Equity Basis Selection in Allocation Environments

Aadland, David and Kolpin, Van

University of Wyoming, University of Oregon

1 March 2008

EQUITY BASIS SELECTION IN ALLOCATION ENVIRONMENTS

Abstract. The successful formation and long-term stability of a cooperative venture is often linked to the perceived fairness of the associated cost or resource allocation. In particular, the effectiveness of such collaborations can be hampered by the lack of a consensus view on what basis should be used for gauging an allocation's "fairness." Standards of equity in traditional cost-sharing applications could be assessed on many dimensions: per capita, per unit of demand, or per unit of revenue, to mention a few. This multiplicity of logically compelling "equity bases" is a feature common to many practical cost-sharing applications. Our analysis shows that features of the allocation environment are capable of explaining a substantial amount of the variation in the equity bases employed in practice and are consistent with the axiomatic principles of collective behavior.

JEL classification numbers: C71, D63, C25

Key words: cooperative games, cost allocation, equity, probit model

David Aadland
Department of Economics and Finance
1000 E. University Avenue
Laramie, WY 82072
aadland@uwyo.edu

and

Van Kolpin[†][*]
Department of Economics
1285 University of Oregon
Eugene, OR 97403-1285
vkolpin@uoregon.edu

[†] We thank Marty Van Cleave, Bob Hill, Jim Kindle, Rick Krannich, Stephanie Kuster, and, of course, our survey respondents.

[*] Corresponding author.

1. Introduction

Successful formation and long-term stability of a cooperative venture are often linked to the perceived fairness of the associated cost or resource allocation. Whether a venture is a simple business partnership or a global collaboration, such as that which led to the Kyoto protocol, its effectiveness can be hampered by the lack of a consensus view on what basis should be used for gauging an allocation's "fairness." Consider, for instance, the classic airport problem in which multiple airlines share a common landing strip (Littlechild and Owen, 1973). Aside from the issue of how costs should be apportioned amongst the set of players is the more fundamental question of what represents the relevant basis over which principles of equity should be applied. Should concern be focused on the distribution of costs across the set of airlines, the set of flights, the set of passengers, the set of revenues, or perhaps some other basis? Although these various bases are intertwined, each offers a different perspective on notions of fair treatment. This multiplicity of logically compelling fairness bases is a feature that is common to many practical cost-sharing applications. For example, participants of international initiatives to mitigate global climate change must agree whether the burden of reducing greenhouse gases should be distributed according to a per capita, per unit of GDP, per unit of wealth, or some other basis (Ashton and Wang, 2003).[1] What then leads to the selection of one basis over another in practice? Is this choice essentially arbitrary or can parameters of the cooperative environment predict the fairness basis that is embraced? Moreover, if such explanatory power exists, is it consistent with theoretical principles of collective behavior? This paper aims to shed light on these puzzles.

---

[1] The issue of the appropriate equity basis has become a major obstacle to the effective implementation of agreements to mitigate global climate change. According to Frank E. Loy, the head of the U.S. delegation to international climate meetings during the Clinton administration and former U.S. Under Secretary of State for Global affairs, "if we're going to start talking about per capita emissions and trying to equalize those, we will never, never, never, never have an international agreement and therefore I think that is a dead-end" (Grossman, 2001).

Much of the theoretical cost-sharing literature employs axiomatic principles to derive a unique cost-sharing rule. This methodology operates under the premise that a collective decision on which allocation rule to employ should depend solely on the mathematical structure of the associated cost-sharing game. While this has proven to be a useful theoretical approach, practical applications often involve a variety of distinct fairness bases, even when their associated cost-sharing games are indistinguishable. We introduce the *benefit inequity principle* which posits that the greater the differences in benefits realized by elements of a given equity basis, the greater the pressure to adopt an alternative basis. Our empirical analysis will test the efficacy of this axiom as a guiding force in the selection of actual cost-sharing procedures.

We use irrigation cost sharing in two neighboring counties of Montana, USA as a context for our study. The context is well suited for our study, thanks in part to the close geographic and social proximity of the sample. This closeness makes it less likely that unobserved variation in cultural conventions, which may in turn influence cooperative decision making, will bias our estimates. Another compelling feature of our data is that the cost-sharing practices observed on these ditches are long-lived, in many cases over a century old. Although disputes between individual ranchers do arise on occasion, the cost-allocation practices have proven to be remarkably stable.

The ditches in our sample share a common physical structure. The “main” ditch begins at the headgate, which diverts water from the source stream, and continues in a sequential path through the users’ properties. Costs for private ditches that branch off the main ditch are covered by their respective owners and are not shared by the group as a whole. Examples of shared costs include headgate repair, silt and debris removal, repair of deteriorating (main) ditch banks, and the like.

The ditches are used to irrigate hay fields and other cash crops, water livestock, and irrigate lawns and gardens, with some variation in these uses across the ditches.

Our data set is constructed from a combination of state and federal sources, as well as our own survey efforts. The data strongly support the conclusion that the benefit inequity principle serves as a guiding force in the selection of an actual equity basis. In particular, the coefficients of our empirical model have signs consistent with this principle and our independent variables exhibit considerable explanatory power.

2. Axiomatic Motivation

Much of the theoretical cost-sharing literature adopts essentially the same methodology as that pursued in the creation of the bargaining solution (Nash, 1950) or the Shapley value (Shapley, 1953). In loose terms, this methodology formulates a mathematical abstraction of the universe of all environments under consideration, identifies desirable properties of a "solution" defined on this universe, and then demonstrates that these properties will in fact characterize a unique solution or class of solutions. For instance, the "Shapley program" considers the universe of all TU (transferable utility) games, defines a solution as a value operator, and then demonstrates that the Shapley value is the only solution to satisfy the anonymity, additivity, and dummy axioms.

The approach adopted in our paper differs from that outlined above in several important respects. First, each irrigation ditch in our universe of cost-sharing environments has not one, but three distinct "populations" that can serve as the "player" set over which equity is assessed – namely, the population of irrigators using the ditch, the population of acres irrigated by the ditch, and the population of water shares distributed on the ditch. (Note that a "water share" represents a share of stock ownership of the ditch.) As each of these populations provides a different basis for equity assessment, we shall refer to these populations generally as *equity bases*, and specifically as the per capita basis, per acre basis, and per water-share basis respectively.[2] Recall, the focus of our paper is to investigate whether features of the cost-sharing environment are able to explain the selection of the equity basis used in practice.

A second dimension where our approach differs from traditional treatments of cost sharing is that we consider the possibility that features of the cost-sharing environment, in particular ones that have absolutely no effect on the values subject to redistribution, may influence the choice of allocation procedure. In our setting, ditch maintenance costs are the only values subject to redistribution. A traditional approach would dictate that the universe of cost-sharing games should be expressed in terms of only those parameters that impact the costs to be shared and the player population over which sharing is to occur. We depart from the traditional approach and hypothesize that the benefits accruing across the ditch may influence the equity basis embraced even though these benefits are not themselves subject to redistribution.

A final dimension of difference in our approach is that we do not to use axiomatic principles to identify a unique cost-sharing rule that *should* be employed. Instead, we seek to examine whether axiomatic principles are consistent with how features of the cost-sharing environment determine the equity bases that *are* employed.

The irrigation cost-sharing environments under consideration are characterized by the maintenance costs incurred on the ditch, the population of users that have access to the ditch (the user basis), the population of acres serviced by the ditch (the acre basis), the population of water shares distributed across users of the ditch (the water-share basis), and the benefits that accrue to

---

[2] In principle, additional equity bases could also be considered. As we were unable to detect the use of alternative equity bases in our sample, we restrict our attention to the bases noted above.

ditch users. A comprehensive description of cost sharing on an irrigation ditch would detail the determining factors of which equity basis is selected as well as the manner in which the costs of each individual maintenance project are allocated. As our focus is solely on the question of how equity bases are determined, we will forego any detailed discussion of how costs are distributed across a given equity basis, e.g., serial versus average cost sharing. The interested reader can turn to Aadland and Kolpin (2004) for a treatment of this latter subject.

A key axiom underlying our analysis is that of the benefit inequity principle. The essential idea behind this principle is that since only costs (not benefits) are subject to redistribution, there may be a fundamental pressure to administer cost sharing over a "level playing field." That is, greater pre-tax benefit inequity from the perspective of a given equity basis may imply a diminished commitment to use that basis to administer costs and assess fairness.

Benefit inequity principle: Greater differences in the irrigation benefits realized by elements of a given equity basis imply greater pressure to adopt an alternative equity basis.

The reader will note that our formulation of the benefit inequity principle does not delineate when inequity pressures reach the point where one equity basis will be chosen over all others. Similarly, the principle does not impose a precise specification for how the benefit differences across an equity basis are to be measured, e.g., by sample variance, Gini coefficient, maximum realized benefit minus minimum realized benefit, or some other statistical measure. This vagueness enables our data to speak to where these lines should be drawn, rather than having rigid specifications that may not be supported by the data.

3. The Data

The cost-sharing agreements within our sample represent a set of stable, yet informal conventions that are "understood" by the ditch users. As these conventions are not documented in publicly available sources, we surveyed the users to obtain information regarding their cost-sharing procedures and the circumstances surrounding their use. Our survey efforts revealed that there are three bases over which costs are shared in our sample. Costs are either shared on a per capita basis, a per acre basis, or a per water-share basis. (Our survey also delved into the issue of whether the group who shares responsibility for costs may differ depending on the nature of the project that induced the costs, the subject of which was the focus of Aadland and Kolpin, 2004. Our focus in the present paper is instead on the determination of an appropriate equity basis.)

Our survey yielded a total of 270 usable responses from 101 of the 169 irrigation ditches in Carbon and Stillwater counties. These ditches service 2,840 individual parcels of irrigated land, comprising a total of 150,000 acres. Three ditches were excluded from the analysis because there was only one reported user and therefore no need for cost sharing. An additional 14 ditches were excluded because the respondents either (*a*) reported no cost sharing or (*b*) they did not select any of the cost-sharing options in the survey, instead choosing "Other" without specifying in their written comments how costs were actually shared. Our final sample therefore includes 84 ditches that can be associated with either a per capita, per acre, or per water-share rule.

As might be expected given the informal nature of the cost-sharing arrangements, the reported rules were not always consistent across users. Apparent inconsistencies in the stated cost-sharing rule were resolved by assigning to each ditch the rule stated by the majority of its respondents and, if a tie were to occur (there were seven ties) the most common rule in the overall data set was selected.

In addition to our survey, we relied on several other sources to construct our data set. First, the Water Resource Division of the Montana Department of Natural Resources and Conservation (DNRC) provided information on the location and size for the parcels of land serviced by the irrigation ditches in our sample, as well as the primary use[3] for the water (Water Resource Division, 2002). Second, we used the Soil Surveys for Carbon and Stillwater counties (USDA, 1975 and 1980) to derive estimates of the irrigated and non-irrigated productivity of the land served by a ditch. These estimates were in turn used to formulate a measure of the benefits bestowed by access to the irrigation ditch. Finally, we use spatial climate maps produced by the Oregon Climate Service to generate measures of expected rainfall along the ditches in our sample, another factor impacting the incremental benefits of irrigation (Oregon Climate Service, 2002).

We now turn to the specification of the variables used in our econometric analysis. Because we only consider three equity bases (i.e., per capita, per acre, and per water-share), it suffices to consider two dependent variables – PC and PA. PC is a ditch-level binary variable that is equal to one if costs are shared on a per capita basis and zero if an alternative basis is employed. Similarly, PA is a ditch-level binary variable that equals one if costs are shared on a per acre basis and zero otherwise. Given that there are three bases that appear in our sample, it follows that if PC=PA=0, then costs are distributed on a per water-share basis.

The motivation underlying the selection of our explanatory variables was to identify factors in the available data that may contribute to benefit inequity in the three equity bases. The first such factor is SIZE, which is defined to be the number of users, or user population size, of the ditch in question. In the majority of the cases we have precise information regarding the number of users on a ditch. However, some ditches are incorporated and filed with a single water right in

---

[3] Examples of primary use include crop irrigation, watering of livestock, and lawn/garden use.

the name of the ditch corporation rather than separate water rights for each user. For the incorporated ditches in our sample, we therefore do not have a useable measure of SIZE. To handle this missing data, we replace the missing observations with a zero and add an additional dummy variable that captures whether information regarding the number of users was available (Cohen and Cohen, 1983).

ATYPICAL represents the fraction of ditch users that employ the water in a nonstandard way. We used the Montana DNRC data to make this determination. A user is assigned as pursuing atypical use if their water right is designated as primarily for something other than crop irrigation or if they irrigate a parcel of land that is less than 1/2 acre (an imperfect proxy for someone who is using the land for purposes other than farming or whose operations may be small in scale).

TOWN is a variable that represents the fraction of ditch users that have at least one field within a mile radius of a town center. This variable serves as a proxy for a landowner who has the potential to develop land to make it suitable for something other than agricultural use, or has done so already.

ACRE DIFF is a variable that represents a quick, back-of-the-envelope calculation of the variation in scale of the operations across irrigators. It is defined to be the difference between the irrigated acres of the biggest and smallest users on the ditch.

RAIN is a variable that captures the minimum expected rainfall on a given ditch. We focus on the minimum rainfall that is expected on land serviced by the ditch for two reasons. First, this value serves as a proxy for the risk associated with low levels of rainfall. Second, some of the ditches are located near steep terrain and thus are home to fields that are located in relatively close geographic proximity to mountainous regions that receive considerably more precipitation than the tillable fields served by the ditch. In such cases, the expected rainfall that the OCS data attributes to some fields is biased upwards relative to expected rainfall elsewhere on the ditch. Our specification of the RAIN variable moderates this bias.

SCARCE represents a measure of the perceived scarcity of irrigation water, which impacts the expected benefits of ditch access. In our original survey, we asked irrigators whether or not the irrigation ditch provides all of the water users need in most years. A clear majority of 81.3% of the respondents indicated that the irrigation ditch is capable of servicing their needs. On some ditches, however, there are indications that scarcity of irrigation water is more problematic. SCARCE is defined as the fraction of respondents who report that the ditch is not generally capable of meetings all users' needs.[4]

YIELD RATIO is defined as the ratio of the expected alfalfa yield (the most common crop) on irrigated land relative to the expected alfalfa yield using dry-land farming methods. Soil types vary across our sample and, as a consequence, so does YIELD RATIO. This variable was constructed using the Carbon and Stillwater Soil Surveys.

Finally, GINI is a variable the measures the dispersion of irrigated acres across users on each unincorporated ditch. GINI is calculated as

$$GINI_i = (2\mu_i n_i^2)^{-1} \sum\nolimits_{j=1}^{n_i} \sum\nolimits_{k=1}^{n_i} | acre_{i,j} - acre_{i,k} |, \qquad (1)$$

where μ_i is the average number of irrigated acres per user on ditch i, n_i is the number of users on ditch i, and $acre_{i,j}$ is the total number of acres irrigated by user j on ditch i. The Gini coefficient

---

[4] There were 25 ditches for which we do not have responses for the water scarcity question. For these ditches, we replace the missing observations for SCARCE with a zero and incorporate an additional dummy variable indicating the missing data.

has a long tradition of measuring the degree of income or wealth inequality (see, for instance, Sen, 1973 and Lambert, 2002). The coefficient ranges from a minimum of zero (when all users have equivalent acreage) to a maximum of one (in an infinite population with all users except one having no acreage).

Table 1 reports the definitions of the variables and various summary statistics. Notice that the sample is unbalanced in the direction of per water-share rules, with the precise distribution being per water-share ($n = 53$), per capita ($n = 19$) and per acre ($n = 12$) rules. Due to the manner in which the data are recorded, we do not have information on SIZE, TOWN, ACRE DIFF and GINI for the 21 incorporated ditches. For the 63 unincorporated ditches, there is an average of approximately seven users with most users living outside town. There is also substantial variation in irrigated acres, either measured by ACRE DIFF with a mean of 207 or by GINI with a mean of 0.363. The ATYPICAL variable indicates that most ditches are comprised of larger users who are irrigating crops – only six ditches have users who are irrigating small acreage or are using the water for stock or domestic purposes. There is substantial variation in RAIN (with the minimum rainfall varying between 108 and 245 millimeters per growing season) and in YIELD RATIO (with a minimum of 1.4 and a maximum of 7.0). Finally, SCARCE indicates that for 43 of the 59 reporting ditches, all of the corresponding survey responses assert that there is sufficient water to meet all irrigation needs. Of the remaining 16 ditches, an average of two-thirds of the respondents report limited water availability.

4. Econometric Analysis

In this section, we introduce the econometric models and the estimation methods. The primary goal of this section is to investigate whether features of the cost-sharing environment can explain the actual choice of equity bases and whether this explanatory power is consistent with benefit inequity, and other axiomatic principles.

We consider two different estimation frameworks. In the first, we treat the choice of equity basis as a two-stage binary decision estimated with sequential probit models. In stage one, the agents choose whether the equity basis should be per capita or selected amongst the remaining two alternatives. If not, then in stage two the agents choose between per acre and per water-share equity bases. An advantage of this approach over the traditional multinomial logit model is that it allows for a more parsimonious model in stage one. Because per capita allocations are transparent and easy to calculate, agents may naturally gravitate toward the per capita basis unless there is sufficient variation in benefits to cause them to consider other bases. Consistent with this “go simple unless there is a compelling reason to the contrary – axiom,” we assume that agents use a small set of simple structural indicators to determine if the per capita basis is an equitable choice. Once this initial choice is determined, ditches that did not select the per capita basis use a larger and more sophisticated set of structural indicators to narrow the choice between the per acre and per water-share variants.

A disadvantage of the two-stage approach is that it restricts decisions to be made sequentially, such that the second-stage decision between per acre and per water-share bases is made without consideration of the per capita basis. With the multinomial logit model, we relax this assumption by allowing agents to consider all three equity bases simultaneously using the full set of structural factors. However, because the multinomial logit model effectively imposes the “independence of irrelevant alternatives” (IIA) assumption, we perform a Hausman test to see if agents do indeed decide between per acre and per water-share bases independent of the per capita rule (Hausman and McFadden, 1984).

4.1 Bivariate Probit

We begin by assigning the irrigation ditch as the unit of observation, which is indexed from $i = 1,\ldots,n$. In our first model, we restrict irrigators to select a per capita basis or to select from the set of all other alternatives. This choice of equity basis is in turn assumed to depend on structural characteristics as indicated by the following equation:

$$PC_i^* = X_i'\beta + \varepsilon_i , \tag{2}$$

where PC_i^* is a latent variable measuring the likelihood of choosing the per capita basis for ditch i, X_i is a column vector of explanatory variables for ditch i thought to influence the choice of equity basis, β is a column vector of coefficients, and ε_i is a normally distributed error term with mean zero. By assuming a normal distribution, we then form the likelihood function conditional on the observed data. Letting F denote the cumulative density function associated with the error term, we can write the probability that the i^{th} ditch chooses the per capita equity basis (indicated by $PC_i = 1$) as:

$$P_i = \Pr(PC_i = 1) = \Pr(PC_i^* > 0) = \Pr(\varepsilon_i > -X_i'\beta) = F(X_i'\beta). \tag{3}$$

The probability that the i^{th} ditch adopts either the per acre or per water share basis ($PC_i = 0$) is therefore given by $1 - P_i = \mathrm{F}(X_i'\beta)$. Assuming independence of error terms, we can then write the (log) likelihood function as

$$\ln(L) = \sum_{i=1}^{n} [PC_i \ln(P_i) + (1 - PC_i)\ln(1 - P_i)]. \tag{4}$$

The problem of forming and maximizing (4) by choosing β, given normally distributed error terms, is referred to as the probit model. This estimation procedure requires nonlinear optimization techniques to generate estimates of the β parameters and the associated marginal effects (Greene, 2008).[5]

In stage two, ditches that did not select the per capita equity basis, then decide between per acre and per water-share bases. This model is given by

$$PA_j^* = Z_j'\gamma + v_j\ , \tag{4}$$

where j indexes all ditches that did not choose the per capita equity basis in stage one, PA_j^* is a latent variable measuring the likelihood of choosing the per acre equity basis, Z_j is a column vector of explanatory variables for ditch j thought to influence the choice of equity basis, γ is a column vector of coefficients, and v_j is a mean-zero, normally distributed error term. The log likelihood function is then formed and maximized to obtain estimates of γ and the associated marginal effects.

4.2 Multinomial Logit

We also consider a model where agents on each ditch choose simultaneously from amongst the per capita, per acre, and per water share equity bases. The model can be written as

---

[5] The estimation was carried out in Gauss 8.0 using the Constrained Maximum Likelihood (CML) module and Newton's method for the nonlinear optimization.

$$PC_i^* = Z_i'\beta_1 + \varepsilon_{1,i} \tag{5.1}$$

$$PA_i^* = Z_i'\beta_2 + \varepsilon_{2,i} \tag{5.2}$$

where the disturbances $\varepsilon_{1,i}$ and $\varepsilon_{2,i}$ are assumed to be independent and follow a type 1 extreme value distribution (Greene, 2008). This leads to the following probabilities

$$P_{1,i} = \Pr(PC_i = 1) = \exp(Z_i'\beta_1)/(1 + \exp(Z_i'\beta_1) + \exp(Z_i'\beta_2) \tag{6.1}$$

$$P_{2,i} = \Pr(PA_i = 1) = \exp(Z_i'\beta_2)/(1 + \exp(Z_i'\beta_2) + \exp(Z_i'\beta_2) \tag{6.2}$$

$$P_{3,i} = \Pr(PWS_i = 1) = 1 - P_{1,i} - P_{2,i} \tag{6.3}$$

and log likelihood function

$$\ln(L) = \sum\nolimits_{i=1}^{n}\left[PC_i \ln(P_{1,i}) + PA_i \ln(P_{2,i}) + PWS_i \ln(P_{3,i})\right]. \tag{7}$$

Because the multinomial logit model assumes independent and homoscedastic error terms, it implies that the log-odds ratio between any two choices does not depend on the third choice. As mentioned above, we test the IIA assumption using a Hausman test.

4.3 Discussion of the Results

The estimation results are presented in Table 2. Let us begin with the stage-one estimation results using all 84 ditches. We consider four explanatory variables: SIZE, ATYPICAL, TOWN and ACRE DIFF. Larger values of SIZE, ATYPICAL, and ACRE DIFF are correlated with greater differences in the benefits realized by irrigators on the ditch and thus the benefit inequity principle would suggest there is pressure to adopt an equity basis other than per capita. The effect of TOWN is not as clear, although we expect that users near town will tend to be more uniform in the benefits received from ditch access. Our priors are supported by the econometric results – large rural ditches with atypical users and greater variation in irrigated acres are less likely to choose the per capita equity basis. The coefficients of ATYPICAL, TOWN, and ACRE DIFF are all statistically significant at the 5% level. In terms of goodness-of-fit, the likelihood ratio test indicates that the model explains a significant amount of the variation in the use of the per capita equity basis. The model correctly predicts 70 of the 84 ditches while a maximum score (MS) estimator, a semi-parametric estimator that directly maximizes the number of correct predictions, predicts 76 of the 84 ditches.[6]

In stage two, the 65 remaining ditches choose between per acre and per water-share bases. Agents on the ditch consider a larger and more sophisticated set of explanatory variables in deciding between these two bases than they did in stage one. In addition to the explanatory variables from stage one, we consider RAIN, SCARCE, YIELD RATIO and GINI. GINI is a more sophisticated measure of the variation in irrigated acres than ACRE DIFF. To avoid multicollinearity issues, we exclude ACRE DIFF from stage two.

We expect that higher values for SIZE and ATYPICAL are likely to be associated with greater variation in the use and quality of the land, both of which suggest greater variation in the benefits received from irrigating a given acre. As such, the benefit inequity principle would tend to decrease the chance that a per acre basis is chosen. Conversely, we expect that land clustered

---

[6] The coefficient estimates for the MS estimator have the same sign as those from the probit model and are available upon request.

near a town will be more uniform in both its quality and use, which in turn leads to a greater likelihood of the per acre basis. Therefore, the coefficient on TOWN is expected to be positive.

RAIN, SCARCE, and YIELD RATIO do not speak directly to variation in the quality of land, but do speak (each in a different way) to the variation in benefits realized by a given distribution in land quality. Ample rainfall tends to lessen the incremental per acre benefits of irrigation, thus reducing the variation in acre benefit and making the per acre basis more likely. Greater water scarcity suggests that the benefits of receiving the water are larger, benefit variation is amplified, and the per acre basis is less likely to be selected. A higher YIELD RATIO tends to indicate lower quality land and thus for a given amount of rain and availability of irrigation water there will be less benefit variation from irrigation on a per acre basis. As such, we expect higher values of YIELD RATIO to lead to a greater likelihood of per acre basis adoption. Finally, values of GINI do not directly speak to variation in the benefits received on individual acres. However, larger values of GINI tend to indicate that variation in benefits can be more fully explained by just the variation in the number of acres, suggesting that a per acre basis is more likely to be adopted. All the coefficient estimates are statistically significant except for SCARCE and have signs consistent with the discussion above. The probit model correctly predicts 55 of the 65 ditches while the MS estimator correctly predicts 59 ditches.

Finally, we discuss the estimation results from the multinomial logit model. The multinomial logit model allows the simultaneous selection from amongst the alternative equity bases while considering the full set of explanatory variables. Overall, the model has significant explanatory power (likelihood ratio statistic is 47.96, significant at the 1% level) and is able to correctly predict 75% (63 of the 84) of the equity bases. We highlight several salient features from the coefficient estimates and marginal effects.

First, the most significant determinants of the per capita basis are TOWN, SCARCE and GINI. The marginal effects indicate, all else equal, that an increase in the fraction of users residing in town from 0.25 to 0.75 will increase the probability of choosing the per capita basis by 0.035 percentage points. Likewise, a decrease in GINI or SCARCE from 0.5 to 0.25 increases the probability of choosing the per capita basis by 0.08 and 0.02 percentage points, respectively. All the coefficients in the PC equation have the expected sign – large ditches in town with more rain have an increased chance of using the per capita basis; while ditches with atypical users, increased water scarcity, and greater variation in irrigated acres are less likely to employ the per capita basis.

Second, the two most significant factors that lead to the choice of a per acre basis are the yield ratio and the variation in irrigated acres. For example, if the ratio of yield from irrigated to non-irrigated land doubled from 1 to 2, the probability of choosing the per acre basis would increase by 0.036 percentage points, all else equal. An increase in the Gini index from 0.25 to 0.5 leads to an increase in the probability of choosing the per acre basis by 0.08 percentage points, all else equal. The coefficient on RAIN is positive as expected and significant at a 15% level.

Third, although the magnitude of the coefficients between the probit and multinomial logit models differs, they are qualitatively similar. This provides a degree of confidence that the two-stage approach and the IIA assumption of the multinomial logit are reasonable. Furthermore, the Hausman statistic to test the IIA assumption in the PA equation of the multinomial logit is 0.132 with chi-squared (df = 7) critical value equal to 14.1. Therefore, we fail to reject the IIA hypothesis and the choice between per acre and per water-share bases can apparently be made independent of the per capita basis.

The empirical results represent a robust relationship between environmental parameters and the observed equity basis selection. We experimented with several alternative explanatory variables and various definitions of the current explanatory variables. For instance, we examined measures of the slope, roughness, and elevation of the land; ditch length; alternative threshold values for ATYPICAL and TOWN; alternative measures of acre variation such as standard deviation and normalized ACRE DIFF; rainfall and yield variation; imputation of missing SIZE observations; and self-reported variation in water usage, to name a few. The coefficients from these various specifications exhibited the expected signs and the models explained a significant amount of the variation in the dependent variables. In sum, the empirical analysis appears to indicate a robust and stable relationship between features of the allocation environment and the chosen equity basis.

5. Conclusion

Cooperative environments are frequently endowed with multiple bases for assessing the "fairness" of a proposed allocation. Rather than simply take this equity basis selection as given, we have sought to determine whether parameters of the cooperative environment could be used to effectively explain this selection. Moreover we have sought to establish whether this explanatory power, to the extent that it existed, was consistent with axiomatic principles of collective behavior. Our results have confirmed that both of these questions can be answered affirmatively. Using irrigation cost-sharing data, we have demonstrated that features of the cost-sharing environment enjoy substantial explanatory power in determining the equity basis embraced in practice. This explanatory power is consistent with the benefit inequity principle – that is, greater pre-tax benefit inequity across the elements of an equity basis is a deterrent for its selection. As such, our empirical results are also supportive of an axiomatic approach to cost allocation applications.

In closing, we note that an important first step in the forging of stable and mutually beneficial cooperative agreements is the selection of an appropriate basis for equity assessment. Our analysis can be viewed as providing guidance on the best way to select such a basis. Indeed, the understanding of how environmental features can be used to explain the foundations for successful, well-established cooperative ventures can in turn be used to help construct the foundations for ventures yet to be undertaken.

6. References

Aadland, D. and V. Kolpin, 2004. Environmental determinants of cost sharing. *Journal of Economic Behavior and Organization*, 53, 495-511.

Ashton J. and X. Wang, 2003. Equity and climate: in principle and practice. In: *Beyond Koyoto: advancing the international effort against climate change*. Pew Center on Global Climate Change, 61-84.

Cohen, J. and P. Cohen, 1983. *Applied multiple regression/correlation analysis in behavioral sciences*. 2nd ed., Erlbaum, Hillsdale, NJ.

Greene, W. H., 2008. *Econometric Analysis*. 6th ed., Prentice Hall, Upper Saddle River, NJ.

Grossman, J., 2001. Blue planet: the Kyoto fairness issue. *UPI Science News*. United Press International.

Hausman, J. and D. McFadden, 1984. Specification tests for the multinomial logit model. *Econometrica*, 52, 1219-40.

Lambert, P, 2002. *The Distribution and Redistribution of Income*. 3rd ed. Manchester University Press, Manchester, UK.

Littlechild, S. C. and G. Owen, 1973. A simple expression for the Shapley value in a special case. *Management Science*, 20, 370-2.

Nash, J. F., 1950. The bargaining problem. *Econometrica*, 28, 155-62.

Oregon Climate Service, 2002. http://www.ocs.orst.edu.

Sen, A., 1973. *On Economic Inequality*. Clarendon Press, Oxford, UK.

Shapley, L. S., 1953. A value for n-person games. In: *Contributions to the Theory of Games, vol II*, H. W. Kuhn and A. W. Tucker, eds. Annals of Mathematics Studies 28. Princeton University Press, Princeton, NJ, pp. 307-17.

USDA Soil Conservation Service and Forest Service, 1975. *Soil Survey of Carbon County Area, Montana.*

USDA Soil Conservation Service, 1980. *Soil Survey of Stillwater County Area, Montana.*

Water Resource Division of the Montana DNRC, 2002. http://www.dnrc.state.mt.us/wrd/home.htm.

Table 1. Variable Definitions and Descriptive Statistics

Variable	Definition	Statistics				
		Mean	Standard Deviation	Minimum	Maximum	Sample Size
PC	1 if cost-sharing rule is specified on a per capita basis, 0 otherwise	0.226	0.421	0.000	1.000	84
PA	1 if cost-sharing rule is specified on a per acre basis, 0 otherwise	0.143	0.352	0.000	1.000	84
PWS	1 if cost-sharing rule is specified on a per water-share basis, 0 otherwise	0.631	0.485	0.000	1.000	84
SIZE	Number of irrigators on unincorporated ditches, 0 for incorporated ditches	7.381	6.781	2.000	33.000	63
ATYPICAL	Fraction of users on unincorporated ditches that use water for stock/domestic purposes or irrigate fields less than half an acre, 0 otherwise	0.190	0.091	0.040	0.333	6
TOWN	Fraction of users on unincorporated ditches with at least one field within a one mile radius of the center of town, 0 for incorporated ditches	0.521	0.358	0.111	1.000	16
ACRE DIFF	Difference between maximum irrigated acres and minimum irrigated acres for unincorporated ditches, 0 for incorporated ditches	207.384	186.862	7.000	993.000	63
RAIN	Minimum rainfall (millimeters) on a ditch parcel during the growing season (May through August)	193.321	30.669	108.440	245.330	84
SCARCE	Fraction of survey respondents reporting insufficient water to meet their needs, 0 otherwise	0.648	0.356	0.100	1.000	16
YIELD RATIO	Ratio of average yield on irrigated to non-irrigated fields	2.153	1.067	1.389	7.000	84
GINI	Gini coefficient for acre dispersion, 0 for incorporated ditches	0.363	0.165	0.013	0.654	63

Notes: For the *ATYPICAL*, *TOWN* and *SCARCE* variables, the reduced sample size reflects only ditches with positive values. The reduced sample sizes for *SIZE*, *ACRE DIFF* and *GINI* reflect only unincorporated ditches. Towns under consideration include Absarokee, Bridger, Columbus, Fromberg, Joliet, Red Lodge, and Roberts.

Table 2. Estimation Results: Choice of Equity Basis for Irrigation Cost-Sharing Rules

Variable	Stage #1. PC vs. PA/PWS Bivariate Probit			Stage #2. PA vs. PWS Bivariate Probit			PC vs. PA vs. PWS Multinomial Logit					
	Dependent Variable = PC			Dependent Variable = PA			Dependent Variable = PC			Dependent Variable = PA		
	Coef.	P Value	ME	Coef.	P Value	ME	Coef.	P Value	ME	Coef.	P Value	ME
Constant	0.602*	0.081	--	-6.393**	0.012	--	-0.329	0.422	--	-7.642**	0.038	--
SIZE	-0.009	0.404	-0.002	-0.123*	0.084	-0.024	0.026	0.283	0.004	-0.093	0.192	-0.004
ATYPICAL	-9.514**	0.029	-1.828	-11.108*	0.085	-2.179	-10.474†	0.117	-0.153	-5.208	0.300	0.091
TOWN	2.041**	0.031	0.392	3.051*	0.072	0.599	4.575**	0.028	0.069	2.192	0.178	-0.042
ACRE DIFF	-0.008***	0.003	-0.002									
RAIN				0.019**	0.045	0.004	0.017	0.170	-6.0e-5	0.021†	0.108	2.4e-4
SCARCE				-0.276	0.362	-0.054	-2.178**	0.047	-0.036	-0.890	0.258	0.025
YIELD RATIO				0.411**	0.019	0.081	-0.647	0.183	-0.033	0.705**	0.026	0.036
GINI				4.066**	0.042	0.798	-8.402***	0.002	-0.319	4.192†	0.102	0.320
LR Statistic	31.74***			17.25*			47.96***					
Hausman Statistic	--			--			0.053			0.132		
Sample Size	84			65			84					

Cost Sharing Type	Actual	Predicted Correct (%)	MS Predicted Correct (%)	Actual	Predicted Correct (%)	MS Predicted Correct (%)	Actual	Predicted Correct (%)	MS Predicted Correct (%)
No. of PC Ditches	19	10 (53%)	15 (79%)	--	--	--	19	10 (53%)	--
No. of PA Ditches	65	60 (92%)	61 (94%)	12	4 (33%)	7 (58%)	12	4 (33%)	--
No. of PWS Ditches				53	51 (96%)	51(96%)	53	49 (92%)	--

Notes. The marginal effects are evaluated at the mean value of the explanatory variables. (†), (*), (**) and (***) indicate significance at the 15, 10, 5, and 1 percent level, respectively. PC = Per Capita; PA = Per Acre; PWS = Per Water-Share; ME = Marginal Effect; MS = Maximum Score Estimator; LR = Likelihood Ratio. The "missing observation" dummy variables are omitted to conserve space.