Do Structural Breaks exist in Okun's Law? Evidence from the Lost Decade in Japan

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Abstract

This paper presents an analysis as to whether or not there has been a structural break of Okun's Law, and if so, how many, when and what kind of break, using Bayesian methods via MCMC (Markov Chain Monte Carlo) simulation for the Japanese data from 1961Q1 to 2001Q1. In addition, it identifies structural breaks both in trends (normal level) and cyclical components (normal level), which are decomposed from output and unemployment rate by new proposed methods as well as widespread methods: H-P filter and Band Pass filter, in contrast to the previous studies of several countries.

JEL cassification : C11, C13, E24, E32

Keyword : Structural Change Test with Unknown Timing, MCMC, Kalman Filter, Hodrick-Prescott Filter, Band Pass Filter, Bayesian inference,

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1 Introduction

Okun's law, an empirical negative relationship between movements of the unemployment rate and the real gross domestic product (GDP) is important issue to understand macroeconomy. This relationship has been considered very stable over time, contrast to unstable Phillips curve. Hence Okun's Law plays significant role to separate cyclical components from gnormalh level of economic activity such as natural unemployment rate and potential growth rate. This paper considers the possibility of structural breaks of Okun's Law using Bayesian methods via MCMC (Markov Chain Monte Carlo) simulation for the Japanese data, and identifies structural breaks in trends (normal level) and cyclical components deviation from normal level), in contrast to the previous studies of Okun's Law in several countries. The Japanese data has two special features contrast to those of other countries. First, the stagnation of Japanese economy in the 1990s has been very serious, and the cause of stagnation has been widely discussed. Among the issues, an important controversy arose how much the potential GDP growth rate falls. Krugman (1998) stated in his article considering gliquidity traph that::

> *If we were to take the average 2.5 percent unemployment rate in the pre-slump period as an estimate of the natural rate, the 3.4 percent unemployment rate in 1997 would therefore seem to imply an output gap of more than 5 percent last year - and with potential output still presumably growing while output slumps, the gap by end-1998 could be as high as 10 percent.*

Second and related, if estimated Okun's Law is stable even in 1990s, the above statement by Krugman is right and output gap in Japan is very large. The previous estimation of Okun's Law, however, even using the Japanese data of pre-bubble period by Hamada and Kurosaka (1985) suggested that the relationship is very unstable. Figure 1 plotted that Okun's Law in Japan using data (a) from 1981 to 1991, which confirms stable relationship as in the Krugman's statement, (b) from 1961 to 2003, which seems unstable, and (c) from 1992 to 2003, which looks shifted to the left hand side and indicated structural breaks.

[insert Figure 1]

Hence, this study examines whether there is structural changes of the Okun's law for the recent forty years, i.e., the first quarter of 1961 through the first quarter of 2001, in the Japanese economy. To this end, we firstly decompose the seasonally adjusted quarterly series of both real GDP and unemployment rate to two factors; cyclical factor and trend factor, using the following four methods. That is, two new techniques proposed by this study, as well as a couple of techniques by earlier studies estimating cyclical component of output; Hodrick-Prescott (1997) decomposition and band pass filter by Baxter and King (1999).

The latter two widespread methods decompose to non-linear cycle and trend components without considering structural breaks. In contrast, the former two new approaches consider the presence of multiple structural breaks for trend, when decomposing to the two factors. Since the Japanese economy has encountered several drastic changes such as high growth in the 1960s, hyper-inflation by the oil shock in the 1970s, stable growth in the 1980s, and the so-called "lost decade" of the 1990s, for these four decades, it is plausible that we add the presence of multiple unknown structural breaks into trend of output and unemployment rate.

The one of new decomposition technique is based on a Bayesian approach via MCMC (Markov Chain Monte Carlo) simulation, which estimates linear trend considering multiple unknown structural breaks. The other extracts stochastic trend with multiple structural breaks, using Kalman filter. And then, we estimate the Okun's coefficient between the decomposed cyclical factors of output and unemployment rate, by three different methods proposed by Weber (1995) and examine whether or not there has been a structural break with unknown timing for the coefficient, following Andrew and Ploberger (1994), and if the breaks are found, how many, when and what kind of break in the Okun's coefficient.

There were several fact-findings from the estimation results. (1) There was slope for trend of the unemployment rate no matter what decomposition methods were used. This change rate, however, was quite tiny. Cyclical components of unemployment rate were clearly negative correlated with cyclical GDP. In addition, there is a gap between the cycles of unemployment rate and GDP by two through three quarters. (2) The range of cyclical component of the unemployment rate might be quite narrow; at most 0.7% through 1.0 %, whereas that of trend component is wide; around 4 %. (3) The structural change of trend of unemployment rate arises one year through three years after that of trend of real GDP. (4) The both trends of GDP and unemployment rate were close to linear until 1998. And after then, both trends became nonlinear because of enlarging the volatility of trends. (5) There might be at least one-time structural change in Okun's coefficient for the forty years. The coefficient changed by twice through fifth times so that cyclical component of unemployment rate has became more sensitive for the fluctuation of the business cycle after a structural change. The time of change, however, could not be identified at pinpoint, and the possibility of time was located between the beginning of 1970s and the end of the 1980s.

To sum up, the bands of fluctuations of the cyclical components in output and unemployment rate were small compared with the bands of their trends in Japan. In contrast, Clark (1989) and Kim and Nelson (1999a) show that their cycles occupied large portion of the band of fluctuation of entire series in the U.S. and European countries. Furthermore, we identified structural breaks both in trends (normal level) and cyclical components (normal level), in contrast to the previous studies of several countries.

Sogner and Stiassny (2002) also studied structural changes of Okun's coefficient using Kalman filter.

Their model assumed the change occurred smoothly and was based on a simple auto regressive model directly using the difference of macroeconomic variables without decomposing into cycle and trend. In contrast to our study, they could not find structural change of Okun'law in the recent Japanese economy. The reason would be that they were confused irregular movement of trend components which accounted for large part of fluctuation in Japan and that they failed to capture the cycle movements.

The organization of this paper is follows. Section 2 describes the four decomposition methods, and then discusses the results of decomposition. In section 3 we estimate the Okun's coefficient based on Weber's methods and the evidence of stability of the coefficient is presented. Section 4 concludes the paper.

2 Cycles and Trends in Output and Unempoyement Rate

Okun's law is known as a rule of thumb denoting an inverse relationship between cyclical fluctuation in output and change in realized unemployment rates from *natural rate of unemployment* (NRU). We give the following empirical framework for the law.

Let y_t and y_t^n represent the logs of observed and potential GDP, respectively. Similarly, let U_t and U_t^n represent the observed unemployment rate and NRU, respectively.

$$y_t^c \equiv y_t - y_t^n \tag{1}$$

$$U_t^c \equiv U_t - U_t^n \tag{2}$$

$$U_t^c = \alpha y_t^c, \quad \alpha < 0 \tag{3}$$

where y_t^c is the cyclical fluctuation in GDP, the log output ratio; U_t^c is the cyclical unemployment rate, the difference between the observed unemployment rate and the natural rate. Thus, equation (3) is regarded as one version of Okun's law, and α is the Okun's coefficient. However, there is one difficulty in estimating equation (3). That is measuring U_t^n and y_t^n, which are unobserved.

For estimating the Okun's coefficient in the U.S. economy, Perron (1989, 1990), Evans (1989) and Weber (1995) used linear trends of y_t and U_t derived from the following equations (4) and (5) in the place of y_t^n and U_t^n. This linear trend was changed at structural break point arisen by the first oil shock in the forth quarter of 1973 in the following equations;

$$y_t = \beta_0 + \beta_1 t + \beta_2 D_t t + \epsilon_{yt} \tag{4}$$

$$U_t = \gamma_0 + \gamma_1 D_t + \epsilon_{ut} \tag{5}$$

where t is a time trend, D_t is a dummy variable eqaul to zero up to and including the forth quarter of 1973 and equal to one afterward, and ϵ_{yt} and ϵ_{ut} are stationary random disturbance terms and regarded as cyclical components, y_t^c and U_t^c, respectively.

Grant (2002) used Hodrick-Prescott (1997) filter as well as linear trend and Perron's trend, to measure the potential output y_t^n. In above studies, a *slope* trend as equation (4) is used as potential output. Meanwhile, a *horizontal* trend as equation (5) is generally used for unemployment rate in earlier studies for the U.S. economy and the European economies, since the level of NRU is thought to be unchanged at unique point, whose size is depended on each country's circumstance, as long as no structural breaks. A slope trend, however, is most likely to be appropriate for NRU in the recent Japanese economy as shown in this section. One possibility of reasons why NRU change over time might be hysteresis effect in unemployment (see e.g. Blanchard and Summers, 1986 and Cross, 1988). Thus, we use a slope trend following the manner of Clark (1989) who estimated the cyclical components of real output and unemployment rate in the developed countries including Japan.

In our study, both trends of unemployment rate and real GDP; U_t^n and y_t^n, are measured using the following four decompositions; (1) Hodrick and Prescott decomposition, (2) Baxter and King decomposition, (3) Linear Trend with Multiple Breaks, and (4) Stochastic Trend with Multiple Breaks. The former two methods do not consider structural breaks of trends, whereas the latter two methods do consider the possibility of them. As described in the introduction, the Japanese economy has experienced various turning points for the postwar period, so it is plausible that U_t^n and y_t^n, have multiple structural breaks for the period. Employed data is seasonally adjusted quarterly series of real GDP and unemployment rate from 1961Q1 to 2001Q1.

2.1 Hodrick and Prescott Decomposition

The work of Hodrick and Prescott (1997) provides a widespread method for breaking a series into trend and cyclical components. Hodrick and Prescott observe that the business cycle deals primarily with high frequency fluctuations of GDP in the neighborhood of 4 - 8 years. Their decomposition allows these *short* wave fluctuations to represent cyclical movements of GDP, while incorporating *long* wave into a flexible trend.

To accomplish this, Hodrick and Prescott consider an economic time series $\{x_t\}$. The unobserved long run stochastic trend of this series is denoted $\{s_t\}$, and the irregular, or cyclical, component of series is defined as $\{x_t - s_t\}$.

To split the series into trend and irregular components, Hodrick and Prescott consider the following optimization framework:

$$\min_{\{s_t\}} H = \left\{ (1/t) \sum_{t=1}^{T} (x_t - s_t) + (\lambda/T) \sum_{t=2}^{T-1} \left[(s_{t+1} - s_t) - (s_t - s_{t-1}) \right] \right\} \tag{6}$$

The first term can be viewed as a limiting factor that penalizes overly large irregular components. The second term represents a penalty for allowing the stochastic trend to change smoothly. The magnitude of penalty hinges critically on the value λ chosen: if λ = 0, all movement of the x_t series is assumed to be generated by permanent fluctuations. As λ approaches infinity, movements in x_t are increasingly assumed to be attributable to transitory disturbances and the trend becomes increasingly smooth. In the limit the stochastic trend becomes a straight line and the problem reduces to the deterministic trend assumption of purely linear model.

This technique was used to solve for the unobserved permanent component s_t of both quarterly series in output and unemployment rate from 1961Q1 to 2001Q1. Following Hodrick and Prescott (1997), the parameter λ is chosen to be 1600. This choice of λ preserves high frequency components above $\pi/16$, corresponding to a cycle length of approximately eight years.

[Insert Table 1]

[Insert Figure 2]

The top and the middle of Figure 2 show the raw series x_t and trend components s_t in real GDP and unemployment rate, respectively. The straight line and dash line denote raw series x_t and trend series s_t, respectively. The filer trend succeed in capturing two productivity slowdowns of the beginning of 1970s and 1990s for real GDP, whereas there were two upward kinks of the trend between 1973Q3 and 2001Q1 sandwiching downward curve in the end of the 1980s for unemployment rate. As can be seen from the bottom of Figure 2, major postwar business are generally well defined from both cyclical components of y_t and U_t. The straight line and dash line denote real GDP and unemployment rate, respectively. (The shade regions of the bottom of Figure 2 represent recessions by the report of Economic Social Research Institution (ESRI).) Here, multiplied by 100 for y_t^c, the cyclical GDP is expressed with the percentage. On the other hand, multiplied by -1 for U_t^c, the cyclical unemployment rate, which is expressed with the percentage, is reversed at the x axis in the bottom of figures in order to specify the sympathy between two cyclical components. (In the cases of the other decompositions described as below, we also transform the both components in the same way.) Table 2 represents correlation coefficients between cyclical output and lags of cyclical unemployment rate. The cyclical unemployment rate was likely to delay two quarters from cyclical output. The correlation coefficient of lag two is -0.662 and the largest of all lag orders.

2.2 Band Pass Filter by Baxter and King

Baxter and King (1999) proposed a specific band pass filter used to capture fluctuations with a period of length 8 to 32 quarters in the U.S. GDP series. The Baxter-King decomposition regards a centered moving average with symmetric weights as a cyclical component x_t^c; that is,

$$x_t^c = \sum_{i=-K}^{K} w_i x_{t-i}. \tag{7}$$

The coefficients w_i of Baxter-King filter are derived under the constraint that the filter gain should be zero as zero frequency. This constraint leads to the requirement that the sum of the filter coefficients must be zero.

In order to capture the major features of business cycles from quarterly data set, Baxter and King (1999) recommended a lead/lag length of K = 12. The Baxter-King (8, 32) filter admits most frequency components between 8 to 32 quarters by removing the low-frequency trend variation and by smoothing high-frequency irregular variation.

[insert Figure 3]

In Figure 3, Baxter-King (8, 32) filter outputs are presented. The bottom of the figure show cyclical components of real GDP and unemployment rate. The cyclical unemployment rate U_t^c is reversed as the same way of the bottom of Figure 2. Like the Hodrick-Prescott filter case, both could capture the business cycle well. However, the change in unemployment rate seems to follow the fluctuation of real GDP. In fact, as Table 1 the negative correlation between lag two of unemployment rate and cyclical GDP account for as high as -0.669 so that we found the delay of two quarters between them as well as the result of the former decomposition. In the top and the middle of the figure, real GDP series and unemployment rate series, which are the difference between the raw series and the cyclical components, $\{x_t - x_t^c\}$, are shown, respectively. Since they are the admixtures of the low-frequency trend factor and high-frequency irregular factor, we do not analyze in detail. But the movements of these series were very close to the trend of Hodrick-Prescott decomposition case.

The Baxter-King filter has been criticized on the ground that it may induce spurious dynamic properties and that its cyclical component fails to capture a significant fraction of the variability in business-cycle frequencies. Nevertheless, both the Hodrick-Prescott and Baxter-King filter have become standard tools in current business cycle studies. Notice that the Baxter-King output is smoother than Hodrick-Prescott output. However, there are 12 missing points at each end of the Baxter-King (8, 32) filter output as shown in Figure 3, since it is a noncausal filter.

2.3 Linear Trend with Multiple Breaks

The trend stationary representation of the real GDP series has received a wealth of recent attention in the literature. Perron (1989) revisit the linear trend hypothesis by accounting for a one-time structural change in the path of trend GDP. By Monte Carlo simulation, he shows that transitional hypothesis tests are biased in favor of a unit root when a series exhibits structural break. His analysis contains important implications for macroeconomist. The unit root hypothesis implies that all shocks permanently affect the variable of interest. But Perron's analysis implies that the only two shocks which have had a permanent impact on output were the Great Depression and the oil shock. Accordingly, Perron adopted linear trend such as equations (4) and (5) considering the presence of structural break. We follow him and built linear trend model with multiple changes.

But, in order to build a model that takes the presence of these structural changes into account, we must first address the following issues: (i) Do structural changes really exist? (ii) How many changes have occurred? (iii) When did these changes occur? To cope with these obstacles, we adopt Bayesian inference and built linear trend models that consider multiple change points, following the manner of Chib (1998)[1]. This is because there are some problems with the tests of unknown timing in the classical framework proposed by Andrews and Ploberger (1994), *etc.*. One of these problems is that the test statistics are based on a nonstandard asymptotic distribution, because the estimated change-point becomes a nuisance parameter that exists only under the alternative hypothesis that structural change occurred. Another problem is that, with only a few exceptions (e.g., Bai and Perron, 1998), theoretical studies have dealt with the issue of test statistics only for one-time structural breaks, but have not yet considered the case of multiple breaks.

In contrast, a Bayesian approach is very applicable to this problem. Koop and Potter (1999) demonstrated that a Bayesian approach is superior to the classical approach for nonlinear models, of which structural break models are a subset. In addition, only a Bayesian approach allows a comparison among models with various numbers of change-points, and a selection of the model with the most appropriate number and timing of such points, using the Bayes factor. (In Bayesian approach, model selection is implemented based on the size of Bayes factor, like using Akaike information criterion for selecting a model in classical approach.) Using the model selection procedure, we specify the number and timing of breaks. As a byproduct of Bayesian inference, posterior distribution of break points can be obtained as shown later.

Let linear trend with multiple breaks be described as the followings.

[1] Works investigating structural change of economic fluctuation based on Chib (1998) are Kim and Nelson (1999b) and Kim, Nelson and Piger (2004). The former dealt with the structural change of business cycle in the U.S. using real GDP quarterly series. The latter investigated the structural change of volatility of many macroeconomic variables in the U.S.

$$x_t = \beta_{0t} + \beta_{1t} z + \epsilon_t, \quad \epsilon_t \sim iidN(0, \sigma^2) \tag{8}$$

$$D_{0t} = \begin{cases} 1 & \text{if } 0 < t < \tau_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$D_{it} = \begin{cases} 1 & \text{if } \tau_i \leq t < \tau_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$D_{Nt} = \begin{cases} 1 & \text{if } \tau_N \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_{0t} = \beta_{00} D_{0t} + \beta_{01} D_{1t} + \cdots + \beta_{0N} D_{Nt} \tag{9}$$

$$\beta_{1t} = \beta_{10} D_{0t} + \beta_{11} D_{1t} + \cdots + \beta_{1N} D_{Nt} \tag{10}$$

where z is time trend and the tick size of z is set with 0.25 since employed data is quarterly. Thus, β_{1t} is expressed as the annual mean of change rate. D_{it} is shift parameter equal to one when a period t is in i-th regime changed by structural breaks and equal to zero for the outside of i-th regime. N is the number of structural changes.

And, the shift parameter D_{it} is determined based on the following Markov process proposed by Chib (1998).

$$\Pr(D_{it} = 0 \mid D_{it-1} = 0, D_{i-1,t-1} = 1) = p_i, \tag{11}$$

$$\Pr(D_{it} = 1 \mid D_{it-1} = 0, D_{i-1,t-1} = 1) = 1 - p_i, \tag{12}$$

$$\Pr(D_{Nt} = 1 \mid D_{Nt-1} = 1) = 1, \tag{13}$$

$$\Pr(D_{Nt} = 0 \mid D_{Nt-1} = 1) = 0, \tag{14}$$

where equations (11), (12), (13) and (14) indicate that the transition probabilities of the shift parameter of i-th regime, D_{it}, depend only on the value of shift parameters of the previous period, D_{it-1}, and $D_{i-1,t-1}$, but not on other factors, and that these probabilities are fixed with respect to time. In addition, they mean that each current regime is irreversible with its previous regime; i-th regime can either stay at its current sate or jump to the next regime, $(i+1)$-th regime. That is, for $i = 1, 2, \cdots, N-1$, equation (11) denotes the probability that $(i-1)$-th regime remains at the period t when the previous period belongs to the $(i-1)$-th regime is constant value p_i, whereas equation (12) denotes the probability changing to the next regime, the i-th regime, at the period t is $1 - p_i$. Equations (13) and (14) express the transition probability in the last regime, N-th regime; the transition probability that N-th regime remains at period t is one, while that of changing to the other regimes is zero.

[Insert Table 2]

[Insert Figure 4]

Following Chib (1995), we calculated Bayes factor for the models with several change points. In real GDP series, the model with three change point was selected, and in unemployment rate series the model with four points was selected. The estimation result was summarized in Table 2. In GDP in order to express the percentage, the observations were multiplied by 100. The top and the middle of Figure 4 draw decomposed trends of GDP and unemployment rate, respectively, with posterior distributions of structural change points. The modes of the posterior distributions of three structural change points in GDP are located in 1974Q1, 1990Q3, and 1996Q1. The annual means of growth rate are 9.1% between 1961Q1 and 1973Q4, 3.8% between 1974Q1 and 1990Q2, nearly 1.0% between 1990Q3 and 1995Q4, and -0.1% between 1996Q1 and 2001Q1, according to the size of β_{1i} for $i = 0, 1, 2, 3$ in Table 2. Meanwhile, the four points are 1974Q4, 1988Q2, 1993Q2, and 1998Q2 in the unemployment rate as the middle of Figure 4. The annual means of change rate of the five regimes in the unemployment rate are very small such as -0.003%, 0.08%, -0.06%, 0.21%, and 0.09%, respectively, from the magnitude of β_{1i} for $i = 0, 1, 2, 3, 4$ in Table 2. In contrast to the change rate, the jumps at structural break points, which are generated by a change of the intercept β_{0i} for $i = 0, 1, 2, 3$, are quite large; at the first break point, the rate jumped by 0.42%, at the second point 0.42% down, at the third break point 0.55% up, and at the forth break point 0.83% up. In the terms of trend, the structural breaks of unemployment rate was very likely to arise one year through three years after the breaks of GDP as can be seen from the posterior distributions of break points in Figure 4.

The bottom of Figure 4 shows the cyclical components of both series decomposed by linear trend with multiple breaks. General speaking, both cyclical factors seems to capture the business cycle well. The lag of unemployment rate is three in Table 1, so that the unemployment rate delays three quarters for the cyclical output. But, the correlation coefficient is not much large (-0.46), compared with the former two decompositions. The reason is that the jumps of trend factor arisen by structural breaks absorbed the fluctuation of the cyclical factors of two series at break points in the model. And so, we consider the model overcoming this drawback in the next subsection.

2.4 Stochastic Trend with Multiple Breaks

In contrast to Perron (1989)'s finding described above, Leybourne, Mills and Newbold (1998) and Leybourne and Newbold (2000) pointed out that Dickey-Fuller tests in which unit root is set as null hypothesis are biased in favor of stationary process when a series exhibits structural breaks. (It is referred to converse Perron phenomenon.) This suggests that we should consider the possibility of non-stationary trend even though Dickey-Fuller tests reject the unit root hypothesis when there is break in the series. In addition, hysteresis hypothesis asserts that the unemployment rate has path dependence and there is long persistence for all shocks. It means that the unemployment rate behaves random walk. Hence, we propose stochastic trend model with multiple breaks and decompose both GDP and the unemployment rate using this model.

Nelson and Plosser (1982) suggested that the non-stationarity in economic activity should be removed by first-differencing rather than linear detrending, making the trend component of real GDP as random walk with drift rather than a deterministic function of time. Noting that annual average blurs the pattern of economic activity apparent in quarterly data, Clark (1989) applies Kalman filter to quarterly real GDP and unemployment rate in order to evaluate the relative importance of the stochastic trend and the stationary cyclical components of economic activity. Here, we apply Clark's unobserved components model to quarterly real GDP and unemployment rate for the period 1961Q1 through 2001Q1.

To distinguish between linear trend and stochastic trend models of real output, Clark (1989) consider an unobserved components model using Kalman filter. His model is applied and extended to a model with multiple breaks as below.

$$x_t = C_t + T_t \tag{15}$$

$$C_t = \alpha_1 C_{t-1} + \alpha_2 C_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim iidN(0, \sigma_\epsilon^2) \tag{16}$$

$$T_t = g_t + T_{t-1} + \nu_t, \quad \nu_t \sim N(0, \sigma_t^2) \tag{17}$$

$$g_t = g_0 D_{0t} + g_1 D_{1t} + \cdots + g_N D_{Nt} \tag{18}$$

$$\sigma_t^2 = \sigma_0^2 D_{0t} + \sigma_1^2 D_{1t} + \cdots + \sigma_N^2 D_{Nt} \tag{19}$$

$$D_{0t} = \begin{cases} 1 & \text{if } 0 < t < \tau_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$D_{it} = \begin{cases} 1 & \text{if } \tau_i \leq t < \tau_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$D_{Nt} = \begin{cases} 1 & \text{if } \tau_N \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

where x_t is macroeconomic variables: the log of real GDP or the unemployment rate, T_t is a stochastic trend component which has drift term g_t, and C_t is a stationary cyclical component; ε_t and ν_t are independent white noise processes. In this model, drift term g_t and diffusion term ν_t are assumed to arise structural changes in equation (17) based on equations (18) and (19). D_{it} is dummy variables equal to one when a period t is in i-th regime changed by structural breaks and equal to zero for the outside of i-th regime. N is the number of structural changes. Drift term g_t expresses the expected quarterly growth rate of trend T_t so that a structural change of g_t derives a change of the mean of growth rate. On the other hand, the change of diffusion term ν_t would effect the shape of trend. When the size of ν_t is very close to zero, the trend seems to be linear. When ν_t is large, it becomes non-linear such as random walk.

Unlike the last model, we could not estimate this model with unknown timing break points. Instead we chose the break points estimated in Section 2.3 as dummy variables D_{it} of this model. Although the four break points were exploited in the unemployment rate in Section 2.3, the estimation of the third point is distributed with quite wide range as the middle of Figure 3. So we omit the third point from selecting dummy variables and then we set 1974Q4, 1988Q2 and 1998Q2 as three structural change points in the above model. For estimating the model, we use Kalman filter by maximum likelihood estimation. In order to express the percentage, the log of real GDP y_t is multiplied by 100.

[Insert Table 3]

Table 3 reports estimation results. Both trends of GDP and unemployment rate are likely to be close to linear up to 1998 since the size of variances of trend σ_0^2, σ_1^2 and σ_2^2 are nearly zero contrast to the variance of cycle σ_ϵ^2. In particular, the variance $\sigma_1^2$of diffusion term in real GDP between 1974Q4 and 1988Q1 and that of unemployment rate between 1961Q1 and 1973Q3 are definitely zero. Thus, the main factor of structural change in permanent component would be the change of its growth rate. In real GDP the quarterly mean g_0 of growth rate was 2.18% up to 1974Q3, and then the slowdown of productivity by the oil shock decreased the mean g_1 and g_2 to 0.74% through 0.85% between 1974Q4 and 1998Q2. Asian financial crisis in 1997 made the quarterly mean g_3 drop down to negative value such as -0.02%. Meanwhile, the unemployment rate had undergone a transition with horizontal trend (almost zero percent growth) until the oil shock increased the mean of growth rate to quite low rate such as 0.03%. Then this rate became more mild (0.01%) between 1988Q2 and 1998Q2. And financial crisis also attacked unemployment rate and rose it up to 0.09% in 1998Q2.@

[Insert Figure 5]

Figure 5 plots the log of real GDP and unemployment rate along their trends and cyclical components implied by the models. The cyclical components of GDP and unemployment rate behave different movement from the former three models. That is, this decomposition method evaluated cyclical factor with large value for around a decade between 1988 and 1997. (This period is known to big economic fluctuation so-called gBubble erah and gBubble burst erah.) Hodrick-Prescott filter and Baxter-King filter described in Sections 2.1 and 2.2 decomposed the two variables to trends as non-linearity and was likely to underestimate cyclical components because of absorbing the fluctuation of this period as nonlinear trend. The linear trend model mentioned in Section 2.3 might absorb the fluctuation as the change of intercept β_{0t} of time trend in equation (8) at a break point and derive to underestimate the cyclical factor at the point. On the other hand, in the model explained in this subsection as in Clark (1989), trend is estimated as almost linearity and a significant portion of the quarter to quarter innovations in real GDP and unemployment rate are cyclical from the model. As Table 1, the cycle decomposed by the forth method of unemployment rate delays three quarters for GDP, and the correlation coefficient takes the highest value -0.875 in the four decompositions.

The advantage of this model is not to over-estimate the cyclical factor at the period immediately before breaks because the almost linear trends kinks at just break points in this model. In the case of Hodrick-Prescott filter and Baxter-King filter, the non-linear trend rounded within the periods immediately before structural breaks and, as a result, under-estimate the portion of the trends and over-estimate that of the cycles for the periods.

2.5 Comparison of results obtained decomposition methods

Notice that we do not have a criterion to measure which decomposition method superiors to others and it is difficult to evaluate the performance of these filters, although we decomposed GDP and unemployment rate to unobserved cyclical and permanent factors using four methods as above. In the next section, the output of each method is equally dealt with and using it the Okuns's coefficient is estimated. Here, we sum up the result of decompositions in this Section. (1) There was slope for trend of the unemployment rate no matter what methods were used. But this change rate was quite tiny. Cyclical components of unemployment rate were clearly negative correlated with cyclical GDP. In addition, cycle of unemployment rate delays two to three quarters for fluctuation of GDP. (2) The range of cyclical component of the unemployment rate might be quite narrow; at most 0.7% through 1.0 %, whereas that of trend component is wide; around 4 %. (3) The structural change of trend of unemployment rate occurred one year through three years after that of trend of real GDP. (4) The both trends of GDP and unemployment rate were close to linear until 1998. And after then, both trends became nonlinear because of enlarging the volatility of trends.

3 Empirical Evidence of Structural Change in Okun's Coefficient

3.1 Methods of Estimating Okun's Coefficient

We use three different methods to estimate Okun's coefficient, following Weber (1995) [2]. Although Weber derived cyclical components y_t^v and U_t^c using linear trend model with one-time break such as equations (4) and (5) and setting the oil shock in 1973Q4 as a break point, we derived these factors from the four decompositions explained in the previous section and compare the coefficient estimated from one decomposition with those of other decompositions. His three methods are described as followings. The first approach is static OLS applied to equation (3). Second, a long-run version of the coefficient can be estimated using a method of Gordon (1984). First, an autoregressive-distributed lag model is estimated for the unemployment rate:

$$U_t^c = \sum_{i=1}^{k} \delta_{1i} U_{t-i}^c + \sum_{i=1}^{k} \delta_{2i} y_{t-i}^c + \varepsilon_{ut} \tag{20}$$

Let d_{1i} and d_{2i} be estimates of δ_{1i} and δ_{2i}. Then, α^{LR} an estimate of the impact of a change in y_t^v on U_t^c in the long run, is

$$\alpha^{LR} = \frac{\sum_{i=1}^{k} d_{2i}}{1 - \sum_{i=1}^{k} d_{1i}} \tag{21}$$

The third method is adapted from Blanchard (1989). This method estimate a version of α relating innovations in cyclical unemployment rate to innovations in cyclical output. A two-step procedure is used: First, estimate a bivariate VAR, including equation (20) and analogous AR-DL equation for y_t^c:

$$y_t^c = \sum_{i=1}^{k} \delta_{3i} U_{t-i}^c + \sum_{i=1}^{k} \delta_{4i} y_{t-i}^c + \varepsilon_{yt} \tag{22}$$

Using e_{ut} and e_{yt}, the estimators of ε_{ut} and ε_{yt}, Okun's coefficient is estimated in the process of retrieving structural innovations from their reduce-form counterparts by estimating

$$e_{ut} = a e_{yt} + u_t \tag{23}$$

where u_t is the portion of cyclical unemployment rate innovation which is orthogonal to the cyclical GDP innovation.

[2]Weber(1995) also proposed a method estimating Okun's coefficient from cointegration coefficient as the forth method. But we have already de-trended macroeconomic variables using different decompositions and make it become stationary. So we thought that it was not necessary to use non-stationary approach and we omitted the method from estimating the coefficient.

[Insert Table 4]

Table 4 summarizes the coefficients estimated for the full sample period based on Weber's three manners. In terms of the dynamic model and Blanchard model, we set the number of lag order as $K = 2$ and $K = 4$ since Table 1 showed the cyclical unemployment rate delayed two through three quarters for the cyclical GDP. Weber (1995) showed the estimators of the coefficients derived from three methods are close to the others in the U.S. economy. On the other hand, in the recent Japanese economy Table 4 shows that the estimators of Blanchard model are completely different from those of the other models: static OLS model and dynamic OLS model, in all of the four decompositions, although the estimators are similar between static OLS model and dynamic OLS model.

Note that the estimators are different against decomposition methods. Focus on the result of static model and dynamic model. The estimators derived from stochastic trend with multiple breaks are between 0.104 and 0.139 and twice or three times larger than the estimator of the rest of decompositions. Meanwhile, in the case of Hodrick-Prescott decomposition those are between 0.049 and 0.069, in Baxter and King decompositions those are between 0.033 and 0.061, and in the case of linear trend model those are the smallest and between 0.026 and 0.033.

These sizes are depended on how much the fluctuation of permanent component absorbs the cyclical component. On one hand, several breaks of permanent component in the linear trend model absorbed part of the cyclical component at break points and thus made it become small as explained in the previous section. In the similar way, the swinging part of flexible trend also absorbed the fluctuation of cyclical factor in both of Hodrick-Prescott and Baxter-King decompositions. On the other hand, trend derived from the stochastic trend model were nearly linear because of small value in diffusion term ν_t in equation (17). And as a result, the cyclical components of GDP and the unemployment rate account for quite large part of these series compared with other three decompositions. In additions, their correlation was the highest in the four decompositions. These things would lead to high value in the Okun's coefficient. It is worth noting that the size of Okun's coefficient depends on decomposition methods and on break points in these methods, although we cannot evaluate which decomposition is superior to the others because of no criterion.

3.2 Structural Change Test of Okun's Coefficient

In order to examine the effect of structural change of the oil shock in 1973 on Okun's coefficient in the U.S. economy, Weber (1995) implemented Chow test for the estimator. However, we have no information on the number and timing of break points in Japan. Thus, we follow the manner of Andrews and Ploberger (1994) who proposed asymptotically optimal test procedure for testing problems in which a nuisance parameter, which corresponds to the time of change point in our study, exists under the alternative hypothesis but not under the null and they applied the test for one-time structural change test with unknown change-point. And we examine whether or not there has been a structural break in Okun's coefficients derived from Weber's three methods described in Section 3.1, and if so, how many, when and what kind of changes.

Firstly, we change the three Weber's models to that including one-time structural break. Accordingly, equations (3), (20) and (23), are reset into equations (24), (25) and (26), respectively.

$$U_t^c = (\alpha_0 + \alpha_1) y_t^c, \tag{24}$$

$$U_t^c = \sum_{i=1}^{k} d_{1i} U_{t-i}^c + \sum_{i=1}^{k} (d_{20i} + d_{21i}) y_{t-i}^c + \varepsilon_{ut} \tag{25}$$

$$e_{ut} = (a_0 + a_1) e_{yt} + u_t \tag{26}$$

where α_0, d_{20} and a_0 denote Okun's coefficient before a structural break, and $\alpha_0 + \alpha_1$, $d_{20i} + d_{21i}$ and $a_0 + a_1$ denote the coefficient after a break.

Secondly, average Lagrange multiplier (LM) test [3], which Andrews and Ploberger (1994) proposed as one kind of asymptotically optimal tests, is applied for the three models. The null hypothesis is that structural change does not occurred, i.e., $H_0 : \alpha_1 = 0$, $H_0 : \delta_{21i} = 0$ or $H_0 : a_1 = 0$. Although the manner of the test is based upon Chi-square test in which the numbers of the degree of freedom are those of changed parameters, this test statistics follows non-standard asymptotic distribution. This is because a change point π, which is a nuisance parameter in our models, does exist only under the alternative hypothesis that a break occurred but not the null. Hence, we test the null using the table of critical value calculated by Andrews and Ploberger (1994, p1401-1402).

The test results are summarized in Table 5. Viewed in their entirely, we verified that a structural change is most likely to exist for Okun's coefficients derived from all decompositions. In particular, in static

[3] In the procedure of average LM test, we iterate LM test of structural change models such as equations (24), (25) and (26), setting each time t as the time of change point π from the beginning of time up to the end of time in the period in which the presence of change point is suspected, and take average for the value of the LM test statistic across over the suspected period. The test statistics is written as the following equation;

$$\begin{aligned} Ave - LM_T &= \frac{1}{1 - 2\pi_0} \int_{\pi_0}^{1-\pi_0} LM_T(\pi) d\pi \\ &= \frac{1}{T(1 - 2\pi_0)} \left[\sum_{t=[T\pi_0]+1}^{T-[T\pi_0]-1} LM_T(t/T) \right. \\ &\left. + \left([T\pi_0] + 1 - T\pi_0 \right) \Big\{ LM_T([T\pi_0]/T) + LM_T\big((T - [T\pi_0])/T\big) \Big\} \right]. \end{aligned}$$

where T is the sample size, π denotes the time of structural change as a fraction of the sample size, and $LM(\pi)$ is Lagrange multiplier test statistics when a change exists at time π. This test examines the presence of change for a range between $T\pi_0$ and $T - [T\pi_0]$. Here we set $\pi_0 = 0.05$ so that the period suspected for change point is between 1962Q3 and 1999Q2.

OLS model there are a structural change for the underlying sample period at 1 % significant level for all decomposition methods. In term of dynamic OLS model ($k = 2, 4$), we saw that there is a break in both of Hodrick-Prescott decomposition and Baxer-King decomposition, whereas the null hypothesis was not significantly rejected for either $k = 2$ or $k = 4$ in linear trend model and stochastic trend model. In Blanchard model, a break is generally likely to exist in all methods except Baxter-King filtering method.

[Insert Table 5]

Next, we turn to estimate how many, when and what kind of changes in Okun's coefficient. To this end, we apply the model estimated by Bayesian method used in Section 2.3 for static OLS model, equation (24), because a structural change significantly exists in static OLS model for all decompositions. The numbers of changes are selected based on the value of Bayes factor of a model with corresponding change points. As the time of change points, the modes of estimated posterior distributions of change points are regarded.

Table 6 summarizes the number and timing of change points in the coefficient for the four methods. Figure 6 draws the posterior distributions of change points of the selected model from Bayes factor. The number of change point might be one in term of Hodrick-Prescott, Baxter-King, Stochastic trend model. The time of change point, however, was distributed with wide range and the modes of the posterior distributions were different from one another for these three models. That is, the timing of point is likely to be 1975Q1 in Hodrick-Prescott filter, 1980Q1 in Baxter-King filer and 1984Q3 in stochastic trend model. In addition, there are bimodal in the posterior distributions in the former two models in spite of selecting the model with one-time change point from Bayes factor. On the other hand, there would be two points in linear trend model; the first point might be 1980Q1, the second point 1984Q4.

[Insert Figure 6]

[Insert Table 6]

Based on the break points estimated in each decomposition method, full sample period 1961Q1 through 2001Q1 is divided into two or three regimes and Okun's coefficient is estimated for each regime as Table 7. In the similar way of Table 4 in Section 3.1, the values of the coefficient are different by kinds of adopted decomposition methods, and the estimators in Blanchard models, in which the coefficient is derived from innovations of unemployment and output, conflicts with those in static OLS and dynamic OLS. From the estimation result of static OLS and dynamic OLS, the structural change raised the size of Okun's coefficient by twice through five times; in the case of liner trend model the first change increased this size by about six times, while the second change dropped it down to the level in the first regime. As explained in Section 2, the size in the third regime is thought to be quite small in liner trend model since the estimated break of trend might have absorbed the fluctuation of cyclical component. Consequently, the time of change point cannot be identified at pinpoint, but it is thought that at least one-time structural break in Okun's coefficient existed across over between the beginning of 1970s and the end of the 1980s, and that the cyclical component of unemployment rate has became more sensitive for the fluctuation of the business cycle after a structural change.

[Insert Table 7]

The estimation results are graphically summarized in Figure 7 along the traditional form of Okun's law, $\Delta U_t^c = \beta_0 + \beta_1 \Delta y_t^c$. Normal growth rate of GDP has been downward shifted (smaller β_0') at several times while Okun coefficient β_1 has been steeper since several structual breaking point.

[Insert Figure 7]

4 Conclusion

This paper presents an analysis as to whether or not there has been a structural break of Okun's Law, and if so, how many, when and what kind of break, using Bayesian methods via MCMC (Markov Chain Monte Carlo) simulation for the Japanese data from 1961Q1 to 2001Q1. In addition, it identifies structural breaks both in trends (normal level) and cyclical components (normal level), which are decomposed from output and unemployment rate by new methods as well as widespread methods: H-P filter and Band Pass filter, in contrast to the previous studies of several countries.

There were several fact-findings from the estimation results.

(1) There was slope for trend of the unemployment rate no matter what methods were used. This change rate, however, was quite tiny. Cyclical components of unemployment rate were clearly negative correlated with cyclical GDP. In addition, cycle of unemployment rate delays two to three quarters for fluctuation of GDP. (2) The range of cyclical component of the unemployment rate might be quite narrow; at most 0.7% through 1.0 %, whereas that of trend component is wide; around 4 %. That is, the bands of fluctuations of the cyclical components in output and unemployment rate were small compared with the bands of their trends in Japan. In contrast, Clark (1989) and Kim and Nelson (1999a) show that their cycles occupied large portion of the band of fluctuation of entire series in the U.S. and European countries. (3) The structural change of trend of unemployment rate arises one year through three years after that of trend of real GDP. (4) The both trends of GDP and unemployment rate were close to linear until 1988. And after then, both

trends became nonlinear because of enlarging the volatility of trends. (5) There might be at least one-time structural change in Okun's coefficient for the forty years. The coefficient changed by twice through fifth times so that cyclical component of unemployment rate has became more sensitive for the fluctuation of the business cycle after a structural change. The time of change, however, could not be identified at pinpoint, and the possibility of time was located between the beginning of 1970s and the end of the 1980s.

In addition, our study suggested that the long relationship is also unstable because the trend of both variables had some breaks as different time. Accordingly, our study is against Krugman's (1998) statement. A reduction of potential output and a steep rise of natural rate of unemployment, which would correspond to their trends, might bring shrinkage of output gap in the 1990s. The true gap is most likely to not as much as Kugman's estimation quoted in the introduction.

And this unstable for long relationship denies the cointegration beween them. Attified and Silverstone (1998) estimated Okun's coefficient by interpreting it as cointegrating coefficient since there has been stable for the long relationship in the U.S. We, however, cannot apply their method to the Japanese economy.

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Table 1.

Correration between GDP and Unemployment Rate in Cycle Components

	Correlation Coefficient				
Decomposition Method	UR(0)	UR(+1)	UR(+2)	UR(+3)	UR(+4)
Hodrick-Prescott filter	-0.544	-0.619	-0.662	-0.644	-0.544
Baxter-King filter	-0.348	-0.550	-0.669	-0.663	-0.526
Linear Trend with Multiple Breaks	-0.392	-0.429	-0.442	-0.459	-0.381
Stochastic Trend with Multiple Breaks	-0.833	-0.857	-0.872	-0.875	-0.853

Table 2.

Linear Trend with Multiple Breaks

	GDP				Unemployment Rate			
parameter	mean	S.D.	95%band		mean	S.D.	95%band	
β_{00}	-16733.75	136.76	(-17013.61	-16485.03)	7.174	8.899	(-9.888	24.518)
β_{01}	-6256.33	100.45	(-6256.59	-6449.09)	-159.074	9.899	(-178.271	-140.102)
β_{02}	-609.85	575.28	(-1762.39	533.23)	130.199	67.649	(3.642	262.369)
β_{03}	1588.74	540.40	(551.50	2676.96)	-425.364	78.386	(-434.467	-551.718)
β_{04}	-	-	(-	-)	-171.351	96.554	(-179.130	-339.600)
β_{10}	9.107	0.069	(8.980	9.247)	-0.0030	0.0452	(-0.0118	0.0057)
β_{11}	3.794	0.051	(3.693	3.890)	0.0814	0.0050	(0.0719	0.0911)
β_{12}	0.959	0.288	(0.385	1.531)	-0.0643	0.0340	(-0.1312	-0.0008)
β_{13}	-0.139	0.270	(-0.687	0.376)	0.2147	0.0392	(0.1366	0.2778)
β_{14}	-	-	(-	-)	0.0880	0.0482	(-0.0141	0.1719)
σ^2	3.035	0.351	(2.433	3.779)	0.015	0.002	(0.012	0.020)
p_0	0.982	0.017	(0.936	0.999)	0.977	0.018	(0.929	0.998)
p_1	0.986	0.013	(0.950	0.999)	0.983	0.017	(0.939	0.999)
p_2	0.964	0.034	(0.876	0.999)	0.961	0.036	(0.866	0.999)
p_3	-	-	(-	-)	0.965	0.033	(0.872	0.999)

Note [a] The first 2,000 iterations of Gibbs sampler are discarded to guarantee convergence and then the next 10,000 iterations are used for calculating the posterior means, the standard deviations (S.D.) of the posterior means, 95% band, and the convergence diagnostic (CD) satistics proposed by Geweke (1992).

[b] See eq.(9), eq.(10),eq.(11) and eq.(12) for the notations of parameters.

[c] The posterior means are computed by averaging the simulated draws.

[d] 95 % bands refers to 95 % posterior probability bands. This bands are calculated using the 2.5-th and 97.5-th percentiles of the simulated draws.

[e] Prior distributions employed;

$\tilde{\beta} \sim N(0', 10I_2), \quad 1/\sigma_1^2 \sim Gamma(1, 1), \quad p_i \sim Beta(9, 0.1)$

Table 3.

Stochastic Trend with Multiple Breaks

	GDP		Unemployment Rate	
parameter	estimator	standard error	estimator	standard error
α_1	0.9838	(0.0734)	1.0503	(0.1120)
α_2	-0.0201	(0.0697)	-0.0614	(0.1121)
g_0	2.18	(0.11)	-0.0022	(0.0112)
g_1	0.85	(0.08)	0.0282	(0.0122)
g_2	0.74	(0.11)	0.0131	(0.0134)
g_3	-0.02	(0.30)	0.0939	(0.0321)
σ_0^2	0.36	(0.3)	0.0000	(0.0001)
σ_1^2	0.00	(0.0)	0.0013	(0.0022)
σ_2^2	0.10	(0.3)	0.0016	(0.0025)
σ_3^2	0.74	(0.6)	0.0086	(0.0061)
σ_ϵ^2	0.72	(0.2)	0.0065	(0.0014)

Note [a] See eq.(16), eq.(18) and eq.(19) for the notations of parameters.

Table 4.

Estimator of Okun's Coefficient (1961Q1-2001Q1)

Estimation Method	Decomposition Method			
	Hodrick-Prescott filter	Baxter-King filter	Linear Trend with Multiple Breaks	Stochastic Trend with Multiple Breaks
Static OLS	-0.0493	-0.0502	-0.0260	-0.1042
	(0.0060)	(0.0058)	(0.0048)	(0.0088)
Dynamic OLS, k = 2	-0.0685	-0.0528	-0.0326	-0.1387
	(0.0)	(0.0)	(0.0)	(0.0)
Dynamic OLS, k = 4	-0.0575	-0.0371	-0.0320	-0.1117
	(0.0)	(0.0)	(0.0)	(0.0)
Blanchard, k = 2	-0.0157	-0.0301	0.0063	-0.0335
	(0.0066)	(0.0079)	(0.0076)	(0.0091)
Blanchard, k = 4	-0.0151	-0.0203	0.0044	-0.0269
	(0.0067)	(0.0064)	(0.0078)	(0.0093)

Note [a] Standard errors in parentheses. For α^{LR} (dynamic OLS), standard errors are approximations calculated using first-oder Taylor expansions as on Greene (1993, p75).

[b] Full sample period of Baxter-King decomposition is between 1961Q1 and 1998Q1, becuase there are twelve missing points at the end of the sample.

Table 5.

Structural Change Test of Okun's Coefficient

Estimation Method	Decomposition Method			
	Hodrick-Prescott filter	Baxter-King filter	Linear Trend with Multiple Breaks	Stochastic Trend with Multiple Breaks
Static OLS	5.73***	10.36***	2.68**	26.43***
Dynamic OLS, k = 2	7.78***	10.95***	2.45	4.24*
Dynamic OLS, k = 4	9.80***	16.45**	8.85**	5.65
Blanchard, k = 2	3.82**	0.81	2.08*	2.76**
Blanchard, k = 4	5.83***	0.90	3.48**	2.11*

Note * 10% significant level, ** 5% significant level, *** 1% significant level. Critical values are used from Table II calculated by Andrews and Ploberger (1994, p1401-1402)

Table 6.
Structural Change Test of Okun's Coefficient

Estimation Method	Decomposition Method: Hodrick-Prescott filter	Baxter-King filter	Linear Trend with Multiple Breaks	Stochastic Trend with Multiple Breaks
Number of Change Points	1	1	2	1
Timing of Change Points	$1975Q1$	$1980Q1$	$1980Q1$ $1984Q4$	$1987Q3$

Note The numbers of change points are selecthed from the size of Bayes factor. The timings of change points are the mode of posterior distribution of change points in Figure 5.

Stractural change model in Okun's coefficient, eqaution (24), was estimated by Bayesian inference via MCMC. Following the model in Section 2.3, the first 2,000 iterations of Gibbs sampler are discarded to guarantee convergence and then the next 10,000 iterations are used for culculating the posterior distributions and their modes.

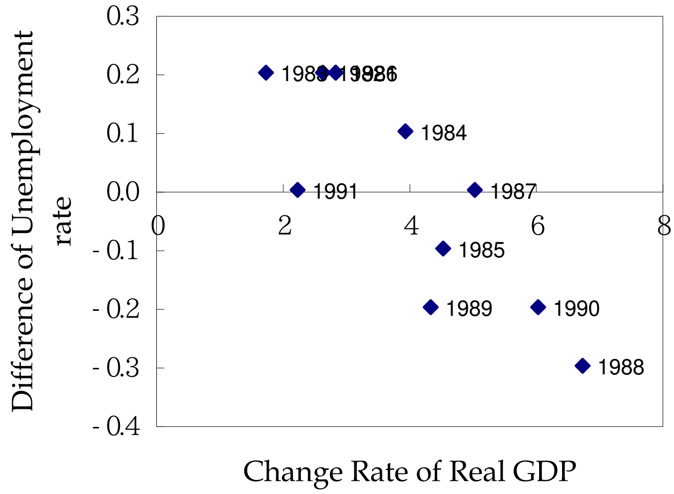
Table 7.
Estimator of Okun's Coefficient (1961Q1-2001Q1)

	Static OLS	Dynamic OLS, k = 2	Dynamic OLS, k = 4	Blanchard, k = 2	Blanchard, k = 4
Hodrick-Prescott					
1961Q1-1975Q1	-0.0283	-0.0403	-0.0603	-0.0245	-0.0288
	(0.0062)	(0.0)	(0.0)	(0.0079)	(0.0082)
1975Q2-2001Q1	-0.0759	-0.0938	-0.1440	-0.0047	-0.0006
	(0.0094)	(0.0)	(0.0)	(0.0103)	(0.0100)
Baxter-King					
1961Q1-1979Q4	-0.0311	-0.0371	-0.0494	-0.0294	-0.0187
	(0.0064)	(0.0)	(0.0)	(0.0098)	(0.0072)
1980Q1-19981Q1	-0.0894	-0.0862	-0.0818	-0.0316	-0.0241
	(0.0089)	(0.0)	(0.0)	(0.0013)	(0.0125)
Linear Trend					
1961Q1-1979Q4	-0.0198	-0.0243	-0.0182	0.0018	-0.0052
	(0.0056)	(0.0)	(0.0)	(0.0108)	(0.0109)
1980Q1-1984Q3	-0.1220	-0.1138	-0.1139	-0.0014	0.0022
	(0.0175)	(0.0)	(0.0)	(0.0469)	(0.0421)
1984Q4-2001Q1	-0.0172	-0.0266	-0.0407	0.0127	0.0161
	(0.0090)	(0.0)	(0.0)	(0.0107)	(0.0111)
Stochastic Trend					
1961Q1-1987Q2	-0.0347	-0.0583	-0.0834	-0.0377	-0.0321
	(0.0101)	(0.0)	(0.0)	(0.0122)	(0.0127)
1987Q3-2001Q1	-0.1591	-0.1821	0.0226	-0.0259	-0.0178
	(0.0112)	(0.0)	(0.0)	(0.0127)	(0.0127)

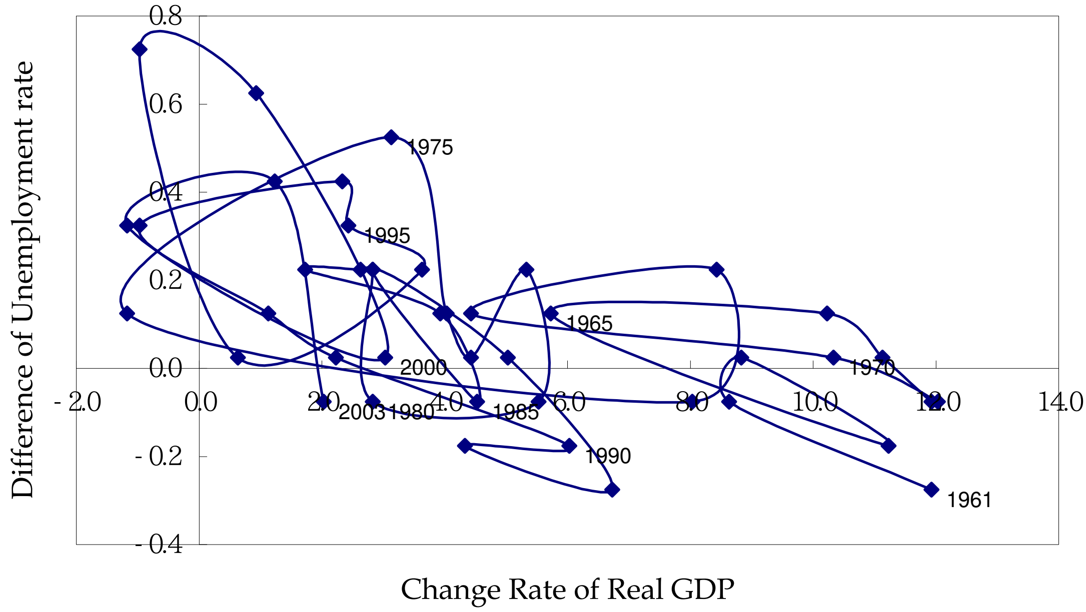
Note ^{a} Standard errors in parentheses. For α^{LR} (dynamic OLS), standard errors are approximations calculated using first-oder Taylor expansions as on Greene (1993, p75).

Figure 1

(a) Okun's law poited by Krugman (1998) 1981-1991



(b) 1961-2003



(c) After bubble period (1992-2003)

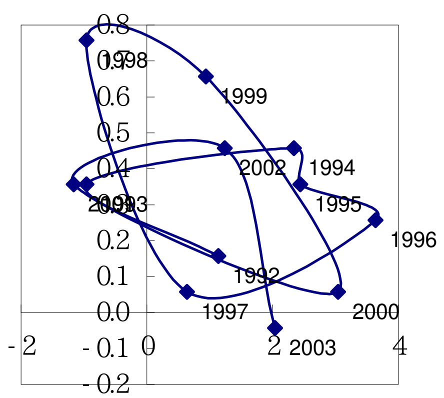
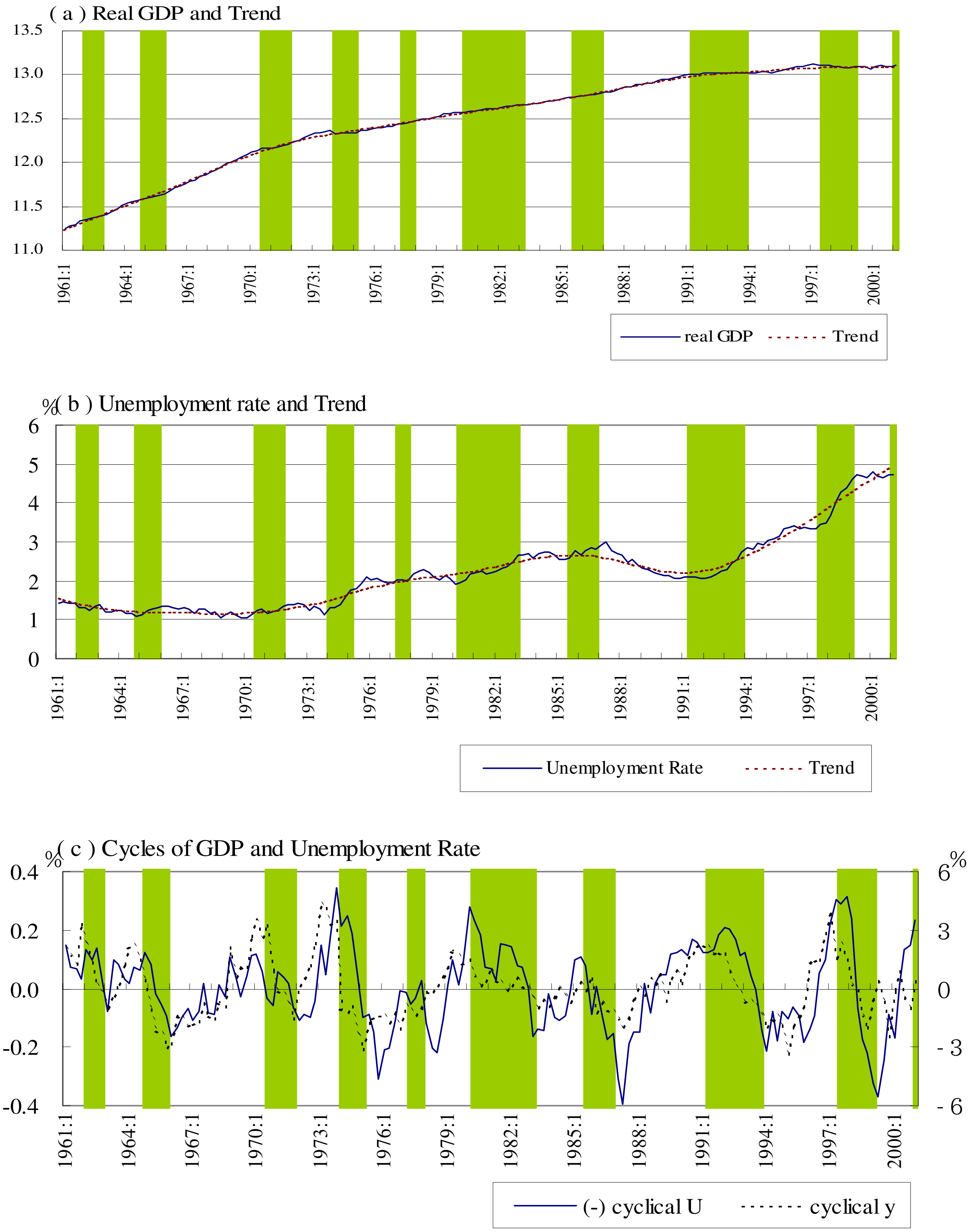
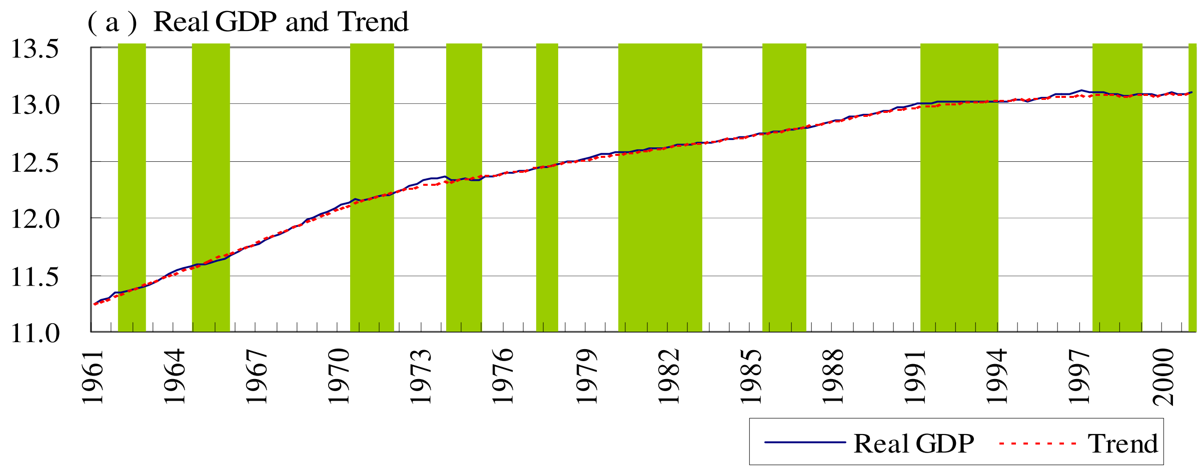


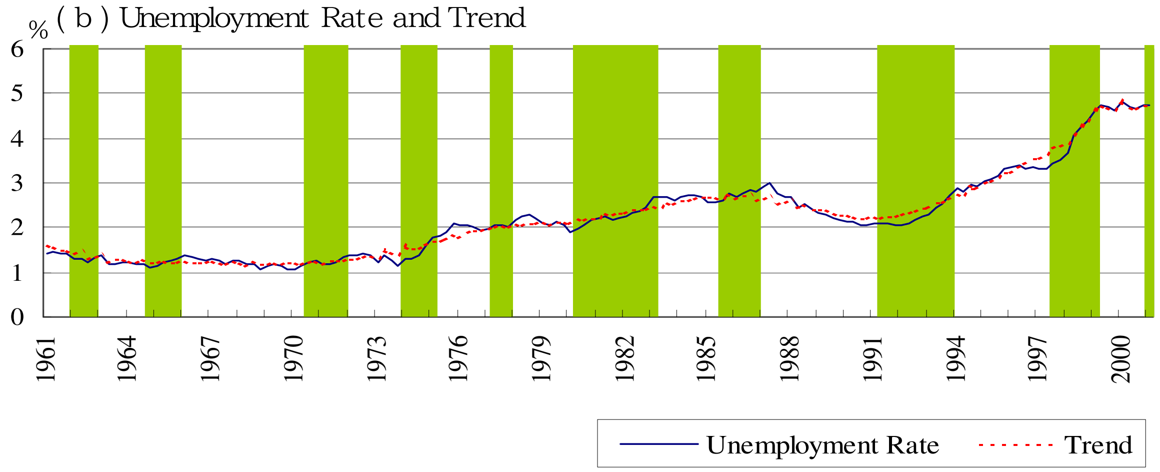
Fig 2.   Hodrick Prescott Filter Trend

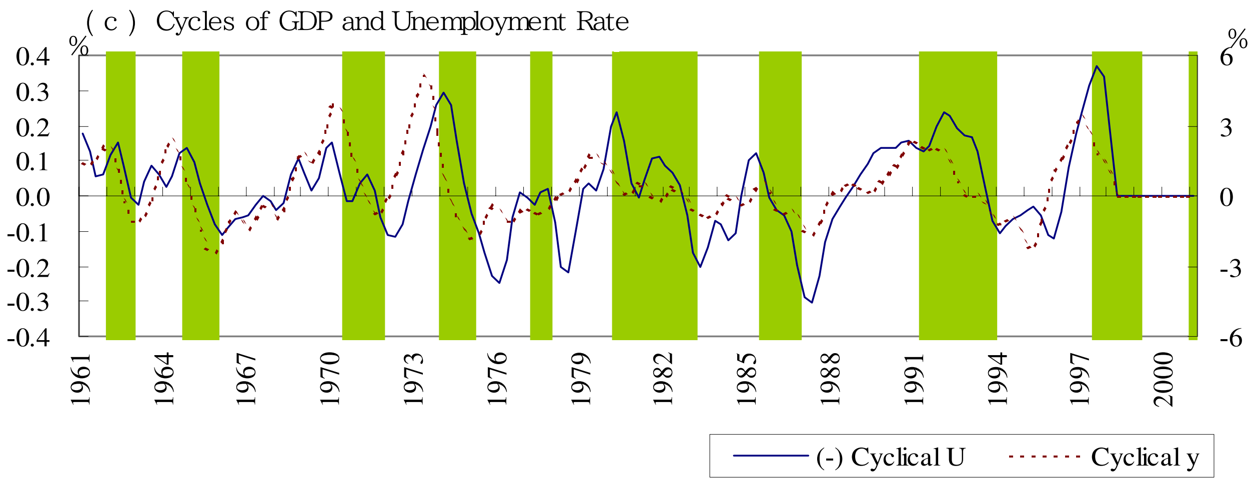


Note: The shade represents recessions reported by the Economic and Social Research Institute of the Cabinet Office of the government.

Fig. 3. Band Pass Filter by Baxter and King (1999)

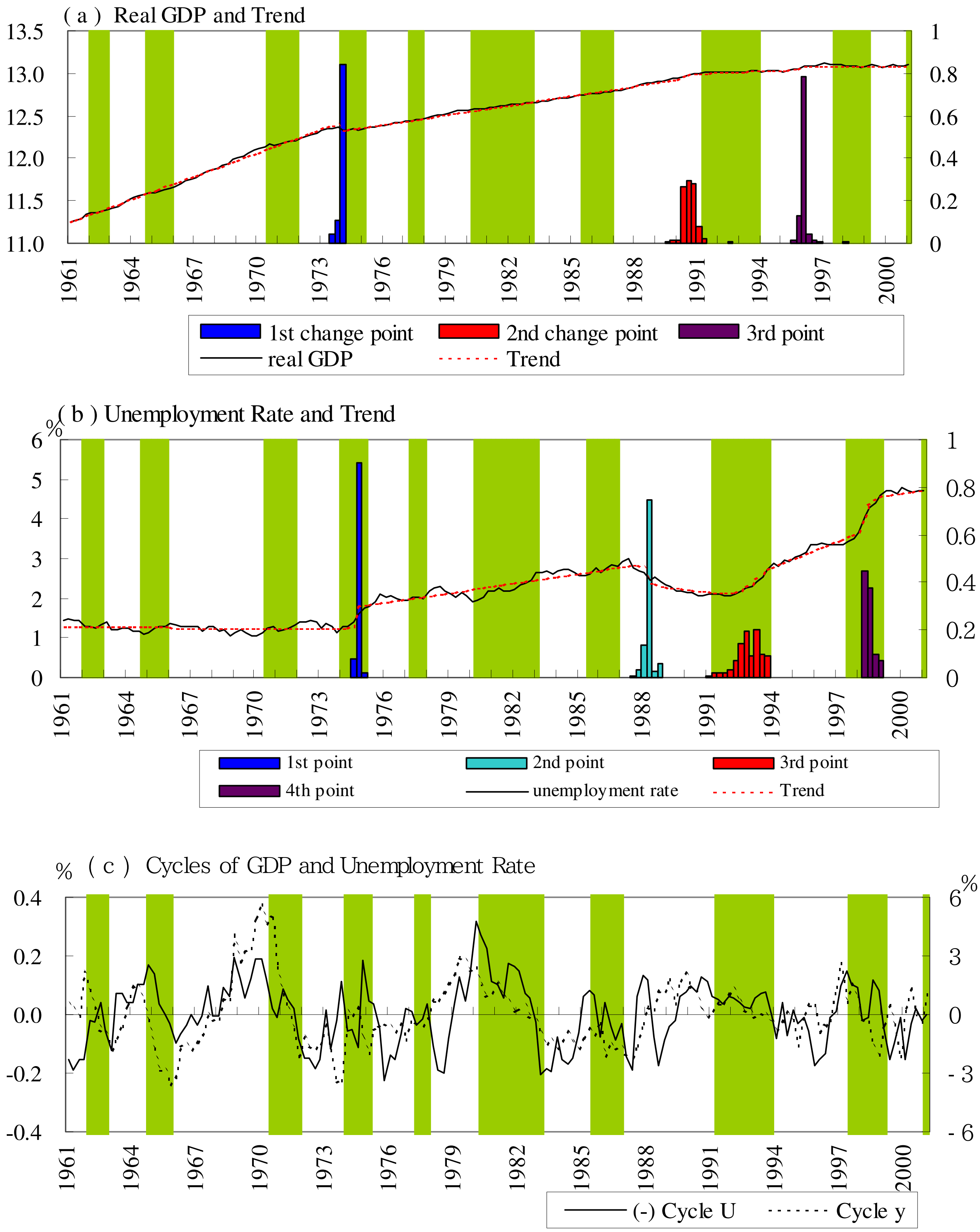






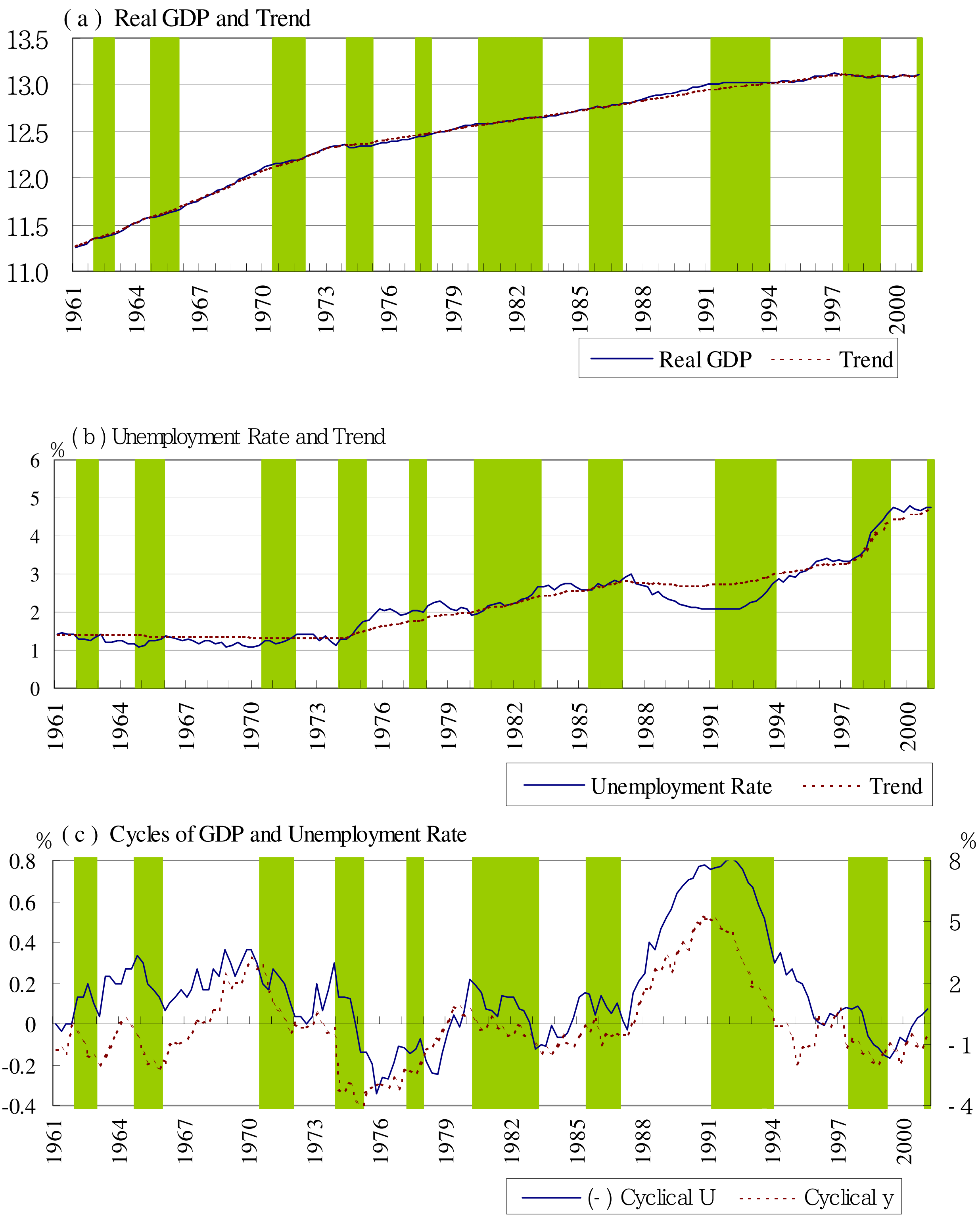
Note: The shade represents recessions reported by the Economic and Social Research Institute of the Cabinet Office of the government.

Fig. 4. Linear Trend with Multiple Breaks



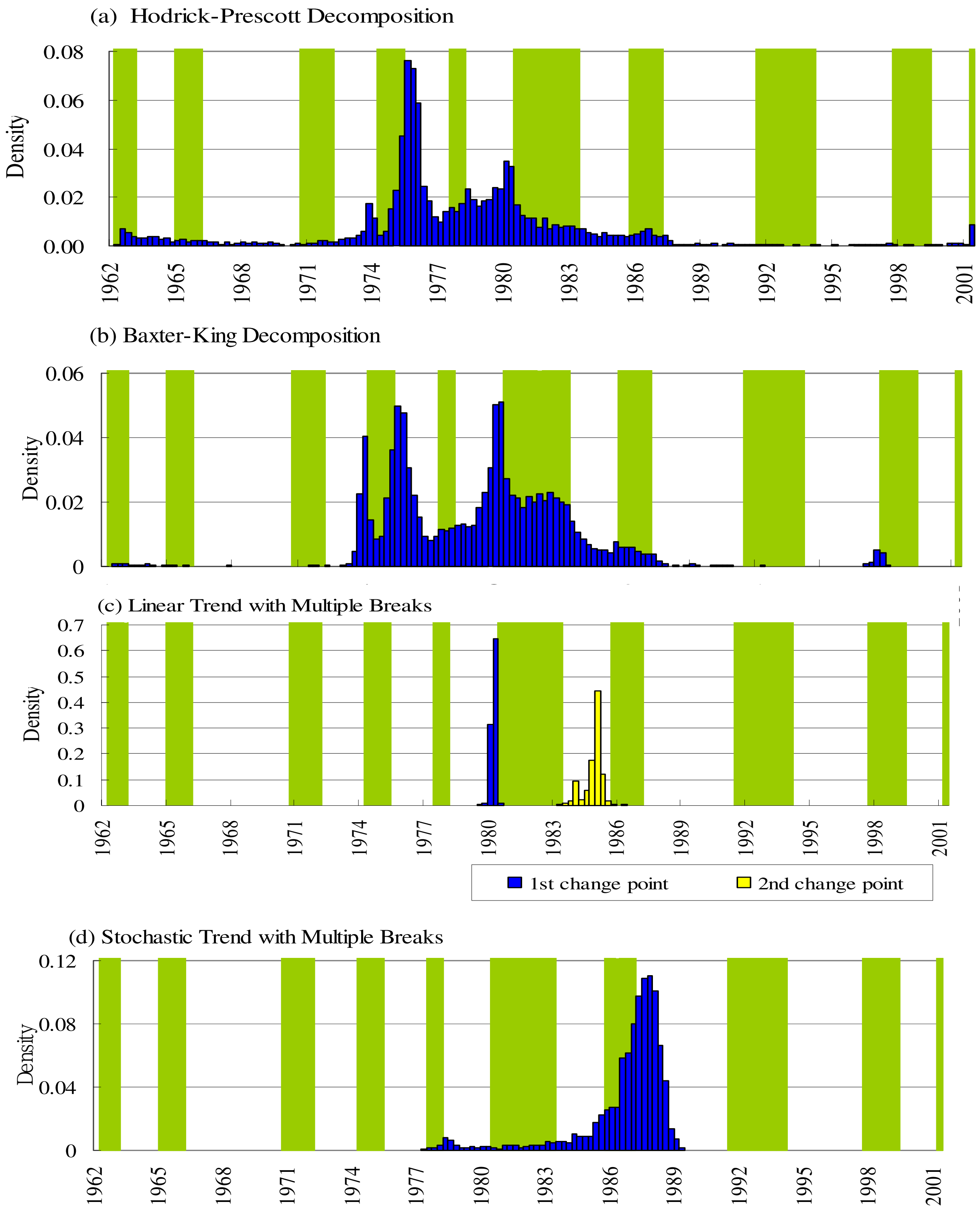
Note: The shade represents recessions reported by the Economic and Social Research Institute of the Cabinet Office of the government.

Fig.5 Stochastic Trend with Multiple Breaks



Note: The shade represents recessions reported by the Economic and Social Research Institute of the Cabinet Office of the government.

Fig 6. Distributions of Structural Change Points of Okun's Coefficient



Note: The shade represents recessions reported by the Economic and Social Research Institute of the Cabinet Office of the government.

Figure 7

Estimation results along the traditional form of Okun's law