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The significance of faith proven by decision theory – Pascal’s wager game is correct and refutes atheism completely

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Abstract

Pascal's wager game showed that the atheistic view is always inferior compared to the theistic. The reason is that an infinitely high reward for the theist is always opposed to a finite payoff for the atheist, given that the existence of God cannot be excluded. However, an atheist can also obtain the infinite reward just by choosing strategy "believe", i.e. by pretending faith, but without really believing. As God may be demanding and grants the reward only those in firm faith, the inspection of faith by God is needed, which is included in the present analysis, i.e. God tests the faith. In addition, the infinitely high reward is removed from the payoffs, i.e. the believer goes out empty, and is even charged cost of faith. It shows that believing in God and worshipping is still the best option regardless, even if we believe that praying is in vain and has absolutely no value. The analysis also shows that the faith should even be so strong that we give our entire life to God and retain nothing for us since this is not only the safe rescue, but the rationally optimal choice of the extent of worship, and thus the unique salvation. Finally, all common criticisms to the wager game have been reinvestigated in detail; logic and mathematical analysis showed that they fail against the consideration of eternal punishment.

Keywords: Pascal’s wager game, decision theory, belief, theism, prayer, atheism, proof of concept

1. The atheist and the believer

Imagine an atheist meets a believer and they begin to argue about the believing in God. Says the atheist: "If there is no God, you have prayed and worshipped in vein, meaning loss of lifetime, not to mention that you made a fool of yourself". Answers the believer: "I better risk to becoming a fool than spending eternity in everlasting hellfire should God exist yet".

The question is: who is right? Blaise Pascal, a mathematician and philosopher living 1623-1662, was the first to look for a rational answer to the question by calculating the odds of a wager, and assigning payoffs to the players' present life and future eternity [1]. He showed that it is better to bet on God as the expected gains from believing are higher than the expected gains from not believing. However, this approach is disputed because the players may receive rewards from eternity, but it is unknown whether God rewards believers, even if there is a God, as discussed in the more recent literature (see Hajek, 2003 [2], and [3]). The argument is as follows: If there is a God who rewards those who use their critical faculty and punishes those with blind faith not really believing brings infinite reward. Thus, the problem may be overcome by removing the infinite reward from the calculation, and including God's hidden decision about the sufficiency of belief, as shown in the following analysis (sections 8 and 9).

Instead of considering a reward from God, the situation is actually better described as follows: the atheist regards praying and following God's commandments as a loss of lifetime, loss of joy, and generally a cost to his life. He regards the whole matter soberly and rationally, and is convinced that the "return on investment" (i.e., the uncertain future reward by God) is not outweighing this cost. So, he better takes what he can get for now rather than waiting for an uncertain future payoff that is worth to him nothing, quite according to the saying "a bird in the hand is worth two in the bush". His position is an expression of his reliance on the non-existence of God and eternal life or damnation. The believer, in contrast, is convinced of the existence of God, and does not argue in economic terms. As a real believer,

he feels obligated and happy to worship. The service to God is not a cost to the believer, but to the atheist. So, there are two very different views.

However, to answer the question for the atheist we need to examine it in rational terms and disregard the joy of worship the believer may have because this is what the atheist does not have, and it can thus not be included in his consideration. So, let us consider the statement of the atheist as the hypothesis which we take as the starting point of our investigation.

2. The hypothesis

Based on the considerations of the atheist, we can state the following hypothesis:

“It is better **not** to pray, but to enjoy life, instead of serving God, whose existence is unlikely, since the prayer is in vein, and you never receive a payoff from it.”

3. The scenario

Let define the following two events: X be the case that God exists, and Y that God does not exist. Let $P(X)$ be the probability for X to come true, C may include all the costs of praying, worshipping, and serving God (what a real believer would do), the maximum payoff for any year lifetime be equal 1, and the average lifetime be 80 years *ceteris paribus* (i.e. irrespective of any other variation in life caused by health state, salary, etc.). While assuming that truly believing saves from punishment, let the payoff from hypothetically spending one year in hell be equal -1, which corresponds to the Gospel in Mark 16, 16 (“He that believeth and is baptized shall be saved; but he that believeth not shall be damned”), as also referred to by Pascal [3]. Then the payoffs from life as an atheist (A) and a believer (B) are as follows:

$$A_A = 80 \times 1 = 80$$

$$A_B = 80 \times (1 - C) = 80 - 80C$$

Note that believing is not a dominant strategy anymore compared to Pascal's wager game since the theist is charged cost C.

4. Proposition 1

Considering the above hypothesis and model we can show the following:

Proposition 1: *If $P(X) > 0$ it is always better to pray, worship, and serve God.*

5. Proof to proposition 1

We can now calculate the expected payoffs $E(X)$:

$$E_B = P(X) [80 - 80C] + [1 - P(X)] [80 - 80C]$$

$$E_A = P(X) [80 - \infty] + [1 - P(X)] 80$$

For the hypothesis to be true we need:

$$E_A > E_B$$

Inserting and rearranging yields:

$$-P(X) \infty + 80C > 0$$

Simplifying to:

$$80 > P(X) \infty$$

However, for any $P(X) > 0$, even if marginally small, the right side of the inequality is always greater than the left side. Thus, we have shown that the hypothesis that it is rationally better not to pray is wrong. It is always better to pray. *Q.E.D.*

6. First extension to the scenario – uncertainty about sufficiency of service

As the atheist is calculating rationally, he is now convinced that he made a mistake not believing in God and not serving God. Therefore, he asks the believer how he can be sure that his extend of service is sufficient to escape eternal damnation, and how much service is actually necessary to meet God's demands, if there is a God. This question was similarly asked by Hayek [2]. The believer answers that God is discerning, prayers in blind faith may not be sufficient, but that he is unsure about the exact extend sufficient to escape damnation. He only knows that $C = 0$ is not sufficient and $C = 1$ is sufficient for sure.

Let $\mu(C)$ be the probability that God accepts the service as sufficient, with $\partial\mu/\partial C > 0$ and $0 \leq \mu \leq 1$. If $C = 0$ then $\mu = 0$ and if $C = 1$ then $\mu = 1$. The second derivation of μ is unknown. This means that the shape of μ is unknown as well. Then, we can state the following proposition:

Proposition 2: *If $P(X) > 0$, and if there is uncertainty about the extent of service necessary to escape infinite punishment, it is better to pray, worship, and serve God.*

7. Proof of proposition 2

Note that $C > 0$ since $C = 0$ leads to $\mu = 0$ with certainty, which resembles the status of the atheist, but not the believer. Then, we can calculate the payoff for the believer and the atheist under the condition of uncertainty about the sufficiency of service necessary to escape infinite punishment:

$$E_B = P(X) [\mu(80 - 80C) + (1 - \mu)(80 - 80C - \infty)] + [1 - P(X)] (80 - 80C) \quad (1)$$

$$E_A = P(X) [80 - \infty] + [1 - P(X)] 80$$

We again need $E_A > E_B$, and we receive after simplification:

$$80 > P(X)\mu\infty$$

However, since $C > 0$, which is self-evident, the right-hand side of the inequality is always larger than the left-hand side. Thus, $E_A < E_B$. *Q.E.D.*

8. Second extension to the scenario - diligence of service

The atheist has now observed that it is always better to pray, even if it is uncertain whether the service is sufficient for salvation. Since both the theist and the atheist wish to do sufficient service, they intend to find out the “break even” value of C , i.e. how much they need to do exactly in order to escape damnation. Given the uncertainty over the sufficiency of service and the above model we can state the following proposition:

Proposition 3: *If $P(X) > 0$, and there is uncertainty about the extent of service necessary to escape infinite punishment, then the only rational choice of the extent of service is $C = 1$.*

9. Proof of proposition 3:

We can calculate the payoff for a person who is attempting to serve God sufficiently under the condition that the true extent of service necessary to escape punishment is unknown. It is the same as presented in (1), namely:

$$E_B = P(X) [\mu(80 - 80C) + (1 - \mu)(80 - 80C - \infty)] + [1 - P(X)] (80 - 80C)$$

Rearranging yields:

$$E_B = (80 - 80C) - P(X) \infty (1 - \mu) \quad (2)$$

Thus, for all $\mu < 1$ the expected value is always $-\infty$, i.e. any extent of service is in vein except for $C = 1$,

where $E_B = 0$. *Q.E.D*

10. Answers to criticisms to Pascal's wager game

To foreclose the answer directly, there is absolutely no criticism countered to Pascal's wager game to date that is applicable or valid. Let us take a look at the different areas of criticism.

10.1 Believing cannot be a rational choice

It is argued that it is impossible to adopt a belief just by rational decision, and that Pascal's wager game does not provide a rationale for believing in God. Clearly, the game is not about how to become a believer, but simply about the superiority of the believer's earnings over that of the atheist.

As explained above, an atheist has a mere rational perspective as he is not so "naïve" to believe in something that he cannot see or measure. However, matter-of-fact, rational calculation has shown to him that he is wrong in that prayer is a waste of time. In fact, he is a victim of misconception that is psychologically called an illusion. Similarly to money illusion, where paying by bank card instead of cash money leads to underestimation of the value of money, the atheist succumbs to the illusion of chance. While facing immediate real cost now, the eternal reward (and even the eternal punishment) is so uncertain and so far in the future that he drastically underestimates it. Considering the atheist's rational view of calculating cost and utility, the outcome clearly tells the atheist about his failure.

Certainly, the atheist may have a problem now: how can he come to belief, while his conviction is just based on rational calculation, without belief? However, it is according to the saying “awareness of failure is the first step towards betterment”. As mentioned above, the atheist needs help from other sources than this rational analysis, i.e. the question of how the atheist can become a theist is out of the scope of this analysis. He could wish to research on the topic and seek help from believers who may explain to him why they believe in God and share the Gospel apart from a rational calculation. The understanding of the small analysis in this paper can also be the first step towards a new recognition. As Peter said, “work out your salvation with fear and trembling” (Philippians 2, 12), confirming that there are steps to be crested on the scale of belief, and that there is uncertainty about the sufficiency of service.

Finally, nobody is born a master, but Jesus Christ alone, who knew the problem of the intensity of belief as He (even) accused the apostles because of their insufficient belief by saying: “O thou of little faith, wherefore didst thou doubt? ... Verily I say unto you, if ye have faith, and doubt not, ye shall not only do this to the fig tree, but also if ye shall say unto this mountain, be thou removed, and be thou cast into the sea, and it shall be done.” (Matthew 14, 31 and 21, 21).

Last but not least, it needs to be mentioned that there is striking evidence for the existence of God today provided by various scientific disciplines including astrophysics, microbiology, and genetics, but there is no evidence that God does not exist. We must not ignore the important findings by these professional faculties, showing non-reducible complexities ruling out evolution, the complexity of ordered information encompassing genome/DNA ruling out accidental occurrence of life, and the materialization of the universe out of nothing ruling out self-occurrence, not even to mention the manifold evidence for the living, acting, and crucifixion of Jesus Christ. No one who is of sound mind can argue today that there is definitely no God and that there is no evidence for God’s existence. Not even the model-atheist Richard Dawkins claims this, as he stated: “I can't be sure God does not exist.”

10.2 The many-gods objection

Criticism also argues that there are different religions, and that decision theory cannot decide which of them is right. Clearly, the game is not about religion, but about the rationale of believing in God. The question of religion and whether one should follow human-made doctrine and religion at all is far beyond the scope of the game. Similarly, one may use epistemic considerations to choose a religion, but this is again out of scope. The criticism is simply not applicable, and thus does not undermine the strength of the result.

10.3 Relativism

The wager game may be unsuitable for today's multi-cultural and technology-enchanted audience. This generation should simply understand that God's word does not change with changes in society. In contrast, it remains unchanged and valid to all ages, as predicted by Jesus Himself: "Heaven and earth shall pass away, but my words shall not pass away." (Matthew 24, 35)

10.4 Inapplicability of decision theory

There is some criticism that decision theory is not applicable to the problem of believing in God because of the eternal reward. In this regard, several examples have been constructed, such as the following which is set up according to the considerations by Duff (1986): If you regularly brush your teeth then there is a chance that you will receive the reward from God by spending eternity in heaven [4]. Clearly, this is nonsense. One can only assign a positive probability to an event if there is any chance of occurrence. For this chance to be considered scientifically we need to have some information of evidence. However, while there is evidence of God's existence, there is no evidence for an eternal reward by brushing teeth. Therefore, we cannot assign any positive probability to this event.

Similarly for the St. Petersburg paradox which is not applicable as it is a different game. The nature of the St. Petersburg game is that you never get the infinite payoff. Instead you may in fact end up with \$2, \$4, or \$8. One would receive infinite payoff if and only if the game lasts for infinite periods, i.e. forever, which means never. Therefore, in contrast to what is stated in discussions on the St. Petersburg game, it is not wise to pay any finite value in order to play the game (see Sorensen, 1994 [5]). One should not play the game. The wager game, in contrast, shows the payoffs as they are, and they are paid. Also, it must be played by everybody whether he wants to do so or not. Therefore, this criticism does not hold any value in relation to the wager game.

10.5 Duff's argument revisited

There is more relevance in Duff's argument [4] than that any unrelated action (like brushing teeth) may have a correlation with receiving the eternal reward by God. While this correlation is of course not supported by sound argumentation, as shown above, there is some merit in the consideration that even a player who decides not to believe and not to serve God now may remain a chance of finding God later in his life, and this may attach a positive probability, even if marginally small, to the outcome of receiving the eternal reward (in heaven). This seems to put the non-believer on a par with the believer, even though the non-believer's probability of receiving the reward is smaller than that of the believer. Therefore, the non-believer would measure up to the believer when including the option of later belief in the game, irrespective of the probability attached to this event, and this is even in accordance with the Scripture, saying: "Take that which is thine, and go your way; I will give unto this last even as unto thee." Matthew 20, 14.

Unfortunately, this argumentation is limb again as it is associated with same failure a recklessly acting atheist is afflicted with; it is deficient of the very realistic probability of eternal punishment which must not be omitted. Of course, without such a punishment, the non-believer may play recklessly, and just

gamble for highest payoffs by choosing not to believe and not to serve God (and maybe serving God later), and he may thus maintain a chance of ending up with the reward. But we cannot disregard the punishment and the risk of missing the moment of repentance in the death bed. We know that it can never be argued that there is no God. Likewise, we cannot argue that there is no such eternal punishment, given that the prophets as well as Jesus Himself mentioned eternal punishment and everlasting hellfire for several times (“And these shall go away into eternal punishment, and the righteous into eternal life.” Matthew 25, 46, and: “Therefore hell hath enlarged itself, and opened its mouth without measure: and their glory, and their multitude, and their pomp, and he that rejoiceth, shall descend into it.” Isaiah 5, 14). Therefore, if we accept a positive probability for the existence of God and heaven, we need to put a positive probability to the existence of everlasting damnation as well.

It is also a matter of logic that eternal damnation must be existent if there is a righteous and holy God as otherwise God’s righteousness would be redundant. In other words, if we were to ignore this logic then anything may be reverse, and God may be not good and holy. Scripture also warns of this conversion by saying: “Woe unto them that call evil good, and good evil; that put darkness for light, and light for darkness; that put bitter for sweet, and sweet for bitter ... which justify the wicked for a reward, and take away the righteousness of the righteous!” Isaiah 5, 20 and 23.

Considering infinite punishment, we can see now that the logic by Duff’s argument is exactly converse, and plays against this argument, namely: while a tiny probability for a non-believer to becoming a believer later in life is sufficient for equaling up with the believer in terms of eternal reward, a tiny probability of not becoming a believer later in life is sufficient for falling apart from the believer’s status by maintaining the risk of infinite punishment. This provides us with the correct and logic insight now that the chance of receiving the infinite reward is counteracted by the risk of ending up in eternal damnation, which is always fatal to the non-believer in contrast to the believer.

One may prove this easily in mathematical terms. Let s be the probability that a non-believer finds God later in life (i.e., after playing the wager game) and thereby spares infinite punishment, s' that he fails to find God, and k be the number of years the non-believer has not served God, then we can set up the expected payoffs as follows:

$$E_A = P(X) [s(80 - (80 - k)C) + s'(80 - \infty)] + [1 - P(X)] [s(80 - (80 - k)C) + s'80]$$

$$E_B = 80 - 80C$$

Whilst the reward can be omitted anyway because of cancelling out, when investigating $E_A > E_B$ again we receive after modification and simplification:

$$80C + kC \frac{1}{s'} > P\infty \quad (3)$$

Which is wrong again. It is thus again always best to serve God because of the risk of ending up with infinite punishment even if we assume that the chance of finding God later in life is very large and the service provided by the present non-believer is tiny (but sufficient to escape punishment). Note that s' is indeed equal to $(1-s)$, but must be regarded as one probability, because otherwise the infinity values may be cancelled out against each other which would be wrong.

Finally, in order to foreclose further argumentation on this account, namely that the believer would also need to be confronted with the risk of infinite punishment (as we have already investigated in sections 8 and 9), the same result as in inequality (3) occurs since the non-believer always faces the additional risk of not coming to belief anymore, which the believer does not face, and thus ending up with negative infinite value anyway. This conforms to the logic that rejecting faith now does not only bear the fatal risk of missing repentance later, but also means to proceed living in willful sin while postponing repentance to nevermore, which also conforms to the Scripture, saying: "For where we sin willfully after receiving

the knowledge of the truth, there no longer remains any sacrifice for sins, but a certain fearful expectation of judgment, and heat of fire about to devour the adversaries.” (Hebrews 10, 26-27).

11. Conclusions

Proposition 1 shows that it is always better to pray and worship God if not believing results in an infinite punishment in the case God exists. The result corresponds to the original Pascal wager game which considered an infinite reward. However, the removal of the reward by God from the analysis means that the believer has nothing to gain from believing. In contrast, he even incurs a loss of lifetime because of the cost of worship. However, serving God saves from punishment. Clearly, if the atheist wishes to escape punishment, what he should do after reading this article, he is reliant on becoming a believer. Thus, he needs help from another side apart from this rational analysis.

Note that believing is a dominant strategy in Pascal’s wager game, but is not one anymore in this game since the believer incurs cost C . However, believing is still the optimal choice.

Proposition 2 tells us that believing and serving God is still the best choice even it is unknown whether God accepts the extent of service applied. The atheist may wonder why he should become a believer now, while it is unknown whether God exists, and he does not even know whether his service will be sufficient. Afterwards, his service may be in vein, even if God exists. Since he may consider that serving God will not be sufficient anyway, being a believer is even more uncomely to him than in the former case where the payoff from believing was certain salvation. However, even if he assumes that God’s existence is unlikely and that his service will presumably not be sufficient, it is still the best answer to pray and serve God, nevertheless. This is according to the scripture: “Work out your salvation with fear and trembling.” (Phil. 2, 12).

This approach clearly brings a solution to the problem associated with the infinite reward, and makes the prayer decision the true winner. One may argue that the infinite reward is just replaced by an infinite punishment, which changes nothing, but this argument is limb. The opposite is true, and makes praying always the best choice, even if the atheist cannot be sure to be saved by a prayer, since he can reduce the probability of ending up in eternal hellfire by every prayer. It is clearly better to do anything not to end up in this place, even up to $C = 1$. If there is no such punishment, one may gamble for highest gains with levity.

Proposition 3 finally tells us that there is only one extent of service that keeps the believer from infinite punishment, namely $C = 1$, which is the only solution to the problem. Equation (2) shows that the first term (representing payoff from life before eternity) is always positive as long as $C < 1$, even for $C = 0.99$, which means that the believer would retain a tiny piece of his life for his own sake, while giving almost all to God. However, in this case, the second term (representing payoff from eternity) is equal to $-\infty$, which resembles infinite punishment or eternal damnation. If $C = 1$, the first term become equal to 0, i.e. the believer retains nothing for his own sake, giving all to God. In this case, which is the only instance, the infinite punishment is avoided.

This result may be somewhat shocking even to the community of believers since it tells us not only that $C = 1$ is the **certain salvation**, but also that it is the **only salvation**. Any extent of service other than $C = 1$ gives infinite negative expected payoff, i.e. infinite punishment. In other words, salvation demands the scarification of the entire life to God. One must not hold back something for his own sake, but **give all to God**. While this may be new and surprising to some people, it is not a new perception, but a principle that Jesus Himself taught us: "For whoever wants to save his life will lose it, but whoever loses his life for My sake will find it" (Matthew 16, 25).

It appears that obtaining salvation is not an easy goal given that many are inflicted with the needs of the world. This is also confirmed in scripture: "For many are called, but few are chosen." (Matthew 20, 16). However, there is facilitation. God knows that the people are imperfect, and that they cannot reach righteousness on their own without help. Therefore, he sent His only Son such "that whosoever believeth in him should not perish, but has eternal life." (John 3, 15). And Jesus Himself taught us: "For your heavenly Father knoweth that ye have need of all these things. But seek ye first the kingdom of God, and his righteousness; and all these things shall be added unto you." (Matthew 6, 32-33). God wants us to ask for His help in reaching righteousness: "Ask, and it shall be given you; seek, and ye shall find; knock, and it shall be opened unto you." (Matthew 7, 7).

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