What we have learnt from financial econometrics modeling?

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WHAT WE HAVE LEARNT FROM FINANCIAL ECONOMETRICS MODELING?

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Abstract

A central issue around which the recent growth literature has evolved is that of financial econometrics modeling. Expansions of interest in the modeling and analyzing of financial data and the problems to which they are applied should be taken in account.This article focuses on econometric models widely and frequently used in the examination of issues in financial economics and financial markets, which are scattered in the literature. It begins by laying out the intimate connections between finance and econometrics. We will offer an overview and discussion of the contemporary topics surrounding financial econometrics. Then, the paper follows the financial econometric modeling research conducted along some different approaches that consist of the most influential statistical models of financial-asset returns, namely Arch-Garch models; panel models and Markov Switching models. This is providing several information bases for analysis of financial econometrics modeling.

Keywords: *financial modeling, Arch-Garch models, panel models, Markov Switching models.*

Classification JEL : C21, C22, C23, C 50, C51, C58.

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1. INTRODUCTION

Today, financial models find broad applications in managing financial and no financial corporations. The importance of those models can only increase over time as both system technologies and financial technologies continue to change our financial markets and the management of firms. Traders apply the financial models to trading securities; analysts use the models to analyze stocks and bonds; portfolio strategies use them to position their investment portfolios; corporations use them to stimulate financial and strategic planning. Financial models seem to be everywhere, but what, exactly, are they? Why do we need them? To answer these questions, we first illustrate the relationship between finance and econometrics models.

2. THE ISSUES AROUND FINANCIAL ECONOMETRICS MODELING

2.1. Financial model: definition and meaning

Ender A. Robinson (1966) states that a model is a simplified and idealized abstraction whose purpose is to approximate the behavior of a system. Edmond Malinvaud (1956) define a model as the formal representation of ideas and knowledge relating to a phenomenon. As a Gini's definition, this is a valid for any field of learning.

Generally, financial modeling is the task of building an abstract representation (a model) of a financial decision making situation.[1]

This is a mathematical model designed to represent the performance of a financial asset or portfolio of a business, project, or any other investment. Financial modeling is a general term that means different things to different users; the reference usually relates either to accounting and corporate finance applications, or to quantitative finance applications. While there has been some debate in the industry as to the nature of financial modeling - whether it is a tradecraft, such as welding, or a science - the task of financial modeling has been gaining acceptance and rigor over the years.[2] Typically, financial modeling is understood to mean an exercise in either asset pricing or corporate finance, of a quantitative nature. In other words, financial modeling is about translating a set of hypotheses about the behavior of markets or agents into numerical predictions[3].

Otherwise, financial modeling is an important methodology used by managers at all levels for the purpose of providing business solutions. The process must begin with understanding the business objectives and specifying the economic concepts on which the model is based.

This paper will cover the most known financial econometrics models. But first we need to define "financial model", since the term is interpreted differently in various contexts. After understanding the business problem, financial modeling starts with specifying the assumptions, implementing the models, and applying the models.

2.1.1. Economic assumptions

[1] http://www.investopedia.com/terms/f/financialmodeling.asp

[2] Nick Crawley (2010). *Which industry sector would benefit the most from improved financial modelling standards?*, fimodo.com.

[3] For example, a firm's decisions about investments (the firm will invest 20% of assets), or investment returns (returns on "stock A" will, on average, be 10% higher than the market's returns).

A model is a simplified of real circumstances. Its intention is to provide insights into the functioning of markets, firms, or financial processes. To make a complex problem abstract, we launch a set of assumptions, a model. Often, these assumptions may not seem realistic. We can separate the assumptions in two categories. On the one hand, some assumptions are technical in nature. They are made to present a concept elegantly and precisely, but they do not significantly affect the basic idea of the model. On the other hand, there are the economic assumptions. Some economic assumptions are held as first a principle, which means that they are fundamental in describing the basic nature of the people and the markets. As a consequence, developing financial models is an art, offering a scientific challenge.

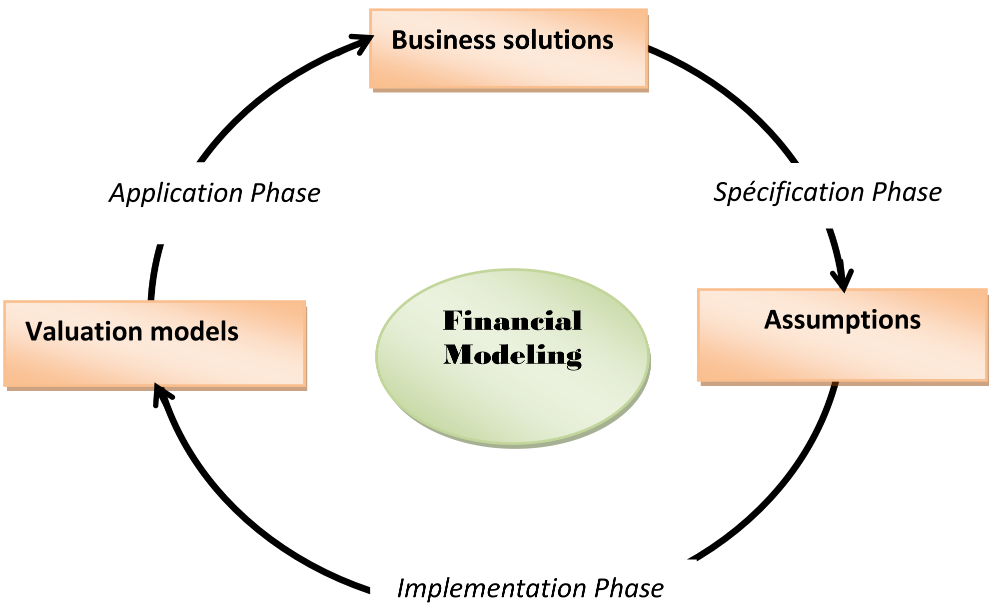
2.1.2. Quantification of the economic models

Financial models must be quantitative. Indeed, theses models are formulated in the way they can use input data to calculate security prices, determine valuations of decisions, and measure the performance of each phase of a business process. In addition, they can be implemented and have direct repercussions for finance in practice. Some economic theories are essential in providing insights to further understand the economy.

2.1.3. Applications of the models

Models are developed to solve specific problems. Their efficacy is therefore measured by their ability to do so. Therefore, in a description of any model, we first have to state the problems that it inquires about to explain and then crack them. In this perspective, we can spell out different types of applications. Some models are used to provide and make availably hypotheses about such cases that can be tested empirically. Other models are proposed as tools for financial management. In addition, there are models that identify the relevant characteristic of complex problems and to point toward how these problems may be solved.

Figure1: Financial Model Cycle



Source: Financial modeling: application fir capital markets, corporate finance, risk managements and financial institutions, Thomas S.Y. Ho & Sang Bin Lee, Oxford University Press, Inc, 2004, p8.

Financial modeling is a course of action to solve the business problems related to the finance. As mentioned above, to address the business problems, we specific the assumptions, implement the models and apply the models to solve the business problems. We note that the process does not bring to an end with the applications. The financial model's results should be checked and the assumptions of the model should be recurrently evaluated. For that reason, financial modeling is a cycle of movements and activities.

2.2. The need for financial models

The purpose of a good financial model is twofold; first, it clarifies the business owner's thoughts and expectations regarding key financial metrics, second, it presents financial data and projections to company executives, lenders and/or outside investors that are both cohesive and supportable. Strategic Growth Concepts associates provide strong financial plans that are powerful, concise, and useful tools for managing your business.

More accurately, the need of financial models is justified by the following reasons:

- Financial models have been developed to determine the relationships of cross currencies, interest rate movements, equity market risks, and a wide range of securities valuation models.
- Financial models can help corporate management to build a framework. For this reason, financial models play a significant role in the optimal functioning of the markets.
- Financial models are crucial to analyze a strategic investment decision.
- These models play a crucial role in measuring the performances of investment or other financial process.
- Their use in the management of a firm, in risk management, and in other business activities can significantly enhance the functioning of a business entity.

So, we should take in account the importance of developing the appropriate financial models for the purpose and understanding the assumptions used in each financial model.

2.3. How to formulate a financial model?

The characteristics and operational uses of every model are determined by the principles upon which the model is built and the objectives which it purses.

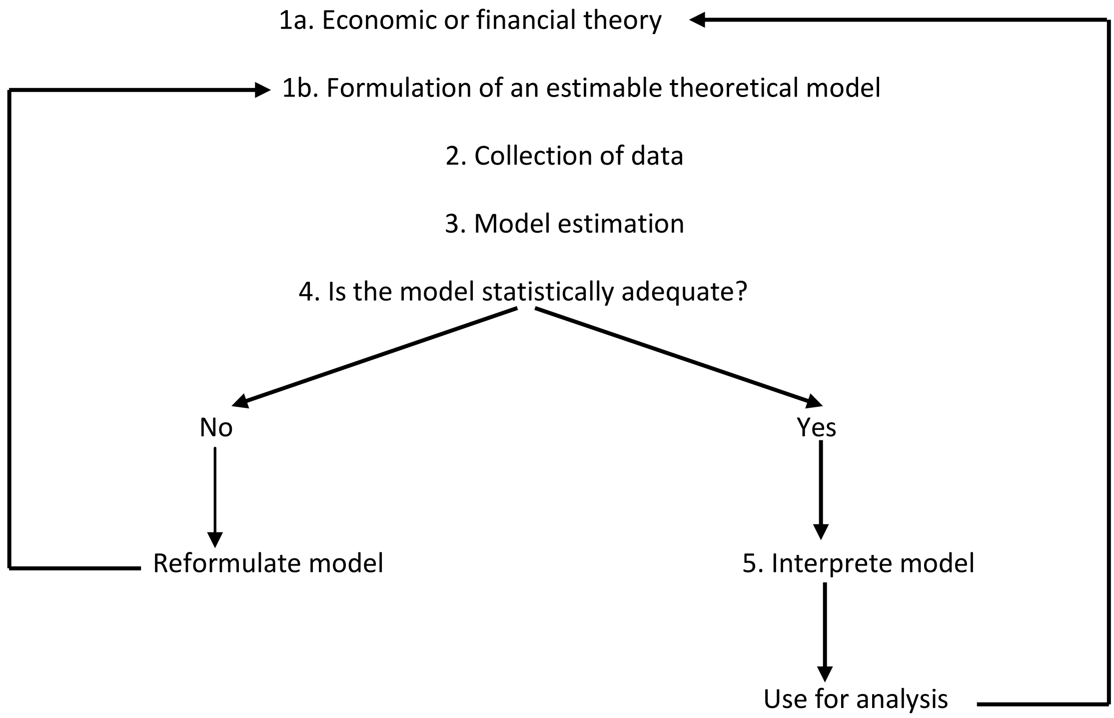
The actual phenomenon[4] is represented by the model in order to explain it, to predict it and to control it, goals corresponding to the three purposes of econometrics, namely structural analysis, forecasting, and policy evaluation.

Also, the model should be realistic in incorporating the main elements of the phenomena being represented. So, it must specifying the interrelationships among the elements of the system in a way that is adequately detailed and explicit so as to make certain that the study of the model will lead to insights. The art of model building involves balancing the often competing goals of realism and manageability.

There are many different ways to go about the process of models building. A logical and valid approach would be to follow the steps described in figure 1.The steps involved in the model construction process are now listed and detailed.

[4] Sometimes the actual system is called real-world system in order to emphasize the distinction between it and the model system that represents it.

Figure 2: Steps to formulate a financial model



-step 1a and 1b: *General statement of the problem*:

A problem statement is a concise description of the issues that need to be addressed by a problem solving team and should be presented to them (or created by them) before they try to solve the problem. When bringing together a team to achieve a particular purpose provide them with a problem statement. This will usually involve the formulation of a theoretical model, or intuition from financial theory that two or more variables should be related to one another in a certain way.

-step2: *Collection of data relevant to the model*:

Data collection is any process of preparing and collection data. A formal data collection is necessary as it ensures that data gathered are both defined and accurate and that subsequent decisions based on arguments embodied in the findings are valid[5].

The data required may be available electronically through a financial information provider, such as Reuters or from published government figures.

-step 3: *Choice of estimation method:*

Estimation is the process of finding an estimate, or approximation, which is a value that is usable for some purpose even if input data may be incomplete, uncertain, or unstable. Typically, estimation involves "using the value of a statistic derived from a sample to estimate the value of a corresponding population parameter".[6]

The choice of the appropriate estimation model depends on a number of factors and terms. For example, is a single equation or multiple equation technique to be used?

[5] Data collection and analysis Dr. Roger Sapsford, Victor Jupp.
[6] Raymond A. Kent. (2001). Estimation, Data Construction and Data Analysis for Survey Research. P.157.

We emphasizes that well specified estimation method is sometimes able to predict with some accuracy how individuals would react in a particular situation

-step 4 and step 5: *Statistical evaluation of the model*:

In order to bring about an operational use for the model, i.e., a use which is aimed at prediction and decision-making, we need to complement this empirical base by the study of the structural permanence of the equations of the model. If we find that our theory is an acceptable approximation of reality then we can apply it towards practical ends (proced to step 5); if not, go back to step 1-3 and either reformulate the model. Collect more data, or select a different estimation technique that has less stringent requirements.

-step 6: *Use of model*:

When a researcher is finally satisfied with the model, it can then be used for testing the theory specified in step 1, or for formulating forecasts or suggested courses of actions.

Models are very useful. First, on a conceptual point of view, since the model list and define the concepts and variables expected to intervene in the functioning of an economic process. Second, it state relations defining the effects of exogenous variables (input data) results (output data) via the endogenous data (data entered into the heart of the model). Finally, in a point of view of economic action, as it is to take advantage of producing simulations and optimizing situations.

Otherwise, the use of a logic model provides support and documented in order to propose economic policies, present reasonable arguments to justify economic policies, explain and influence business strategies and provide advice to households; plan and allocate resources, and improve business management.

2.4. Econometric packages for modeling financial data

Widely used for more than a decades as a high-powered statistical system, econometric package's capabilities have steadily grown.

These packages dramatically raise the bar for statistical computation, making possible a vastly higher level of integration of the complete analysis workflow, as well as providing major extensibility beyond the specific functions implemented in particular statistical systems.

Advances in computing have integrated decision support, to develop field tools (including software experts) to help a decision maker to analyze a problem or situation, and to provide solutions, possibly prioritized on the basis of logical criteria it has selected.

As the same suggest, we can enumerate various computer packages that may be employed to estimate econometric models (confers to annex). The number of available packages is large, and over time, all packages have improved in breadth of available packages in large, and over time, all packages have improved in breadth of available techniques, and have also converged in terms of what is available in each package.

Examples of econometric package for modeling finance with their fundamental characteristics are described in the same annex.

2.5. What can explain the remarkable growth of financial modeling?

There are many factors which can explain the significant growth of financial modeling. These factors lie in the confluence of three parallel developments in the last half century.

The first is the fact that the financial system has become more complex over time and the benefits of more highly developed financial technology become greater and greater and, ultimately, indispensable. This is an obvious consequence of general economic growth and development in which the number of market participants, the variety of financial transactions, and the sums involved has also grown.

The second factor is the set of breakthroughs in the quantitative modeling of financial markets, e.g., financial technology. Pioneered over the past three decades by the giants of financial economics Fischer Black, John Cox, Eugene Fama, John Lintner, Harry Markowitz, Robert Merton, and others their contributions laid the remarkably durable foundations on which all of modern quantitative financial analysis is built. Financial econometrics is only one of several intellectual progeny that they have sired.

The third factor is a recent set of breakthroughs in computer technology, including hardware, software, and data collection and organization. The advent of powerful desktop microcomputers and machine-readable real-time and historical data breathed life into financial econometrics; irrevocably changing the way finance is practiced and taught. Concepts like alpha, beta, R^2, correlations, and cumulative average residuals have become concrete objects to be estimated and actively used in making financial decisions.

3. THE MOST WIDESPREAD FINANCIAL ECONOMETRICS MODELS

We will examine some of the common, generic, approaches we will encounter in financial models today, with a view to understanding the technical background.

Otherwise, the financial world is an uncertain universe where events take place every day, every hour, and every second. Information arrives randomly and so do the events. Nonetheless, there are regularities and patterns in the variables to be identified, effect of a change on the variables to be assessed, and links between the variables to be established. In this perspective, financial econometrics attempts to perform the analysis of these kinds through employing and developing various relevant statistical procedures.

3.1. The three types of economic and financial variables

There are generally three types of economic and financial variables: the rate variable, the level variable and the ratio variable. The first category measures the speed at which, for example, wealth is generated, or savings are made, at one point of time (continuous time) or in a short interval of time (discrete time). The rate of return on a company's stock or share is a typical rate variable. The level variable works out the amount of wealth, such as income and assets, being accumulated over a period (continuous time) or in a few of short time intervals (discrete time).[7] The ratio variable consists of two sub-categories, one is the type I ratio variable or the component ratio variable, and the other is the type II ratio, the contemporaneous relativity ratio variable

3.2. Basic concepts of price and returns

A significant management issue that we will discuss concerning financial models in valuation. It is essential to determine the present and future value of investments.

3.2.1. Time value of the money

[7] A firm's assets and a country's GDP are typical level variables, though they are different in a certain sense in that the former is a stock variable and the latter is a flow variable.

The time value of money is the value of money figuring in a given amount of interest earned over a given amount of time. The time value of money is the central concept in finance theory.[8]

Consider an amount $V invested for n years at a simple interest rate of r per annum the year, the future value after n years is:

$$FV_n = V \times (1+r)^n \qquad (1)$$

If interest is paid m times per year then the future value after n years is:

$$FV_n^m = V \times \left(1+\frac{r}{m}\right)^{m\times n} \qquad (2)$$

The Time Interval Is 1 Year and the Interest Rate is 10% per Annum.
As m, the frequency of compounding, increases the rate becomes continuously compounded and it can be shown that the future value becomes:

$$FV_n^m = \lim_{n\to\infty} V \times \left(1+\frac{r}{m}\right)^{m\times n} = V \times \exp(r+n) \qquad (3)$$

Where exp(.) is the exponential function.

3.2.2. Financial assets

A financial asset is an intangible asset that derives value because of a contractual claim. Examples include bank deposits, bonds, and stocks. Financial assets are usually more liquid than tangible assets, such as land or real estate, and are traded on financial markets.[9] According to the International Financial Reporting Standards (IFRS), a financial asset is defined as one of the following:

- Cash or cash equivalent;
- Equity instruments of another entity;
- Contractual right to receive cash or another financial asset from another entity or to exchange financial assets or financial liabilities with another entity under conditions that are potentially favorable to the entity;
- Contract that will or may be settled in the entity's own equity instruments and is either a non-derivative for which the entity is or may be obliged to receive a variable number of the entity's own equity instruments, or a derivative that will or may be settled other than by exchange of a fixed amount of cash or another financial asset for a fixed number of the entity's own equity instruments.[10]

In finance, a bond is an instrument of indebtedness of the bond issuer to the holders. It is a debt security, under which the issuer owes the holders a debt and, depending on the terms of the bond, is obliged to pay them interest (the coupon) and/or to repay the principal at a later date, termed the maturity.[11]

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[8] For example, $100 of today's money invested for one year and earning 5% interest will be worth $105 after one year. Therefore, $100 paid now or $105 paid exactly one year from now both have the same value to the recipient who assumes 5% interest; using time value of money terminology, $100 invested for one year at 5% interest has a *future value* of $105

[9] www.investopedia.com Financial Asset

[10] International Accounting Standard (IAS) 32.11.

[11] O'Sullivan, Arthur; Sheffrin, Steven M. (2003). Economics: Principles in action. Upper Saddle River, New Jersey 07458: Pearson Prentice Hall. pp. 197, 507

At the time of issue of the bond, the interest rate and other conditions of the bond will have been influenced by a variety of factors, such as current market interest rates, the length of the term and the creditworthiness of the issuer.

Now, we try to determine the value of a bond. We note that there is two types of bond: the zero-coupon bond and the coupon bond.

(i) Zero-coupon Bond (discount bond).[12] A zero-coupon bond with maturity date T provides a monetary unit at date t. at date t with $t \leq T$, the zero-coupon bond has a residual maturity of $H= T-t$ and a price of $B(t,H)$ (or $B(t,T-t)$), which is the price at time t,

$$B(t, T)= \begin{cases} (1+r)^{-(T-t)} \\ \left(1+\frac{r}{m}\right)^{-m(t-T)} \\ \exp\left(-r(T-t)\right) \end{cases} \quad (4)$$

Where r is the interest rate, in particular, B(0,T) is the current, time 0 of the bond, and $B(T,T)=1$ is equal to the face value, which is a certain amount of money that issuing institute (for example, a government, a bank or a company) promises to exchange the bond for.

(ii) Coupon Bond. Bonds promising a sequence of payments called coupon bonds is traded at any date t between 0 and the maturity date T differs from the issuing price p_0.

3.2.3. Statistical features *Prices*

In finance, there are many types of prices such as closing prices in stock market[13]; currency exchange rates[14], option prices[15], more,…

Frequency of observations

It depends on the data available and the questions that interest a researcher. The price interval between prices should be sufficient to ensure that trade occurs in most intervals and it is preferable that the volume of trade is substantial. Daily data are fine for most of the applications. Also, it is important to distinguish the price data indexed by transaction counts from the data indexed by time of associated transactions.

Definition of returns

The statistical inference on asset prices is complicated because asset price might have nonstationary behavior (upward and downward movements). One can transform asset prices into returns, which empirically display more stationary behavior. Also, returns are scale-free and not limited to the positiveness.

In finance, rate of return (ROR), also known as return on investment (ROI), rate of profit or sometimes just return, is the ratio of money gained or lost (whether realized or unrealized) on an investment relative to the amount of money invested. ROI is usually expressed as a percentage.

There are five types of returns of a financial asset:

[12] For more details about bonds, see the book by Capinsky and Zastawaniak (2003).

[13] It is the price of the last transaction completed during a day's trading session

[14] It is the rate at which one currency can be exchanged for another.

[15] It is the amount per share that an option buyer pays to the seller.

(i) Returns of a financial asset (stock) which price P_t at date t that produces no dividends over the period (t, t+H) is defined as:

$$r(t,t+H) = \frac{P_{t+H} - P_t}{P_t} \quad (5)$$

Very often, we will investigate returns as a fixed unitary horizon. In this case H=1 and return is defined as:

$$(t,t+1) = \frac{P_{t+1} - P_t}{P_t} = \frac{P_{t+1}}{P_t} - 1 \quad (6)$$

Returns $t,t+H$ and $(t,t+1)$ in (5) and (6) are sometimes called the simple net returns. Very often, $r(t,t+1)$ is simply denoted r_{t+1}. The simple gross return is defined as:

$$r(t,t+H) = \frac{p_{t+H}}{P_t} + 1 + r(t,t+H) \quad (7)$$

Since $\frac{p_{t+H}}{P_t} = \frac{p_{t+H}}{P_{t+H-1}} \frac{p_{t+H-1}}{P_{t+H-2}} \times \ldots c$ the $R(t,t+H)$ can be rewritten as:

$$\begin{aligned} R(t,t+H) &= \frac{p_{t+H}}{P_{t+H-1}} \frac{p_{t+H-1}}{P_{t+H-2}} \times \ldots \times \frac{p_{t+H}}{P_{t+H-1}} \frac{p_{t+H-1}}{P_{t+H-2}} \\ &= R(t+H-1,t+H) \times R(t+H-2,t+H-1) \times \ldots \times R(t,t+1) \\ &= \prod_{j=1}^{H} R(t+H-j,H+1-j) \quad (8) \end{aligned}$$

The simple gross return over *H* periods is the product of one period returns.
The formula in (6) is often replaced by the following approximation:

$$r(t,t+1) = r_{t+1} \approx \ln(P_{t+1}) - \ln(P_t) = ln\left(\frac{P_{t+1}}{P_t}\right) = \ln\big(R(t,t+1)\big) \quad (9)$$

The return in (9) is also known as continuously compounded return or log return. To see why $r(t,t+1)$ is called the continuously compounded return, take the exponential of both sides of (9) and rearing we get:

$$P_{t+1} = P_t \exp\big(r(t,t+1)\big) \quad (10)$$

By comparing (10) with (3) one can see that $r(t,t+1)$ is the continuously compounded growth rate in prices between months $t-1$ and t. Rearranging (9) one can show that:

$$r(t,t+H) = \sum_{j=1}^{H} r(t+H-j,t+H+1-j) \quad (11)$$

(ii) return of a financial asset (stock) with price P_t at date t that produces dividends D_{t+1} over the period (t, t+1) is defined as:

$$r(t,t+H) = \frac{P_{T+1} + D_{t+1} - P_t}{P_t} + \frac{P_{T+1} - P_t}{P_t} + \frac{D_{t+1}}{P_t} \quad (12)$$

Where $\frac{D_{t+1}}{P_t}$ is the ratio of dividend over price (d-p ratio), which is a very important financial instrument for studying financial behavior.

(iii) Spot currency returns.

Suppose that P_t is the dollar price in period *t* for one unit of foreign currency (say, euro). Let i^*_{t-1} be the continuously compounded interest rate for deposits in foreign currency from time t-1 until time t. then one dollar used to buy $1/P_{t-1}$ euros in period t-1, which are sold with accumulated interest in period t, gives proceeds equal to P_t*exp(i^*_{t-1})/ $P_{t-1}$and the return is:

$$r_t = \log(P_t) - \log(P_{t-1}) + i^*_{t-1} = p_t - p_{t-1}+i^*_{t-1} \quad (13)$$

In practice, the foreign interest rate is ignored because it is very small compared with the magnitude of typical daily logarithmic price change.

(iv) Futures returns.

Suppose $F_{t,T}$ is the futures price in period *t* for delivery or cash settlement in some later period *T*. As there are no dividends payouts on futures contracts, the futures return is defined as:

$$r_t = \log(F_{t,T}) - \log(F_{t-1,T}) = f_{t,T} - f_{t-1,T} \quad (14)$$

Where $f_{t,T} = \log(F_{t,T})$.

(v) Excess return

It is defined as the difference between the asset's return and the return on some reference asset. The reference asset is usually assumed to be riskless and in practice is usually a short-term Treasury bill return. Excess is defined as:

$$z(t, t+1) = z_{t+1} = r(t, t+1) - r_0(t, t+1) \quad (15)$$

Where $r_0(t, t+1)$ is the reference return from period *t* to period *t*+1 .

3.3. Arch-Garch models

Recent developments in financial econometrics suggest the use of nonlinear time series structures to model the attitude of investors toward risk and expected return. For example, Bera and Higgins (1993, p.315) remarked that "a major contribution of the ARCH literature is the finding that apparent changes in the volatility of economic time series may be predictable and result from a specific type of nonlinear dependence rather than exogenous structural changes in variables."

Campbell, Lo, and MacKinlay (1997, p.481) argued that "it is both logically inconsistent and statistically inefficient to use volatility measures that are based on the assumption of constant volatility over some period when the resulting series moves through time." In the case of financial data, for example, large and small errors tend to occur in clusters, i.e., large returns are followed by more large returns, and small returns by more small returns. This suggests that returns are serially correlated.

3.3.1. Properties and Interpretations of ARCH Models

Let Ω_1 and Ω_2 are two sets of random variables such that $\Omega_1 \subseteq \Omega_2$
Lest Y be scalar random variable.
Then, $E[Y|\Omega_1]=E[E[Y|\Omega_2]|\Omega_1]$
if $\Omega_1=\emptyset$, then $E[E[Y|\Omega_2]]=E[y]$.
Without loss of generality, lest a ARCH(1) process be represented by

$$u_t= \varepsilon_t\sqrt{\alpha_0+\alpha_1 u^2_{t-1}} \quad (16)$$

Where $\{\varepsilon_t\}_{t=0}^{\infty}$ is a white noise process. Johansen and DiNardo (1997) briefly mention the flowing properties of Arch Models:

- u_t have mean zero

Proof:

$u_t = \varepsilon_t\sqrt{\propto_0+\propto_1 u_{t-1}^2}$

$E_{t-1}[u_t] = E_{t-1}[\varepsilon_t] \sqrt{\propto_0+\propto_1 u_{t-1}^2}$

or $E_{t-1}[\varepsilon_t] = 0$

so $E_{t-2}E_{t-1}[u_t] = 0$ (17)

$(\ldots)$

$= 0$

- u_t have conditional variance given by $\sigma_t^2 = \propto_0+\propto_1 u_{t-1}^2$

Proof

$u_t^2 = \varepsilon_t^2 [\propto_0+\propto_1 u_{t-1}^2]$

$E_{t-1}[u_t^2] = \sigma_\varepsilon^2 [\propto_0+\propto_1 u_{t-1}^2]$ (18)

$= 1[\propto_0+\propto_1 u_{t-1}^2]$

$=\sigma_t^2$

- u_t =have unconditional variance given by $\sigma^2= \frac{\propto_0}{1-\propto_1}$

  $proof$

  $E_{t-2}E_{t-1}[u_t^2] = E_{t-2}[\propto_0+\propto_1 u_{t-1}^2]$

  $= \propto_0+ \propto_1 E_{t-2}[u_{t-1}^2]$

  $= \propto_0+\propto_0\propto_1+ \propto_1^2 u_{t-2}^2$

  $E_{t-3}E_{t-2}[u_t^2] = E_{t-3}[\propto_0+\propto_0+\propto_0\propto_1+\propto_1^2 u_{t-2}^2]$

  $= \propto_0+ \propto_0\propto_1\propto_1^2 E_{t-3}[u_{t-2}^2]$

  $= \propto_0+\propto_0\propto_1+ \propto_0 \propto_1^2 +\propto_1^3 u_{t-3}^2$

$(\ldots)$

$E_0E_1E_2(\ldots)E_{t-2}E_{t-1} [u_t^2] = \propto_0 (1+\propto_1+\propto_1^2+\ldots+\propto_1^{t-1} + \propto_1^t u_0^2$

$= \frac{\propto_0}{1-\propto_1}$

$=\sigma^2$ (19)

Therefore, unconditionally the process is homoscédastic.

- u_t have zero autocovariances

Proof

$E_{t-1}[u_t - u_{t-1}] = u_{t-1}\text{-}E_{t-1}[u_t]=0$ (20)

Regarding Kurtosis and Higgins (1993) show that the process has a heavier tail than the Normal distribution, given that

$$\frac{E[\varepsilon_t^4]}{\sigma_t^4} = 3\,(\frac{1-\propto_t^2}{1-3\propto_t^2} > 3 \qquad (21)$$

3.3.2. Estimating and Testing ARCH Models

Johnston and DiNardo (1997) suggest a very simple test for the presence of ARCH problems. The basic menu (step-by-step) is:

-Regress y on x by OLS and obtain the residuals $\{\varepsilon_t\}$.

-Compute the OLS regression $\varepsilon_t^2 = \widehat{\alpha_0} + \widehat{\alpha_1}\varepsilon_{t-1}^2 + \cdots + \widehat{\alpha_p}\varepsilon_{t-p}^2 + error$ (22)

-Test the joint significance of $\widehat{\alpha_0}$, $\widehat{\alpha_p}$.

In case that any of the coefficients are significant, a straight-forward method of estimation (correction) is provided by Green (1997). It consists in four-step FGLS:

-Regress y on x using least square to obtain $\hat{\beta}$ and ε vectors.
-Regress ε_t^2 on a constant and ε_{t-1}^2 to obtain the estimates of $\propto_0$ and $\propto_1$, using the whole sample (T). Denote $[\widehat{\alpha_0}, \widehat{\alpha_1}] = \vec{\propto}$.

-Compute $f_t = \widehat{\alpha_0} + \widehat{\alpha_1}\varepsilon_{t-1}^2$. Then compute the asymptotically efficient estimate $\widehat{\propto}$, $\widetilde{\propto} = \vec{\propto} + d_\alpha$, where d_α is the least square coefficient vector in the regression.

-Recompute f_t using $\widetilde{\propto}$, then compute

$$r_t = \left[\frac{1}{f_t} + 2\left(\frac{\widetilde{\propto_1}\ \varepsilon_t}{f_{t+1}}\right)^2\right]^{1/2}$$

$$s_t = \frac{1}{f_t} - \frac{\widetilde{\propto_1}}{f_{t+1}}\left[\frac{\varepsilon_{t+1}^2}{f_{t+1}} - 1\right] \qquad (23)$$

Compute the estimate $\tilde{\beta} = \vec{\beta} + d_\beta$, where d_β is the least squares coefficient vector in the regression$\left[\frac{\varepsilon_t s_t}{r_t}\right] = \widehat{w} x_t r_t + error\ \ (24)$

The asymptotic covariance matrix for $\tilde{\beta}$ is given by $(\widehat{w}'\widehat{w})^{-1}$, where $\widehat{w}$ is the regressor vector on the equation above.

Remarque:
*In this section, we discuss univariate ARCH and GARCH models. Because in this chapter we focus on financial applications, we will use financial notation.

3.3.3. Why ARCH/GARCH?

The ARCH/GARCH framework proved to be very successful in predicting volatility changes. Empirically, a wide range of financial and economic phenomena show evidence of the clustering of volatilities. As we have seen, ARCH/ GARCH models describe the time evolution of the average size of squared errors, that is, the evolution of the magnitude of uncertainty. Despite the empirical success of ARCH/GARCH models, there is no real consensus on the economic reasons why uncertainty tends to cluster. That is why models tend to perform better in some periods and worse in other periods.

In financial models, the future is always uncertain but over time we learn new information that helps us forecast this future. As asset prices reflect our best forecasts of the future profitability of companies and countries, these change whenever there is news. In this perspective, ARCH/GARCH models can be interpreted as measuring the intensity of the news process. Volatility clustering is most easily understood as news clustering.

3.4. Panel data models

Dynamic panel models play an increasingly prominent role in corporate finance research. Empirically understanding payout policy, capital structure, or investment decisions arguably requires the use of firm fixed effects to control for unobserved, time-invariant differences across firms.[16]

[16] For example, see Andres et al. (2009), Bond and Meghir (1994), Dittmar and Duchin (2010), Lemmon, Roberts, and Zender (2008), Loudermilk (2007), Machin and van Reenen (1993), and Ozkan (2000).

The situation often arises in financial modeling where we have data comprising both time series and cross-sectional elements, and such a dataset would be known a panel of data or longitudinal data. A panel of data will embody information across both time and space. Importantly, a panel keeps the same individuals or subjects (henceforth we will call these "entities") and measures some quantity about them over time. This (chapter) will present and discuss the techniques used to model such data.

Econometrically, the setup we may have is as described in the following equation

$$y_{it} = \propto + \beta x_{it} + u_{it} \qquad (24)$$

Where y_{it} is the dependant variable, $\propto$ is the intercept term, β is $k \times 1$ vector of parameters to be estimated on the explanatory variables, $t = 1, \ldots., T; i = 1, \ldots\ldots, N$. [17]

3.4.1. The fixed effects model

To see how the fixed effects model works, we can take equation (24) above and decompose the disturbance term, u_{it}, into an individual specific effect, μ_i, and the "remainder disturbance" , ϑ_{it} , that varies over time and entities (capturing everything that isleft unexplained about y_{it}).

$$u_{it} = \mu_i, + \vartheta_{it} \quad (25)$$

So we could rewrite equation (1)(24) by substituting in for u_{it} from (2) (25) to obtain

$$y_{it} = \propto + \beta x_{it} + \mu_i, + \vartheta_{it} \qquad (26)$$

We can think of μ_i as encapsulating all of the variables that affect y_{it} cross-sectionally but do not vary over time-for example, the sector that a firm operates in, a person's gender, or the country where a bank has its headquarters, etc. This model could be estimated using dummy variables, which would be termed the least squares dummy variable (LSDV) approach

$$y_{it} = \beta x_{it} + \mu_1 D1_i + \mu_2 D2_i + \mu_3 D3_i + \cdots + \mu_N DN_I + \vartheta_{it} \quad (27)$$

Where $D1_i$ is a dummy variable that takes the value 1 fot all observations on the first entity and zero otherwise, $D2_i$ is a dummy variable that takes the value 1 for all observations on the second entity and zero otherwise, and so on.

When the fixed effects model is written in this way, it is relatively easy to see how to test for whether the panel approach is really necessary at all. This test would be a slightly modified version of the Chow test, and would involve incorporating the restriction that all of the intercept dummy variables have the same parameter (i.e. H_0: $\mu 1 = \mu_2 = \cdot\cdot\cdot = \mu_N$). If this null hypothesis is not rejected, the data can simply be pooled together and OLS employed. If this null is rejected, however, then it is not valid to impose the restriction that the intercepts are the same over the cross-sectional units and a panel approach must be employed.

In order to avoid the necessity to estimate so many dummy variable parameters, a transformation is made to the data to simplify matters. This transformation, known as the *within transformation*, involves subtracting the time-mean of each entity away from the values of the variable.

[17] *K* represent the number of slope parameters to be estimated (rather than the total number of parameters as it is elsewhere), which is equal to the number of explanatory variables in the regression model.

So define $\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$ as the time-mean of the observations on y for cross-sectional unit i, and similarly calculate the means of all of the explanatory variables. Then we can subtract the time-means from each variable to obtain a regression containing demeaned variables only.

Note that again, such a regression does not require an intercept term since now the dependent variable will have zero mean by construction.
The model containing the demeaned variables is

$$y_{it} - \bar{y}_i = \beta(x_{it} - \bar{x}_i) + u_{it} - \bar{u}_i \quad (28)$$

Which we could write as

$$\ddot{y}_{it} = \beta\ddot{x}_{it} + \ddot{u}_{it} \qquad (29)$$

where the double dots above the variables denote the demeaned values.

An alternative to this demeaning would be to simply run a cross sectional regression on the time-averaged values of the variables, which is known as the *between estimator.*
The regression on the time-demeaned variables will give identical parameters and standard errors as would have been obtained directly from the LSDV regression, but without the hassle of estimating so many parameters!
A major disadvantage of this process, however, is that we lose the ability to determine the influences of all of the variables that affect y_{it} but do not vary over time.

3.4.2. The random effects models

An alternative to the fixed effects models is the random effects model, which is sometimes also known as the error components model. The random effects approach proposes different intercept terms for each entity and again these intercepts are constant over time, with the relationships between the explanatory and explained variables assumed to be the same both cross sectionally and temporally.

However, the difference is that under the random effects model, the intercepts for each cross-sectional unit are assumed to arise from a common intercept $\propto$ (which is the same for all cross-sectional units and over time), plus a random variable ϵ_i that varies cross sectionally but is constant over time. ϵ_i measures the random deviation of each entity's intercept term from the "global" intercept term $\propto$. We can write the random effects panel model as:

$$y_{it} = \propto + \beta x_{it} + w_{it}\,, w_{it} = \epsilon_i + \vartheta_{\mathrm{it}} \qquad (30)$$

Where x_{it} is still a $1 \times k$ vector of explanatory variables, but unlike the fixed effects model, there are no dummy variables to capture the heterogeneity (variation) in the cross-sectional dimension. Instead, this occurs via the ϵ_i terms. Note that this framework requires the assumptions that the new cross-sectional error term, ϵ_i, has zero mean, is independent of the individual observation error term (ϑ_{it}), has constant variance σ_ϵ^2 and independent of the explanatory variables (x_{it}).

The parameters (α and the β vector) are estimated consistently but inefficiently by OLS, and the conventional formulae would have to be modified as a result of the cross-correlations between error terms for a given cross-sectional unit at different points in time. Instead, a generalised least squares procedure is usually used.
θ will be a function of the variance of the observation error term, σ_ϑ^2, and of the variance of the entity-specific error term, σ_ϵ^2

$$\theta = 1 - \frac{\sigma_\vartheta}{\sqrt{T\sigma_\epsilon^2 + \sigma_\epsilon^2}} \quad (31)$$

This transformation will be precisely that required to ensure that there are no cross-correlations in the error terms, but fortunately it should automatically be implemented by standard software packages.

Just as for the fixed effects model, with random effects it is also conceptually no more difficult to allow for time variation than it is to allow for cross-sectional variation. In the case of time variation, a time period specific error term is included

$$y_{it} = \propto + \beta x_{it} + \omega_{it}, \omega_{it} = \epsilon_t + \vartheta_{\mathrm{it}} \qquad (32)$$

3.4.3. Fixed or random effects?

It is often said that the random effects model is more appropriate when the entities in the sample can be thought of as having been randomly selected from the population, but a fixed effect model is more plausible when the entities in the sample effectively constitute the entire population (for instance, when the sample comprises all of the stocks traded on a particular exchange).

A test for whether this assumption is valid for the random effects estimator is based on a slightly more complex version of the Hausman test. If the assumption does not hold, the parameter estimates will be biased and inconsistent.
To see how this arises, suppose that we have only one explanatory variable, x_{2it} , that varies positively with y_i t and also with the error term, ω_{it}. The estimator will ascribe all of any increase in y to x when in reality some of it arises from the error term, resulting in biased coefficients.

3.4.4. The advantages of panel data

If we are fortunate enough to have a panel of data at our disposal, there are important advantages to making full use of this rich structure. First, and perhaps most importantly, we can address a broader range of issues and tackle more complex problems with panel data than would be possible with pure time-series or pure cross-sectional data alone.

Second, it is often of interest to examine how variables, or the relationships between them, change dynamically (over time). To do this using pure time-series data would often require a long run of data simply to get a sufficient number of observations to be able to conduct any meaningful hypothesis tests. But by comparing cross-sectional and time series data, one can increase the number of degrees of freedom, and thus the power of the test, by employing information on the dynamics behavior of a large number of entities at the same time. The additional variation.

3.5. Markov switching models

Recent renewed interests in Markov chain processes and Markov switching models are largely fascinated by Hamilton (1989, 1994). While the major contributors with economic significance to the popularity of this family of models are the intensified studies in business cycles in the last two decades in the frontier of macro and monetary economics, and the proliferating use of mathematical tools in the exploitation of excess returns in a seemingly efficient while volatile financial market. The regime shift or state transition features of Markov switching, when applied properly, are able to illustrate and explain economic fluctuations around boom–recession or more complicated multi-phase cycles. In financial studies, the state transition process can be coupled with bull–bear market alternations, where regimes are less clearly defined but appear to have more practical relevance.

However, estimation of Markov switching models may be technically difficult and the results achieved may be sensitive to the settings of the procedure. Probably, rather than producing a set of figures of immediate use, the approach helps improve our understanding about an economic process and its evolving mechanism constructively, as with many other economic and financial models.

3.5.1. The Markov chains

A Markov chain is defined as a stochastic process $\{S_t, t = 0,1, \dots\}$ that takes finite or countable number of integer values denoted by *i,j*, and that the probability of any future value of S_{t+1} equals *j*, i.e., the conditional distribution of any future state S_{t+1}, given the past state S_0, S_1, …, S_{t-1} and the present state S_t, is only dependent on the present state and independent of the past states. That is:

$$P\{S_{t+1} = j|S_t = i_t, S_{t-1} = i_{t-1}, S_1 = i_1, S_0 = i_0\} = P\{S_{t+1} = j|S_t = i_t\} = p_{ij}$$

p_{ij} is the probability that the state will next be *j* when the immediate preceding state is *i*, and can be called the transition probability from *i* into *j*. suppose there are N states, then all the transitions can be expressed in a transition matrix:

$$P = \begin{bmatrix} P_{11} & P_{12} \cdots & P_{1N} \\ P_{21} & P_{22} \cdots & P_{2N} \\ \vdots & \vdots & \vdots \\ P_{N1} & P_{N2 \dots} & * P_{NN} \end{bmatrix}$$

The probability is non-negative and the process must transit into some state, including the current state itself, so that:

$$\sum_{j=1}^{N} P_{ij} = 1, i = 1, 2, \dots, N$$

Above are one –step transition probabilities. It is natural for us to extend the one- step case and consider n-step transitions that are clearly functions and results of several one-step transitions. For example, a two-step transition $P\{S_{t+2} = j|\, S_t = i\}$ probabilities is the summation of the probabilities of transitions from state i into all the states, then from all the states, then from all the states into *j*:

$$\sum_{j=1}^{N} P = \{S_{t+2} = j|\, S_t = k\} P\{S_{t+1} = K|\, S_t = i\}$$

More generally, define the n-step transition probability as:

$$P\{S_{t+n} = j|\, S_t = i\} = p_{ij}^{n}$$

A formula called the Chapman-Kolmogorov equation holds for calculating multistep transition probabilities:

$$P_{ij}^{m+n} = \sum_{K=1}^{N} P_{ik}^{n} P_{kj}^{m}, i = j = 1,2, \dots N$$

3.5.2. The Estimation of a Markov Chain

The estimation of a Markov chain process or Markov switching model is achieved, naturally, by considering the joint conditional probability of each of future states, as a function of the joint conditional probabilities of current states and the transition probabilities. This procedure is called filtering: the conditional probabilities of current states are input, passing through or being filtered by the system of dynamic transformation that is the transition probability matrix, to produce the conditional probabilities of future states as output. The conditional likelihood function can be obtained in the meantime, and the parameter can be estimated accordingly.

Suppose there is a simply two-state Markov chain process:

$$y_t = \mu_1 S_1 + \mu_2 S_2 + \varepsilon_t$$

Where $S_{1=1}$when in state 1 and 0 otherwise, $S_2 = 1$ when in state 2 and 0 otherwise, and ε_t is a white noise residual. We are interested to know how the joint probability of $y_t$and S_t transit over time. This can be achieved in two major steps.

The first is to have an estimate of the conditional $(S_t = s_t | y_{t-1})$, i.e., the probability of being in state s_t, based on information available at time $t - 1$.

According to the transition probability and property, that is straightforward. The second is to consider the joint probability density distribution of $y_t$and S_t, so the probability of being in state s_t is updated to $P(s_t = s_t | y_t)$, using information available at time t. The procedure is as follows:

(i) Estimating the probability of being in stae s_t, conditional on information at $t - 1$:

$$P(S_t = s_t | y_{t-1}) = P(S_t = s_t | S_{t-1} = s_{t-1}) \times P(S_{t-1} = s_{t-1} | y_{t-1}$$

(ii) (a) calculating the joint density distribution of $y_t$and S_t:

$$f(y_t, S_t = s_t | y_{t-1})$$
$$= f(y_t | S_t = s_t, y_{t-1}) \times P(S_t = s_t | y_{t-1})$$
$$= f(y_t | S_t = s_t, y_{t-1}) \times P(S_{t-1} = s_{t-1}) \times P(s_{t-1} | y_{t-1})$$

(b) calculating the density distribution of y_t

$$f(y_t | y_{t-1}) = \sum_{st=1}^{2} f(y_t, S_t = s_t | y_{t-1})$$

(c) calculating the following:

$$P(S_t = s_t | y_t) = \frac{f(y_t, S_t = s_t | y_{t-1})}{f(y_t | y_{t-1})}$$

That is the updated joint probability of y_t and S_t.

4. PROSPECTS FOR THE DEVELOPMENT OF FINANCIAL ECONOMETRICS MODELING

It is of course, difficult to predict with accuracy what will be the new and important econometric models of tomorrow. However, there are of course topics that are currently 'hot' and which are likely to see continued interest in the future. A non-exhaustive selection of these is discussed below.

4.1. Tail models

It is widely known that financial asset returns do not follow a normal distribution, but rather they are almost always leptokurtic, or fat-tailed. This observation has several implications for econometric modeling. First, models and inference procedures are required that are robust to non-normal error distributions. Second, the riskiness of holding a particular security is probably no longer appropriately measured by its variance alone.

4.2. Copulas and quantile regressions

New types of assets and structures in finance have led to increasingly complex dependencies that cannot be satisfactorily modelled in the classical framework. *Copulas* provide an alternative way to link together the individual (*marginal*) distributions of series to model their joint distribution. One attractive feature of copulas is that they can be applied to link together any marginal distributions that are proposed for the individual series. The most commonly used copulas are the Gaussian and Clayton copulas. They are particularly useful for modelling the relationships between the tails of series, and find applications in stress testing and simulation analysis[18].

The possibility of application in the risk management arena has also stimulated renewed interest in another rather old technique, which has now become fashionable, known as *quantile regression*. Dating back to Koenker and Bassett (1978), quantile regression involves constructing a set of regression curves each for different quantiles of the conditional distribution of the dependent variable. So, for example, we could look at the dependency of y on x in the tails of y's distribution. This set of regression estimates will provide a more detailed analysis of the entire relationship between the dependent and independent variables than a standard regression model would[19].

4.3. Higher moment models

Research over the past two decades has moved from examination purely of the first moment of financial time series (i.e. estimating models for the returns themselves), to consideration of the *second moment* (models for the variance).

An extension of the analysis to moments of the return distribution higher than the second has also been undertaken in the context of the capital asset pricing model, where the conditional co-skewness and cokurtosis of the asset's returns with the market's are accounted for (e.g., Hung *et al*., 2004). A recent study by Brooks *et al*. (2006) proposed a utility based framework for the determination of optimal hedge ratios that can allow for the impact of higher moments on the hedging decision in the context of hedging commodity exposures with futures contracts.

5. CONCLUSIONS

Expansion of the interest in the modeling and analyzing of financial data and the problems to which they are applied should be taken in account. In this way, this paper was provided a structural and conceptual basis for financial modeling.

Indeed, we presented a variety of techniques that are commonly used for the analysis of financial data, including topics that would usually be treated only in a mathematically advanced way.

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[18] For introductions to this area and applications in finance and risk management. see Nelsen (2006), Alexander (2008, chapter 4) and Embrechts et al. (2003).
[19] See Koenker, 2005.

The article put forward an outline of some stylized characteristics of finance and described some econometric software packages that is widely employed for the financial data exploration.

As the final aspect of the financial models is their application to finance, this paper presented a set of financial models used in capital market and corporate finance by assuming that each model has the same consistent framework of analysis, which will enable readers to gain insights into financial systems in one coherent view. The techniques and models presented included basic concepts of price and returns, ARCH Garch models, Panel data models and Markov Switching models.

Finally, we reported that financial econometrics modeling will have a good prospective in the future especially with the expected developments of the tail models, Copulas and quantile regressions and the higher moment models.

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Annex: Comparison of statistical packages for finance

Product	Developer	Latest vesrsion	Interface	Rgression												
				OLS	WLS	2SLS	NLSS	Logistic	GLM	LAD	Stepwise	Quantile	Probit	Cox	Poisson	MLR
EVIEWS	IHS	November 2011	CLI/GUI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		Yes	Yes
GAUSS	Aptech systems	October 2011	CLI/GUI							No		No				
MATLAB	MathWorks	New releases twice per year	CLI/GUI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		Yes	Yes	Yes	Yes
RATS	Estima	October 1, 2010	CLI/GUI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		Yes	Yes
SAS	SAS Institute	December 2011	CLI/GUI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SHAZAM	SHAZAM Analytics Ltd	June 2012	CLI/GUI	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		Yes	Yes
SIGMAXL	SigmaXL Inc	January 18, 2011	GUI	Yes				Yes								Yes
SPSS	IBM	2011	CLI/GUI	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes					
TSP	TSP International	September 2009	CLI	Yes	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes		Yes	Yes

Product	Time Series Analysis						Staistical charts and diagrams			
	ARIMA	**GARCH**	**Unit root test**	**Cointegration test**	**VAR**	**Multivariate GARCH**	**Correlogram**	**Histogram**	**Scatter-plots**	
EVIEWS	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
GAUSS	Yes	Yes			Yes	Yes		Yes	Yes	
MATLAB	Yes	No	No	No	No	No	Yes	Yes	Yes	
RATS	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
SAS	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
SHAZAM	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	
SPSS	Yes						Yes	Yes	Yes	
TSP	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	