collusion, restricting attention only to direct collusion-proof mechanisms would be never optimal. This paper proves that, the indirect and sophisticated mechanism performs better than the direct and simple mechanisms in fighting collusion, which appeals to rethink the methodology of the literature.

The theory of collusion-proof mechanism design has grown rapidly since the seminal paper by Tirole (1986). Under the framework of adverse selection, Laffont and Martimort (1997) have shown that the optimal outcome can be made collusion-proof at no cost to the principal if the agents' types are uncorrelated; however, Laffont and Martimort (2000) have also shown that preventing collusion entails strict cost to the principal with correlated types. More recently, Che and Kim (2005) develop a general method for collusion-proof mechanism design, which utilizes the idea of "selling the firm to the agents". They show that any payoff attainable by the principal in the absence of collusion can be attained in the presence of collusion, in a large class of environments with risk neutral agents for both uncorrelated and correlated types cases whenever there are more than two agents. However, when there are only two agents whose types are correlated, collusion-proof entails strictly costs under their mechanism.

Under moral hazard settings, Varian (1990) has shown that preventing collusion is costly to the principal if agents' actions are privately observable. However, when agents can mutually observe each others' actions, Itoh (1993) shows that the principal can benefit from agents' side contracting and mutually monitoring, whereas the first-best allocations are unattainable due to perfect collusion. When outputs are unverifiable, preventing collusion becomes more difficult under rank-order tournaments, in the sense that, efficient effort levels is impossible to be implemented under collusion, as shown by Ishiguro (2004).

The basic framework of collusion-proof mechanism design is built on contract theory, which allows to integrates collusion as part of the general mechanism design under a hierarchy of organizations. As a shortcut of the methodology, it is assumed that a coalition is formed through a side contract, which is designed and enforced by a benevolent third party called side mediator.

The so-called "full-side-contracting" assumption plays a key role in collusion-proof mechanism design; it provides a simple and neat approach in modeling coalition formation and thus avoids the problem of finding an extensive form for describing the collusive game between agents. The third party paradigm can be seen as a black box for the repeated interaction by which collusion emerges, and it characterizes the upper bound that can be achieved by the coalition.

While the literature restricts attention to the public enforcement of grand contract, the relative situation between the principal and collusive agents is not characterized adequately, as argued by Chen (2006b). It is assumed that the side contract is enforced privately, as a black box which cannot be observed by the principal; while the grand contract is enforced publicly, as a transparent box observed by the coalition. Then, the situation between the principal and collusive agents is asymmetric in the sense that, the coalition is entitled more contracting power than the principal who is restricted only to public contracting.

Chen (2006b) then introduce the an new approach of private enforcement to fight collusion, in a framework of moral hazard with mutually observed actions a la Itoh (1993); and show that first-best allocation can be implemented through an indirect mechanism which incorporates secret reporting and private enforcing of transfers.

The rest of this paper is organized as follows. In section 2, a simple tournament model is presented. As a benchmark, in section 3, we derive the optimal tournament contract in the absence of collusion and then illustrate the impossibility result of implementing efficient effort pair. In section 4, we introduce a biased promotion rule to make efforts pair-wise identifiable, and shows that it is not sufficient to implement efficient effort pair. We then develop the sophisticated mechanism with manipulated information in section 5 and show the main theme which asserts that efficient effort pair is possible to be implemented. We finally conclude in section 5.

## 2. THE MODEL

A risk-neutral principal has two independent tasks delegated to two identical and risk-neutral agents,  $A_1$  and  $A_2$ , whose effort levels are only privately observable. Agents are recruited from a competitive labor market and protected by limited liability with reservation salary 0. Each agent  $i, i = 1, 2$ , can choose an effort level  $e^i \in \{1, 0\}$ , namely "work" or "shirk", at a cost  $\psi(e^i)$  which satisfies  $\psi(1) = \psi > 0$  and  $\psi(0) = 0$ . For a given effort level, each independent production process generates one of the two possible outcomes  $q \in \{\bar{q}, \underline{q}\}$ , namely, "success" and "failure", with  $\bar{q} > \underline{q}$ . Let  $\pi(e^i) = \pi_1$  (resp.  $\pi(e^j) = \pi_0$ ) denote the probability of success given an agent works (resp. shirks), with  $1 > \pi_1 > \pi_0 > 0$ .

Assume that neither absolute nor relative performance is verifiable<sup>5</sup>. The principal can design only a rank-order tournament with fixed prizes, which assigns two positions, high and low with prize  $T$  and 0, to the winner and loser respectively.

The principal benefits only from the success of production which generates a revenue  $R$  to him. His expected utility is expressed by:

$$V(e) = [\pi(e^i) + \pi(e^j)]R - T,$$

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<sup>5</sup>This assumption is less restrictive than Ishiguro (2004) who assumes that the relative performance, or rank-orders are verifiable. As we will see later, when the relative performance is not verifiable, Ishiguro's discrimination mechanism cannot be employed.

For any effort pair  $e = (e^i, e^j)$ , denote by  $p_i(e)$  agent  $i$ 's probability of winning. Agent  $i$ 's expected utility is given by:

$$U_i(e) = p_i(e)T - \psi(e^i).$$

The following notation is useful for simplicity:

$$\underline{e} = (0, 0), \hat{e} = (1, 0), \tilde{e} = (0, 1), \bar{e} = (1, 1),$$

where the first variable stands for agent  $A_i$ 's effort level and second for his peer's.

Let  $\Delta\pi = \pi_1 - \pi_0$  denote the probability premium of success when an agent works rather than shirks. Assume the benefits from agents success are so large that the principal always prefers agents' working; this allows us to focus on the problem of implementing efficient effort pair  $\bar{e}$ .

### 3. BENCHMARK: SIMPLE TOURNAMENT MECHANISM

In the absence of collusion, the second-best tournament prize of implementing efficient effort pair  $\bar{e}$  can be derived from solving the following optimization program:

$$(P.1) \text{ Min } T$$

Subject to:

$$(IR_i) U_i(\bar{e}) = p_i(\bar{e})T - \psi \geq 0, \quad i = 1, 2.$$

$$(IC_i) U_i(\bar{e}) \geq p_i(\tilde{e})T, \quad i = 1, 2.$$

Where  $(IR_i)$  is agent  $A_i$ 's participation constraint and  $(IC_i)$  is his incentive compatibility constraint of working.

Suppose the promotion rule is fair and symmetric: if agent  $A_i$  has better performance than his peer, then he gets the high position and his peer gets the low position; if both agents have the same performance, then ties are broken by tossing an unbiased coin which assigns each agent probability of winning  $\frac{1}{2}$ . Given an effort pair  $e = (e^i, e^j)$ , this promotion rule yields the following probability of winning:

$$p_i(e^i, e^j) = \Pr\{q_i > q_j | e\} + \frac{1}{2} \Pr\{q_i = q_j | e\} = \pi(e^i)(1 - \pi(e^j)) + \frac{1}{2}[\pi(e^i)\pi(e^j) + (1 - \pi(e^i))(1 - \pi(e^j))]$$

Under this promotion rule, an agent works rather than shirks if and only if his expected utility of working  $\frac{1}{2}T - \psi$  exceeds his expected utility of shirking  $[\pi_0(1 - \pi_1) + \frac{1}{2}\pi_0\pi_1 + \frac{1}{2}(1 - \pi_0)(1 - \pi_1)]T$ , given that his peer works. Hence, the optimal prize for the high position can be derived from the binding incentive compatibility constraint  $(IC_i)$ :

$$T^* = \frac{2\psi}{\Delta\pi}.$$

However, this simple mechanism is not robust to collusion. Suppose both agents collude to take lower effort pair  $\underline{e}$ . The relative performance under effort pair  $\underline{e}$  is the same as under effort pair  $\bar{e}$ , which implies that each agent can benefit from saving his effort cost  $\psi$  by collectively shirking. Therefore, the agents have incentives to form coalition. To characterize collusive behavior, we assume that a coalition is sustained by a side contract:

**Assumption 1.** (full side contracting) *A coalition formed by agents is sustained through a side contract, which is designed and enforced by a benevolent side mediator.*

Assumption 1, which is also called the full-side-contract assumption, allows us to deal with coalition formation under a neat and simple framework of contract theory. Under this assumption, the problem of coalition formation is regarded as an optimization program of the side mediator, whose objective is to maximize the

total welfare of coalition members, subject to coalition incentive compatibility constraints and coalition participation constraints. In other words, we assume that the constrained Pareto efficient allocations can be attained in the coalition. According to Myerson (1982), Assumption 1 then warrants the Revelation Principle in side contracting: without loss of generality, the side mediator can restrict attention to incentive compatible direct mechanisms.

Under assumption 1, consider the following simple side contract: whoever wins the prize must pay a side payment  $s = \frac{1}{2}T$  to his losing peer. Under this side contract, both agents are fully insured under collusion, and thus have no incentives to work at all. Therefore, this side contract induces the low effort pair  $\underline{e}$ , as expressed in the following proposition:

**PROPOSITION 1.** *In the presence of collusion, only low effort levels can be implemented under the simple tournament mechanism.*

*Proof.* Denote by  $\widehat{U}_i(e)$  agent  $i$ 's utility level under collusion, as expressed by:  
 $\widehat{U}_i(e) = p_i(e)(T - s(e)) + (1 - p_i(e))s(e) - \psi(e^i)$ .

Given any tournament contract, consider the following optimization program of the side mediator:

$$\begin{aligned}
[P.1]: \quad & \max_e T - [\psi(e^i) + \psi(e^j)] \\
\text{Subject to:} \quad & \text{for } i = 1, 2, \\
(CIR_i) \quad & \widehat{U}_i(e) \geq U_i(\bar{e}) = \frac{1}{2}T - \psi \\
(CIC_i) \quad & \widehat{U}_i(e) \geq \widehat{U}_i(\widehat{e}^i, e^j) = p_i(\widehat{e}^i, e^j)(T - s(e)) + (1 - p_i(\widehat{e}^i, e^j))s(e) - \psi(\widehat{e}^i), \\
& \forall \widehat{e}^i \neq e^i. \\
(CLL) \quad & 0 \leq s(e) \leq T.
\end{aligned}$$

Where  $(CIR_i)$  is the coalition participation constraint of agent  $A_i$  which brings stakes of collusion;  $(CIC_i)$  is the coalition incentive compatibility constraint which resolves agent  $A_i$ 's moral hazard in the coalition; and  $(CLL)$  is the coalition limited liability constraint imposed on side payments.

It is easy to check that side contract  $s(\underline{e}) = \frac{1}{2}T$  solves the optimization program [P.1] which implements the low effort pair  $\underline{e}$ . Q.E.D. ■

Proposition 1 asserts that efficient effort pair  $\bar{e}$  is impossible to be implemented under full side contracting. Ishiguro (2004) also proves this impossibility result in a different setting of tournament. Moreover, when agents' identities and relative performance are verifiable such that unanonymous incentive schemes are feasible, Ishiguro (2004) shows that a discriminatory scheme based on agents' identities is sufficient to implement asymmetric effort pair  $\widehat{e} = (1, 0)$  which induces the favored agent to work and the disfavored to shirk in spite of collusion. However, under the assumption of nonverifiable relative performance, Ishiguro (2004) discrimination mechanism cannot be applied, which implies that only low effort levels can be induced under simple tournament mechanisms.

#### 4. TOURNAMENT MECHANISM WITH BIASED PROMOTION RULE

The intuition behind this impossibility theorem can be further illustrated. Under the simple tournament mechanism, the promotion rule is unbiased, that is, given any symmetric effort pair  $e = (e^i, e^j)$  with  $e^i = e^j$ , each agent has equal probability of winning:  $p_i(\bar{e}) = p_i(\underline{e}) = \frac{1}{2}, i = 1, 2$ . In other word, effort pair  $\bar{e}$  and  $\underline{e}$  are not distinguishable statistically. Hence, the expected rewards of the agents when both work are the same as when both shirk, which brings the stakes of collusion.

Therefore, to break down this impossibility result, the principal must design a biased promotion rule which generates different probability distributions for different effort pairs. Consider the following biased promotion rules:

Agent  $A_i$  obtains the high position if he has better performance than his peer, namely,  $q_i > q_j$ . When both agents have the same performance, different promotion rules are employed. If  $q_i = q_j = \underline{q}$ , ties are broken by an unbiased coin; if  $q_i = q_j = \bar{q}$ , then ties are broken through a biased lottery which assigns probability of winning  $1 > v > \frac{1}{2}$  to the favored agent and  $1 - v < \frac{1}{2}$  to the disfavored agent.

To avoid discrimination, the favored agent is selected by the principal through an unbiased coin at the beginning of the game, which ensures that both agents have equal chance to be favored. The lucky (unlucky) agent who is selected to be favored (disfavored) by chance is denoted by  $A_l$  ( $A_u$ ). For any effort pair  $e = (e^i, e^j)$ , the probability of winning for the lucky and unlucky agent, as denoted by  $p_l$  and  $p_u$  respectively, are expressed by:

$$\begin{aligned} p_l(e^i, e^j) &= \Pr\{q_i > q_j | e\} + v \Pr\{q_i = q_j = \bar{q} | e\} + \frac{1}{2} \Pr\{q_i = q_j = \underline{q} | e\} \\ &= \pi(e^i)(1 - \pi(e^j)) + v\pi(e^i)\pi(e^j) + \frac{1}{2}(1 - \pi(e^i))(1 - \pi(e^j)) \\ p_u(e^i, e^j) &= \Pr\{q_i > q_j | e\} + (1 - v) \Pr\{q_i = q_j = \bar{q} | e\} + \frac{1}{2} \Pr\{q_i = q_j = \underline{q} | e\} \\ &= \pi(e^i)(1 - \pi(e^j)) + (1 - v)\pi(e^i)\pi(e^j) + \frac{1}{2}(1 - \pi(e^i))(1 - \pi(e^j)) \end{aligned}$$

Let  $\Delta p(e^i, e^j) = p_l(e^i, e^j) - p_u(e^i, e^j)$  denote the probability premium of winning that measures the difference of winning probability between the lucky and unlucky type; the biased promotion rule then yields  $\Delta p(e^i, e^j) = (2v - 1)\pi(e^i)\pi(e^j)$ , which implies  $\Delta p(\bar{e}) > \Delta p(e^i, e^j)$ ,  $\forall e = (e^i, e^j) \neq \bar{e}$ . Hence, the probability premium under efficient effort pair  $\bar{e}$  is greater than under low effort pair  $\underline{e}$ , which provides more incentives of working for the lucky type.

While naked discrimination is prohibited, biased promotion rules which are difficult to verify, are quite common in bureaucracy, as widely discussed in the literature of labor economics. Under the biased promotion rule, the agents are discriminated and thus have different status quo utility when they participate in the coalition. The favored agent who has higher status quo utility level will claim more stakes of collusion than the disfavored agent, which causes unequal allocation of stakes in the coalition. If the identity of the favored agent is common knowledge, the side mediator can then design an unanonymous side contract based on agents' identities. Allocating stakes through unanonymous side transfers entails no costs under full side contracting, which implies that there is no loss of efficiency in the coalition. Hence, the simple tournament mechanism with such a biased promotion rule is not sufficient to implement efficient effort pair if the agents' identities of being favored or not are common knowledge, as concluded by:

**PROPOSITION 2.** *Under simple tournament mechanisms with biased promotion rule, only low effort levels can be implemented due to collusion.*

*Proof.* Given any tournament contract with biased promotion rule illustrated above, denote by  $S = \{s_l, s_u\}$  the side contract which specifies side transfers for the lucky and unlucky type respectively. Consider the following optimization program of the side mediator:

$$\begin{aligned} [P.1]: \quad & \max_e T - [\psi(e_l) + \psi(e_u)] \\ \text{Subject to:} \quad & \text{for } i = 1, 2, \\ & (CIR_l) \quad \widehat{U}_l(e) = p_l(e)(T - s_l) + (1 - p_l(e))s_u - \psi(e_l) \geq U_l(\bar{e}) = p_l(\bar{e})T - \psi \\ & (CIR_u) \quad \widehat{U}_u(e) = p_u(e)(T - s_u) + (1 - p_u(e))s_l - \psi(e_u) \geq U_u(\bar{e}) = p_u(\bar{e})T - \psi \\ & (CIC_l) \quad \widehat{U}_l(e) \geq \widehat{U}_l(\widehat{e}_l, e_u) = p_l(\widehat{e}_l, e_u)(T - s_l) + (1 - p_l(\widehat{e}_l, e_u))s_u - \psi(\widehat{e}_l), \forall \widehat{e}_l \neq e_l \end{aligned}$$

$$\begin{aligned} (CIC_u) \quad \widehat{U}_u(e) &\geq \widehat{U}_u(\widehat{e}_u, e_l) = p_l(\widehat{e}_u, e_l)(T - s_u) + (1 - p_u(\widehat{e}_u, e_l))s_l - \psi(\widehat{e}_u), \\ \forall \widehat{e}_u &\neq e_u \\ (CLL) \quad 0 &\leq s_l, s_u \leq T \end{aligned}$$

Where  $(CIR_l)$  ( $(CIR_u)$ ) is the coalition participation constraint of agent  $A_l$  ( $A_u$ ) which brings stakes of collusion;  $(CIC_l)$  ( $(CIC_u)$ ) is the coalition incentive compatibility constraint which resolves agent  $A_l$ 's ( $A_u$ ) moral hazard in the coalition; and  $(CLL)$  is the coalition limited liability constraint imposed on side payments.

It is easy to check that side contract  $s_l = p_u(\bar{e})T$ ,  $s_u = p_l(\bar{e})T$  solves the optimization program [P.1]. This simple side contract yields the virtual incentive power  $T - s_l - s_u = 0$ , hence both agents are fully insured under this side contract and thus have no incentives to work at all, which implements the low effort pair  $\underline{e}$ . Q.E.D ■

## 5. SOPHISTICATED MECHANISM WITH MANIPULATED INFORMATION

### 5.1. Mechanism and Timing

Proposition 2 asserts that, due to perfect collusion, implementing efficient effort pair is impossible under simple and direct tournament mechanisms, even though unanonymous and biased promotion rules are taken into account.

We argue that, while the Revelation Principle fails in the grand mechanism due to collusion, restricting attentions only to direct mechanisms would be never optimal. Therefore, to implement efficient effort pair, indirect and sophisticated mechanisms should be considered. According to the Principle of divide and conquer, to undermine the coalition, the principal should create asymmetric information between agents. Indeed, after selecting a favored agent, the principal can hide the information about who is favored or not, and then reveals it only to one agent, say agent  $A_1$ . This brings asymmetric information into the coalition. The timing of this sophisticated mechanism with manipulated information is illustrated as follows:

#### [Timing of the mechanism]

1. The principal proposes a grand mechanism  $G$ , which is approved if no agent vetoes.
2. The principal tosses a coin to select a favored agent, and then reveals the outcome only to the informed agent.
3. The side mediator proposes a side mechanism  $S$ , which is ratified if no agent vetoes.
4. Both agents take efforts simultaneously, which are only privately observed.
5. Outcomes realize; both grand and side contract are enforced.

*Remark 1.* By assumption  $v < 1$ , which implies that the lottery for the tie-broken is fully stochastic. Hence, agents' identities cannot be revealed through ex post information, which implies that the side contract cannot be contingent on ex post information.

Under this sophisticated mechanism with manipulated information, asymmetric information between agents causes conflicts in the coalition. The side mediator now faces two kinds of agents: the informed agent with hidden information as well as hidden actions, and the uninformed agent with hidden actions only. By the Revelation Principle, without loss of generality, the side mediator can design a direct side mechanism  $S = \{(E_l, S_l), (E_u, S_u)\}$  to extract the hidden information of informed agent:

$$E_l = (e_l^1, e_u^2), E_u = (e_u^1, e_l^2), S_l = (s_l^1, s_u^2), S_u = (s_u^1, s_l^2)$$

Where the subscripts  $l$  and  $u$  stand for lucky and unlucky type. Denote by  $E = (E_l, E_u)$  effort pairs of the coalition; with a bit abuse of notation, let  $C(E)$  denote the coalition induces this effort pair.

Under this side mechanism, the side mediator requires the informed agent  $A_1$  to report his type, that is, lucky or unlucky, then chooses a side contract according to his report. For instance, if agent  $A_1$  reports his type "lucky", then side contract  $(E_l = (e_l^1, e_u^2), S_l = (s_l^1, s_u^2))$  will be chosen, which assigns agent  $A_1(A_2)$  effort level  $e_l^1(e_u^2)$  and side payment to his peer  $s_l^1(s_u^2)$  if he wins respectively.

## 5.2. Possibility of Implementing Efficient Effort Pair

Under this sophisticated grand mechanism with manipulated information, the program of side contracting becomes much complicated, which involves in the incentive problem of adverse selection with countervailing incentives, mixed with moral hazard.<sup>6</sup>

In this paper, we are not motivated to solve the optimal collusion-proof tournament contract which implements the efficient effort pair. To prove that the efficient effort pair is possible to be implemented, it is sufficient to show that there exists trade-off between rent extraction and efficiency which causes conflicts between coalition truth-telling constraint and coalition participation constraints. The full characterization of the program of side contracting is presented in appendix A.

First of all, consider the interim coalition participation constraints of the informed agent, for the lucky and unlucky type respectively:

$$[CIR_l^1(E)]: \widehat{U}_l^1(e_l^1, e_u^2) \geq U_l(\bar{e})$$

$$[CIR_u^1(E)]: \widehat{U}_u^1(e_u^1, e_l^2) \geq U_u(\bar{e})$$

And the ex ante coalition participation constraint of the uninformed agent:

$$[CIR^2(E)]: \widehat{U}_l^2(e_l^2, e_u^1) + \widehat{U}_u^2(e_u^2, e_l^1) \geq U_l(\bar{e}) + U_u(\bar{e})$$

Which can be rewritten by:

$$[CIR^2(E)]: \frac{1}{2}[\widehat{U}_l^1(e_l^1, e_u^2) + \widehat{U}_u^1(e_u^1, e_l^2)] \leq T - \frac{1}{2}[U_l(\bar{e}) + U_u(\bar{e})] - \Psi$$

Where  $\Psi = \frac{1}{2}[\psi(e_l^1) + \psi(e_u^1) + \psi(e_l^2) + \psi(e_u^2)]$  is the expected effort cost.

Under the biased promotion rule, agents have different status quo utility levels when participating into the coalition, which implies that they will claim different stakes of collusion. Let  $\Delta\widehat{U}(E) = \widehat{U}_l^1(e_l^1, e_u^2) - \widehat{U}_u^1(e_u^1, e_l^2)$  denote the spread of utilities between lucky and unlucky agents,  $\Delta p(e) = p_l(e) - p_u(e)$  denote the probability premium of winning. By combing two coalition participation constraints  $[CIR_l^1(E)]$  and  $[CIR^2(E)]$ , yields the lower bound of  $\Delta\widehat{U}(E)$  determined by the difference of the status quo utility:

$$\Delta\widehat{U}(E) \geq \Delta p(\bar{e})T - (4\psi - 2\Psi).$$

The spread of utilities  $\Delta\widehat{U}(E)$ , which brings incentives for the unlucky type to mimic the lucky type, increases with tournament prize  $T$ . Hence, the problem of countervailing incentives arises as  $T$  becomes sufficiently large, which implies that the truth-telling constraint of the unlucky type is relevant:

$$[CTT_u^1(E)]: \widehat{U}_u^1(e_u^1, e_l^2) \geq \widehat{U}_{u,l}^1(e_l^1, e_u^2) = \widehat{U}_l^1(e_l^1, e_u^2) - \Delta p(e_l^1, e_u^2)T_l^1$$

Where  $T_l^1 = T - s_l^1 - s_u^2$  denotes the virtual incentive power of the informed lucky agent, which measures the difference of the payments between high and low positions under collusion. Note that, the utility level of the unlucky type when he mimics the lucky type  $\widehat{U}_{u,l}^1(e_l^1, e_u^2)$  can be decomposed into two terms: the first term  $\widehat{U}_l^1(e_l^1, e_u^2)$  is the utility level of the lucky type, and the second term  $\Delta p(e_l^1, e_u^2)T_l^1$

<sup>6</sup>See Laffont and Martimort (2002) for detailed discuss.

is the expected cost of his misreporting, which is the product of the probability premium times the virtual incentive power of the lucky type.

To prevent the unlucky type from mimicing the lucky type, the side mediator has to make  $s_l^1 = s_u^2 = 0$  such that  $T_l^1 = T$ , and thus ensures that the expected costs of misreporting exceeds the benefits of lying:

$$\Delta p(e_l^1, e_u^2)T \geq \Delta p(\bar{e})T - (4\psi - 2\Psi).$$

However, by the biased promotion rule yields  $\Delta p(\bar{e}) = (2v-1)\pi_1^2 > \Delta p(e_l^1, e_u^2) = (2v-1)\pi(e_l^1)\pi(e_u^2)$  for any  $E_l = (e_l^1, e_u^2) \neq \bar{e}$ . This implies that the constraint above cannot hold when  $T \geq T^m(v) = \frac{3\psi}{(2v-1)\pi_1\Delta\pi}$ , where  $T^m(v) = \max \frac{4\psi - 2\Psi}{[\Delta p(\bar{e}) - \Delta p(e_l^1, e_u^2)]}$  is the maximal threshold of tournament prize.

The side mediator must raise the virtual incentive power of the lucky type to extract information rents of the unlucky type, which brings a trade-off between rent-extraction and efficiency as the tournament prize  $T$  increases. When the tournament prize  $T \geq T^m(v)$ , the contradiction between coalition truth-telling constraint of the unlucky type and the coalition participation constraints flunks the program of side contracting for implementing effort pair  $E_l = (e_l^1, e_u^2) \neq \bar{e}$ . To resolve this contradiction, the side mediator has to implement  $E_l = \bar{e}$  under the informed agent is lucky type. On the other hand, no trade-off causes the distortion of coalition effort levels when the informed agent is unlucky type, which implies  $E_u = \underline{e}$ . Hence, the efficient effort pair can be implemented in probability  $\frac{1}{2}$ , as concluded in the following proposition:

**PROPOSITION 3.** *Under the sophisticated mechanism with manipulated information, the efficient effort pair is possible to be implemented in spite of collusion.*

*Proof.* The analysis above shows that, given tournament prize  $T^m(v)$ , only  $E_l = \bar{e}$  is feasible when the informed agent is lucky type. To induce  $(E_l = \bar{e}, E_u = \underline{e})$ , consider the side contract  $S$  which satisfies:  $s_l^1 = s_u^2 = 0$ ,  $s_u^1 = s_l^2 = \frac{1}{2}T^*(v)$ . This side contract yields the virtual incentive power  $T_l^1 = T^m(v)$  and  $T_u^1 = T - s_u^1 - s_l^2 = 0$ . It is easy to check that all the coalition constraints are satisfied under this side contract, inducing effort pairs  $(E_l = \bar{e}, E_u = \underline{e})$  in the coalition. **Q.E.D** ■

*Remark 2.* In the program of side contracting, we exclude the possibility that the side mediator may design a pooling side contract to induce low effort levels, since the pooling side contract is inefficient and thus cannot implement any collusive effort pair  $e \neq \bar{e}$ , as shown in the appendix B.

*Remark 3.* Under this mechanism,  $T^m(v) = \frac{3\psi}{(2v-1)\pi_1\Delta\pi} > T^*$ , hence implementing efficient effort pair entails strictly higher costs than the optimal tournament prize in the absence of collusion. In this paper, however, we only focus on the problem of implementing the efficient effort pair under perfect collusion, and ignore the problem of seeking for an optimal collusion-proof mechanism.

By manipulating information of agents' identities, the principal brings asymmetric information between agents, which causes the trade-off between rent extraction and efficiency in the coalition. Under the biased promotion rule, the lucky type has higher status quo utility than the unlucky type and the spread of status quo utilities between two agents increases with tournament prize. When joining the coalition, the lucky type must be allocated more stakes of collusion to compensate his higher participation costs, which brings incentives for the unlucky type to mimic the lucky type. To prevent the unlucky type from lying, the side mediator must raise the virtual incentive power of the lucky type such that the cost of misreporting exceeds

the benefit of lying which also increases with tournament prize. However, due to limited liability constraint, the virtual incentive power is bounded above to be less than the tournament prize, which imposes an upper bound on the unlucky type's cost of misreporting. As a result, the side mediator has to raise the coalition effort levels when the informed agent is lucky type, which leads to a loss of efficiency in the coalition. As the tournament prize becomes large enough, only high effort levels can be implemented when the informed agent is lucky type.

*Remark 4.* Under this program of side contracting, the problem of moral hazard has no bites to the coalition, since it can be easily resolved through side transferring which entails no costs. Hence, only asymmetric information makes sense in fighting collusion under this mechanism.

## 6. CONCLUDING REMARKS

Tackling collusion is a permanent challenge faced by politicians and scholars, and therefore deserves more attention in both practice and theory. The principle of divide and conquer has been developed in the practice of anti-collusion for a long history with a great success, however, it has been greatly ignored in economic theory of collusion-proof mechanism design.

This paper highlights the principle of divide and conquer by showing how the principal can benefit from manipulating information which creates conflicts in the coalition and thus undermine its efficiency. Hence, it bridges the practice of anti-collusion to the theory. Although the main theme is based on a simple tournament model, unquestionably, the basic idea and new concept can be applied to other settings, which leaves many tasks to further research in the future.

## 7. APPENDIX A: FULL CHARACTERIZATION OF SIDE CONTRACTING PROGRAM

Given the side contract described above, the agents' utility levels in the coalition are expressed by:

$$\begin{aligned}\widehat{U}_l^1(e_l^1, e_u^2) &= p_l(e_l^1, e_u^2)(T - s_l^1) + [1 - p_l(e_l^1, e_u^2)]s_u^2 - \psi(e_l^1) \\ \widehat{U}_u^1(e_u^1, e_l^2) &= p_u(e_u^1, e_l^2)(T - s_u^1) + [1 - p_u(e_u^1, e_l^2)]s_l^2 - \psi(e_u^1) \\ \widehat{U}_l^2(e_l^2, e_u^1) &= p_l(e_l^2, e_u^1)(T - s_l^2) + [1 - p_l(e_l^2, e_u^1)]s_u^1 - \psi(e_l^2) \\ \widehat{U}_u^2(e_u^2, e_l^1) &= p_u(e_u^2, e_l^1)(T - s_u^2) + [1 - p_u(e_u^2, e_l^1)]s_l^1 - \psi(e_u^2)\end{aligned}$$

Suppose the informed agent is lucky type. Let  $\widehat{U}_{l,u}^1(e_u^1, e_l^2)$  denote his utility level when he misreports his type "unlucky",  $\widehat{U}_l^1(\widehat{e}_l^1, e_u^2)$  denote his utility level when he takes effort level  $\widehat{e}_l^1$  rather than  $e_l^1$ , and  $\widehat{U}_{l,u}^1(\widehat{e}_u^1, e_l^2)$  denote his utility level when he misreports as well as misbehaves. To form coalition  $C(E)$ , the following constraints of the informed agent must be satisfied:

$$\begin{aligned}[CTT_l^1(E)]: \quad &\widehat{U}_l^1(e_l^1, e_u^2) \geq \widehat{U}_{l,u}^1(e_u^1, e_l^2) \\ [CTM_l^1(E)]: \quad &\widehat{U}_l^1(e_l^1, e_u^2) \geq \widehat{U}_{l,u}^1(\widehat{e}_u^1, e_l^2) \\ [CIC_l^1(E)]: \quad &\widehat{U}_l^1(e_l^1, e_u^2) \geq \widehat{U}_l^1(\widehat{e}_l^1, e_u^2) \\ [CIR_l^1(E)]: \quad &\widehat{U}_l^1(e_l^1, e_u^2) \geq U_l(\bar{e}) \\ [CLL(E)]: \quad &0 \leq s_l^1, s_u^2 \leq T\end{aligned}$$

Where  $[CIR_l^1(E)]$  is the coalition participation constraint that guarantees his stakes of collusion;  $[CTT_l^1(E)]$  is the coalition truth-telling constraint that warrants

his truth-telling;  $[CIC_l^1(E)]$  is the coalition incentive compatibility constraint to prevent him from misbehaving; and  $[CTM_l^1(E)]$  is the coalition truth-telling and right-behaving constraint which ensures that he has no incentive to misreport as well as misbehave. Moreover,  $[CLL(E)]$  is the coalition limited liability constraint that ensures non-negative utility levels under collusion.

By the same reason, the following constraints must be met when the informed agent is unlucky type:

$$\begin{aligned} [CTT_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq \hat{U}_{u,l}^1(e_l^1, e_u^2) \\ [CTM_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq \hat{U}_{u,l}^1(\hat{e}_l^1, e_u^2) \\ [CIC_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq \hat{U}_u^1(\hat{e}_u^1, e_l^2) \\ [CIR_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq U_u(\bar{e}) \\ [CLL(E)]: \quad & 0 \leq s_u^1, s_l^2 \leq T \end{aligned}$$

We now consider the program of the uninformed agent  $A_2$ . First of all, the ex ante coalition participation constraint must be satisfied since he is uninformed:

$$[CIR^2(E)]: \quad \hat{U}_l^2(e_l^2, e_u^1) + \hat{U}_u^2(e_u^2, e_l^1) \geq U_l(\bar{e}) + U_u(\bar{e})$$

After the informed agent reporting his type, agents' identities become common knowledge. This implies that the following interim coalition incentive compatibility constraint of the lucky and unlucky type must be met:

$$\begin{aligned} [CIC_l^2(E)]: \quad & \hat{U}_l^2(e_l^2, e_u^1) \geq \hat{U}_l^2(\hat{e}_l^2, e_u^1) \\ [CIC_u^2(E)]: \quad & \hat{U}_u^2(e_u^2, e_l^1) \geq \hat{U}_u^2(\hat{e}_u^2, e_l^1) \end{aligned}$$

We can now characterize the optimization program of side contracting as follows: Program [P.2]:

$$\begin{aligned} \max_{\{S\}} \hat{U}^1(E) + \hat{U}^2(E) &= \frac{1}{2}[\hat{U}_l^1(e_l^1, e_u^2) + \hat{U}_u^1(e_u^1, e_l^2) + \hat{U}_l^2(e_l^2, e_u^1) + \hat{U}_u^2(e_u^2, e_l^1)] \\ &= T - \frac{1}{2}[\psi(e_l^1) + \psi(e_u^1) + \psi(e_l^2) + \psi(e_u^2)] \end{aligned}$$

Subject to all constraints above.

Denoting by  $T_l^1 = T - s_l^1 - s_u^2$  and  $T_u^1 = T - s_u^1 - s_l^2$  the virtual incentive power of the informed lucky agent and informed unlucky agent respectively, which measures the difference of the payments between high and low positions under collusion; denoting by  $\Psi = \frac{1}{2}[\psi(e_l^1) + \psi(e_u^1) + \psi(e_l^2) + \psi(e_u^2)]$  the expected effort costs, we can rewrite the program as follows:

$$\text{Program [P.2]: } \max_{\{S\}} \hat{U}^1(E) + \hat{U}^2(E) = T - \Psi$$

Subject to:

$$\begin{aligned} [CTT_l^1(E)]: \quad & \hat{U}_l^1(e_l^1, e_u^2) \geq \hat{U}_u^1(e_u^1, e_l^2) + [p_l(e_u^1, e_l^2) - p_u(e_u^1, e_l^2)]T_u^1 \\ [CTT_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq \hat{U}_l^1(e_l^1, e_u^2) - [p_l(e_l^1, e_u^2) - p_u(e_l^1, e_u^2)]T_l^1 \\ [CTM_l^1(E)]: \quad & \hat{U}_l^1(e_l^1, e_u^2) \geq \hat{U}_u^1(e_u^1, e_l^2) + [p_l(\hat{e}_u^1, e_l^2) - p_u(e_u^1, e_l^2)]T_u^1 + \psi(e_u^1) - \psi(\hat{e}_u^1) \\ [CTM_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq \hat{U}_l^1(e_l^1, e_u^2) - [p_l(e_l^1, e_u^2) - p_u(\hat{e}_l^1, e_u^2)]T_l^1 + \psi(e_l^1) - \psi(\hat{e}_l^1) \\ [CIR_l^1(E)]: \quad & \hat{U}_l^1(e_l^1, e_u^2) \geq U_l(\bar{e}) \\ [CIR_u^1(E)]: \quad & \hat{U}_u^1(e_u^1, e_l^2) \geq U_u(\bar{e}) \\ [CIR^2(E)]: \quad & \frac{1}{2}[\hat{U}_l^1(e_l^1, e_u^2) + \hat{U}_u^1(e_u^1, e_l^2)] \leq T - \frac{1}{2}[U_l(\bar{e}) + U_u(\bar{e})] - \Psi \\ [CIC_l^1(E)]: \quad & [p_l(\hat{e}_l^1, e_u^2) - p_l(e_l^1, e_u^2)]T_l^1 \leq \psi(\hat{e}_l^1) - \psi(e_l^1) \\ [CIC_u^1(E)]: \quad & [p_u(\hat{e}_u^1, e_l^2) - p_u(e_u^1, e_l^2)]T_u^1 \leq \psi(\hat{e}_u^1) - \psi(e_u^1) \\ [CIC_l^2(E)]: \quad & [p_l(\hat{e}_l^2, e_u^1) - p_l(e_l^2, e_u^1)]T_l^1 \leq \psi(\hat{e}_l^2) - \psi(e_l^2) \\ [CIC_u^2(E)]: \quad & [p_u(\hat{e}_u^2, e_l^1) - p_u(e_u^2, e_l^1)]T_u^1 \leq \psi(\hat{e}_u^2) - \psi(e_u^2) \\ [CLL(E)]: \quad & -T \leq T_l^1, T_u^1 \leq T \end{aligned}$$

## 8. APPENDIX B: POOLING SIDE CONTRACT

PROPOSITION 4. *Given tournament prize  $T = T^m(v)$ , only efficient effort pair  $\bar{e}$  can be implemented under pooling side contract.*

*Proof.* Suppose the side mediator proposes a pooling side contract  $S^P(e) = \{e = (e^1, e^2), s = (s^1, s^2)\}$  which assigns the informed agent  $A_1$  a pair of effort level and side payment  $(e^1, s^1)$ , and uninformed agent  $A_2$  a pair  $(e^2, s^2)$ . Then the following coalition constraints must be satisfied:

$$\begin{aligned} [CIR_l^1(e)]: & \hat{U}_l^1(e^1, e^2) \geq U_l(\bar{e}) \\ [CIC_l^1(e)]: & \hat{U}_l^1(e^1, e^2) \geq \hat{U}_l^1(\hat{e}^1, e^2) \\ [CIR_u^1(e)]: & \hat{U}_u^1(e^1, e^2) \geq U_u(\bar{e}) \\ [CIC_u^1(e)]: & \hat{U}_u^1(e^1, e^2) \geq \hat{U}_u^1(\hat{e}^1, e^2) \\ [CIR^2(e)]: & \hat{U}_l^2(e^2, e^1) + \hat{U}_u^2(e^2, e^1) \geq U_l(\bar{e}) + U_u(\bar{e}) \\ [CIC^2(e)]: & \hat{U}_l^2(e^2, e^1) + \hat{U}_u^2(e^2, e^1) \geq \hat{U}_l^2(\hat{e}^2, e^1) + \hat{U}_u^2(\hat{e}^2, e^1) \\ [CLL(e)]: & 0 \leq s^1, s^2 \leq T \end{aligned}$$

Rewrite these constraints as follows:

$$\begin{aligned} [CIR_l^1(e)]: & (1 - p_l(e))s^2 - p_l(e)s^1 \geq (p_l(\bar{e}) - p_l(e))T + \psi(e^1) - \psi \\ [CIR_u^1(e)]: & (1 - p_u(e))s^2 - p_u(e)s^1 \geq (p_u(\bar{e}) - p_u(e))T + \psi(e^1) - \psi \\ [CIR^2(e)]: & (2 - p_l(e) - p_u(e))s^1 - (p_l(e) + p_u(e))s^2 \geq (1 - p_l(e) - p_u(e))T + \\ & 2\psi(e^2) - 2\psi \\ [CIC_l^1(e)]: & (p_l(\hat{e}^1, e^2) - p_l(e))(s^1 + s^2) \geq (p_l(\hat{e}^1, e^2) - p_l(e))T + \psi(e^1) - \psi(\hat{e}^1) \\ [CIC_u^1(e)]: & (p_u(\hat{e}^1, e^2) - p_u(e))(s^1 + s^2) \geq (p_u(\hat{e}^1, e^2) - p_u(e))T + \psi(e^1) - \psi(\hat{e}^1) \\ [CIC^2(e)]: & (p_l(e^1, \hat{e}^2) + p_u(e^1, \hat{e}^2) - p_l(e) - p_u(e))(s^1 + s^2) \geq (p_l(e^1, \hat{e}^2) + \\ & p_u(e^1, \hat{e}^2) - p_l(e) - p_u(e))T + 2\psi(e^2) - 2\psi(\hat{e}^2) \\ [CLL(e)]: & 0 \leq s^1, s^2 \leq T \end{aligned}$$

First of all, consider coalition  $C(\underline{e})$  induced by a side contract  $S^P(\underline{e}) = \{\underline{e} = (0, 0), s = (s^1, s^2)\}$ . The relevant constraints are expressed by:

$$\begin{aligned} [CIR_l^1(\underline{e})]: & p_u(\underline{e})s^2 - p_l(\underline{e})s^1 \geq (p_l(\bar{e}) - p_l(\underline{e}))T - \psi \\ [CIR^2(\underline{e})]: & s^1 - s^2 \geq -2\psi \\ [CIC_l^1(\underline{e})]: & s^1 + s^2 \geq T - \frac{\psi}{p_l(\bar{e}) - p_l(\underline{e})} \end{aligned}$$

From  $[CIR_l^1(\underline{e})]$  and  $[CIR^2(\underline{e})]$  we get:

$$s^1 + s^2 \leq \frac{4\psi}{p_l(\underline{e}) - p_u(\underline{e})} - \frac{p_l(\bar{e}) - p_l(\underline{e})}{p_l(\underline{e}) - p_u(\underline{e})} T \quad (\#)$$

It is easy to check that, given  $T = T^m(v)$ , this constraint contradicts to  $[CIC_l^1(\underline{e})]$  for any pair of side payments  $(s^1, s^2)$ , which implies that coalition  $CE(\underline{e})$  cannot be sustained.

In a similar way, one can check coalition  $C(\hat{e})$  ( $C(\tilde{e})$ ) which induces the informed agent to work (shirk) and uninformed to shirk (work) are not sustainable. Hence only null coalition  $C(\bar{e})$  can be implemented when the side mediator restrict to pooling side contract. Q.E.D ■

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