Optimal Taxation and (Female)-Labor Force Participation over the Cycle

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Abstract

Optimal labor tax results over the cycle are, quantitatively, typically driven by an estimate of the intratemporal elasticity of substitution that governs the reaction of hours worked to business cycle shocks and tax rate changes. A recent literature tries to decompose this intratemporal elasticity into its main components, the "Ins and Outs of Unemployment" (Shimer (2007)) to emphasize the importance of the extensive margin. This paper provides a model that a.) endogenizes all transition rates including firings and quits on the job as well as movements in and out of inactivity, b.) explains the fluctuations in these rates quantitatively while allowing for differences across gender and c.) remains tractable and open to Ramsey-optimal policy.

We estimate the model on US-data for the years 1970:1 to 2004:4 and show that the model predicts all labor market flows very well. We apply our model to show that observed labor tax rates over the cycle correspond fairly closely to the implied Ramsey-optimal ones.

*JEL Classification System: E31,E32,E24,J64

Keywords: search theory, unemployment, hours worked

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1 Introduction

Optimal labor tax results over the cycle are, quantitatively, typically driven by an estimate of the intratemporal elasticity of substitution that governs the reaction of hours worked to business cycle shocks and tax rate changes. A recent literature has emerged that has emphasized the important role of the extensive margin, in particular the high volatility of unemployment rates, for business cycle dynamics. The focus has shifted away from the neoclassical labor-leisure trade-off to a decomposition of the "Ins and Outs of Unemployment", see Shimer (2007). These labor market transitions decompose the standard hour’s worked choice of the neoclassical model into its components and can potentially provide us with a better understanding of labor market reactions to business cycle shocks. To date much attention has been paid to the job-finding probability given that the results in Shimer (2007) suggest that this probability is the main driving force for (un)-employment fluctuations, explaining about 50% of the unemployment volatility. However, his findings do not necessarily imply that labor force participation rates or firing rates are unimportant. This is particularly true for issues concerning optimal fiscal policy over the cycle given that his results, at face value, describe the percentage share of actual employment rate behavior under a given policy. Without a structural model not much can be learned about the importance of different flows under alternative policies.

The purpose of this paper is therefore to provide a structural general equilibrium model with an extensive and an intensive margin that a.) endogenizes all transition rates including firings and quits on the job as well as movements in and out of inactivity, b.) explains the fluctuations in these rates quantitatively and c.) remains tractable and open to Ramsey-optimal policy.

To this end we proceed in 4 steps:

First, we utilize a discrete choice framework that allows us to deal in a straightforward manner with the particular truncations that typically complicate endogenous entry/destruction models. Under particular distributional assumptions this choice structure allows us to obtain closed form solution for the truncation points which in turn allows us to introduce endogenous choice probabilities while essentially keeping the representative agent structure of the model. The model remains continuously differentiable and open to Ramsey optimal policy using standard tools.

Second, our model explicitly allows for the different labor market behavior across gender. This might be important, quantitatively, when discussing the distortive element of labor taxes given that females comprise a much bigger inactive group than males. Most studies on the intratemporal substitution elasticity also suggest that females have a significantly higher intratemporal elasticity than males, see Alesina and Ichino (2007). These gender differences can be important for optimal tax rate policy given that females are likely to react stronger to tax-incentives over the cycle than their male counterparts.
We highlight in the companion paper, see Jung (2008), that even with the same preference parameters for the labor-leisure choice in our model the implied intratemporal elasticity of substitution (or a corresponding concept adapted to the participation decision) for females will be higher than for males due to technological and distributional differences.

In a third step, we use observed measures for taxes, governmental expenditures and a TFP-measure as exogenous driving forces to compare the predictions of the model to the data for the years 1970:1 to 2004:4. Using the estimated exogenous shock processes as inputs we show that the model predicts all transition flows and other labor market variables fairly well. In particular we are able to show that firing/separation rates and the job-finding probability, which we both predict almost perfectly, are driven by the same common factor, the profit of the firm, as suggested already by the standard Mortensen-Pissarides framework and elaborated in den Haan, Ramey, and Watson (2000). Our prediction of the time-series path of quits allows us to provide a structural interpretation on the importance of firing rates and job-finding probabilities, that has been debated between Shimer (2007) and Fujita and Ramey (2007). Given that both rates are driven by the identical force, the data-driven counterfactuals suggested in Shimer (2007) and Fujita and Ramey (2007) lose part of their controversy.

The quantitative impact of labor taxes suggests that the government can, potentially, use its influence on this margin to smooth out cyclical variation in employment to stabilize the business cycle. We apply our model, in a final step, to the Ramsey problem of optimal labor tax rates over the cycle, which is typically infeasible in endogenous destruction or entry models that rely on a numerical evaluation of the cut-off level. We use this tool to ask the counterfactual question how the labor market had looked like if the US-policy had followed a Ramsey optimal path. We show that the government by encouraging or discouraging workers to actively search for a job, mainly influence the behavior of females by adjusting labor tax rates over the cycle. However, our analysis suggests that it is not optimal to follow a very aggressive stabilization policy given that there are substantial switching cost involved that need to be internalized.

Our paper fits into the tradition of the neoclassical Ramsey-taxation literature. The optimal tax policy has been extensively studied, see i.e. Atkeson, Chari, and Kehoe (1999)\textsuperscript{1}. Benigno and Woodford (2006b) and Benigno and Woodford (2006a) provide a time-consistent solution method to the basic stochastic neoclassical Ramsey-optimal policy model by restricting the choice set at period zero in a particular way. Their results can be viewed as a benchmark case which with to compare labor market matching models. As shown in the companion paper, models with labor market frictions substantially

\textsuperscript{1}see Chari and Kehoe (1999) for a survey of optimal taxation in the neoclassical model, and Judd (1985) and Chamley (1986) as the classical references.
deviate from the neoclassical world given that labor taxes add an additional intertemporal distortion to the model that is absent in the neoclassical world. Therefore, in the absence of additional instruments, capital tax rates will, in general, be positive.

The extension to labor force participation within the Mortensen-Pissarides framework has been analyzed in Garibaldi and Wasmer (2005) for average transition rates and in Haefke and Reiter (2006) for cyclical variations. We extend part of their insights to a considerably richer general equilibrium framework in which we can conduct Ramsey-optimal policy. Quantitatively our model relies on the ability of the Mortensen-Pissarides framework in explaining the "Ins and Outs" of unemployment, see Shimer (2007). Our model builds upon the literature that tries to explain the puzzle observed in Shimer (2005) and on the resolutions that have been proposed in, among many others, Hall and Milgrom (2007), Costain and Reiter (2005a), Shimer (2005), Jung (2005) and Hagedorn and Manovskii (2006). Our mechanisms utilizes a sticky wage mechanisms similar to Hall and Milgrom (2007) that allows us to resolve the Shimer-puzzle without resorting to the particular small surplus calibration of Hagedorn and Manovskii (2006) which would be particularly troublesome in models that discusses tax rate changes.

To our knowledge this paper is the first that provides a structural framework in which all transition flows are addressed and estimated jointly.\footnote{We build upon the data provided by Shimer (2007) and Fallick and Fleischman (2004) and the discussion of the properties in, among others, Fujita and Ramey (2007).}

The paper proceeds as follows: In section I we presents the main model and derive some non-standard feature due to the discrete choice structure of the model. Section II explains our calibration strategy and shows how the model performance. Section III presents our results on optimal taxation. Section IV concludes.

2 Description of the Model

Before giving the precise mathematical structure of the model we shall shortly outline the new elements we wish to stress in this paper with respect to labor market decisions.

Time is discrete. There is a measure one of male and female workers in society. Males and females face identical decision processes and have identical functional forms (but might differ in parameters). Figure (1) plots the decision tree of the model with respect to the elements of the labor market. A typical worker can be in three states, employed, unemployed or out of the labor force, denoted by $\tilde{e} \in \{e,u,o\}$, respectively. Let gender $\tilde{g} \in \{m,f\}$ denote male and female.
When employed the worker bargains with her employer, at the beginning of the period, about her wage and hours worked and produces a labor good according to her linear (in hours worked) production technology. At the end of the period she might move to a new employment state or remains in her current job-relation. If she receives a favorable alternative offer, she might move to a new firm with probability $q_{t, \tilde{g}}$. If she does not break up voluntarily, the firm might decide to fire her and does so with probability $f_{t, \tilde{g}}$. If she has not been fired or has voluntarily quit the job she might decide to drop out of the labor force and stay at home next period. She does so with probability $q_{t,eo,\tilde{g}}$. We employ a hierarchical decision process to avoid tie-breaking assumptions that might occur if workers would like to quit and be fired at the same instance. This also allows us to work with a decision process over two alternatives only. However, our results are not sensitive to the ordering and can be generalized to a higher stage decision process.\(^3\)

When unemployed, the worker might work at home and potentially receives unemployment benefits. She can either decide to stop searching for a job and drop out of the labor force with probability $q_{t,uo,\tilde{g}}$ or pay a search cost and enter the matching market. If she is lucky, she is matched with a firm and starts producing next period, otherwise she remains unemployed. Correspondingly, when out of the

\(^3\)We also do not consider a joint bargaining process over firing and quitting decisions. Instead we assume a unilateral process, given that we believe that private information would make a joint bargaining infeasible. However, it is straightforward to extend the equations to a decision process where workers and firms make their decisions jointly efficient. The resulting choice probabilities would be functions of the total match surplus instead of the share that accrues to the worker or the firm. Given the high correlation, our assumption will likely not affect the results significantly.
labor force, she again might spend time working at home. She can decide to enter the search pool after paying a setup cost with probability \( q_{t,ou,g} \) and enters the unemployment pool next period.

**Discrete Choice Setup:** Whenever the worker has a (discrete) choice she obtains an i.i.d. shock \( \epsilon = \epsilon_2 - \epsilon_1 \) reflecting the utility difference between the two choice states, say employed (\( \epsilon_2 \)) and inactivity (\( \epsilon_1 \)). We assume that \( \epsilon_1 \) and \( \epsilon_2 \) are i.i.d. double exponentially distributed with zero mean, such that \( \epsilon \) is logistic distributed with zero mean and variance \( \frac{\psi^2 \pi^2}{3} \). At the beginning of the period, before these shocks are materialized the worker decides on the cut-off level \( \xi \). Whenever the shock realizes above this level, she decides to switch from inactivity to employment receiving \( \epsilon_2 \), while she remains inactive otherwise and obtains a utility of \( \epsilon_1 \). Note that for welfare considerations just the utility difference between the two alternatives matters.

To see this point more clearly, consider the following, for the moment ad hoc\(^4\), value functions of a risk neutral worker, where the worker has a choice from moving to employment to earn a net wage \( w(1 - \tau_l) \) with associated value \( V_e \) or to remain inactive and to earn an outside option value \( \tilde{A}w \) assumed proportional to wages with associated value \( V_o \):

\[
V_e = w(1 - \tau_l) + \beta(1 - \lambda)V'_e + \beta\lambda V'_o
\]

\[
V_o = \tilde{A}w + E\max\{\beta V'_e + \epsilon_1, \beta V'_o + \epsilon_2\}
\]

From standard computations of an expected value of a maximum of extreme-value distribution we know that optimal choices are given by:

\[
\pi_{oe} = \frac{1}{(1 + e^{\frac{\beta V'_e - V'_o}{\psi}})}
\]

and the value can be expressed as:

\[
V_o = \tilde{A}w + \psi \log(e^{\frac{\beta V'_e}{\psi}} + e^{\frac{\beta V'_o}{\psi}})
\]

as shown in Anderson, de Palma, and Thisse (1992). Alternatively, we can rewrite the above program

\(^4\)It turns out that these value functions capture the essence of the friction we wish to stress in this paper by noticing that the worker receives a net wage when working while he receive a gross wage when inactive due to the assumption that the government cannot tax home production. This wedge is important for the government when choosing tax rates optimally.
as choosing the cut-off value \( \xi \):

\[
V_e = w(1 - \tau l) + \beta(1 - \lambda) V'_e + \beta \lambda V'_o
\]

\[
V_o = \max \{ \frac{-A w + \pi_{oe}\beta \epsilon + E(\epsilon_1|e) + (1 - \pi_{oe}) \beta \epsilon + \xi V'_e + \beta \lambda V'_o}{\xi} \}
\]

\[
E^* \epsilon = -[\psi(1 - \pi_{oe}) \ln(1 - \pi_{oe}) + \psi \pi_{oe} \ln(\pi_{oe})] > 0
\]

\[\pi_{oe} \equiv \frac{1}{1 + e^{-\psi}} \]

\[\xi = \beta(V'_e - V'_o)\]

First order conditions of the alternative program immediately reveal the equivalence of the solution.\(^5\)

The truncated expectation \( E^* \epsilon \) and, by the law of large numbers, its average realization, is concave in \( \pi_{oe} \) given that \( \frac{\partial E^* \epsilon}{\partial \pi_{oe}} = \psi \log \left( \frac{1 - \pi_{oe}}{\pi_{oe}} \right) \) and \( \frac{\partial E^* \epsilon}{\partial \pi_{oe} \pi_{oe}} < 0 \). This means that even though the idiosyncratic shocks have a zero mean by assumption, the fact that the workers have a choice between the two alternatives will lead to a positive valuation due to the truncation. The option value of this choice depends on the variance \( \psi \). The new program is continuously differentiable and can be handled much easier.

\(^5\)To see this more easily, note that \( \pi_{oe} \equiv \tilde{F}(\xi) \equiv \Pr(\epsilon_2 - \epsilon_1 \leq \xi) = \frac{1}{1 + e^{-\psi}} \) by the definition of a logistic random variable, where \( \epsilon_1 \) and \( \epsilon_2 \) are extreme valued i.i.d. random variables with zero mean. We denote the distribution function of the extreme-value distributed variables \( \epsilon_1 \) by \( F_i(x) \) and the corresponding density by \( f_i(x) \). So, given a cut-off level \( \xi \) the expected value of the shock is given by:

\[
E\epsilon_1 = \int_{-\infty}^{\infty} x f_1(x) F_2(x + \xi) dx
\]

\[
F_2(x + \xi) = e^{-e^{-x+\xi}}
\]

\[
f_1(x) = \frac{1}{\psi} ke^{-\xi} e^{-ke^{-\xi}}
\]

\[
k = e^{\frac{\psi}{\xi}}
\]

Following Anderson, de Palma, and Thisse (1992), a change of variable gives :

\[
E\epsilon_1 = -\int_{0}^{\infty} \psi \log(t) e^{-t(1+e^{-\xi})} - \int_{0}^{\infty} \psi \log(k) e^{-t(1+e^{-\xi})}
\]

Using:

\[
\int_{0}^{\infty} e^{-st} \log(t) dt = -\frac{\log(s)}{s} - \gamma; \gamma = \text{Euler constant}
\]

\[s = (1 + e^{-\xi})\]

\[k = e^{\frac{\psi}{\xi}}\]
easier than the original problem, given that we can let the family choose, by monotonicity and the law of large numbers, the probabilities directly. In the sequel, we shall rely on this equivalence extensively. To see how these choice probabilities map into elasticities define

$$\frac{\partial \pi_{oe,t}}{\partial w_{t+1}} = (1 - \pi_{oe,t}) \beta \left(1 - \tau_t - \bar{A}_t\right) w_{t+1}$$

Two main factors\(^6\) will govern this elasticity: The first factor is the underlying heterogeneity in idiosyncratic preferences. The higher the underlying variance of the idiosyncratic utility distribution or say the search cost is, the smaller will be the impact of changes (and the smaller will be the distortion on capital taxes). Intuitively \(\psi\) governs the mass of workers around the cut-off level and is therefore an ultimate driving force for the reaction of the probabilities to business cycle shocks (who implicitly change net wages) as well as for the reaction to tax rate changes. We identify this margin by the volatility of the corresponding transition rates driven by aggregate shocks. The second crucial margins, driven by technology, is given by the efficiency distance between home-production and the market good \(\bar{A}\) and the associated tax distortion. We will, once we have specified a general equilibrium version with endogenous hours choice, identify this margin by using the hour choice differentials across gender and employment status using the time-use survey of Aguiar and Hurst (2007), given that standard intratemporal hours worked decision across states will be a monotone function of the differences in technology.

We obtain using the zero mean assumption:

$$E\epsilon_1 = \psi \log \left(1 + e^{-\epsilon_2}\right) + \gamma + \eta \frac{\pi_{oe}}{1 + e^{-\epsilon_2}}$$

$$= \psi \log(1 + e^{-\epsilon_2})$$

$$= \frac{1 - \pi_{oe}}{\pi_{oe}}$$

$$E\epsilon_1 = -\psi \pi_{oe} \log(\pi_{oe})$$

$$E\epsilon_2 = -\psi (1 - \pi_{oe}) \log(1 - \pi_{oe})$$

where the last step follows from the fact that the worker does not switch if \(\epsilon_1 - \epsilon_2 \leq 0\) and a symmetric argument. So by properties of conditional expectations, \(E\epsilon_1 = \pi_{oe} E(\epsilon_1|\xi)\) and we can define the option value of having a choice as \(E\epsilon_1 + E\epsilon_2 = E^*\epsilon\), which gives the entropy type expectation:

$$E^*\epsilon = -[\psi (1 - \pi_{oe}) \ln(1 - \pi_{oe}) + \psi \pi_{oe} \ln(\pi_{oe})] > 0$$

\(6\)In fact, in the fully developed general equilibrium version of the model the elasticity will also depend importantly on preferences, i.e. the general equilibrium valuation of the home-produced good \(\bar{A}\) and the market produced substitute. To clarify the effect, our example abstracts from this dimension right now.
We now embed this labor market structure into a general equilibrium model to discuss optimal taxation in a meaningful way. To keep the model tractable we employ the standard family structure that pools idiosyncratic uncertainty and assumes that unemployment risk is insured by the typical family. This assumption allows us to stay within a representative agent framework and discuss Ramsey optimal policy.

2.1 Firms

We introduce 4 different sectors into the economy. This structure is needed to endogenize home-production, given that the decision to enter the labor market is a crucial ingredient of our model. As shown in a companion paper, see Jung (2008) the interaction of the tax system with this margin is quantitatively important.

The first three sectors are perfectly competitive and are used to ease aggregation. The final good sector combines capital and a labor input good to produce the numeraire output good. This good $Y_1$ is used for investment and for government spending:

$$Y_{1,t} = e^{\alpha t} K_t^{\alpha} L_{1,t}^{1-\alpha}$$

(6)

where $K_t$ is aggregate capital, $L_1$ is the amount of the labor good demanded in final production and $a_t$ is a technology shock assumed to follow an AR(1) process with normal increments.

The service good $Y_{2,t}$ is also produced competitively and is labor intensive, relative to the final good sector:

$$Y_{2,t} = L_{2,t}$$

(7)

We view this as a close substitute to home-produced goods. $L_{2,t}$ is the demand for the labor good produced in the service sector.

The third sector, again perfectly competitive, simply aggregates the labor good, produced in the main labor firm industry sector.

$$L_t = e_{t,m} l_{t,e,m} + e_{t,f} l_{t,e,f}$$

(8)

Let gender $\tilde{g} \in [m, f]$ denote male and female respectively and denote the three labor market states $\tilde{e} \in [e, u, o]$ as employment, unemployment and out of the labor force, respectively. Then $m_{\tilde{g}}$ are the measures of males and females in the society, $e_{t,\tilde{g}}$ are the percentage of employed males and females
and \( l_{t, \bar{g}} \) is the labor good produced in the labor service industry, where each employment relationship is viewed as a particular match between one worker and one firm.

The technology when employed or working at home is given by the homogenous production function:

\[
l_{t, \bar{e}, \bar{g}} = A_{\bar{e}, \bar{g}} h_{t, \bar{e}, \bar{g}}
\]  

(9)

where \( h_{t, \bar{e}, \bar{g}} \) denotes the amount of hours supplied by males and females and \( A_{\bar{e}, \bar{g}} \) denotes the technology level of a typical males or female in each of the three labor market states. We allow this level to differ across sex and employment state.

Aggregate prices from the final goods perfect competitive firms behavior are given by the standard conditions:

\[
\begin{align*}
r_{t, \bar{g}} &= \alpha \frac{Y_{1,t}}{K_t} \\
p_{t,w} &= (1 - \alpha) \frac{Y_{1,t}}{L_{1,t}}
\end{align*}
\]  

(10)  

(11)

where \( r \) denotes the interest rate and \( p_w \) denotes the price of the labor good. Given perfect competition \( p_w \) is also the price charged by the service firms.

Market clearing requires that:

\[
L_t = L_{1,t} + L_{2,t}
\]  

(12)

2.2 Laws of motion:

The state variables of the economy are, for each gender, the measure of unemployed \( u_{\bar{g}} \) and employed workers \( e_{\bar{g}} \), as well as the aggregate capital stock \( K \) and the exogenous shock processes described below. Let \( \delta \) denote the depreciation rate and \( I \) denote aggregate investment, then the respective laws of motion are given by:

\[
K_{t+1} = (1 - \delta)K_t + I_t
\]  

(13)

\[
e_{t+1, \bar{g}} = (1 - q_{t, \bar{g}})(1 - f_{t, \bar{g}})(1 - q_{t,eo, \bar{g}})e_{t, \bar{g}} + q_{t, \bar{g}}e_{t, \bar{g}} + (1 - q_{t,ou, \bar{g}})\pi_{t,ue, \bar{g}}u_{t, \bar{g}}
\]  

(14)

\[
u_{t+1, \bar{g}} = u_{t, \bar{g}}(1 - (1 - q_{t,ou, \bar{g}})\pi_{t,ue, \bar{g}} - q_{t,uo, \bar{g}}) + e_{t, \bar{g}}(1 - q_{t, \bar{g}})f_{t, \bar{g}} + u_{t, \bar{g}}\pi_{t,ou, \bar{g}}
\]  

(15)
and

\[ o_{t+1,\tilde{g}} = o_{t,\tilde{g}}(1 - \pi_{t,ou,\tilde{g}}) + e_{t,\tilde{g}}(1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})q_{t,eo,f} + q_{t,uo,\tilde{g}}u_{t,\tilde{g}} \]

where the inactivity state is redundant given that we have the accounting identity \( o_{t,\tilde{g}} = 1 - u_{t,\tilde{g}} - e_{t,\tilde{g}} \).

Some remarks about the setup are in order: First, note that we do not allow direct flows from inactivity to employment. By definition of unemployment as a state of search any flow from inactivity to employment must go through the unemployment state, even for a short time interval. Our model treats this decision process as lasting at least a month. When mapping the model to the data, a time-aggregation bias as highlighted in Shimer (2007) arises. Second, our timing convention is, obviously, somewhat arbitrary. We have chosen a sequential structure to ease the already quite complex structure, and to avoid tie-breaking situations where a worker has to decide to quit to a new job or become inactive. The sequential structure insures that the choice problem is binary, and therefore easier to handle. ⁷

### 2.3 Family

To avoid complications with respect to the wage bargaining that arises in incomplete market setting of the model, we assume the existence of a continuum of identical families that fully insures the consumption risk of their individual members and decides on aggregate savings. Given the separability of the preferences this implies that all individuals consume the same.⁸ In turn, however, the individual gives up his bargaining rights, or, alternatively, evaluates the stream of utility flows from the perspective of the aggregate family. This assumption, basically, ensures a common discount kernel, implying a common valuation of future streams of utility and allows us to neglect the distribution of capital holdings as an additional (infinite dimensional) state variable.⁹

---

⁷We have experimented by flipping part of the decision processes, but the particularities of the choices are not driving our results. Note that all workers are, conditional on their states, identical, and switching probabilities are quite small. So the mass of workers that stochastically first decide not to quit, and are fired afterwards, or are first not fired and then decide to quit, is almost identical. Even though the decision process is influenced by the aggregate shock, this influence is typically small quantitatively.

⁸see Jung (2005) for a treatment of non-separable utility in a labor market context

⁹The main complication that would arise otherwise is not so much the handling of a measure over capital holdings, given that standard tools could be used, but the fact that the wage bargaining would induce individual wages to a be a function of individual capital holding. This fact would considerably complicates the numerical analysis, or would need to be complemented by assumptions how average wages are set. Also optimal policy would even be harder to do, see Costain and Reiter (2005b) for an attempt, that does not deliver Ramsey optimal rules. These authors run precisely into the problem that labor tax rates influence, in a first order sense, the bargaining surplus of the average worker which
The representative family aggregates individual consumption and maximizes, given the law of motion of the state variables, sequences for wages and hours (derived below) and its budget constraint, consumption allocations and transition probabilities according to the inter-temporal utility function:

$$U \equiv \max_{\{c_1, c_2, c_3, I, \tilde{g}, \tilde{e}, \tilde{g}, \pi_{ou, \tilde{g}}, \pi_{e, \tilde{g}}, \tilde{h}_{u, \tilde{g}}, \tilde{h}_{u, \tilde{g}}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} E^{t-t_0} \left[ 2u(c_1, c_2, c_3) - \sum_{\tilde{g}} e_{t, \tilde{g}} \psi h_{t, \tilde{e}, \tilde{g}} - u_{t, \tilde{g}} \psi h_{t, \tilde{u}, \tilde{g}} - \alpha_{t, \tilde{g}} \varphi h_{t, \tilde{o}, \tilde{g}} \right] + \text{discrete choices} \right]$$

\[ (16) \]

s.t.

Budget Constraint

Initial Conditions and Laws of Motion for states

Given sequences of $w_{\tilde{g}}, h_{\tilde{e}, \tilde{g}}$

\[ (17) \]

Here $u(c_1, c_2, c_3)$ is a consumption aggregator over the final good $c_1$, the service good $c_2$ and the home-produced good $c_3$.

$$u(c_1, c_2, c_3) = \log(c_1, c_2, c_3)$$

with $c$ being a consumption aggregator

$$c = (\gamma c_1^{\gamma_1} + (1 - \gamma)(c_2^{\gamma_2} + c_3^{\gamma_2})^{\frac{\gamma_1}{\gamma_2}})^{\frac{1}{\gamma_1}}$$

\[ (18) \]

The parameters $\gamma_1, \gamma_2$ govern the substitution elasticity across goods and $\gamma$ gives the share of the final good relative to the labor intensive service good.

Here $h_{t, \tilde{e}, \tilde{g}}$ is the amount of hours supplied by a worker, with gender-status $\tilde{g}$ and employment status $\tilde{e}$. Note that workers who are not employed, still work in the sense of using time in producing at home. $\varphi_{\tilde{g}}$ is a normalizing constant scaling hours worked and $\frac{1}{\varphi_{\tilde{g}}}$ is the intratemporal elasticity of substitution.

The term *discrete choices* is a short hand for the mean cost and the option value of having a (discrete) is a function of aggregate capital holding in their model. This in turn delivers their strong results of tax rates on the unemployment rate, which appears to be an artifact of their wage bargaining modeling.
choice to change their respective labor states at the end of this period.

\[
discrete choices = \sum_{\tilde{g}} -e_{\tilde{g}}(q_{t,\tilde{g}}q_{t,\tilde{g}}) + \varphi_{\tilde{g}}\epsilon_{t,\tilde{g}}(1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})q_{t,\tilde{g}}
\]

\[
- E^*\epsilon_{t,ee,\tilde{g}}(1 - q_{t,\tilde{g}})(1 - f_{t,\tilde{g}})E^*\epsilon_{t,\tilde{g}}
\]

\[
- u_{t,\tilde{g}}(q_{t,\tilde{g}}\epsilon_{t,uo,\tilde{g}}(1 - \pi_{uo,\tilde{g}}) - E^*\epsilon_{t,uo,\tilde{g}})
\]

\[
- o_{t,\tilde{g}}(\pi_{t,ou,\tilde{g}}\epsilon_{t,ou,\tilde{g}} - E^*\epsilon_{t,ou,\tilde{g}})
\]

We express the fixed costs associated with each choice in units of average hours worked. So, for example, the utility cost of quitting an employment relation and moving to a new employer, potentially in a different town, say, are denoted by \(s_{q,\tilde{g}}\) and are suppose to capture the potentially important cost of moving, finding new friends and colleagues or being away from home. The notation for all other cost are correspondingly. For example, the cost \(s_{uo,\tilde{g}}\) captures the setup cost of staring to search for a job.

The budget constraint of the family is given by:

\[
(m_m + m_f)c_{1,t} + (m_m + m_f)p_{w,t}c_{2,t} = m_m w_{t,m} h_{t,m} e_{t,m}(1 - \tau_{t,\tilde{m}}) + m_m b_{t,m} u_{t,m}
\]

\[
+ m_m e_{t,m} q_{t,m} E_d t+1 \Pi m.t+1
\]

\[
+ m_f w_{t,f} h_{t,f} e_{t,f}(1 - \tau_{t}) + m_f b_{t,f} u_{t,f}
\]

\[
+ m_f e_{t,f} q_{t,f} E_d t+1 \Pi f.t+1
\]

\[
+ K_t + (r_t - \delta)(1 - \tau_{t,c})K_t - K_{t+1} + D_t + Tr_t
\]

\[
(m_m + m_f)c_{3,t} = m_f A_{t,u,f} h_{t,u,f} u_{t,f} + m_m A_{t,u,m} h_{t,u,m} u_{t,m}
\]

\[
+ m_f A_{t,o,f} h_{t,o,f} o_{t,f} + m_m A_{t,o,m} h_{t,o,m} o_{t,m}
\]

Here \((r - \delta)(1 - \tau_c)\) is the after-depreciation, after tax capital income, \(D\) are aggregate profits that might arise in the economy and are distributed back to the owners, \(Tr\) denotes lump sum transfers or taxes from or to the government and \(\tau_c, \tau_l\) denotes the capital and labor tax while \(K\) is the aggregate capital stock. Unemployment benefits are denoted by \(b_m\) for males and \(b_f\) a for females, which we allow to be different from their male counterpart in general. The unemployment rates and out of the labor force rates are taken with respect to the mass of males and females, not with respect to total population.

Having endogenous firing rates in the model, we decided to endogenize quits on the job as well to capture the observed empirical regularity taken for example from the JOLST database, that overall destruction rates are rather acyclical, while firing rates are countercyclical and quantitatively smaller than overall destruction rates. Procyclical quits, as documented in Shimer (2007) basically dampens
fluctuations in the overall destruction rate and capture the important transitions from employment to employment, with a potentially one month intervening spell of inactivity as documented in Nagypal (2005). Given that we did not want to increase the already quite complex structure of the model by introducing separate matching functions for quits on the job\(^{10}\) we model the incentives to quit as receiving an initial “golden hand check” \(E_d t+1 \Pi_{t+1}\) when she decides to switch her job. To mimic the behavior of on-the-job matching markets we assume that, on the job, workers can search and contact other firms much more easily than unemployed workers.\(^{11}\) This implies that an infinite number of firms stand ready to hire these workers without having to pay vacancy posting cost. We therefore have to redistribute the profits that arise. We assume that these profits are paid lump sum up front to the newly hired worker and that the standard bargaining starts from that point on, insuring that the wage each worker receives is the same and that firms hiring workers on the job make zero profits.\(^{12}\) The amount of this extra payment is the probability adjusted saved amount of vacancy posting cost the firm would incur when hiring an unemployed worker, or the value of a match from the firms perspective, so our assumptions insures that firms still make zero profits on average.

The first order conditions for the different consumption and capital choices are:

\[
\frac{\partial u}{\partial c_{1,t}} = \lambda_{1,t} \tag{22}
\]

\[
\frac{\partial u}{\partial c_{2,t}} = \lambda_{1,t} p_{w,t} \tag{23}
\]

\[
\frac{\partial u}{\partial c_{3,t}} = \lambda_{2,t} \tag{24}
\]

\[
\lambda_{1,t} = E \beta \lambda_{1,t+1} (1 + (r_{t+1} - \delta)(1 - \tau_{c,t+1})) \tag{25}
\]

\[
\beta \lambda_{1,t}^{t+1} = d_{t+1} \tag{26}
\]

where \(\lambda_{1,t}\) is the lagrange multiplier on the budget constraint, \(\lambda_{2,t}\) is the lagrange multiplier on the home-produced good constraint, and \(d_{t+1}\) is the definition of the discount kernel. Finally, the first order conditions on the hours choice when working at home is given by:

\[
A_{t,u,\tilde{g}} \lambda_{2,t} = \varphi_{h}^{\tilde{g}_{t,u,\tilde{g}}^{t-1}} \tag{27}
\]

\[
A_{t,o,\tilde{g}} \lambda_{2,t} = \varphi_{h}^{\tilde{g}_{t,o,\tilde{g}}^{t-1}} \tag{28}
\]

The hour choice when employed is derived within the bargaining setup explained next.

\(^{10}\)In fact it is straightforward to do and would also lead to pro-cyclical quitting behavior.

\(^{11}\)That is they might have access to the companies intra-net and can switch jobs within their own company or have contact to different companies through their job.

\(^{12}\)This assumption allows us to remain within a representative agent framework. We assume that wages are continuously re-bargained, and that all payments made before are sunk. This exclude the possibility of providing an optimal contract over time and shifts the extra surplus as a lump-sum payment to the beginning of the job.
2.4 Flow values and Bargaining

We now turn to a discussion of the matching markets, where the main labor friction is located. We first derive the standard flow values of individual matches from the worker/family point of view and then from the firms point of view. These values are needed to derive the bargaining solution in the next section.

2.4.1 Flow Values

Given our assumption of perfect insurance the family values of the stream of labor income at the beginning of the period (that is before individual utility draws are made, but after the aggregate shock is observed) from an employed or unemployed female is given as the marginal utility of having one worker employed rather than unemployed. Define the lagrange multipliers on the law of motion conditions with respect to next period employment states, we obtain\(^\text{13}\):

\[
\Delta_{t,eu,\tilde{g}} = V_{t,e,\tilde{g}} - V_{t,u,\tilde{g}} \\
\Delta_{t,eo,\tilde{g}} = V_{t,e,\tilde{g}} - V_{t,o,\tilde{g}} \\
\Delta_{t,uo,\tilde{g}} = V_{t,u,\tilde{g}} - V_{t,o,\tilde{g}}
\]

\[
V_{e,\tilde{g}} = \lambda_{t,w} H_{t,\tilde{g}} (1 - \tau_{t,l}) \varphi^{\omega}_{t,\tilde{g}} - \varphi^{\pi}_{t,\tilde{g}} q_{t,\tilde{g}} - (1 - f_{t,\tilde{g}}) q_{t,eo,\tilde{g}} \varphi^{\rho}_{t,eo,\tilde{g}} + E^{*} \tilde{\varepsilon}_{t,ee,\tilde{g}} \\
+ \lambda_{t,m} e_{t,\tilde{g}} q_{t,\tilde{g}} E_{t+1} I_{t+1} \\
+ E \beta (1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}}) (1 - q_{t,eo,\tilde{g}}) V_{t+1,ee,\tilde{g}} \\
+ E \beta q_{t,\tilde{g}} V_{t+1,e,\tilde{g}} + E \beta (1 - q_{t,\tilde{g}}) f_{t,\tilde{g}} V_{t+1,u,\tilde{g}} \\
+ E \beta (1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}}) q_{t,eo,\tilde{g}} V_{t+1,o,\tilde{g}} + ((1 - q_{t,\tilde{g}}) (1 - f_{t,\tilde{g}})) E^{*} \varepsilon_{t,eo,\tilde{g}}
\]

\[
V_{l,u,f} = \lambda_{t,b} h_{t,\tilde{g}} + \lambda_{t,p} p_{t,w} A_{t,u,\tilde{g}} h_{t,u,\tilde{g}} - \varphi^{\omega}_{t,u,\tilde{g}} (1 - \pi_{t,u,\tilde{g}}) - \varphi^{\rho}_{t,u,\tilde{g}} + E^{*} \tilde{\varepsilon}_{t,uo,\tilde{g}} \\
+ E \beta (1 - q_{t,uo,\tilde{g}}) \pi_{t,ue,\tilde{g}} V_{t+1,e,\tilde{g}} + E \beta (1 - q_{t,uo,\tilde{g}}) (1 - \pi_{t,ue,\tilde{g}}) V_{t+1,u,\tilde{g}} + q_{t,uo,\tilde{g}} E \beta V_{t+1,o,\tilde{g}}
\]

\[
V_{l,o,f} = \lambda_{t,A} A_{t,o,\tilde{g}} h_{t,o,\tilde{g}} - \varphi^{\omega}_{t,o,\tilde{g}} - \pi_{t,ou,\tilde{g}} \varphi^{\rho}_{t,ou,\tilde{g}} + E^{*} \tilde{\varepsilon}_{t,ou,\tilde{g}} + \varphi^{\rho}_{t,o,\tilde{g}} E \beta V_{t,uo,\tilde{g}} + (1 - \pi_{t,ou,\tilde{g}}) E \beta V_{t+1,o,\tilde{g}}
\]

To reiterate, the expectation (emphasized with \(E^{*}\)) with respect to the idiosyncratic utility components is a particular truncated value, and is derived below. Note that all wage terms are deflated by the

\(^{13}\)Again, say \(\Delta_{t,eu,\tilde{g}}\) is redundant. One can, after some algebra, rewrite the system below, as is done in our coding, in terms of, say, \(\Delta_{t,uo,\tilde{g}}\) and \(\Delta_{t,eo,\tilde{g}}\) alone, replacing all \(V_{e}\) terms. However, the decomposition below is much easier to interpret because they relate back to the flow value function equations of a standard labor market model.
lagrange multipliers (which one can think of as \( \frac{1}{c} \)). This essentially converts utility flows to the same units as valuations of goods and services.

Turning to individual matches from the firms perspective, firms profit values are given by:

\[
\Pi_{t, g} = p_{t, w} A_{t, g} h_{t, g} - w_{t, g} h_{t, g} + Ed_{t+1}(1 - f_{t, g})(1 - q_{t, g})(1 - q_{t, eo, g})\Pi_{t+1, g} \\
- (1 - q_{t, g}) f_{F, g} \kappa_{F, g} + (1 - q_{t, g}) E^* \tilde{\varepsilon}_{t, \Pi, g}
\]  

(31)

Here \( \kappa_{F, g} \) denote firing costs and the idiosyncratic shock is interpreted as a random fixed cost, say energy cost, that has to be paid if the match is prolonged to the next period and is denominated in units of the final output good. We account for it in the aggregate resource constraint.\(^{14}\)

### 2.5 Bargaining and Wage Setting

It remains to specify how wages and hours are set in this economy. A standard approach would look at the Nash-bargaining over the surplus from employment to unemployment, and is given by:

\[
(V_{e, f} - V_{u, f})^\mu (\Pi_f)^{1-\mu}
\]

Standard first order condition would be given by:

\[
\frac{\lambda_{1, t} \mu (1 - \tau_{l, t})}{(V_{t, e, f} - V_{t, u, f})} = \frac{1 - \mu}{\Pi_{f, t}}
\]

As argued in the introduction, for optimal taxation the form of the wage setting is crucial. In particular, in our model the Nash-bargaining rule would have a first order effect on the profit-share of firms, which in turn would strongly influence unemployment volatility over the cycle. Additionally, the resolution of the Shimer-puzzle within this wage setting mechanism leads to a very small match surplus and potentially counterfactual reactions to changes in unemployment benefits, as argued in Costain and Reiter (2005a).

To provide a benchmark where these effect do not occur, we follow ideas from Hall and Milgrom (2007) who argue that the threat point of the worker is essentially independent of unemployment benefits. In accordance with this view, we assume that the threat point of the worker is not unemployment, but delay or strike. That is if bargaining breaks down, the worker strikes for a month and next period will restart his bargaining. So his outside option is given by the amount of strike money she receives from her family or union supporting her strike, or the gains she has from delaying:

She might still decide to quit during the month if she receives an alternative offer and might still be fired at the end of the month (even though we need to assume here that this decision would be

\(^{14}\)At chosen parameter values, the overall fixed cost in the economy are below 0.5%.
independent of her being at strike). Both the worker and the firm take future decisions as given, so we heavily rely on a Markov-perfect game-theoretic structure. Her utility difference when striking is given by:

$$V_{t,e,\tilde{g}} - \tilde{V}_{t,e,\tilde{g}} = \lambda_{1,t} w_t \tilde{g}_t h_{t,e,\tilde{g}} (1 - \tau_l) - \varphi h_{t,e,\tilde{g}}^\theta - \lambda_{1,t} strike_{t,\tilde{g}}$$

while the threat point of the firm is given by

$$\tilde{\Pi}_{t,\tilde{g}} = p_{t,w} A_{t,\tilde{g}} h_{t,e,\tilde{g}} - w_t \tilde{g}_t h_{t,e,\tilde{g}}$$

We therefore obtain, using again a Nash-bargaining procedure, the static wage condition:

$$w_t,\tilde{g} = \mu_{\tilde{g}} p_{t,w} A_{t,\tilde{g}} (1 - \tau_l) + (1 - \mu_{\tilde{g}}) [\frac{strike_{t,\tilde{g}}}{h_{t,e,\tilde{g}} (1 - \tau_l)} + \frac{\varphi h_{t,e,\tilde{g}}^\theta - 1}{\lambda_{1,t} (1 - \tau_l)}]$$

and the standard first order conditions for the choice of hours worked (independent of the assumed threat point) in all three sectors is given by:

$$p_{w} A_{t,\tilde{g}} (1 - \tau_l) \lambda_{1,t} = \varphi h_{t,e,\tilde{g}}^\theta - 1$$

due to the joint efficiency of the bargaining process for workers on the job, and the standard hours choice for inactive workers.

Note that average wages per hour will not depend on changes in taxes directly, if we assume that the strike value is a fraction of average net wage $w h$. 

$$strike_{t,\tilde{g}} = strike_{\tilde{g}} w_t h_{t,e,\tilde{g}} (1 - \tau_l)$$

This assumption shields the wage from being affected by tax rate changes or changes in unemployment benefits. Given that $\Sigma$ is arbitrary and being never played, it is unobserved.\(^{15}\)

This wage setting mechanism is ideally suited for our purposes in terms of long run taxation effects, given that it shields the wage from any non-proportional effects of labor tax-rates, as it is assumed in Arsenau and Chugh (2006) and as would be the case if we relied on the full Nash-bargaining solution with unemployment as the outside option\(^{16}\). We view this equation as the long run steady state

\(^{15}\)The literature has not converged on a particular wage setting mechanism. Many different forms have been employed. It is fairly easy to provide, for each of this mechanisms, an underlying bargaining game that can "rationalize" or "microfound" the particular equation in use. The empirical content of such games appear to be an open question.

\(^{16}\)We experimented with this wage rule quite a bit. However, to resolve the Shimer puzzle we had to rely on the small surplus parameterization as in Jung (2005) and Hagedorn and Manovskii (2006). The model is much harder to calibrate and has some undesirable properties.
relation, that generates a constant profit markup unaffected by fiscal policy given that labor tax rates cancel out as can be seen from the above equations.

However, to resolve the Shimer puzzle in a dynamic version of the model, we either need to assume that wages are sticky as we do here or profits are additionally strongly procyclical due to an additional technology effect, as is done in Costain and Reiter (2005a). As shown in Jung (2005) the above wage setting rule could not, for any parameterization of the model, generate an unemployment volatility in line with the data. A standard way to model wage stickiness, see Shimer (2005) or Hall and Milgrom (2007), would be to assume a constant outside option:

\[
strike_{t,\bar{g}} = \frac{\text{strike}_{\bar{g}} w_{t,e,\bar{g}} h_{t,e,\bar{g}} (1 - \tau_l)}{A_t^x}
\]  

(38)

However, this assumption would lead to a positive first order effect of taxes on unemployment. Given the arbitrariness of the wage setting assumption in these class of models we rule these effects out by assumption. We resolve the dilemma between business cycle fluctuations and long run influence of taxes by assuming the following form, similar to the one proposed in Blanchard and Gali (2008):

\[
strike_{t,\bar{g}} = \frac{\text{strike}_{\bar{g}} w_{t,e,\bar{g}} h_{t,e,\bar{g}} (1 - \tau_l)}{A_t^x}
\]  

(38)

Given that the technology level A is normalized to one in steady state, this assumption does not influence long run optimal taxation results as shown in Jung (2008). By choosing \(x\) appropriately, we can mimic the behavior of a sticky wage result as close as desired.\(^{17}\)

It is worth pointing out that our sticky wage mechanism does not depend on a small surplus of the match, as in Jung (2005) or Hagedorn and Manovskii (2006), (even though, of course, the underlying economic argument is identical given that the surplus with respect to the cost of delaying is, of course, extremely small), nor does it imply problematic responses of unemployment to governmental interventions because, by assumption, we have ruled out any first order effects on profits.

2.5.1 Truncation

As shown above the option value is given by:

\[
E^*\varepsilon_{t,e,\bar{g}} = -[\psi q_{t,\bar{g}} (1 - q_{t,\bar{g}}) \ln (1 - q_{t,\bar{g}}) + \psi q_{t,\bar{g}} \ln (q_{t,\bar{g}))]
\]

\(^{17}\)A potential drawback, which is also true for the countercyclical bargaining power model of Shimer (2005) and the procyclical technology model of Costain and Reiter (2005a) is the fact, that the model does not generalize to a higher dimensional shock structure easily without imposing even stronger assumption on the joint process. However, almost all of our results can be generated by assuming a constant outside option given that the choice of \(x\) turns out to deliver precisely that.
where $\psi_x$ parameterizes the variances of the different distributions. All other truncated expectation follow the identical line of reasoning.

### 2.6 Discrete Choices

In this section we describe the discrete choices made by the agents/family. In all cases, the choices are made after the worker’s binding wage and hour contracts for this period have been made. We assume in each case, that the bargained contract from the beginning of the period is binding and can not be renegotiated. Given that we work for tractability with i.i.d. shocks the definition of firings in this context are problematic, because in our framework it would be beneficial not to fire the worker but to lay her off and wait one period.\(^\text{18}\) Also, if renegotiation is allowed, the firm might want to compensate the worker for a bad shock to save new vacancy posting cost. Our discrete choice assumption essentially means that when making their decisions workers and firms do not look at their joint surplus, but just at their own utility differences. Even though the shocks are i.i.d. by assumption they are denoted in terms of the total value of a state. That is we view them as a summary statistic over the beliefs of the worker of how much pleasure a particular work environment will deliver to him. To obtain the discrete choice probabilities we let the family/worker choose the cut-off level which is equivalent to choosing the transition probabilities directly. Note that we have assumed that, say, the firm has a right to manage and does the firing decision not taking into account the harm it inflicts upon the worker. Vice versa, the worker does not take into account, when quitting, that the firm takes a loss. So the decisions are not jointly efficient. Technically the case can be easily handled by including the decisions in the bargaining choices and let workers and firms choose them jointly. Given that the idiosyncratic shocks are hard to verify we felt that it is much more sensible to assume that workers and firms make these decisions separately.\(^\text{19}\)

Using the first order conditions and the definitions of the utility flows above we get the following quitting decision:

$$
q_{t,\tilde{g}} = \frac{\pi_{e,e}}{1 + e^{\left(\frac{-\kappa_{11} E\Pi_{t+1,\tilde{g}}^{h_1,1} + \phi_{\tilde{g},\tilde{g}} N - E\beta f_{s,\tilde{g}} \Delta_{t,\epsilon_0,\tilde{g}}^{h_1,1} - E\beta (1-f_{s,\tilde{g}}) \epsilon_{t,\epsilon_0,\tilde{g}}^{h_1,1} + (1-f_{s,\tilde{g}}) E\beta \epsilon_{t,\epsilon_0,\tilde{g}}^{h_1,1} - (1-f_{s,\tilde{g}}) \epsilon_{t,\epsilon_0,\tilde{g}}^{h_1,1}}{\phi_{\tilde{g},\tilde{g}}}ight)}}
$$

\(^{18}\)This fact also holds true for the standard endogenous destruction model of den Haan, Ramey, and Watson (2000), where firings are also essentially lay-offs. This property is owned to the i.i.d. assumption, but we do not view this property as particularly problematic.

\(^{19}\)The resulting equations, say for firings, would include the flow values of the worker as well and would essentially be a function of the total match surplus. Given that firms profit and total match surplus move in the same direction the time series properties will likely not change much.
where $\pi_{e,e}$ denotes the (exogenous) probability of receiving an outside offer\textsuperscript{20}.

The firing decision follows a similar line of reasoning.

$$f_{t,\tilde{g}} = \frac{1}{E\beta(1-\pi_{e,e})\Pi_{t+1,m}+\kappa_{F}}$$  \hspace{1cm} (40)

Here firms trade off the cost of keeping the worker while having to retrain him and pay a fixed cost or simply firing him and pay some firing cost.

The decision to quit out of labor force is given by the utility difference between the two options adjusted for the quitting cost, given that it is the last decision made in this stage.

$$q_{t,e_{o},\tilde{g}} = \frac{1}{E\beta(1-\pi_{e,e})\Pi_{t+1,m}+\kappa_{F}}$$  \hspace{1cm} (41)

Similarly, the decision to quit into inactivity when currently searching for a job is given by:

$$q_{t,u_{o},\tilde{g}} = \frac{1}{E\beta(1-\pi_{e,e})\Pi_{t+1,m}+\kappa_{F}}$$  \hspace{1cm} (42)

She weights the expected gains from searching against the value of being out of the labor force.

Finally, the decision to enter into the unemployment pool is given by:

$$\pi_{t,u_{o},\tilde{g}} = \frac{1}{E\beta(1-\pi_{e,e})\Pi_{t+1,m}+\kappa_{F}}$$  \hspace{1cm} (43)

where the worker again trades-off the gain in searching and potentially receiving a job with the setup cost of doing so. Each of these shocks is parameterized by its own parameter $\psi$ governing the idiosyncratic shock distribution.

### 2.7 Matching markets

There are separate matching markets for males and females. That is firms can observe sex and can create different markets for both types. Given that males and females generate, potentially, a different surplus once the match is formed this assumption avoids to model the decision of firms when faced with a male and a female application, which might affect the shape of the matching function. It also allows for different employment probabilities across sex, while a common matching market might

\textsuperscript{20}We experienced with matching markets for employed and standard vacancy postings and matching decisions that endogenize this probability. Given that the cyclical properties are very similar we decided to use the most simple framework.
not. On the other hand discrimination legislation might suggest a common matching market. We feel that the hiring process can likely be influenced in a discriminatory way, so our treatment might not conflict with this view. In any case, this assumption is not crucial for our results, given that a common matching market does work rather similar.

To close the matching market we use the standard matching functions separated by sex:

\[
\begin{align*}
M_{t,\bar{g}} &= \kappa_{\bar{g}}(u_{t,\bar{g}}(1 - q_{t,u,\bar{g}}))^{\varsigma}v_{t,m}^{1-\varsigma} \\
M_{t,\bar{g}} &= \theta_{t,\bar{g}} = \kappa_{\bar{g}} \frac{1}{x_{t,\bar{g}}} \\
\frac{M_{t,\bar{g}}}{v_{t,\bar{g}}}(1 - q_{t,u,\bar{g}}) &= \pi_{t,ue,\bar{g}} = \kappa_{\bar{g}}x_{t,\bar{g}}^{1-\varsigma} \\
x_{t,\bar{g}} &= \frac{v_{t,\bar{g}}}{u_{t,\bar{g}}(1 - q_{t,u,\bar{g}})}
\end{align*}
\]

We allow \(\kappa\), the normalizing matching function coefficient, to be different for males and females.

Finally the free entry condition for males and females reads as:

\[
\frac{(\kappa_{\bar{g}})}{\theta_{t,\bar{g}}} = Ed_{t+1}\Pi_{t+1,\bar{g}}
\]  

2.8 Government and Exogenous

Finally we assume a balanced budget rule for the government, such that

\[
G_t + Tr_t = m_m w_{t,m} h_{t,m} e_{t,m} \tau_{t,l} + m_m b_{t,m} \ast u_{t,m} + (r_t - \delta) \tau_{t,c} K_t, + m_f w_{t,f} h_{t,f} e_{t,f} \tau_{t,l} + m_f b_{t,f} \ast u_{t,f}
\]  

The net replacement rates for males and females is given by

\[
b_{t,\bar{g}} = \tilde{b}_{\bar{g}} w_{\bar{g},t} h_{\bar{g},\bar{e},t}(1 - \tau_{t,l})
\]

where \(\tilde{b}_{\bar{g}} \in [0,1]\) and is assumed to be proportional to net wages, such that it does not induce by itself a reason for a positive capital taxation.

Before turning to results on optimal taxation we need to specify the behavior of the actual US government. For the exogenous shock processes we assume that \(a_t, G_t, \tau_{t,l}, \tau_{t,c}\) follow exogenous AR(1)
processes\textsuperscript{21}, which we directly estimate from the data.

\[ G_t = \vartheta e^{gt} \]  
\[ g_{t+1} = \rho_g g_t + \epsilon_{g,t+1}, \quad \epsilon_g \sim N(0, \sigma_g^2) \]  
\[ \tau_{t,l} = \tau_{t,l}^0 e^{\tilde{\tau}_t} \]  
\[ \tilde{\tau}_{t+1,l} = \rho_{\tau} \tilde{\tau}_{t,l} + \epsilon_{\tau,t+1}, \quad \epsilon_{\tau} \sim N(0, \sigma_{\tau}^2) \]  
\[ \tau_{t,c} = \tau_{t,c}^0 e^{\tilde{\tau}_{t,c}} \]  
\[ \tilde{\tau}_{t+1,c} = \rho_{\tau} \tilde{\tau}_{t,c} + \epsilon_{\tau,c,t+1}, \quad \epsilon_{\tau,c} \sim N(0, \sigma_{\tau,c}^2) \]  
\[ a_{t+1} = \rho a_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_a) \]  

We let the direct transfers \( Tr_t \) pick up the residual in the balanced budget rule. When discussing optimal policy, transfers are set to zero.

For completeness we state the aggregate resource constrained of this economy for all periods (defining aggregate consumption \( C_{1,t} \equiv (m_m + m_f) c_{1,t} \)):

\[ C_{1,t} + p_w C_{2,t} + I_t + G_t = Y_{1,t} + p_w Y_{2,t} - m_m \kappa_m u_{t,m} x_{t,m} - m_f \kappa_f u_{t,f} x_{t,f} - m_m \epsilon_{t,m} \kappa_{F,\tilde{g}} f_{t,m} (1 - q_{t,m}) - m_f \epsilon_{t,f} \kappa_{F,\tilde{g}} f_{t,f} (1 - q_{t,f}) + m_m \epsilon_{t,m} (1 - q_{t,m}) E^* \tilde{\varepsilon}_{t,\Pi,m} + m_f \epsilon_{t,f} (1 - q_{t,f}) E^* \tilde{\varepsilon}_{t,\Pi,f} \]  

\[ (58) \]

\subsection*{2.9 Ramsey Problem}

The Ramsey problem of the above economy is defined as a planners problem that maximize families utility over all endogenous taking the above described decentralized equilibrium constrains into account. We follow the setup of Benigno and Woodford (2006a) closely.

That is we define governmental problem as choosing the evolution of the state vector \( z_t \) according to:

\[ W = \max_{\{z_t\}} E_0 \sum_{t=t_0}^{\infty} E^{\beta^{t-t_0}} U(z_{t-1}, z_t, S_t) \]
\[ s.t.: \quad E_t f(z_{t-1}, z_t, z_{t+1}, S_t, S_{t+1}) = 0 \]  
\[ E_t f(z_{t_0-1}, z_{t_0}, z_{t_1}, S_t, S_{t+1}) = \overline{f}_{t_0} \]  

\[ (59), \quad (60) \]

Here \( f \) collects the system of equation that characterize the above described decentralized economy, \( z \) collects the vector of all endogenous variables of the model and \( S \) collects the exogenous shock processes. We further assume that the initial pre-commitment rule \( \overline{f}_{t_0} \) is self-consistent in the sense

\textsuperscript{21}Obviously, \( G_t, \tau_{t,l} \) and \( \tau_{t,c} \) do not need to be independent, so we can allow for correlation across the shocks.

21
defined in Benigno and Woodford (2006a) such that the resulting government problem becomes recursive.\textsuperscript{22} Given the assumption of a balanced budget rule, implying the abstraction from government debt, our steady state equilibrium is unique also for other, more standard, restrictions on the initial choice set of the government. It also does not drive our finding that capital taxes in this economy are, typically, positive as long as the instrument set of the government is restricted to capital and labor tax rates due to the assumed friction that the government cannot tax home-production. We derive the Lagrange constraints of the government problem using symbolical derivatives and then solve for the steady state numerically.\textsuperscript{23} To obtain time-series properties of the model we perturb the steady state using a first order approximation to the lagrangian problem evaluated at the optimal steady state which was shown to be equivalent to a second order approximation to welfare in Benigno and Woodford (2006a). When looking at simple tax rules and applying the Kalman filter we use a first order approximation relying, as before, on the code provided by Gomme and Klein (2006). A second order approximation has been used to see how important non-linearities might be.\textsuperscript{24}

3 Results

This section provides a quantitative evaluation of the properties of the model. We start out by describing how we choose parameters and obtain the processes for the exogenous driving forces of the model. Taking these processes as inputs we use the model to predict all labor market series and compare them to the actual data. Given the fit of the model we then turn to a discussion of optimal taxation results over the cycle.

\textsuperscript{22}The concept of a precommitment rule is, mathematically, not entirely convincing as specified in Benigno and Woodford (2006a). Given our balanced budget assumption we have ruled out governmental debt rules that typically create non-uniqueness in the steady states of the Ramsey problem. In our case, therefore, the resulting time-inconsistency in period zero will not affect our long run steady state behavior. Essentially we assume that the world starts, initially, very close or in the steady state.

\textsuperscript{23}The system has, in our coding around 68 variables (this is not the minimum, because we included some identities in our code) and the Ramsey problem has, due to the fact that many lagged variables become part of the states, around 147 equations. We therefore do not give the entire system here.

\textsuperscript{24}Given that the system is already quite big we rely on perturbation techniques when bringing the model to the data. Labor market models do have non-linearities that can only be seen when using global methods. However, given that we are mainly concerned with correlations and standard deviations over the cycle and our steady state discussion does not require perturbations, we chose to proceed with perturbation methods.
3.1 Calibration and Estimation

The appendix lists the parameters of the model together with the chosen targets. Most parameters can be set in a standard fashion, so our discussion will focus on the complications that arise due to the new features of the model.

**Exogenous processes:** The period of the model is one month which we aggregate to quarterly frequency when discussing cyclical properties. We report results for the period 1970:1 to 2004:4.\(^{25}\) To obtain sequences for the four exogenous series, we estimate the model with a Kalman filter using GDP, labor tax rates and government expenditure to obtain estimates of the underlying shock variances of the TFP shock, government expenditure and tax rates shocks respectively.\(^{26}\) The standard approach to obtain an exogenous TFP shock, see Gomme and Rupert (2007) can not be used in our model, given that we have a two good structure for measured output and the labor share is not equal to the wage share. We want our model to be consistent with aggregate output and evaluate all other series relative to this. Most labor market series, in particular labor force participation show strong long run trends that must have affected output as well. We therefore choose a low frequency HP filter \((\lambda = 100000)\) to de-trend all series.\(^{27}\) Given the estimates of the Kalman filter for the exogenous states, we can predict all endogenous using the model, taking the entire time-path prediction as our measure of fit.\(^{28}\)

**Calibration of Parameters - Basics:** Our parameters are detailed in table (2).\(^{29}\) With respect

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\(^{25}\)The CPS labor market series separated by sex obtained from Shimer (2007) start in 1976 only. We use his merged series for the years before to obtain a longer time-series. The mean rates are calibrated to the years 1976:1 to 2004:12, see the appendix for the details.

\(^{26}\)We initialize our Kalman filter in 1948:1 to obtain some estimates of the initial state.

\(^{27}\)It is well known that this filtering procedure causes problems in obtaining "true" innovations given that the HP-filter will uses forward looking variables. However, if we use a linear trend in the Kalman estimation the results on the frequency we consider are almost unchanged. In predicting all variables, on business cycle frequency, if \((\lambda = 1600)\) or using a BP-filter of Christiano-Fitzgerald for 2 to 8 years, the model would predict even better, so the low order HP-filter makes it harder given the strong trends in the data for certain labor market series. Our model is not designed to explain this trends within a business cycle estimation procedure.

\(^{28}\)The fact that we basically employ a RBC interpretation using (essentially) one technology shock does not imply that we believe other shocks to be unimportant. However the estimation results in Jung and Kuester (2007) based on a simplified labor market model with monetary and wage frictions suggest, that the basic mechanism of strongly pro-cyclical profits driven by a wage friction as the driving force for labor market volatility does survive the extension to a more elaborate model featuring a more realistic set of shocks. In addition a parsimonious shock structure simplifies the analysis on where and how the basic mechanism works and where it can be improved.

\(^{29}\)Box 1 standard aggregate macroeconomic targets we wish our model to match, box 2 lists preference related parameters, box 3 list parameters related to the government, box 4 describes the technology parameters detailed in table(1),
to standard labor market parameters, we set the elasticity of the matching function to .5 as in Hall (2005), set the vacancy posting cost to 11% of the average monthly productivity of the match and the hiring probability to 70%, as argued for in Hagedorn and Manovskii (2006). The bargaining power and the strike value are not jointly identifiable in our framework, so we set the bargaining power arbitrarily and innocently to 50% and choose the strike value such that the unemployment rate is 5.4% for males and 6% for females as in the data. We target the right amount of unemployment volatility by setting $x$, the sticky-wage parameter in our wage setting equation to match the volatility of the unemployment to population rate, side-stepping the debate surrounding Shimer’s work. Note that our choice of $x$ is almost equivalent to a fixed option outside in a standard model, given that procyclical wage income is counteracted by procyclical technology, making the strike-value acyclical. Our results still hinge on a small surplus calibration. In our calibration vacancy posting cost and thereby profits from a match from the firms perspective are small, implying that the resulting wedge is small and wages are close to the competitive equilibrium outcome.\footnote{Small surplus calibrations have been criticized forcefully, on a priori grounds. This might be correct but it implies also that labor markets were, on average, far away from the competitive outcome. Scholars arguing in favor of substantial profits must acknowledge that the world is substantially away from the neoclassical competitive benchmark.} Note, however, that the surplus for the family (and thereby the total match surplus) is substantial in our calibration. The family would pay 3.1 times the average monthly net wage rate (or, equivalently one quarterly wage), to shift a male worker directly from inactivity to work, and 1 month net wage for a female.\footnote{Note that our model can sustain a substantial surplus relative to the calibration of say Hagedorn and Manovskii (2006) because our bargaining threat point is an abstract strike value that reflects labor market inefficiencies without relating them to the real surplus. This trick, used in Hall and Milgrom (2007), of course implies that the surplus relative to the threat point is still minuscule.}

**Calibration of Parameters - New Features:** We now turn to a discussion of the new parameters in our model: We harmonize as many preference parameters across males and females as possible. The first set of new parameters are related to the utility cost parameter $\tilde{\gamma}$ and the variances $\psi^{\tilde{\gamma}}$ of the discrete choice distributions. We target these parameters by the mean transition rates separated by sex and the variances of the choice probabilities obtained from our estimation procedure.\footnote{Given that our parameter setting is subject to the constraint that the value of employment must be bigger than the value of unemployment and the value of inactivity, the range of possible parameters is limit given all other targets. We were unable to generate enough volatility in the transition rate from inactivity to unemployment and from unemployment to inactivity. We chose the parameter governing search-cost $s_{ou}$ to be zero, the boundary value. We varied the distribution parameter of $\psi_{\pi_{ou,m}}$ within the range of permissible values such that the value of employment is bigger than the value of inactivity and picked the local maximum. However, the parameter is hard to identify and does not influence the cyclical}

box 5 lists standard labor market parameters and in box 6 parameters related to the discrete choices are discussed.
sex and let the differences between sex be captured by the underlying distributions of the shock. As discussed in the intuition section the variances are informative about the key elasticities that will drive the reaction of the model to changes in the tax rate. The second moments of these time-series appear therefore to be a natural choice to identify these elasticities.

**Time-Aggregation:** An immediate problem of targeting the mean rates is the fact that our model does not have a direct transition from inactivity to employment, which is quantitatively non-negligible. By definition these flows do not exist\(^{33}\) and are an immediate consequence of the time-aggregation bias highlighted in Shimer (2007). However, in our view the main problem is not so much the aggregation from a continuous time framework to a discrete time framework, but the treatment of the flows from employment to inactivity and vice versa. As shown in Nagypal (2005) around 40% of the transitions from employment to inactivity are followed by a flow to employment with one intervening month. Naturally what might happen is that workers have searched on the job, obtained a new job but due to some time frictions the starting date is postponed by a month. Potentially the worker has to move, or the worker simply takes a vacation. In fact we view part of these flows as quits on the job. We deal with this problem by looking at the net flows from employment to inactivity. According to the data of Fallick and Fleischman (2004) the flows from employment to inactivity are roughly 2 times as large as the reversed flows. So we treat half of the flows from employment to inactivity as real quits, and the other flows as part of the job-to-job transitions. Table (3) lists our chosen targets. The stars indicates that these variables are not freely chosen but need to adjust to fulfill the requirements of a probability transition matrix. Note that our job-finding probability, while at odds with the unadjusted monthly rates, is close to the average job-finding probability reported in Shimer (2005), who argues that this rate was on average 45% over the last 60 years. This is a consequence of our treatment of the net-flows from inactivity to employment.

**Technology across state and sex:** The second set of non-standard parameters are related to the productive capability across sex on the job relative to the home-market. This margin is, as discussed shortly when defining the elasticity, of crucial importance in identifying the reaction of the participation decision of in particular female workers. In our benchmark scenario we chose the productivity differences between males and females working in the market. We use the time-use survey data of Aguiar and Hurst (2007) and compare the total amount of time spend working of an employed worker relative to a non-employed worker using the definitions of work/leisure as highlighted properties much.

\(^{33}\)By definition a worker who obtains a job must have been matched to an employer, and must have exerted a time-search cost, at least by answering the phone or appearing to an interview. So, in any case, there must have been a time where he was actively searching, which implies that he must have been unemployed in between.
in Aguiar and Hurst (2007). We find, that on average, a female supplies .77 as much time as their male counterpart to market work. However, employed females do not enjoy more leisure, but use the time-difference mainly working at home. More crucially for our purpose is the relative amount of time supplied when out of the labor force compared to the employment state. While the classification, what accounts as leisure, and what accounts as work, is somewhat arbitrary, see the critic by Ramey (2007), this effect is reduced by looking at the quotient, given that a miss-specification shows up both in the numerator and the denominator. In our benchmark calibration inactive males spent roughly 50% less time in undertaking productive activities (or enjoy 50% more leisure $\frac{h_{o,m}}{h_{m}} = 1/2$), while females enjoy roughly 1/3 more leisure ($\frac{h_{o,f}}{h_{f}} = 2/3$)). Given that employed females work, on the job and at home, jointly as much as males when employed, this implies that inactive females do devote a substantial amount of time to activities that should be economically valued by society. In terms of efficiency units ($A_{\tilde{g},\tilde{e}}$) this implies that females are roughly 40% as productive at home than their employed counterparts, while males are only 26% as efficient at home as on the job.\footnote{The fact that females work less hours in the market than males implies in our model that females are less efficient. We normalize $A_{m} = 1$ and the hour differential (and also the wage differential) between males and females implies that $A_{f} = .68$. What matters for our purposes is not so much the absolute number, but the relative distance to home-production. Here $A_{e,m} = A_{w,m} = .26$ and $A_{e,f} = A_{w,f} = .28$.}

**Preferences:** The third set of parameters is related to the utility specification. As discussed in the intuition section, our basic mechanism supposes that, say a female, when making her entry decision into the labor market, evaluates her time spend in the home market at the opportunity cost of time, which is the market wage. We therefore work with a log utility specification of overall consumption to be consistent with balanced growth, and choose, as a benchmark, the case of perfect substitutability between the home produced good and the market service good. We set the elasticity between the final good and the service good ($\gamma_1$) to .4 in line with the findings of McGratten, Rogerson, and Wright (1997) and set the share $\gamma$ to .5, which implies that 10% of the working population works in sectors with a high substitution elasticity to home-produced goods. Our results are not sensitive to the choice of these last two parameters. Perfect substitutability might appear as a strong assumption given the findings. Note however, that McGratten, Rogerson, and Wright (1997) look at a two good economy, while our argument relies strongly on the ability of say a female to buy child care in the market.\footnote{As we discuss in Jung(2008) the parameter is important for determining long run optimality results.}

**Government:** Finally, with respect to parameters influencing governmental policy, we set the labor tax rate to 24% as observed in the data, choose a government expenditure to output rate of 20% and set the capital tax residually (28.7%).\footnote{The data would suggest a rate of around 30% for the before depreciation capital tax, and around 50% for the after depreciation capital tax, see Gomme and Rupert (2007).} In our model unemployment benefits payed to the unemployed...
would work as a subsidy to search, given that our model does not differentiate properly between unemployed coming from employment state and unemployed coming from inactivity.\textsuperscript{37} We therefore treat unemployment benefits as transfers to the family that do not influence the decision of starting to search when inactive.\textsuperscript{38}

We estimate the autocorrelation and variances of our AR(1)-processes directly from the data and let the cyclical residual be picked up by lump-sum transfers to fulfill the balanced budget.

3.1.1 Evaluation of the Model

This section summarizes the properties of the model with respect to key labor market variables. Before going into the details of the estimation results, it is helpful to recall what ultimately drives our results and which feature of the model is responsible for the successes and failures highlighted below. First, any labor market model of the Mortensen-Pissarides type has to embed a mechanism that resolves the Shimer puzzle. While there are conflicting views on this issue, the ultimate driving force of any mechanism proposed that we are aware of, and that entertains the free entry condition for vacancies, is an argument why profits in a boom increase strongly over-proportional relative to wages. For example, Costain and Reiter (2005a) argue that newly hired workers receive an additional productivity gain correlated, fairly ad hoc, with the business cycle that introduces additional profits into the model. Hagedorn and Manovskii (2006), Jung (2005) and Hall and Milgrom (2007) argue, in one way or the other, that the surplus of the match relative to a particular outside option is small, such that equilibrium profits boost, in percentage terms, over-proportional relative to wages. The same argument underlies the ideas in Shimer (2007) who makes the bargaining power (negatively) correlated with the business cycles. Other ideas, like Blanchard and Gali (2008) and Jung and Kuester (2007), make the outside option negatively correlated with the shocks. Given that the literature has not converged to a particular, convincing, resolution, potentially due to the fact, that the amount of wage setting mechanisms proposed are huge, and wage payments are badly pinned down even in the neoclassical model, we rely on a sticky wage mechanism that essentially induces a countercyclical outside option. More important for our purpose is the fact that labor tax rate changes or unemployment benefit changes do not have any first order effects on the wage setting equation. This avoids the critic raised by Costain and Reiter (2005a) who question the equilibrium response of the

\textsuperscript{37}To capture this effect the model would have to introduce an additional state and additional flows making the model even more complex. Given that the role of unemployment benefits is not the main focus of the paper we chose to assume it away.

\textsuperscript{38}Obviously, the model can easily be recalibrated and unemployment benefits can be included. None of our basic results would change.
small surplus calibration of Hagedorn and Manovskii (2006) and shields the unemployment rates from first order effects of tax rate changes (or unemployment benefit changes)\(^{39}\), which we feed into the model exogenously. Only general equilibrium effects remain, which are small in nature. We therefore evaluate the success of the model relative to a mechanisms that induces the right volatility into the job-finding probability. Second, as is well known since the work of den Haan, Ramey, and Watson (2000), endogenous firings, and by extension, all endogenous choice probabilities, are driven by the mass of workers around the cut-off level. If shocks are proportional to productivity as in den Haan, Ramey, and Watson (2000), the idiosyncratic variance of the, typically log-normal shocks, drives the volatility of the rates almost one for one. In our case, the means and variance of the additive shocks are driven by the underlying extreme value distribution that in turn determines the mass of workers that are affected by a change in the endogenous together with the fixed cost that jointly determine the means and variances. Again, no particular success can be claimed along these dimensions. In this paper we precisely use this connection to pin down the underlying heterogeneity in preferences or life-circumstances, the model generously has abstracted from, as part of our calibration strategy. The success and failure of the model lies therefore in the correlation across all variables which is neither targeted nor is there an immediately obvious relation that drives our results. Note that all of our probabilities, due to the discrete choice structure, are ultimately functions of the share of the match surplus that goes to workers and firms, and therefore induces strong equilibrium restrictions on the correlations across variables. To highlight how these correlations evolve, we report the entire time-path prediction for all variables. This gives additional valuable information on the failure and success of the basic mechanisms, because it captures, essentially, all lead and lag cross-correlations across variables.

Table (4) gives the basic moments of the model and Figure(9) to Figure(4) visualize the performance of the model for selected series:\(^{40}\)

\(^{39}\)In fact, unemployment rates are, to a first order, unaffected to changes in unemployment benefits, such that the semi-elasticity highlighted in Costain and Reiter (2005a) would be zero. Given that benefits would induce a change in search activities for workers out of the labor force it would have, as a second order effect due to general equilibrium, have an effect on unemployment rates by making search, or entry into the search pool, more attractive. This effect, of course, is due to the fact that our model does not differentiate the state of unemployment as coming from employment, or coming from out of the labor force. A four state model would be needed that separates these effects. However, the model would have to keep track of the entire distribution of labor market states, so we choose to ignore this complication.

\(^{40}\)All data are in logs, given that our mean rates do not always correspond to the data-means and might otherwise have a feedback effect into the standard deviation.
The figure shows the prediction of the model relative to the data for the employment to population rate of males and females as well as the unemployment rate. The blue solid lines are the data, while the red dashed lines are the predicted series. All data and predictions are in logs and are HP-filtered with $\lambda = 100000$.

**Employment States:**
The first fact to highlight is that the model predicts the unemployment rate very well, as figure (2) makes clear. The actual series correlates with .92 to the predicted series. Given that unemployment and output are highly correlated and we targeted the relative standard deviation of unemployment to output in the calibration, this result is not entirely surprising, though still a reconfirming feature of the model. It shows that the Mortensen-Pissarides model with a wage-frictions is an excellent model for discussing cyclical properties of labor market variables, in contrast to recent claims in the literature.\footnote{This does not mean that the, ad hoc, wage friction is a satisfying modeling device, but it does mean, that alternative...} The model overpredicts the volatility in the employment-population rate for both males...
and females and overpredicts the correlation for females while it matches almost perfectly the reaction for males. This is mainly due to an overshooting in the oil-price crises in the eighties and at the end of the sample, which might be driven by the strong labor tax decline, that might be miss-measured.

**Nipa and BLS-aggregates:**

The model predicts, see figure (3), investment almost perfectly, while it under-estimates the volatility in consumption, but captures the correlation correctly. The model fails to match the wage per hour series obtained from the BLS for non-farm business almost completely, at least during the late eighties up to now, even though the correlation-coefficient comes close. The model correlates very well with the total hour measure from the BLS, but underpredicts the volatility. This is mainly due to an underprediction of the average hours worked per person, given that total employment is matched rather accurately.

Two remarks about this failure are in order: first, note that Hagedorn and Manovskii (2006) use the role of wages over the cycle as a target to explain the success of the Mortensen-Pissarides framework in explaining unemployment volatility. We caution to judge the ability of the Mortensen-Pissarides model to generate unemployment volatility by a matched wage-elasticities alone, either by claiming a success or by claiming a failure, given the picture provided above. As the prediction makes clear the correlation-coefficient (and likely the elasticity) is not a sufficient statistic for capturing the wage dynamics.

However, second, it is unlikely that any standard RBC model will be able to explain the behavior of the above wage series given that between 1994 and 2000, say, common believe, our TFP-shock and our hours worked measure suggest a substantial boom of the US economy, while wages suggest a severe depression. If these data are actually correct, our model is unable to explain them and the wage friction we highlight is problematic. However, the wage and hour concepts used by the BLS is clearly

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42 Note that in principle, given that we use government expenditure and output in our estimation procedure and match investment by implication, consumption should me matched perfectly as well, being the residual. Of course, given that we filter all series seperately the HP-filter distorts the identity, so does foreign trade.

43 Note, however, that the model makes strong assumption how wages are split, but this must not correspond to wages payed. The timing of wage payments is undefined given that both the family and the firm would be willing to shift the discounted flows back and force in time. Some small cost involved in changing payed wages over the quarter could give raise to very different observed wage-payments. However, obviously, this argument is a cheap way out of the problem, making the theory not falsifiable.

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The figure shows the prediction of the model relative to the data for total hours worked, investment and wages per hour. Hours and wage per Hours are taken from the BLS, while investment is NIPA investment plus consumer durables. The blue solid lines are the data, while the red dashed lines are the predicted series. All data and predictions are in logs and are HP-filtered with $\lambda = 100000$. 

not consistent with other wage and hour series used in the literature, so some caution is appropriate. This point is emphasized in ? who argues that Nipa compensation is seriously miss-measured.

**Labor Market Probabilities:**

As mentioned in the calibration section, we were unable to generate enough volatility for the unemployment to inactivity flows and vice versa. 44 However, given that we understate both rates, part of the failure might actually cancel out, given the noisy estimates of these flows. As the picture, see figure (4), and the table(4), make clear the model captures the correlation to output as well as the entire cyclical movements very well.

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44This is due to the fact that we reached the lower bound of the search cost, setting them to zero, so we did not have the degree of freedom to jointly match the mean rates and the volatility. This is in part due to our ignorance with respect to flows from inactivity to employment, that we channel through transitions from inactivity to unemployment.
The figure shows the prediction of the model relative to the data for the average transition flows. The data are taken from Shimer (2007). The blue solid lines are the data, while the red dashed lines are the predicted series. All data and predictions are in logs and are HP-filtered with $\lambda = 100000$.

In fact, it is interesting to note that transition rates from unemployment to inactivity are positively correlated with the business cycle, while the transition rates from inactivity to unemployment is negatively correlated with the business cycle, in the data and the model. Intuitively one might have expected that in a boom more, say, females would start searching, not less. Apparently the wealth effect overturns this result, given that the family as a whole is richer in a boom and does not need to force that many females with a potentially negative shock realization to enter the search pool. Flows from inactivity to employment are procyclical in the data, which we capture in our model by strongly procyclical quits. Quits to inactivity are countercyclical in the model and the data though our model overpredicts the correlation. The model explains firings very convincingly, if we abstract from high-frequency noise that surrounds the estimate. In our model, firing rates (or separations from
employment to unemployment) and the job-finding probability are driven by the same variable, \( \Pi \), the surplus of the match from the firms perspective, which corresponds to the prediction of the standard model. Given the picture the standard insights offered from the Mortensen-Pissarides framework appear to be correct. This also implies that the debate on the importance of the "Ins" versus the "Outs" appears less severe given that both rates are driven by the identical force.

Finally, it is worthwhile to mention that the model generates the right Beveridge-curve despite the fact that firing rates are endogenous.\(^{45}\)

To summarize, the model, driven by a parsimonious shock structure does capture the basic movements and correlation signs of all probabilities correctly, and predicts the time-path of all labor market transitions and states in our view to a reasonable extent. A richer shock structure as well as an interaction with some standard macroeconomic frictions not considered in this model might enhance the fit. The price to pay though is the loss of a parsimonious shock structure. Also, a much better understanding of the behavior of hours worked and wages per hour is needed to claim a real success.

### 3.2 Optimal Fluctuations

So far we have treated tax policy as exogenous. In the companion paper, see Jung (2008), we show that Ramsey-optimal labor and capital tax rates are not too far from observed US values. The friction that drives our result is the assumed inability of the government to tax home-production. Labor tax rates generate a distortion in particular for females to enter the main labor market. The resulting wedge needs to be counteracted, in the absence of additional instruments, by positive capital taxes. Our results are driven by differences in the implied intra-temporal elasticity of substitution which is, in our model, not driven by the hours worked margin, but mainly driven by participation friction. To see the quantitative impact of labor tax rates figure (7) plots the impulse response function to a unit shock in labor tax rates.

We see that the government has, by setting labor tax rates over the cycle, a potentially strong instrument to influence employment. Our wage setting assumption though rules out a direct effect of tax rate changes on the profit margin of firms, a typical mechanism highlighted in Costain and Reiter (2005b), who find strong dampening forces of labor taxes to the job-finding probability, thereby stabilizing unemployment. In our model the government has an effect on the participation decision of workers. By setting pro-cyclical labor tax rates the government can create an incentive in particular

\(^{45}\)It is interesting to note that the argument of Shimer (2005), that fluctuating destruction rates would destroy the Beveridge-curve is not valid in our model. Even if firings are double or triple as volatile as quits on the job, and, correspondingly, destruction rates were counter-cyclical, we would still obtain a negative correlation between unemployment rates and vacancies. What is crucial is the endogeneity of both rates, while Shimer makes them exogenously fluctuating.
for females to enter the labor market in recessions and can thereby stabilize employment and output. As the figure makes clear, an increase in labor tax rates encourage, at the margin, in particular females to exit the unemployment search pool and to move into inactivity. This effect translates into a decline in employment rates and a corresponding increase in non-participation rates. The effect on unemployment is non-monotonic. Initially more workers get discouraged and drop out of the search pool into non-participation, which translates into an increase in matching success for the remaining workers. However, this effect is quickly dominated by the general equilibrium effects of a decline in hours worked on the job and a reduction in output, making it less attractive for workers to start searching and increases the cost of existing matches.

The question remains whether the government should use this described impact on employment by stabilizing output fluctuations over the cycle?

**Ramsey-Optimal labor tax path:**

To address this question we use the estimated exogenous TFP shock and the governmental expenditure shock as given, and ask how the economy would have evolved if US taxes had followed a Ramsey optimal rule. To this end we re-solve the model around the optimal labor and capital tax rates implied by our model and perturb the Ramsey-optimal steady state following the approach outlined in Benigno and Woodford (2006a). As shown in Jung (2008) the optimal capital and labor tax rates evaluated at the above discussed steady state values is given by 25.8% and 23.1% respectively. Given that actual fiscal policy does not coincide with the optimal policy implied by our model, the steady states across our two exercises do therefore change. However, as we show in Jung (2008), the implied optimal labor tax rate in the steady state is not far away from the optimal tax rules, and the general equilibrium changes in endogenous variables are very modest. A comparison of the cyclical properties across the two steady states is therefore still meaningful.

We find that the logarithm of the optimal labor tax rate should have been half as volatile than the measured actual tax rate. However, the cyclical properties are fairly similar. Figure (5) plots the predicted tax series as well as the predicted employment sequences for males and females under an optimal rule.

We see that the access volatility in employment rates is reduced under the optimal one compared to the volatility we showed above under an exogenous policy rule, but the reduction is only modest.\(^{46}\)

\(^{46}\)The actual tax rate shows high volatility in the end of the sample, in line with the behavior of our wage per hour measure (and also overall compensation measure from which labor taxes have been inferred) measure. Again, even though we followed a standard procedure from macroeconomics, see Gomme and Rupert (2007), in constructing our labor tax
The figure shows the optimal labor tax path implied by the model (dashed red line) vs. the actual tax rate path (solid blue line), as well as predicted paths for selected labor market states.

In fact part of our "excess volatility" on employment rates might have been driven by "labor tax rate shocks" that might have been fully anticipated by the agents as part of a governmental rule or that might have been simple measurement error due to the problematic construction of average tax rates out of macroeconomic aggregates. In any case, our model suggests that the government should not excessively use its influence on employment to smooth out cycles. The reasons for this result lies in the fact that our model imposes considerable cost for workers to move across employment states. A change in behavior induces cost, either moving cost or search cost, which we have, partly endogenized in this paper. A social (Ramsey) - planner would and should take these cost into account.

Can we therefore conclude that the explicit modeling of the inactivity margin is unimportant for cyclical policy?

**What can go wrong?**

To show how important endogenous transition rates might be figure (6) plots the behavior of the model when we fix capital taxes at the steady state value and let labor tax rates only balance the budget. This happens in some models who specify, say, a total income tax rule (typically jointly with a debt stabilization rule).

In this case, labor taxes are counter-cyclical! Even though overall volatility of tax rates increases only modestly compared to the actual and unemployment volatility is not strongly affected, we see a strong impact on employment rates. This is due to a strong cyclical behavior of the probability to enter

measure it is clear that actual tax rates might have behaved rather differently given that NIPA compensation might be seriously miss-measured. The time-series are, therefore, at best a crude approximation, but the best that was available to us for the time period considered. In particular we suspect that the volatility is strongly overstated.
The figure shows the labor tax path implied by the model (dashed red line) vs. the actual tax rate path (solid blue line) under a hypothetical policy rule, as well as predicted paths for selected labor market states.

the market. The search flow is quickly absorbed into employment rates and employment volatility strongly increases, so does output volatility. Counter-cyclical labor tax rate were a disaster. This example makes clear that even though observed transition rates might not have contributed much to the fluctuation in the employment to population ratio as claimed in Shimer (2007), this should, in our view, not lead to the conclusion of not modeling these transitions carefully when conducting policy experiments. A wrong policy can have very bad consequences along these margins.

4 Conclusion

The paper presents a model that endogenizes all labor market transition rates in a tractable fashion. We show that the model is able to replicate key facts of the labor market dynamics when estimated on a parsimonious shock structure. The government has, by influencing the entry margin, a strong policy instrument to influence employment fluctuations. The fact that the government cannot tax home-production easily leads naturally to an inter-temporal distortion of labor taxes that is absence in the neoclassical world. This distortive effect mainly affects the participation decision of the worker. However, in equilibrium, this influence comes at a considerable cost. To rationalize observed average transition rates males and females must face significant entry cost that need to be accounted for in general equilibrium. It is therefore not optimal for the government to smooth out transition rates to strongly. Whether this prescription holds when we enrich the model along sticky prices and other demand side frictions has yet to be demonstrated. Potentially interesting trade-offs between fiscal and monetary policy might emerge.
References


5 Appendix

The appendix summarizes our calibration procedure and shows the predictions of our model.

5.1 Calibration

We first describe the data used in the estimation procedure. Table (1) shows total hours worked obtained from the American Time Use Survey, table (2) gives the parameter values chosen together with the description of the targets and table (3) gives the average monthly probabilities we target.

5.1.1 Data

Aggregate data are nominal output, consumption (non-durable and services), investment (plus durable consumption) and government expenditure taken from the NIPA and deflated by the GDP-deflator and expressed in per capita terms. Average labor and capital taxes are obtained using the procedure outlined in Gomme and Rupert (2007). Total Hours worked and wages per hour in non-farm business are taken from the BLS as well as aggregate labor market states. Labor market probabilities are taken from Shimer (2007). In the plots we use his aggregate measure and his time-aggregation correction downloaded from his homepage. For the HP-filter employed this measure turns out to be very similar with respect to its cyclical properties as the monthly unadjusted rates. The data from June 1967 and December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley. The monthly rates separated by sex are taken from the CPS micro-data using the (adjusted) code and were constructed by Robert Shimer.

5.1.2 American Time Use Survey

Table 1: Time-Use-Survey

<table>
<thead>
<tr>
<th>Year</th>
<th>Work-Male (Emp)</th>
<th>Work-Female (Emp)</th>
<th>Work-Male (Inactive)</th>
<th>Work-Female (Inactive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975</td>
<td>51.55</td>
<td>53.47</td>
<td>22.7</td>
<td>39.04</td>
</tr>
<tr>
<td>1985</td>
<td>53.39</td>
<td>52.82</td>
<td>27.95</td>
<td>36.78</td>
</tr>
<tr>
<td>1993</td>
<td>53.63</td>
<td>52.97</td>
<td>28.64</td>
<td>35.15</td>
</tr>
<tr>
<td>2003</td>
<td>54.15</td>
<td>52.2</td>
<td>24.13</td>
<td>33.37</td>
</tr>
</tbody>
</table>

Notes: The data have been made publicly available by Aguiar and Hurst (2007) and we follow their definitions. We drop all people under the age of 21 and over 65 as well as retired workers from the sample. We then condition on age (not reported), sex and employment status. We do not use fixed sample weights controlling for changes in demographics across surveys as in Aguiar and Hurst (2007) but report averages across our demographic cells, given that changes in these cells are our primary focus. Our definition of work uses their definition: work = work core + home production
work core = regular-work + working-at-home + overtime + moonlighting
home production = meals + housework + home car maintenance + home other + garden - pet + obtaining - goods
### 5.1.3 Calibration - Parameters

#### Table 2: Basic Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.997</td>
<td>Capital-output-ratio=4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.400</td>
<td>Capital share=.4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.004</td>
<td>Investment to output ratio=2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.950</td>
<td>.95 (quarterly)</td>
</tr>
<tr>
<td>$m_m$</td>
<td>1.000</td>
<td>Population normalization</td>
</tr>
<tr>
<td>$m_f$</td>
<td>1.000</td>
<td>Population normalization</td>
</tr>
<tr>
<td>Box 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.000</td>
<td>Balanced Growth</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.000</td>
<td>McGratten, Rogerson, and Wright (1997)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.400</td>
<td>10% in labor sector in the market</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>1.000</td>
<td>$h_m = 3$</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>1.000</td>
<td>Equal across sex</td>
</tr>
<tr>
<td>$\phi$</td>
<td>2.500</td>
<td>Micro-estimates</td>
</tr>
<tr>
<td>Box 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_m$</td>
<td>0.000</td>
<td>Treated as Transfers</td>
</tr>
<tr>
<td>$b_f$</td>
<td>0.000</td>
<td>Treated as Transfers</td>
</tr>
<tr>
<td>$\tau_l$</td>
<td>0.240</td>
<td>$\tau_l = 24$</td>
</tr>
<tr>
<td>Box 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_m$</td>
<td>1.000</td>
<td>Aguiar and Hurst (2007) time use survey</td>
</tr>
<tr>
<td>$A_f$</td>
<td>0.676</td>
<td>Aguiar and Hurst (2007) time use survey</td>
</tr>
<tr>
<td>$\mu_{m, f}$</td>
<td>0.269</td>
<td>Aguiar and Hurst (2007) time use survey</td>
</tr>
<tr>
<td>$\mu_{m, f}$</td>
<td>0.275</td>
<td>Aguiar and Hurst (2007) time use survey</td>
</tr>
<tr>
<td>Box 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi_{q, m}$</td>
<td>0.599</td>
<td>$\psi_{q, m}$</td>
</tr>
<tr>
<td>$\psi_{q, f}$</td>
<td>0.598</td>
<td>$\psi_{q, f}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.316</td>
<td>Vacancy posting Cost</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.190</td>
<td>Vacancy posting Cost</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.500</td>
<td>Matching Elasticity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.500</td>
<td>Matching Elasticity</td>
</tr>
<tr>
<td>$\mu_{m}$</td>
<td>0.500</td>
<td>Set to Hosios (Bargaining Power)</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>0.500</td>
<td>Set to Hosios (Bargaining Power)</td>
</tr>
<tr>
<td>Box 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.637</td>
<td>Normalized (Constant in Matching fct.)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.693</td>
<td>Normalized (Constant in Matching fct.)</td>
</tr>
<tr>
<td>$\psi_{q, m}$</td>
<td>1.006</td>
<td>Average $q_m$</td>
</tr>
<tr>
<td>$\psi_{q, f}$</td>
<td>1.046</td>
<td>Average $q_f$</td>
</tr>
<tr>
<td>$\psi_{f, m}$</td>
<td>0.947</td>
<td>Average $f_m$</td>
</tr>
<tr>
<td>$\psi_{f, f}$</td>
<td>0.852</td>
<td>Average $f_f$</td>
</tr>
<tr>
<td>$\psi_{q, o, m}$</td>
<td>3.676</td>
<td>Average $q_{o, m}$</td>
</tr>
<tr>
<td>$\psi_{q, o, f}$</td>
<td>0.925</td>
<td>Average $q_{o, f}$</td>
</tr>
<tr>
<td>$\psi_{q, o, m}$</td>
<td>12.45</td>
<td>* Std($\psi_{q, o, m}$)</td>
</tr>
<tr>
<td>$\psi_{q, o, f}$</td>
<td>7.960</td>
<td>* Std($\psi_{q, o, f}$)</td>
</tr>
<tr>
<td>$\psi_{q, o, m}$</td>
<td>2.817</td>
<td>Average $q_{o, m}$</td>
</tr>
<tr>
<td>$\psi_{q, o, f}$</td>
<td>1.951</td>
<td>Average $q_{o, f}$</td>
</tr>
<tr>
<td>$\psi_{o, m}$</td>
<td>1.645</td>
<td>Average $o_m$</td>
</tr>
<tr>
<td>$\psi_{o, f}$</td>
<td>1.369</td>
<td>Average $o_f$</td>
</tr>
<tr>
<td>$\psi_{o, m}$</td>
<td>4.000</td>
<td>Std($\psi_{o, m}$)</td>
</tr>
<tr>
<td>$\psi_{o, f}$</td>
<td>4.000</td>
<td>Harmonized</td>
</tr>
<tr>
<td>$\psi_{o, m}$</td>
<td>6.670</td>
<td>Std($\psi_{o, m}$)</td>
</tr>
<tr>
<td>$\psi_{o, f}$</td>
<td>6.670</td>
<td>Harmonized</td>
</tr>
<tr>
<td>$\psi_{f, o, m}$</td>
<td>3.400</td>
<td>Std($\psi_{f, o, m}$)</td>
</tr>
<tr>
<td>$\psi_{f, o, f}$</td>
<td>3.400</td>
<td>Harmonized</td>
</tr>
<tr>
<td>$\psi_{q, o, m}$</td>
<td>0.000</td>
<td>*Std($\psi_{q, o, m}$)</td>
</tr>
<tr>
<td>$\psi_{q, o, f}$</td>
<td>0.000</td>
<td>*Boundary</td>
</tr>
</tbody>
</table>

**Notes:** This table gives the numerical parameters used and the attempted targets the particular parameter influences the most. The stars indicate that we were unable to achieve this target. We set $\psi_{o, m}$ to the boundary value 0. Given all other targets the influence of $\psi_{q, o, m}$ on the variance is small and we were unable to generate enough volatility there, given that again we hit the boundary that the value of employment must be bigger than the value of inactivity on average. We varied the parameter in the allowable range and set it to the value that gave the maximal variance, even though the impact of this parameter on the results is rather small.
## 5.1.4 Average Probabilities

The following table gives the average probabilities of our model in our benchmark steady state.

**Table 3: Mean-Probabilities**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Model</th>
<th>CPS Monthly</th>
<th>Shimer (2007)</th>
<th>Fallick et al. (94)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job to Job (Male)</td>
<td>0.025</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job to Job (Female)</td>
<td>0.025</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job to Job (Average)</td>
<td>0.025</td>
<td>0.025</td>
<td>-</td>
<td>0.025</td>
</tr>
<tr>
<td>Firing (Male)</td>
<td>0.017</td>
<td>0.017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firing (Female)</td>
<td>0.013</td>
<td>0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firing (Average)</td>
<td>0.015</td>
<td>0.015</td>
<td>0.021</td>
<td>0.013</td>
</tr>
<tr>
<td>Quit E to Inactiv (Male)</td>
<td>0.011</td>
<td>0.021</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quit E to Inactiv (Female)</td>
<td>0.019</td>
<td>0.039</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quit E to Inactiv (Average)</td>
<td>0.014</td>
<td>0.029</td>
<td>0.029</td>
<td>2.700</td>
</tr>
<tr>
<td>Job-Finding (Male)*</td>
<td>0.480</td>
<td>0.245</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job-Finding (Female)*</td>
<td>0.500</td>
<td>0.283</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job-Finding (Average)*</td>
<td>0.489</td>
<td>0.265</td>
<td>0.369</td>
<td>0.280</td>
</tr>
<tr>
<td>Quit U-Inactivity (Male)</td>
<td>0.170</td>
<td>0.171</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quit U-Inactivity (Female)</td>
<td>0.270</td>
<td>0.275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quit U-Inactivity (Average)</td>
<td>0.215</td>
<td>0.219</td>
<td>0.311</td>
<td>0.230</td>
</tr>
<tr>
<td>Inactivity-U (Male)*</td>
<td>0.058</td>
<td>0.033</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactivity-U (Female)*</td>
<td>0.043</td>
<td>0.022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactivity-U (Average)*</td>
<td>0.048</td>
<td>0.026</td>
<td>0.044</td>
<td>0.024</td>
</tr>
<tr>
<td>Inactivity-E (Male)</td>
<td>-</td>
<td>0.055</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactivity-E (Female)</td>
<td>-</td>
<td>0.042</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inactivity-E (Average)</td>
<td>-</td>
<td>0.046</td>
<td>0.044</td>
<td>0.048</td>
</tr>
<tr>
<td>E (Male)</td>
<td>0.710</td>
<td>0.711</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U (Male)*</td>
<td>0.041</td>
<td>0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O (Male)</td>
<td>0.250</td>
<td>0.242</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E (Female)</td>
<td>0.526</td>
<td>0.525</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U (Female)*</td>
<td>0.034</td>
<td>0.036</td>
<td></td>
<td></td>
</tr>
<tr>
<td>O (Female)</td>
<td>0.440</td>
<td>0.440</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The table documents our calibration targets for the average monthly transition probabilities. The stars above indicate that these variables can not be chosen freely, but have to fulfill the requirements of a transition matrix.
5.2 Properties of the Model

Table 4: Basic Properties (HP=100000)

<table>
<thead>
<tr>
<th>Log:</th>
<th>Relative Std to Y</th>
<th>Corr with Output</th>
<th>Corr(Model,Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td></td>
<td>Std</td>
<td>Corr(Y,x)</td>
<td>Corr(Y,x)</td>
</tr>
<tr>
<td>Y</td>
<td>0.022</td>
<td>0.022</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>0.344</td>
<td>0.690</td>
<td>0.771</td>
</tr>
<tr>
<td>I</td>
<td>3.857</td>
<td>3.551</td>
<td>0.965</td>
</tr>
<tr>
<td>G</td>
<td>1.427</td>
<td>1.427</td>
<td>0.227</td>
</tr>
<tr>
<td>τl</td>
<td>1.550</td>
<td>1.550</td>
<td>0.332</td>
</tr>
<tr>
<td>τc</td>
<td>2.208</td>
<td>2.208</td>
<td>0.424</td>
</tr>
<tr>
<td>Hours</td>
<td>0.791</td>
<td>1.218</td>
<td>0.915</td>
</tr>
<tr>
<td>Wage per Hour</td>
<td>0.259</td>
<td>0.724</td>
<td>0.375</td>
</tr>
<tr>
<td>Employed (Male)</td>
<td>0.854</td>
<td>0.683</td>
<td>0.904</td>
</tr>
<tr>
<td>Employed (Female)</td>
<td>0.841</td>
<td>0.665</td>
<td>0.866</td>
</tr>
<tr>
<td>Unemployed (Male)</td>
<td>7.887</td>
<td>8.043</td>
<td>-0.905</td>
</tr>
<tr>
<td>Unemployed (Female)</td>
<td>5.271</td>
<td>5.532</td>
<td>-0.904</td>
</tr>
<tr>
<td>Out of labor (Male)</td>
<td>1.328</td>
<td>0.567</td>
<td>-0.780</td>
</tr>
<tr>
<td>Out of labor (Female)</td>
<td>0.714</td>
<td>0.435</td>
<td>-0.709</td>
</tr>
<tr>
<td>Transition Probabilities:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-Rate</td>
<td>7.034</td>
<td>6.986</td>
<td>-0.914</td>
</tr>
<tr>
<td>Vacancies</td>
<td>9.658</td>
<td>8.581</td>
<td>0.799</td>
</tr>
<tr>
<td>EU</td>
<td>3.435</td>
<td>3.671</td>
<td>-0.934</td>
</tr>
<tr>
<td>EN</td>
<td>1.977</td>
<td>1.931</td>
<td>0.658</td>
</tr>
<tr>
<td>UE</td>
<td>7.829</td>
<td>6.016</td>
<td>0.934</td>
</tr>
<tr>
<td>UN</td>
<td>1.911</td>
<td>4.847</td>
<td>0.630</td>
</tr>
<tr>
<td>NU</td>
<td>0.282</td>
<td>3.094</td>
<td>-0.631</td>
</tr>
<tr>
<td>NE</td>
<td>-</td>
<td>2.566</td>
<td>-</td>
</tr>
<tr>
<td>Q</td>
<td>2.145</td>
<td>-</td>
<td>0.931</td>
</tr>
</tbody>
</table>

Notes: The table documents the performance of the model. The standard deviation is expressed relative to output except for output itself, which is given in raw terms. Here Y denotes output, C consumption, I investment and G government expenditure. EU denotes the the flow probability from employment to unemployment, and all other follow a corresponding notation. All data are HP-filtered with \( \lambda = 100000 \).
5.3 Impulse Response Functions

The figure shows the Impulse Response function for selected variables of the model. Here m denotes male, f denotes female, and u,o and e are the employment states. The other variables denote the transition flows across labor market states.
The figure shows the Impulse Response function for selected variables of the model. Here m denotes male, f denotes female, and u,o and e are the employment states. The other variables denote the transition flows across labor market states.
The graphs show the exogenous processes where government expenditure is taken from the NIPA, labor and capital taxes are created using NIPA data following the methodology of Gomme and Rupert (2007) and the technology process is estimated using a Kalman filter on output given the model.

Figure 9: Exogenous Shock Processes

- TFP
- Labor Tax
- Capital Tax
- Government Expenditure