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12 June 2018

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June 12, 2018

Abstract

We investigate the endogenous order of moves in a price-setting mixed oligopoly model, comprising two private firms and a public firm. We show that sequential moves emerge as the equilibrium in the observable delay game. Specifically, one of the private firms and the public firm set their prices in period 1, and the other private firm does so in period 2, in equilibrium, if their goods are not significantly differentiated. This is a clear contrast to a mixed duopoly where a simultaneous move game is a unique equilibrium. We also discuss a number of extensions and the robustness of our result.

JEL classification numbers: H44; L13

Key words: Mixed Markets; Endogenous Timing; Stackelberg

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*We are grateful to Toshihiro Matsumura for his helpful advice. We also thank to Susumu Cato, Hikaru Ogawa, Dan Sasaki, and Daisuke Shimizu for their comments. Any remaining errors are our own.

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1 Introduction

The literature on oligopoly theory contains numerous studies of firms' endogenous order of moves. Depending on the context, there is a first-mover or second-mover advantage through strategic effects,[1] which firms try to obtain in advance. In a seminal paper on this issue, Hamilton and Slutsky (1990) first allowed firms to choose the order of moves in an observable delay game and showed that a simultaneous (resp. sequential) move game occurs in equilibrium if the firms' reaction functions slope up (down). Prior work has provided valuable insights by analyzing the endogenous choice of the leader/follower role in several different settings. In particular, a market with private firms and a state-owned public firm, called a mixed market, has received substantial interest from economists.[2] The difference between a private and a public firm can be summed up as a divergence between their objectives: private firms only consider their own profit, while public firms pursue a non-profit goal, such as public interest. Although the public sector's real goal is controversial, the types of firms may change their behavior.[3]

Pal (1998) showed, revisiting the observable delay game formulated by Hamilton and Slutsky (1990) in a quantity-setting mixed oligopoly, that a public firm follows all private firms in equilibrium. This result contrasts with private oligopoly, where all firms set their output simultaneously. More recently, Bárcena-Ruiz (2007), which is closely related to this paper, examined firms' order of moves in a price-setting mixed duopoly and showed that simultaneous moves occur in equilibrium. This result is also contrary to private duopoly, where firms set prices sequentially. The author's analysis, however, is limited to the case of a duopoly. One natural question is whether or not the simultaneous move structure is robust to a change in the number of private firms.

[1]See Gal-Or (1985) or Amir and Stepanova (2006), for example.

[2]Examples of public enterprises are the United States Postal Service, Areva, Nippon Telecom and Telecommunication , Volkswagen, Renault, Électricité de France, Enel SpA, Tokyo Electric Power Co. Holdings, Inc., Japan Postal Bank, and the Development Bank of Japan.

[3]In the literature on mixed oligopolies, private firms simply maximize their own profit, but public firms care about the social surplus. For recent developments in this field, see Chen (2017), Heywood and Ye (2017), and Lee, Matsumura and Sato (2017).

In this paper, we study the endogenous order of moves in a price-setting mixed oligopoly with differentiated products. To begin with, suppose one welfare-maximizing public firm competes with two private firms that maximize their own profits. We find the emergence of sequential moves, called a Stackelberg game in this paper, in which the public firm and one of two private firms, in equilibrium, choose its prices in period 1 and the other private firm does so in period 2. This is a clear contrast to Bárcena-Ruiz (2007), which presents the simultaneous move game as an equilibrium structure.

Furthermore, we discuss a multi-period model.[4] We show that a so-called hierarchical Stackelberg game (another type of sequential move game), where firms choose their prices sequentially (the public firm in the first period, one private firm in the second period, and the other private firm in the third period), never occurs in an equilibrium, but the Stackelberg game does so. Finally, we discuss the case of an arbitrary number of private firms. Given that any number of private firms would raise the analytical complexity of finding equilibrium patterns, we present a numerical example. Our result shows that as the number of privates increases, the simultaneous move game, which is the equilibrium in a mixed duopoly (Bárcena-Ruiz, 2007), is less likely to occur.

The contributions of this paper are twofold. First, because most markets are oligopolistic, our result suggests that a Stackelberg game (a sequential game with multiple leaders) occurs under price competition even when a public firm exists. This is directly linked to the issue of endogenous order of moves and to the problem of identifying the market structure for competition policies. Second, this paper's findings emphasize the importance of whether firms set price or quantity variables to consider the equilibrium market structure in a mixed oligopoly.

For clarity, it would be useful to mention what happens in the endogenous price-quantity model formulated by Singh and Vives (1990), where each firm can commit to a quantity or price strategy before the competition stage. They showed that in a private duopoly, a quantity contract is the

[4] See Matsumura(1999).

dominant strategy for both firms. Matsumura and Ogawa (2012) and Haraguchi and Matsumura (2016) have analyzed endogenous competition structure in a mixed oligopoly with differentiated products. The first (second) work shows that in the case of a mixed duopoly (oligopoly), the private firm chooses prices (may choose quantities) endogenously. These results show the importance of the number of private firms in an endogenous competition structure in mixed oligopolies. This study is related to Bárcena-Ruiz (2007) in the same way as Haraguchi and Matsumura(2016) is to Matsumura and Ogawa (2012). There are major difference between Haraguchi and Matsumura (2016) and our study. The former endogenized a price-quantity choice, but this study investigates endogenous timing. In addition, the endogenous timing model can extend the aforementioned hierarchical Stackelberg model, but the endogenous price-quantity model cannot have a corresponding application in this discussion.

The rest of the paper is organized as follows. Section 2 presents the model, while Section 3 analyzes the endogenous order of moves for a two-period case. Section 4 checks the robustness of our main result and Section 5 concludes .

2 The Model

We adopt a standard differentiated oligopoly model with a linear demand (Dixit, 1979). The quasi-linear utility function of the representative consumer is

$$U(q_i) = \alpha \sum_{i=0}^{n} q_i - \beta(\sum_{i=0}^{n} q_i^2 + \delta \sum_{i=0}^{n} \sum_{i \neq j} q_i q_j)/2 + y,$$

where q_i is the consumption of good i, produced by firm i, and y is the consumption of an outside good that is provided competitively (with a unit price). The parameters α and β are positive constants, and $\delta \in (0, 1)$ represents the degree of product differentiation: a smaller δ indicates a larger degree of product differentiation. The demand functions are given by

$$q_i = \frac{\alpha(1-\delta) - p_i + \delta \sum_{j \neq i} p_j}{\beta(1-\delta)(1+2\delta)} \quad (i \neq j; \quad i, j = 0, 1, 2, ..., n). \tag{1}$$

For simplicity, we assume all firms share the same constant marginal cost of production of firms, given by $c \in [0, \alpha)$.[5] The profit function of firm i is given by:

$$\pi_i = (p_i - c)q_i \quad (i \neq j; \quad i, j = 0, 1, 2, ..., n).$$

Firm i $(i = 1, 2, ..., n)$ is a private and it chooses the price p_i, which maximizes its own profit. Firm 0 is a state-owned public enterprise and chooses the price p_0, which maximizes social welfare, which is given by

$$\begin{aligned} SW &= \sum_{i=0}^{n}(p_i - c)q_i + \left[\alpha \sum_{i=0}^{n} q_i - \frac{\beta\left(\sum_{i=0}^{n} q_i^2 + \delta \sum_{i=0}^{n} \sum_{i \neq j} q_i q_j\right)}{2} - \sum_{i=0}^{n} p_i q_i\right] \quad (2) \\ &= (\alpha - c)\sum_{i=0}^{n} q_i - \frac{\beta\left(\sum_{i=0}^{n} q_i^2 + \delta \sum_{i=0}^{n} \sum_{i \neq j} q_i q_j\right)}{2}. \quad (3) \end{aligned}$$

Following Hamilton and Slutsky (1990), we consider a two stage game. In the first stage, the public and private firms simultaneously decide whether to set prices in period 1 or in period 2.[6] Let $t_i \in \{1, 2\}$ denote the time period chosen by firm i $(i = 0, 1, 2, ..., n)$. After observing their decisions, the firms set their prices according to the commitments in the second stage. If all firms choose the same period, the firms make price decisions simultaneously. If the timing decisions differ among firms, they move sequentially in the price-setting period. To obtain a sub-game perfect Nash-equilibrium, we solve this game backwards.

3 Results

For simplicity, we assume that the number of private firms is two (hereafter, a mixed triopoly).[7] Before characterizing a Nash equilibrium of the full game, we explore the second stage given the order of moves, (t_0, t_1, t_2).

[5] The results in this paper holds even in the case of a cost asymmetry between the public and private firms, e.g., $c_0 > c$.

[6] We extend our baseline model, in section 4, to one with more than two periods.

[7] We discuss the case of an arbitrary number of private firms in section 4.

3.1 Simultaneous game

First, we discuss a simultaneous game, in which all firms choose their prices in the same period. The first-order conditions for the public and private firms are, respectively,

$$\begin{aligned} \frac{\partial SW}{\partial p_0} &= \frac{(1-\delta)c - p_0(1+\delta) + \delta \sum_{i=1}^{2} p_i}{\beta(1+2\delta)(1-\delta)} = 0, \\ \frac{\partial \pi_i}{\partial p_i} &= \frac{(1-\delta)\alpha + (1+\delta)c - 2(1+\delta)p_i + \delta \sum_{j \neq i} p_j}{\beta(1+2\delta)(1-\delta)} = 0 \quad (i \neq 0). \end{aligned}$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions for the public and private firms, respectively:

$$R_0(p_i) = \frac{(1-\delta)c + \delta \sum_{i=1}^{2} p_i}{1+\delta}, \tag{4}$$

$$R_i(p_j) = \frac{(1-\delta)\alpha + (1+\delta)c + \delta \sum_{j \neq i} p_j}{2(1+\delta)} \quad (i \neq 0) \tag{5}$$

These functions lead to the equilibrium prices of the public and the private firms as follows:

$$\begin{aligned} p_0^N &= \frac{2\delta(1-\delta)\alpha + (2+\delta+\delta^2)c}{2+3\delta-\delta^2}, \\ p^N &= \frac{(1-\delta^2)\alpha + (1+3\delta)c}{2+3\delta-\delta^2}, \end{aligned}$$

where superscript N denotes the respective equilibrium outcomes of the simultaneous move game. Substituting these equilibrium prices into the payoff functions, we obtain the resulting welfare and profit of the private firm:

$$SW^N = \frac{(\alpha-c)^2(10+34\delta+21\delta^2-16\delta^3-\delta^4)}{2\beta(1+2\delta)(2+3\delta-\delta^2)^2}, \tag{6}$$

$$\pi^N = \frac{(\alpha-c)^2(1-\delta)(1+\delta)^3}{\beta(1+2\delta)(2+3\delta-\delta^2)^2}. \tag{7}$$

3.2 Sequential games

In this subsection, we consider four different Stackelberg games: (i) the public firm as a Stackelberg leader (i.e., $(t_0, t_1, t_2) = (1, 2, 2)$), (ii) the public firm as a Stackelberg follower (i.e., $(t_0, t_1, t_2) =$

(2, 1, 1)), (iii) one private firm as a Stackelberg leader (i.e., $(t_0, t_1, t_2) = (2, 1, 2)$ or $(2, 2, 1)$), and (iv) one private firm as a Stackelberg follower (i.e., $(t_0, t_1, t_2) = (1, 1, 2)$ or $(1, 2, 1)$). Let superscripts LFF, FLL, FLF, and LLF denote the equilibrium outcome in cases (i), (ii), (iii), and (iv), respectively.[8] We use the subscript $\{l, f\}$ to indicate a private leader and private follower, respectively. With the straightforward derivation, the equilibrium outcomes for these four games can be obtained, which is provided in Appendix A. Table 1 presents the set of equilibrium payoffs for each case.

Table 1: Payoff matrix for 2 periods case

0\1	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	(SW^N, π^N, π^N)	$(SW^{LLF}, \pi_f^{LLF}, \pi_l^{LLF})$
$t_0 = 2$	$(SW^{FLL}, \pi^{FLL}, \pi^{FLL})$	$(SW^{FLF}, \pi_f^{FLF}, \pi_l^{FLF})$

Firm 2 chooses $t_2 = 1$.

0\1	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	$(SW^{LLF}, \pi_l^{LLF}, \pi_f^{LLF})$	$(SW^{LFF}, \pi^{LFF}, \pi^{LFF})$
$t_0 = 2$	$(SW^{FLF}, \pi_l^{FLF}, \pi_f^{FLF})$	(SW^N, π^N, π^N)

Firm 2 chooses $t_2 = 2$.

3.3 Equilibrium outcomes in the observable delay game

In this subsection, we analyze the first stage and characterize a Nash equilibrium of our full game. For the analysis of the first stage, we start with a welfare comparison among the five games.

Lemma 1 *The equilibrium welfare has the following relationships:*

(i) $SW^{FLL} < SW^N < SW^{LFF}$ *and (ii)* $SW^{FLF} < SW^{LLF}$.

Proof See Appendix B.

We provide the intuition behind Lemma 1-(i). When the private firms move in period 1 and act as a leader in the second stage, they set higher prices than they do in the simultaneous game.

[8]For instance, in case (i), the public firm is the *leader* and the private firms are *followers*, so we denote case (i) by LFF.

This leads to a decline in total production so that social welfare is worse, $SW^{FLL} < SW^N$. When the public firm is the leader, it becomes more aggressive and sets a lower price than it does in the simultaneous move game. Through the strategic effect, equilibrium prices decrease and total production increases. This improves social welfare, $SW^N < SW^{LFF}$. These two effects yield Lemma 1-(ii). From Lemma 1, we obtain the following lemma:

Lemma 2 *The dominant strategy of the public firm is to choose* $t_0 = 1$*.*

It implies that the public firm never accepts the role of a follower. More importantly, however, the observable delay game, in which one public and two private firms play, can be reduced to a game with only private firms, given $t_0 = 1$. This reduces the complexity of the observable delay game played by more than two players.

Next, we compare the equilibrium profit. From Lemma 2, the possible equilibria in the first stage are a simultaneous move game, case (i), and case (iii). Focusing on these three cases, we obtain the following lemma:

Lemma 3 *The equilibrium profit has the following relationships:*
(i) $\pi_f^{LLF} < (>) \pi^N$ *if* $\delta < (>) \delta^*$, *(ii)* $\pi^{LFF} < \pi_l^{LLF}$ *and (iii)* $\pi_l^{LLF} < \pi_f^{LLF}$,
where $\delta^* \simeq 0.699417$.

Proof See Appendix C.

We provide the intuition behind Lemma 3. As mentioned above, when the public firm is the leader, it sets a lower price to induce the private firms to lower the equilibrium price. Thus, the private firm avoids being a follower, $\pi_f^{LLF} < \pi^N$ and $\pi^{LFF} < \pi_l^{LLF}$. However, the extent of competition between the private firms is another aspect related to private profit. It is well established in the literature on first/second-mover advantage that a sequential move game relaxes industry competition and a second mover advantage occurs in price competition. Comparing π_f^{LLF} with π^N, we find the threshold $\delta^* \in (0, 1)$ such that $\pi_f^{LLF} = \pi^N$. This means that the effect of the second mover's advantage under price competition offsets the effect of aggressive behavior by the public firm. A sufficiently large degree of product differentiation causes the profit of the private follower to increase, $\pi_f^{LLF} > \pi^N$. Lemma 3-(*iii*) represents the second-mover advantage in the sequential move game.

Before characterizing the Nash equilibrium of the full game, we state the result of endogenous timing in a price-setting mixed duopoly to emphasize our result.[9]

Result *In a mixed duopoly equilibrium, a simultaneous move game occurs, where both the public and a private firm set prices in period 1.*

In a mixed duopoly, firms set prices simultaneously in equilibrium. The intuition is as follows: (i) If the public firm is a leader, the public firm commits to a lower price than in the simultaneous case to induce aggressive pricing by the private firm in the second period. Lower pricing by the public firm increases welfare but decreases private profit. Thus, the public firm would want to be a leader. (ii) If the private firm is the leader, it would set a higher price than in the simultaneous case to induce less aggressive pricing by the public firm in the second period. The higher price raises the private firm's profit but lowers the welfare. Therefore, the private firm would also want to be a leader. Finally, a simultaneous move game appears in a mixed duopoly.

Now, we present our main results for the two time periods. The results of Lemmas 2 and 3 allow us to derive the equilibrium outcomes in the first stage. From Lemma 2, the public firm always selects $t_0 = 1$. Thus, the equilibrium outcome depends only upon the choice of the private firms. From Lemma 3, we obtain the following proposition.

Proposition 1 *In a mixed oligopoly with one public firm and two private firms,*
(i) a simultaneous move game occurs, where all firms choose their prices in period 1, if $\delta \in (0, \delta^*)$*;*
(ii) a Stackelberg game (a sequential game) occurs, where the public firm and one private firm choose their prices in period 1 and the other private firm does so in period 2, if $\delta \in (\delta^*, 1)$.

[9] Proposition 2 in Bárcena-Ruiz (2007) shows this result.

Proof

Given $t_0 = 1$, from Lemma 3, neither private firm sets its price in period 2 if $\delta \in (0, \delta^*)$. This leads to (i). From Lemma 3, only one private firm sets the price in period 2 if $\delta \in (\delta^*, 1)$. This implies (ii). ■

In the mixed duopoly case, a simultaneous move in which all firms move in period 1 is the unique equilibrium. In the mixed triopoly case, however, a simultaneous move game may not provide an equilibrium outcome. We explain the intuition behind the results. By Lemma 2, the public firm's dominant strategy is to choose $t_0 = 1$. This implies that the public firm never accepts the role of a follower. On the other hand, the private firms may have an incentive to be the followers if their goods are not sufficiently differentiated. In such a case, the firms engage in intense competition in the same period because their goods are substantially substitutable. This situation is favorable for the public firm but lowers the private firms' profit. As discussed immediately after Lemma 3, the sequential move structure under price competition relaxes industry competition through its strategic effect. Thus, one of the private firms might move in equilibrium at period 2. The incentive for deviating from the simultaneous move game would be much greater with a larger the number of private firms in the market. Most markets are oligopolistic and include more than one private firm. This implies that the Stackelberg game prevails under price competition even when a public enterprise exists. Note that the private firms would not become followers at the same time. If both the private firms are followers, there would be no strategic interaction between them to soften market competition, and the public firm would become aggressive, inducing lower pricing by the private firms.

4 Robustness

In this section, we discuss two extensions of our model: a mixed triopoly with more than two periods and the case of an arbitrary number of private firms.

4.1 More than two periods

With more than two periods, a hierarchical Stackelberg game can occur where all firms sequentially set their price in an equilibrium. In such a situation, each firm plays the role of leader, follower, or intermediate. We consider an oligopoly game with three firms so that there is at most one intermediate. There are different kinds of sequential move games in addition to the games defined in Section 3.2: (v) public firm as the Stackelberg leader in a hierarchical Stackelberg game (i.e., $(t_0, t_1, t_2) = (1, 2, 3)$ or $(1, 3, 2)$), (vi) public firm as the intermediate in a hierarchical Stackelberg game (i.e., $(t_0, t_1, t_2) = (2, 1, 3)$ or $(2, 3, 1)$), (vii) public firm as the Stackelberg follower in a hierarchical Stackelberg game (i.e., $(t_0, t_1, t_2) = (3, 1, 2)$ or $(3, 2, 1)$). Let the superscript LMF, MLF, and FLM denote the equilibrium outcome in cases (v), (vi), and (vii), respectively.[10] We use the subscript $\{l, m, f\}$ to indicate the private leader, private intermediate, and private follower, respectively. With the straightforward derivation, the equilibrium outcomes for these four games can be obtained, which is described in Appendix D.

We begin with a welfare comparison to analyze the first stage

Lemma 4 *The equilibrium welfare has the following relationships:*
(i) $SW^{FLM} < SW^{LLF} < SW^{LMF}$ *and (ii)* $SW^{MLF} < SW^{LLF}$.

Proof See Appendix E.

The intuition behind Lemma 4 is similar to explanation for Lemma 1. On the one hand, when a private firm is the leader in the second stage, it takes into account the reaction of the follower firm and sets a higher price to increase its profit. This decreases total output and, in turn, social welfare. The existence of a private intermediate accelerates the distortion of private leadership because the private intermediate adopts the same strategy. Moreover, the private leader predicts the intermediate's reaction and raises its price as well, $SW^{FLM} < SW^{LLF}$. On the other hand,

[10] For instance, in case (v), the public firm is *leader*, one private is the *intermediate*, and the other private is the *follower* so that we denote case (v) by LMF.

when the public firm is the leader in the second stage, it charges a lower price to induce the follower firm to set a lower price. This increases total production and improves social welfare. With a private intermediate, the positive effect of the public firm's leadership accumulates as if the private leader were on the opposite side, $SW^{LLF} < SW^{LMF}$. If the public firm is the intermediate, it can induce the private follower to lower its prices, while a private leader would induce higher prices. The latter effect diminishes the positive effect of the former, $SW^{MLF} < SW^{LLF}$.

From Lemmas 1 and 4, we obtain the following:

Lemma 5 *Suppose a mixed triopoly with three periods. The public firm's dominant strategy is to choose* $t_0 = 1$.

The result is fairly straightforward, but it is worth noting that the public firm's preference for the order of moves is robust to the existence of an intermediate. As seen in Section 3, this result reduces the number of candidates for a subgame perfect equilibrium. Indeed, the only remaining candidate is case (v), in which public firm is the Stackerberg leader in a hierarchical Stackelberg game.

Next, we find the property of the resulting profit by additionally emphasizing case (v). Then, we obtain the following lemma:

Lemma 6 *The equilibrium profit has the following relationships:*
(i) $\pi_f^{LMF} < (>) \pi_l^{LLF}$ *if* $\delta < (>) \delta^{**}$ *and (ii)* $\pi_m^{LMF} < \pi_l^{LLF}$,
where $\delta^{**} \simeq 0.968794$.

Proof See Appendix F.

We provide the intuition behind Lemma 6. As we have seen above, when the public firm is the leader, it sets a lower price to increase total outputs. The effect of the public firm's leadership accelerates in the presence of an intermediate so that the equilibrium profits of the private firms decrease, $\pi_f^{LMF} < \pi_l^{LLF}$. However, if the products are not significantly differentiated, the first-order effect of competition in the same period is so great that it offsets the effect of aggressive behavior by the public firm. Comparing π_f^{LMF} with π_l^{LLF}, we find the threshold $\delta^{**} \in (0,1)$ such that $\pi_f^{LMF} = \pi_l^{LLF}$. It means that, if the products are close to homogeneous goods, the private firms prefer to move to a different period in which there are no other firms, $\pi_f^{LMF} > \pi_l^{LLF}$. If a private firm is an intermediate, it would induce the follower firm to raise its price, while the public firm's leadership hinders this effect. Thus, the profit of the private leader is definitely larger than that of the private intermediate, $\pi_m^{LMF} < \pi_l^{LLF}$.

Lastly, we analyze the first stage of this game. From Lemma 5, the public firm always prefers to choose $t_0 = 1$. Thus, the equilibrium outcome in the first stage depends only upon the private firm's behavior. From Lemmas 2,3, and 6, we obtain the following proposition.

Proposition 2 *Suppose a three period case as follows:*

(i) a simultaneous move game occurs, where all firms choose their prices in period 1, if $\delta \in (0, \delta^*)$*,*

(ii) a sequential move game occurs, where the public firm and one private firm set their prices in period 1 and the other private firm does so in the following period, if $\delta \in (\delta^*, \delta^{**})$*, and*

(iii) a sequential move game occurs, where the public firm and one private firm set their prices in period 1 and the other private does so in period 3, if $\delta \in (\delta^{**}, 1)$*.*

Proof

From Lemmas 2 and 6, both private firms set their prices in period 1 if $\delta \in (0, \delta^*)$. This implies (i). From Lemmas 3 and 6, one private firm becomes the follower if $\delta \in (\delta^*, \delta^{**})$. This implies (ii). From Lemmas 3 and 6, one private follower sets its prices in period 3 if $\delta \in (\delta^{**}, 1)$. This implies (iii). ■

We show that a sequential move equilibrium still emerges but a hierarchical Stackelberg equilibrium never occurs in an equilibrium. We discuss the intuition of our result. By Lemma 5, the public firm strictly prefers to be a leader. We explain in detail the private firms' incentive for the order of moves. There are two opposite effects in the three period case:(i) As in the two-period model, the public firm becomes aggressive with its leadership role, decreasing the private firms' profits. (ii) As is well established in the price competition literature, both leader and follower firms' profits in Stackelberg competition are higher than in simultaneous competition because of the strategic effect. The effect is stronger when the products are not differentiated. Thus, either of the private firms becomes a follower for $\delta > \delta^*$, resulting in a Stackelberg game in equilibrium. The positive effect also leads to a hierarchical Stackelberg game. However, the effect of aggressive public pricing reduces the profit-improving effect in that game. Therefore, the profit of a private intermediate in a hierarchical Stackelberg game cannot be higher than the private leader's profit in a Stackelberg game. Thus, a hierarchical Stackelberg equilibrium never occurs.

4.2 More than two private firms

In our analysis, we have assumed two private firms. In this subsection, we discuss the case of n private firms. The larger the number of private firms, the greater would be the analytical complexity of finding equilibrium patterns. Thus, we present a numerical example.

Figures 1 and 2 describe the relationship between the equilibrium pattern and the degree of product differentiation. Figures 1 and 2 depict cases with 3 and 4 private firms, respectively. In Figure 2, $LLLL$ represents that all the firms set their prices in the first period, and $LLLF$ indicates that firm 0,1, and 2 set their prices in the first period and firm 3 sets its price in the second period, and so on.

These results show that as the number of privates increases, the simultaneous move game that result in an equilibrium in a mixed duopoly (Bárcena-Ruiz, 2007) is less likely to occur. This is because competition increases with a large number of private firms, and each of them has an incentive to quickly distance itself from such acute competition. Our analysis also suggests that at least one private firm continues to be a leader. This implies that the public firm's marginal cost pricing is always unacceptable to private firms.

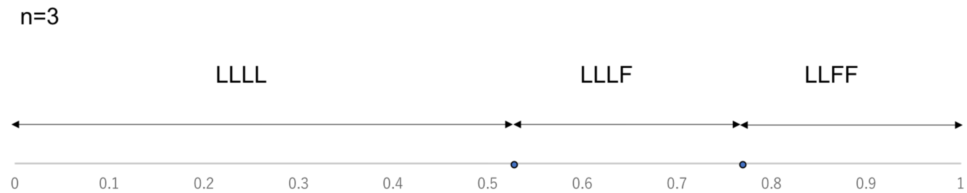

Figure 1: Equilibrium patterns (n=3).

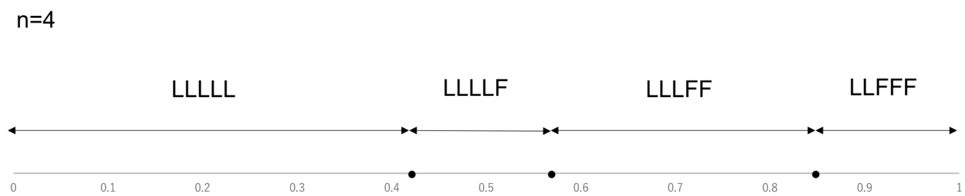

Figure 2: Equilibrium patterns (n=4).

5 Concluding Remarks

We investigate an endogenous order of moves in a price-setting mixed oligopoly. The study shows that firms may set prices sequentially in a mixed oligopoly, but not in a mixed duopoly. This result suggests that the number of private firms is important in a price-setting mixed market. We also study a multi-period case and show that firms set prices sequentially where the public firm leads the private firms. However, the public firm follows the private firms in a quantity-setting competition. These results suggest that the role of the public sector depends on whether the competition is based on price or quantity. We also show that as the number of private firms increases, each private firm has a greater incentive to become a follower. Thus, the larger the number of private firms, the more the private followers in the equilibrium, when the product is sufficiently differentiated.

In mixed oligopolies, the nationality of private firms is important.[11] For instance, Matsumura(2003)

[11] See Corneo and Jeanne (1994), Fjell and Pal(1996), Pal and White (1998), Cato and Matsumura(2012), and Lin and Matstumura(2012).

showed that Pal's(1998) finding depends on the nationality of the private firm and that the public leader is endogenously determined. From this result, the nationality of private firms in a timing game is important. Further extensions in this direction are forthcoming challenges.

Appendix A

Public firm as a Stackelberg leader: $(t_0, t_1, t_2) = (1, 2, 2)$

First, we discuss a Stackelberg model in which the public firm chooses its price in period 1 and the private firms do so in period 2. That is, the public firm is a leader, and the private firms are followers. We solve the game backwards and start with the followers' problems. The private firms choose their actions after observing the leader's action. Their strategy is (5). Anticipating their actions, the public firm chooses its action to maximize social welfare, $SW(p_0, R_i(p_0), R_j(p_0))$. The first-order condition of the public firm is given by

$$\begin{aligned}\frac{\partial SW}{\partial p_0} &= \sum_{i=0}^{2}(p_i - c)\left(\frac{\partial q_i}{\partial p_0} + \sum_{k=1}^{2}\frac{\partial q_i}{\partial p_k}\frac{\partial R_k(p_0)}{\partial p_0}\right) \\ &= \frac{2\alpha\delta(1-\delta^2) + c(4+6\delta-\delta^2-\delta^3) - (4+8\delta-\delta^2-3\delta^3)p_0}{\beta(1-\delta)(2+\delta)^2(1+2\delta)} = 0.\end{aligned}$$

We obtain the equilibrium price of the public firm:

$$p_0^{LFF} = \frac{2\alpha\delta(1-\delta^2) + c(4+6\delta-\delta^2-\delta^3)}{4+8\delta-\delta^2-3\delta^3}.$$

Substituting it into (5), we obtain the equilibrium price of the private firm

$$p^{LFF} = \frac{\alpha(4+4\delta-7\delta^2-\delta^3) + c(4+12\delta+5\delta^2-5\delta^3)}{2(4+8\delta-\delta^2-3\delta^3)}.$$

In this game, let SW^{LFF} and π^{LFF} be the equilibrium welfare and profit of the private firm, respectively. We obtain

$$SW^{LFF} = \frac{(\alpha-c)^2(36+88\delta+15\delta^2-43\delta^3)}{8\beta(1+2\delta)(4+8\delta-\delta^2-3\delta^3)}, \quad (8)$$

$$\pi^{LFF} = \frac{(\alpha-c)^2(1-\delta^2)(4+8\delta+\delta^2)^2}{4\beta(1+2\delta)(4+8\delta-\delta^2-3\delta^3)^2}. \quad (9)$$

Public firm as a Stackelberg follower: $(t_0, t_1, t_2) = (2, 1, 1)$

We discuss a Stackelberg model in which the public firm chooses its price in period 2 and all private firms do so in period 1. We solve the game backwards and start with the follower's problem. The public firm chooses its price after observing the leaders' actions. Its strategy is (4). The leader firms, that is, the private firms, choose their actions to maximize their own profits, $\pi_i(R_0(p_i,p_j),p_i,p_j)$ $(i \neq j;\ \ i,j=1,2)$. The first-order condition of firm i is given by

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{\partial R_0(p_i,p_j)}{\partial p_i} + \frac{\partial q_i}{\partial p_i}\right) + q_i \\ &= \frac{\alpha(1-\delta^2) + c(1+3\delta-\delta^2) - 2(1+2\delta)p_i + \delta(1+2\delta)p_j}{\beta(1-\delta^2)(1+2\delta)} = 0.\end{aligned}$$

We obtain the equilibrium price of the private firm:

$$p^{FLL} = \frac{\alpha(1-\delta^2) + c(1+3\delta-\delta^2)}{(2+\delta)(1+2\delta)}.$$

Substituting it into (4), we obtain the equilibrium price of the public firm:

$$p_0^{FLL} = \frac{2\delta(1-\delta)\alpha + (2+\delta)c}{(2+\delta)(1+2\delta)}.$$

In this game, let SW^{FLL} and π^{FLL} be the equilibrium welfare and profit of the private firm, respectively. We obtain

$$SW^{FLL} = \frac{(\alpha-c)^2(10+14\delta-19\delta^2+4\delta^3)}{2\beta(2+\delta)^2(1+2\delta)^2}, \tag{10}$$

$$\pi^{FLL} = \frac{(\alpha-c)^2(1-\delta^2)}{\beta(2+\delta)^2(1+2\delta)^2} \tag{11}$$

One private firm as a Stackelberg leader: $(t_0,t_1,t_2)=(2,1,2)$ **or** $(2,2,1)$

We discuss a Stackelberg model in which one private firm sets its price in period 1 and the others in period 2. The followers choose their price after observing the leader's action. From (4) and (5), we obtain

$$\begin{aligned}p_0(p_i) &= \frac{\alpha\delta(1-\delta) + c(2+\delta-\delta^2) + \delta(2+3\delta)p_i}{2+4\delta+\delta^2}, \\ p_j(p_i) &= \frac{\alpha(1-\delta^2) + c(1+3\delta) + \delta(1+2\delta)p_i}{2+4\delta+\delta^2} \quad (i \neq j;\ \ i,j=1,2).\end{aligned}$$

The leader firm, private firm i, chooses its price to maximize own profits, $\pi_i(p_0(p_i), p_i, p_j(p_i))$. The first-order condition of firm i is given by

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{\partial p_0(p_i)}{\partial p_i} + \frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial p_j(p_i)}{\partial p_i}\right) + q_i \\ &= \frac{\alpha(2+3\delta-2\delta^2-3\delta^3) + c(2+9\delta+6\delta^2+5\delta^3) - 4(1+3\delta+\delta^2-2\delta^3)p_i}{\beta(1-\delta)(1+2\delta)(2+4\delta+\delta^2)} \\ &= 0.\end{aligned}$$

We obtain the equilibrium price of the private leader:

$$p_l^{FLF} = \frac{\alpha(2+3\delta-2\delta^2-3\delta^3) + c(2+9\delta+6\delta^2+5\delta^3)}{4(1+3\delta+\delta^2-2\delta^3)}.$$

Substitute it into (12) and (12), we obtain

$$\begin{aligned}p_0^{FLF} &= \frac{\alpha\delta(8+20\delta-3\delta^2-24\delta^3-\delta^4) + c(8+32\delta+40\delta^2+15\delta^3-4\delta^4-7\delta^5)}{4(1+3\delta+\delta^2-2\delta^3)(2+4\delta+\delta^2)}, \\ p_f^{FLF} &= \frac{\alpha(4+6\delta-5\delta^2-6\delta^3+\delta^4) + c(4+18\delta+17\delta^2-6\delta^3-5\delta^4)}{4(1+\delta-\delta^2)(2+4\delta+\delta^2)}.\end{aligned}$$

In this game, let SW^{FLF}, π_l^{FLF}, and π_f^{FLF} be the equilibrium welfare, the private leader's profit, and private follower's profit, respectively. We obtain

$$SW^{FLF} = \frac{(\alpha-c)^2 H_1}{32\beta(1+2\delta)^2(1+\delta-\delta^2)^2(2+4\delta+\delta^2)^2}, \tag{12}$$

$$\pi_l^{FLF} = \frac{(\alpha-c)^2(1-\delta)(2+5\delta+3\delta^2)^2}{8\beta(1+2\delta)^2(1+\delta-\delta^2)(2+4\delta+\delta^2)}, \tag{13}$$

$$\pi_f^{FLF} = \frac{(\alpha-c)^2(1-\delta)(1+\delta)^3(4+6\delta-\delta^2)^2}{16\beta(1+2\delta)^2(1+\delta-\delta^2)^2(2+4\delta+\delta^2)^2} \tag{14}$$

where $H_1 = 160 + 1344\delta + 4272\delta^2 + 5836\delta^3 + 1547\delta^4 - 4004\delta^5 - 3102\delta^6 + 268\delta^7 + 627\delta^8 + 108\delta^9$.

One private firm as a Stackelberg follower : $(t_0, t_1, t_2) = (1, 1, 2)$ **or** $(1, 2, 1)$

We discuss a Stackelberg model in which both the public firm and one private firm set their price in period 1 and the other private does so in period 2. The follower, private firm j ($j \neq i$; $i, j = 1, 2$), chooses its price after observing the leaders' actions. The follower's strategy is given by

(5). The leader firms, the public and private firm i, choose their prices to maximize their payoff, $SW(p_0, p_i, R_j(p_0, p_i))$ and $\pi_i(p_0, p_i, R_j(p_0, p_i))$, respectively. The first-order conditions are given by

$$\begin{aligned}
\frac{\partial SW}{\partial p_0} &= \sum_{k=0}^{2}(p_k - c)\left(\frac{\partial q_k}{\partial p_0} + \frac{\partial q_k}{\partial p_j}\frac{\partial R_j(p_0,p_i)}{\partial p_0}\right) \\
&= \frac{\alpha\delta(1-\delta) + c(4+3\delta-5\delta^2) - (4+8\delta+\delta^2)p_0 + \delta(4+7\delta)p_i}{4\beta(1-\delta)(1+\delta)(1+2\delta)} = 0 \\
\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial R_j(p_0,p_i)}{\partial p_i}\right) + q_i \\
&= \frac{\alpha(1-\delta)(2+3\delta) + c(2+5\delta+2\delta^2) - 2(2+4\delta+\delta^2)p_i + \delta(2+3\delta)p_0}{2\beta(1-\delta)(1+\delta)(1+2\delta)} \\
&= 0.
\end{aligned}$$

Solving the system of equations, we obtain the equilibrium prices of the public firm and the private leader:

$$\begin{aligned}
p_0^{LLF} &= \frac{\alpha\delta(12+22\delta-11\delta^2-23\delta^3) + c(16+52\delta+46\delta^2+9\delta^3+4\delta^4)}{16+64\delta-68\delta^2-2\delta^3-19\delta^4} \\
p_l^{LLF} &= \frac{2\alpha(4+10\delta-11\delta^3-3\delta^4) + c(8+44\delta+68\delta^2+20\delta^3-13\delta^4)}{16+64\delta-68\delta^2-2\delta^3-19\delta^4}.
\end{aligned}$$

Substituting it into (5), we obtain the equilibrium price of the private follower:

$$p_f^{LLF} = \frac{\alpha(8+28\delta+18\delta^2-24\delta^3-25\delta^4-5\delta^5) + 2c(4+26\delta+57\delta^2+45\delta^3+2\delta^4-7\delta^5)}{(1+\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)}$$

In this game, let SW^{LLF}, π_l^{LLF}, and π_f^{LLF} be the equilibrium welfare, private leader's profit, and private follower's profit, respectively. We obtain

$$\begin{aligned}
SW^{LLF} &= \frac{(\alpha-c)^2 H_2}{2\beta(1+\delta)(1+2\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)^2}, && (15) \\
\pi_l^{LLF} &= \frac{2(\alpha-c)^2(1-\delta)(2+3\delta)^2(2+4\delta+\delta^2)^3}{\beta(1+\delta)(1+2\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)^2}, && (16) \\
\pi_f^{LLF} &= \frac{(\alpha-c)^2(1-\delta)(8+36\delta+54\delta^2+30\delta^3+5\delta^4)^2}{2\beta(1+\delta)(1+2\delta)(16+64\delta-68\delta^2-2\delta^3-19\delta^4)} && (17)
\end{aligned}$$

where $H_2 = 640+6016\delta+22896\delta^2+43920\delta^3+40672\delta^4+7508\delta^5-16109\delta^6-10161\delta^7+285\delta^8+1107\delta^9$.

Appendix B

Proof of Lemma 1

From (6), (8), (10), (12), and (15), we can directly compare the equilibrium welfare. We obtain

$$\begin{aligned}
SW^{LFF} - SW^{N} &= \frac{2(\alpha - c)^2(1-\delta)^3\delta^2\,(1+2\delta)}{\beta(-3\delta^3 - \delta^2 + 8\delta + 4)\,(\delta^2 - 3\delta - 2)^2} > 0, \\
SW^{N} - SW^{FLL} &= \frac{(\alpha - c)^2(1-\delta)^2\delta^2\left(-3\delta^3 + 3\delta^2 + 10\delta + 4\right)}{\beta(2-\delta)^2(2\delta+1)^2\,(\delta^2 - 3\delta - 2)^2} > 0, \\
SW^{LLF} - SW^{FLF} &= \frac{(\alpha - c)^2(1-\delta)\delta^2 H_3}{32\beta(1+\delta)(2\delta+1)^2\,(\delta^2 - \delta - 1)^2\,(\delta^2 + 4\delta + 2)^2\,(H_4)^2} > 0
\end{aligned}$$

where

$H_3 = 3564\delta^{15} + 37731\delta^{14} + 191374\delta^{13} + 631828\delta^{12} + 911470\delta^{11} - 845859\delta^{10} - 51358848\delta^{9} - 7489972\delta^{8} - 35134086\delta^{7} + 3560208\delta^{6} + 6991488\delta^{5} + 5439936\delta^{4} + 2464512\delta^{3} + 677632\delta^{2} + 105472\delta + 7168,$

$H_4 = -19\delta^4 - 2\delta^3 + 68\delta^2 + 64\delta + 16.$

Appendix C

Proof of Lemma 3

From (7), (9), (16), and (17), we directly compare the equilibrium profit. We obtain

$$\begin{aligned}
&\pi^{N} - \pi_f^{LLF} \\
&= \frac{(\alpha - c)^2(1-\delta)\delta^2(3\delta+2)\left(-7\delta^3 - 9\delta^2 + 4\delta + 4\right)\left(-8\delta^5 - 13\delta^4 + 39\delta^3 + 102\delta^2 + 72\delta + 16\right)}{\beta(1+\delta)\,(\delta^2 - 3\delta - 2)^2\,(19\delta^4 + 2\delta^3 - 68\delta^2 - 64\delta - 16)^2}.
\end{aligned}$$

$\pi^{N} - \pi_f^{LLF}$ is positive if and only if $(-7\delta^3 - 9\delta^2 + 4\delta + 4)$ is positive and

$$-7\delta^3 - 9\delta^2 + 4\delta + 4 > (<)\;\; 0 \text{ if } \delta < (>)\delta^* \simeq 0.699417.$$

We also have

$$
\begin{aligned}
& \pi_l^{LLF} - \pi^{LFF} \\
= & \frac{(\alpha-c)^2(1-\delta)\delta^2 H_5}{\beta(\delta+1)(2\delta+1)\left(3\delta^3+\delta^2-8\delta-4\right)^2\left(19\delta^4+2\delta^3-68\delta^2-64\delta-16\right)^2} > 0,
\end{aligned}
$$

$$
\begin{aligned}
& \pi_f^{LLF} - \pi_l^{LLF} \\
= & \frac{(\alpha-c)^2(1-\delta)\delta^3(7\delta+4)\left(\delta^2+4\delta+2\right)^2}{\beta(\delta+1)(2\delta+1)\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2} > 0
\end{aligned}
$$

where $H_5 = -199\delta^{12} + 3636\delta^{11} + 17056\delta^{10} + 13344\delta^9 - 45349\delta^8 - 105024\delta^7 - 65832\delta^6 + 42128\delta^5 + 98064\delta^4 + 72704\delta^3 + 28416\delta^2 + 5888\delta + 512$.

Appendix D

Public firm as a Stackelberg leader a hierarchical Stackelberg game: $(t_0, t_1, t_2) = (1, 2, 3)$ **or** $(1, 3, 2)$

First, we discuss a Stackelberg model in which the public firm chooses its price in period 1, one private firm in period 2, and the other private firm in period 3. That is, the public firm is a leader, one private firm is the intermediate, and the other private firm is the follower. The follower firm, firm j ($j \neq i$; $i, j = 1, 2$), choose its action after observing the leader's and intermediate's actions. Thus, its strategy is (5). Anticipating the follower's action, the private firm i as an intermediate chooses its action to maximize its own profit, $\pi_i(p_0, p_i, R_j(p_0, p_i))$. The first-order condition of the private intermediate is given by

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_i} &= (p_i - c)\left(\frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial R_j(p_0, p_i)}{\partial p_i}\right) + q_i \\
&= \frac{c\left(2\delta^2+5\delta+2\right) + (3\delta+2)(\alpha(1-\delta)+\delta p_0) - 2\left(\delta^2+4\delta+2\right)p_i}{2\beta(1-\delta)(\delta+1)(2\delta+1)} = 0.
\end{aligned}
$$

We obtain the optimal strategy of the intermediate:

$$
R_i(p_0) = \frac{(3\delta+2)(\alpha(1-\delta)+\delta p_0) + c\left(2\delta^2+5\delta+2\right)}{2\left(\delta^2+4\delta+2\right)}.
$$

The public firm chooses its action to maximize social welfare, $SW\left(p_0, R_i(p_0), R_j\left(p_0, R_i\left(p_0\right)\right)\right)$, anticipating their actions. The first-order condition of the public firm is given by

$$\frac{\partial SW}{\partial p_0} = \sum_{k=0}^{2}(p_k - c)\left(\frac{\partial q_k}{\partial p_0} + \frac{\partial q_k}{\partial p_i}\frac{\partial R_i(p_0)}{\partial p_0} + \frac{\partial q_k}{\partial p_j}\left(\frac{dR_j\left(p_0, R_i\left(p_0\right)\right)}{dp_0}\right)\right) = 0.$$

We obtain the equilibrium price of the public firm:

$$p_0^{LMF} = \frac{H_6}{71\delta^6 + 292\delta^5 + 128\delta^4 - 576\delta^3 - 800\delta^2 - 384\delta - 64}$$

where $H_6 = \alpha\delta(43\delta^5 + 153\delta^4 + 84\delta^3 - 120\delta^2 - 128\delta - 32) + c(28\delta^6 + 139\delta^5 + 44\delta^4 - 456\delta^3 - 672\delta^2 - 352\delta - 64)$.

Substitute it into $R_i\left(p_0\right)$ and $R_j\left(p_0, R_i\left(p_0\right)\right)$, we obtain the equilibrium prices of the private intermediate and follower: $p_m^{LMF} = R_i\left(p_0^{LMF}\right)$ and $p_f^{LMF} = R_j\left(p_0^{LMF}, R_i\left(p_0^{LMF}\right)\right)$. In this game, let SW^{LMF}, π_m^{LMF}, and π_f^{LMF} be the equilibrium welfare, the profit of the private intermediate, and the profit of the private follower, respectively. We obtain

$$\begin{aligned} & SW^{LMF} \\ = \; & \frac{(\alpha - c)^2\left(333\delta^7 + 1347\delta^6 + 1177\delta^5 - 1827\delta^4 - 4324\delta^3 - 3360\delta^2 - 1184\delta - 160\right)}{2\beta(\delta+1)(2\delta+1)\left(71\delta^6 + 292\delta^5 + 128\delta^4 - 576\delta^3 - 800\delta^2 - 384\delta - 64\right)}, \quad (18) \\ & \pi_m^{LMF} \\ = \; & \frac{2(\alpha - c)^2\left(-\delta^3 - 3\delta^2 + 2\delta + 2\right)\left(-21\delta^5 - 2\delta^4 + 116\delta^3 + 168\delta^2 + 88\delta + 16\right)^2}{\beta(\delta+1)(2\delta+1)\left(-71\delta^6 - 292\delta^5 - 128\delta^4 + 576\delta^3 + 800\delta^2 + 384\delta + 64\right)^2}, \quad (19) \\ & \pi_f^{LMF} \\ = \; & \frac{(\alpha - c)^2(1-\delta)\left(-35\delta^6 - 50\delta^5 + 192\delta^4 + 536\delta^3 + 504\delta^2 + 208\delta + 32\right)^2}{\beta(\delta+1)(2\delta+1)\left(-71\delta^6 - 292\delta^5 - 128\delta^4 + 576\delta^3 + 800\delta^2 + 384\delta + 64\right)^2}. \quad (20) \end{aligned}$$

Public firm as an intermediate a hierarchical Stackelberg game: $(t_0, t_1, t_2) = (2, 1, 3)$ **or** $(2, 3, 1)$

We discuss a Stackelberg model in which the public firm chooses its price in period 2, one private firms does so in period 1, and the other private firm in period 3. That is, the public firm is an intermediate, one private firm is the leader, and the other private fir is the follower. The follower firm,

firm j ($j \neq i$; $i, j = 1, 2$), choose its action after observing the leader's and intermediate's actions. Thus, its strategy is (5). Anticipating the follower's action, the public firm, as an intermediate, chooses its action to maximize social welfare, $SW(p_0, p_i, R_j(p_0, p_i))$. The first-order condition of the public intermediate is given by

$$\frac{\partial SW}{\partial p_0} = \sum_{k=0}^{2}(p_k - c)\left(\frac{\partial q_k}{\partial p_0} + \frac{\partial q_k}{\partial p_j}\frac{\partial R_j(p_0)}{\partial p_0}\right) = 0$$

We obtain the optimal strategy of the public intermediate:

$$R_0(p_i) = \frac{c\left(-5\delta^2 + 3\delta + 4\right) + \delta(\alpha(1-\delta) + (7\delta+4)p_i)}{\delta^2 + 8\delta + 4}.$$

The private leader chooses its action to maximize its own profit, $\pi_i\left(R_0(p_i), p_i, R_j\left(R_0\left(p_i\right), p_i\right)\right)$, anticipating others' actions. The first-order condition of the private firm is given by

$$\frac{\partial \pi_i}{\partial p_i} = (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{\partial R_0(p_i)}{\partial p_i} + \frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{dR_j\left(R_0\left(p_i\right), p_i\right)}{dp_i}\right) + q_i = 0.$$

We obtain the equilibrium price of the private leader:

$$p_l^{MLF} = \frac{\alpha\left(3\delta^4 + 11\delta^3 - 10\delta - 4\right) + c\left(17\delta^4 + 3\delta^3 - 30\delta^2 - 22\delta - 4\right)}{2(\delta+1)(2\delta+1)\left(5\delta^2 - 4\delta - 4\right)}$$

Substitute it into $R_0\left(p_i\right)$ and $R_j\left(R_i\left(p_i\right), p_i\right)$, we obtain the equilibrium prices of the public intermediate and private follower: $p_0^{MLF} = R_0\left(p_l^{MLF}\right)$ and $p_f^{MLF} = R_j\left(R_0\left(p_l^{MLF}\right), p_l^{MLF}\right)$. In this game, let SW^{MLF}, π_l^{MLF}, and π_f^{MLF} be the equilibrium welfare, the profit of the private leader, and of private follower, respectively. We obtain

$$SW^{MLF} = \frac{(\alpha-c)^2 H_7}{4\beta(\delta+1)^2(2\delta+1)^2\left(-5\delta^2+4\delta+4\right)^2\left(\delta^2+8\delta+4\right)}, \tag{21}$$

$$\pi_l^{MLF} = \frac{(\alpha-c)^2(1-\delta)\left(3\delta^3+14\delta^2+14\delta+4\right)^2}{4\beta(\delta+1)^2(2\delta+1)^2\left(\delta^2+8\delta+4\right)\left(-5\delta^2+4\delta+4\right)}, \tag{22}$$

$$\pi_f^{MLF} = \frac{4(\alpha-c)^2(1-\delta)\left(\delta^4+\delta^3-12\delta^2-14\delta-4\right)^2}{\beta(\delta+1)(2\delta+1)\left(-5\delta^2+4\delta+4\right)^2\left(\delta^2+8\delta+4\right)^2}. \tag{23}$$

where $H_7 = 333\delta^9 + 2925\delta^8 + 1996\delta^7 - 8324\delta^6 - 11888\delta^5 + 806\delta^4 + 11080\delta^3 + 8488\delta^2 + 2688\delta + 320$.

Public firm as follower in a hierarchical Stackelberg game: $(t_0, t_1, t_2) = (3, 1, 2)$ **or** $(3, 2, 1)$

We discuss a Stackelberg model in which the public firm chooses its price in period 3, one private firms does so at period 1, and the other private firm in period 2. That is, the public firm is a follower, one private firm is the leader, and the other private firm is the intermediate. The public firm choose its action after observing the leader's and intermediate's actions. Thus, its strategy is (4). Anticipating the follower's action, the private firm j ($j \neq i; \quad i, j = 1, 2$) as an intermediate chooses its action to maximize own profit, $\pi_j(R_0(p_i, p_j), p_i, p_j)$. The first-order condition of the private intermediate is given by

$$\frac{\partial \pi_j}{\partial p_j} = (p_j - c)\left(\frac{\partial q_j}{\partial p_j} + \frac{\partial q_j}{\partial p_0}\frac{\partial R_0(p_i, p_j)}{\partial p_j}\right) + q_j = 0$$

We obtain the optimal strategy of the private intermediate:

$$R_j(p_i) = \frac{(1-\delta^2)\alpha - (\delta^2 - 3\delta - 1)c + (2\delta^2 + \delta)p_i}{4\delta + 2}.$$

The private leader chooses its action to maximize own profit, $\pi_i\left(R_0\left(p_i, R_j\left(p_i\right)\right), p_i, R_j(p_i)\right)$, anticipating their actions. The first-order condition of the private firm is given by

$$\frac{\partial \pi_i}{\partial p_i} = (p_i - c)\left(\frac{\partial q_i}{\partial p_0}\frac{dR_0\left(p_i, R_j\left(p_i\right)\right)}{dp_i} + \frac{\partial q_i}{\partial p_i} + \frac{\partial q_i}{\partial p_j}\frac{\partial R_j(p_i)}{\partial p_i}\right) + q_i = 0.$$

We obtain the equilibrium price of the private leader:

$$p_l^{FLM} = \frac{\alpha\left(-\delta^3 - 2\delta^2 + \delta + 2\right) + c\left(-3\delta^3 + 7\delta + 2\right)}{2(2\delta + 1)\left(2 - \delta^2\right)}.$$

Substitute it into $R_0\left(p_i, R_j\left(p_i\right)\right)$ and $R_j(p_i)$, we obtain the equilibrium prices of the private follower and private intermediate: $p_0^{FLM} = R_0\left(p_l^{FLM}, R_j\left(p_l^{FLM}\right)\right)$ and $p_f^{FLM} = R_j(p_l^{FLM})$. In this game, let SW^{FLM}, π_l^{FLM}, and π_m^{FLM} be the equilibrium welfare, the profit of the private leader, and the profit of the private intermediate, respectively. We obtain

$$SW^{FLM} = \frac{(\alpha - c)^2 \left(5\delta^6 + 92\delta^5 + 11\delta^4 - 380\delta^3 - 128\delta^2 + 384\delta + 160\right)}{32\beta(2\delta + 1)^2 \left(\delta^2 - 2\right)^2}, \tag{24}$$

$$\pi_l^{FLM} = \frac{(1-\delta)(\delta+1)(\delta+2)^2(\alpha - c)^2}{8\beta(2\delta+1)^2 \left(2 - \delta^2\right)}, \tag{25}$$

$$\pi_m^{FLM} = \frac{(1-\delta)(\delta+1)\left(\delta^2 - 2\delta - 4\right)^2 (\alpha - c)^2}{16\beta(2\delta+1)^2 \left(\delta^2 - 2\right)^2}. \tag{26}$$

Appendix E

From (8), (15), (18), (21), and (24), we can compare the equilibrium welfare. We obtain

$$\begin{aligned}
& SW^{LLF} - SW^{FLM} \\
= \ & \frac{(\alpha - c)^2 \delta^2 H_8}{32\beta(\delta+1)(2\delta+1)^2 \left(\delta^2 - 2\right)^2 \left(-19\delta^4 - 2\delta^3 + 68\delta^2 + 64\delta + 16\right)^2} > 0, \\
& SW^{LMF} - SW^{LLF} \\
= \ & \frac{2(\alpha - c)^2 \delta^2 \left(2\delta^3 + \delta^2 - 2\delta - 1\right)\left(-51\delta^4 - 52\delta^3 + 36\delta^2 + 56\delta + 16\right)^2}{\beta \left(-19\delta^4 - 2\delta^3 + 68\delta^2 + 64\delta + 16\right)^2 \left(71\delta^6 + 292\delta^5 + 128\delta^4 - 576\delta^3 - 800\delta^2 - 384\delta - 64\right)} > 0, \\
& SW^{LLF} - SW^{MLF} \\
= \ & \frac{(\alpha - c)^2 \delta^2 (3\delta+2)^2 H_9}{4\beta(\delta+1)^2(2\delta+1)^2 \left(-5\delta^2 + 4\delta + 4\right)^2 \left(\delta^2 + 8\delta + 4\right)\left(-19\delta^4 - 2\delta^3 + 68\delta^2 + 64\delta + 16\right)^2} > 0,
\end{aligned}$$

where

$H_8 = -1805\delta^{13} + 27\delta^{12} - 4823\delta^{11} - 73127\delta^{10} - 64732\delta^9 + 271340\delta^8 + 460240\delta^7 - 89584\delta^6 - 627520\delta^5 - 346944\delta^4 + 152576\delta^3 + 227072\delta^2 + 86016\delta + 11264$,

$H_9 = -1057\delta^{13} - 18391\delta^{12} - 68012\delta^{11} - 13540\delta^{10} + 251952\delta^9 + 319880\delta^8 - 114184\delta^7 - 473928\delta^6 - 274112\delta^5 + 90208\delta^4 + 183424\delta^3 + 93696\delta^2 + 22016\delta + 2048$.

Appendix F

Proof of Lemma 6

From (16), (19), and (20), we directly compare the equilibrium profit. We obtain

$$\pi_l^{LLF} - \pi_f^{LMF} > (<) \ 0 \text{ if } \delta < (>) \delta^{**} \simeq 0.968794.$$

The exact difference between the resulting profits is too complex. The inequality and the threshold value are obtained by Mathematica (available upon request). We also have

$$\begin{aligned} & \pi_l^{LLF} - \pi_m^{LMF} \\ = & \frac{8(\alpha - c)^2(1-\delta)\delta^2(3\delta+2)^3\left(\delta^2+4\delta+2\right)H_{10}}{\beta\left(-19\delta^4-2\delta^3+68\delta^2+64\delta+16\right)^2\left(-71\delta^6-292\delta^5-128\delta^4+576\delta^3+800\delta^2+384\delta+64\right)^2} \\ > & \ 0 \end{aligned}$$

where $H_{10} = -527\delta^{11} + 5237\delta^{10} + 24561\delta^9 + 14658\delta^8 - 63676\delta^7 - 115848\delta^6 - 45632\delta^5 + 58496\delta^4 + 79680\delta^3 + 41088\delta^2 + 10240\delta + 1024$.

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