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Labor Market Distortions under Sovereign Default Crises

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Abstract

Risk of sovereign debt default has frequently affected emerging market and developed economies. Such financial crisis are often accompanied with severe declines of employment that are hard to justify using a standard dynamic stochastic model. In this paper, I document that a labor wedge deteriorates substantially around swift reversals of current accounts or default episodes. I propose and evaluate two different explanations for these movements by linking the wedges to changes in labor taxes and in the cost of working capital. By adding these two features in a dynamic model of equilibrium default I am able to replicate the behavior of the labor wedge observed in the data around financial crisis. In the model, higher interest rates are propagated into larger costs of hiring labor through the presence of working capital. As an economy is hit with a stream of bad productivity shocks, the incentives to default become stronger, thus increasing the cost of debt. This reduces firm demand for labor and generates a labor wedge. A similar effect is obtained with a counter-cyclical income tax rate policy. The model is used to shed light on the recent events of the Euro Area debt crisis and in particular of the Greek default event.

JEL classification: F32, F34, F41, E62

Keywords: Sovereign default, labor markets, distortionary taxation, external debt, debt renegotiation

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1 Introduction

Sovereign default crises are events usually associated with large economic costs. In particular, countries that experience such events typically experience large drops in both output and employment. [Mendoza and Yue \(2012\)](#) notes, by looking at a set of emerging economies, that default events are associated with deep recessions where employment falls of average by about 15% lower compared with pre-crisis levels. More recently, the same kind of patterns have been observed in the advanced economies that were in the epicenter of the Euro Area sovereign debt crisis, namely Greece, Portugal, and Ireland. One natural question to ask is, therefore, if the observed fall in employment in these events is unusual at the light of standard economic theory (*e.g.* [Chari, Kehoe, and McGrattan, 2007](#)) given an equally large fall in output. Showing evidence suggesting that the answer to this question is negative, this paper advances with an explanation where labor markets distortions arise as a consequence of limited access to credit markets by governments on the onset of a sovereign debt crisis.

To motivated the claim that labor market distortions are increasing during episodes of sovereign default, the Euro Area sovereign debt crisis is used as a main source of empirical evidence. One main advantage of using this set of countries resides on the availability of high frequency data for series such as output, consumption, and employment. The data analysis suggests that European countries that were close or defaulted during the crisis are also the ones where the long-term, otherwise, statistically stable correlation between employment/output growth - traditionally coined as Okum's law ([Okun, 1962](#)) - breaks down on the aftermath of the crisis. Additionally, using the [Chari, Kehoe, and McGrattan, 2007](#) accounting methodology, it is shown that measured labor wedges, often associated with distortions, deteriorated much faster for countries that were more severely affected by the debt crisis. When regressed against a set of controls, worse labor wedges wedges are statistically associated with higher government interest rate spreads thus suggesting a channel over which adverse government credit conditions spillover into labor market distortions.

These empirical observations are rationalized with a dynamic stochastic model of default with endogenous labor supply. As standard in sovereign default models of the [Eaton and Gersovitz \(1981\)](#) type, the government has limited commitment in honoring its debt contracts, implying that interest rates on borrowing include a premium over the risk-free rate demanded by international investors to compensate for the risk of default. Because the government is assumed to be impatient and debt is non-contingent, the interest rate spread

displays a counter-cyclical as the debt burden become more onerous to the government when the economy is in recession with a consequent fall in revenue. Given these interest rate spread dynamics, a financial crisis is associated with sharp tightness of credit market access triggered by a sequence of negative productivity shocks. To link these financial crises with the labor market, the model adds two additional features. First, the government only has access to distortionary taxation and debt to finance public consumption. This implies that distortionary taxes have to be raised when access to credit markets becomes constrained thus distorting the household labor supply decision. Second, the model also assumes that firms are required to keep working capital to finance their salary payments. Additional, and in line with the literature (Neumeyer and Perri, 2005; Uribe and Yue, 2006; Arellano and Kocherlakota, 2014), it is assumed that high government interest rates spill over to high corporate interest rates. This creates an additional source of distortions on labor markets from the demand end. In summary, both the distortionary taxation and working capital requirements work as frictions that distort labor markets when the government is close to default. As a result the fall in employment is larger than what would be without these frictions. Under this framework, default entails costs in terms of loss productivity and financial market access. On the reserve side, repudiating debt can release resources to both public and private consumption. At the same time, more resources also relaxes the fiscal constraint and allows the government to stop tax increases. In every period the government evaluates the costs and benefits of defaulting and acts accordingly.

A simulated version of this model is computed to match the economy of Greece. Several features of the Greek business cycle moments are captured in the model. In particular the model generates counter-cyclical interest spreads or tax rates, a characteristic usually associated to emerging market economies (Cuadra et al., 2010; Vegh and Vuletin, 2012). In this paper, counter-cyclical tax rates arise due to imperfect credit market access. It is also due to these dynamics of interest and tax rates that explain the main results of the simulation: on the path to default employment falls substantially and is followed by increased distortions in the labor markets. The model accounts for a 15% decline of employment from 3 years before default (against an observation of 17% for Greece) from which 3pp is accounted by distortionary taxes and an additional 4pp from working capital constraints.

This paper is builds on the literature on endogenous default risk. Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), or Arellano (2008) developed sovereign default models where the probability of default is increasing when debt is high or income low. However, these papers assume that the government is able to transfer resources to households in a non-distortionary fashion, thus abstract from fiscal constraints. In Cuadra et al. (2010), a

similar model of sovereign default extended to include endogenous labor and distortionary consumption and conclude that under imperfect credit access tax rates become counter-cyclical. [Arellano and Bai \(2013\)](#) use a similar paper to obtain the result that exogenously raising labor taxes may be self-defeating in the sense that the impact of distortions may reduce the revenue base used by the government to repay debt. Despite these important conclusions, neither paper attempt to quantitatively account for the labor market implications of a pro-cyclical tax policy.

Also related is the literature that uses interest rate shocks as a main source of fluctuations in emerging economies. [Neumeyer and Perri \(2005\)](#) and [Uribe and Yue \(2006\)](#) present models where firms require working capital to pay salaries in advance financed by external debt. This imply that labor demand is reduced when interest rates are high. An important setback of these models relies on the fact that interest rates are completely exogenous and, for that reason, disconnected from the level of government indebtedness. Using a model of sovereign default, [Mendoza and Yue \(2012\)](#), assume that some imported inputs require working capital financing, thus providing a channel over which endogenous interest rate fluctuations affect firms decisions. However, this paper also abstracts from fiscal constraints and the model cannot generate a fall in labor at defaults as seen in the data.

Finally, the literature that uses the accounting methodology developed by [Chari, Kehoe, and McGrattan \(2007\)](#) to study labor market distortions as measured by labor wedges. For example, [Karabarbounis \(2014\)](#), using a set of developed and emerging economies points out the the labor wedge is in general pro-cyclical, that is, it deteriorates when output is in recession, while [Ohanian et al. \(2007\)](#) regress labor wedges against a set of regressors and conclude that labor taxes affect negatively this wedge. Neither of these authors relates their wedges with financial crisis. An exception comes from [Pratap and Quintin \(2011\)](#) that associate distortions in labor markets to increases in interest rates and tax rates in the Mexican crisis of 1994, however, in their model, interest are exogenously determined and taxes play no role.

The rest of this paper is organized in the following way. Section 2 presents empirical evidence on labor market distortions arising from the Euro Area debt crisis; section 3 presents a model that rationalizes such evidence; section 4 calibrates the model for the Greek economy and run some robustness checks; and section 5 concludes. The appendix A presents some additional evidence and describes the computational methods used.

2 Empirical Evidence

The EZ crisis starting in 2008 is marked by a large heterogeneity in the macroeconomic responses within each country. Table 1 shows the length and depth of the recession for the different countries composing the European monetary union at the beginning of 2008¹. It is quite evident that some countries experienced large and prolonged recessions while others did not. For the case of Greece, the recession is even comparable to the US great depression with a length of more than 6 years followed by a 27% fall in output and an 20pp increase in unemployment (as in figure 1). With the exception of Italy and Slovenia where labor markets stayed more contained, this was also the pattern observed for the other countries that experienced prolonged recessions. Given that Greece, Portugal, Ireland, Spain all received financial assistance from official creditors, a natural question to ask is if that recession, apart from its length depth, had any unusual features.

Table 1: 2008 Recession cycle of Euro Area countries

	Peak	Trough	Diff	ΔY (%)	ΔC (%)	ΔU (pp)	ΔE (pp)
Austria	2008q1	2009q2	5	-5.8	1.7	1.2	-0.5
Belgium	2008q2	2009q1	3	-4.2	-0.6	1.1	-0.3
Germany	2008q1	2009q1	4	-6.9	1.3	-0.2	0.3
Spain	2008q2	2013q2	20	-8.0	-8.6	15.8	-9.4
Finland	2007q4	2009q2	6	-9.6	-0.4	1.9	-2.0
France	2008q1	2009q2	5	-4.0	0.8	2.0	-0.7
Greece	2007q2	2013q4	26	-27.4	-22.2	19.2	-11.0
Ireland	2007q4	2012q2	18	-9.7	-9.9	10.0	-9.9
Italy	2008q1	2014q4	27	-9.6	-5.9	6.4	-3.0
Netherlands	2008q1	2009q2	5	-4.2	0.5	0.4	0.1
Portugal	2008q1	2013q1	20	-9.6	-10.4	9.5	-8.3
Slovenia	2008q2	2012q4	18	-11.2	-2.7	5.3	-4.3

Data source: OECD. Output and consumption are OECD volume estimates. As in [Harding and Pagan \(2002\)](#), peak and trough turning points are determined using the following methodology: (1) output is measured in logs at quarterly frequency; (2) peaks are selected when $y_t = \max\{y_{t-2}, y_{t-1}, y_t, y_{t+1}, y_{t+2}\}$ and troughs when $y_t = \min\{y_{t-2}, y_{t-1}, y_t, y_{t+1}, y_{t+2}\}$; (3) censoring rules apply where peaks and troughs have to alternate and the minimum phase is 2 quarters with a 5 quarters minimum cycle.

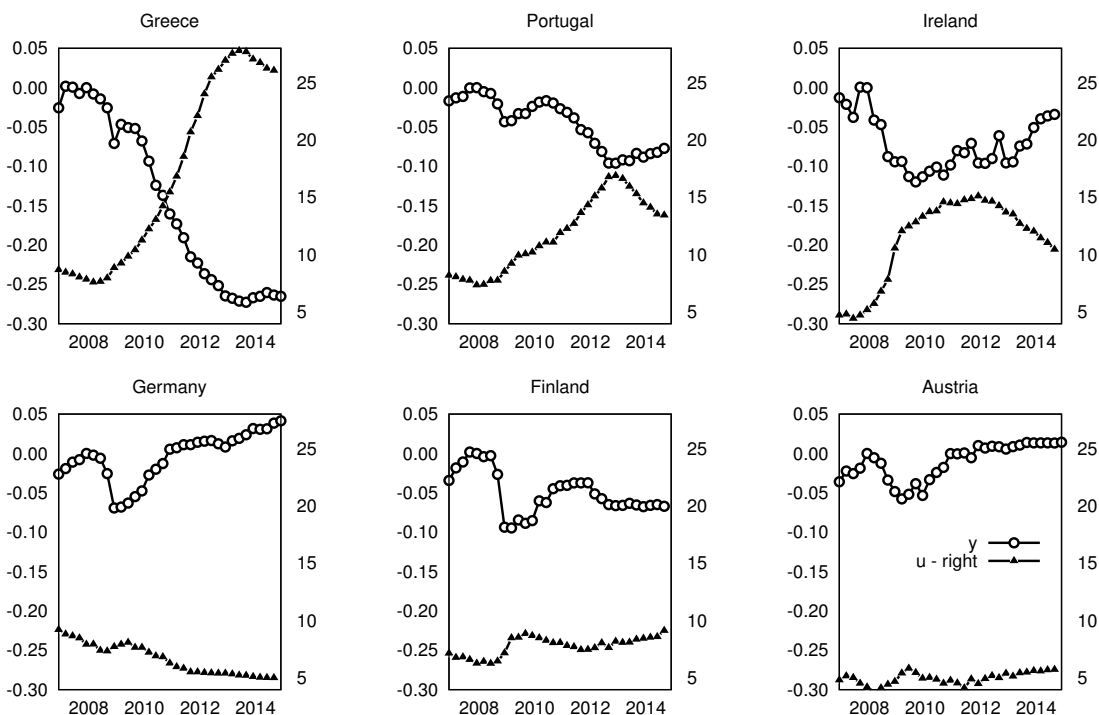
A simple way to analyze this question is look for deviations of the historical relationship between employment and output change. Figure 1 shows very clearly how output is negatively correlated with unemployment, a well identified relationship known as Okun's law. Recent studies ² indicate that such relationship has been kept strong and stable for most countries, even after including the 2008 Great Recession. Typical analysis point that

¹Luxembourg has a population of 0.5 millions and because of such small size it is not included in this analysis.

²Examples include [Ball et al. \(2013\)](#) or [Elsby, Hobijn, Şahin, Valletta, Stevenson, and Langan \(2011\)](#).

different linear slopes characterizing the relationship employment/output in different countries are due to different labor market frictions or market structures. However, if such environments are invariable during the business cycles, then the slope of the relationship absorbs such institutional features. In this sense, systematic deviations of that long-term relationship can be indicative of a breakdown.

Figure 1: Output, consumption, and unemployment dynamics around Euro debt crises for selected countries



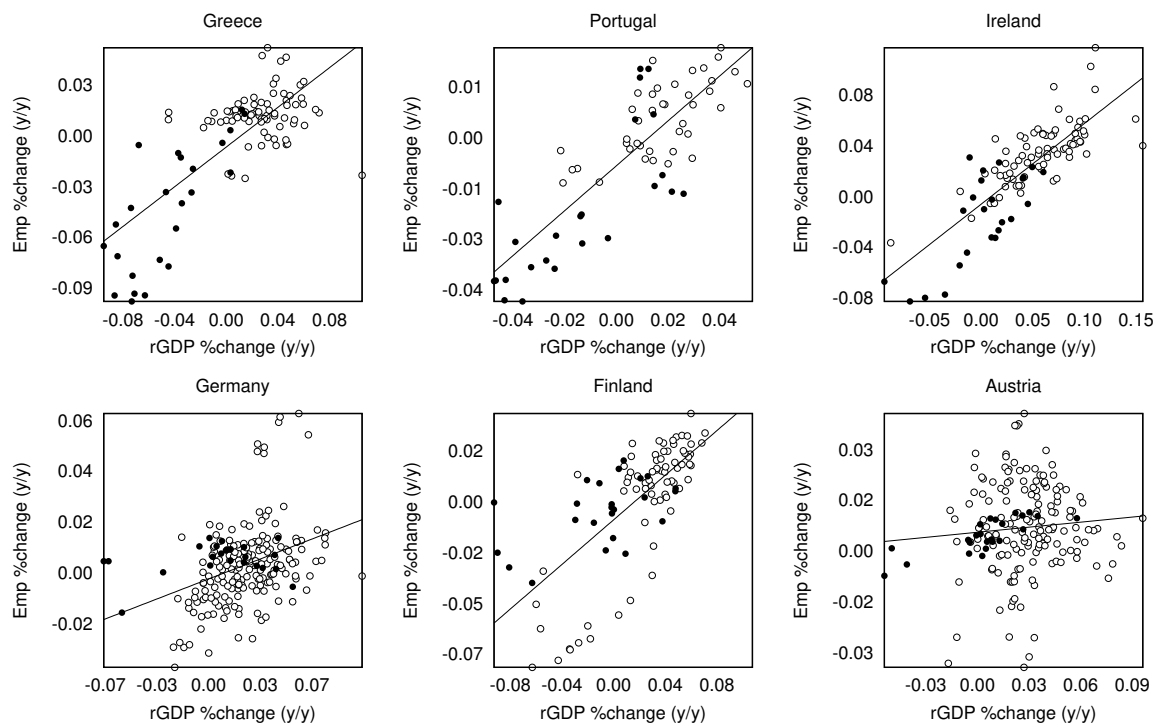
Data source: OECD. The variable y in the figures stands for the cyclical component of the quarterly GDP using an HP filter with 1600 smooth parameter.

In order to study the impact of the European debt crisis on the stability of the Okun's law, figure 2 plots a scatter of employment changes against output changes. The figure suggests that the employment has fallen faster relatively to output during the period of 2009-2014 for the countries represented at the top panels as the red dots are substantially below the historical trend line. This indicates that over the period, for each one percent of output drop, employment fell more than the historical average.

A structural break of the relationship can be tested using a Chow test which consists in estimating for each country the following regression using OLS

$$\Delta E_t = \beta_0 + \beta_1 \Delta Y_t + \alpha_0 D_t^{2008} + \alpha_1 \Delta Y_t \cdot D_t^{2008} + u_t$$

Figure 2: Employment and output correlation for selected countries (solid circles for after 2008)



Data source: OECD. The solid circles represent observations after 2008.

where ΔE_t , ΔY_t , and D_t^{2008} are change in employment, change in employment, and a dummy variable taking one when $t > 2008$. The test consists in evaluating the null hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 0$ using a Wald statistic. The results of applying such tests for all member countries of the EZ can be found in table 2. Five out of 12 countries present evidence statistical evidence that the Okun's law has kept stable over the period. For the remaining countries, Germany, France, and Netherlands show an improvement of the relationship in the sense that less employment fall is associated with an 1 percent fall in output, that is, either the slope decreases or the intercept increases. As for Spain, Greece, Ireland, Italy and Portugal, the statistical evidence supports the idea that there was a structural break in 2009 that aggravated labor market conditions.

A different approach to inspect labor market conditions makes use of the accounting methodology developed by Chari et al. (2007). This derives a wedge that reflects the difference of what is observed in the data and the prediction of a neoclassical growth model in some key variables. The difference measured in the wedge accounts for unknown factors that are unaccounted by a frictionless standard economic model. Specifically, the

Table 2: Okun's Law structural break

	Before 2009				After or 2009				Chow (p-val)
	b_1	b_0	R^2	N	b_1	b_0	R^2	N	
Austria	0.20	0.005	0.20	52	0.23	0.006	0.69	25	0.47
Belgium	0.55	-0.002	0.31	48	0.30	0.002	0.37	24	0.25
Germany	0.37	0.000	0.26	56	0.05	0.005	0.06	24	0.00
Spain	1.34	-0.009	0.65	36	1.50	-0.012	0.96	24	0.71
Finland	0.29	0.00	0.14	40	0.27	0.00	0.52	24	0.12
France	0.72	-0.005	0.77	56	0.50	-0.002	0.89	25	0.01
Greece	0.12	0.010	0.03	56	0.79	-0.005	0.59	24	0.00
Ireland	0.42	0.014	0.46	56	0.70	-0.018	0.55	24	0.00
Italy	0.19	0.010	0.18	40	0.16	-0.004	0.19	24	0.00
Netherlands	0.49	0.003	0.40	40	-0.24	-0.006	0.15	24	0.00
Portugal	0.46	-0.002	0.48	40	0.68	-0.012	0.58	24	0.01
Slovenia	0.26	-0.005	0.08	52	0.05	-0.010	0.03	23	0.04

Notes: Data source OECD. The coefficients b_1 and b_0 are the slope and intercept coefficient of the corresponding regressions. A standard dummy variables method is used to perform the Chow's test assuming the break occurs at the first quarter of 2009.

methodology presents a labor market equilibrium equation of the form:

$$u_{lt}/u_{ct} = \omega_t \cdot y_t/h_t \quad (1)$$

where u_{lt} and u_{ct} are the marginal utility of labor and consumption at time t ; y_t the output, h_t hours; and ω_t represents the labor wedge. Because different utility functions have different functional forms, the wedge measurement will also differ. In practice, for this exercise, the two most widely used utility functions in the literature are chosen³:

$$u^{CRRRA}(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \Gamma \cdot \frac{h^{1+\gamma}}{1+\gamma}$$

$$u^{GHH}(c, h) = \left(c - \Gamma \cdot \frac{h^{1+\gamma}}{1+\gamma} \right)^{1-\sigma} / (1-\sigma)$$

Applying these utility functions to (1) imply the following wedges measurement:

$$\hat{\omega}_t^{CRRRA} = (1+\gamma) h_t - \hat{y}_t + \sigma \hat{c}_t$$

$$\hat{\omega}_t^{GHH} = (1+\gamma) h_t - \hat{y}_t$$

where a variable with an hat represents a log deviation from steady state⁴.

³The parameters σ , γ , and Γ regulate an economic agent's preferences for consumption and hours of work.

⁴Specifically, for a level variable x , $\hat{x}_t = \log x_t - \log \bar{x}_t$ and \bar{x}_t is trend component of x_t at time t .

Figure 3: Labor wedges for selected countries

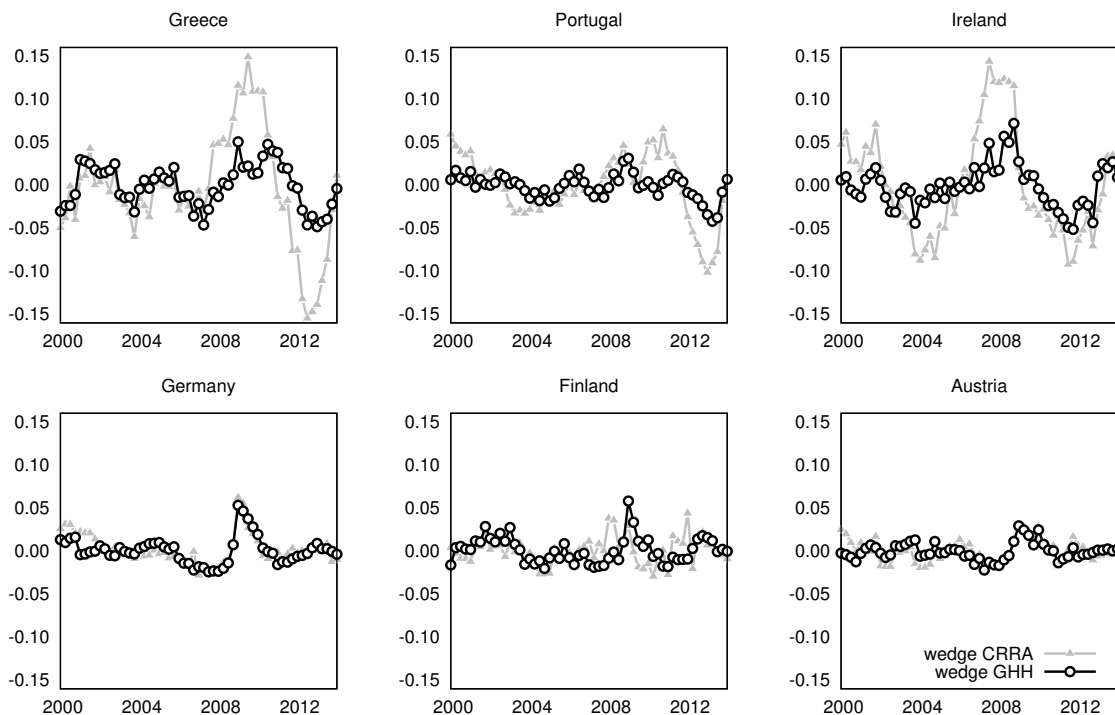


Figure 3 measures the cyclical component of these labor wedges for a subset of countries in the EZ for a particular choice of the parameter values⁵. Both measures indicate that countries in the EZ were differently affected by the recession. One can easily see that the wedges deteriorated substantially for countries such as Greece, Portugal, or Ireland, while were very stable for Germany, Finland, or Austria. This observation seem to suggest that the severity of the financial crisis may be responsible for the adverse behavior in labor markets. To better understand what is driving the labor wedge, the following panel data regression is estimated for every quarter t and every country i from the EZ:

$$\hat{\omega}_{it} = \beta_i + \beta_t + \beta_1 \hat{y}_{it-1} + \beta_2 \text{spread}_{it-1} + u_{it}$$

where spread_{it} is the spread between the 10 year government yield of country i and Germany for the quarter t . This variable is used as a proxy for the country specific severity of the financial crisis. Table 3 shows the results. Across specifications and for both wedge measures, the results are consistent at showing that the labor wedge is negatively correlated

⁵The parameter values are $\sigma = 2$ and $\gamma = 0.5$, which are exactly the same as the ones used in section 4, and close to the ones used, for example, in Neumeier and Perri (2005).

with both the interest rate spread and the output gap. The last result is consistent with previous studies (Karabarbounis, 2014) and simply states that the labor wedge is countercyclical. The results also show a negative correlation between the labor wedge and the domestic financial conditions as captured by the interest rate spread. These, suggest that times of large spreads are associated with labor markets that are more distorted and are also consistent with the results presented in tables 1 and 2. That is, the unusual drop in employment, observed mainly in southern European countries and Ireland, may be related with the sharp increase in interest rate motivated by a sovereign debt crisis.

Table 3: Labor wedge panel regressions

	CRRA		GHH	
interest rate spread	-0.78** (0.06)	-0.80** (0.07)	-0.23** (0.04)	-0.19** (0.04)
output gap	-0.69** (0.20)	-0.40** (0.12)	-0.65** (0.13)	-0.40** (0.12)
time effects	No	Yes	No	Yes
N	569	569	569	569
R^2	0.17	0.34	0.18	0.31

Notes: Data source OECD and ECB. All regressions are estimated using fixed effects and the standard errors, presented in parenthesis, are heteroskedastic robust. The statistical models without fixes constrains $\beta_t = 0$ for all t . ** significance at 1%; *significance at 5%.

3 Model Economy

The previous section suggests that labor markets and credit access are closely related. Here, a typical sovereign default model is presented to account for that relationship. The model economy is one based on Eaton and Gersovitz (1981) where a sovereign government borrows or saves from international markets in order to maximize consumers utility. Because the government cannot commit to honor its debts contracts, international investors demand an interest rate premium over the risk free rate to account a default probability. The main departure from the Eaton and Gersovitz (1981) model resides on the labor market. On one hand consumers supply labor based on the income tax rate determined by the government. On the other hand, firms demand labor under a working capital constrain. The first feature links labor supply to credit conditions as a debt constrained government, in need to finance public expenditure, has to increase taxes that distorts consumers decisions. As for the second feature, under the assumption that higher government interest rates spillover to corporate higher interest rates, then, due to the working capital constrain,

harsher credit conditions affects firms inducing them to hire less labor. The details of this model are outlined in the following sections.

3.1 Household

A representative household is infinitely lived, valuing consumption and labor accordingly to:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \quad (2)$$

where \mathbb{E} is the expectation operator, β denotes the discount factor, and the period utility $u : \mathbb{R}_+ \times [0, 1] \rightarrow \mathbb{R}$ is: continuous, differentiable and concave in both arguments; increasing in c and decreasing in h . Maximization of lifetime utility (2) is subjected to the following budget constraint:

$$c_t = (1 - \tau_t) \cdot (w_t h_t + \pi_t + e_t) \quad (3)$$

Consumption c_t equals income provided by wage income derived from supplying h_t hours at a wage rate of w_t , firms' profits π_t and interest earnings e_t , all income taxed at rate τ_t . Optimality from maximization of (2) subjected to (3) imply the following first order condition:

$$\frac{\partial u(c_t, h_t) / \partial h_t}{\partial u(c_t, h_t) / \partial c_t} = (1 - \tau_t) w_t \quad (4)$$

That is, marginal rate of substitution of hours to consumption equals the wage rate net of taxes. Equation (4) together with (3) characterize simultaneously this household labor supply and demand for consumption goods. Similar optimality conditions were used within the framework of sovereign default models, for example, in [Cuadra, Sanchez, and Sapriza \(2010\)](#) and [Arellano and Bai \(2013\)](#). Implicitly in this environment is the restriction that the household cannot directly access external borrowing.

3.2 Firm

Final consumption goods are produced by firms using labor services as inputs. The production function $f(h)$ - continuous, differentiable, concave, and satisfying Inada's con-

ditions - is also subjected to a multiplicative stochastic productivity shock z_t that follows a Markov process. As in [Neumeyer and Perri \(2005\)](#) or [Uribe and Yue \(2006\)](#), profits equal the difference between revenues and costs that include both the wage bill and working capital costs. This means that in order to pay wages to workers, firms need to set aside a fraction $\theta \in [0, 1]$ of the wage bill immediately after the beginning of the period in order to pay workers before the end of the the same period. However, because production is only available at the end of the period, firms need to borrow an amount equal to $\theta w_t h_t$ from households at a gross interest rate of $R_t \geq 1$ ⁶. As such, at the end of time t , profits are given by:

$$\pi_t = z_t f(h_t) - w_t h_t - (R_t - 1) \cdot \theta w_t h_t \quad (5)$$

It follows that profit maximization implies the following equation that characterizes demand for labor:

$$w_t = \frac{1}{1 + (R_t - 1) \cdot \theta} \cdot z_t \frac{\partial f(h_t)}{\partial h_t} \quad (6)$$

Note that the wage rate is equated to the product of the marginal product of labor and a term bounded between 0 and 1. It follows that, from the firm's perspective, an increase in its cost of financing is equivalent to a negative productivity shock.

3.3 Government

The sovereign government finances public consumption using income taxes or by issuing non-contingent new long-term debt at international markets price of q_t . In order to introduce long-term debt, I follow a similar approach to [Chatterjee and Eyigungor \(2012\)](#) and [Hatchondo, Martinez, and Sosa-Padilla \(2016\)](#) by assuming that the debt stock matures at any given period with probability λ and, if it does, pays a coupon equal to c . This implies that given new issuances of debt I_t , debt accumulates accordingly to:

$$D_{t+1} = (1 - \lambda) D_t + I_t \quad (7)$$

Because of the inability of international investors to enforce contracts, the government has the ability to repudiate its debt liabilities. However, under the case of default, the government acquires a bad credit history and becomes excluded from borrowing for for a random number of periods. At the end of that period the government has the option of

⁶A different way to interpret working capital requirements is that, due to some friction in the technology for transferring resources, workers demand a fraction θ of the wage payment before production takes place. For that reason, firms need to borrow in advance.

restructuring its debt by paying a fraction φ of what is due. If that's the case a good credit standing status is regained. Also, while in financial exclusion, firms productivity is negatively affected becoming $\tilde{z} = z - l(z)$ where $l(z)$ is an increasing loss function. Given this elements, the government budget constrain is given by:

$$\tau_t \cdot (w_t h_t + \pi_t + e_t) = g_t + q_t [D_{t+1} - (1 - \lambda) D_t] - D_t [\lambda + (1 - \lambda) c] \quad \text{if repay} \quad (8)$$

$$\tau_t \cdot (w_t h_t + \pi_t + e_t) = g_t \quad \text{if default} \quad (9)$$

It follows that the problem for the government, when the credit history is good, resides on choosing the tax rate τ_t , next period debt D_{t+1} , and whether or not to repay current debt in order to maximize the household's lifetime utility (2) subjected to equations (8) and (9), together with (3) and (4). These last two constraints are due to the fact that the government has no access to *lump sum* tax instruments and has to operate under a competitive labor market equilibrium.

3.4 International Investors

If the government has a good credit history, then it can issue debt in international markets where risk neutral investors charge a debt price q_t that compensates them for the opportunity cost of alternative investments with a certain rate of return of $1/\bar{q} - 1$ to which adds the risk that the government defaults on its debt. Under default, debt grows at the risk free rate, so $D_{t+1} = D_t/\bar{q}$. Also under default the government receives a restructuring shock with probability ζ that reduces the debt it owes by a fraction $\varphi \in [0, 1]$ and allows it to regain access to credit markets. Letting $\mathbb{I}(D, z)$ be an indicator function taking 1 whenever the government decides to default, then international investors price new issuances of debt, $q_t \equiv q(D_{t+1}, z_t)$, accordingly to:

$$q_t = \bar{q} \int \left\{ (1 - \mathbb{I}_{t+1}) (\lambda + (1 - \lambda) (c + q_{t+1})) + \mathbb{I}_{t+1} q_{t+1}^{def} \right\} dF(z_{t+1}|z_t) \quad (10)$$

where $F(z_{t+1}, |z_t)$ is the process governing productivity, $q_{t+1} \equiv q(D_{t+2}, z_{t+1})$ is the bond price in the following period without default, and $q_{t+1}^{def} \equiv q^{def}(D_{t+1}/\bar{q}, z_{t+1})$ is the bond price under default, defined recursively as:

$$q_t^{def} = \bar{q} \int \left\{ (1 - \zeta) q_{t+1}^{def} + \zeta \varphi \left((1 - \mathbb{I}_{t+1}) [\lambda + (1 - \lambda) (c + \tilde{q}_{t+1})] + \mathbb{I}_{t+1} \tilde{q}_{t+1}^{def} \right) \right\} dF(z_{t+1}|z_t) \quad (11)$$

and $\tilde{q}_{t+1} \equiv q [\varphi D_{t+1}/\bar{q}, z_{t+1}]$ is the price of restructured debt under repayment, while $\tilde{q}_{t+1}^{def} \equiv q^{def} [\varphi D_{t+1}/\bar{q}, z_{t+1}]$ is the price of restructured debt under default.

The debt price schedule (10) is composed of 3 main elements: the first one, \bar{q} , is the price for risk-free investments; the second, $\int [1 - \mathbb{I}_{t+1}] dF(z_{t+1}, |z_t)$, is the probability that the government honors the contract; and the last, $\int [\mathbb{I}_{t+1} \tilde{q}_{t+1}^{def}] dF(z_{t+1}, |z_t)$, reflects the expected recovery rate of an international investor. That recovery depends on the parameter φ .

Finally, as pointed by [Hatchondo et al. \(2016\)](#), models with long-term debt and positive recovery rate can give the government incentives to issue large amounts of debt just before defaulting, which could allow for a large increase in consumption. These authors solve this issue by preventing the government from issuing debt at certain low prices. In this model, the same kind of restriction is used where the government cannot issue new debt at a price lower than \underline{q} . In the calibration section, a value of \underline{q} is chosen such that this constraint is rarely binding, and still allows for debt issuances at the sovereign spreads that are observed in the data.

3.5 Recursive Formulation

The timing of events for a government with good credit history is summarized as following:

- the government enters a period t with debt D_t and productivity z_t is realized and observed.
- if the government decides to repay maturing debt λD_t and the coupons on non-maturing debt $(1 - \lambda) c D_t$, it then chooses current tax rate τ_t , and new debt issuances I_t to finance public consumption g_t .
 - at the beginning of the period, households decide on labor supply and firms on labor demand. Labor market equilibrium implies that for a wage rate w_t , $h_t^s = h_t^d = h_t$. Simultaneously, households also decide on their consumption schedules.
 - production follows and, towards the end of the period, profits π_t and interest earnings $e_t = (R_t - 1) \cdot \theta w_t h_t$ are transferred to the household and consumption follows.

- next period $t + 1$ the government keeps its good credit history and starts the period with D_{t+1} .
- if the government decides to default then credit history becomes bad, productivity suffers a loss equal to $l(z_t)$ and the government chooses taxes τ_t to finance g_t .
 - a similar chain of events as above determines consumption and labor, c_t and h_t respectively.
 - while in default the government's debt accumulates interests at the risk-free rate: $D_{t+1} = D_t/\bar{q}$.
 - with probability ζ the government receives a debt restructuring shock and its current debt is reduced by φ . The government can decide to repay that new amount of debt, thus regaining a good credit history, or to remain in default.

The remaining object to be define is the interest rate firms face regarding their working capital requirements. As in existing models of business cycles for small open economies with working capital (Neumeyer and Perri, 2005, Uribe and Yue, 2006, or Mendoza and Yue (2012)), the interest rate that affects firms is a function of the interest rate on sovereign debt:

$$R = m(1/q) \tag{12}$$

where m is an increasing function.

The structure described above implies that the government's problem admits a recursive formulation, where (2) is maximized subjected to equations (4) to (10). Letting $v(D, z)$ be the value of the government with good credit history, then the problem can be represented as

$$v(D, z) = \max_{\mathbb{I} \in \{1,0\}} \{(1 - \mathbb{I}) \cdot v^{rep}(D, z) + \mathbb{I} \cdot v^{def}(z)\} \tag{13}$$

where the value of repayment is defined as⁷:

$$\begin{aligned}
v^{rep}(D, z) &= \max_{D' \geq 0, \tau} \{u(c, h) + \beta E_z [v(D', z')]\} & (14) \\
st & \\
c &= (1 - \tau) \cdot (wh + \pi + e) \\
\pi &= zf(h) - wh - (R - 1) \cdot \theta wh \\
e &= (R - 1) \cdot \theta wh \\
\frac{u_h(c, h)}{u_c(c, h)} &= (1 - \tau)w \\
w &= \frac{1}{1 + \theta(R - 1)} \cdot zf_h(h) \\
\tau \cdot (wh + \pi + e) &= g + D[\lambda + (1 - \lambda)c] - q(D', z) \cdot [D' - (1 - \lambda)D] \\
R &= m(1/q(D', z))
\end{aligned}$$

Note that, because the government decides on debt and taxes under a labor market equilibrium⁸, once new debt is chosen, taxes become determined by (8). Also, constraints in (14) can be substituted into one and another yielding, after some algebra, a simplified representation:

$$\begin{aligned}
v^{rep}(D, z) &= \max_{D'} \{u(c, h) + \beta E_z [v(D', z')]\} & (15) \\
st &
\end{aligned}$$

$$c = zf(h) - g + \Psi' \quad (16)$$

$$\frac{u_h(c, h)}{u_c(c, h)} = zf_h(h) \cdot \frac{1 - \frac{g - \Psi'}{zf(h)}}{1 + \theta(m[1/q(D', z)] - 1)} \quad (17)$$

where $\Psi' = q(D', z) \cdot [D' - (1 - \lambda)D] - D[\lambda + (1 - \lambda)c]$ are net external inflows. Equation (16) can be interpreted as the usual resources constraint, and (17) as an implementability constraint.

⁷ $u_c(c, h)$, $u_h(c, h)$, and $f_h(h)$ stands for $\frac{\partial u(c, h)}{\partial c}$, $\frac{\partial u(c, h)}{\partial h}$, and $\frac{\partial f(h)}{\partial h}$ respectively

⁸That is, it represents the set of competitive allocations (c, h) such that both consumers and firms are optimizing given prices and taxes.

Similarly, the value of default is defined as:

$$v^{def}(D, z) = u(c, h) + \beta E_z [\zeta \cdot v(\varphi(D/\bar{q}) D/\bar{q}, z') + (1 - \zeta) v^{def}(D/\bar{q}, z')] \quad (18)$$

st

$$c = [z - l(z)] f(h) - g \quad (19)$$

$$\frac{u_h(c, h)}{u_c(c, h)} = [z - l(z)] f_h(h) \cdot \frac{1 - \frac{g}{[z - l(z)] f(h)}}{1 + \theta (m [1/q^{def}(D/\bar{q}, z)] - 1)} \quad (20)$$

3.6 Recursive Equilibrium

With the above model economy description, a Markov Perfect Equilibrium can be defined. This is an equilibrium notion requiring that, at every possible state, agents' beliefs over other agents are specified. Given these beliefs, each agent must choose actions that are the best responses to the strategies of the other agents. The government and international investors only use stationary Markov strategies.

Definition 1. A recursive equilibrium is a set of:

- i) Value function: $v(D, z)$
- ii) Debt price functions: $q(D', z)$ and $q^{def}(D', z)$

such that

- a) Given the debt price functions $q(D', z)$ and $q^{def}(D', z)$, the value function $v(D, z)$ solves the government problem (13)
- b) Given the value function $v(D, z)$, the debt price functions $q(D', z)$ and $q^{def}(D', z)$ are consistent with the lenders zero profit condition in (10).

Condition (a) requires that the government's default and borrowing decisions are optimal given the debt price schedule. Condition (b) requires the equilibrium debt prices that determine country risk premia to be consistent with optimal lender behavior. Moreover, given that allocations satisfy equations (4) to (10), then these are consistent with a competitive equilibrium in the labor market and satisfy the economy's resources constraint. A solution to this recursive equilibrium includes solutions for new debt $D'(D, z)$, consumption $c(D, z)$, hours $h(D, z)$, taxes $\tau(D, z)$, and default sets $\mathbb{I}(D, z)$.

4 Calibration and Quantitative Analysis

The quantitative implications of the model outlined are studied using numerical simulations at a quarterly frequency and using a baseline calibration. In order to proceed in that way, different functional forms are selected.

4.1 Functional Forms

The household utility function is a GHH after [Greenwood, Hercowitz, and Huffman \(1988\)](#) and has a long tradition in literature studying business cycles in small open economies ([Mendoza, 1991](#); [Neumeyer and Perri, 2005](#); [Aguiar and Gopinath, 2007](#)). This utility function has the advantage of shutting down the wealth effect on labor supply, therefore shocks in the productivity process have an output response of the same signal. This is of particular relevance for the present exercise that focus on labor response to a sovereign default crisis. For example, with a common CRRA utility function⁹, a strong negative income shock would generate a counter-factual increase in labor supply due to strong wealth effects. For that reason, instead, the following function is used:

$$u(c, h) = \frac{1}{1 - \sigma} \cdot \left(c - \Gamma \frac{h^{1+\gamma}}{1 + \gamma} \right)^{1-\sigma} \quad (21)$$

For the productivity loss function under default $l(z)$, a non-linear specification that is increasing with the the level of productivity:

$$l(z) = \max \{0, z - d_1\} \quad (22)$$

implying that for defaults that occur with $z > d_1$, the loss penalty becomes proportional to the productivity. This functional form is similar to the one used in [Chatterjee and Eyigungor \(2012\)](#) or in [Arellano \(2008\)](#). These authors showed that an increasing loss function in productivity is important to generate realistic default frequencies¹⁰. Such loss function enables some additional contingency to the government by penalizing less severely if a default occurs in a low productivity state of the world. The model proposed by [Mendoza and Yue \(2012\)](#) endogenizes this loss function using a model of trade credit.

As for the gross interest rate on working capital, a simple identity function is used for

⁹A constant relative risk aversion (CRRA) utility function has the following functional form: $u(c, h) = c^{1-\sigma} / (1 - \sigma) - \Gamma h^{1+\gamma} / (1 + \gamma)$.

¹⁰This is in contrast with a proportional one as, for example, in [Aguiar and Gopinath \(2006\)](#).

the equivalent zero coupon rate \tilde{r} associated with the long-term bond with price q :¹¹

$$R = m(1/q) = 1 + [\lambda + (1 - \lambda)(c + q)]/q$$

Note that different models of small open economies with working capital (Neumeier and Perri, 2005; Uribe and Yue, 2006) assume that interest rates on sovereign debt and working capital are equal. Here, the same assumption is made. In Mendoza and Yue (2012), it is argued that the strong correlation between the two interest rates emerge since the government can confiscate firm repayments at default. Arellano and Kocherlakota (2014) finds evidence of positive co-movement between private and sovereign interest rates. Also, in the appendix A.1, some evidence for Euro Area countries is provided where, using the interest rates for new loans to non-financial corporations as a proxy of corporate interest rates, it is showed that this correlation can be as large as 0.83 for the case of Greece.

Finally, the productivity process is modeled as a log-normal $AR(1)$, with

$$\log z' = \rho_z \log z + \epsilon', \quad \epsilon' \sim N(0, \sigma_z)$$

The numerical computation of the model uses value function iteration with finite element method, where optimal policies are search using grid search. Details of the algorithm used can be found in the appendix A.2.

4.2 Model Calibration

The model is computed at a quarterly frequency targeting the Greek economy for some key data moments. As already mentioned, Greece provides a recent example of the dramatic impact of of a financial crisis on labor markets. Greece announced its default in the last quarter of 2011 and, between 2009 and 2012, the unemployment rate jumped from 7.96% to 22.1%, a 14.2pp increase while, during the same period, real GDP fell by 16.3%. Mendoza and Yue (2012) shows that these observations are not uncommon for previous default episodes on emerging economies. High frequency data readily available provides an additional advantage of focusing this study to Greece.

With the numerical solution at hand, the model is then used to interpret the macroeconomic dynamics of variables of interest around default, such as output, consumption, and labor, while other features of simulated economy closed with other features of the observed

¹¹As shown in Chatterjee and Eyigungor (2012), that interest rate can be found by solving $1 + \tilde{r} = [\lambda + (1 - \lambda)(c + q)]/q$.

data. Observed data for output, consumption, employment, and trade balance are seasonally adjusted quarterly real series obtained from OECD from 1990 to 2015. Government debt is taken from the Eurostat. The tax series, also taken from the Eurostat, refers the annual average income tax of a single person with no children and 100% of the average income received by a worker in Greece. Output and consumption are in logs while trade balance is presented as a percentage of GDP. All series are filtered using a Hodrick–Prescott filter with a 1600 smooth parameter with the exception of the yearly tax series that uses 100.

The fixed parameters used in the calibration can be found in table 4. The risk aversion on consumption $\sigma = 2$ is adopted from [Uribe and Yue \(2006\)](#). The Frisch elasticity of $1/\gamma = 2$ is higher than some used in the literature however is not uncommon to see even larger values be some authors¹². In the above model, a large Frisch elasticity enables to model to generate a strong response of labor supply to shocks affecting the marginal product of labor¹³. The labor income share of 0.5 is taken directly from averaging one of AMECO’s adjusted wage share series. Parameter $\zeta = 0.083$ ([Richmond and Dias, 2009](#)) implies an average market exclusion of 3 years and is consistent with evidence presented in [Gelos, Sahay, and Sandleris \(2011\)](#) finding that debt restructuring have become faster in recent decades. Given that evidence is scarce on the importance of working capital, previous values used in the literature are used for guidance on the determination of θ . Because [Neumeier and Perri \(2005\)](#) uses a value of 1 and [Mendoza and Yue \(2012\)](#) of 0.7, an intermediate value of $\theta = 0.85$ is used in the model’s calibration implying that firms hold about 3.5 months of wages in working capital. The parameter φ that governs the international lenders in case of default is set to 0.5, which is in accordance with the restructuring observed in the Greek default as documented by [Zettelmeyer et al. \(2013\)](#). The parameter Γ is calibrated to deliver a mean labor supply in the model of 1 (normalized) and the government consumption g is set to generate a relative government consumption to GDP of 20%, as observed on average on Greece. Finally the parameters governing debt issuance are set in the following way: Germany’s yield is used to set the risk-free debt price to \bar{q} to 0.995, targeting an annual rate of 2%; the maturity parameter λ captures an average 7.4 years debt maturity as reported by the Eurostat; the debt coupon c is normalized to be $1 - \bar{q}$ so as to deliver a maximum debt price of \bar{q} ¹⁴; and the minimum issuance debt price is set to 0.65 which prevents the

¹²[Shimer \(2009\)](#), uses a Frisch elasticity of 4 in his model in order to justify some large labor movements in some European countries.

¹³Note that with a Frisch elasticity of 0, changes in the marginal product of labor would imply no change in hours supplied.

¹⁴The maximum debt price, that is, when the probability of repayment is certain, is given by $q^{max} =$

government from issuing debt at interest rates larger than 11%¹⁵.

Table 4: Fixed parameter values

Parameter		Value	Target
Risk aversion on consumption	σ	2	(standard in the literature)
Inverse Frisch elasticity	γ	0.5	(standard in the literature)
Risk free debt price	\bar{q}	1/1.005	Germany's government interest rate
Minimum issuance debt price	\underline{q}	0.65	Maximum annual issuance interest rate of 11%
Output elasticity of labor	α	0.5	Labour income share in GDP
Redemption probability	ζ	0.083	12 quarters of market exclusion
Share of working capital	θ	0.85	(standard in the literature)
Recovery parameter	φ	0.5	Greek 50% debt restructuring
Debt maturity	λ	0.034	Greek debt maturity of 7.4 years
Government consumption	g	0.2	Greek 20% government consumption to GDP
Labor disutility	Γ	0.4	Mean labor supply of 1

All the remaining 4 parameters $\{\beta, \sigma_z, \rho_z, d_1\}$ are jointly estimated using simulated method of moments where the following data statistics are targeted: standard deviation and autocorrelation of output; the ratio of the standard deviation of trade balance with respect to the standard deviation of output; and interest rate spread standard deviation. The results of this estimation process can be found in tables 5 and 6.

Table 5: Estimated parameter values

Parameter		Value
Discount rate	β	0.9715
Standard deviation of error	σ_z	0.005
Persistency of productivity	ρ_z	0.95
Productivity loss	d_1	0.923

Some comments are in order. First, the the choice for these moments attempts to bring some discipline to the model in the sense that the simulated economy resembles Greece. Second, the specific the Greek default event is being targeted with the choice of some targeted moments, namely the 50% recovery rate offered to Greek bondholder, and some un-targeted moments such as the fall in employment¹⁶. Third, one should note that under

$\bar{q}(\lambda + (1 - \lambda)c) / (1 - \bar{q}(1 - \lambda))$. Therefore, when $c = 1 - \bar{q}$, then $q^{max} = \bar{q}$.

¹⁵The conversion of a debt price q to an annual interest rate r^{annual} is given by the following expression: $r^{annual} = [1 + (\lambda + (1 - \lambda)c - \lambda q)/q]^4 - 1$.

¹⁶This choice for the recovery rate and the evaluation of the employment fall are not that far from the ones observed in the history of sovereign default. Benjamin and Wright (2009) document that on average

default, the government interest rate is given by equation (11), which implies an annualized corporate interest rate of 23% under the model simulation, consistent with the evidence from the 2002 Argentinean default where the average lending rate and deposit rate increased to above 50% (Arellano and Kocherlakota, 2014). Finally, as can be seen in the top panel of table 6, the simulated moments are in quite close to the data targets although the difference is not exactly zero.

Table 6: Targeted and non-targeted moments: simulation and data

Targeted moments		Data	Model
standard deviation of output	$stdev(y)$	1.9	1.9
autocorrelation of output	$corr(y_t, y_{t-1})$	0.80	0.79
standard deviation of the spread	$stdev(i^{spread})$	1.6	1.6
relative volatility of trade balance to output	$stdev(TB/Y)/stdev(y)$	0.4	0.4
Non-targeted moments			
relative volatility of consumption to output	$stdev(c + g)/stdev(y)$	1.2	1.3
correlation of output with trade balance	$corr(TB/Y, y)$	-0.45	-0.74
mean spread	$mean(i^{spread})$	3.2%	2.3%
correlation of output with spread	$corr(y, i^{spread})$	-0.58	-0.79
correlation of trade balance with spreads	$corr(TB/Y, i^{spread})$	0.44	0.65
correlation of employment with spread	$corr(i^{spread}, h)$	-0.74	-0.76
correlation of employment with tax rate (annual)	$corr(\tau, h)$	-0.49	-0.89
correlation of output with tax rate (annual)	$corr(\tau, y)$	-0.37	-0.78
employment drop from 3 years before default	$mean(h_t/h_{t-12}) - 1$	-17%	-15%
Mean debt to output (annual)	$mean(D/Y)$	110%	42%
Default rate (annual)	$default\ rate$	1-4%	2.5%

Notes: data from OECD, ECB, and Eurostat and refers to Greece. The quarterly data spans from 1994 to 2012 in order to exclude the default period. All data series are log-detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600 for quarterly data and of 100 for yearly data (the trade balance is detrended in levels). The interest spread spread is computed using the yield long-term government bonds in Greece and Germany. The model statistics are calculated using 50 simulation samples, each with 3000 periods (quarters).

Table 6 presents the simulation results for both targeted and non-targeted moments. The model maintain most of the discipline imposed by the targeted moments. Within the non-targeted moments, it should be underlined that the model is able to generate a fall in employment of the same magnitude as observed in the data. A large degree of proximity with the observed data is also achieved along the typical moments that are usually studied in the literature. In particular, volatility of consumption is larger than output and both spreads and trade balance are negatively correlated with output. Given recovery rates amount to about 60% and Mendoza and Yue (2012) present evidence showing that, on average, employment is 15% lower than in the three years prior to default events.

the focus of the model on other features, such as employment and taxes, moments regarding these dimension, usually not analyzed in previous literature, can also be computed. The results shows that, at least qualitatively, the model mimics well the data observations. In particular, tax rates are counter-cyclical and also negatively associated with employment. The default rate is in line with other studies of sovereign default, with a non-targeted annual frequency of 2.5%. The main discrepancy in this calibration is related with the average debt to output where the model generates 42% while in the data shows that Greece has a level of government debt of more that 100% of output. With this respect, [Aguiar and Gopinath \(2006\)](#) extend the Lucas example to show that financial autarky is not a harsh punishment enough to sustain large amounts of debt in equilibrium, so the results from the simulation are not inconsistent with those findings.

These results indicate that austerity policies, that is, tax policies that tend to aggravate output fluctuations, are a consequence rather than an active choice policy of governments. When an economy is hit by a recession, harder credit conditions forces a government to increase tax rates under an impossibility of reducing public consumption. Such increases affect negatively consumers that respond by lowering their market supply of labor. Since firms are also exposed to similar credit conditions as the government, they also reduce labor demand when faced with higher interest rates. Note that these adverse effects would not be present under an alternative economy without working capital constraints and where lump sum fiscal policies are available to the government. Thus, such movements in hours can be interpreted as a labor wedge that falls when the economy is in recession. Under a default event, the effects previously described become even more pronounced.

4.3 Policy and Impulse Response Functions

To facilitate the the understanding of the mechanism underlying the results presented in the previous section, figure 4 plots the computed model policy and debt price functions, as well as the default set. The figure shows that new debt issuances, as defined in equation (7), is a decreasing function of current debt but increasing in productivity; the debt price schedule is downward sloping in current debt and increasing with productivity. This reflect the fact that indebted governments have higher probability of default, and this probability is smaller when the economy is growing (as depicted in the right panel of figure 4).

Alternatively, impulse response functions can be plotted to analyze the optimal government policy when the economy is hit by external shocks. Figure 5 shows the economy's response when the productivity suddenly drops by 1.5% from its average.

Figure 4: Policy functions, price schedules, and default set

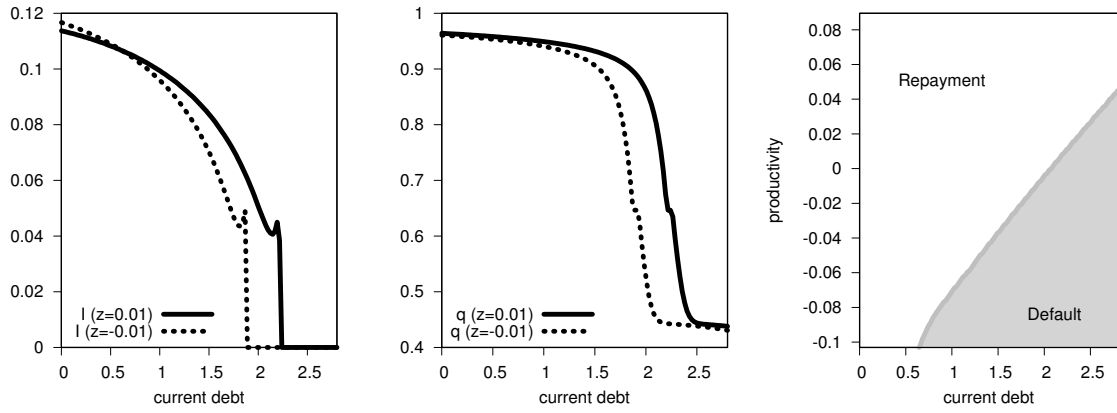
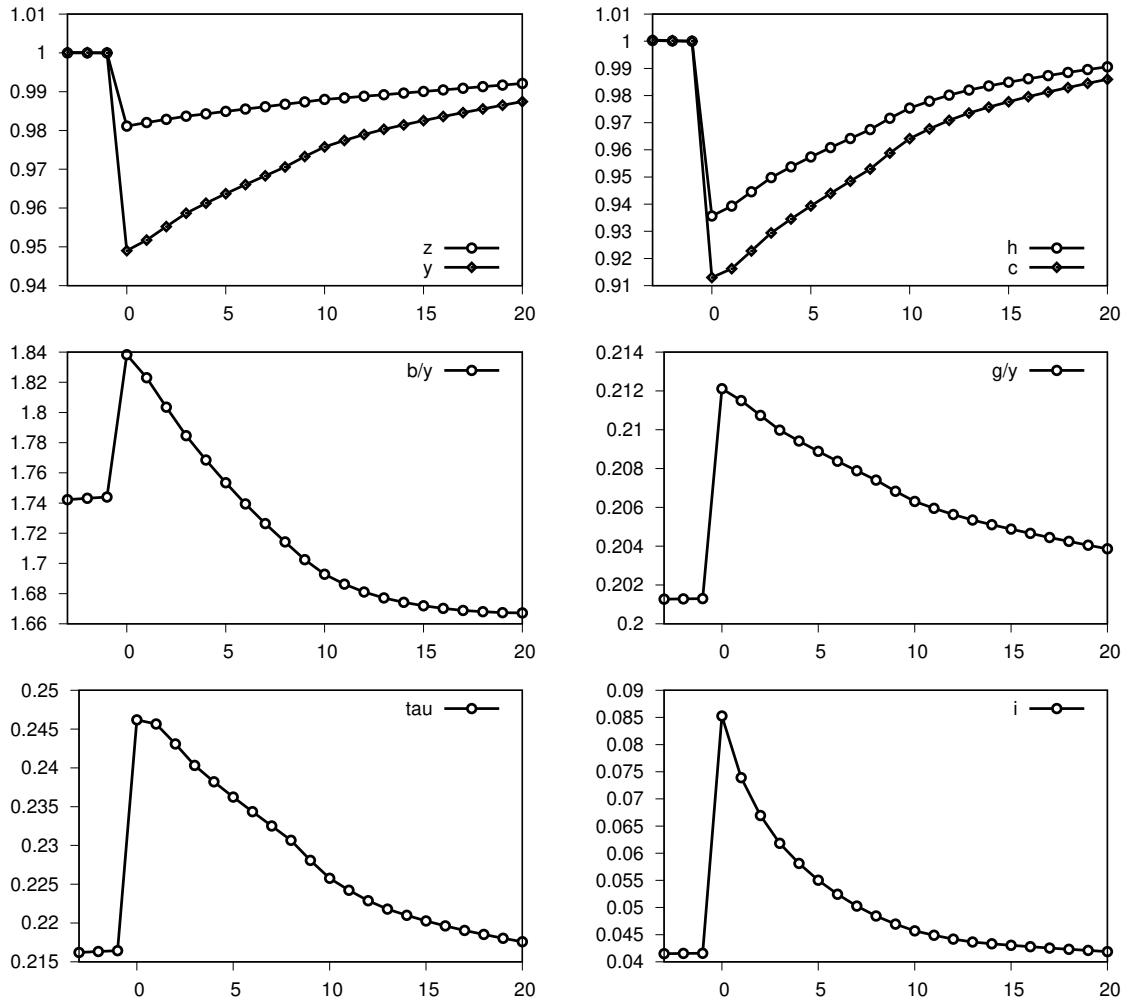


Figure 5: Impulse response functions when productivity falls by 1.5%



One first observation is that, although productivity falls by 1.5%, output falls by about 5%. This happens due to an endogenous response of labor that retraces by about 6%. The figure also shows that, in response to the fall in productivity, the government increases the tax rate from 21.5% to 24.5%. Additionally, on impact, the interest rate increases substantially and that is due to the fact that, given the current debt level, the government is more likely to enter in default. Because of this price increase, the government tries to reduce its debt exposure allowing for a normalization of the interest rates during the subsequent periods. The gradual reduction in the interest rates, lowers the working capital costs for firms, allowing them to increase production. This expansion increases the taxation base which also allows the government to reduce tax rates.

To better understand the effect of taxes and the interest rate spreads on labor dynamics, note that equation (4) and (6) can be combined into the following expression:

$$\frac{\partial u(c_t, h_t)/\partial h_t}{\partial u(c_t, h_t)/\partial c_t} = (1 - \tau_t) \frac{1}{1 + (R_t - 1) \cdot \theta} \cdot z_t \frac{\partial f(h_t)}{\partial h_t}$$

Which can be re-written as:

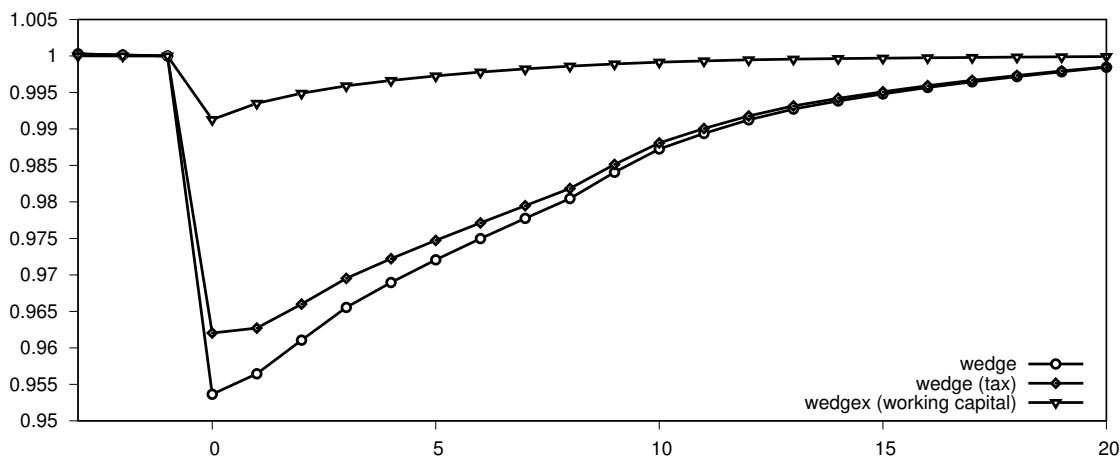
$$\frac{\partial u(c_t, h_t)/\partial h_t}{\partial u(c_t, h_t)/\partial c_t} = \omega_t \cdot z_t \frac{\partial f(h_t)}{\partial h_t} \quad (23)$$

Here ω_t is a labor wedge: the combined distortion of income taxes and working capital constraints on the labor market equilibrium. Applying a log-linearization to (23) near the values $(\bar{\tau}, \bar{R})$ implies:

$$\hat{\omega}_t = -\frac{\bar{\tau}}{1 - \bar{\tau}} \hat{\tau}_t - \frac{\theta \bar{R}}{1 + (\bar{R} - 1) \theta} \hat{R}_t \quad (24)$$

where the first term in the right hand side is the contribution of the income tax to changes in the wedge and the second term is contribution of the interest rate. Such decomposition is depicted in figure 6 when productivity z drops 1.5%. One can see that from the almost 5% fall in the labor wedge, 4% can be accounted due to an increase in the income tax while 1% is due to an increase of interest rates through working capital constraints. One can conclude that both channels play an important role to explain the magnitude of the labor wedge.

Figure 6: Labor wedge and contributions when productivity falls by 1.5%



4.4 Event Analysis - the Greek crisis

To evaluate the performance of the model, the Greek recent crisis is used as an event analysis. That is, the Greek GDP path between 2006-2012 is used to feed the simulated model using a sequence of productivity shocks $\{z_t\}$. In this sense, the GDP path of the model is matched to what is observed in the data and the remaining variables series, not targeted directly, can be evaluated. The result of this exercise can be seen in figure 7.

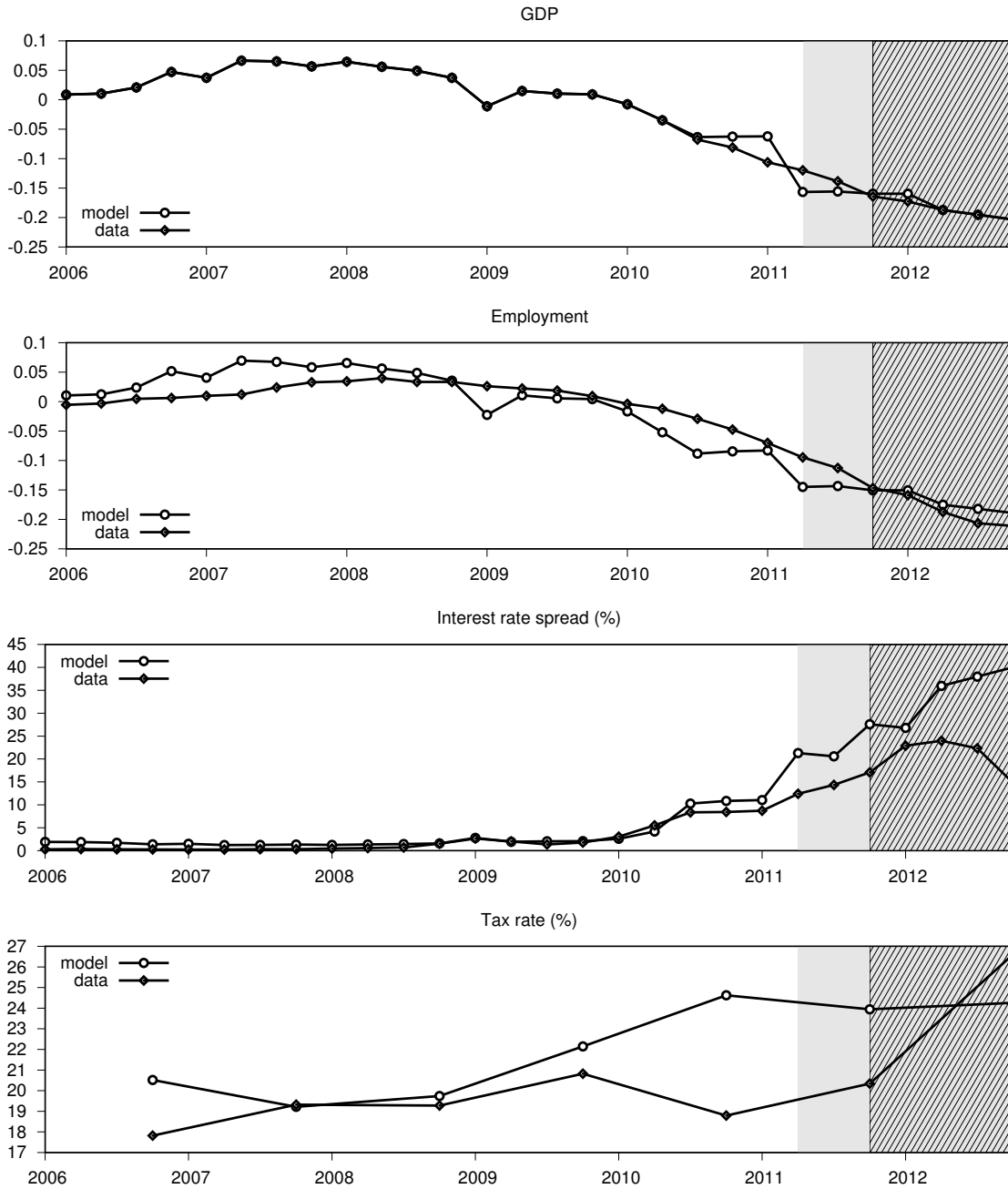
The figure plots, for both data and model, time series for GDP, hours¹⁷, the interest rate spread for government bonds, and the tax rate¹⁸. In the model, a default is triggered on the 1st quarter of 2011, 2 quarters earlier from the default event observed during in the Greek crisis¹⁹.

¹⁷Due to data limitations, Greek employment is being used in this panel and any fluctuations in average hours is not being considered.

¹⁸The tax series is taken from the Eurostat and refers the annual average income tax of a single person with no children and 100% of the average income received by a worker in Greece. To increase comparability the model counterpart averages the generated tax rate for the same years.

¹⁹It should be noted however that before the its default event, Greece negotiated a bailout rescue loan with the IMF and the European Union institutions in the 2nd quarter of 2010. Without such loan it is uncertain if the government would be able to honor its debt obligations.

Figure 7: Model and data time series for the Greek crisis

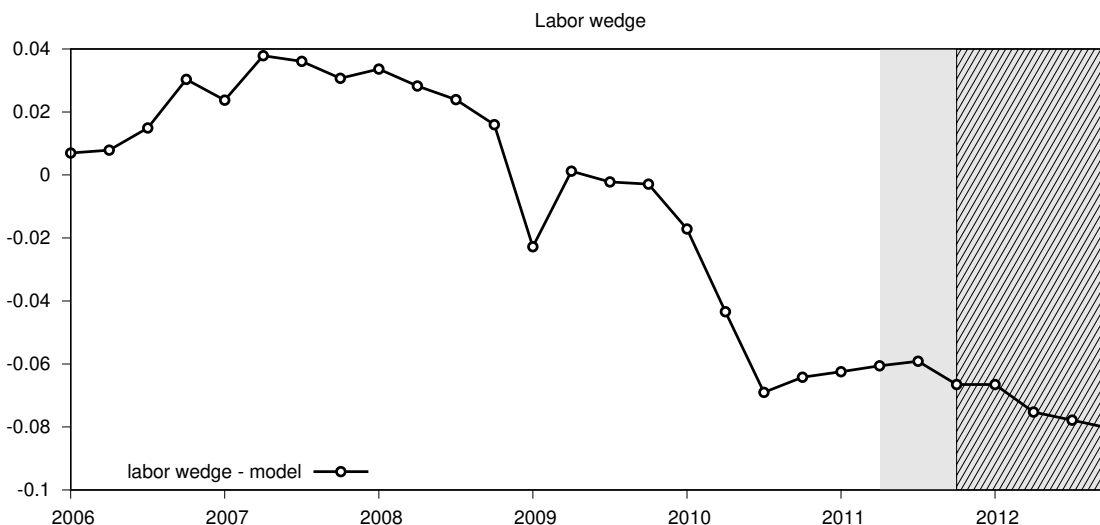


Notes: The shaded area in gray corresponds to a default period generated in the model, while the pattern shaded area corresponds to the Greek economy default period.

The top panel of the figure shows that the model can capture well the evolution of GDP during the Greek crisis with the exception of a few periods around the default decision. This is because of the productivity loss function (22) that is used. With this function, when a default occurs, productivity drops immediately to d_1 , reducing the flexibility of the

model to match the output path. That is also why employment, the interest rate spread, and the tax rate jump at the precise default quarter. The second panel in the figure shows that the employment time series path of the model captures the same dynamics that are observed in the data. The last two panels shows the main drivers of these dynamics. Both the interest rate spread and the tax rate are increasing in the model as well in the data. The government in the model has to raise taxes to finance an inelastic public expenditure thus affecting the household supply of labor. At the same time, the prevalence of negative productivity shocks affects the sovereign ability to fulfill its debt obligations, implying that interest rates continue to increase and that, in turn, affects firms given their working capital requirements. These movements combined add pressure on labor markets that subsequently collapse at default with a 15% fall in hours when compared with pre-crisis values.

Figure 8: Labor wedge time series in the model simulation



Notes: The shaded area in gray corresponds to a default period generated in the model, while the pattern shaded area corresponds to the Greek economy default period.

As mentioned before, both the effects of the interest rate spread and taxes affect negatively the labour market. It should be noted that such impact is on top of an already adverse path of successive negative productivity shocks. Contrary to a situation where, with full access to credit markets, a government facing a recession would lower distortionary taxes (Lucas and Stokey, 1983), tax rates are countercyclical in this model. Limited commitment in debt contracts implies that the interest rate is increasing when the economy is in recession. Because credit markets become more restricted the government is forced to raise taxes. This provides a channel between the possibility of government default and labor

market distortions induced by countercyclical tax and interest rates as shown in the bottom panels of figure 7. Using the measure of labor market distortion introduced in equations (23) and (24), figure 8 plots the labor wedge that this event analysis produces. As should be clear, the labor wedge is decreasing, implying stronger labor market distortions, as the Greek economy moves into default, consistent with the evidence presented in section 2.

4.5 Robustness and Experiments

In order to isolate the non-standard features of the baseline model outline before, this section re-evaluates the baseline model under different scenarios. Three alternative versions of the model are computed under the same calibration summarized in tables 4 and 5:

Lump sum taxation The model is recomputed assuming that the government has access to non-distortionary tax policies. That is, the new consumer budget constraint is now given by:

$$c_t = (1 - \bar{\tau}) \cdot (w_t h_t + \pi_t + e_t) - T_t$$

where T_t is a non-distortionary tax (which can be positive or negative) and $\bar{\tau}$ is the average distortionary income tax rate generated in the baseline model²⁰. The inclusion of $\bar{\tau}$ attempts to increase comparability between this version and the baseline model by imposing the same level of average distortions in the labor markets. Given the new consumer budget constraint, the counterparts of (16) and (17) simplify to:

$$c = z f(h) + q(D', z) \cdot [D' - (1 - \lambda) D] - D[\lambda + (1 - \lambda) c] - g$$

$$\frac{u_h(c, h)}{u_c(c, h)} = \frac{(1 - \bar{\tau})}{1 + \theta(m[1/q(D', z)] - 1)} \cdot z f_h(h)$$

Hence, the recursive problem of the government becomes exactly defined in the same way as before with the only difference in the above 2 equations.

Lump sum taxation with no working capital This formulation is exactly defined as the previous one imposing that $\theta = 0$, that is, in firms have no working capital requirements.

External impact of working capital In the formulation of the baseline model, the impact of the the government policy choices on the working capital requirements of firms through R is properly internalized. That is, the government understands that

²⁰In this exercise $\bar{\tau}$ is calibrated to be 0.217.

different prices for debt q affect firms decisions through different corporate interest rates R under the function $m(1/q)$. Under this experiment, instead, such impact is external for the government. The recursive problem for the government becomes (letting $\Psi' = q(D', z) \cdot [D' - (1 - \lambda) D] - D [\lambda + (1 - \lambda) c]$):

$$\begin{aligned}
 v^{rep}(D, z, R) &= \max_{D'} \{u(c, h) + \beta E_z [v(D', z', R')]\} & (25) \\
 \text{st} & \\
 c &= zf(h) + q(D', z) \cdot [D' - (1 - \lambda) D] - D [\lambda + (1 - \lambda) c] - g \\
 \frac{u_h(c, h)}{u_c(c, h)} &= zf_h(h) \cdot \frac{1 - \frac{g - \Psi'}{zf(h)}}{1 + \theta(R - 1)} \\
 R' &= \Psi(D', z')
 \end{aligned}$$

and the consistency condition $R' = \Psi(D', z')$ is such that $\Psi(D, z) = m(1/q(D', z))$ where $D'(D, z)$ comes from the optimal debt policy of the decision-maker given the state variables. The formulation of this problem, is analogous to [Kim and Zhang \(2012\)](#) that study government default decisions when borrowing is decentralized. It should be noted that because the government's problem changes, so does the computational method to solve it²¹.

Endogenous government expenditure This experiment endogenizes government consumption. To implement it, the following utility function is used²²:

$$u(c, g, h) = \frac{1}{1 - \sigma} \cdot \left(c - \Gamma \frac{h^{1+\gamma}}{1 + \gamma} \right)^{1-\sigma} + \Upsilon \cdot \frac{g^{1-\sigma}}{1 - \sigma}$$

It follows that, after some algebra, that the recursive problem for the government can

²¹The formal definition of the problem and algorithm used can be found in the appendix [A.3](#)

²²When this model is computed, the parameter Υ is calibrated to generate an average government to output similar to the data.

be summarized by (letting $\Psi' = q(D', z) \cdot [D' - (1 - \lambda) D] - D [\lambda + (1 - \lambda) c]$):

$$\begin{aligned}
v^{rep}(D, z) &= \max_{D'} \{u(c, g, h) + \beta E_z [v(D', z')]\} \\
st \\
c &= zf(h) - g - D + q \cdot D' \\
\frac{u_h(c, g, h)}{u_c(c, g, h)} &= zf_h(h) \cdot \frac{1 - \frac{g - \Psi'}{zf(h)}}{1 + \theta (m [1/q(D', z)] - 1)} \\
u_c(c, g, h) &= u_g(c, g, h)
\end{aligned} \tag{26}$$

and a similar formulation is used for the value of default v^{def} . The only difference with respect to the baseline model resides in equation (26) that simply states that the marginal utility of private consumption equals the marginal utility of public consumption.

Table 7 shows the results of the computational results of the different alternatives against the baseline model. The results highlight the relevance of two channels introduced in this paper to explain the large fluctuations of employment during default events. In particular, when the sovereign has access to lump sum taxes, in the column “Lump sum”, the fall of employment during a default is reduced by 3pp to 12%, lower than what is observed in the data. This happens since, under such environment, the government is not pushed into raising income taxes when the economy is in recession. For that reason, part of the tax distortions that increase the instability in labor markets disappear and, as a consequence, the the output volatility becomes smaller. When on top of lump sum policies, working capital requirements are withdrawn, under column “No working capital”, the employment drop is reduced by some additional 4pp to 8%. These effects highlight importance of the two transmission mechanism of labor market distortions.

Table 7: Model experiments and robustness checks

Targeted moments	Data	Baseline	Lump sum	No work cap	External	Endog g
$stdev(y)$	1.9	1.9	1.4	1.3	1.9	1.7
$corr(y_t, y_{t-1})$	0.80	0.79	0.83	0.83	0.79	0.79
$stdev(i^{spread})$	1.6	1.6	1.7	1.9	1.6	1.7
$stdev(TB/Y)/stdev(y)$	0.4	0.4	0.8	0.8	0.4	0.6
Non-targeted moments						
$stdev(c + g)/stdev(y)$	1.2	1.3	1.5	1.5	1.4	1.5
$corr(TB/Y, y)$	-0.45	-0.74	-0.52	-0.47	0.74	-0.72
$mean(i^{spread})$	3.2%	2.3%	2.6%	3.0%	2.3%	2.7%
$corr(y, i^{spread})$	-0.58	-0.79	-0.84	-0.81	-0.79	-0.81
$corr(TB/Y, i^{spread})$	0.44	0.65	0.66	0.58	0.67	0.69
$corr(i^{spread}, h)$	-0.74	-0.76	-0.89	-0.89	-0.76	-0.76
$corr(\tau, h)$	-0.49	-0.89	-	-	-0.89	-0.65
$corr(\tau, y)$	-0.37	-0.78	-	-	-0.78	-0.54
$mean(h_t/h_{t-12}) - 1$	-17%	-15%	-12%	-8%	-15%	-11%
$mean(D/Y)$	110%	42%	47%	44%	40%	42%
$default\ rate$	1-4%	2.5%	3.0%	3.6%	2.7%	3.0%

The column “External” shows that, when the working capital channel is not internalized by government, most of the dynamics of the model are kept constant. The main difference is that default becomes slightly more frequent²³. Given the small magnitude of this effect, it does not seem unreasonable to think that a government may not take into consideration that its own borrowing decisions affect firms credit conditions, similarly to what is implicitly assumed in previous papers that use working capital requirements to generate a channel between interest rates and labor demand (Neumeyer and Perri, 2005; Uribe and Yue, 2006)²⁴.

Finally, in the last column of table 7, government consumption is endogeneized to allow for the fact that government consumption usually falls during default episodes (Cuadra et al., 2010). Qualitatively, this alternative model behaves similarly to the baseline model. However, a few important quantitative differences should be noted. First the correlation of the tax rate with output falls to levels more close to what is observed with the data. This happens because, as productivity falls, both private and public consumption follow, implying that income taxes don’t need to be raised as faster. However, this also implies that

²³A similar result is found in Kim and Zhang (2012)

²⁴Alternative parameterizations that imply larger default frequencies generate also larger differences between the “Baseline” and the “External” models where the later generates larger default probabilities and larger standard deviations of the spread.

the cost of a default becomes less severe than in the baseline model and this increases the default frequency. Given that the model performs better in some dimensions while worse in others, a more flexible, but also more complex approach may be necessary to properly match the data²⁵.

5 Conclusion

New evidence supporting the idea that financial crises, arriving in the form of sovereign default, generate distortions in labor markets can be seen in the recent Euro Area debt crisis, where countries most affected, such as Greece, Portugal, or Ireland, were also the same where employment suffered the most dramatic reduction from pre-crisis levels. Because standard economic theory is unable to account reasonably for these drops in employment, an alternative explanation relies on un-modeled market distortions. Using a standard labor wedge decomposition to quantify distortions, it was shown that these are positively correlated with government interest rate spreads. Increasing government interest rates, are in fact indicative that credit is less available. For that reason, governments need to rely more on taxation, often distortionary, to finance public expenditures. At the same time, a close link between government and corporate interest rates provides an additional source of through which distortions may affect labor markets when firms are required to maintain working capital.

This paper studied the extent to which these two channels can account for labor dynamics in countries affected by sovereign debt risk using a simple but realistic economic model that highlights in a transparent and intuitive fashion the economic relationship of these dynamics. An otherwise standard sovereign default model is augmented to include distortionary income taxation and firm's working capital requirements. Calibrated to match several data moments from Greece, the model is able to deliver simulated moments that are close to the observations from the data. In particular, the model generates an endogenous increase in tax and interest rates when productivity falls, thus implying a deterioration of the labor wedge. The model can account for an average 15% fall in employment with respect to pre-crisis levels at a default event, a fall comparable to what is observed in the data. From these 15%, 3pp can be associated with the an increase of the tax rate, and an additional 4pp due to a spill-over of high interest rates from the government to corporations. It can be concluded that these channels provide an important quantitative explanation for

²⁵A potential source of additional costs of default may be related with political uncertainty that such events produce as in, for example, [Hatchondo, Martinez, and Saprizza \(2009\)](#).

the labor market dynamics that precede a default event.

Because the model was purposely design to be simple, several dimensions were overlooked that can also be important, in particular international trade. As underlined by [Gorodnichenko, Mendoza, and Tesar \(2012\)](#), a sudden change in import conditions, for example through terms of trade shocks, can impose an important adjustment cost to firms that can be protracted. Relating sovereign debt crises with firm's international trade conditions can be a source improvement of the explanations provided in this paper, especially if there are strong complementarities between imported goods and labor in firm's production function. These ideas should be integrated in future research.

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A Appendix

A.1 Corporate and Sovereign Interest Rates in Greece

Using data from the ECB, two measures of interest rate spreads are used:

1. Government interest rate spreads, defined as the difference between 10 year government bond yield of each country against Germany;
2. Corporate interest rate spreads, defined as the difference between average corporate yields charged on new loans in each country against Germany²⁶;

Comparisons between these 2 indicators should be cautious for a variety of reasons: government bonds don't have any collateral associated with it, while corporate bonds may have; heterogeneous institutional frameworks across countries should impact the yields charged in banks, for example, stricter enforceability laws against default should imply different interest rates; a selection effect where only healthy firms have access to credit markets may contaminate cross-country comparisons of corporate bond yields.

Given these caveats, the following table present simple correlations between the two indicators for Euro Area countries

Table 8: Simple correlation between government and corporate yield spreads for 2002 to 2012

	correlation	95% CI	
Austria	-0.305	-0.540	-0.026
Belgium	-0.085	-0.358	0.201
Germany	-	-	-
Spain	0.766	0.618	0.862
Finland	-0.116	-0.384	0.171
France	0.625	0.417	0.771
Greece	0.827	0.711	0.899
Ireland	0.179	-0.107	0.438
Italy	0.684	0.498	0.809
Netherlands	0.128	-0.159	0.395
Portugal	0.865	0.772	0.922
Slovenia	0.570	0.345	0.734

Notes: data source from the ECB.

²⁶The ECB defines these yields as a composite cost-of-borrowing indicator for new loans to non-financial corporations (percentages per annum, rates on new business).

A.2 Computation and Algorithm of Baseline Model

The solution of the dynamic problem follows similar papers, e.g. [Hatchondo et al. \(2016\)](#), and approximates continuously the problem using *b-splines* of degree 3 for the state space of D and 2nd degree *b-splines* for z . The code is implemented in Fortran imposing 31 grid points for the state variable z and 59 for D . The state space of D ranges between $[0; 1.5]$, and z is given by $\log z \in \left[-6.5 \cdot \sigma_z / \sqrt{1 - \rho_z^2}; 6.5 \cdot \sigma_z / \sqrt{1 - \rho_z^2}\right]$. The algorithm uses value function iteration with the following structure:

1. Guess the value functions $v^{rep,0}(D, z)$ and $v^{def,0}(z)$; debt price functions $q^0(D', z)$ and $q^{def,0}(D', z)$; and debt policy function $D'^0(D, z)$
2. Use $v^{rep,0}(D, z)$ and $v^{def,0}(z)$ to solve (15) and (18) using a global optimizer (NEWUOA from Powell) over the space of D :
 - (a) labor equilibrium is obtained using a non-linear equation solver (Brent method)
 - (b) the resulting functions of this maximization step are $v^{rep,1}(D, z)$, $v^{def,1}(z)$, and $D'^1(D, z)$
3. With $v^{rep,1}(D, z)$ and $v^{def,1}(z)$ find the optimal debt prices that are consistent with (10): $q^1(D', z)$ and $q^{def,1}(D', z)$
4. Evaluate $\max \{ ||v^{rep,1}(D, z) - v^{rep,0}(D, z)||, ||v^{def,1}(z) - v^{def,0}(z)|| \}$; if it's larger than ϵ_v iterate on (1) using $v^{rep,0}(D, z) := v^{rep,1}(D, z)$, $v^{def,0}(D, z) := v^{def,1}(D, z)$, $D'^0(D, z) = D'^1(D, z)$, $q^0(D', z) = q^1(D', z)$ and $q^{def,0}(D', z) = q^{def,1}(D', z)$ until convergence is achieved

The maximum error allowed is $\epsilon_v = 10^{-6}$.

A.3 Definition and Algorithm of Model with External Impact of Working Capital

To implement an externality from a spill-over of the government interest rate spread to corporate interest rates, the definition of the equilibrium changes from a Markov Perfect Equilibrium to a Recursive Competitive Equilibrium as in [Kim and Zhang \(2012\)](#):

Definition. A recursive competitive equilibrium is a set of:

- i) Value function: $v(D, z, R)$

- ii) Debt price function: $q(D', z)$
- iii) Law of motion: $R' = \Psi(D', z')$
- iv) Policy function: $D' = \chi(D, z, R)$

Such that

- a) Given the debt price function $q(D', z)$ and law of motion $R' = \Psi(D', z')$, the value function $v(D, z, R)$ solves the government problem and yields $D' = \chi(D, z, R)$
- b) Given the value function $v(D, z, R)$, the debt price function $q(D', z)$ is consistent with the lenders zero profit condition
- c) The law of motion is consistent with firms interest rate: $\Psi(D, z) = l(1/q(\chi(D, z, R), z))$

Noting that the consistency of the law of motion implicitly defines R as a function of z and D , the value function $v(D, z, R)$ can also be implicitly defined by only D and z . Defining this function as $\tilde{v}(D, z)$, an algorithm to solve the model can be schematized as:

1. Guess the value functions $\tilde{v}^{rep,0}(D, z)$, $\tilde{v}^{def,0}(z)$, and a function $\Omega^0(D, z)$
2. Use $\tilde{v}^{rep,0}(D, z)$, $\tilde{v}^{def,0}(z)$, and $R = \Omega^0(D, z)$ to solve (25) and then update the value of default $\tilde{v}^{rep,1}(D, z)$ and $\tilde{v}^{def,1}(z)$ using an optimizer over the space of D :
 - (a) labor equilibrium is obtained using a non-linear equation solver
 - (b) the resulting functions of this maximization step are $\tilde{v}^{rep,1}(D, z)$, $\tilde{v}^{def,1}(z)$ and $D' = \chi(D, z)$
 - (c) compute $\Omega^1(D, z) = l(1/q(\chi(D, z), z))$
3. Evaluate $\max \{ ||\tilde{v}^{rep,1}(D, z) - \tilde{v}^{rep,0}(D, z)||, ||\tilde{v}^{def,1}(z) - \tilde{v}^{def,0}(z)|| \}$; if it's larger than ϵ_v iterate on (1) using $\tilde{v}^{rep,0}(D, z) := \tilde{v}^{rep,1}(D, z)$, $\tilde{v}^{def,0}(D, z) := \tilde{v}^{def,1}(D, z)$, and $\Omega^0 = \Omega^1$ until convergence is achieved