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Abstract

This paper presents the business cycle model without using assumptions of general equilibrium. We use agent-based models, risk assessments and economic space as ground for modelling business cycles. All economic agents are at risk but not for all agents risk assessments are performed. We propose that for each agent risk assessment can be performed and suggest treat risk ratings $\boldsymbol{x}$ of agents as their coordinate $\boldsymbol{x}$ on economic space. Agents fill economic domain bounded by most secure and most risky agents. Economic processes, exogenous or endogenous shocks induce evolution of agent's risk coordinates. We show how risk motions of agents on the bounded economic domain induce the business cycle. We derive the system of economic equations that describe macroeconomic evolution and the business cycle on economic space. As example, we study simple model that describe relations between macro Assets $A(t,\boldsymbol{x})$ and Revenue-on-Assets $B(t,\boldsymbol{x})$. To show how economic equations describe the business cycle we obtain from them the system of ordinary differential equations that describes business cycle time fluctuations of macroeconomic Assets $A(t)$ and Revenue-on-Assets $B(t)$.

Keywords: Business cycle; Agent-Based Models; Risk Assessment; Economic Space

JEL: C00, C60, E32, F44, G00

1. Introduction

Modeling, forecasting and managing business cycles for years remain core problem of macroeconomics. Since Mitchel's remark that: "In a business cycle, the order of events is crisis, depression, revival, prosperity, and another crisis (or recession)". (Mitchell, 1927, p.79) not much was changed in treatment of business cycles. "The incorporation of cyclical phenomena into the system of economic equilibrium with which they are in apparent contradiction, remains the crucial problem of Trade Cycle Theory" (Hayek, 1933, quoted by Lucas, 1995). "Why aggregate variables undergo repeated fluctuations about trend, all of essentially the same character? Prior to Keynes' General Theory, the resolution of this question was regarded as one of the main outstanding challenges to economic research, and attempts to meet this challenge were called business cycle theory" (Lucas, 1995). Questions on properties and modeling of business cycles remain relevant now and will attract researchers for years. Studies of business cycle since Mitchell (1927), Tinbergen (1935) and Schumpeter (1939) were supported by hundreds publications and we mention studies by (Lucas, 1980; 1982; Kydland and Prescott 1982; 1991; Zarnowitz 1992; Lucas 1995; Diebold and Rudebusch 1999; Rebelo 2005; Kiyotaki 2011; Diebold and Yilmaz 2013; Jorda, Schularick and Taylor 2016; Huggett 2017) as just a small part of the business cycle research. "Theories of business cycles should presumably help us to understand the salient characteristics of the observed pervasive and persistent non seasonal fluctuations of the economy" (Zarnowitz, 1992). "One of the most controversial questions in macroeconomics is what explains business-cycle fluctuations?" (Huggett, 2017).

Problems of sources of business cycle fluctuations (Shapiro and Watson, 1988; Shea, 1999; Andrle, Brůha and Solmaz, 2017) and impact of risk assessment are treated as part of business cycle studies and count hundreds of publications (Alvarez and Jermann, 1999; Tallarini, 2000; Pesaran, Schuermann and Treutler, 2007; Mendoza, Yue, 2012; Christiano, Motto, and Rostagno, 2013). Selected papers on financial risks over 100 years of research (Diebold, 2012) outlines a special chapter on "Financial Risk And The Business Cycle". Risk measurements, concepts, techniques and tools (McNeil, Frey and Embrechts, 2005) and relations to macro modeling (Brunnermeier and Krishnamurthy, 2014) present only top slice of risk impact on business cycle studies.

Current modeling of risk impact on business cycles follow general equilibrium framework (Lucas, 1975; Kydland and Prescott, 1982; 1991; Mullineux and Dickinson, 1992; Kiyotaki, 2011; Mendoza and Yue, 2012) and describe endogenous business cycle models within general equilibrium framework

(Grandmont, 1985; Farmer and Woodford, 1997; Farmer, 2016; Bilbiie, Ghironi and Melitz, 2012; Growiec, McAdam and Mućk, 2015). The general equilibrium is treated as ground for business cycle. "Business deterministic cycles will be shown to appear in a purely endogenous fashion under *laisser faire*. Markets will be assumed to clear in the Walrasian sense at every date, and traders will have perfect foresight along the cycles". (Grandmont, 1985). "Real business cycle models view aggregate economic variables as the outcomes of the decisions made by many individual agents acting to maximize their utility subject to production possibilities and resource constraints. More explicitly, real business cycle models ask the question: How do rational maximizing individuals respond over time to changes in the economic environment and what implications do those responses have for the equilibrium outcomes of aggregate variables?" (Plosser, 1989). "The real business cycle theory is a business cycle application of the Arrow-Debreu model, which is the standard general equilibrium theory of market economies" (Kiyotaki, 2011).

However above remark by Hayek that "cyclical phenomena are in apparent contradiction with economic equilibrium" as well as complexity and variability of business cycle properties along with evolution of economy may indicate that business cycle modeling requires new approaches and approximations beyond the concept of general equilibrium.

In this paper we present the business cycle model without using assumptions of the general equilibrium framework (Arrow and Debreu, 1954; Arrow, 1974; Kydland and Prescott, 1982; Lucas, 1975; Gintis, 2007; Ohanian, Prescott, Stokey, 2009; Starr 2011; Cardenete, Guerra, Sancho, 2012; Del Negro, et.al., 2013). Well-known Occam's razor principle (Baker, 2007) states: "Entities are not to be multiplied beyond necessity". In other words: the less initial assumptions – the better. In this paper we show that description of business cycle don't necessarily requires usage of market equilibrium and other assumptions that establish general equilibrium theory. Business cycles are very complex processes and different approximations should be developed to describe their various properties. This paper presents "simple" approximation of business cycles that don't use behavioral motivations (Sterman and Mosekilde, 1993; Lucas, 1995; Jaimovich and Rebelo, 2007) and expectations (Grossman, 1980; Lucas, 1980; Taylor, 1984). We are planning to enhance our approach and develop the business cycle model that takes into account expectations and behavioral motivations in forthcoming publications.

This paper develops the business cycle model on base of econometric observations and risk assessment. Let us regard all participants of economic processes as banks, corporations, small firms and households and etc., as economic agents. All economic agents are at risk but econometrics don't assess risk ratings for all agents. Let's propose that econometrics can deliver sufficient data to provide assessment of economic and financial risks for all agents and use agent's risk ratings as their coordinates on economic space (Olkhov, 2016a-b, 2017a-f) *alike to* coordinates of physical particles. In other words: we suggest treat risk assessments *alike to* measurements of agent's coordinates. As we show below that can help model macroeconomic evolution and describe business cycle fluctuations without usage of economic equilibrium framework.

Each agent has a lot of extensive economic and financial variables as Assets and Debts, Credits, Loans, Production Function, Demand and Consumption and so on. We develop transition from description of extensive variables of separate agents to description of cumulative macro variables with risk ratings $\boldsymbol{x}$. Main idea: let's rougher description of extensive economic variables of separate economic agents by description of cumulative variables associated with groups of agents. In other words, let's roughen economic space and define macro variables at point $\boldsymbol{x}$ that belong to all agents in small domain near point $\boldsymbol{x}$. Such a roughening is already used in economics. For example aggregation (without doubling) of all Credits provided by agents of entire economics define macro Credit $C(t)$ provided in macroeconomics at moment t. Cumulative Investment of all agents equals macroeconomic Investment $I(t)$ and so on. We propose develop intermediate aggregation of variables of agents near each risk point $\boldsymbol{x}$. Such approximation can be treated as intermediate between description of extensive economic variables of separate agents on one hand and description of variables time-series obtained as aggregation of extensive variables of all agents of entire economics on other hand. For example, aggregation of Assets $A(t,\boldsymbol{x})$ at point $\boldsymbol{x}$ is intermediate approximation of Assets $A_j(t,\boldsymbol{x})$ of separate agents j, $j=1,2,\ldots$ with risk ratings $\boldsymbol{x}$ and time-series function $A(t)$ of total macroeconomic Assets. We don't specify type of Assets as our approach can be used for any type of Assets. Transition from description of Assets of separate agents to description of assets distribution $A(t,\boldsymbol{x})$ as function of risk $\boldsymbol{x}$ on economic space, allows define mean Assets risk $\boldsymbol{X}_A(t)$ (4.3) as mean risk coordinates $\boldsymbol{x}$ weighted by Assets $A(t,\boldsymbol{x})$ on economic space. Mean Assets risk $\boldsymbol{X}_A(t)$ can be treated *alike to* mass center $\boldsymbol{X}_A(t)$ of a body with total Assets mass $A(t)$ *(4.1)*. Mean Assets risk $\boldsymbol{X}_A(t)$ is not a constant and can change due to variation of Assets risks and variations of Assets at point $\boldsymbol{x}$. Borders of economic domain (1.1) reduce motion of mean Assets risk $\boldsymbol{X}_A(t)$ and hence its evolution should follow complex fluctuations on bounded economic domain (1.1) on economic space.

We state that mean Assets risk $\boldsymbol{X}_A(t)$ fluctuates along risk axes of economic space and that reflect business cycle and accompanied by fluctuations of total macroeconomic assets $A(t)$. We show (Olkhov, 2017f) that motion of mean Assets risk $\boldsymbol{X}_A(t)$ is governed by (5.1.1-5.1.3; 5.2; 5.3) economic equations that describe evolution of assets $A(t,\boldsymbol{x})$. Mean risks are different for different economic and financial variables and their mutual motions and interactions are very complex. Fluctuations of mean risks of economic or financial variables reflect complex business cycle processes and are accompanied by fluctuations of macro variables like Assets $A(t)$, Investment $I(t)$ and etc. Fluctuations of mean risks are induced by change of agent's risk ratings that can have exogenous or endogenous origin, can be induced by monetary, fiscal, policy or technology shocks or etc. Recent reviews of macroeconomic shocks that drive economic fluctuations see (Ramey, 2016). We don't describe origin of risk rating change but show that collective evolution of agent's risk coordinates on economic space can describe the business cycle.

The rest of the paper is organized as follows. In Section 2 we argue the model setup and economic space (Olkhov, 2016a-b, 2017a-b). In Section 3 we discuss business cycle fluctuations in terms of economic space model and explain dependence of business cycles on economic equations. As example, we present a simple model interactions between Assets A(t,x) and Revenue-on-Assets B(t,x). We show that solutions for aggregate macro Assets A(t) and Revenue-on-Assets B(t) follow the business cycle time fluctuations. The same considerations can be applied to any time-series of extensive macroeconomic and financial variables as Credits, Investment, Demand and etc. Conclusions are in Section 4. In Appendix we present derivation of the system of ordinary differential equations that describe time fluctuations of macro Assets A(t) and Revenue-on-Assets B(t).

2. Model setup

Main issue of our approach – risk assessment of economic agents can be treated *alike to* measurements of coordinates of physical particles. That requires development of risk assessment methodologies and econometric data. Up now rating agencies (Chane-Kon, et.al. 2010; Kraemer and Vazza, 2012; Metz and Cantor, 2007) provide risk assessment for huge banks and corporations and indicate them by risk ratings. Let's assume that current assessment methodology can be extended to provide risk assessment for all agents of entire economics. Let's propose that it is possible to assess all economic and financial risks those affect macroeconomics. Let's use agent's risk ratings as their coordinates. Let's call a space that imbed agent's risk ratings as coordinates – economic space. Below we introduce economic space modeling relations according to (Olkhov, 2016a-b, 2017a-b).

2.1. Economic space

Let's regard extensive (additive) economic variables as Assets, Debts, Consumption and Investment, Credits and Loans and etc. All extensive macro variables are determined as aggregates of corresponding variables of economic agents. For example macro Investment of entire economics are defined as sum of Investment of all agents. Cumulative Credits of agents define macroeconomic Credits and macro Assets are determined as cumulative Assets of all agents. Thus growth and fluctuations of extensive macro variables are determined by growth and fluctuations of corresponding extensive variables of economic agents. To describe evolution of macro variables one should model corresponding evolution of agent's economic variables. To correspond complexity of economic processes we introduce economic space notion and describe economic and financial variables as functions of time and coordinates (Olkhov, 2016a-b; 2017a-b). That vastly increases and enhances methods for economic modeling. Our approach is completely different from agent-based economics (Tesfatsion and Judd, 2005) and spatial economics (Fujita, 2010).

Introduction of economic space as foundation for modeling evolution of agent's extensive variables allows describe dynamics of agents *alike to* description of multi particle systems. We outline vital differences between economic and physical systems but show that usage of similar concepts allow develop useful parallels between description of multi-particles systems and physics of fluids on one hand and business cycle fluctuations on other hand.

Risk rating agencies (Chane-Kon et all., 2010; Metz, 2007; Kraemer and Vazza, 2012) estimate risk ratings of huge corporations and banks. These ratings take values of risk grades and noted as *AAA, BB, C* and so on. Let us treat risk grades *AAA, BB, C* as points $x_1, x_2,.. x_m$ of discrete space. Let's propose, that risk assessments methodologies can be extended to estimate risk ratings for all agents of entire economics. Let's regard grades of a single risk as points of one-dimensional space and simultaneous assessments of n different risks as measurements of agent's coordinates on n-dimensional economic space. Let's propose, that risk assessments methodologies can be generalized so that risk grades can take continuous values on space R. Thus risk grades of n different risks can be defined as points on R^n.

It is impossible take into account all economic and financial risks. Description of agents of entire macroeconomics on economic space R^n requires choice of n major risks that cause major effects on economic and financial processes. To determine economic space one should estimate current risks and select two, three major risks as main factors affecting economic system. That establishes economic space with two or three dimensions. To select most valuable risks one should compare impact of different risks on economic and financial processes and chose few most valuable. Selection of n major risks defines a separate and very tough problem for future. Below we develop business cycle models on economic space R^n in the assumption that macroeconomics and economic agents are under permanent action n risks. Risk ratings of agents play role of their coordinates on economic space R^n.

2.2. Economic hydrodynamic-like approximation

Introduction of economic space permits allocate agents by their risk ratings $\boldsymbol{x}$ as coordinates $\boldsymbol{x}$. Economic agents can move on economic space *alike to* particles. Such motion reflects change of agent's risk ratings and causes evolution of economic and financial variables of agents. Agents of entire economics fill certain economic domain that is bounded by minimum or most secure and maximum or most risky grades. Let's assume that coordinates $\boldsymbol{x}=(x_1,\ldots x_n)$ of economic domain (1.1) on n-dimensional economic space are determined by relations

$$0 < x_i < X_i \,, i = 1, \ldots n \tag{1.1}$$

Here $x=0$ define most secure area and X_i define most risky area on economic space. Let us assume that a unit volume $dV(\boldsymbol{x})$ near point $\boldsymbol{x}$ contains many separate agents but scales dV_i of a unit volume are small to compare with scales X_i of economic domain.

$$dV_i \ll X_i \,, i = 1, \ldots n \;\; ; \quad dV = \prod\nolimits_{i=1,..n} dV_i \tag{1.2}$$

Let us aggregate extensive variables of agents inside a unit volume $dV(\boldsymbol{x})$. For example, aggregation (without doubling) of Credits provided by agent inside unit volume $dV(\boldsymbol{x})$ allows define Credits as function of time t and coordinate $\boldsymbol{x}$ on economic space. Such approximation is intermediate between description of extensive variables of separate agents on one hand and description of time-series of extensive macro variables on the other hand. Indeed, time-series of macro variables are defined as aggregates of corresponding variable for all economic agents. For example, macro Investment time-series are defined as aggregate (without doubling) of Investment of all economic agents. Macro Credits time-series are defined as aggregate of Credits provided by all agents, and etc. Economic space permits define intermediate approximation of extensive economic and financial variables as cumulative variables of agents with risk rating in a unit volume $dV(\boldsymbol{x})$ near point $\boldsymbol{x}$. Such approach to economic modeling is *alike* to hydrodynamic approximation in physics: hydrodynamic scales are small to compare with macro scales of fluid but contain a lot of molecules (Landau and Lifshitz, 1987). Let's call such intermediate approximation as economic hydrodynamic-like approximation. This approximation presents transition from description of numerous separate agents and their variables to description that neglects granularity of agents and model economic variables alike to *"economic fluids"* (Olkhov, 2016a-b, 2017a-b). Below we present formal derivation of economic hydrodynamic-like approximation. For brevity let's further call economic agents as economic particles or *e-particles* and economic space as *e-space*.

Each e-particle has many extensive economic and financial variables like Assets and Debts, Investment and Savings, Credits and Loans and etc. Let's assume that for all e-particles sum of extensive variables of any group of agents equals same variable of entire group. For example, aggregation of Assets of e-particles with coordinates $\boldsymbol{x}$ on e-space define Assets as function of time t and $\boldsymbol{x}$. Integral of Assets $A(t,\boldsymbol{x})$ over e-space equals aggregate Assets $A(t)$ of entire macroeconomics. We mention Assets as example of extensive macro variable only and our considerations are valid for any extensive economic variables. To aggregate Assets of separate e-particles at point $\boldsymbol{x}$ let's collect Assets of e-particles with risk rating $\boldsymbol{x}$. This sum will be random due to random number of e-particles and random values of Assets of e-particles at point $\boldsymbol{x}$. To obtain regular value of Assets $A(t,\boldsymbol{x})$ at point $\boldsymbol{x}$ let's average this sum by probability distribution f. Let's state that distribution f define probability to observe $N(\boldsymbol{x})$ e-particles with values of Assets equal $a_1,\dots a_{N(x)}$. That determine Assets $A(t,\boldsymbol{x})$ at point $\boldsymbol{x}$ on e-space (Eq.(2.1) below). Assets $A(t,\boldsymbol{x})$ as function of time t and coordinate $\boldsymbol{x}$ behave *alike to* mass density of fluids in hydrodynamics and we shall call $A(t,\boldsymbol{x})$ as density of Assets "economic *fluid"*. We underline that we use these notions to outline *parallels to* hydrodynamics only. Nature and properties of Assets $A(t,\boldsymbol{x})$ have nothing common with properties of hydrodynamics. To describe evolution of Assets fluid let's define velocity of such a fluid. Let's mention that velocities of e-particles are not additive variables and their sum doesn't define velocity of Assets motion. To define correctly velocities of Assets fluid one should define additive variable - "*Assets impulses*" $\boldsymbol{p_i}$ at point $\boldsymbol{x}$ as product of Assets a_i of e-particle i and velocity $\boldsymbol{v_i}$ (Eq.(2.2) below). "*Assets impulses*" $\boldsymbol{p_i}$ describes flow of Assets a_i with velocity $\boldsymbol{v_i}$. Such "Assets impulses" $\boldsymbol{p_i} = a_i\,\boldsymbol{v_i}$ - are additive variables and sum of "Assets impulses" can be averaged by similar probability distribution f. Assets impulses $\boldsymbol{P}(t,\boldsymbol{x})$

permit define velocities $\boldsymbol{v}$ of Assets fluid (Eq.(2.3) below). Different economic and financial fluids are determined by different extensive variables and can flow with different velocities. For example flow of Capital fluid on e-space can have velocity higher then flow of Assets fluid, nevertheless they are determined by motion of same e-particles. Interactions between numerous "*economic fluids*" describe macroeconomics in hydrodynamic-like approximation and uncover hidden complexity of economic modeling. Let's present above issues in a more formal way.

Let's assume that each e-particle on e-space R^n at moment t is described by l extensive variables $(u_1,\dots u_l)$ and there are $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$. Let's state that velocities of e-particles at point $\boldsymbol{x}$ equal $\boldsymbol{v}=(\boldsymbol{v}_1,\dots\ \boldsymbol{v}_{N(x)})$. Velocity of e-particle on e-space describes change of risk rating of e-particle per unit of time. Let's assume that values of variables equal $u=(u_{1i},\dots u_{li}),\ i=1,..N(\boldsymbol{x})$. Each extensive variable u_j at point $\boldsymbol{x}$ defines macro variable U_j as sum of variables u_{ji} of $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$

$$U_j = \sum_i u_{ji}\ ; \quad j = 1,..l\ ; \quad i = 1,\dots N(\boldsymbol{x}) \tag{1.3}$$

To describe motion of variable U_j (1.3) let's establish additive variable $\boldsymbol{p}_{ji}$ alike to impulse in physics. For e-particle i let's define impulses $\boldsymbol{p}_{ji}$ (1.4) as product of extensive variable u_j that takes value u_{ji} and velocity $\boldsymbol{v}_i$:

$$p_{ji} = u_{ji}v_i \tag{1.4}$$

Economic meaning of impulses $\boldsymbol{p}_{ji}$: they describe flows or amount of extensive variables u_{ji} those change their risk coordinates with velocities $\boldsymbol{v}_i$. For example if Assets of e-particle i take value a_i and velocity of e-particle i equals $\boldsymbol{v}_i$ then impulse $\boldsymbol{p}_{ai}$ of Assets of e-particle i equals $\boldsymbol{p}_{ai} = a_i\boldsymbol{v}_i$. Thus if e-particle has l extensive variables $(u_1,\dots u_l)$ and velocity $\boldsymbol{v}$ then it has l impulses $(p_1,p_2,..p_l)=(u_{11}\boldsymbol{v},\dots u_{lN(x)}\boldsymbol{v})$. Let's define impulse $\boldsymbol{P}_j$ (1.5) of macro variable U_j as

$$\boldsymbol{P}_j = \sum_i u_{ji} \cdot \boldsymbol{v}_i\ ; \quad j = 1,..l\ ; \quad i = 1,\dots N(\boldsymbol{x}) \tag{1.5}$$

Impulse $\boldsymbol{P}_j$ (1.5) for variable U_j describes collective impulse of all e-particles at point $\boldsymbol{x}$. Values of U_j and $\boldsymbol{P}_j$ are random due to random values of u_{ji} and $\boldsymbol{v}_i$. To define regular variables let's introduce economic distribution function $f=f(t,\boldsymbol{x};U_1,..U_l,\ \boldsymbol{P}_1,..\boldsymbol{P}_l;N)$ that define probability to observe $N=N(t,\boldsymbol{x})$ e-particles with aggregate variables U_j (1.3) and impulses $\boldsymbol{P}_j$ (1.5) at point $\boldsymbol{x}$ at time t. Averaging of U_j and $\boldsymbol{P}_j$ (2.1; 2.2) within economic distribution function $f(t,\boldsymbol{x};U_1,..U_l,\ \boldsymbol{P}_1,..\boldsymbol{P}_l;N)$ establish transition from approximation that takes into account variables of separate e-particles to economic *hydrodynamic-like* approximation that neglect granularity of e-particles and describe macro variables *alike to "economic fluids*". Let's define $U_j(t,\boldsymbol{x})$:

$$U_j(t,\boldsymbol{x}) = \sum_N \int U_j\, f(t,\boldsymbol{x},U_1,\dots U_l,\boldsymbol{P}_1,..\boldsymbol{P}_l;N)\, dU_1..dU_l d\boldsymbol{P}_1..d\boldsymbol{P}_l \quad (2.1)$$

and impulse functions $\boldsymbol{P}_j(t,x)$

$$\boldsymbol{P}_j(t,\boldsymbol{x}) = \sum_N \int \boldsymbol{P}_j\, f(t,\boldsymbol{x},U_1,\dots U_l,P_1,..P_l;N)\, dU_1..dU_l d\boldsymbol{P}_1..d\boldsymbol{P}_l \quad (2.2)$$

Relations (2.1; 2.2) define e-space velocities υj(t,x) of "economic fluids" Uj(t,x) as

$$\boldsymbol{P_j}(t,\boldsymbol{x}) = U_j(t,\boldsymbol{x})\boldsymbol{v_j}(t,\boldsymbol{x}) \quad (2.3)$$

Relations (2.1-2.3) describe averaging of variables U_j and their impulses $\boldsymbol{P}_j$ over unit volume *dV*. Let propose that unit volume *dV* contains a lot of e-particles but its scales dV_i are small to compare with scales of economic domain (1.1;1.2). Economic variables $U_j(t,\boldsymbol{x})$ describe cumulative value of e-particles inside a unit volume *dV*. Impulses $\boldsymbol{P}_j(t,\boldsymbol{x})$ (2.2) of economic variables $U_j(t,\boldsymbol{x})$ describe flow of aggregate variable $U_j(t,\boldsymbol{x})$ (2.1) with aggregate velocity $\boldsymbol{v_j}(t,\boldsymbol{x})$ (2.3). For example (see below: 2.4-2.6) Assets impulse $\boldsymbol{P}(t,\boldsymbol{x})$ (2.5; 2.6) describes flow of Assets *fluid* $A(t,\boldsymbol{x})$ (2.4) with velocity $\boldsymbol{v}(t,\boldsymbol{x})$ (2.6). Velocities $\boldsymbol{v}_j(t,\boldsymbol{x})$ of *"economic fluids"* $U_j(t,\boldsymbol{x})$ describe collective change of risk coordinates of economic variables $U_j(t,\boldsymbol{x})$ per time term *dt*. Functions $U_j(t,\boldsymbol{x})$ can describe Investment and Loans, Assets and Debts and so on. For example, Assets $A(t,\boldsymbol{x})$, impulse $\boldsymbol{P}(t,\boldsymbol{x})$ and velocity $\boldsymbol{v}(t,\boldsymbol{x})$ can be defined as

$$A(t,\boldsymbol{x}) = \sum_N \int a\, f(t,\boldsymbol{x},a,\boldsymbol{P};N)\, da d\boldsymbol{P} \quad (2.4)$$

$$\boldsymbol{P}(t,\boldsymbol{x}) = \sum_N \int \boldsymbol{P}\, f(t,\boldsymbol{x},a,\boldsymbol{P};N)\, da d\boldsymbol{P} \quad (2.5)$$

$$A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x}) = \boldsymbol{P}(t,\boldsymbol{x}) \quad (2.6)$$

Here a and $\boldsymbol{P}$ denote sum of Assets and impulses of $N(\boldsymbol{x})$ e-particles with coordinates $\boldsymbol{x}$ in a unit volume *dV*. Economic distribution $f(t,\boldsymbol{x},a,\boldsymbol{P};N)$ describes probability to observe $N=N(\boldsymbol{x})$ e-particles with aggregate assets a and aggregate impulse $\boldsymbol{P}$ at point $\boldsymbol{x}$ at moment t. To describe evolution of macro variables let's take as example Assets $A(t,\boldsymbol{x})$ and Assets impulse $\boldsymbol{P}(t,\boldsymbol{x})$ (2.4-2.5) and derive economic equations for these variables.

2.3. Economic equations

Derivations of hydrodynamic equations in physics are presented in numerous studies (Huang, 1963; Resibois and De Leener, 1977; Landau and Lifshitz, 1987; Smits, 2017). Economics is completely different from physical systems but certain parallels between them allow derive economic equations and describe evolution of "*economic fluids*" *alike to* hydrodynamics. Meaning of economic equations on Assets is very simple (Olkhov, 2016a-b, 2017a-b): they describe balance between left side that describe change of Assets $A(t,\boldsymbol{x})$ and its impulse $\boldsymbol{P}(t,\boldsymbol{x})$ in a unit volume $dV(\boldsymbol{x})$ and right hand side that describe factors that can affect these changes. Let's regard evolution of Assets density $A(t,\boldsymbol{x})$ in a unit volume $dV(\boldsymbol{x})$. There are two factors that can change $A(t,\boldsymbol{x})$ in a unit volume dV. Its value can be changed in time and can be changed due to flow of Assets "*economic fluid*" trough borders of a unit volume dV. Change in time can be presented as

$$\frac{\partial}{\partial t}A(t,\boldsymbol{x})$$

Flow of Assets "*economic fluid*" trough borders of a unit volume dV is described by surface integral of flux of Assets $\boldsymbol{P}(t,\boldsymbol{x}) = A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$ of a unit volume:

$$\oint dS\, \boldsymbol{P}(t,\boldsymbol{x}) = \oint dS\, A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$$

Well known Divergence theorem (Strauss 2008, p.179) states that surface integral of flux through surface of a unit volume $dV(\boldsymbol{x})$ equals volume integral of divergence:

$$\oint dS\, A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x}) = \int dV\, \nabla \cdot \big(A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})\big)$$

Thus total change of Assets $A(t,\boldsymbol{x})$ in a unit volume $dV(\boldsymbol{x})$ takes form

$$\frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla \cdot (A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})) \tag{3.1}$$

Let's denote as Q_1 factor that balance change of Assets $A(t,\boldsymbol{x})$ in a unit volume at point $\boldsymbol{x}$ described by (3.1) and take economic equation on Assets density $A(t,\boldsymbol{x})$ as:

$$\frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla \cdot \big(A(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})\big) = \frac{\partial}{\partial t}A(t,\boldsymbol{x}) + \nabla \cdot \boldsymbol{P}(t,\boldsymbol{x}) = Q_1 \tag{3.2}$$

Direct form of factor Q_1 is determined by particular economic model. Left side of equation (3.2) describes two variables: $A(t,\boldsymbol{x})$ and its impulse $\boldsymbol{P}(t,\boldsymbol{x})$. Thus one needs equation that describes Assets impulse $\boldsymbol{P}(t,\boldsymbol{x})$. Due to same reasons economic equation on impulse $\boldsymbol{P}(t,\boldsymbol{x})$ takes same form as (3.2):

$$\frac{\partial}{\partial t}\boldsymbol{P}(t,\boldsymbol{x}) + \nabla \cdot \big(\boldsymbol{P}(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})\big) = \boldsymbol{Q}_2 \tag{3.3}$$

$\boldsymbol{P}(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})$ mean flow of Assets impulse $\boldsymbol{P}(t,\boldsymbol{x})$ through surface of a unit volume dV. For each component i of n-dimensional vector P_i , $i=1,..n$ equation (3.3) takes:

$$\frac{\partial}{\partial t}P_i(t,\boldsymbol{x}) + \nabla \cdot \big(P_i(t,\boldsymbol{x})\boldsymbol{v}(t,\boldsymbol{x})\big) = Q_{2i} \;\; ; i = 1, \dots n \tag{3.4}$$

$\boldsymbol{Q}_2$ describes factors those impact evolution of Assets impulse $\boldsymbol{P}(t,\boldsymbol{x})$ and are determined by particular economic model. Thus economic meaning of equations (3.2; 3.3; 3.4) and usage of these equations for description of particular economic problem is defined by right hand side factors Q_1 and $\boldsymbol{Q_2}$. We underline that economic equations (3.2-3.4) have nothing common with Hydrodynamic Navier-Stokes Equations that describe physics of fluids (Resibois and Leener, 1977; Landau and Lifshitz, 1987; Smits, 2017). It seems that equations (3.2; 3.3) are simple. But one should remember that enormous number of economic and financial variables and diversity of possible interactions between them makes entire description of interactions between economic variables much more complex then any problem of physical hydrodynamics. Equations similar to (3.2-3.4) can be used to describe evolution of extensive variables as Investment $I(t,\boldsymbol{x})$, Credits $C(t,\boldsymbol{x})$, Production function $F(t,\boldsymbol{x})$ and etc.

Let's outline one important issue. Extensive variables of e-particles are changed due to transactions between e-particles. For example, Investment transactions from e-particle 1 to e-particle 2 change total Investment of e-particle 1 and total Debts of e-particle 2. This paper studies simplest assumptions on form of transactions between e-particles. In this paper we assume that e-particles at point $\boldsymbol{x}$ enter into transactions with e-particles with same coordinates $\boldsymbol{x}$ only. Such simplification can be treated as *a local approximation* of economic and financial transactions between e-particles and allow describe factors Q_1 and $\boldsymbol{Q_2}$ by simple linear differential operators (Olkhov 2016a-b; 2017a; 2017c). We use *local approximation of transactions* in the next Section. More general and more complex economic models with *non-local approximations* or "*action-at-a-distance*" transactions between e-particles with risk coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ on e-space are presented in (Olkhov 2017b; 2017 d-e).

3. The Business Cycle Model

In this Section we present the business cycle model that respond to the question: "Why aggregate variables undergo repeated fluctuations about trend, all of essentially the same character?" (Lucas 1995). Definitions of economic and financial densities (2.1-2.6) as functions of time t and coordinate $\boldsymbol{x}$ allow define time-series for corresponding macro variables as integrals over e-space. For example integral of Assets density $A(t,\boldsymbol{x})$ over e-space:

$$A(t) = \int d\boldsymbol{x}\ A(t,\boldsymbol{x}) \tag{4.1}$$

equals macro Assets $A(t)$ of entire economics. Same relations define all aggregate macroeconomic and financial variables of entire economics as function of time t only. Investment, Credits, Taxes etc., can be presented similar to (4.1). Time evolutions and business cycles fluctuations of macro variables in time are determined by dynamics of corresponding economic and financial variables on e-space.

For example, time evolution of macro Assets $A(t)$ is determined by Assets $A(t,\boldsymbol{x})$. Now let's introduce notion of mean Assets risk $X_A(t)$. Similar mean risks can be determined for all economic and financial variables. As example let's assume that e-particle 1 with risk coordinate $\boldsymbol{x}$ at moment t has Assets $A_1(t,\boldsymbol{x})$ and e-particle 2 with risk coordinate $\boldsymbol{y}$ at moment t has Assets $A_2(t,\boldsymbol{y})$. Let's define risk coordinates of Assets composed by e-particle 1 and 2. Sum of their Assets equal $A_1(t,\boldsymbol{x})+ A_2(t,\boldsymbol{y})$ and we define mean Assets risk $\boldsymbol{X}_{A1,2}(t)$ as:

$$\boldsymbol{X}_{A1,2}(t)\big(A_1(t,\boldsymbol{x})+A_2(t,\boldsymbol{y})\big)=\boldsymbol{x}A_1(t,\boldsymbol{x})+\boldsymbol{y}A_2(t,\boldsymbol{y}) \tag{4.2}$$

Relations (4.2) define mean Assets risk $\boldsymbol{X}_{A1,2}(t)$ as average of risk weighted by value of Assets of e-particles 1 and 2. Relations (4.2) are *alike to* center of Assets mass $\boldsymbol{X}_{A1,2}(t)$ of two physical particles with mass $A_1(t,\boldsymbol{x})$ at point $\boldsymbol{x}$ and mass $A_2(t,\boldsymbol{y})$ at point $\boldsymbol{y}$. For distribution of Assets $A(t,\boldsymbol{x})$ let's define mean Assets risk $\boldsymbol{X}_A(t)$ similar to (4.2) as integral over economic domain (1.1) taking into account total Assets $A(t)$ (4.1):

$$A(t)\boldsymbol{X}_A(t)=\int d\boldsymbol{x}\;\;\boldsymbol{x}\,A(t,\boldsymbol{x}) \tag{4.3}$$

Mean Assets risk $\boldsymbol{X}_A(t)$ equals mean risk of macroeconomic Assets $A(t)$. It is *alike to* center of mass $\boldsymbol{X}_A(t)$ of a body with total mass $A(t)$ and mass density $A(t,\boldsymbol{x})$. We propose that evolutions of mean risks of different macro variables describe business cycle fluctuations of these variables. For example, mean Assets risk $\boldsymbol{X}_A(t)$ is changing due to variations of coordinates $\boldsymbol{x}$ and changing of Assets at point $\boldsymbol{x}$. As we show below mean Assets risk $\boldsymbol{X}_A(t)$ governed by economic equations (3.2;3.4 and 5.3; 5.4) that describe evolution of Assets $A(t,\boldsymbol{x})$ and assets impulse $\boldsymbol{P}(t,\boldsymbol{x})$. Mean Assets risk $\boldsymbol{X}_A(t)$ and mean risks of other macro variables can't grow up or diminish steadily along each risk axes as their values are bounded on economic domain (1.1). Values of mean risks and value of Assets mean risk $\boldsymbol{X}_A(t)$ in particular should oscillate along each risk axes between their minimum and maximum values. Dynamics of these fluctuations can be very complex. We propose that business cycles are related with fluctuations of mean risks of macro variables and fluctuations of Assets mean risk $\boldsymbol{X}_A(t)$ are linked with fluctuations of macroeconomic Assets $A(t)$. We repeat main statement: business cycles can be treated as fluctuations of mean risks of macro variables. Below we show how economic equations (3.2- 3.4) describe business cycle fluctuations.

3.1. Economic equations and business cycles

To develop simple business cycle model let's take economic equations (3.2-3.3) on Assets density $A(t,\boldsymbol{x})$. Let's assume that right hand side factors Q_1 and $\boldsymbol{Q_2}$ depend on one economic variable - Revenue-on-Assets $B(t,\boldsymbol{x})$ and its impulse $\boldsymbol{PB}(t,\boldsymbol{x})$. Returns on Assets are more widespread variable but it is intensive (not additive) variable. Revenue-on-Assets is extensive (additive) variable and relations between these variables are well known. To make our model self-consistent let's assume that Revenue-on-Assets $B(t,\boldsymbol{x})$ and impulse $\boldsymbol{PB}(t,\boldsymbol{x})$ follow same equations (3.2-3.3) and Q-factors in their right hand side depend on Assets $A(t,\boldsymbol{x})$ and its impulse $\boldsymbol{PA}(t,\boldsymbol{x})$ only. It is obvious that we simplify relations between Assets and Revenue-on-Assets and neglect all other economic factors that affect their evolution. We present very simple model to show two things. First – business cycle fluctuations exist even in such simple models. Second – derivation of business cycle equations even for such simple models is sufficiently complex problem.

To define Q-factors in the right side of equations (3.2-3.3) let's assume that Assets $A(t,\boldsymbol{x})$ and impulse $\boldsymbol{PA}(t,\boldsymbol{x})$ at moment t depend on Revenue-on-Assets $B(t,\boldsymbol{x})$ and its impulse $\boldsymbol{PB}(t,\boldsymbol{x})$ at moment t and vice versa. Let's assume that factor Q_{1A} for Continuity Equation (3.2) on Assets density $A(t,\boldsymbol{x})$ is proportional to square of risk coordinate x^2 and to divergence of Revenue-on-Assets impulse:

$$Q_{1A} = a\, x^2 \nabla \cdot \boldsymbol{PB}(t,\boldsymbol{x}) \tag{5.1}$$

Let's assume that similar relations define factor Q_{1B} for Continuity Equation (3.2) on Revenue-on-Assets density $B(t,\boldsymbol{x})$

$$Q_{1B} = b\, x^2 \nabla \cdot \boldsymbol{PA}(t,\boldsymbol{x}) \tag{5.2}$$

Here a and b – const. Economic meaning of (5.1; 5.2) is as follows. We propose that Q_{1A} is proportional to square of risk coordinates x^2 and divergence of Revenue-on-Assets impulse $\nabla \cdot \boldsymbol{PB}(t,\boldsymbol{x})$. Positive divergence of Revenue-on-Assets impulse $\nabla \cdot \boldsymbol{PB}(t,\boldsymbol{x}) = \nabla \cdot \big(\boldsymbol{v}(t,\boldsymbol{x})A(t,\boldsymbol{x})\big) > 0$ means that there is a source of additional Revenues flow at point $\boldsymbol{x}$. Such a source of Revenues can increase growth of Assets at that point proportionally to square of risk x^2. Indeed, Investor's risk-taking objectives are always supported by Investor's expectations of additional Revenues. Due to similar reasons let's assume that growth of Revenues $B(t,\boldsymbol{x})$ is determined by factor Q_{1B} at point $(t,\boldsymbol{x})$ proportional to square of risk coordinates x^2 and divergence of Assets impulse $\nabla \cdot \boldsymbol{PA}(t,\boldsymbol{x})$. Negative divergence of Assets impulse $\nabla \cdot \boldsymbol{PA}(t,\boldsymbol{x}) < 0$ means loss and run-off Assets flow at point $\boldsymbol{x}$ due to excess bankruptcy for example and that may cause reduction of Revenues-on-Assets. Assumptions (5.1-5.2) neglect time gaps between moment of Investments into Assets at point $\boldsymbol{x}$ and moment of Revenue-on-Assets received. We neglect other factors that may impact Assets allocations at point $\boldsymbol{x}$ and Revenue-on-Assets to simplify relations between them. Thus let's take equations (3.2) on Assets density $A(t,\boldsymbol{x})$ and on Revenue-on-Assets density $B(t,\boldsymbol{x})$ as

$$\frac{\partial A}{\partial t} + \nabla \cdot (\boldsymbol{v}A) = a\, x^2 \nabla \cdot \boldsymbol{PB}(t,\boldsymbol{x}) \quad ; \quad \frac{\partial B}{\partial t} + \nabla \cdot (\boldsymbol{u}B) = b\, x^2 \nabla \cdot \boldsymbol{PA}(t,\boldsymbol{x}) \tag{5.3}$$

Let's identify Equations of Motion (3.3) on impulses $\boldsymbol{PA}(t,\boldsymbol{x})$ and $\boldsymbol{PB}(t,\boldsymbol{x})$. Let's assume that $\boldsymbol{Q}_{2A}$ for equations on Assets impulse $\boldsymbol{PA}(t,\boldsymbol{x})$ is proportional to Revenue-on-Assets impulse $\boldsymbol{PB}(t,\boldsymbol{x})$. In other words let's assume that growth of Assets impulse $\boldsymbol{PA}(t,\boldsymbol{x})$ is proportional to Revenue-on-Assets impulse $\boldsymbol{PB}(t,\boldsymbol{x})$ and vice versa. Let's take Equations of Motion (3.3) as:

$$\frac{\partial \boldsymbol{PA}}{\partial t} + \nabla \cdot (\boldsymbol{v}\, \boldsymbol{PA}) = c\, \boldsymbol{PB}(t,\boldsymbol{x}) \ ; \ \frac{\partial \boldsymbol{PB}}{\partial t} + \nabla \cdot (\boldsymbol{u}\, \boldsymbol{PB}) = d\, \boldsymbol{PA}(t,\boldsymbol{x}) \tag{5.4}$$

Here c and d are constants. Economic meaning of (5.4) is following. E-particles of entire economics fill economic domain (1.1) on n-dimensional e-space that is bounded by minimum or most secure and maximum or most risky grades. Aggregate macro Assets impulse $\boldsymbol{PA}(t)$ and Revenue-on-Assets impulse $\boldsymbol{PB}(t)$ are defined alike to aggregate Assets $A(t)$ (4.1):

$$\boldsymbol{PA}(t) = \int d\boldsymbol{x}\ \boldsymbol{PA}(t,\boldsymbol{x}) = A(t)\boldsymbol{v}(t) \ ; \ \ \boldsymbol{PB}(t) = \int d\boldsymbol{x}\ \boldsymbol{PB}(t,\boldsymbol{x}) = B(t)\boldsymbol{u}(t) \tag{5.5}$$

Macro Assets $A(t)$ (4.1) describe value of total Assets of economy. Macro Assets impulse $\boldsymbol{PA}(t)$ defines flow of macro Assets $A(t)$ of entire economy with velocity $\boldsymbol{v}(t)$ (5.5). Thus velocity $\boldsymbol{v}(t)$ describe motion of macro Assets $A(t)$ of entire economy on economic domain (1.1) that is bounded by minimum and maximum risk grades. Thus positive velocity $\boldsymbol{v}(t)$ describes motion of Assets $A(t)$ from secure to risky area and that increase total macro Assets risk coordinates. Such a motion along each risk axis is reduced by max limits X_i of economic domain those define most risky grades. Thus motion of Assets $A(t)$ with positive velocity $\boldsymbol{v}(t)$ in the direction from low to high-risk area should be changed by opposite motion with negative velocity $\boldsymbol{v}(t)$ in the direction from high to low risk area on economic domain. Equations (5.4) describe such case with repeated tides of Assets flows $\boldsymbol{PA}(t)$ fluctuations and describe simplest oscillations of aggregate impulses $\boldsymbol{PA}(t)$ and $\boldsymbol{PB}(t)$. However, evolution of aggregate Assets $A(t)$ and aggregate Revenue-on-Assets $B(t)$ don't depend directly on dynamics of impulses $\boldsymbol{PA}(t)$ and $\boldsymbol{PB}(t)$ but contingent upon other hidden economic factors (see Appendix: A.3; A.6; A7). The system of ordinary differential equations that describe time evolution of aggregate macro Assets *A(t)* and Revenue-on-Assets *B(t)* and other macro variables (A.3; A.6; A7) is derived in Appendix.

3.2. Business cycle fluctuations of Assets A(t) and Revenue-on Assets B(t)

Equations (5.3-5.4) on Assets *A(t,**x**)* and Revenue-on-Assets *B(t,**x**)* allow derive closed system of ordinary differential equations that describes business cycle fluctuations of Assets *A(t)* and Revenue-on-Assets *B(t)* and takes form (A.10-13; Appendix):

$$\frac{d}{dt}A(t) = -2a\, YPB(t) \quad ; \quad \frac{d}{dt}B(t) = -2b\, XPA(t)$$

$$\frac{d^2}{dt^2}XPA(t) + \omega^2 XPA(t) = \frac{d}{dt}EA(t) + c\, EB(t)$$

$$\frac{d^2}{dt^2}YPB(t) + \omega^2 YPB(t) = \frac{d}{dt}EB(t) + d\, EA(t)$$

$$\frac{d^2}{dt^2}EA(t) - \gamma_e{}^2\, EA(t) = 0 \quad ; \quad \frac{d^2}{dt^2}EB(t) - \gamma_e{}^2\, EB(t) = 0$$

Economic factors *XPA(t), YPB(t), EPA(t)* and *EPB(t)* are determined by (A.3; A.6; A.7) (Appendix). Here we briefly argue main results. Fluctuations of Assets *A(t)* depend on hidden variables *XPA(t)* and *YPB(t)* defined by (A.3), *EA(t)* (A.6) and *EB(t)* (A.7) (see Appendix). Equations (A.5) on variables *XPA(t)* and *YPB(t)* are derived from Equations of Motion (5.4). *XPA(t)* is defined by (A.3) as integral over e-space of a scalar product of vector ***x*** and impulse ***PA**(t,**x**)* on e-space. Due to (5.5) variables *XPA(t)* follow oscillations with frequency *ω* (A.12). Economic factor *EA(t)* is proportional to product of Assets *A(t)* and square of velocity $v^2(t)$ (A.6) and thus has *appearance* of Assets energy because it *looks like* as energy of "*particle with mass*" *A(t)* with velocity squared $v^2(t)$. Same reasons allow regard factor *EB(t)* (A.7) *alike to* Revenue-on-Assets energy. However we underline that economic factors *EA(t)* and *EB(t)* have *only appearance* of energy but don't have any meaning similar to meaning of energy. To derive equations on Assets *A(t)* in a closed form one should complement Continuity Equations (5.3) and Equations of Motion (5.4) with additional equations (A.8) on factors *EA(t,**x**)* and *EB(t,**x**)*. Equations (A.8) cause that *"Assets energy" EA(t)* and *"Revenue-on-Assets energy" EB(t)* can grow up in time as exponent (A.13). Equations of Motion (5.4) on impulses ***PA**(t)* and ***PB**(t)* cause equations (A.11-12) on *XPA(t)* and *YPB(t)* that describe their fluctuations with frequency *ω* (A.12) around exponential growth trend determined by factors *EA(t)* and *EB(t)* and equations (A.13). Fluctuations of *XPA(t)* and *YPB(t)* around exponential trend cause fluctuations of aggregate assets *A(t)* and Revenue-on-Assets *B(t)* with frequency *ω* near exponential growth trend. Simplest business cycle time fluctuations of aggregate Assets $A(t)$ take form (A.17):

$$A(t) = A(1) + A(2)\cos\omega t + A(3)\sin\omega t + A(4)\,exp(\gamma_e t) + A(5)\,exp(-\gamma_e t)$$

with constants *A(j), j=1,..5* determined by initial values (A.14-A.16). Simplest business cycle time fluctuations of aggregate Revenue-on-Assets *B(t)* take similar form (A.18).

4. Conclusion

Complexity of the business cycle modeling is multiplied by complexity of risk assessments. Description of complex phenomena requires complex methods but should avoid ad-hoc assumptions like assumptions of the general equilibrium. To develop macroeconomic and the business cycle model we assume that it is possible to assess risk ratings for all economic agents. Description of agent's variables under variations of their risk ratings determines evolution of macro variables and properties of business cycles. Origin of agent's risk rating evolution can be different. Economic agents can change their risk ratings due to exogenous or endogenous shocks, monetary, fiscal or technology shocks and under economic or financial processes. We show that collective evolution of agents risk ratings on economic space causes the business cycle fluctuations. We propose transition from modeling separate agents to approximation that neglect granularity of agents and describe macro variables as continuous economic media or "*economic fluids*". Such roughness allows neglect excess details and granulation of agent-based modeling and describes economic variables as aggregates over macro risk scales. This approximation has *parallels to* hydrodynamics but nature and properties of "*economic fluids*" has nothing common with physical fluids.

Our approach is based on economic equations (3.2; 3.3) and can describe business cycles with any number of interacting economic and financial variables. This paper presents *local* approximation for transactions between economic agents at point $\boldsymbol{x}$ that causes *local* interactions between macro variables. We use simple model equations (5.3; 5.4) to show advantages of our approach to study relations between Assets $A(t,\boldsymbol{x})$ and Revenue-on-Assets $B(t,\boldsymbol{x})$ that lead self-consistent business cycle fluctuations. Similar economic equations can describe interactions between three-four or more variables. Huge amount of variables that impact economic and financial evolution makes description of the real business cycle extremely difficult. In Olkhov (2017b;d;e) we describe *non-local* approximation of economic transactions between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ on e-space. *Non-local* approximation allows describe business cycles induced by evolution of economic transactions on $2n$-dimensional e-space (Olkhov, 2017e).

We propose that economic nature of business cycle fluctuations of macro variables is determined by repeated collective motion of agents and their variables from low to high-risk area on economic space and back. Collective fluctuations of agent's positions between low and high-risk areas induce oscillations of macro variables. To model such fluctuations we use simple equations (5.4) that induce simplest harmonique oscillations of economic factors *XPA(t), YPB(t)* (A.3) with frequencies (A.12). Interactions between numerous economic and financial variables cause complex structure for fluctuating frequencies and explain variety scales of aggregate fluctuations. Growth of hidden variables *EA(t)* and *EB(t)* (A.6; A.7) induces growth of macro Assets *A(t)* (4.1) and Revenue-on-Assets *B(t)* with increments γ^2_e (A.8). We present simplest *self-consistent* model (5.3; 5.4) for two variables – Assets $A(t,\boldsymbol{x})$ and Revenue-on-Assets $B(t,\boldsymbol{x})$ to demonstrate advantages of our approach and derive the system of ordinary differential equations that describe business cycle time fluctuations of macro variables *A(t)* and *B(t).* Even such simple model shows certain complexity and indicates that attempts to describe mutual relations between four-five and more variables require modeling huge system of economic equations and computer simulation. The most interesting issue concerns structure of variables that define fluctuations of macro Assets *A(t).* Nevertheless we model interaction between two variables - Assets $A(t,\boldsymbol{x})$ and Revenue-on-Assets $B(t,\boldsymbol{x})$ economic equations (5.3;5.4), evolution of macro Assets *A(t)* depend on additional variables *EA(t)* (A.6) and *EB(t)* (A.7). Economic variables *EA(t)* are proportional to product of Assets *A(t)* and squares of assets velocity $\upsilon^2(t)$ (A.6). Such factors *look like* energy of physical particle as product of mass and velocity squared υ^2. Such sameness allows call factors *EA(t)* as "*Assets energy*" (A.6) and *EB(t)* as "*Revenue-on-Assets energy*" (A.7). However no substantive parallels between *"economic energy"* of Assets flow (A.6) and properties of energy of physical fluid exist. Anyway, usage of this hidden variable extends parallels between economics and hydrodynamics.

Up now no there are no sufficient econometric data required for economic modeling of the business cycle on economic space. Thus our description of the business cycle remains pure theoretical and can't be verified by econometrics now. However it can be treated as respond to Lucas (1995) question: "Why aggregate variables undergo repeated fluctuations about trend, all of essentially the same character?". We are sure that all econometric risk assessment problems with accuracy required for business cycle modeling on economic space can be solved. We hope that our approach can help

Economic Regulators and Central Banks, Market Authorities, Academic and Business Researchers improve modeling, forecasting and management of macroeconomics and the business cycle.

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Appendix

Equations on Macro Assets A(t) and Revenue-on-Assets B(t)

Let us study equations (5.3; 5.4) on *n*-dimensional e-space and use standard notations: bold letters like ***PA, v, x, y*** define vectors and roman *C, CL, X,…* - scalars. To derive equations on macro Assets *A(t)* let's start with (4; 5.3):

$$\frac{d}{dt}A(t) = \int dx\ \frac{\partial}{\partial t}A(t,\boldsymbol{x}) = -\int dx\ \nabla\cdot(\boldsymbol{v}A(t,\boldsymbol{x})) + a\ \int dx\, x^2\ \nabla\cdot\boldsymbol{PB}(t,\boldsymbol{x}) \qquad \text{(A.1)}$$

Due to (1.1) economic domain on e-space is reduced by most secure and most risky risk grades. Hence first integral of divergence over e-space in the right side due to Divergence theorem (Strauss, 2008, p.179) equals integral of flux through surface and that equals zero, as no economic or financial fluxes exist far from boundaries (1.1). Second integral in the right side:

$$\int dx\, x^2\ \nabla\cdot\boldsymbol{PB}(t,\boldsymbol{x}) = \int dx\ \sum_i x_i^{\,2}\ \nabla\cdot\boldsymbol{PB}$$

For each *i* obtain:

$$\int dx_i\, x_i^{\,2}\int dx_{j\neq i}\,\nabla_{j\neq i}\cdot PB_j + \int dx_{j\neq i}\int dx_i\, x_i^{\,2}\nabla_i\cdot PB_i\ \ ;\ \ \int dx_{j\neq i}\,\nabla_{j\neq i}\cdot PB_j = 0$$

due to same reasons as above - integral of divergence over e-space equals zero. Second integral by parts gives:

$$\int dx_i\, x_i^{\,2}\nabla_i\cdot PB_i = -2\int dx_i\, x_i\cdot PB_i\ \ ;\ \int dx\, x^2\ \nabla\cdot\boldsymbol{PB}(t,\boldsymbol{x}) = -2\int dx\, x_i PB_i(t,\boldsymbol{x})$$

Thus equations (A.1) take form (A.2)

$$\frac{d}{dt}A(t) = -2a\,YPB(t)\,;\ \frac{d}{dt}B(t) = -2b\ PA(t) \qquad \text{(A.2)}$$

$$XPA(t) = \int dx\,\boldsymbol{x}\ \cdot\boldsymbol{PA}(t,\boldsymbol{x})\,;\ \ YPB(t) = \int dx\,\boldsymbol{x}\ \cdot\boldsymbol{PB}(t,\boldsymbol{x}) \qquad \text{(A.2.1)}$$

Equations on macro Assets impulse ***PA****(t)* (5.5) can be derived from (5.4):

$$\frac{d}{dt}\boldsymbol{PA}(t) = \int dx\ \frac{\partial}{\partial t}\boldsymbol{PA}(t,\boldsymbol{x}) = -\int dx\ \nabla\cdot(\boldsymbol{vPA}(t,\boldsymbol{x})) + c\int dx\,\boldsymbol{PB}(t,\boldsymbol{x})$$

Taking into account Divergence theorem (Strauss, 2008, p.179) obtain:

$$\frac{d}{dt}\boldsymbol{PA}(t) = c\boldsymbol{PB}(t)\ \ ;\ \frac{d}{dt}\boldsymbol{PB}(t) = d\boldsymbol{PA}(t) \qquad \text{(A.3.1)}$$

For $\omega^2 = -cd > 0$ equations on macro impulses take form:

$$\left(\frac{d^2}{dt^2} + \omega^2\right)\boldsymbol{PA}(t) = 0\ \ ;\ \left(\frac{d^2}{dt^2} + \omega^2\right)\boldsymbol{PB}(t) = 0 \qquad \text{(A.3.2)}$$

and describe simplest harmonic oscillations of impulses ***PA****(t)* and ***PB****(t).* As we show below, frequencies *ω* of their oscillations describe frequencies of business cycle fluctuations of macro Assets

A(t) and macro Revenue-on-Assets *B(t).* If *cd>0* then impulses ***PA****(t)* and ***PB****(t)* should grow up by exponent or should decline by exponent in time. Due to bounded economic domain (1.1) impulses can't grow up by exponent. If they decline by exponent in time then economic should reach state with macro Assets impulse ***PA****(t)=0* and we don't study it here. Below we assume that $\omega^2 = -cd > 0$. Equations on mean Assets risk $\boldsymbol{X}_A(t)$ due to (4.3) can be derived similar to (Olkhov, 2017e-f):

$$\frac{d}{dt}\boldsymbol{X}_A(t)A(t) = \int dx\,\boldsymbol{x}\,\frac{\partial}{\partial t}A(t,\boldsymbol{x}) = -\int dx\,\boldsymbol{x}\,\nabla\cdot\big(\boldsymbol{PA}(t,\boldsymbol{x})\big) + a\int dx\,\boldsymbol{x}\,x^2\,\nabla\cdot\boldsymbol{PB}(t,\boldsymbol{x})$$

$$\int dx\,\boldsymbol{x}\,\nabla\cdot\big(\boldsymbol{PA}(t,\boldsymbol{x})\big) = -\boldsymbol{PA}(t)$$

$$\frac{d}{dt}\boldsymbol{X}_A(t)A(t) = \boldsymbol{PA}(t) + a\,\boldsymbol{X}PB(t)\ ;\ \frac{d}{dt}\boldsymbol{X}_B(t)B(t) = \boldsymbol{PB}(t) + b\,\boldsymbol{X}PA(t) \quad \text{(A.3.3)}$$

$$\boldsymbol{X}x^2PA(t) = \int dx\,\boldsymbol{x}\,x^2\,\nabla\cdot\boldsymbol{PA}(t,\boldsymbol{x})\ ;\ \boldsymbol{X}x^2PB(t) = \int dx\,\boldsymbol{x}\,x^2\,\nabla\cdot\boldsymbol{PB}(t,\boldsymbol{x}) \quad \text{(A.3.4)}$$

To avoid excess complexity we don't derive equations on mean Assets risk $\boldsymbol{X}_A(t)$ in a closed form but indicate that for *a=b=0* equations (A.3.3) describe fluctuations of factor $\boldsymbol{X}_A(t)A(t)$ induced by fluctuations of impulses ***PA****(t)* and ***PB****(t)* (A.3.2). Action of factors $\boldsymbol{X}x^2PA(t)$ and $\boldsymbol{X}x^2PB(t)$ makes these relations mode complex. We underline that business cycle fluctuations of macro Assets *A(t)* don't repeat fluctuations of mean Assets risk $\boldsymbol{X}x^2PA(t)$. Their relations are more complex but their mutual dependence determined by equations (5.3; 5.4). To derive equations on *XPA(t)* и *YPB(t)* let's use equation (5.4):

$$\frac{d}{dt}XPA(t) = \int dx\,\boldsymbol{x}\cdot\frac{\partial}{\partial t}\boldsymbol{PA}(t,\boldsymbol{x}) = -\int dx\,\boldsymbol{x}\cdot(\nabla\cdot(\boldsymbol{v}\,\boldsymbol{PA})) + c\,YPB(t) \quad \text{(A.4)}$$

Let's take first integral in the right hand by parts and take into account that integral by divergence equals zero:

$$-\int dx\,\boldsymbol{x}\cdot(\nabla\cdot(\boldsymbol{v}\,\boldsymbol{PA})) = \int dx\ \ A(t,\boldsymbol{x})v^2(t,\boldsymbol{x})$$

$$\frac{d}{dt}XPA(t) = EA(t) + c\,YPB(t)\quad ;\quad \frac{d}{dt}YPB(t) = EB(t) + d\,XPA(t) \quad \text{(A.5)}$$

$$EA(t) = A(t)v^2(t) = \int dx\ \ A(t,\boldsymbol{x})v^2(t,\boldsymbol{x}) = \int dx\ \ A(t,\boldsymbol{x})\sum_i v_i^2(t,\boldsymbol{x}) \quad \text{(A.6)}$$

$$EB(t) = B(t)u^2(t) = \int dx\ \ B(t,\boldsymbol{x})u^2(t,\boldsymbol{x}) = \int dx\ \ B(t,\boldsymbol{x})\sum_i u_i^2(t,\boldsymbol{x}) \quad \text{(A.7)}$$

It is amazing that equations on aggregate Assets *A(t)* and Revenue-on-Assets *B(t)* require factors (A.6; A.7). Assets "energy" $EA(t) = A(t)v^2(t)$ *looks like* energy of physical particle with mass *A(t)* and velocity squared v^2. It is obvious that Assets "*energy" EA(t)* has only formal resemblance with energy of physical particle but as well it underlines very important issue: adequate modeling of macro variables can't be based on relations between time-series of usual economic variables only but require wide range of derivative factors like (A.3; A.6; A.7). To describe aggregate Assets *A(t)* in a closed form let's take equations on Assets *"energy" EA(t,$\boldsymbol{x}$)* and Revenue-on-Assets *"energy" EB(t,$\boldsymbol{x}$)* as:

$$\frac{\partial EA}{\partial t} + \nabla \cdot (\boldsymbol{v}EA) = c_e\, EB(t,\boldsymbol{x}); \frac{\partial EB}{\partial t} + \nabla \cdot (\boldsymbol{u}EB) = d_e\, EA(t,\boldsymbol{x})\,;\ \gamma_e{}^2 = c_e d_e \quad \text{(A.8)}$$

Economic meaning of (A.8) is as follows. Flows of Assets *A(t,$\boldsymbol{x}$)* from low risk area to high risk area and back are accompanied with growth of Assets *"energy"* proportional to growth of product of Assets A(t) and square of velocity $v^2(t)$. The same happens to flows of Revenue-on-Assets *"energy" EB(t,$\boldsymbol{x}$)*. Assumptions $\gamma^2_e > 0$ for (A.8) mean that Assets *"energy" EA(t)* and Revenue-on-Assets *"energy" EB(t)* grow up as exponent and (A.8) give equations on economic *"energies" EA(t)* and *EA(t)*:

$$\frac{d}{dt}EA(t) = c_e\, EB(t) \quad ; \ \frac{d}{dt}EB(t) = d_e\, EA(t) \qquad \text{(A.9)}$$

That close sequence of ordinary differential equations on macro Assets *A(t)*. Equations (A.2; A.5; A9) form a closed system of ordinary differential equations that describe Assets *A(t)* and Revenue-on-Assets *B(t)* and can be presented as:

$$\frac{d}{dt}A(t) = -2a\, YPB(t) \quad ; \quad \frac{d}{dt}B(t) = -2b\, XPA(t) \qquad \text{(A.10)}$$

$$\frac{d^2}{dt^2}XPA(t) + \omega^2 XPA(t) = \frac{d}{dt}EA(t) + c\, EB(t) \qquad \text{(A.11)}$$

$$\frac{d^2}{dt^2}YPB(t) + \omega^2 YPB(t) = \frac{d}{dt}EB(t) + d\, EA(t)\ ;\ \omega^2 = -cd > 0 \qquad \text{(A.12)}$$

$$\frac{d^2}{dt^2}EA(t) - \gamma_e{}^2\, EA(t) = 0 \ ; \ \frac{d^2}{dt^2}EB(t) - \gamma_e{}^2\, EB(t) = 0\,;\ \gamma_e{}^2 = c_e d_e > 0 \quad \text{(A.13)}$$

Equations (A.13) describe exponential growth or decline. Initial values of variables:

$$A(0) = \int dx\, A(0,\boldsymbol{x})\,;\ \ B(0) = \int dx\, B(0,\boldsymbol{x})\,; \qquad \text{(A.14)}$$

$$XPA(0) = \int dx\, \boldsymbol{x} \cdot \boldsymbol{v}(0,\boldsymbol{x})A(0,\boldsymbol{x})\,;\ \ YPB(0) = \int dx\, \boldsymbol{x} \cdot \boldsymbol{u}(0,\boldsymbol{x})B(0,\boldsymbol{x}) \qquad \text{(A.15)}$$

$$EA(0) = \int dx\, v^2(0,\boldsymbol{x})A(0,\boldsymbol{x})\,;\ \ EB(0) = \int dx\, u^2(0,\boldsymbol{x})B(0,\boldsymbol{x}) \qquad \text{(A.16)}$$

It is easy to show that initial values (A.14 - 16) define solution for aggregate Assets *A(t)* of equations (A.10-13) as:

$$A(t) = A(1) + A(2)\cos\omega t + A(3)\sin\omega t + A(4)\, exp(\gamma_e t) + A(5)\, exp(-\gamma_e t) \qquad \text{(A.17)}$$

A(j) for *j=1,..5* are constants that are defined by (A.10-A.12). Solutions of Equations (A.2; A.5; A9) are simple and we omit here exact derivation for brevity. Aggregate Revenue-on-Assets *B(t)* takes form similar to (A.13) with its own constants *B(j)*.

$$B(t) = B(1) + B(2)\cos\omega t + B(3)\sin\omega t + B(4)\, exp(\gamma_e t) + B(5)\, exp(-\gamma_e t) \qquad \text{(A.18)}$$

Main conclusion: simple assumptions on exponential growth of Assets *EA(t)* and Revenue-on-Assets *EB(t)* "energy" with increment γ_e and oscillations of *XAP(t)* and *YAP(t)* with frequency ω induce fluctuations of macro Assets *A(t)* and Revenue-on-Assets *B(t)* with frequency ω around exponential growth trend. Business cycle Assets fluctuations and growth don't depend directly on Revenue-on-Assets, but on hidden variables *XAP(t), YAP(t)* (A.2.1) and *EA(t), EB(t)* that are *alike to* "energies".

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