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Fingleton, Bernard

University of Cambridge

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Bernard Fingleton*

Department of Land Economy, University of Cambridge, United Kingdom

Abstract

Starting with a reduced form derived from standard urban economics theory, this paper estimates the possible job-shortfall across UK and EU regions using a time-space dynamic panel data model with a Spatial Moving Average Random Effects (SMA-RE) structure of the disturbances. The paper provides a logical rational for the presence of spatial and temporal dependencies involving the endogenous variable, leading to estimates based on a dynamic spatial Generalized Moments (GM) estimator proposed by Baltagi, Fingleton and Pirotte (2018). Given state-of-the art interregional trade estimates, the simulations are based on a linear predictor which utilizes different regional interdependency matrices according to assumptions about interregional trade post-Brexit.

Keywords: Brexit; Interregional trade; Urban economics theory; Panel data; Spatial lag; Spatio-temporal lag; Dynamic; Spatial moving average; Prediction; Simulation.

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JEL classification: C23,C33,C53,E27,F10,J21,R12.

*Corresponding author. Bernard Fingleton. *E-mail:* bf100@cam.ac.uk

1 Introduction

What will be the impact across the EU regions[1] of a shock to the UK and EU economy following Brexit? What I want to do is show possible, speculative, predictions of the impact of Brexit on employment across the EU. The Brexit effect will not be confined to the UK, but how far will it spread, and how long will it last? I am using a state of the art modelling approach, but the predictions, like all predictions, can only be very imprecise. Nevertheless, we can look at the outcome qualitatively. The main outcome of my effort is that spatial and temporal spillovers are a significant feature. In other words, the impact of Brexit will extend beyond the shores of the UK and persist into the future.

I emphasize that these are speculative predictions regarding the impact of Brexit across the EU and UK regions. The predictions are presented with a high level of caution, so the numbers SHOULD NOT be taken too literally. They are naturally dependent on the assumptions underpinning the model. Different assumptions regarding model structure, relevant data and estimation techniques can all potentially affect the predictions of the Brexit impact or of what would otherwise happen had Brexit not been the result of the UK referendum. Nevertheless the predicted % impacts in terms of job-shortfall are robustly predictable given what is assumed regarding the overall % reduction in trade between UK and EU regions. Also it is reasonably insensitive to what is assumed about future paths of drivers of employment. This suggests that the Brexit effect is perhaps somewhat immune to how we predict it, within certain reasonable parameters naturally. If the Brexit effect is in a sense neutral to the modelling and prediction effect, then we might be able to take the predictions a bit more seriously than would otherwise be the case. Of course, as always, extreme caution should be exercised.

The simulations in the paper are the outcomes of assumed changes in across-EU region connectivity due to trade barriers erected on exit from the EU. Using state-of-the-art trade interregional flow estimates, the level of trade between post-Brexit EU regions and UK regions is reduced compared with the trade flows that would otherwise exist. The simulated impact of Brexit is an estimate of the % job-shortfall that would occur were there to be no additional stimulus, in the form of changed levels of output and

[1] Although Switzerland and Norway are formally outside the EU, they are an integral part of the analyis. Their regions are referred to as EU regions for convenience.

capital in response to new trade deals as mooted by the pro-Brexit lobby. In reality it might be the case that the UK and EU's employment shortfall, as predicted by this analysis, will be offset by the introduction of new trade deals with the rest of the World, resulting in a boost to output and capital investment and consequently employment. Moreover, a boost to the UK economy could increase demand across Europe and lead to an increase in employment, contrary to what is predicted here which is based on a reduction in trade. However this is very much speculation and there is much uncertainty as to the real outcome as a result of Brexit. What is attempted here is a simulation of outcomes under strict assumptions, which may or may not hold.

More specifically, in order to estimate of the Brexit effect, I estimate employment levels across $N = 255$ EU regions both with and without Brexit. The explicit drivers of employment are output and capital, which are approximated by Gross Value Added (GVA) and a function of Gross Fixed Capital Formation (GFCF) respectively. Estimation of a model with employment related to output and capital is based on a viable data series over the period 2001 to 2010[2]. Different assumptions can be made about post 2011 paths for GVA and GFCF, given that accessible data with the same geography are not available over the more recent period. One assumption might be that post 2011 paths are driven by each region's average growth over the period 1991 to 2011. A second assumption might be to hold GVA and GFCF at the 2011 levels through into the future. Interestingly different assumptions have relatively little effect on outcomes.

2 The Model

The reduced form used as a basis to simulate the Brexit effect assumes that employment partly depends on level of output, as measured by GVA (Gross Value Added), denoted by $\mathbf{q}_t$, and (a proxy for) the level of capital within the region, based on GFCF (Gross Fixed Capital Formation), which is denoted by $\mathbf{k}_t$. To show this we start with the theoretical model given as equation (1), which is based on equation (30) given in the Appendix. The N by 1 vector $\mathbf{e}_t^d$ is the density of employment per unit area, and $\mathbf{a}_t$ is the level of

[2] Data for 2011 and 2012 are not used in estimation but held for one- and two-step ahead prediction.

efficiency of labour at time t, so that the product $\mathbf{e}_t^d\mathbf{a}_t$ is the number of labour efficiency units. This is related to $\widetilde{\mathbf{q}}_t$, which is a measure of output in the competitive final goods and services sector in each region at time t,via the constant parameters ϕ and $\widetilde{\gamma}$,thus

$$\widetilde{\mathbf{q}}_t = \phi(\mathbf{e}_t^d\mathbf{a}_t)^{\widetilde{\gamma}} \tag{1}$$

In order to obtain total output $\mathbf{q}_t$, it is assumed that $\widetilde{\mathbf{q}}_t = \boldsymbol{\pi}\mathbf{q}_t$, in which $\boldsymbol{\pi}$ is an N by 1 vector giving the share of total output in each region that is competitive final goods and services output. For simplicity of estimation it is assumed that $\boldsymbol{\pi}$ is constant over time. Also the employment levels are $\mathbf{e}_t = \mathbf{h}\mathbf{e}_t^d$ in which $\mathbf{h}$ is the area of land in each region. Taking logs gives

$$\ln\boldsymbol{\pi} + \ln\mathbf{q}_t = \ln\phi + \widetilde{\gamma}\ln\mathbf{e}_t + \widetilde{\gamma}\ln\mathbf{a}_t - \widetilde{\gamma}\ln\mathbf{h} \tag{2}$$

Rearranging (2) gives

$$\ln\mathbf{e}_t = \frac{1}{\widetilde{\gamma}}(\ln\boldsymbol{\pi} + \ln\mathbf{q}_t - \ln\phi) - \ln\mathbf{a}_t + \ln\mathbf{h}$$

To obtain (3) I assume that labour efficiency $\mathbf{a}_t = \frac{\mathbf{q}_t}{\widetilde{\mathbf{k}}_t}$, with more efficient labour having a higher level of output per unit of capital $\widetilde{\mathbf{k}}_t$. As shown below in equation (13), an approximation to the log level of capital is $\ln\widetilde{\mathbf{k}}_t = -\ln\widetilde{a} + \widetilde{b}\ln\mathbf{k}_t$, hence $\ln\mathbf{a}_t = \ln\mathbf{q}_t + \ln\widetilde{a} - \widetilde{b}\ln\mathbf{k}_t$, and from this

$$\ln\mathbf{e}_t = \frac{1}{\widetilde{\gamma}}(\ln\boldsymbol{\pi} + \ln\mathbf{q}_t - \ln\phi) + \ln\mathbf{h} - \ln\mathbf{q}_t - \ln\widetilde{a} + \widetilde{b}\ln\mathbf{k}_t \tag{3}$$

Collecting together constants as c and reorganising gives

$$\ln\mathbf{e}_t = c + \frac{1-\widetilde{\gamma}}{\widetilde{\gamma}}\ln\mathbf{q}_t + \widetilde{b}\ln\mathbf{k}_t + \boldsymbol{\varepsilon}_t \tag{4}$$

in which the error term $\boldsymbol{\varepsilon}_t$ captures the time-invariant regional heterogeneity in land $\mathbf{h}$ and in shares $\boldsymbol{\pi}$, which are unobservable, as given in equation (11).

In the dynamic context, it is reasonable to assume that disparities in employment levels across locations will persist as an equilibrium outcome to unchanging and fundamental causes. We therefore assume that (log) employment levels across regions, denoted by the N by 1 vector $\ln\mathbf{e}_t$ at time t will persist at dynamically stable levels so that $\ln\mathbf{e}_t = \ln\mathbf{e}_{t-1}$ unless there are changes in the levels of $\mathbf{q}_t$ or $\mathbf{k}_t$, or changes in interregional trade, or changes in unobservable effects. If such a disturbance occurs at time t and is ephemeral, then $\ln \mathbf{e}_t \neq \ln \mathbf{e}_{t-1}$ but over a subsequent period of quiescence $t \to T$ then once again we expect employment levels to converge on a new equilibrium at which $\ln \mathbf{e}_T = \ln \mathbf{e}_{T-1}$. Assume data are observed where $\ln \mathbf{e}_t \neq \ln \mathbf{e}_{t-1}$ but tending to converge, so that $\ln \mathbf{e}_t = f(\ln \mathbf{e}_{t-1})$, and an autoregressive process is assumed, hence

$$\ln \mathbf{e}_t = \varsigma + \gamma \ln \mathbf{e}_{t-1} \tag{5}$$

in which ς is an N by 1 vector and γ is a scalar parameter. In the long-run with $abs(\gamma) < 1$, and with no subsequent disturbances, the process converges to $\ln \mathbf{e}_T = \frac{\varsigma}{(1-\gamma)}$.

Consider next connectivity between regions in the form of a matrix $\mathbf{W}_N^*$,which is is a time-invariant N by N matrix where N is the number of regions. For purposes of interpreting parameter estimates we normalize by dividing $\mathbf{W}_N^*$ by the maximum eigenvalue of $\mathbf{W}_N^*$ to give[3] $\mathbf{W}_N$. Using this normalization, the maximum eigenvalue of $\mathbf{W}_N$ is 1, and the continuous range for which $(\mathbf{I}_N - \rho_1 \mathbf{W}_N)$ is nonsingular is $\frac{1}{\min(eig)} < \rho_1 < \frac{1}{\max(eig)} = 1$,in which $\rho_1$is a scalar spatial autoregressive parameter.

Given (5), logic dictates that

$$\rho_1 \mathbf{W}_N \ln \mathbf{e}_t = \rho_1 \mathbf{W}_N \varsigma + \rho_1 \mathbf{W}_N \gamma \ln \mathbf{e}_{t-1} \tag{6}$$

Subtracting (6) from (5) leads to another logically consistent expression in which the spatial dependence implied by (6) can be seen in (7) as an explicit cause of variation in $\ln \mathbf{e}_t$.Thus

$$\ln \mathbf{e}_t - \rho_1 \mathbf{W}_N \ln \mathbf{e}_t = \varsigma + \gamma \ln \mathbf{e}_{t-1} - (\rho_1 \mathbf{W}_N \varsigma + \rho_1 \mathbf{W}_N \gamma \ln \mathbf{e}_{t-1})$$

$$(\mathbf{I}_N - \rho_1 \mathbf{W}_N) \ln \mathbf{e}_t = (\gamma \mathbf{I}_N - \rho_1 \gamma \mathbf{W}_N) \ln \mathbf{e}_{t-1} + (\mathbf{I}_N - \rho_1 \mathbf{W}_N) \varsigma$$

[3] The matrix $\mathbf{W}_N$ retains (scaled) absolute levels rather than shares as the basis of interregional connectivity, and we make the standard assumptions for a weights matrix, that it comprises fixed (non-stochastic) non-negative values with zeros on the leading diagonal and its row and column sums are uniformly bounded in absolute value, and maintain the same assumption for $\mathbf{B}_N^{-1} = (\mathbf{I}_N - \rho_1 \mathbf{W}_N)^{-1}$ (Elhorst, 2014, p. 99).

Writing $\theta = -\rho_1\gamma$ gives

$$\ln \mathbf{e}_t = \mathbf{B}_N^{-1}\left[\mathbf{C}_N \ln \mathbf{e}_{t-1} + \mathbf{B}_N\varsigma\right] \tag{7}$$

in which $\mathbf{B}_N = (\mathbf{I}_N - \rho_1\mathbf{W}_N), \mathbf{C}_N = (\gamma\mathbf{I}_N + \theta\mathbf{W}_N)$ and $\mathbf{I}_N$ is an identity matrix of order N. In order to solve equation (7), given appropriate parameter restrictions, equation (7) converges to $\ln \mathbf{e}_T = (\mathbf{B}_N - \mathbf{C}_N)^{-1}\mathbf{B}_N\varsigma$.

Introducing additional covariates by writing $\mathbf{B}_N\varsigma = (c + \mathbf{x}\boldsymbol{\beta})$, in which c is a constant N by 1 vector, $\mathbf{x}$ is an N by k matrix and $\boldsymbol{\beta}$ is a k by 1 vector, gives

$$\ln \mathbf{e}_t = \mathbf{B}_N^{-1}\left[\mathbf{C}_N \ln \mathbf{e}_{t-1} + c + \mathbf{x}\boldsymbol{\beta}\right]$$

In order to maintain dynamically stable simulations, following Elhorst (2001,2014, p. 98), Parent and LeSage (2011, p. 478, 2012, p. 731) and Debarsy, Ertur and LeSage (2012, p. 162), requires the largest characteristic root ($e_{\max}$) of $\mathbf{B}_N^{-1}\mathbf{C}_N$ to be less than 1. This restriction ensures that employment converges to equilibrium levels $\ln \mathbf{e}_T = (\mathbf{B}_N - \mathbf{C}_N)^{-1}(c + \mathbf{x}\boldsymbol{\beta})$.

Additional realism is introduced in three ways. First, the restriction that $\theta = -\rho_1\gamma$ is removed since ρ_1 and γ are unknown, so that θ is free to vary. However we anticipate that $\widehat{\theta} \approx -\widehat{\rho}_1\widehat{\gamma}$. Second, taking account of the variables in equation (4), the time invariant matrix $\mathbf{x}$ is replaced by time-varying matrix[4] $\mathbf{x}_t$. Third, spatially dependent unobservables are represented by the error term ε_t. Although the system may, depending on $\mathbf{B}_N^{-1}\mathbf{C}_N$, still tend towards equilibrium, equilibrium will be continuously disturbed and new equilibrium levels established as t varies. For simplicity of estimation, inter-regional connectivity is assumed to remain constant over the estimation period. These considerations lead to the model given in equations (8,9,10,11), which is a time-space dynamic panel data model, thus

$$\ln \mathbf{e}_t = \mathbf{B}_N^{-1}\left[\mathbf{C}_N \ln \mathbf{e}_{t-1} + c + \mathbf{x}_t\boldsymbol{\beta} + \boldsymbol{\varepsilon}_t\right] \tag{8}$$

Given $\mathbf{x}_{1t} = \ln \mathbf{q}_t$, $\mathbf{x}_{2t} = \ln \mathbf{k}_t$, $\mathbf{x}_t = [\mathbf{x}_{1t}\ \mathbf{x}_{2t}]$ and $\boldsymbol{\beta} = [\ \beta_1\ \beta_2\ ]^T$, equation(8) can be stated more explicitly as

[4]We assume that the elements of $\mathbf{X}_t$ are uniformly bounded in absolute value.

$$
\begin{aligned}
\ln \mathbf{e}_t &= c + \gamma \ln \mathbf{e}_{t-1} + \rho_1 \mathbf{W}_N \ln \mathbf{e}_t + \beta_1 \ln \mathbf{q}_t + ... \quad (9) \\
& \quad \beta_2 \ln \mathbf{k}_t + \theta \mathbf{W}_N \ln \mathbf{e}_{t-1} + \boldsymbol{\varepsilon}_t \\
\boldsymbol{\varepsilon}_t &= \mathbf{u}_t - \rho_2 \mathbf{M}_N \mathbf{u}_t \quad (10) \\
u_{it} &= \mu_i + \nu_{it} \qquad i = 1, ..., N, t = 1, ..., T \quad (11) \\
\mu_i &\sim iid(0, \sigma_\mu^2) \\
\nu_{it} &\sim iid(0, \sigma_\nu^2)
\end{aligned}
$$

The disturbances $\boldsymbol{\varepsilon}_t$ capture the effects of the spatially dependent unobserved variables, with a compound structure (11) comprising time-invariant unobserved interregional heterogeneity represented by μ_i with $i = 1, ..., N$ and unobserved idiosyncratic shocks represented by $\nu_{it}; i = 1, ..., N, t = 1, ..., T$. These are assumed to be independent of each other and are collectively represented by u_{it}. It is important to recognize that the $\mu_i s$ represent unobserved factors creating interregional heterogeneity perhaps as a result of differences in industrial structure and physical and sociocultural environment, which in the short run can be treated as time-invariant. In the longer run, one might introduce a factor structure in which $\mu_{it} = \lambda_i^T F_t$ where λ_i is a (r x 1) vector of factor loadings and F_t is a (r x 1) vector of common factors, such as shocks to industrial structure and environment, with heterogenous regional effects λ_i,in which case $\mu_{it} = \lambda_{i1} F_{1t} + ... + \lambda_{ir} F_{rt}$. One might assume that in the short run the common factors do not vary over time, so that $F_t = F$. Hence $\mu_i = \mu_{it} = \lambda_{i1} F_1 + ... + \lambda_{ir} F_r$.

Most usually the assumption is that spatial dependence is an autoregressive (SAR-RE) process, such that $\boldsymbol{\varepsilon}_t = \rho_2 \mathbf{M}_N \boldsymbol{\varepsilon}_t + \mathbf{u}_t$.However in this paper the assumption for the error process is a spatial moving average process (SMA-RE) as in equation (10), thus $\boldsymbol{\varepsilon}_t = \mathbf{G}_N \mathbf{u}_t$, where $\mathbf{G}_N = (\mathbf{I}_N - \rho_2 \mathbf{M}_N)$. This means that the error process is such that a shock in a region affects only neighbouring regions as defined by a row standardized interregional contiguity matrix[5] $\mathbf{M}_N$. In contrast, an SAR-RE process would entail shocks affecting all regions.There are two reasons for this. First, Assuming SMA-RE rather than SAR-RE errors improves the predictive performance of the

[5] The matrix $\mathbf{M}_N$ has $\widetilde{\widetilde{e}}_{\max}^{-1} = 1$,where $\widetilde{\widetilde{e}}$ is the vector of purely real characteristic roots of $\mathbf{M}_N$. We assume that $\mathbf{M}_N$ has the same properties as $\mathbf{W}_N$, and with the restriction that $\widetilde{\widetilde{e}}_{\min}^{-1} < \lambda < \widetilde{\widetilde{e}}_{\max}^{-1} = 1$ one guarantees the invertibility of $\mathbf{G}_N$ as in equation(21).

estimator, as described in Section 6. Secondly, SMA-RE errors might proxy for omitted spillovers, which otherwise might be captured by the spatial lags $\mathbf{W}_N\mathbf{x}_t$. This is pertinent since the presence of $\mathbf{W}_N\mathbf{x}_t$ on the right hand side of (8) could adversely affect estimation. As explained by Fingleton, Le Gallo and Pirotte(2017) and Baltagi, Fingleton and Pirotte(2018), an SMA-RE error specification 'mitigates against the problem for instrumental variable estimation identified by Pace et al. (2012)'. In two-stage least squares (2SLS) estimation, the instrument set should comprise the 'exogenous' variables ($\mathbf{x}_t$) and their spatial lags ($\mathbf{W}_N\mathbf{x}_t$), and kept to a low order to avoid linear dependence and retain full column rank for the matrix of instruments (Kelejian and Prucha 1998, 1999) . The performance of the estimation procedure could be suboptimal, as explained by Pace et al. (2012), by including $\mathbf{W}_N\mathbf{x}_t$ among the set of explanatory variables. This is because with spatial lags ($\mathbf{W}_N\mathbf{x}_t$) among the set of regressors, then spatial lags of the spatial lags ($\mathbf{W}_N^2\mathbf{x}_t, \mathbf{W}_N^3\mathbf{x}_t, \ldots$) feature among the instruments, and this could lead to a weak instrument problem. To avoid this, SMA-RE errors are adopted as an alternative way to capture local spillovers.

3 Data

In estimating equation (9), data for employment ($\mathbf{e}_t$), output as measured by Gross Value Added (GVA, $\mathbf{q}_t$) and capital as proxied by a function of Gross Fixed Capital Formation (GFCF, $\mathbf{k}_t$), both denominated in €2005m, are taken from the Cambridge Econometrics European Regional Economic database, with observations over the 10 year period 2001 to 2010 used to estimate the model. Data are also available for $2011 - 2012$, but are held back to allow out-of-sample prediction tests of the model and some rivals. $\mathbf{k}_t$ is used to reflect capital stock $\widetilde{\mathbf{k}}_t$, for which data are unavailable, on the basis of a simple relationship which is assumed to exist between the two variables. $\mathbf{k}_t$ measures gross net investment (acquisitions minus disposals of produced fixed assets) in fixed capital assets and so provides an indicator of changes to the stock of capital. The assumption is that $\mathbf{k}_t$ is a non-linear function of a constant fraction $\widetilde{a}$ of $\widetilde{\mathbf{k}}_t$ so that

$$\mathbf{k}_t = \left(\widetilde{a}\widetilde{\mathbf{k}}_t\right)^{\frac{1}{\widetilde{b}}} \tag{12}$$

hence

$$\widetilde{\mathbf{k}}_t = \frac{1}{\widetilde{a}}\left(\mathbf{k}_t^{\widetilde{b}}\right) \tag{13}$$

As a test of the viability of this approximation, assume a standard model for the evolution of capital stock which is depreciating at a constant rate $\widetilde{d}$ so that

$$\widetilde{\mathbf{k}}_t = \mathbf{k}_t + (1-\widetilde{d})\widetilde{\mathbf{k}}_{t-1}; t = 2, ..., T \tag{14}$$

in which T is a large number. One problem with (14) is that it requires the initial capital stock at time $t = 1$,i.e. $\widetilde{\mathbf{k}}_1$. However given arbitrary values for $\widetilde{\mathbf{k}}_1$ and $\widetilde{d}$, values for $\widetilde{a}$ and $\widetilde{b}$ can be found whereby (13) provides a reasonable approximation to the outcome of iterations (14). A more realistic test is provided by the existence of both (albeit experimental estimates of) capital stock[6] (Derbyshire et al, 2010) and of well-founded GFCF data. Using the latest available data for both $\mathbf{k}_t$ and $\widetilde{\mathbf{k}}_t$, which is for the year, $t = 2008$, and taking logs of (13), leads to a loglinear regression of $\ln \widetilde{\mathbf{k}}_t$ on $\ln \mathbf{k}_t$ which gives OLS estimates of the constant $\ln \widetilde{a}^{-1} = 2.4546$ ($t\ ratio = 13.5628$) and slope $\widetilde{b} = 1.0195$($t\ ratio = 50.8118$),with $R^2 = 0.8888$.The plot of $\ln \mathbf{k}_t$ against $\ln \widetilde{\mathbf{k}}_t$ shows a significant linear relationship and no evidence of outliers or of heteroscedasticity It thus appears that the model given as equation (12) provides a good approximation. The estimated $\widetilde{a} = 0.0859$ suggests the approximate proportion of the capital stock that is invested, and, by comparison, $\sum \mathbf{k}_t / \sum \widetilde{\mathbf{k}}_t = 0.0686$.

The matrix $\mathbf{W}_N$ is based on estimated bilateral trade flows between EU NUTS2 regions. The data come from the PBL (the Netherlands Environmental Assessment Agency)[7] who developed a new methodology which is close to that of Simini et al (2012). Details of the methodology are given in Thiessen et al. (2013,2013a,2013b), see also Gianelle(2014). The method follows a top-down approach and therefore is consistent with the national accounts of the different countries. Given the total international exports and imports on the country level, interregional trade flows are derived using data on business travel (services) and on freight transport (goods). Additionally, exports that went to EU destination countries' final demand[8] were also included. Trade

[6] I am grateful to Cambridge Econometrics for providing this data.

[7] We are grateful to Mark Thiessen, who kindly provided the data. The data can be visualized at http://themasites.pbl.nl/eu-trade/index2.html?vis=net-scores

[8] 1.Final consumption expenditure by households and non-profit organisations

flows involving regions of non-EU countries Switzerland and Norway were obtained on the basis of interregional trade flows estimated by the best linear disaggregation method of Chow and Lin(1971), which was initially used to break down annual time series into quarterly series (see Abeysinghe and Lee, 1998, Doran and Fingleton, 2014). In this, commencing with aggregate trade values[9] between 21 EU counties, these were allocated to the NUTS2 regions. A parallel approach has been used by Polasek, Verduras and Sellner (2010), Vidoli and Mazziotta (2010), and Fingleton, Garretsen and Martin (2015). More detail of the method is provided in the Appendix. Finally, OLS regression of the log PBL trade flows on log Chow-Lin trade flows produced parameters used to predict the missing PBL regional trade flows for Switzerland and Norway using the values for these regions obtained via the Chow-Lin approach. For estimation, the start-of-period trade flows for the year 2000 is used. This year is chosen because it is the earliest available, so it is treated as exogenous to $\mathbf{e}_t$,$\mathbf{q}_t$ and $\mathbf{k}_t$, for $t = 2001$ to 2010. Prediction is based on the 2010 trade flows supplemented in the same way by Chow-Lin data. Estimates are also given in Appendix Table A3 based on a $\mathbf{W}_N$ matrix constructed entirely from the Chow-Lin trade flows. These simply use great circle distances and year 2000 GVA levels, and so are also be assumed to be exogenous. The comparative predictive performance of each set of estimates is discussed in Section 6.

4 Estimator for the time-space dynamic panel data model

Comprehensive overviews of spatial panel econometrics are given by Pesaran(2015, Chapters 29 and 30) and Baltagi(2013, Chapter 13) which highlight its growing importance for the applied econometrician. The estimator used in this paper, introduced by Baltagi et al(2018), adds to the available methodology by allowing a wider range of spatial interaction effects which include the spatial lag of the temporal lag of the dependent variable $W_N \ln \mathbf{e}_{t-1}$,

2.Final consumption expenditure by government
3.Net capital formation
4.Inventory adjustment
[9]They are downloadable from http://cid.econ.ucdavis.edu/data/undata/undata.html, see also Feenstra et al. (2005).

thus avoiding bias due to constraints necessary for dynamic stability and stationarity, and also by allowing spatial moving average compound error dependence rather than the usual autoregressive compound error process found in the majority of spatial econometric models. The estimator, which is applied to equation (8), is based on the earlier paper by Baltagi et al. (2014), which extends the approach of Arellano and Bond (1991) by the introduction of extra moments in line with the presence and availability of spatial lags (see also Bouayad-Agha and Védrine, 2010). Since the estimator is described elsewhere, a simple outline sketch is provided here focussing on the treatment of regressors as predetermined rather than exogenous[10]. Hence in equation (9), $\ln \mathbf{q}_t$ and $\ln \mathbf{k}_t$ are considered to be predetermined alongside endogenous left hand side variables $\ln \mathbf{e}_{t-1}, \mathbf{W}_N \ln \mathbf{e}_{t-1}$and $\mathbf{W}_N \ln \mathbf{e}_t$.

Focussing on the endogenous dependent variable $\ln \mathbf{e}_t$, the instruments include $\ln \mathbf{e}_t$ lagged by two periods, and its spatial lag $\mathbf{W}_N \ln \mathbf{e}_t$ also lagged by two periods, so that the moments equations (15,16) hold assuming ν_{it} is serially uncorrelated and $E(\Delta\nu_{it}, \Delta\nu_{it-2}) = 0$.Thus following Baltagi et al(2007), with, we have

$$
\begin{aligned}
E\left(\ln e_{il} \Delta\nu_{it}\right) &= 0 \quad \forall i, l = 1, 2, ..., T-2; t = 3, 4, ...T \qquad (15)\\
E(\sum_{i \neq j} w_{ij} \ln e_{il} \Delta\nu_{it}) &= 0 \quad \forall i, l = 1, 2, ..., T-2; t = 3, 4, ...T \qquad (16)
\end{aligned}
$$

in which E denotes the expectation. Also, if we were to assume exogenous rather than predetermined regressors $(\mathbf{x}_1, \mathbf{x}_2)$ this leads to (17)

$$
\mathbf{Z}_t = \begin{pmatrix} \ln \mathbf{e}_1, ..., \ln \mathbf{e}_{t-2,} \mathbf{W}_N \ln \mathbf{e}_1, ..., \mathbf{W}_N \ln \mathbf{e}_{t-2}, \mathbf{x}_{11}, ..., \mathbf{x}_{1T}, \\ \mathbf{x}_{21}, ..., \mathbf{x}_{2T}, \mathbf{W}_N \mathbf{x}_{11}, ..., \mathbf{W}_N \mathbf{x}_{1T}, \mathbf{W}_N \mathbf{x}_{21}, ..., \mathbf{W}_N \mathbf{x}_{2T} \end{pmatrix} \qquad (17)
$$

for $t = 3, ..., T$.Given that in (17) the regressors $(\mathbf{x}_1, \mathbf{x}_2)$ are exogenous, the moments equations are satisfied including the entire set

$\mathbf{x}_{11}, ..., \mathbf{x}_{1T}, \mathbf{x}_{21}, ..., \mathbf{x}_{2T}, , \mathbf{W}_N \mathbf{x}_{11}, ..., \mathbf{W}_N \mathbf{x}_{1T}$ and $\mathbf{W}_N \mathbf{x}_{21}, ..., \mathbf{W}_N \mathbf{x}_{2T}$ regardless of time t. As explained in Baltagi et. al.(2018), additional instruments can be generated via the matrix $\mathbf{W}_N^2$,but for simplicity these are omitted from the estimators used in the current paper.

[10]We show below that the assumption of predetermined regressors produces superior one-step ahead predictions compared with assuming exogenous regressors.

Strict exogeneity rules out any feedback from past shocks to current values of the variable, and the need to accommodate feedback leads to the preferred estimator based on predetermined regressors (see Bond, 2002, Pesaran, 2015). Predetermined regressors are contemporaneously uncorrelated, so that $corr(\mathbf{x}_t, \boldsymbol{\nu}_t) = 0$, but do depend on earlier shocks so that, for example, $corr(\mathbf{x}_t, \boldsymbol{\nu}_{t-1}) \neq 0$.This means that an adjustment to $\ln \mathbf{e}$,which embodies $\boldsymbol{\nu}$, at time t does not have an instantaneous effect on output and capital investment time t but takes effect at $t+1$ and later. This allows an extension to the set of instruments (compared with assuming endogeneity, where all endogenous variables are lagged by two periods), by the inclusion of $\mathbf{x}_{1t-1}, \mathbf{x}_{2t-1}, \mathbf{W}_N \mathbf{x}_{1t-1}$and $\mathbf{W}_N \mathbf{x}_{2t-1}$ so that

$$\mathbf{Z}_t = \begin{pmatrix} \ln \mathbf{e}_1, ..., \ln \mathbf{e}_{t-2,} \mathbf{W}_N \ln \mathbf{e}_1, ..., \mathbf{W}_N \ln \mathbf{e}_{t-2}, \\ \mathbf{x}_{11}, ...\mathbf{x}_{1t-2}, \mathbf{x}_{1t-1}, \mathbf{x}_{21}, ...\mathbf{x}_{2t-2}, \mathbf{x}_{2t-1}, \\ \mathbf{W}_N \mathbf{x}_{11}, ...\mathbf{W}_N \mathbf{x}_{1t-2}, \mathbf{W}_N \mathbf{x}_{1t-1}, \mathbf{W}_N \mathbf{x}_{21}, ...\mathbf{W}_N \mathbf{x}_{2t-2}, \mathbf{W}_N \mathbf{x}_{2t-1} \end{pmatrix} \tag{18}$$

Given the set of instruments as in equation (18), these are used to obtain initial estimates of γ, ρ_1, θ, β_1 and β_2, having first differenced the data to eliminate the time invariant individual effects $\boldsymbol{\mu}$ which are correlated with the lagged dependent variable. The resulting estimates are then used to give estimated errors which lead to estimates of the parameters of the spatial moving average error process, namely ρ_2, σ^2_μ and σ^2_ν using moments equations given in Fingleton(2008). Given these, preliminary one-stage consistent spatial GM estimates are obtained, followed by the two-stage Spatial GM estimates of γ, ρ_2, θ and $\boldsymbol{\beta}$ based on a robust version of the variance-covariance matrix.

5 Estimates

Table 2: Estimates of equation (8)		
parameter	param. est.	st. error
γ	0.6525	0.003769
ρ_1	0.5353	0.01045
β_1	0.1272	0.002116
β_2	0.02636	0.0006568
θ	-0.4006	0.008868
ρ_2	-0.7975	
σ_μ^2	0.0753	
σ_ν^2	0.0003	

Table 2 shows that the θ estimate for the spatial lag of the temporal lag ($\mathbf{W}_N \ln \mathbf{e}_{t-1}$) is not dissimilar to $-\widehat{\gamma}\widehat{\rho}_1$, in line with expectation stemming from an equilibrium process. Also the Table 2 estimates are stationary and dynamically stable, as shown by the largest characteristic root of $\mathbf{B}_N^{-1}\mathbf{C}_N$ which is equal to 0.6757, and the stationary bounds for ρ_2 are $\widetilde{e}_{\min}^{-1} = -1.1239 < \rho_2 < \widetilde{e}_{\max}^{-1} = 1$. Observe that the negative values of $\widehat{\rho}_2$ imply positive spatial dependence among the errors. Appendix Table A give the estimates of some rival estimators, including one with SMA-RE errors but assuming exogenous regressors (Table A1), and with SAR-RE errors assuming predetermined regressors (Table A2). As noted in Section 6, the predictive ability of these rivals is not as good as obtained via the preferred estimates summarised in Table 2.

6 Prediction

In order to support the preferred model summarised by Table 2, a cross-validation strategy is employed to assess the performance of competing estimators 'by comparing their predictive ability on data which have not been used in model estimation' (Anselin, 1988). Out-of-sample predictions of the level of employment across regions are obtained for the years 2011 and 2012 using 2011 and 2012 data combined with the parameter estimates obtained for data over the estimation period 2001 to 2010.

Following Chamberlain (1984), Sevestre and Trognon (1996), and Baltagi et. al.(2014), the linear predictor is

$$E\left[\ln \mathbf{e}_t\right] = \mathbf{B}_N^{-1}\left[\mathbf{C}_N E\left[\ln \mathbf{e}_{t-1}\right] + \mathbf{x}_t\boldsymbol{\beta} + c + \mathbf{G}_N E\left[\mathbf{u}_t\right]\right] \tag{19}$$

in which $E\left[.\right]$ denotes the expectation, so this can be seen to be identical to equation (8) but with expectations. With regard to the estimate of the time-invariant component of the error term $\boldsymbol{\mu}$, assuming a spatial moving average error process gives equation (8) rewritten thus

$$\begin{aligned}
\boldsymbol{\varepsilon}_t &= \mathbf{B}_N \ln \mathbf{e}_t - \mathbf{C}_N \ln \mathbf{e}_{t-1} - \mathbf{x}_t\boldsymbol{\beta} - c \\
\mathbf{G}_N\mathbf{u}_t &= \mathbf{B}_N \ln \mathbf{e}_t - C_N \ln \mathbf{e}_{t-1} - \mathbf{x}_t\boldsymbol{\beta} - c \qquad (20) \\
\mathbf{u}_t &= \boldsymbol{\mu} + \boldsymbol{\nu}_t \\
\boldsymbol{\mu}^{(t)} &= \mathbf{G}_N^{-1}\left(\mathbf{B}_N \ln \mathbf{e}_t - \mathbf{C}_N \ln \mathbf{e}_{t-1} - \mathbf{x}_t\boldsymbol{\beta} - c\right) - \boldsymbol{\nu}_t \qquad (21)
\end{aligned}$$

In order to obtain estimates $\widehat{\boldsymbol{\mu}}^{(t)}$ estimates $\widehat{\mathbf{G}}_N = \left(\mathbf{I}_N - \widehat{\rho}_2\mathbf{M}_N\right), \widehat{\mathbf{B}}_N = \left(\mathbf{I}_N - \widehat{\rho}_1\mathbf{W}_N\right), \widehat{\mathbf{C}}_N = \left(\widehat{\gamma} + \widehat{\theta}\mathbf{W}_N\right)$ and $\widehat{c}$ and $\widehat{\boldsymbol{\beta}}$ are used along with random draws from $\boldsymbol{\nu}_t \sim N(0, \widehat{\sigma}_\nu^2)$. We then take the mean over time of the $\widehat{\boldsymbol{\mu}}^{(t)}s$ for $t = 2, ..., T$, subsequently scaling so that it has variance equal to $\widehat{\sigma}_\mu^2$,thus giving the estimate $\widehat{\boldsymbol{\mu}}$ of the time-invariant error component. The outcome is the prediction equation (22) for $T + 1 = 2011$, in which $\mathbf{x}_{1T+1} = \ln \mathbf{q}_{T+1}$and $\mathbf{x}_{2T+1} = \ln \mathbf{k}_{T+1}, t = 1, ..., T$.

$$\ln \widehat{\mathbf{e}}_{T+1} = \widehat{\mathbf{B}}_N^{-1}\left[\widehat{\mathbf{C}}_N \ln \widehat{\mathbf{e}}_T + \mathbf{x}_{T+1}\widehat{\boldsymbol{\beta}} + \widehat{c} + \widehat{\mathbf{G}}_N\widehat{\boldsymbol{\mu}}\right] \tag{22}$$

For two step ahead[11], $\mathbf{x}_{1T+2} = \ln \mathbf{q}_{T+2}$and $\mathbf{x}_{2T+2} = \ln \mathbf{k}_{T+2}$.Figure 1 shows a close correlation between predicted log employment $\ln \widehat{\mathbf{e}}_{T+1}$and observed log employment, suggesting that the preferred estimator giving the Table 2 estimates would be a good basis for simulating the impact on employment following Brexit.

[11] Data limitations mean that for 2012, $\mathbf{k}$ in each region is estimated using each region's previous growth rate.

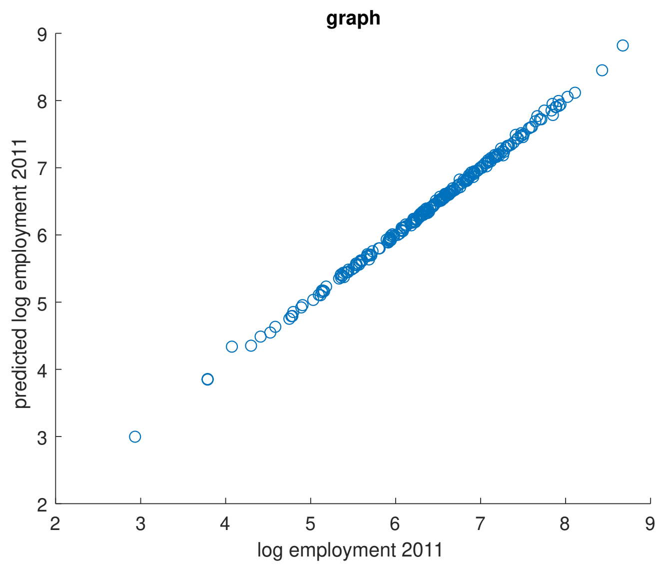

Figure 1 : Out of sample prediction for 2011

The preference for the Table 2 estimates is based on the mean of the $RMSE = \sqrt{\sum_{i=1}^{N} \left(\ln e_{i,T+s} - \ln \widehat{e}_{i,T+s}\right)^2 / N}$ for $s = 1, 2$,denoted by $\overline{RMSE}$. In the case of Table 2, $\overline{RMSE} = 0.0721$.Rival estimators (Appendix Table A), give less accurate one- and two-step ahead predictions. In the case of assuming SMA-RE errors and exogenous regressors, $\overline{RMSE} = 0.1791$. Assuming SAR-RE errors with predetermined regressors gives $\overline{RMSE} = 0.2890$. Note that in the case of SAR-RE errors, $\widehat{\mathbf{G}}_N = (\mathbf{I}_N - \widehat{\rho}_2 \mathbf{M}_N)^{-1}$ in equations (19, 20,22). Table A also gives estimates relating to SMA-RE errors and predetermined regressors, but are based on $\mathbf{W}_N$ derived using the Chow-Lin approach. In this case $\overline{RMSE} = 0.2529$,providing support for the choice of $\mathbf{W}_N$ based on the PBL trade data. Table A also give estimates based on SMA-RE errors and predetermined (and exogenous) regressors, but with the additional variables $\mathbf{W}_N \mathbf{x}_1$,the spatial lag of $\ln \mathbf{q}$, with parameter β_3,and $\mathbf{W}_N \mathbf{x}_2$,the spatial lag of $\ln k$, with parameter β_4, with $\mathbf{W}_N$ given by the PBL trade data. This is thus a form of spatial Durbin specification, but with regressors $\mathbf{x}_t = (\mathbf{x}_{1t}, \mathbf{x}_{2t}, \mathbf{W}_N \mathbf{x}_{1t}, \mathbf{W}_N \mathbf{x}_{2t})$ the additional covariates evidently cause a problem of weak instruments, giving dynamically unstable nonstationary estimates, as reflected by the largest characteristic root of $\mathbf{B}_N^{-1} \mathbf{C}_N$ equal to 1.0663 (1.9041) and, with $\mathbf{x}$ in equation (21) and (22) $\overline{RMSE}$

= 7.4403 (3.0918). The same spatial Durbin specification again assuming predetermined regressors but with ρ_2 restricted to zero gives a largest characteristic root equal to 1.1127 and $\overline{RMSE}$ = 3.3017. The same spatial Durbin specification assuming exogenous regressors and with a spatial autoregressive (SAR) error process gives a largest characteristic root equal to 2.489 and $\overline{RMSE} = 20.9333$.These results point to the viability of the Table 2 estimates for prediction purposes.

7 Simulating the Brexit effect

The approach adopted is to use the parameters estimates in Table 2 to predict the impact on employment of presumably reduced trade between the UK and the remaining EU regions in the year 2020 and beyond. Attention is focussed on 2020 and later, given that the UK's formal exit from the EU is scheduled for the first half of 2019, so 2020 will the first full year outside the EU. Given a lack of appropriate and accessible data, for instance with the same geography as up to 2011, beyond 2011 employment could be predicted on the basis of alternative assumptions about the level of $\mathbf{q}$ and $\mathbf{k}$ in 2020. One assumption might be that $\mathbf{q}$ and $\mathbf{k}$ in 2020 are at the same level as observed in each region in 2011. An alternative assumption could be that from 2011 onwards they grow at their historical rates, taken over the period 1991 to 2011 in each region. On this basis on average the level of $\mathbf{q}$ and $\mathbf{k}$ in 2025 is approximately 25% more than the 2011 levels. However it is easy to show by simulation, using alternative levels of $\mathbf{x}_{T+s}$ in equation (23) that the assumption made about the future path of $\mathbf{q}$ and $\mathbf{k}$ only has a minor effect on the equilibrium % job shortfall across regions. For example, a 100% increase in the 2011 levels of $\mathbf{q}$ and $\mathbf{k}$ only increases the job shortfall in London from -1.829% to -1.927%, a factor of approximately 1.054, assuming a 2% reduction in trade. Similarly assuming a 16% reduction in trade in increases London's job shortfall from -14.58% to -15.36%. Table 3 summarizes the outcomes for various trade reductions. Commensurate effects occur across all regions, as exemplified by the outcomes for Paris in Table 4. The conclusion is that irrespective of the % reduction in trade, doubling the 2011 levels of $\mathbf{q}$ and $\mathbf{k}$ only increases the % job shortfall by a factor of approximately 1.05.

$$\ln \widehat{\mathbf{e}}_{T+s} = \widehat{\mathbf{B}}_N^{-1} \left[\widehat{\mathbf{C}}_N \ln \widehat{\mathbf{e}}_{T+s-1} + \mathbf{x}_{T+s} \widehat{\boldsymbol{\beta}} + \widehat{c} + \widehat{\mathbf{G}}_N \widehat{\boldsymbol{\mu}} \right] \quad (23)$$

Regardless of the assumptions regarding future levels of $\mathbf{q}$ and $\mathbf{k}$, from $\tau = 2020$ onwards there are two scenarios, on based on the trade flows assuming no Brexit effect, and the other assuming a Brexit effect on trade flows, and the difference between them is taken as the Brexit effect. Regarding the no-Brexit effect scenario, this applies matrix $\mathbf{W}_N$,which is based on the latest available trade flows pertaining to the year 2010. The prediction is then given by the solution to equation (24) with $\widehat{\mathbf{B}}_N = (\mathbf{I}_N - \widehat{\rho}_1 \mathbf{W}_N)$, $\widehat{\mathbf{C}}_N = \left(\widehat{\gamma} + \widehat{\theta}\mathbf{W}_N\right)$ and $\widehat{\mathbf{G}}_N = (\mathbf{I}_N - \widehat{\rho}_2 \mathbf{M}_N)$. Also $\mathbf{x}_\tau$ is an (N by 2) matrix containing the forward projections $\ln \mathbf{q}_\tau$ and $\ln \mathbf{k}_\tau$, thus

$$\ln \widehat{\mathbf{e}}_\tau = \widehat{\mathbf{B}}_N^{-1} \left[\widehat{\mathbf{C}}_N \ln \widehat{\mathbf{e}}_{\tau-1} + \mathbf{x}_\tau \widehat{\boldsymbol{\beta}} + \widehat{c} + \widehat{\mathbf{G}}_N \widehat{\boldsymbol{\mu}}\right] \tag{24}$$

The second scenario is to assume that bilateral trade between the UK regions and the (remaining) EU regions is, for example, 2% lower than it would otherwise be. Thus of the $N = 255$ UK plus EU regions, there are $N^2 - N = 64,770$ bilateral trade flows in any one year involving the regions. With 37 UK regions and 218 EU regions, (2 x 37 x 218) = $16,132$ interregional trade flows are assumed to be 2% smaller than under an assumption of no Brexit effect. This Brexit-affected trade flow matrix is denoted by $\widetilde{\mathbf{W}}_N$ which leads to $\widetilde{\mathbf{B}}_N = \left(\mathbf{I}_N - \widehat{\rho}_1 \widetilde{\mathbf{W}}_N\right)$, $\widetilde{\mathbf{C}}_N = \left(\widehat{\gamma} + \widehat{\theta}\widetilde{\mathbf{W}}_N\right)$ and the prediction equation

$$\ln \widetilde{\mathbf{e}}_\tau = \widetilde{\mathbf{B}}_N^{-1} \left[\widetilde{\mathbf{C}}_N \ln \widetilde{\mathbf{e}}_{\tau-1} + \mathbf{x}_\tau \widehat{\boldsymbol{\beta}} + \widehat{c} + \widehat{\mathbf{G}}_N \widehat{\boldsymbol{\mu}}\right]$$

Thus the % job shortfall is $\ln \widetilde{\mathbf{e}}_\tau - \ln \widehat{\mathbf{e}}_\tau$.

The assumption of a 2% reduction in trade is of course an arbitrary one, and influenced at the time of writing by negotiations, debates and proposals relating to the possibility of a transition period in which the pattern of trade might not be too dissimilar to the typical pattern assumed for the pre-Brexit period. Any % change could be assumed, and indeed in an ideal world one might wish to make changes to trade on an individual region by region and sector by sector basis rather than assume that trade reduces by the same amount across all regions and all sectors. However this is very much the unknown, although some sectorally specific estimates are given below which also have geographically nuanced outcomes. Initially the simulation adopts the simple assumption that all trade between EU and UK regions is reduced by 2%, although the resulting geographical pattern is robust to the % reduction assumed. This is exemplified by Tables 3 and 4. These show that doubling the reduction in trade approximately doubles the % job shortfall. Also these ratios hold regardless of the different assumptions one makes about the future path of **q** and **k**. For example, assuming 2011 levels, the equilibrium % job shortfall in London goes from -7.304% assuming an 8% trade reduction, to -14.578% assuming a 16% trade reduction, and from -7.697% to -15.362% if one doubles the 2011 levels of **q** and **k**. A similar pattern is shown for Paris in Table 4, although the % job shortfalls are about 50% of those for London.

Table 3 : London - doubling trade reduction and regressor levels

trade reduction	2%	4%	8%	16%			
	job shortfall				ratios		
2011 levels	-1.829	-3.656	-7.304	-14.578	1.999	1.998	1.996
2x 2011 levels	-1.927	-3.853	-7.697	-15.362	1.999	1.998	1.996
ratios	1.054	1.054	1.054	1.054			

Table 4 : Paris - doubling trade reduction and regressor levels

trade reduction	2%	4%	8%	16%			
	job shortfall				ratios		
2011 levels	-0.917	-1.832	-3.656	-7.281	1.998	1.996	1.991
2x 2011 levels	-0.967	-1.932	-3.856	-7.679	1.998	1.996	1.991
ratios	1.055	1.055	1.055	1.055			

The total impact of Brexit can be broken down into the impact of restricted trade in manufactured goods[12], the impact of restricted trade in services[13], and the impact of restricted trade in other sectors[14]. These sectoral trade patterns have different geographies and therefore the impacts have different geographical distributions to the outcomes assuming a global reduction across all sectors. In order to simulate the impact of restricted trade in manufactured goods, the employment levels assuming no disruption to trade across all sectors is compared to the levels of employment in which overall trade is reduced solely as a result of the reduction by 2% in manufactured goods trade between EU regions and UK regions. A similar comparison is made for employment based on no trade reduction versus employment based on reduced overall trade as a consequence of trade between EU and UK regions involving services being reduced by 2%.

[12] Including construction but excluding energy supply and transport equipment.

[13] Excluding non-market services.

[14] Agricultural goods, mining, non-market services, quarrying and energy supply.

8 Results

The initial outcomes relate to a reduction in EU-UK trade of 2% across all sectors. I predict % change in employment across the EU and UK regions assuming the 2011 levels for $\mathbf{q}$,and $\mathbf{k}$. On this basis Figure 2 shows the dynamic paths for each region to 2050, with convergence to steady state occurring after 2030. From this it is evident that the maximum equilibrium job shortfall is -1.829%,in the case of Inner London,with most other regions falling below 1%. Figure 3 shows the geographical pattern of the Brexit impact equal to $\ln \widetilde{\mathbf{e}}_\tau - \ln \widehat{\mathbf{e}}_\tau$ for $\tau = 2025$, indicating a maximum shortfall by 2025 of -1.72% (Inner London). The picture which emerges from the simulation is that the negative Brexit impact is bilateral, with both UK regions and EU regions likely to see an employment shortfall. Figure 3 shows larger negative impacts in regions with strong trading links to the UK, most notably in the Paris region (-0.87%), the Southern and Eastern region of Ireland (-1.05%), and the Oberbayern region centred on Munich(-0.75%). Figure 4 gives the frequency distribution of the Figure 3 data, highlighting the fact that despite some large impacts, for about 160 of the 255 regions, Brexit is likely to have close to zero effect on employment. Figure 5 shows that within the UK, Inner (-1.72%) and Outer London (-0.99%) are expected to have the biggest % shortfall by 2025, with impacts generally higher along the Thames valley in Berkshire, Bucks and Oxfordshire (-0.83) towards Gloucestershire, Wiltshire and North Somerset (-0.9%). Generally %s are higher around the Greater South East and in some of the large conurbations (Birmingham -0.6%, Manchester -0.57%, West Yorkshire -0.51%) than in more rural and peripheral regions. Figure 6 gives the frequency distribution of the Figure 5 data, emphasizing the Inner London outlier, with the majority of regions having a job shortfall of less than -0.5%. As noted above, if one were to assume different reductions in trade other than 2%, the outcomes for employment would be different, but proportional to the 2% impact, and the geographical pattern would be essentially identical.

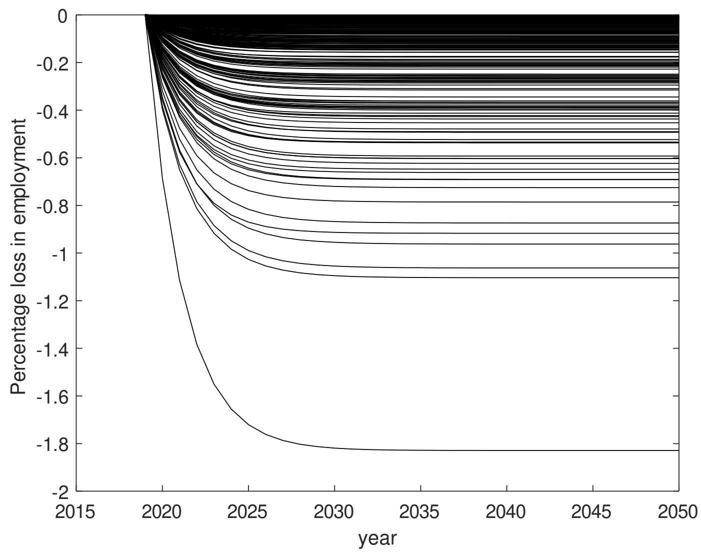

Figure 2 : Dynamic paths for % employment shortfall

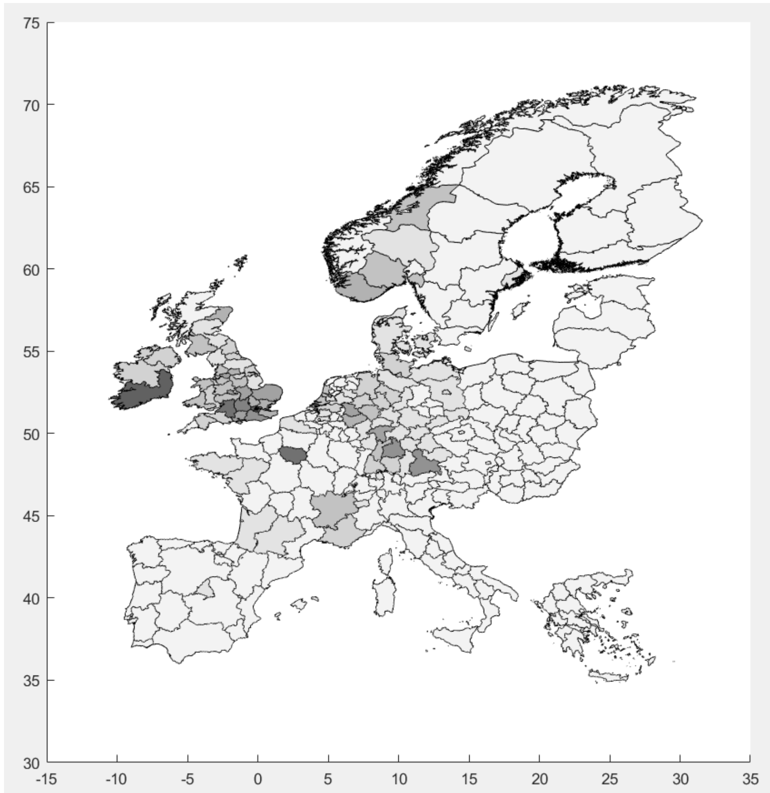

Figure 3 : % employment shortfall across 255 EU regions

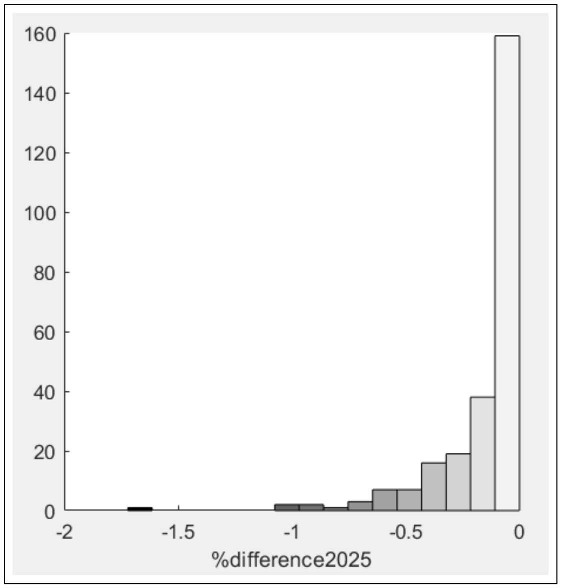

Figure 4 : % employment shortfall 2025

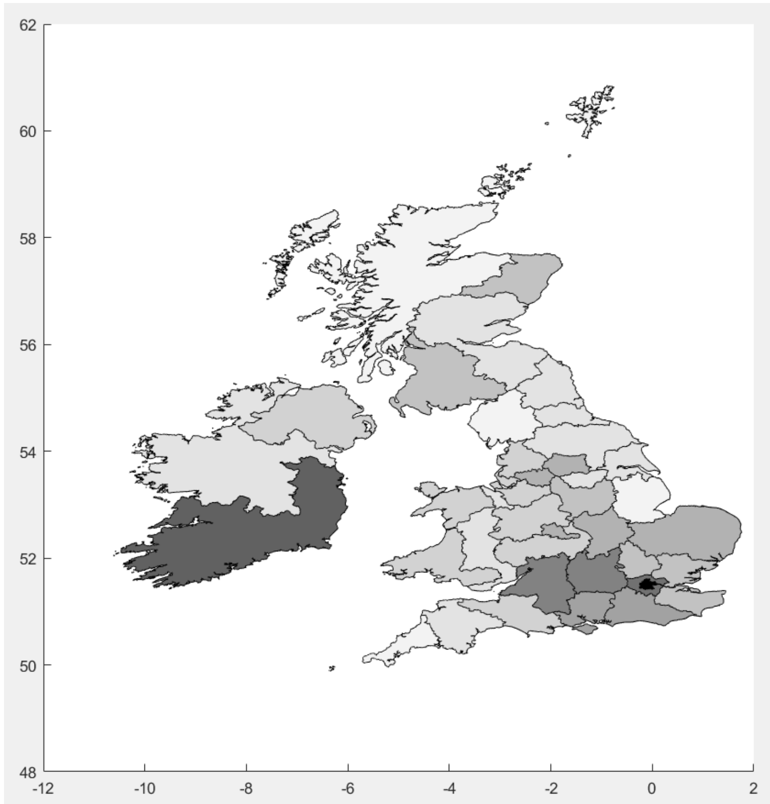

Figure 5 : % employment shortfall UK and Ireland

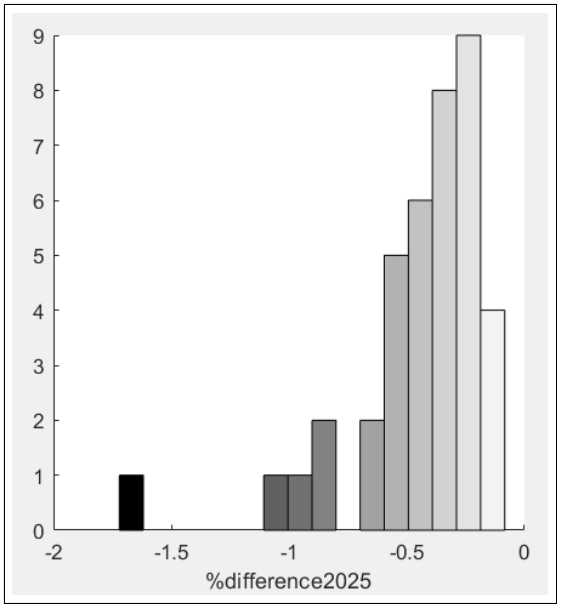

Figure 6 : % employment shortfall

Next consider the separate impacts on employment of restricted trade in manufactures. Simulating on the same basis, region paths converge to equilibrium levels as with Figure 2, although the equilibrium levels differ from those of Figure 2. We take a snapshot across the dynamic paths in Figures 7 to 10, showing the % shortfall in employment in manufactures by region for the year 2025. Figure 7 shows that the geography of the impact due to 2% less industry trade is very similar to the overall pattern shown in Figure 3. However a comparison of Figures 4 and 8 emphasizes the differences in the levels of impact, with the maximum level, equal to −0.86% for Inner London. Figures 9 and 10 show the % job shortfall in the UK and Ireland. Again the impact in the South and East of Britain and especially London is clear, and the effect on Ireland remains pronounced. Compared with industry, the impact of reduced trade in services is much less symmetrical, with the bulk of the job shortfall occurring in Britain and Ireland. This is clear from Figures 11 and 12. Inner London clearly stands out as having the most significant impact compared with all other EU regions, and apart from the South and

Eastern region of Ireland, almost all non-zero employment shortfalls occur in Great Britain. Figures 13 and 14 emphasize the polarized effect of service trade reduction, with Inner London standing out with a job shortfall of -0.62%. Outer London is predicted to see a job shortfall of -0.29% by 2025, but Southern and Eastern Ireland see a comparable effect, equal to -0.3%. All other regions have predictions of -0.2% or less.

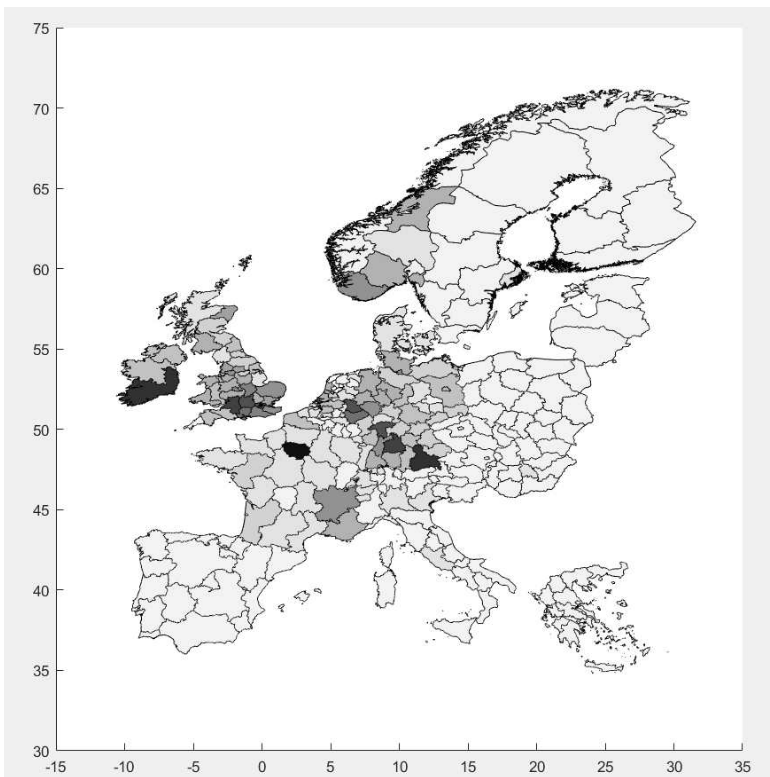

Figure 7 : Industry impact: % employment shortfall across 255 EU regions

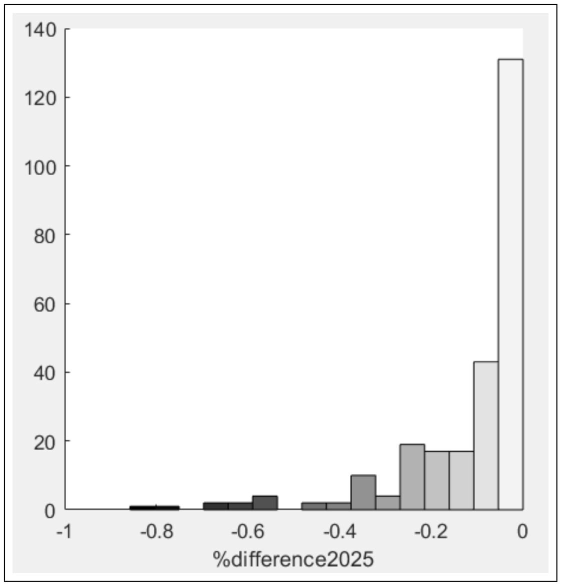

Figure 8 : % employment shortfall due to Industry

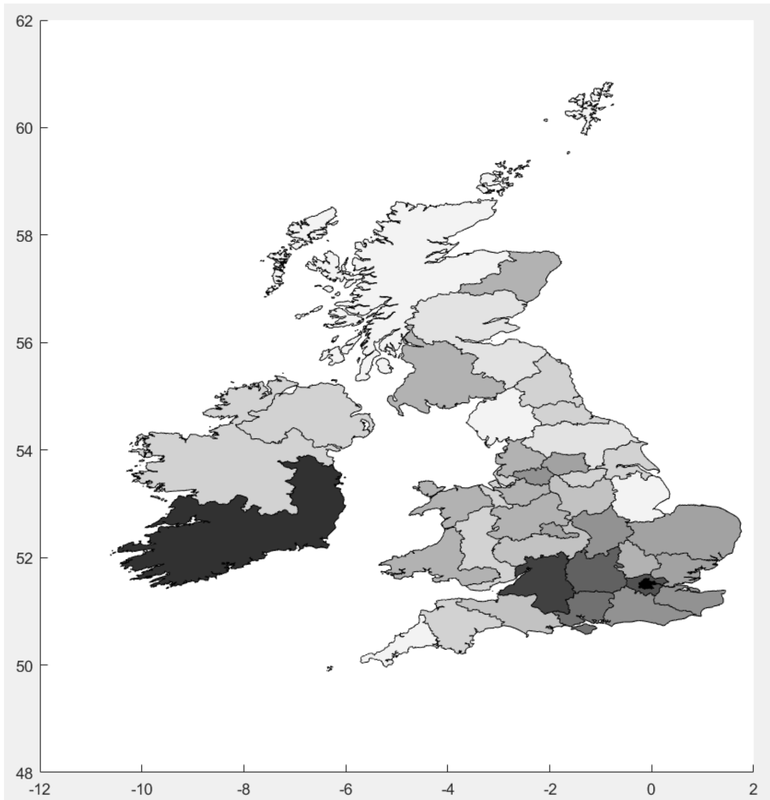

Figure 9 : % employment shortfall UK and Ireland due to Industry

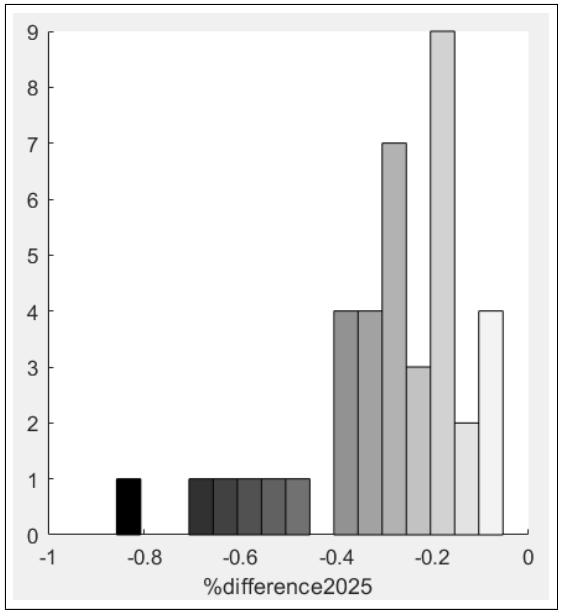

Figure 10 : % employment shortfall due to Industry

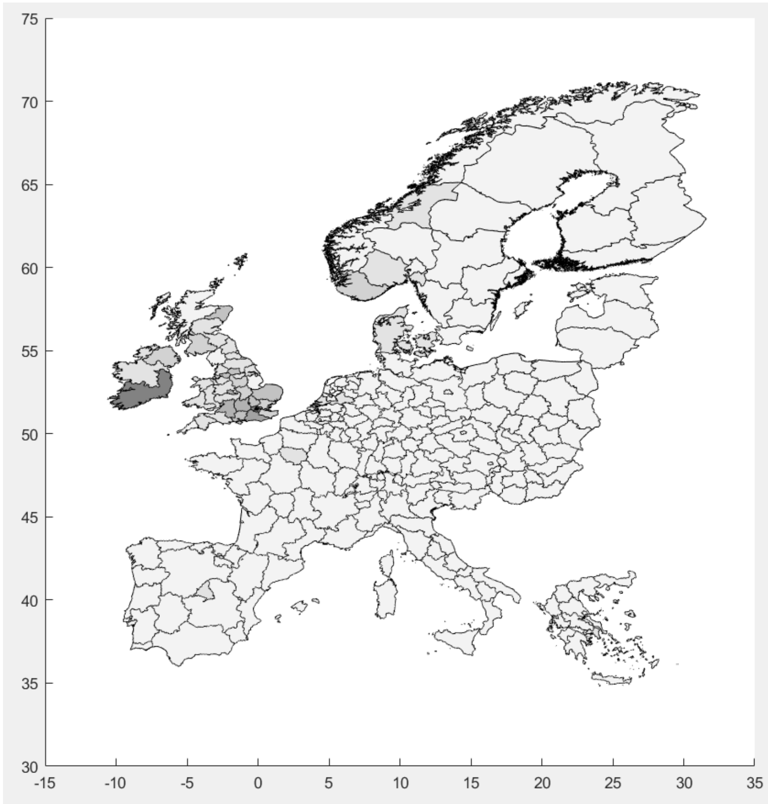

Figure 11: Services impact: % employment shortfall across 255 EU regions

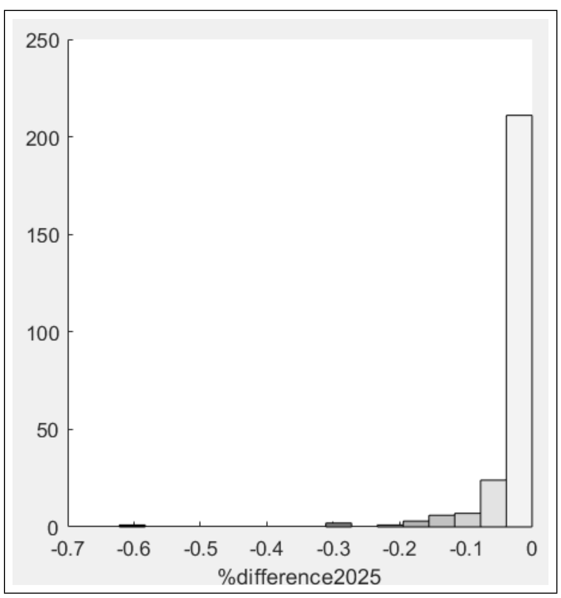

Figure 12 : % employment shortfall due to Services

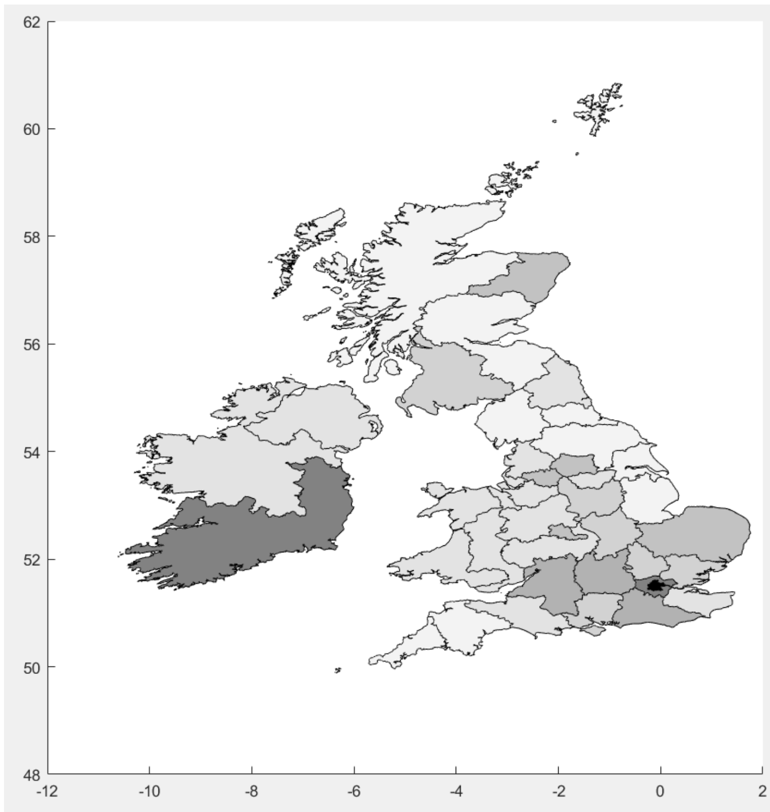

Figure 13: % employment shortfall UK and Ireland due to Services

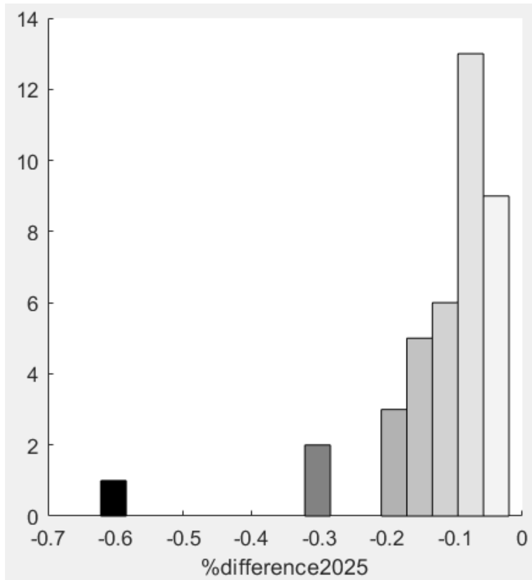

Figure 14 : % employment shortfall due to Services

9 Conclusion

In the paper the impact of Brexit is measured in terms of the job-shortfall, which is the reduction in the number of jobs in each region due to Brexit assuming no alternative sources of employment are put in place. This of course might be a false assumption, as the pro-Brexit lobby has consistently emphasized the potential stimulus of new trade deals with other non-EU countries. Therefore our impact as reflected in the maps of job-shortfall indicate those regions which will be in greatest need of alternative employment sources to compensate for the job-shortfall likely as a result of Brexit. Thus the paper is not predicting a job-loss per se, simply a potential job loss without successful alternative trade arrangements post-Brexit. The paper shows negative impacts on employment which affect not only the UK regions but also employment levels in EU regions, especially those which are close trading partners. This pan-European interregional interdependency is captured in the model by spatial and temporal interactions based on state-of-the-art trade flow estimates which determine the strength of interdependence. This means that employment within a region depends on the levels of output and capital within the region, but also on demand coming from other regions which are trading partners. The approach adopted has been to assume a reduction in trade between EU and UK regions which gives a corresponding reduction in demand. While the impact levels are proportional to the trade reduction assumed, the model implies that the geography of Brexit impact is robust to the assumptions made. In summary, within the UK the predicted job-shortfall by 2025 tends to be concentrated in the greater South East in and around London, plus some major conurbations. Within the EU regions, while the impact is spread more widely and for most regions is quite limited, Southern and Eastern Ireland, Paris and Oberbayern, Stuggart and Dusseldorf stand out as regions likely to see significant job shortfalls. It would seem that not only is it important for the UK to seek new trade agreements, but also the EU. Breaking down the impact sectorally, it appears that the major part of the overall job shortfall is attributable to a loss of manufacturing trade. The impact of reduced trade in services appears to be less, but interestingly is much less symmetrically distributed across EU and UK regions than for manufacturing, with the bulk of the job-shortfall focussed on UK regions, especially London, but also on Ireland.

These are speculative simulations, conditional on important assumptions, so the numbers SHOULD NOT be taken too literally. Great caution is needed in interpreting the validity and value of any 'prediction' effort. It is worth recalling the words of George Box : "All models are wrong but some are useful". David Spiegelhalter, Professor of the Public Understanding of Risk at the University of Cambridge, refers to Donald Rumsfeld as the patron saint of Risk Analysis. He will be remembered for famously saying that "but there are also unknown unknowns. There are things we do not know we don't know". We should therefore put forward predictions with all due humility, but clearly and without fear, because we don't want to come across as 'dithering scientists'. In defence of the approach adopted, there is support from the words of Pesaran(1990), who points out that 'Econometric models are important tools for forecasting and policy analysis, and it is unlikely that they will be discarded in the future. The challenge is to recognise their limitations and to work towards turning them into more reliable and effective tools. There seem to be no viable alternatives'.

10 Appendices

10.1 Theoretical background

The theoretical background to the model specification commencing with equation (1) is derived from standard urban economics as given by Abdel-Rahman and Fujita(1990), Ciccone and Hall(1996) and Fujita and Thisse(2002), among others. The theory commences with a constant returns to scale Cobb-Douglas production function for the output $\widetilde{q}$ of the competitive final goods and services sector

$$\widetilde{q} = \left(m^{\widetilde{\beta}} i^{1-\widetilde{\beta}}\right)^{\alpha} h^{1-\alpha}$$

in which m denotes sector-specific labour efficiency units and the level of composite services is given by i,and h is area of land. Assume $h = 1$ then

$$\widetilde{q} = \left(m^{\widetilde{\beta}} i^{1-\widetilde{\beta}}\right)^{\alpha} \tag{25}$$

Assume that the equilibrium output of each service firm is i_e and there are g firms, depending on the total services effective labour. We obtain g by dividing the total services effective labour by the effective labour per firm, thus

$$g = \frac{(1-\widetilde{\beta})\widetilde{e}}{ai_e + s} \tag{26}$$

In (26), $(1-\widetilde{\beta})$ equals the services labour share of total effective labour $\widetilde{e}$ under competitive equilibrium in the labour market.For each services firm, we have internal returns to scale, with s denoting the fixed labour requirement and a the marginal labour requirement, so that $ai_e + s$ is the effective labour per firm at equilibrium. From the CES production function we obtain

$$i = g^{\widetilde{\mu}} i_e \tag{27}$$

where $\widetilde{\mu}$ is a measure of monopoly power in the monopolistically competitive services sector, and $\frac{\widetilde{\mu}}{\widetilde{\mu}-1}$ is the constant elasticity of substitution. Substituting i into equation (25) gives

$$\widetilde{q} = \left(m^{\widetilde{\beta}} g^{\widetilde{\mu}-\widetilde{\mu}\widetilde{\beta}} i_e^{1-\widetilde{\beta}}\right)^{\alpha} \tag{28}$$

Substituting for g in equation (28) gives

$$\widetilde{q} = \widetilde{e}^{\alpha\left(\widetilde{\beta}+\widetilde{\mu}-\widetilde{\mu}\widetilde{\beta}\right)}\widetilde{\beta}^{\widetilde{\alpha\beta}}\left(ai_e+s\right)^{\alpha\widetilde{\mu}\left(\widetilde{\beta}-1\right)}i_e^{\alpha\left(1-\widetilde{\beta}\right)}\left(1-\widetilde{\beta}\right)^{-\widetilde{\alpha\mu}\left(\widetilde{\beta}-1\right)} \tag{29}$$

which simplifies to

$$\widetilde{q} = \phi\widetilde{e}^{\alpha\left(1+\left(1-\widetilde{\beta}\right)(\widetilde{\mu}-1)\right)} = \phi\widetilde{e}^{\widetilde{\gamma}} \tag{30}$$

This shows that with $\alpha = 1$ there are increasing returns $(\widetilde{\gamma} > 1)$ if service firms are relevant to output in the competitive sector $\left(\widetilde{\beta} < 1\right)$ and also possess monopoly power $(\widetilde{\mu} > 1)$. However $\alpha < 1$ indicates that land is also a relevant factor, and depending on the value of α any tendency to increasing returns could be offset by the congestion effect caused by the restriction of production to a unit of land, leading to diminishing returns.

10.2 Other estimates

Table A: Estimates of equation (8)					
Parameters	A1	A2	A3	A4	A5
γ	0.5634 (0.001425) (395.2)	0.6908 (0.003387) (204.0)	0.7102 (0.005737) (123.8)	0.3357 (0.00209) (160.6)	0.5673 (0.002129) (266.4)
ρ_1	0.5909 (0.006908) (85.54)	0.7355 (0.02388) (30.8)	0.6539 (0.01081) (60.48)	1.516 (0.01156) (131.2)	1.1990 (0.01358) (88.3)
β_1	0.1787 (0.0006129) (291.5)	0.1493 (0.001614) (92.46)	0.1577 (0.002976) (52.98)	0.3604 (0.001307) (275.6)	0.1897 (0.001245) (152.4)
β_2	0.01966 (0.0001589) (123.7)	0.002392 (0.0008091) (2.956)	0.01858 (0.001353) (13.73)	−0.03702 (0.0007463) (−49.61)	0.01581 (0.0002156) (73.34)
β_3	.....	.....	.....	0.2910 (0.005024) (57.91)	0.3576 (0.006044) (59.18)
β_4	.....	.....	.....	−0.3930 (0.003494) (−112.5)	−0.3025 (0.002825) (−107.1)
θ	−0.3807 (0.005913) (−64.39)	−0.6739 (0.02267) (−29.73)	−0.7551 (0.01528) (−49.42)	−0.8863 (0.008452) (−104.9)	−0.9465 (0.009802) (−96.56)
λ	−0.6545	0.7119	−0.9456	−0.3079	−0.5852
σ_μ^2	0.2786	0.0539	0.1851	0.0097	0.4663
σ_v^2	0.0003	0.0692	0.0005	0.0006	0.0003
max char. root	0.5845	0.7595	0.8665	1.0663	1.9041
Forecasting RMSE					
2011	0.1413	0.2248	0.2343	7.4094	3.0746
2012	0.2168	0.3532	0.2715	7.4711	3.1091
Average	0.1791	0.2890	0.2529	7.4403	3.0918

A1 : Assuming exogenous regressors, SMA-RE errors

A2 : Assuming predetermined regressors, SAR-RE errors

A3 : Assuming predetermined regressors, SMA-RE errors, Chow-Lin $\mathbf{W}_N$

A4 : Spatial Durbin, predetermined regressors, SMA-RE errors

A5 : Spatial Durbin, exogenous regressors, SMA-RE errors

10.3 The Chow-Lin approach

This commences with aggregate trade values between $\widetilde{N} = 21$ EU counties (denoted by the square $(\widetilde{N} \times \widetilde{N})$ matrix $\mathbf{T}_{\widetilde{N}}$, in which the subscript $\widetilde{N}$ means national). There are $\widetilde{N} = 21$ unobserved intra-country trade flows, thus giving $p = 420$ observations for the year 2000. The $\widetilde{N}^2 = 441$ 'observations' are the dependent variable in a Weighted Least Squares regression model. All observed trade flows are given the weight 1, and those unobserved weighted zero. In terms of parameter estimates, an entirely equivalent procedure is to estimate the 420 observed trade flows by OLS. In the regression, the explanatory regressors are great circle distances between country pairs ($\mathbf{G}_{\widetilde{N}}$), and the product of each pair of country's national GVA level ($\widetilde{\mathbf{q}}_{\widetilde{N}}$) in the year 2000, so that given the $(\widetilde{N} \times 1)$ vector $\widetilde{\mathbf{q}}_{\widetilde{N}}$, $\mathbf{Q}_{\widetilde{N}} = \widetilde{\mathbf{q}}_{\widetilde{N}}\widetilde{\mathbf{q}}_{\widetilde{N}}^{\top}$ is an $(\widetilde{N} \times \widetilde{N})$ matrix. Subsequently $\mathbf{Q}_{\widetilde{N}}, \mathbf{G}_{\widetilde{N}}$ and $\mathbf{T}_{\widetilde{N}}$ are reshaped as $(\widetilde{N}^2 \times 1)$ vectors $\mathbf{q}_{\widetilde{N}}$, $\mathbf{g}_{\widetilde{N}}$ and $\mathbf{t}_{\widetilde{N}}$, and together with $\mathbf{c}_{\widetilde{N}}$ which is an $(\widetilde{N}^2 \times 1)$ vector of ones, these variables provide the input for the regression denoted by equation (31), thus giving the estimates $\widehat{\boldsymbol{\beta}}_{\widetilde{N}}$. Interregional trade estimates are based on the (national level) regression parameter estimates $\widehat{\boldsymbol{\beta}}_{\widetilde{N}}$ and on the estimated regression residuals $\widehat{\mathbf{e}}_{\widetilde{N}}$. Thus, trade flows between regions, denoted by $\mathbf{t}_{\widetilde{R}}$, are obtained by applying the national level estimates $\widehat{\boldsymbol{\beta}}_{\widetilde{N}}$ to the regressors measured at the regional level, denoted by $\mathbf{g}_{\widetilde{R}}$ and $\mathbf{q}_{\widetilde{R}}$. Also, an equal share of the national level residuals $\widehat{\mathbf{e}}_{\widetilde{N}}$ is added to regions corresponding to country pairs. This process is summarised by the two equations:

$$\ln \mathbf{t}_{\widetilde{N}} = \beta_{\widetilde{N},1}\mathbf{c}_{\widetilde{N}} + \beta_{\widetilde{N},2} \ln \mathbf{g}_{\widetilde{N}} + \beta_{\widetilde{N},3} \ln \mathbf{q}_{\widetilde{N}} + \mathbf{e}_{\widetilde{N}} \tag{31}$$

with log bilateral interregional trade flows $\ln \mathbf{t}_{\widetilde{R}}$ then being obtained using

$$\ln \mathbf{t}_{\widetilde{R}} = \beta_{\widetilde{N},1}\mathbf{c}_{\widetilde{R}} + \beta_{\widetilde{N},2} \ln \mathbf{g}_{\widetilde{R}} + \beta_{\widetilde{N},3} \ln \mathbf{q}_{\widetilde{R}} + \widehat{\mathbf{e}}_{\widetilde{R}}. \tag{32}$$

To obtain the $(\widetilde{R}^2 \times 1)$ vector $\widehat{\mathbf{e}}_{\widetilde{R}}$, we calculate the $(\widetilde{R} \times \widetilde{R})$ matrix $\widehat{\mathbf{V}}_{\widetilde{R}} = \mathbf{h}\widehat{\mathbf{R}}_{\widetilde{N}}\mathbf{h}^{\top}$ where $\mathbf{h} = \mathbf{d}^{\top}(\mathbf{d}\mathbf{d}^{\top})^{-1}$and $\mathbf{d}$ is an $(\widetilde{N} \times \widetilde{R}\ )$ indicator matrix with ones in each country's row indicating which regions are within that country, and zeros indicating those which are not. The $(\widetilde{N} \times \widetilde{N})$ matrix of residuals $\widehat{\mathbf{R}}_{\widetilde{N}}$ is formed by reshaping the $(\widetilde{N}^2 \times 1)$ vector $\widehat{\mathbf{e}}_{\widetilde{N}}$ so that $\widehat{\mathbf{R}}_{\widetilde{N}}(1 : \widetilde{N}, 1) = \widehat{\mathbf{e}}_{\widetilde{N}}(1 : \widetilde{N}), \widehat{\mathbf{R}}_{\widetilde{N}}(1 : \widetilde{N}, 2) = \widehat{\mathbf{e}}_{\widetilde{N}}(\widetilde{N}+1 : 2\widetilde{N}), ..., \widehat{\mathbf{R}}_{\widetilde{N}}(1 : \widetilde{N}, \widetilde{N}) = \widehat{\mathbf{e}}_{\widetilde{N}}(\widetilde{N}^2 - \widetilde{N}+1 : \widetilde{N}^2)$.

The outcome $\widehat{\mathbf{V}}_{\widetilde{R}}$ is an $(\widetilde{R} \times \widetilde{R})$ matrix containing the inter-country residuals $\widehat{\mathbf{e}}_{\widetilde{N}}$ allocated equally to all regions corresponding to country pairs. Also there are block diagonal zeros as a result of the unobserved intra-country trade flows, and zeros across 4 rows representing 4 regions equal to the countries Estonia, Lithuania, Latvia and Luxembourg which were excluded from the initial regression. Finally, $\widehat{\mathbf{V}}_{\widetilde{R}}$ is reshaped as the $(\widetilde{R}^2 \times 1)$ vector $\widehat{\mathbf{e}}_{\widetilde{R}}$ of equation (32) to give $\ln \mathbf{t}_{\widetilde{R}}$. The resulting $\left(\widetilde{R} \times \widetilde{R}\right)$ matrix of interregional trade flows is $\mathbf{W}_N^* = \mathbf{T}_{\widetilde{R}}$.

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