A note on re-switching and the neo-Austrian concept of the average period of production

Saverio M. Fratini

Roma Tre University

June 2018
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Saverio M. Fratini*

Università di Roma Tre

There are certain unsettled questions in economic theory that have been handed down as a sort of legacy from one generation to another. ... Not unfrequently the discussion is carried far beyond the limits of weariness and satiety, so that it may well be regarded as an offence against good taste to again recur to so well-worn a theme. And yet these questions return again and again, like troubled spirits doomed restlessly to wander until the hour of their deliverance shall appear. (Böhm-Bawerk 1884, p. 149)

Abstract

The neo-Austrian average period of production is calculated by taking the shares of costs referable to each period out of the total amount of costs as weights. Once this notion had been introduced, its inverse relationship with the rate of interest prompted some scholars to believe that it could serve as a good measure of capital intensity.

As will be shown, however, this new average period poses some problems. On the one hand, the inverse relationship mentioned above does not preclude the re-switching of production methods. On the other, if re-switching occurs, the most roundabout method may paradoxically be the one that gives the smallest net output per worker.

This result can affect the revival of the Austrian business-cycle theory.

Keywords: average period of production; degree of roundaboutness; capital; re-switching; Austrian business-cycle theory

JEL codes: B25; B53; D24; D33, E32

1. Introduction

The marginalist theory of distribution was initially grounded on the idea that distribution variables are the prices that firms pay for use of the factors of production: labour, land and capital. In particular, the rate of interest was understood as the price firms have to pay on the amount of capital employed. Accordingly, because of factor substitutability, a decrease in the rate of interest was thought, ceteris paribus, to bring about the adoption of more capital-intensive methods of production.

* Thanks are due to C. Gehrke, G.C. Harcourt and H.D. Kurz for comments and suggestions. The usual caveat applies.
On this reasoning, the employment of capital – both in the production of a certain commodity and in the economy as a whole – was often regarded as measurable by the average length of the production process: more roundabout processes would require more capital but would also be more productive.¹

This vision collapsed in the 1960s as a result of the Cambridge capital controversy. As Samuelson wrote:

the simple tale told by Jevons, Böhm-Bawerk, Wicksell, and other neoclassical writers – alleging that, as the interest rate falls in consequence of abstention from present consumption in favor of future, technology must become in some sense more ‘roundabout,’ more ‘mechanized,’ and ‘more productive’ – cannot be universally valid. (Samuelson 1966, p. 568.)

In particular, it became clear that the rate of interest cannot be understood as the price paid for the use of capital. As explicitly stated by Bliss in his important book on capital theory:

The value which accrues from a sale is the product of price and quantity sold. Hence if the rate of interest is the price of capital, the quantity of capital must be the wealth on which an interest yield is calculated. It will be shown shortly why this view is incorrect, but to cut a long story short, the conclusion may be announced at once. The rate of interest is not the price of capital. (Bliss 1975, pp. 6-7.)

Despite these clear statements, however, the temptation to regard capital as a factor of production and the rate of interest as the price for its use resurfaces periodically. This may be due to the fact that applied economists have never stopped regarding income distribution as regulated by ‘factor productivity’ and theoretical economists are thus tempted to indulge in this vision. In particular, the propositions that tend to reappear concern a) the equality between the rate of interest and the ‘marginal product of capital’,² and b) the inverse relationship between the rate of interest and the ‘degree of roundaboutness’ of the production processes in use. Attention will be focused here on the latter.

¹ In Böhm-Bawerk’s words:
The adoption of capitalist methods of production is followed by two consequences, equally characteristic and significant. One is an advantage, the other a disadvantage. The advantage [...] consists in the greater technical productiveness of those methods. With an equal expenditure of the two originary productive forces (that is to say, labour and valuable natural powers) more or better goods can be produced by a wisely chosen capitalist process than could be by direct unassisted production. [...] The disadvantage connected with the capitalist method of production is its sacrifice of time. The roundabout ways of capital are fruitful but long: they procure us more or better consumption goods, but only at a later period of time. (Böhm-Bawerk 1891, p. 82.)

² While this is not the goal of the present paper, a couple of brief remarks can be made on this point. First, since capital is not a factor of production, it cannot have a real marginal product. Second, since the rate of interest is not the price for the use of capital, its equality with an alleged ‘marginal product of capital’ is essentially meaningless.
The ‘degree of roundaboutness’ in question is the neo-Austrian average period of production, first introduced\(^3\) by Hicks in *Value and Capital* (1946 [1939]) and further developed in Sargan (1955), von Weizsäcker (1971 and 1977) and Malinvaud (1986 and 2003).\(^4\) This new concept of the average period is grounded on the idea of using the shares of costs anticipated at each date out of the total cost of production as weights instead of the shares of labour performed in each period out of the total labour embodied. Once this notion has been adopted, if a fall in the rate of interest brings about a change in the method of production of a certain commodity, then the incoming method is more ‘roundabout’ than the outgoing.

Cachanosky and Lewin (2014) have shown more recently that this conception of the degree of roundaboutness corresponds to the notion of ‘duration’ used for the analysis of financial investment projects. They also claim that the adoption of this new average period of production ‘adds plausibility’ (p. 661) to the Austrian business-cycle theory,\(^5\) which they describe as follows:

a monetary policy that reduces interest rates, increases the ‘average period of production’, or the degree of ‘roundaboutness,’ of the ‘structure of production,’ that is out of sync with consumer preferences, thus creating unsustainable imbalances in that structure. The increase in ‘roundaboutness,’ followed by its reduction when the monetary authority revises interest rates upward, is what constitutes the boom and bust in this business-cycle theory. (Cachanosky and Lewin, 2014, p. 648.)

This paper seeks to ascertain whether the new neo-Austrian concept of the average period of production can actually prove useful in arguments of this kind. To this end, it starts from the traditional concept of the average period of production under the assumption of simple interest (section 2) and goes on to introduce compound interest and the new concept of the average period (section 3). The effect of re-switching on the results obtained by this new idea of roundaboutness is then addressed in section 4 and some conclusions concerning the working of the Austrian business-cycle theory are drawn in section 5.

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\(^3\) See Lewin and Cachanosky (2018a) for a reconstruction of the history of the concept of the average period of production.

\(^4\) For some critical assessments of these contributions, see Hagemann and Kurz (1976), Orosel (1979 and 1990), Howard (1980) and Fratini (2013 and 2014).

\(^5\) Similar claims can also be found in Cachanosky and Lewin (2016) and Braun (2017).
2. The old concept of the average period of production

An example will help to clarify the traditional interpretation of the average period of production as a measure of the ‘capital intensity’ of different methods of production. Let us assume that a final commodity such as corn can be obtained by two different and alternative methods of production: method A and method B.

With method A, one unit of corn delivered at the end of period $t$ can be obtained as a result of the employment of $a_3$ units of labour during period $t-2$, $a_2$ units during $t-1$, and $a_1$ during $t$. Similarly, $b_3$, $b_2$ and $b_1$ represent the employment of labour in the three periods in order to produce one unit of corn by method B. A typical way to represent these processes is shown in fig. 1.\(^6\)

Fig. 1. Employment of labour per period. Graphical representation.

\(^6\)All the figures presented in the present paper are based on the following numerical example. Method A: $a_1 = 2$; $a_2 = 7$; $a_3 = 1$. Method B: $b_1 = 8$; $b_2 = 2$; $b_3 = 2$. Readers can use these coefficients to obtain the results discussed here. Similar examples can be found, for instance, in Samuelson (1966, p. 569) and Harcourt (1972, pp. 151–52).
The amount of labour embodied in one unit of corn with method A is $a_1 + a_2 + a_3$. Similarly, $b_1 + b_2 + b_3$ is the amount of labour embodied in one unit of corn with method B. It is assumed in the example in fig. 1 that $a_1 + a_2 + a_3 < b_1 + b_2 + b_3$.

As regards the average period of production, this is regarded according to the traditional conception as the weighted average of its length, in which the weights are the shares of labour employed in each period. For methods A and B, we therefore have:

$$T_a = \frac{a_1 + 2a_2 + 3a_3}{a_1 + a_2 + a_3}$$

(1)

$$T_b = \frac{b_1 + 2b_2 + 3b_3}{b_1 + b_2 + b_3}.$$  

(2)

With reference to fig. 1, the average period of each method is equal to the total area of the rectangles divided by amount of labour embodied. Hence, $T_a > T_b$.

Let us now assume the following: A.1. wages are paid at the beginning of each period; A.2. value is measured in terms of labour commanded and, hence, the wage rate is set equal to 1; A.3. interest is determined by means of the simple interest formula at the rate $r$. On these assumptions, the following propositions hold.

**Proposition 1.** For a given method, the amount of interest paid per unit of labour is proportional to the average period of production.

**Proposition 2.** When a rise (or fall) in the rate of interest entails a change of the method in use, the incoming method has an average period of production shorter (or longer) than the outgoing one.

These propositions can be easily proved. Let us denote by $p_a$ and $p_b$ the unit costs of production (in terms of labour commanded) of one unit of corn with the two methods. Under the assumptions A.1–A.3, we have:

$$p_a = a_1 (1 + r) + a_2 (1 + 2r) + a_3 (1 + 3r) = (a_1 + a_2 + a_3) (1 + T_a r)$$

(3)

$$p_b = b_1 (1 + r) + b_2 (1 + 2r) + b_3 (1 + 3r) = (b_1 + b_2 + b_3) (1 + T_b r).$$

(4)

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As is known, 'labour commanded' is the measure of value adopted by Adam Smith. The value of a commodity in terms of labour commanded is the amount of labour that can be purchased with one unit of this commodity.
The RHS of equations (3) and (4) immediately proves proposition 1. As for proposition 2, since \( a_1 + a_2 + a_3 < b_1 + b_2 + b_3 \), if the rate of interest is zero,\(^8\) then the unit cost of corn with method A is certainly lower than with method B, namely \( p_a < p_b \). As the rate of interest increases, however, \( T_a > T_b \) implies that \( p_a \) grows faster than \( p_b \). Therefore, there is an interest rate level \( r^* \) such that the two methods are equally convenient, i.e. \( p_a = p_b \). Then, for interest rate levels above \( r^* \), the use of method B becomes more convenient than method A, i.e. \( p_a > p_b \). All this is graphically represented in fig. 2.

\[ \text{Fig. 2. Unit cost functions with simple interest.} \]

For interest rate levels below \( r^* \), method A is cost minimizing and, then, it is the method in use. As the rate of interest increases and crosses level \( r^* \), there is a change in the method in use: method A is discontinued and method B comes into operation. As a result, there is a decrease in the average period of production, since \( T_a > T_b \).

\(^8\) Since we are studying price-taker firms’ decisions, the rate of interest can be considered as an exogenous variable and we can give it arbitrary values.
This result is, however, bound up with the adoption of the simple interest formula (assumption A.3). As we shall see in the following sections, adoption of the compound interest formula instead leads to different conclusions.

3. The new concept of the average period of production

If the compound interest formula is adopted, as it must be, equations (3) and (4) no longer express the unit costs of production of corn with the two methods. These unit costs must be redefined as follows:

\[ p_a = a_1 (1 + r) + a_2 (1 + r)^2 + a_3 (1 + r)^3 = (1 + r) [a_1 + a_2 (1 + r) + a_3 (1 + r)^2] \]  
\[ p_b = b_1 (1 + r) + b_2 (1 + r)^2 + b_3 (1 + r)^3 = (1 + r) [b_1 + b_2 (1 + r) + b_3 (1 + r)^2]. \]

Once equations (5) and (6) have been introduced, it is clear that the interest burden with each method is no longer proportional to the corresponding average period of production. In other words, given a level of the interest rate, method A (the one with the longest average period) is not necessarily the one that entails the greatest interest charge. As a result, the average periods \( T_a \) and \( T_b \) do not make it possible to predict how the relative costs of the two methods are affected by a rise in the rate of interest.

In order to restore the connection between interest cost and average period of production, Hicks and the other scholars mentioned in section 1 proposed a new formulation of the latter. This new average period of production is still the weighted average of the length of the process, but the weights are now the shares of costs referable to each period out of the total cost of production. In more precise terms:

\[ \Theta_a = \frac{a_1 (1+r)+2a_2(1+r)^2+3a_3(1+r)^3}{a_1(1+r)+a_2(1+r)^2+a_3(1+r)^3} \]
\[ \Theta_b = \frac{b_1 (1+r)+2b_2(1+r)^2+3b_3(1+r)^3}{b_1(1+r)+b_2(1+r)^2+b_3(1+r)^3} \]

Once this new definition has been adopted, the connection between the average period of production and the rate of interest is twofold. On the one hand, \( \Theta_a \) and \( \Theta_b \) depend on the rate of interest; on the other, they express the elasticity of the unit costs \( p_a \) and \( p_b \) with respect to the interest factor. In fact, on positing \( R \equiv (1 + r) \), because of equations (5) and (6), we have:
\[
\frac{dp_a}{dR} = \frac{a_1 + 2a_2 R + 3a_3 R^2}{a_1 + a_2 R + a_3 R^2} = \Theta_a \tag{9}
\]

\[
\frac{dp_b}{dR} = \frac{b_1 + 2b_2 R + 3b_3 R^2}{b_1 + b_2 R + b_3 R^2} = \Theta_b. \tag{10}
\]

The relative cost of the method with the longest average period of production therefore rises as the rate of interest increases.9

As pointed out by various scholars,10 this result has an interesting corollary.

**Proposition 3.** Let \( R^* \) be an interest factor such that \( p_a(R^*) - p_b(R^*) = 0 \). In a small neighbourhood of \( R^* \), if \( dp_a/dR - dp_b/dR \neq 0 \), then a rise in the rate of interest entails a change of the method in use and the incoming method has an average period of production shorter than the outgoing one.

**Proof.** Since the two methods entail the same unit cost at \( R^* \) and \( dp_a/dR - dp_b/dR \neq 0 \), then \( R^* \) is a switching point. In a small neighbourhood of \( R^* \), the method in use depends on the sign of the difference \( dp_a/dR - dp_b/dR \). In particular, there are two possible cases: i) if \( dp_a/dR - dp_b/dR > 0 \), then method A is adopted for interest factor levels just below \( R^* \) and method B for levels just above \( R^* \); ii) if \( dp_a/dR - dp_b/dR < 0 \), then method B is adopted for interest factor levels just below \( R^* \) and method A for levels just above \( R^* \). Now, since \( p_a(R^*) = p_b(R^*) \), equations (9) and (10) imply that \( \text{sign}[dp_a/dR - dp_b/dR] = \text{sign}[\Theta_a(R^*) - \Theta_b(R^*)] \). Therefore, in a small neighbourhood of \( R^* \), an increase in the rate of interest brings about the use of the method with the shortest average period of production. 

The argument can also be presented graphically, as in fig. 3. In a neighbourhood of the interest factor \( R^* \), \( \Theta_a > \Theta_b \) implies that the difference \( p_a - p_b \) rises as the interest factor increases. Therefore, the rise of the interest factor leads to the abandonment of method A and the use of method B. Similarly, \( \Theta_a < \Theta_b \) means that the difference \( p_a - p_b \) falls as the interest factor increases and method B is therefore abandoned in favour of method A.

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9 In fact: \( d(p_a/p_b)/dR = [(dp_a/dR) p_b - (dp_b/dR) p_a]/p_b^2 = (p_a/p_b)(\Theta_a - \Theta_b)/R \).

10 See, for instance, von Weizsäcker (1971, p. 65) and Malinvaud (2003, p. 518).
This result might give the impression that the new average period of production is a good measure of the capital-intensity of production methods, a measure of their degree of roundaboutness. As should be understood, however, this is necessarily a false impression because what this average period of production is thought to be measuring does not exist. As Samuelson pointed out more than fifty years ago, there turns out to be no unambiguous way of characterising different processes as more ‘capital-intensive’ or more ‘roundabout’ (cf. Samuelson 1966, p. 582). In actual fact, as we shall see in the next section, the interpretation of the neo-Austrian average period as a measure of the quantity employed of a factor of production may lead to paradoxical conclusions.

4. Re-switching and the new concept of the average period of production

Once the compound interest formula is adopted, the unit cost functions defined by equations (5) and (6) – unlike the situation with equations (3) and (4) – are not linear with respect to the rate of interest. This feature has major implications for the choice of the method of production.
In order to show this, let us calculate the difference in unit costs from equations (5) and (6):

\[ p_a - p_b = R \left[ (a_1 - b_1) + (a_2 - b_2) R + (a_3 - b_3) R^2 \right]. \tag{11} \]

Now, since \( R \geq 1 \), the sign of the difference \( p_a - p_b \) depends on the sign of the expression in the square brackets. In particular, as this expression is a second-degree polynomial, there can be up to two different interest-factor levels for which \( p_a - p_b = 0 \), i.e. two 'switching points.'

The situation is described in fig. 4.\(^{11}\) For \( R \) close to 1 – i.e. for the rate of interest close to zero – \( p_a \) is smaller than \( p_b \) because we are assuming \( a_1 + a_2 + a_3 < b_1 + b_2 + b_3 \). A is therefore the cost-minimising method at first. When the rate of interest passes the first switching point \( R' \), B becomes the cost-minimising method, i.e. \( p_a > p_b \). Since there is a second switching point, however, method A comes back into use when the rate of interest passes through the level \( R'' \).

This re-switching of the methods of production is perfectly consistent with proposition 3. In fact, in a neighbourhood of the first switching point \( R' \), the curve is increasing, and this means \( \Theta_a > \Theta_b \). By contrast, in a neighbourhood of the second switching point \( R'' \), we have \( \Theta_a < \Theta_b \) since the curve is decreasing. At both the switching points, the rise in the interest factor therefore brings the method with the shortest average period of production into use. The

\(^{11}\) Similar reasoning and figure can be found in Sraffa (1960, p. 38).
problem is that the same method – let us say method B – can be the one with the shortest average period in \( R' \) and the one with the longest average period in \( R'' \), and proposition 3 therefore does not rule out the possibility of re-switching.

The possibility of re-switching, combined with proposition 3, allows us to show that the interpretation of the neo-Austrian conception of average period of production as a measure of the capital-intensity of the production methods may lead to paradoxical results. According to the traditional Austrian conception of capital, more roundabout methods of production are also more productive, in the sense that they give a higher amount of final output per worker. With the new concept, methods with longer average periods can instead, paradoxically, give a smaller final output per worker.

If \( y_a \) and \( y_b \) are the corn obtained per worker employed respectively with method A and B, then:

\[
y_a = \frac{1}{a_1+a_2+a_3} \\
y_b = \frac{1}{b_1+b_2+b_3}
\]

\( a_1 + a_2 + a_3 < b_1 + b_2 + b_3 \) therefore implies \( y_a > y_b \). This is consistent with the traditional idea of the degree of roundaboutness, since \( T_a > T_b \). In other words, method A is the most roundabout and gives the greatest final output per worker.

By contrast, if we adopt the new concept of the average period of production and take for our calculation an interest factor in a neighbourhood of \( R'' \), then \( \Theta_a < \Theta_b \), as stated above. We are thus faced with the following results:

i) For interest factors slightly below \( R'' \), B is the cost-minimising method even though it is the least productive and entails the longest average period of production.

ii) In a neighbourhood of \( R'' \), even though an increase in the rate of interest brings the method with the shortest period into use, it entails not a fall but a rise in the final output per worker.

The first point would be paradoxical for anyone really believing that the neo-Austrian average period of production represents a measure of the employment of capital. In order to avoid the paradox, it must be admitted that \( \Theta_a \) and \( \Theta_b \) represent nothing other than the elasticities of the costs of production \( p_a \) and \( p_b \) with respect to the interest factor \( R \), as emerges from equations (9) and (10).
The total employment of labour being fixed, the second point means that an increase in the rate of interest brings about an expansion of total output while a decrease entails a contraction. Is this result consistent with the working of the Austrian business-cycle theory?

5. Conclusions

According to some recent re-interpretations, the Austrian business-cycle theory must be freed from the conception of capital as a factor of production, i.e. as an amalgam of heterogeneous capital goods (cf. Braun 2018 and Lewin and Cachanosky 2018b). These studies suggest that this fallacious concept can be replaced with the new conception of the average period of production discussed here. While capital is certainly not a factor of production and capital goods are certainly not its physical form, as established beyond all reasonable doubt in the 1960s controversy, the new average period does not, however, appear to provide the Austrian business-cycle theory with the required support.

According to Cachanosky and Lewin (2014, p. 648, and 2016, p. 25), the Austrian business-cycle theory is grounded on the following relationships: (a) an inverse relationship between the rate of interest and the degree of roundaboutness of the production processes; (b) a direct relationship, *ceteris paribus*, between the degree of roundaboutness of the production processes and the level of output. If (a) and (b) hold, then a fall in the rate of interest due to the adoption of an expansive monetary policy prompts an artificial boom and the return toward the equilibrium position leads to a bust.

On the one hand, it has been proved in section 3 that relationship (a) always holds when the new concept of the average period is adopted (with compound interest capitalisation). On the other, however, it has been shown in section 4 that relationship (b) fails in the event of re-switching, as the most roundabout method of production gives the smallest output per worker in a neighbourhood of the second switching point. In this case, a fall in the rate of interest would bring about an increase in the degree of roundaboutness but not a boom. Contrary to what the Austrian business-cycle theory predicts, it would entail a bust.

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12 It can be incidentally stressed that, in Samuelson's view, the possibility of an 'unconventional relation' between the rate of interest and the final output is 'the single most surprising revelation from the re-switching discussion' (1966, p. 577, footnote 6).

13 For an in-depth reconstruction of the working of the Austrian business-cycle theory and the Hayek-Keynes-Sraffa controversy on it, we can refer the reader to Kurz (2000 and 2015).
In the light of this result, the average period of production considered in these pages does not appear to provide adequate support for the Austrian business-cycle theory. In particular, this business-cycle theory would actually appear to require the faulty conception of capital that these neo-Austrian scholars are rightly endeavouring to avoid.

References


