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Abstract

This study investigates the effects of monetary policy on economic growth and social welfare in a Schumpeterian economy with an upstream and a downstream sector in which the R&D investment of these sectors is subject to a cash-in-advance (CIA) constraint. We show that a higher nominal interest rate reallocates labor from a more cash-constrained R&D sector to a less one, which could generate an inverted-U effect on economic growth. In addition, we examine the necessary and sufficient conditions for the (sub)optimality of the Friedman rule by relating the underinvestment and overinvestment of R&D in the decentralized economy, and find that this relationship is crucially determined by the presence of CIA constraints, the relative productivity between upstream R&D and downstream R&D, and the strength of markup.

JEL classification: O30; O40; E41.

Keywords: CIA constraint; Endogenous growth; Monetary policy; Two R&D sectors

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1 Introduction

In this study, we explore the growth and welfare implications of monetary policy in a Schumpeterian economy with two vertically related sectors in which the R&D investment of these sectors are subject to a cash-in-advance (CIA) constraint. In the well-established tradition of R&D-based endogenous growth, such as Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), firms engage in research activities in either the downstream sector (i.e., the final-goods sector) or the upstream sector (i.e., the intermediate-goods sector). In their line of argument, policy instruments can only affect the resource allocation in R&D for either of these sectors with manufacturing. However, existing evidence strongly supports the fact that both downstream and upstream firms invest in R&D activities to develop innovations that upgrade their technologies. For example, the early study by Nelson (1986) finds that both upstream and downstream industries have significant contributions to the US R&D intensity. A recent study by Pillai (2013) shows that upstream semiconductor equipment firms like Nikon, Canon, Applied Materials and ASML invent new generations of capital equipment to allow microprocessor firms like Intel and AMD to invent higher performance microprocessors. More recently, Yang (2017) shows that in the US smartphone market, upstream chipset firms (such as Qualcomm) invest to conduct innovations, with which downstream handset markers (such as Samsung) innovate to improve their hardware. Therefore, the feature of R&D sectors for vertical industries needs stressing in the R&D-based growth model.

Furthermore, another important feature of the latest literature on money and R&D is that R&D investment is heavily affected by liquidity requirements. Brown *et al.* (2012) find that the increase in corporate cash flow in 1990s is the result of firms' smoothing of R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Falato and Sim (2014) use the US firm-level data to demonstrate that firms hold cash to finance their R&D investment due to the presence of financing frictions. Brown and Petersen (2015) reveal that firms tend to use cash to finance investment in R&D but not in capital. A recent study by Lyandres and Palazzo (2016) finds that the sharp increase in the the average cash-to-assets ratio for US firms since the mid-1980s is driven almost only by firms which invest heavily in R&D. Therefore, the presence of a CIA constraint on R&D also needs stressing in the R&D-based growth model.

To take into account the above two features on R&D, we build up a scale-invariant version of the quality-ladder model in which both the upstream and downstream sectors devote resources in R&D activities and money demand is incorporated into the model through a CIA constraint on R&D investments in these vertically related sectors. Our main results can be summarized as follows. Under the CIA constraints on the upstream and downstream R&D, there are two effects of an increase in the nominal interest rate on economic growth. First, a higher nominal interest rate would generate a growth-retarding effect through decreasing the sectoral levels of R&D because the cost of borrowing for R&D investments rises. Second, a higher nominal interest rate would generate a growth-enhancing effect through reallocating the resources (i.e., labor in this study) from the more-cash-constrained R&D sector to the less one (i.e., a cross-sector effect to simulate economic growth). As long as the markup and/or the strength of the CIA constraint on the less constrained R&D sector is relatively large, the growth-enhancing effect will dominate the growth-retarding effect at low levels of nominal interest rate. Nevertheless, as the nominal interest rate increases, this domination will be gradually mitigated and finally reversed. Hence, the nominal interest rate would have an overall inverted-U effect on economic growth, as documented in recent empirical studies, such as López-Villavicencio and Mignon (2011) and Eggoh and Khan (2014).[1]

In addition, we explore the long-run implications of monetary policy on social welfare by comparing our results to the monetary policy rule proposed by Friedman (1969) in which the optimal nominal interest rate is zero. There has been a large body of literature that analyzes the optimality of the Friedman rule in different economic environments.[2] However, until recent, several industrialized countries announced a very low (close to zero) level of nominal interest rate target,[3] and a growing number of studies attempt to incorporate money demand into R&D-based growth models to address this stylized fact by finding the condition for the suboptimality of the Friedman rule, such as Chu and Cozzi (2014) and Hori (2017). In this paper, we assume that both vertically related industries conduct R&D activities and reexamine the necessary and sufficient conditions for the (sub)optimality of the Friedman rule by relating the underinvestment and overinvestment of R&D in the decentralized economy. We find that this relationship is crucially determined by the presence of CIA constraints, the relative productivity between upstream R&D and downstream R&D, and the strength of markup. In particularly, in the presence of the CIA constraint on a sectoral R&D, when the relative productivity of this sectoral R&D is low (high), then R&D overinvestment/underinvestment is sufficient/necessary (necessary/sufficient) for the Friedman rule to be suboptimal/optimal. Nonetheless, in the presence of the CIA constraints with equal strength on both sectoral R&D, R&D overinvestment (underinvestment) becomes sufficient (necessary) for the Friedman rule to be suboptimal (optimal), regardless of the relative productivity of the sectoral R&D is low or high. Moreover, the strength of markup determines the degree of R&D over- or under-investment, when it is a necessary condition, to generate the suboptimality of the Friedman rule.

We also perform a quantitative analysis in which the current two-R&D-sector model is calibrated to the US economy to evaluate the growth and welfare effects of monetary policy. The benchmark case shows that a zero-interest-rate policy (i.e., the Friedman rule) maximizes economic growth and social welfare, and the welfare gain by optimizing the nominal interest rate from the decentralized equilibrium can be as large as 1% of consumption. Furthermore, to be consistent with more empirical evidence and to test the robustness of the analytical results as well, we conduct several sensitivity checks by varying the CIA parameters, markup, and the R&D labor ratio, to show the cases in which the growth-maximizing or welfare-maximizing rate of nominal interest becomes positive (e.g., the Friedman rule can be suboptimal).

This study contributes to the growth-theoretic literature in R&D and monetary policy that features CIA requirements. The pioneer work by Marquis and Reffett (1994) firstly introduces a CIA constraint on consumption into the Romer (1990) type variety expanding model to investigate

[1] See Chu *et al.* (2017) for an analysis of the CIA constraint on R&D in a Schumpeterian growth model with endogenous entry of heterogeneous firms, which also yields an inverted-U effect of monetary policy on economic growth.

[2] See the discussion regarding the optimality of the Friedman rule in, for example, Gahvari (2007, 2012) for overlapping generations models, Ho *et al.* (2007) for inflation taxation, and Lai and Chin (2010) for an open economy.

[3] Huang *et al.* (2017) document the facts for a low level of nominal interest rate target in the large economies such as US, Japan, and China during 2008–2015.

the effects of monetary policy, which proves that the Friedman rule is optimal. Unlike their model, the current study considers a Schumpeterian growth model and analyzes the effects of monetary policy via a CIA constraint on R&D. Our study is closely related to Chu and Cozzi (2014), who show that the overinvestment of R&D in the market economy is the necessary condition for the suboptimality of the Friedman rule in a Schumpeterian growth model with CIA on R&D. However, their result is based on the setting with R&D activities in only the intermediate-goods sector (i.e., the downstream sector in this study), so raising the nominal interest rate yields only a reallocating effect on resources from R&D to consumption and manufacturing. The current study complements their study by considering a more realistic setting in which R&D activities are engaged in both the intermediate-goods and final-goods sectors. Under this setting, in addition to the reallocative effect as in Chu and Cozzi (2014), raising the nominal interest rate generates a *cross-R&D-sector* effect between the upstream and downstream R&D sectors, which makes both R&D overinvestment and underinvestment possible to lead the optimal nominal interest rate to be positive. Moreover, Huang *et al.* (2015) and Chu and Ji (2016) explore the effects of monetary policy via CIA constraints on R&D in a Schumpeterian growth with endogenous market structure, but they do not consider vertical sectors in which firms invest in R&D as in the present study. Therefore, to the best of our knowledge, this is the first study that analyzes monetary policy in a growth-theoretic framework featuring R&D activities for vertical industries in a cash-in-advance economy.

Additionally, this study contributes to a small but growing literature that investigates the effects of government policy in endogenous growth models with two R&D sectors. For example, as for R&D subsidies, Li (2000) analyzes the effectiveness of R&D subsidies on stimulating economic growth in a two-R&D-sector model with both fully endogenous growth and semi-endogenous growth, whereas Segerstrom (2000) completely characterizes the effects of R&D subsidies on long-run growth in an endogenous growth model with both vertical R&D and horizontal R&D. As for patent policy, Goh and Olivier (2002) explore optimal patent protection in a variety expanding growth model where firms in both the upstream and downstream sectors engage in R&D, whereas Chu (2011) addresses a similar issue in a quality-ladder growth model where firms in two horizontal final-goods sectors (i.e., the downstream sectors) engage in R&D. Consequently, our paper complements the above interesting studies by focusing on the role of monetary policy in a two-R&D-sector economy.[4]

The rest of this paper is organized as follows. Section 2 presents the model setup. Section 3 characterizes the decentralized equilibrium and analyzes the growth effect of monetary policy. Section 4 explores the welfare effects of monetary policy and derives the optimal monetary policy. Section 5 provides a quantitative analysis. Section 6 concludes the study.

2 Model

In this section, we present the monetary Schumpeterian growth model. We extend a version of the quality-ladder model in Grossman and Helpman (1991) by allowing for firms to invest in R&D to develop innovations in both upstream and downstream sectors as in Goh and Olivier (2002)

[4] Zheng *et al.* (2018) also examine the growth and welfare implications of monetary policy in a Schumpeterian growth model with two R&D sectors. Nevertheless, R&D investments are conducted to develop vertical and horizontal innovations in their study instead of upstream and downstream industries in the current study.

and by introducing money demand via CIA constraints on R&D investments as in Chu and Cozzi (2014). Moreover, we allow for population growth and remove scale effects in the Schumpeterian model by incorporating a dilution effect on R&D productivity as in Laincz and Peretto (2006). The nominal interest rate serves as the instrument for monetary policy and the effects of monetary policy are examined by considering the implications of altering the rate of nominal interest on economic growth and social welfare.

2.1 Households

At time t, each household has a population size of N_t, which grows at the rate of $n \geq 0$ such that $\dot{N}_t = nN_t$. There is a unit continuum of identical households, and the lifetime utility function of each member is given by

$$U = \int_0^\infty e^{-\rho t} u_t dt, \tag{1}$$

where $\rho > 0$ represents the discount rate and u_t denotes the instantaneous utility. The law of motion for assets of each household member is

$$\dot{a}_t + \dot{m}_t = (r_t - n)a_t + w_t + i_t b_t + \tau_t - e_t - (\pi_t + n)m_t, \tag{2}$$

where a_t is the asset value, r_t is the real interest rate, and each individual inelastically supplies one unit of labor at the rate w_t. e_t is the expenditure on consumption. τ_t denotes the lump-sum transfer from the government, π_t is the inflation rate the reflects the cost of holding money, and m_t is the money balance that the household member holds in order to facilitate entrepreneurs' loans b_t, which finance the R&D investment on the return rate of i_t. Therefore, the cash-in-advance (CIA) constraint is $b_t \leq m_t$.

The static utility function is $u_t = \ln c_t$, where c_t is the level of consumption whose price is normalized to unity. Consumption is derived by aggregating a unit continuum of differentiated final goods $y_t(j)$ indexed by $j \in [0, 1]$ such that

$$c_t = \exp\left(\int_0^1 \ln y_t(j) dj\right). \tag{3}$$

There are two stages for utility maximization. In the first stage, each household member decides the allocation of expenditure across the unit measure of final products. Each household's expenditure is given by

$$e_t = p_t c_t, \tag{4}$$

where p_t is the price of c_t. Given this price, each individual chooses an optimal level of $y_t(j)$ to maximize the payoffs of deriving consumption. Solving the static optimization problem gives rise to the demand for the differentiated final goods $y_{j,t}$ as follows:

$$y_t(j) = e_t / p_{y,t}(j), \tag{5}$$

where $p_{y,t}(j)$ is the price of $y_t(j)$.

In the second stage, the household member allocates her expenditure across the planning horizon. The optimal problem is to maximize the discounted utility in (1) subject to the budget constraint in (2) and the CIA constraint. The optimality condition for the household's consumption yields the familiar Euler equation

$$\frac{\dot{e}_t}{e_t} = r_t - \rho - n. \tag{6}$$

Finally, using the optimality condition for real money balance m_t, it is straightforward to derive the Fisher equation such that $i_t = \pi_t + r_t$.

2.2 Final Goods

As for each differentiated final good j, the total demand equals its supply such that

$$Y_t(j) = N_t y_t(j). \tag{7}$$

In addition, the aggregate amount of the final goods in this economy is given by a unit continuum of the differentiated final good $Y_t(j)$:

$$Y_t = \exp\left(\int_0^1 \ln Y_t(j) dj\right). \tag{8}$$

The differentiated final goods in each industry j is produced by a monopolistic leader who holds a patent on the latest innovation and uses a unit continuum of intermediate goods indexed by $k \in [0, 1]$. This leader's products are replaced by the products of a new entrant who has a more advanced innovation due to the *Arrow replacement effect.* The current leader's production function is

$$Y_t(j) = z^{q_{y,t}(j)} \exp\left(\int_0^1 \ln x_t(j,k) dk\right), \tag{9}$$

where the parameter $z > 1$ measures the step size of each quality improvement, $q_{y,t}(j)$ denotes the number of innovations between time 0 and time t, and $x_t(j, k)$ is the quantity of intermediate good k used for final good j. Thus, the marginal cost of producing each final goods is

$$mc_{y,t}(j) = \frac{P_{x,t}}{z^{q_{y,t}(j)}}, \tag{10}$$

where $P_{x,t} \equiv \exp\left(\int_0^1 \ln p_{x,t}(k) dk\right)$ is the price index for the intermediate goods and $p_{x,t}(k)$ is the price of intermediate good k.

In each industry of differentiated final goods, the current and previous leaders engage in Bertrand competition. Following previous studies such as Goh and Olivier (2002), Iwaisako and Futagami (2013), and Yang (2018), this model assumes that intellectual property rights protect inventions in the form of incomplete patent breadth. Therefore, patent breadth enables the current leader to charge a markup $\mu > 1$ over the marginal cost, and the profit-maximizing price is given by

$$p_{y,t}(j) = \mu \frac{P_{x,t}}{z^{q_{y,t}(j)}}. \tag{11}$$

The monopolistic profit of each differentiated final-goods producer is identical and is given by

$$\Pi_{y,t} = \Pi_{y,t}(j) = (\mu - 1)Y_t(j)\frac{p_{y,t}(j)}{\mu} = \left(\frac{\mu - 1}{\mu}\right) e_t N_t, \tag{12}$$

where the second equality and the third equality are obtained by using (12) and (5), respectively. Then, using the definition of price index $P_{x,t}$ along with (10), (11), and (12) yields the demand for each intermediate good k:

$$x_t(k) = \int_0^1 x_t(j,k)dj = \frac{e_t N_t}{\mu p_{x,t}(k)}. \tag{13}$$

2.3 Intermediate Goods

The environment of the intermediate-goods sector is similar to that of the final-goods sector. Each industry in this sector is temporarily dominated by a monopolist holding the latest innovation, and the industry leadership is replaced by an entrant who holds a new invention. However, the production structure of this sector is different from the previous sector since intermediate goods are produced by manufacturing labor. Specifically, the production function for the current intermediate-goods producer in industry k is given by

$$x_t(k) = z^{q_{x,t}(k)} L_{x,t}(k), \tag{14}$$

where the step size of quality improvement is identical to that in the final-goods sector, $q_{x,t}(k)$ is the number of innovation as of time t, and $L_{x,t}(k)$ is the employment level of production in industry k. Given the pricing strategy of the current leaders in this sector, the profit-maximizing price is again a constant markup over the marginal cost such that

$$p_{x,t}(k) = \mu mc_{x,t}(k) = \mu \frac{w_t}{z^{q_{x,t}(k)}}. \tag{15}$$

Accordingly, the monopolistic profit for each intermediate-goods producer is

$$\Pi_{x,t} = \Pi_{x,t}(k) = \frac{(\mu - 1)w_t}{z^{q_{x,t}(k)}} x_t(k) = \left(\frac{\mu - 1}{\mu}\right)\frac{e_t N_t}{\mu}, \tag{16}$$

where the second equality and the third one are obtained by using (15)-(16) and (5), respectively. Moreover, it is straightforward to show the production-labor income in the industry of intermediate k:

$$w_t L_{x,t}(k) = \frac{1}{\mu} p_{x,t}(k) x_t(k) = \frac{e_t N_t}{\mu^2}, \tag{17}$$

and therefore the labor demand for the intermediate good k is given by

$$L_{x,t}(k) = \frac{e_t N_t}{\mu^2 w_t}. \tag{18}$$

2.4 Innovations and R&D

The environments for R&D firms that conduct innovations for the final-goods sector and the intermediate-goods sector are the same as follows. The expected value of owning the most recent innovation in the industry j (k) of the final- (intermediate-) goods sector is denoted as $v_{y,t}(j)$ ($v_{x,t}(k)$). Following the standard literature, we focus on a symmetric equilibrium (see, for example, Cozzi *et al.* (2007)). This implies that given $\Pi_{y,t}(j) = \Pi_{y,t}$ ($\Pi_{x,t}(k) = \Pi_{x,t}$), it follows that $v_{y,t}(j) = v_{y,t}$ ($v_{x,t}(k) = v_{x,t}$). Denote by $\lambda_{y,t}$ ($\lambda_{x,t}$) the *aggregate-level* Poisson arrival rate of innovations for final- (intermediate-) goods. Then, the Hamilton-Jacobi-Bellman (HJB) equation for $v_{y,t}$ ($v_{x,t}$) is

$$r_t v_{s,t} = \Pi_{s,t} + \dot{v}_{s,t} - \lambda_{s,t} v_{s,t}, \tag{19}$$

which is the no-arbitrage condition for the value of the asset in sector $s = y, x$, respectively. In equilibrium, the return on the asset $r_t v_{s,t}$ equals the sum of the flow profits $\Pi_{s,t}$, the capital gain $\dot{v}_{s,t}$, and the potential losses $\lambda_{s,t} v_{s,t}$ when creative destruction takes place.

New innovations in each industry of the final- and intermediate-goods sectors are invented by a unit continuum of R&D firms indexed by $\theta \in [0, 1]$ and $\vartheta \in [0, 1]$, respectively, and each of the R&D firms in sectors y and x employs R&D labor $L^y_{r,t}(\theta)$ and $L^x_{r,t}(\vartheta)$ for producing inventions. This study follows the existing literature, such as Chu and Cozzi (2014), Huang *et al.* (2015), and Huang *et al.* (2017) to incorporate a CIA constraint on R&D investment at time t, such that households lend the θ-th (ϑ-th) entrepreneur an amount $B^y_t(\theta) = b^y_t(\theta)N_t$ ($B^x_t(\vartheta) = b^x_t(\vartheta)N_t$) of money, which finances the wage payment for the downstream-R&D labor $w_t L^y_{r,t}(\theta)$ (for upstream-R&D labor $w_t L^x_{r,t}(\vartheta)$) on the return rate of i_t. Thus, the expected profit of the χ-th R&D firm, where $\chi = \theta, \vartheta$, is

$$\Pi^s_{r,t}(\chi) = v_{s,t}\lambda_{s,t}(\chi) - (1 + \xi_s i_t) w_t L^s_{r,t}(\chi), \tag{20}$$

where $\xi_s = \xi_y, \xi_x \in [0, 1]$ is the respective strength of the CIA constraint on downstream (upstream) R&D. Moreover, we formulate the *firm-level* arrival rate of innovations $\lambda_{s,t}(\chi)$ to be

$$\lambda_{s,t}(\chi) = \bar{\varphi}_{s,t} L^s_{r,t}(\chi) = \frac{\varphi_s}{N_t} L^s_{r,t}(\chi), \tag{21}$$

where the specification $\bar{\varphi}_{s,t} = \varphi_s / N_t$ captures the dilution effect that removes scale effects as in Laincz and Peretto (2006) and $\varphi_s = \varphi_y, \varphi_x$ is the productivity parameter for downstream and upstream R&D, respectively. In equilibrium, the aggregate-level arrival rate of innovations is thus given by $\lambda_{s,t} = \int_0^1 \lambda_{s,t}(\chi) d\chi = \varphi_s L^s_{r,t} / N_t$. Then, free entry into the R&D sectors implies the following zero-expected-profit condition:

$$v_{s,t}\lambda_{s,t} = (1 + \xi_s i_t) w_t L^s_{r,t}. \tag{22}$$

This equation is a condition that pins down the allocation of labors between production and R&D.

2.5 Monetary Authority

Denote the nominal money supply by M_t and its growth rate by $\Phi_t \equiv \dot{M}_t/M_t$, respectively. Accordingly, the real money balance is given by $m_t N_t = M_t/p_t$, where p_t is the price of consumption. Then, consider that the growth rate of money supply Φ_t serves as a policy instrument that can be controlled by monetary authority. In this case, the rate of inflation is determined by $\pi_t = \Phi_t - \dot{m}_t/m_t - n$. Additionally, combining this condition with the Fisher equation (i.e., $i_t = \pi_t + r_t$) yields the one-to-one relationship between the nominal interest rate and the nominal money supply, such that[5]

$$i_t = \Phi_t + \rho. \tag{23}$$

Given this result, throughout the rest of this study, we will use i_t to represent the instrument of monetary policy for simplicity. Finally, the monetary authority redistributes the increase in money supply as a lump-sum transfer to the households, namely, $\tau_t N_t = \dot{M}_t/p_t = \Phi_t m_t N_t = [(\pi_t + n)m_t + \dot{m}_t]N_t$.

3 Decentralized Equilibrium

An equilibrium consists of

(a) a sequence of allocations $[c_t, m_t, Y_t(j), y_t(j), x_t(k), L_{x,t}(k), L^y_{r,t}(\theta), L^x_{r,t}(\vartheta)]^\infty_{t=0,j,k,\theta,\vartheta\in[0,1]}$ and

(b) a sequence of prices $[r_t, p_{y,t}(j), p_{x,t}(k), w_t, v_{y,t}, v_{x,t}]^\infty_{t=0,j,k\in[0,1]}$. Moreover, at each instance of time,

- households produce $[c_t]$ using $[y_t(j)]$ as inputs to maximize payoffs taking $[p_{y,t}(j)]$ as given;
- households choose $[c_t]$ to maximize their utility taking $[r_t, i_t, w_t]$ as given;
- monopolistic leaders for final goods produce $[Y_t(j)]$ and choose $[p_{y,t}(j)]$ to maximize profits taking $[p_{x,t}(k)]$ as given;
- monopolistic leaders for intermediate goods produce $[x_t(k)]$ and choose $[p_{x,t}(k), L_{x,t}(k)]$ to maximize profits taking $[w_t]$ as given;
- competitive downstream-R&D firms choose $[L^y_{r,t}(\theta)]$ to maximize profits taking $[w_t, v_{y,t}]$ as given;
- competitive upstream-R&D firms choose $[L^x_{r,t}(\vartheta)]$ to maximize profits taking $[w_t, v_{x,t}]$ as given;
- the labor market clears such that $L_{x,t} + L^y_{r,t} + L^x_{r,t} = N_t$;
- the innovations value adds up to households' asset value such that $v_{x,t} + v_{y,t} = a_t N_t$;
- the R&D entrepreneurs finance their wage payments through borrowing such that $\xi_y w_t L^y_{r,t} + \xi_x w_t L^x_{r,t} = b_t N_t$; and
- monetary authority balances its budget such that $\tau_t N_t = (i_t - \rho) m_t N_t$.

3.1 Balanced Growth Path

This section characterizes the decentralized equilibrium for this model and show that the economy grows in a unique and stable balanced growth path (BGP). To facilitate this result, we first

[5] On the balanced growth path, which will be shown in Section 3.1, c_t, e_t, and m_t grow at the same rate of $r_t - \rho - n$ according to the Euler equation.

derive the growth rate of aggregate technology g_t. Using (9), (10), and (15) yields $Y_t = Z_{y,t} Z_{x,t} L_{x,t}$, where $Z_{y,t}$ and $Z_{x,t}$ are defined as the level of technology in the downstream sector and in the upstream sector, where $\ln Z_{y,t} \equiv \ln z \int_0^1 q_t(j)dj = \ln z \int_0^t \lambda_{y,\iota} d\iota$ and $\ln Z_{x,t} \equiv \ln z \int_0^1 q_t(k)dk = \ln z \int_0^t \lambda_{x,\kappa} d\kappa$, respectively, and the second equalities are obtained by the law of large numbers. Then, differentiating these two equations with respect to time yields the growth rate of aggregate technology given by

$$g_t = \frac{\dot{Z}_{y,t}}{Z_{y,t}} + \frac{\dot{Z}_{x,t}}{Z_{x,t}} = (\varphi_y l^y_{r,t} + \varphi_x l^x_{r,t}) \ln z, \tag{24}$$

where the second equality is obtained by using (21) and $l^y_{r,t} \equiv L^y_{r,t}/N_t$ and $l^x_{r,t} \equiv L^x_{r,t}/N_t$ are defined as downstream-R&D labor per capita and upstream-R&D labor per capta, respectively. Similarly, $l_{x,t} \equiv L_{x,t}/N_t$ is defined as manufacturing labor per capita.

For an arbitrary path of the nominal interest rate $[i_t]_{t=0}^{\infty}$, we obtain the following result.

Proposition 1. *Holding constant i, the economy jumps to a unique and stable balanced growth path.*

Proof. See Appendix A. □

3.2 Equilibrium Allocations and the Growth Effect

As implied by Proposition 1, given a constant i, the equilibrium labor allocations $\{l_x, l^y_{r,t}, l^x_{r,t}\}$ are stationary along the BGP. Using the zero-expected-profit condition for upstream R&D (22) and the production-labor income in the intermediate-goods sector (17) yields $v_{y,t}\lambda_{y,t} = (1+\xi_y i)N_t e_t/(\mu^2 l_{x,t})$ implying $\dot{v}_{y,t}/v_{y,t} = \dot{e}_t/e_t + n$. Combining this result with (19) and (7) and imposing the BGP implies $\Pi_y/v_y = \rho + \lambda_y$. Then, substituting this equation into (22) and applying (12) and (21) derives the relationship between l^y_r and l_x such that

$$l^y_r = \frac{\mu(\mu-1)l_x}{1+\xi_y i} - \frac{\rho}{\varphi_y}, \tag{25}$$

which is the first equation to solve for $\{l_x, l^y_{r,t}, l^x_{r,t}\}$. Following a similar logic, we can use (7), (16), (17), (19), (21), and (22) to derive the second equation, which is the relationship between l^x_r to l_x given by

$$l^x_r = \frac{(\mu-1)l_x}{1+\xi_x i} - \frac{\rho}{\varphi_x}. \tag{26}$$

The last equation is the labor-market-clearing condition such that

$$l_x + l^x_r + l^y_r = 1. \tag{27}$$

Thus, solving (25)-(27) yields the equilibrium labor allocations as follows:

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{1 + \frac{\mu-1}{1+\xi_x i} + \frac{\mu(\mu-1)}{1+\xi_y i}}, \tag{28}$$

$$l_r^y = \frac{\frac{\mu(\mu-1)}{1+\xi_y i}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)}{1+\frac{\mu-1}{1+\xi_x i}+\frac{\mu(\mu-1)}{1+\xi_y i}} - \frac{\rho}{\varphi_y}, \tag{29}$$

$$l_r^x = \frac{\frac{\mu-1}{1+\xi_x i}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)}{1+\frac{\mu-1}{1+\xi_x i}+\frac{\mu(\mu-1)}{1+\xi_y i}} - \frac{\rho}{\varphi_x}. \tag{30}$$

In these equilibrium labor allocations, (28) shows that production labor l_x is increasing in the nominal interest rate i, because a higher i raises the cost of borrowing for R&D investments, which reallocates the labor from R&D to manufacturing. Nevertheless, (29) reveals that there are two effects of i on the downstream-R&D labor l_r^y. On the one hand, i has a negative effect on l_r^y due to the reallocation of labor to production as aforementioned. On the other hand, i has a negative (positive) effect on l_r^y if ξ_y is greater (smaller) than ξ_x, namely downstream R&D is more (less) constrained to CIA than upstream R&D. This creates another re-allocative effect of labor between the two R&D sectors (i.e., a cross-R&D-sector effect). Whether a higher i increases or decreases l_r^y depends on the relative magnitude of ξ_y and ξ_x and the level of markup μ. Specifically, when $\xi_y \geq \xi_x$, the second effect is negative and thus reinforces the first effect, making l_r^y to be decreasing in i. When $\xi_y < \xi_x$, the second effect becomes positive. In this case, if μ is sufficiently large (small), markup strengthens (dampens) the second positive effect and induces it to dominate (to be dominated by) the first negative effect. As a result, l_r^y is increasing (decreasing) in i. Furthermore, (30) shows that i has two effects on the upstream-R&D labor l_r^x, and the analysis of the overall impact is analogous to that for the impact of i on l_r^y.[6]

Therefore, the above result and (24) imply that the effect of the nominal interest rate i on the growth rate of technology $g = \ln z(\varphi_y l_r^y + \varphi_x l_r^x)$ is mixed, and it is determined by the impacts of i on the levels of labor l_r^y and l_r^y in the two R&D sectors in addition to the productivity parameters φ_y and φ_x. Using (28)-(30) and differentiating (24) with respect to i yields

$$\begin{aligned}\frac{\partial g}{\partial i} &= \left(\varphi_y \frac{\partial l_r^y}{\partial i} + \varphi_x \frac{\partial l_r^x}{\partial i}\right)\ln z \\ &= \frac{(\mu-1)\ln z}{\Lambda^2}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)\begin{Bmatrix} \varphi_y \mu[(\mu-1)\xi_x - (\mu+\xi_x i(2+\xi_x i))\xi_y] \\ +\varphi_x[\mu(\mu-1)\xi_y - (\mu(\mu-1)+(1+\xi_y i)^2)\xi_x] \end{Bmatrix},\end{aligned} \tag{31}$$

where $\Lambda \equiv (1+\xi_y i)(1+\xi_x i) + (\mu-1)(1+\xi_y i) + \mu(\mu-1)(1+\xi_x i)$. It can be see that $\partial g/\partial i$ is decreasing in i, namely g is a concave function of i. In particular, if $\partial g/\partial i|_{i=0} = \mu\varphi_y[(\mu-1)\xi_x - \mu\xi_y] + \varphi_x\left[\mu(\mu-1)\xi_y - (\mu^2-\mu+1)\xi_x\right] > 0$, then the growth rate of aggregate technology g and the nominal interest rate i exhibit an inverted-U relationship. Intuitively, R&D labor in the less CIA constrained sector tends to be increasing in the nominal interest rate i in the support of a large markup. Together with a large sectoral productivity parameter, the less constrained sector yields a sufficiently high (positive) growth rate of technology that can dominate the (negative) growth rate of technology in the more constrained sector where R&D labor is decreasing in i.

[6] When $\xi_x \geq \xi_y$, l_r^x is decreasing in i, whereas when $\xi_x < \xi_y$, l_r^x is increasing (decreasing) in i if μ is sufficiently large (small).

Thus, monetary policy is growth-enhancing in this case. Notwithstanding, as i increases, the above domination of the less constrained sector becomes weaker since the reallocation of labor from R&D to production keeps strengthening the negative growth rate of technology in the more constrained sector. Then, monetary policy turns to be growth-retarding. Notice that if the markup is small and/or the productivity parameter in the less constrained sector is small, the growth-increasing part of monetary policy will be invalid; g can only be decreasing in i.

Proposition 2. *The growth rate of aggregate technology is concave in the nominal interest rate. If $\mu\varphi_y[(\mu-1)\xi_x - \mu\xi_y] + \varphi_x\left[\mu(\mu-1)\xi_y - (\mu^2-\mu+1)\xi_x\right] > 0$ holds, then the nominal interest rate generates a non-monotonic effect on economic growth.*

Proof. Proven in the text. □

3.3 Socially Optimal Allocations

Imposing balanced growth on (1) yields

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right) \tag{32}$$

where $c_0 = Z_{x,0}Z_{y,0}l_x$ and $g = \ln z(\varphi_y l_r^y + \varphi_x l_r^x)$. Dropping the exogenous terms $Z_{x,0}$ and $Z_{y,0}$ and maximizing (32) subject to the resource constraint for labor $l_x + l_r^y + l_r^x = 1$ yields the first-best allocations denoted with a superscript asterisk:

$$l_x^* = \min\left[\frac{\rho}{\varphi_y \ln z}, \frac{\rho}{\varphi_x \ln z}\right], \tag{33}$$

$$l_r^{y*} = \max\left[1 - \frac{\rho}{\varphi_y \ln z}, 0\right], \tag{34}$$

$$l_r^{x*} = \max\left[1 - \frac{\rho}{\varphi_x \ln z}, 0\right]. \tag{35}$$

The socially optimal outcome implies that technology advances in the downstream and upstream sectors are perfectly substitutable. Therefore, a corner solution arises for the first-best labor allocations in the sense that the social optimum only allocates labor to the R&D sector with a higher level of productivity.[7] Specifically, suppose $\varphi_y > \varphi_x$, that is, innovative activities in the downstream sector are more productive, then devoting all R&D labor to this sector is socially optimal. As a result, the labor in the upstream R&D sector is zero, implying that economic growth in the social optimum only depends on downstream innovations. In contrast, for $\varphi_y < \varphi_x$, the situation reverses such that R&D labor is only allocated to the upstream sector but none to the downstream sector.

[7] Suppose that the productivity is identical between the two R&D sectors such that $\varphi_x = \varphi_y = \varphi$, then this implies that the first-best production labor $L_x^* = \rho/(\varphi \ln z)$. In this case, one condition is lost to determine the relationship between the two first-best R&D labor allocations L_r^{y*} and L_r^{x*}. In other words, any combination of $\{L_r^{y*}, L_r^{x*}\}$ satisfying $L_r^{y*} + L_r^{x*} = 1 - \rho/(\varphi \ln z)$ can be a solution. Without loss of generality, it is assumed that φ_y and φ_x differs to facilitate the welfare analysis that follows, and thus, the possibility of $\varphi_y = \varphi_x$ is excluded.

4 Optimal Monetary Policy and the Friedman Rule

In this section, we analyze optimal monetary policy and examine the conditions under which the Friedman rule is (sub)optimal. In Sections 4.1 and 4.2, we consider the cases in which the CIA constraint only on upstream R&D or downstream R&D is present, respectively. In Section 4.3, we consider the case in which the model features an equal CIA constraint on both R&D sectors. We use i^* to denote the optimal rate of nominal interest, which maximizes social welfare.

4.1 CIA Constraint on Downstream R&D

In this subsection, we consider optimal monetary policy with the CIA constraint only downstream R&D, which is captured by $\xi_x = 0$. We also normalize the CIA constraint on upstream R&D to unity (i.e., $\xi_y = 1$) for simplicity in this case. Furthermore, we consider this case by taking into account both $\varphi_y > \varphi_x$ and $\varphi_y < \varphi_x$, since the relative R&D productivity between downstream R&D and upstream R&D matter in the existence of the labor allocations of R&D from the social perspective.

Imposing $\xi_x = 0$, the equilibrium labor allocations are simplified to

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{\mu + \frac{\mu(\mu-1)}{1+i}}, \tag{36}$$

$$l_r^y = \frac{\frac{\mu(\mu-1)}{1+i}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{\mu + \frac{\mu(\mu-1)}{1+i}} - \frac{\rho}{\varphi_y}, \tag{37}$$

$$l_r^x = \frac{(\mu-1)\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{\mu + \frac{\mu(\mu-1)}{1+i}} - \frac{\rho}{\varphi_x}. \tag{38}$$

From (36)-(38), it is easy to see that production labor l_x is increasing in the nominal interest rate i, whereas downstream- (upstream-) R&D labor is decreasing (increasing) in i. Given the fact that downstream R&D is more constrained by CIA than upstream R&D (i.e., $\xi_y = 1 > \xi_x = 0$), the effect of i now operates through the CIA constraint on the downstream R&D sector so that a higher i increases the cost of downstream R&D relative to upstream R&D, leading to a labor reallocation from downstream R&D to upstream R&D and manufacturing.

Accordingly, substituting (36)-(38) into (32) yields the lifetime utility U along the BGP. Then, to examine the (sub)optimality of the Friedman rule in this case, differentiating U with respect to i and evaluating $\partial U/\partial i$ at $i = 0$ yields

$$\text{sign}\left(\frac{\partial U}{\partial i}|_{i=0}\right) = \text{sign}\left\{1 - \frac{\ln z}{\mu^2 \rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)[\varphi_x + \mu(\varphi_y - \varphi_x)]\right\}, \tag{39}$$

which can be either positive or negative depending on parameter values. It can be seen that the relative magnitude of downstream-R&D productivity φ_y and upstream-R&D productivity φ_x is crucial to determine the sign of (39). Therefore, we analyze the following two cases.

4.1.1 Low Relative Productivity of Downstream R&D

As for the case in which downstream R&D is less productive than upstream R&D, namely $\varphi_y < \varphi_x$, comparing the first-best production labor (33) and the equilibrium production labor (36) evaluated at $i = 0$, we find that the equilibrium at $i = 0$ features R&D overinvestment (i.e., $l_x|_{i=0} < l_x^*$) such that $(1 + \rho/\varphi_x + \rho/\varphi_y) < (\mu^2\rho)/(\varphi_x \ln z)$. Using this result in the right-hand side of (39) implies

$$\frac{\partial U}{\partial i}|_{i=0} > 1 - \frac{\varphi_x \ln z}{\mu^2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right) > 0. \tag{40}$$

Therefore, for $\varphi_y < \varphi_x$, R&D overinvestment in equilibrium is sufficient for the Friedman rule to be suboptimal, because by raising i to be positive, the welfare gains from enhancing consumption through the increase in l_x and from stimulating growth through the increase in l_r^x (i.e., the cross-R&D-sector effect) reinforce the welfare gain from overcoming R&D overinvestment through the decrease in l_r^y. Nevertheless, it is easy to see that R&D overinvestment is not necessary for the suboptimality of the Friedman rule; R&D underinvestment implies that $\partial U/\partial i|_{i=0}$ is greater than a negative value, and a relatively large (small) markup would also enable R&D underinvestment to make a positive nominal interest rate (a zero nominal interest rate) optimal.[8] This implies that the suboptimality of the Friedman rule can be supported by R&D underinvestment if the welfare gain from reallocating labor through the increase in l_x and l_r^x dominates the welfare cost from worsening R&D underinvestment through the decrease in l_r^y.[9] The above analysis is equivalent to show that R&D underinvestment is necessary for the Friedman rule to be optimal.

4.1.2 High Relative Productivity of Downstream R&D

In contrast, as for the case in which downstream R&D is more productive than upstream R&D, namely $\varphi_y > \varphi_x$, comparing (33) and (36) at $i = 0$ shows that R&D underinvestment arises in the equilibrium where $i = 0$ (i.e., $l_x|_{i=0} > l_x^*$) such that $(1 + \rho/\varphi_x + \rho/\varphi_y) > (\mu^2\rho)/(\varphi_y \ln z)$ holds. Hence, we obtain

$$\frac{\partial U}{\partial i}|_{i=0} < 1 - \frac{\varphi_y \ln z}{\mu^2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right) < 0, \tag{41}$$

which implies that R&D underinvestment in equilibrium is sufficient (but not necessary) for the Friedman rule to be optimal. In this case, by reducing i to zero, the welfare gain from overcoming R&D underinvestment through the increase in l_r^y overwhelms the welfare cost from reallocating labor through the decrease in l_x and l_r^x. In other words, this result also implies that R&D overinvestment is necessary for the suboptimality of the Friedman rule. However, the degree of R&D overinvestment has to be sufficiently high in order to make a positive rate of nominal interest optimal, and again, this can be achieved under a large markup;[10] otherwise, R&D overinvestment would also lead the zero

[8] Specifically, if $\mu > (<) \bar{\mu} = -\frac{\ln z}{2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)(\varphi_x - \varphi_y) + \sqrt{\left[\frac{\ln z}{2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)(\varphi_x - \varphi_y)\right]^2 + \frac{\varphi_x \ln z}{\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}$, then i^* will become positive (zero) even with R&D underinvestment.

[9] Note that the welfare gain from the cross-R&D-sector effect through the increase in l_r^x also includes the one from mitigating the problem of R&D underinvestment.

[10] If the degree of R&D overinvestment $l_x^* - l_x|_{i=0}$ is greater than the threshold value, then the LHS of (39) will suffice to be positive. Suppose $1 - l_x|_{i=0} = 1 - l_x^* + \chi \Leftrightarrow l_x^* = l_x|_{i=0} + \chi$, where $\chi > 0$. There thus exists a threshold value $\bar{\chi}$ such

nominal interest rate to be optimal. Intuitively, this case implies that by raising i to be positive, two layers of welfare gains arise from overcoming R&D overinvestment through the decrease in l_r^y and from enhancing consumption through the increase in l_x. Nonetheless, the cross-R&D-sector effect of reallocating l_r^y to l_r^x yields a welfare cost, since this effect tends to depress growth through the labor shift from a more productive R&D engine to a less productive one. Therefore, the suboptimality of the Friedman rule can be supported by R&D overinvestment if the welfare gains dominate the welfare cost. Notice that this welfare cost is absent in Subsection 4.1.1 since in that case, a higher i always shifts labor from a less productive R&D engine to a more productive one.

Also, the analytical solution for the optimal interest rate i^* is found to exist. Using the first-order condition of U with respect to i, we derive i^* for $\xi_x = 0$ given by

$$i^* = \max\left[\frac{\mu - 1}{\frac{\ln z}{\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)\left(\varphi_y - \varphi_x\left(\frac{\mu-1}{\mu}\right)\right) - 1} - 1, 0\right], \tag{42}$$

and the value of i^* is chosen based on the sign of $\partial U/\partial i|_{i=0}$ in (39), as analyzed in Subsections 4.1.1 and 4.1.2. The above results are summarized in Proposition 3.

Proposition 3. *Suppose that only the CIA constraint on downstream R&D is present. Then the optimal nominal interest rate i^* is given by (42). Furthermore, when $\varphi_x > \varphi_y$ ($\varphi_x < \varphi_y$), R&D overinvestment/underinvestment in the zero-nominal-interest-rate equilibrium is sufficient/necessary (necessary/sufficient) for the Friedman rule to be suboptimal/optimal.*

Proof. Proven in the text. □

As for the comparative statics of i^* when it is positive, the analysis is similar to those in Chu and Cozzi (2014), and the intuition is as follows. First, a higher discount rate ρ implies a worsening of the intertemporal spillover effect, which is a negative externality that leads R&D overinvestment to be more likely to occur. Thus, i^* is increasing in ρ. This model features a (positive) negative surplus-appropriability effect on the productivity in the (downstream) upstream R&D, in the sense that a higher φ_x/φ_y is more likely to cause R&D overinvestment. Thus, i^* is increasing in φ_x/φ_y. Additionally, a larger patent breadth μ strictly increases downstream R&D and thus R&D overinvestment is more likely to occur. At the same time, a larger μ reinforces the double marginalization problem in the upstream R&D sector and thus enlarges the welfare gain of raising i through the increase in the upstream-R&D labor l_r^x even if R&D is underinvested. Thus, i^* is increasing in μ. Finally, a smaller step size z of innovation implies a negative externality that makes R&D overinvestment more possible, so that i^* is decreasing in z.

4.2 CIA Constraint on Upstream R&D

Then, we turn to consider optimal monetary policy in a case where the CIA constraint only on upstream R&D is present. We also normalize the CIA constraint on downstream R&D to unity

that if and only if $\chi > \bar{\chi}$, then $\partial U/\partial i|_{i=0} > 0$. Furthermore, $\bar{\chi}$ is given by $(\mu-1)(\varphi_y - \varphi_x)(1+\rho/\varphi_y + \rho/\varphi_x)/(\varphi_y \mu^2)$, which is decreasing in μ. So the degree of R&D overinvestment is more likely to dominate the threshold value under a large markup.

for simplicity in this case. Consequently, imposing $\xi_y = 0$ and $\xi_x = 1$ into (28)-(30) yields the equilibrium labor allocations such that

$$l_x = \frac{1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}}{1 + \mu(\mu - 1) + \frac{\mu-1}{1+i}}, \tag{43}$$

$$l_r^y = \frac{\mu(\mu-1)\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \mu(\mu - 1) + \frac{\mu-1}{1+i}} - \frac{\rho}{\varphi_y}, \tag{44}$$

$$l_r^x = \frac{\frac{\mu-1}{1+i}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)}{1 + \mu(\mu - 1) + \frac{\mu-1}{1+i}} - \frac{\rho}{\varphi_x}. \tag{45}$$

(43) to (45) show that under $\xi_y = 0$, a higher rate of nominal interest i continues to increase the manufacturing labor l_x and decrease (increase) the labor in the constrained (unconstrained) R&D sector, i.e., the upstream R&D labor l_x^r (the downstream R&D labor l_x^r). In addition, substituting (43) to (45) into (32) and evaluating $\partial U/\partial i$ at $i = 0$ yields

$$\text{sign}\left(\frac{\partial U}{\partial i}|_{i=0}\right) = \text{sign}\left\{1 - \frac{\ln z}{\mu^2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)[\varphi_x + \mu(\mu-1)(\varphi_x - \varphi_y)]\right\}, \tag{46}$$

which again can be either positive or negative. Analogous to the previous section, we analyze the sign of (46) by considering the case of $\varphi_y > \varphi_x$ and of $\varphi_y < \varphi_x$, respectively.

4.2.1 Low Relative Productivity of Upstream R&D

As for the case of $\varphi_y > \varphi_x$, comparing the first-best production labor (33) and the equilibrium production labor (43) at $i = 0$ implies that there is R&D overinvestment in the equilibrium with $i = 0$ (i.e., $l_x|_{i=0} < l_x^*$) since $(1 + \rho/\varphi_x + \rho/\varphi_y) < (\mu^2\rho)/(\varphi_y \ln z)$ holds. In this case, using (46), we obtain

$$\frac{\partial U}{\partial i}|_{i=0} > 1 - \frac{\varphi_y \ln z}{\mu^2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right) > 0. \tag{47}$$

Therefore, for $\varphi_y > \varphi_x$, R&D overinvestment in equilibrium is sufficient for the Friedman rule to be suboptimal, but similar to Subsection 4.1.1, this condition is not necessary. This result implies that the R&D underinvestment may also cause the suboptimality of the Friedman rule, however, it is necessary for the optimality of the Friedman rule.

4.2.2 High Relative Productivity of Upstream R&D

As for the case of $\varphi_y < \varphi_x$, comparing (43) and (33) at $i = 0$ yields $(1 + \rho/\varphi_x + \rho/\varphi_y) > (\mu^2\rho)/(\varphi_x \ln z)$, which means that R&D underinvestment arises in the equilibrium where $i = 0$ (i.e., $l_x|_{i=0} > l_x^*$). Then, we can use (46) to obtain

$$\frac{\partial U}{\partial i}|_{i=0} < 1 - \frac{\varphi_x \ln z}{\mu^2\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right) < 0. \tag{48}$$

Similar to Subsection 4.1.2, R&D underinvestment in equilibrium is a sufficient but not necessary condition for the Friedman rule to be optimal. Hence, R&D overinvestment continues to be necessary for the Friedman rule to be suboptimal, and the suboptimality of the Friedman rule is guaranteed by a sufficiently high degree of R&D overinvestment; otherwise, a low degree of R&D overinvestment may lead the Friedman rule to be optimal.

Notice that the intuition for the relationship between the (sub)optimality of the Friedman rule and R&D overinvestment and underinvestment in Subsections 4.2.1 and 4.2.2 is analogous to the counterpart in Subsection 4.1.1 and 4.1.2, which depends on the interaction between the welfare gains and costs of labor reallocation, and thus it is omitted to conserve space. Moreover, differentiating U with respect to i yields the analytical solution for the optimal nominal interest rate i^* for $\xi_y = 0$ given by

$$i^* = \max\left[\frac{1}{\frac{\ln z}{\rho}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)\left(\varphi_x\left(\mu+\frac{1}{\mu-1}\right)-\varphi_y\mu\right)-\left(\mu+\frac{1}{\mu-1}\right)}-1, 0\right], \tag{49}$$

and the selection of the value for i^* follows the analysis in Subsections 4.2.2 and 4.2.2. We summarize the above results in Proposition 4.

Proposition 4. *Suppose that only the CIA constraint on upstream R&D is present. Then the optimal nominal interest rate i^* is given by (49). Furthermore, when $\varphi_y > \varphi_x$ ($\varphi_y < \varphi_x$), R&D overinvestment/underinvestment in the zero-nominal-interest-rate equilibrium is sufficient/necessary (necessary/sufficient) for the Friedman rule to be suboptimal/optimal.*

Proof. Proven in the text. □

Again, investigating (49) shows that i^*, when it is positive, is increasing in ρ and φ_y/φ_x, but decreasing in z. The intuitions for these results are similar to that in Section 4.1. Nevertheless, in this case, i^* can be increasing or decreasing in μ. The reason that i^* increases in response to a larger μ also follows from the counterpart in Section 4.1. The difference is that the CIA constraint is now imposed on the upstream R&D sector. This makes the double marginalization problem less severe when the level of μ becomes higher, which mitigates the welfare effect of increasing i. And if this welfare effect is significantly attenuated, it is also possible for i^* to decrease in response to a larger μ.

4.3 CIA Constraints on Two Sectors

In this section, we consider that both downstream and upstream R&D sectors are subject to CIA constraints, and the strength of the CIA constraints on the two sectors is equal, namely $\xi_x = \xi_y = \xi$, where we normalize $\xi = 1$ for analytical simplicity in this case as in the previous two subsections. For a complete analysis, we later on allow the strength of the CIA constraints on the two sectors to differ in the numerical exercise. Under this setting, the equilibrium labor allocations are given by

$$l_x = \frac{1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}}{1+\frac{\mu^2-1}{1+i}}, \tag{50}$$

$$l_r^y = \frac{\frac{\mu(\mu-1)}{1+i}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)}{1+\frac{\mu^2-1}{1+i}} - \frac{\rho}{\varphi_y}, \tag{51}$$

$$l_r^x = \frac{\frac{\mu-1}{1+i}\left(1+\frac{\rho}{\varphi_x}+\frac{\rho}{\varphi_y}\right)}{1+\frac{\mu^2-1}{1+i}} - \frac{\rho}{\varphi_x}. \tag{52}$$

Substituting (50)-(52) into (32) and evaluating it at $i = 0$ yields

$$\text{sign}\left(\frac{\partial U}{\partial i}|_{i=0}\right) = \text{sign}\left\{1 - \frac{1}{\mu^2} - \frac{\ln z(\mu-1)}{\mu^4}\left(1+\frac{\rho}{\varphi_y}+\frac{\rho}{\varphi_x}\right)[\mu\varphi_y+\varphi_x]\right\}. \tag{53}$$

In contrast to the previous cases for the CIA parameters imposition, under $\xi_x = \xi_y = 1$, a higher rate of nominal interest i increases the manufacturing labor l_x by decreasing the R&D labor l_r^y and l_r^x in both upstream and downstream sectors. Similarly, we then separate the analysis of the sign in (53) by $\varphi_y > \varphi_x$ and $\varphi_y < \varphi_x$, respectively.

4.3.1 Low Relative Productivity of Upstream R&D

As for the case $\varphi_y > \varphi_x$, comparing l_x^* in (33) and l_x in (50) at $i = 0$, we find that the equilibrium features R&D overinvestment since $(1+\rho/\varphi_x+\rho/\varphi_y) < (\mu^2\rho)/(\varphi_y \ln z)$. Therefore, using (52) implies

$$\frac{\partial U}{\partial i}|_{i=0} > \frac{\mu-1}{\mu^2}\left(1-\frac{\varphi_x}{\varphi_y}\right) > 0. \tag{54}$$

In other words, R&D overinvestment in equilibrium is sufficient for the Friedman rule to be suboptimal, and obviously, this condition is not necessary. Again, as in the previous subsections, R&D underinvestment may also make the Friedman rule suboptimal, and it is the necessary condition for the optimality of the Friedman rule.[11]

4.3.2 High Relative Productivity of Upstream R&D

As for the case $\varphi_y < \varphi_x$, comparing l_x^* in (33) and l_x in (50) at $i = 0$ yields $(1+\rho/\varphi_x+\rho/\varphi_y) < (\mu^2\rho)/(\varphi_x \ln z)$, implying that there is also R&D overinvestment in the equilibrium. Therefore, we obtain

$$\frac{\partial U}{\partial i}|_{i=0} > \frac{\mu(\mu-1)}{\mu^2}\left(1-\frac{\varphi_y}{\varphi_x}\right) > 0. \tag{55}$$

As a result, R&D overinvestment in equilibrium continues to be a sufficient (but not necessary) condition that guarantees the suboptimality of the Friedman rule; R&D underinvestment in this case is necessary for the Friedman rule to be optimal.

The above analysis implies that $\xi_x = \xi_y = 1$, when the zero-nominal-interest-rate equilibrium features R&D overinvestment, by raising i to be positive, the welfare gain from mitigating R&D

[11] Analogously, a large (small) value of markup μ can ensure the (sub)optimality of the Friedman rule under $\varphi_y > \varphi_x$ when $\xi_y = \xi_x = 1$.

overinvestment through the decrease in l_r^y and l_r^x reinforces the welfare gain from reallocating labor through the increase in l_x. By contrast, when the zero-nominal-interest-rate equilibrium features R&D underinvestment, if the welfare gain from reallocating labor through the increase in l_x dominates the welfare cost from exacerbating R&D underinvestment through the decrease in l_r^y and l_r^x, R&D underinvestment would also make the Friedman rule suboptimal; otherwise, the Friedman rule becomes optimal. This result is invariant to the relative productivity between the sectoral R&D.

Additionally, we derive the optimal nominal interest rate i^* for $\xi_x = \xi_y = 1$ by differentiating U with respect to i, which is given by

$$i^* = \max\left[\frac{\mu^2 - 1}{\frac{\ln z}{\rho}\left(1 + \frac{\rho}{\varphi_x} + \frac{\rho}{\varphi_y}\right)(\varphi_x + \varphi_y \mu)\frac{1}{1+\mu} - 1} - 1, 0\right], \tag{56}$$

and the value for i^* is chosen according to the analysis in Subsections 4.3.1 and 4.3.2. Then, we summarize the above results in Proposition 5.

Proposition 5. *Suppose that the CIA constraints on both R&D sectors are equally present. Then the optimal nominal interest rate i^* is given by (56). Furthermore, R&D overinvestment (underinvestment) in the zero-nominal-interest-rate equilibrium is sufficient (necessary) for the Friedman rule to be suboptimal (optimal), regardless of the relative productivity of upstream R&D is low or high.*

Proof. Proven in the text. □

Notice that the comparative statics of i^* in (56) on ρ and z are similar to the previous sections. Moreover, i^* is increasing in μ. This is because now both R&D sectors are subject to an equal CIA constraint, and an increase in μ will increase l_r^x and l_r^y together. Therefore, i^* must increase unambiguously to reduce the R&D labor in the two sectors in response. What is different here is that a higher φ_y/φ_x can either increase or decrease i^*, and it depends on the level of μ. Specifically, if μ is large (small), then raising φ_y/φ_x will decrease (increase) i^*; the double-marginalization problem will intensify (attenuate) the effect of a relatively low level of φ_x on the upstream-R&D labor l_r^x, so R&D underinvestment (overinvestment) is more likely to occur and i^* would response to decline (rise).

5 Numerical Analysis

In this section, we calibrate this two-R&D-sector model to the US economy to quantitatively analyze the growth and welfare effects of monetary policy.

5.1 Calibration

To perform this numerical analysis, the strategy is to assign steady-state values to the following structural parameters $\{\rho, \mu, \xi_x, \xi_y, z, \varphi_x, \varphi_y\}$. We follow Acemoglu and Akcigit (2012) to set the discount rate ρ to 0.05. As for the level of patent breadth, we set $\mu = 1.1$ as the baseline to capture the empirical finding from Jones and Williams (2000) (i.e., $1.05 - 1.4$) and Laitner and Stolyarov (2004) (i.e., $1.09 - 1.1$). As for the strength of the CIA constraints on upstream and downstream R&D activities, we first choose $\{\xi_x = \xi_y = 1\}$ to correspond to the case as in Section 4.3 where the CIA constraints on both sectors are identical, and thereafter choose the groups $\{\xi_x = 0, \xi_y = 1\}$ and $\{\xi_x = 1, \xi_y = 0\}$, which are the cases where a CIA constraint is only present in the downstream (i.e., Section 4.1) and upstream sector (i.e., Section 4.2), respectively.

Furthermore, we use the arrival rate of innovation and the equilibrium growth rate to calibrate the quality step size of innovation. The arrival rate of innovation is set to 6.5%, which is a reasonable value consistent with empirical estimates.[12] According to the Conference Board Total Economy Database, the growth rate of total factor productivity (TFP) for the US economy from 1990-2016 is approximately 0.5%. Thus, we consider $g = 0.005$ as the benchmark value and the quality step size is then calibrated to $z = 1.08$.[13] To calibrate the remaining R&D productivity parameters φ_x and φ_y, in addition to the benchmark growth rate, we use another empirical moment: the ratio of scientists and engineers engaged in R&D over manufacturing labor force.[14] The average ratio is around 5.7%, indicating $(1 - l_x)/l_x = 0.057$ and then the aggregate R&D labor ratio $l_r^y + l_r^x = 1 - l_x$ is around 0.05.[15]

The population growth rate is set to $n = 0.01$ to correspond the data during 1990-2016 according to the Conference Board Total Economy Database. The market-level nominal interest rate i is calibrated by targeting at $\pi = 2.5\%$, which is the average annual inflation rate of the US economy within this period according to Bureau of Labor Statistics. Hence, the benchmark value of the nominal interest rate is given by $i = r + \pi = g + \rho + n + \pi = 0.09$. Table 1 summarizes the values of parameters and variables in this quantitative exercise.[16]

Table 1: Parameter Values in Baseline Calibration

Targets				r	$l_r^y + l_r^x$	g	π
				0.065	0.05	0.005	0.025
Parameters	ξ_y	ξ_x	z	μ	φ_y	φ_x	i
	1	1	1.08	1.1	0.5265	1.3136	0.09

[12] In the literature, studies have considered different values for the arrival rate of innovations. For example, Caballero and Jaffe (2002) estimate a mean rate of creative destruction of roughly 4% and Laitner and Stolyarov (2013) find 3.5% per year of the rate of creative destruction, while Lanjouw (1998) estimates the probability of obsolescence is the range of $7\% - 12\%$. Here, we consider an intermediate value of 6.5% within this range.

[13] The quality step size is calibrated according to $z = \exp(0.005/0.065) \approx 1.08$.

[14] The number of scientists and engineers engaged in R&D for 1990-1997 are from Science and Engineering Indicators 2000 (Appendix Tables 3-25) published by National Science Foundation. The data of manufacturing labor employee is obtained from the Bureau of Labor Statistics.

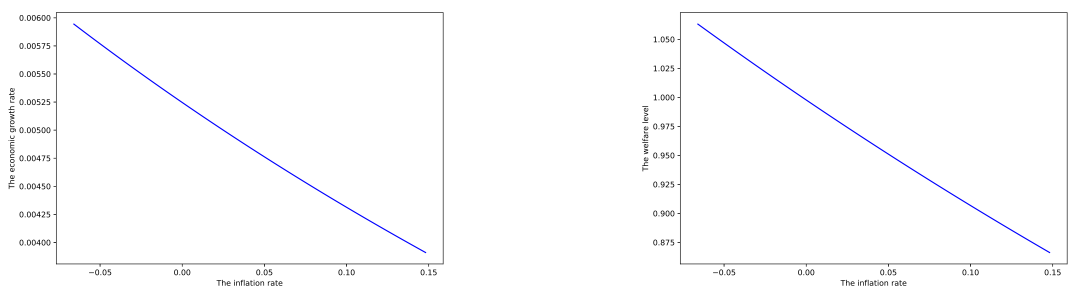
[15] When using the data for a even longer period such as 1980-1997, the average ratio remains almost unchanged, although a slightly increasing trend of R&D labor ratio can be observed. Thus, we leave the robustness check on a larger R&D labor ratio in the sensitivity analysis by taking into account the possibly increasing trend of R&D labor ratio.

[16] When solving for the values of φ_x and φ_y, we restrict to positive values ensuring that the R&D labor allocations of l_r^y and l_r^x are positive.

5.2 Growth and Welfare Implications of Monetary Policy

This subsection evaluates the growth and welfare effects of inflation in the case of an equal CIA constraint on both R&D sectors.

Fig.1a displays that for $\xi_x = \xi_y = 1$, the growth rate of technology is monotonically decreasing in the inflation rate. According to (51) and (52), a higher nominal interest rate i raises the cost of both downstream and upstream R&D due to CIA constraints in these sectors. A negative growth effect emerges since labor is reallocated from R&D to manufacturing. As discussed in Subsection 3.2, a positive growth effect of a higher nominal interest rate may arise given the presence of the cross-R&D-sector effect; R&D labor is shifted from a more cash-constrained sector to the less-constrained one, but this potentially growth-enhancing effect does not appear when the strength of CIA constraints is equal.[17] As a result, a higher nominal interest rate (and inflation rate) leads to a monotonic decrease in the rate of economic growth.



(a) (b)

Fig. 1. (a) Inflation and economic growth ($\xi_x = \xi_y$) (b) Inflation and social welfare ($\xi_x = \xi_y$)

Moreover, the level of steady-state welfare is also decreasing in the inflation rate for $\xi_x = \xi_y = 1$ as shown in Fig.1b. According to (32), the effects of a higher nominal interest rate i on steady-state welfare U operate through the steady-state level of consumption c_0, determined by the production labor l_x, and the growth rate of technology g, determined by the R&D labors in two sectors l_r^x and l_r^y. From (28)-(30), it can be seen that a higher i leads to a rise in l_x and then c_0, indicating a positive welfare effect, whereas it leads to a decline in l_r^x and l_r^y and then g, implying a negative welfare effect. In this case, the latter negative welfare effect always dominates the former positive one. In addition, when expressing it as the usual equivalent variation in consumption flow $\kappa \equiv \exp(\rho \Delta U) - 1$, the welfare gain can be as large as approximately 1% of consumption per annum when moving the equilibrium at $i = 0.2140$ ($\bar{\pi} = 0.15$, the upper bound of inflation rate)[18] to the second-best outcome

[17] The quantitative results indicate that both downstream and upstream R&D labors are decreasing in the nominal interest rate, and, of course, the manufacturing labor is increasing in it in response.

[18] Throughout the quantitative analysis, we restrict our analysis on an empirically realistic case of the inflation rate $\pi \leq 0.15$. According to Bureau of Labor Statistics, the maximum of quarterly and annual inflation rate for US economy are 0.136 and 0.124, respectively, from 1958 (when the indicator is available) to 2016. Thus, to cover these empirical moments, we consider a slightly large value 0.15 as the upper bound of inflation rate. Correspondingly, the upper bound of nominal interest rate is around 0.2140 in the benchmark case by solving the function of $i - g(i) - \rho - n = 0.15$.

(i.e., the equilibrium at $i = 0$).

To see how R&D overinvestment or underinvestment in the zero-nominal-interest-rate equilibrium (i.e., $i = 0$) relates to the optimality or suboptimality of Friedman rule, we compare the socially optimal manufacturing labor l_x^* and the market equilibrium manufacturing labor $l_x|_{i=0}$. The calibrated productivity parameters, $\varphi_y = 0.5265$ and $\varphi_x = 1.3136$, indicate a high relative productivity of upstream R&D. Thus, using (33)-(35), the socially optimal labor allocations are given by $l_x^* = 0.4946$, $l_r^{y*} = 0$, and $l_r^{x*} = 0.5054$, respectively. Evaluating (50) at $i = 0$ yields the equilibrium manufacturing labor of $l_x = 0.9364$, which implies $l_x|_{i=0} > l_x^*$. This shows that relative to the first-best allocation, much less R&D labor is assigned in equilibrium. According to (56), the optimal nominal interest rate is therefore given by $i^* = 0$, which is the lower bound on the nominal interest rate. In this case, the Friedman rule is optimal for which R&D underinvestment is the necessary condition, as shown in Proposition 5, but setting i to its optimal level cannot achieve the first-best labor allocations.

5.3 Sensitivity Analysis

In this subsection, we undertake sensitivity checks to test the robustness of our quantitative results. Specifically, we first examine the cases of only one CIA constraint is present in either sector to correspond to our analytical analysis in Subsection 4.1 and Subsection 4.2, respectively.[19] We also consider a less severe CIA constraint equally imposed in both sectors. Thereafter, sensitivity exercises on markup and R&D labor ratio are conducted, respectively. Before closing the analysis, we show that the inverted-U growth effect of inflation rate, as indicated in Proposition 2, is quantitatively available when the markup and CIA parameters are simultaneously adjusted.

Table 2: Sensitivity analysis

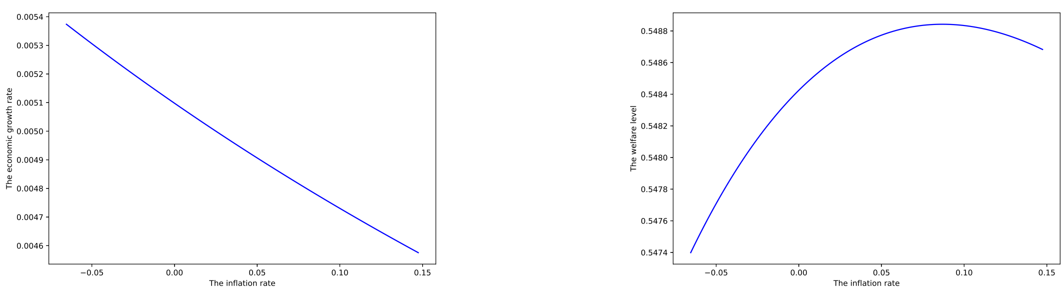
Parameters	ξ_y	ξ_x	μ	φ_y	φ_x	$l_r^y + l_r^x$
	1	0	1.1	0.7268	1.0404	0.07‡
	-	-	-	1.0348	0.7296	0.07
	0	1	1.1	0.5265	1.3136	0.05
	0.5	0.5	1.1	0.5265	1.3136	0.05
	1	1	1.2	0.5265	1.3136	0.05
	1	1	1.26	0.5265	1.3136	0.05
	1	1	1.14	0.7311	0.5857	0.095§
	1	0.01	1.69	0.5265	1.3136	0.05

Note: ‡ and § represent the sensitivity checks in which the parameters φ_y and φ_x are re-calibrated to match the R&D intensity as indicated. Additionally, in the case of ‡, there are two groups of productivity parameters generated.

[19] We have also considered the case that CIA constraint is exclusively present on consumption. Unsurprisingly, when labor is elastically supplied, the presence of CIA constraint on consumption leads a higher nominal interest rate to reduce economic growth rate and social welfare through a conventional effect of decreasing the aggregate labor supply as well as labor supplies in both R&D sectors.

5.3.1 CIA parameters

First, in the case of a CIA constraint on downstream R&D sector only (i.e., $\xi_x = 0$, $\xi_y = 1$), the benchmark value of aggregate R&D labor ratio is adjusted to a slightly larger one, such as 7%.[20] The two sectoral productivity parameters are then re-calibrated and yielded, $\varphi_y = 0.7268 < 1.0404 = \varphi_x$ and $\varphi_y = 1.0348 > 0.7296 = \varphi_x$. Fig.2a and Fig.3a display the growth effects of inflation for $\varphi_y < \varphi_x$ and $\varphi_y > \varphi_x$. When the CIA constraint is only present in the downstream R&D sector, raising the nominal interest rate increases the R&D cost for only the downstream sector and hence shifts its R&D labor to its upstream and manufacturing counterparts. However, the growth-enhancing effect of inflation brought about by the increase in upstream R&D labor is unable to dominate the growth-retarding effect because only a small fraction of l_r^y that is shifted out flows to upstream R&D sector. Therefore, the growth rate of technology is decreasing in the inflation rate as the benchmark case, regardless of high or low relative productivity of downstream R&D. It is useful to note that the magnitudes of growth-decreasing effect of inflation differ slightly because when only downstream R&D suffers from cash constraint, a higher nominal interest rate would strengthen the growth-retarding effect when its own productivity is higher than the upstream R&D.



(a): $\varphi_y < \varphi_x$ (b): $\varphi_y < \varphi_x$

Fig. 2. (a) Inflation and economic growth ($\xi_x < \xi_y$) (b) Inflation and social welfare ($\xi_x < \xi_y$)

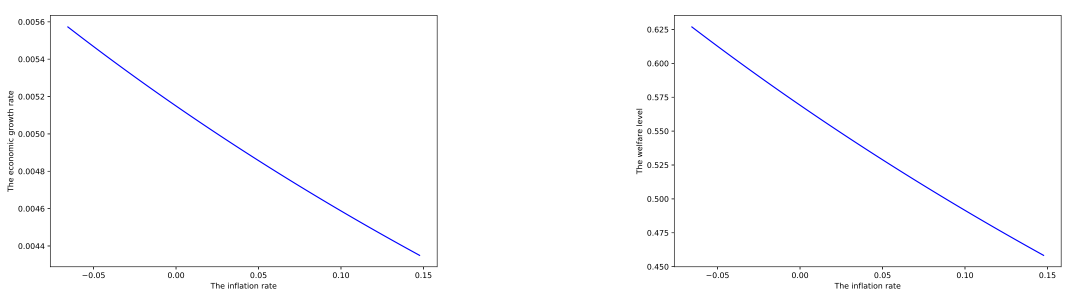
By contrast, the welfare effects of inflation differ in the cases of $\varphi_y < \varphi_x$ and $\varphi_y > \varphi_x$. When low relative productivity of downstream R&D occurs in the economy, the level of social welfare is now a concave function of inflation rate, displayed in Fig.2b, unlike the baseline case. As mentioned above, the welfare effect of inflation relies on its impacts on l_x and g. When only the downstream sector suffers from the CIA constraint, the positive welfare effect from an increase in l_x initially dominates the negative effect because of a decrease in g. However, as the welfare-increasing effect becomes weaker, it is eventually dominated by the welfare-decreasing effect when the inflation rate is above the threshold value of $\pi = 8.68\%$ ($i = 0.1516$). In this case, raising the nominal interest rate from $i = 0$ to the second-best equilibrium ($i = 0.1516$) yields a marginal welfare gain of $\kappa = 0.0072\%$.

To examine the Proposition 3, we show that the first-best labor allocations are $l_x^* = 0.6245$,

[20] We recalibrate the productivity parameters by using an alternative R&D labor ratio because in the case of $\xi_x = 0$ and $\xi_y = 1$, using the benchmark values would lead the downstream R&D labor l_r^y to be negative when $\pi \geq 0.028$, which strictly restricts our analysis.

$l_r^{y*} = 0$, and $l_r^{x*} = 0.3755$, given that the downstream R&D sector is less productive herein (i.e., $\varphi_y = 0.7268 < 1.0404 = \varphi_x$). The equilibrium manufacturing labor at $i = 0$ is $l_x = 0.9230$ according to (36), implying $l_x|_{i=0} > l_x^*$. Therefore, again, R&D underinvestment occurs in the zero-nominal-interest-rate equilibrium. Nevertheless, according to Proposition 3, the Friedman rule can be suboptimal under a sufficiently large markup. Since the markup $\mu = 1.1$ in this case is larger than the threshold value of markup ($\bar{\mu} = 1.0947$) as given in footnote 8, a positive welfare-maximizing inflation rate is present.

In the alternative case of a high relative productivity of downstream R&D, $\varphi_y = 1.0348 > \varphi_x = 0.7296$, social welfare becomes monotonically decreasing in inflation, although R&D underinvestment is still the case, $l_x|_{i=0} = 0.9230 > l_x^* = 0.6278$. It is coherent with analytical findings in Proposition 3 that R&D underinvestment in the case of high relative productivity of downstream R&D suffices to ensure the optimality of Friedman rule. Thus, the largest welfare difference between the equilibrium at $i = 0.2140$ (corresponding to the upper bound of $\pi = 0.15$), and the second-best equilibrium (the equilibrium at $i = 0$ again) is $\kappa = 0.8531\%$. This result is new to the implication of monetary policy regarding the relationship with the underinvestment versus overinvestment of R&D and complements Chu and Cozzi (2014) who consider the welfare effect of monetary policy in a Schumpeterian economy with a single R&D sector.

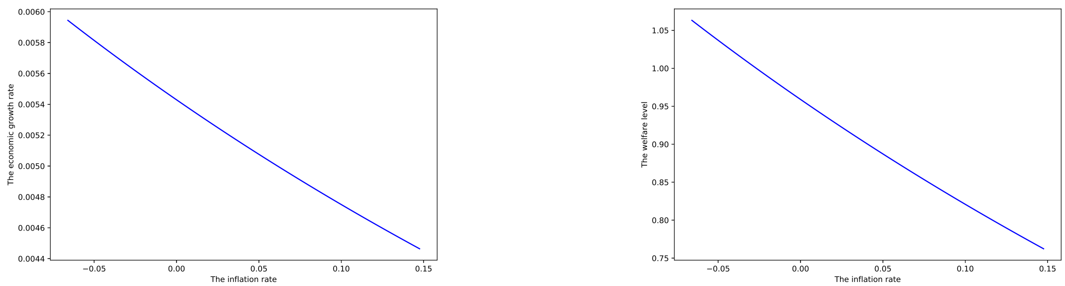


(a): $\varphi_y > \varphi_x$ (b): $\varphi_y > \varphi_x$

Fig. 3. (a) Inflation and economic growth ($\xi_x < \xi_y$) (b) Inflation and social welfare ($\xi_x < \xi_y$)

Second, Fig.4a illustrates the growth effect of inflation when only the upstream R&D activities are subject to a CIA constraint (i.e., $\xi_x = 1$, $\xi_y = 0$). Other parameters remain unchanged as those in the baseline case. According to Proposition 2, given a low relative sectoral productivity in downstream, an increase in the inflation rate is not able to yield a sufficiently positive growth rate arising from the less constrained R&D sector (the upstream sector herein) to dominate the negative growth rate arising from the more constrained R&D sector (the downstream sector herein). Because of this weak cross-sector effect, an increase in the inflation rate results in a lower rate of technology growth. In addition, a comparison between Fig.1a and Fig.4a indicates that the absence of the CIA constraint in the downstream R&D sector mitigates the growth-retarding effect of a higher inflation and therefore leads to a smoother relationship between the growth rate and the inflation rate.

The welfare effect of inflation for the case with $\xi_x = 1$ and $\xi_y = 0$ is illustrated in Fig.4b. It is obvious that an increase in the inflation rate causes welfare losses in this case. When the nominal



(a) (b)

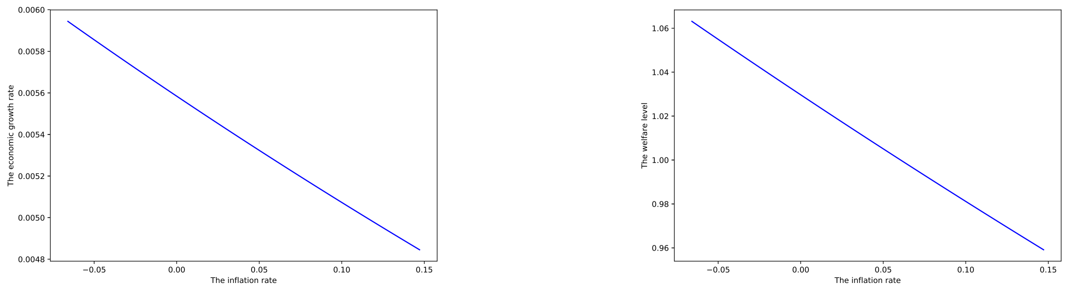
Fig. 4. (a) Inflation and economic growth ($\xi_x > \xi_y$) (b) Inflation and social welfare ($\xi_x > \xi_y$)

interest rate i increases, the positive welfare effect from an increase in the steady-state level of consumption c_0 through raising the labor employment of manufacturing l_x is completely dominated by the negative welfare effect from the growth-decreasing effect. The welfare gain between the equilibrium at $i = 0.2145$[21] and the welfare-maximizing outcome (the equilibrium at $i = 0$) in this case is $\kappa = 1.531\%$, implying a more severe welfare loss as compared to the baseline case. It arises because given the low relative downstream productivity, when it no longer suffers from cash constraint (i.e., $\xi_y = 0$), a higher i always increases the downstream R&D labor whereas the upstream R&D labor, also the high relative productive, decreases more than the benchmark case. As a consequence, the welfare loss rises because of the even more inefficient labor reallocation. Furthermore, we obtain $l_x|_{i=0} > l_x^*$, which, again, implies R&D underinvestment. Hence, the quantitative result is consistent with the Proposition 4 in the sense that R&D underinvestment is a sufficient condition for the optimality of Friedman rule.

Finally, we perform a sensitivity analysis by considering the two equal CIA constraint parameters at a less severe level, i.e., $\xi_x = \xi_y = 0.5$. The growth and welfare effects of increasing the inflation rate are similar to those in the case of $\xi_x = \xi_y = 1$, except for the magnitudes. Specifically, when the extents of the cash constraint are simultaneously lowered, an increase in the inflation rate generates a more moderate effect on the labor reallocation among the R&D and manufacturing sectors. As a result, an increase in the nominal interest rate from 0 to 0.2140 leads to a smaller decline in the growth rate in Fig.5a and the welfare level in Fig.5b, respectively, as compared to Fig.1a and Fig.1b. The largest welfare gain between the equilibrium at $i = 0.2149$[22] and the second-best outcome (the equilibrium at $i = 0$) in this case is $\kappa = 0.5281\%$, a less welfare loss as compared the benchmark. It is understandable that since the second-best outcome is the same with the case of $\xi_x = \xi_y = 1$, a less severe CIA constraint alleviates welfare losses due to an increase in the nominal interest rate.

[21]The upper bound of inflation rate of 0.15 corresponds to $i = 0.2145$ in this case. It is obtained by solving the function indicated in footnote 18.

[22]The upper bound of inflation rate of 0.15 corresponds to $i = 0.2149$ in this case.

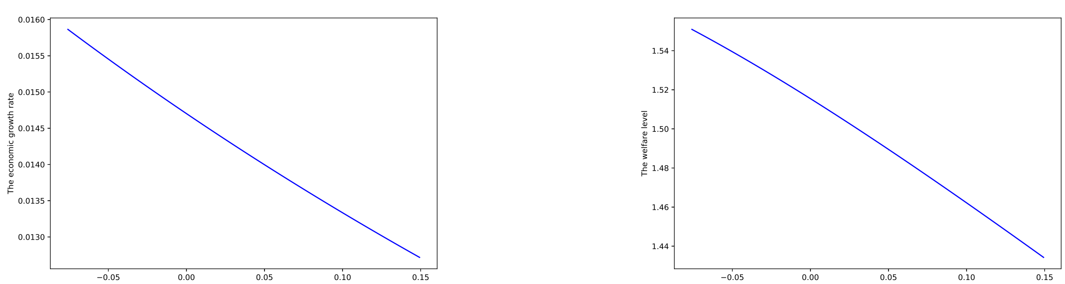


(a) (b)

Fig. 5. (a) Inflation and economic growth ($\xi_x = \xi_y$) (b) Inflation and social welfare ($\xi_x = \xi_y$)

5.3.2 Markup

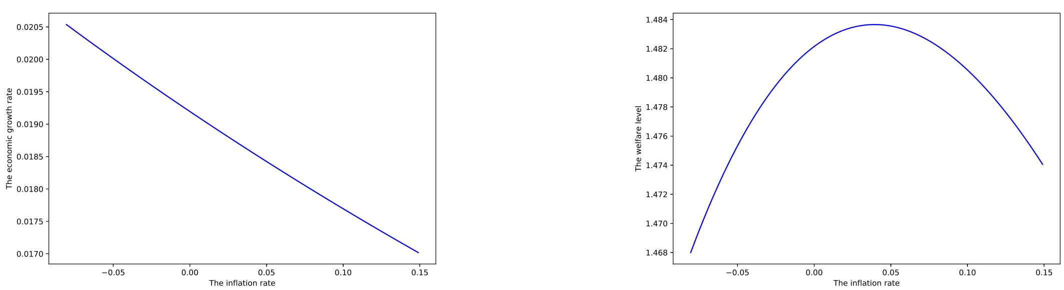
To assess the role of markup in the growth and welfare effect of inflation, we first consider a larger markup, i.e., $\mu = 1.2$, remaining other parameters unchanged as in the baseline case. The growth-retarding and welfare-decreasing effect of inflation are still found in Fig.6a and Fig.6b, respectively. A comparison to Fig.1a reveals that raising the markup increases the level of growth rate. It works through promoting downstream R&D labor allocations and thus spurring the growth rate of technology, although a smaller magnitude of growth-retarding effect arises as well by decreasing upstream R&D labor employment. Fig.1b and Fig.6b together shows that a higher markup increases the welfare level through raising the level of growth rate, whereas it does not alter the relationship between inflation and social welfare. Thus, the welfare-maximizing equilibrium is still at $i = 0$, and the welfare losses between it and the equilibrium $i = 0.2228$, referring to the upper bound of $\pi = 0.15$ in this case, is $\kappa = 0.5872\%$.



(a) (b)

Fig. 6. (a) Inflation and economic growth ($\mu = 1.2$) (b) Inflation and social welfare ($\mu = 1.2$)

Interestingly, when continuing to raise the markup to a even larger value such that $\mu = 1.26$, we find that although the growth rate of aggregate technology is still monotonically decreasing in inflation rate, social welfare becomes a concave function of inflation, with a welfare-maximizing inflation rate $\pi = 3.94\%$ ($i = 0.1180$). It is consistent with our analytical results shown in 4.3.2 in the sense that the welfare-maximization nominal interest rate i^* is increasing in μ, and R&D underinvestment could also lead to the suboptimality of Friedman rule in the support of a higher markup.[23] As shown in 4.3.2, a higher i^* is always required to counteract the increase in both R&D labor because of a larger markup. Consequently, with a sufficiently large markup, the optimal nominal interest rate turns positive. The quantitative results are reported in Fig.7a and Fig.7b. The level of growth rate and social welfare certainly rise as higher markups strengthen the effect of inflation on labor allocations, bringing about the possibility that the increase in l_x due to a higher i, contributing to a higher c_0, can dominate the welfare-decreasing effect for lower growth rates. Thus, the welfare gain by increasing $i = 0$ to the welfare-maximizing $i = 0.1180$ is $\kappa = 0.078\%$.



(a) (b)

Fig. 7. (a) Inflation and economic growth ($\mu = 1.26$) (b) Inflation and social welfare ($\mu = 1.26$)

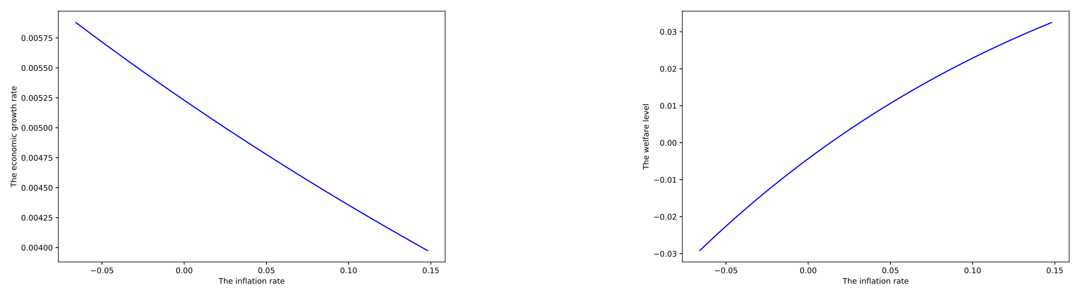
5.3.3 R&D labor ratio

As shown in the above quantitative results, the ratio of R&D labor over manufacturing labor matters in the growth and welfare effects of inflation. Here, we use a larger value of R&D labor ratio, i.e. 9.5%, to examine the change of results in comparison to the benchmark case. We are especially interested in the case of R&D overinvestment, and find that when simultaneously adjusting to a larger markup (i.e., 1.14), R&D overinvestment emerges. The productivity parameters in both R&D sectors are re-calibrated to match this new R&D labor ratio, $\varphi_y = 0.7311 > 0.5857 = \varphi_x$.[24] The growth and welfare effect of inflation are reported in Fig.8a and Fig.8b, respectively. The growth effect of inflation under a higher R&D intensity is similar with the benchmark case, which can be seen by comparing Fig.8a with Fig.1a. Nevertheless, the Friedman rule is no longer optimal and it is new that R&D overinvestment arises. Given the high relative productivity of downstream R&D, the social optimal labor allocations are $l_x^* = 0.8886$, $l_r^{y*} = 0.1114$, and $l_r^{x*} = 0$, and the manufacturing labor allocation at $i = 0$ is $l_x|_{i=0} = 0.8878 > l_x^*$, indicating that R&D overinvestment at zero-nominal-interest-rate equilibrium. According to 4.3.1, in the case of low relative productivity of

[23]In this case, R&D is underinvested such that $l_x|_{i=0} = 0.7137 > l_x^* = 0.4946$.

[24]When recalibrating the model, another productivity group is also generated, $\varphi_y = 0.5537 < 0.7880 = \varphi_x$. In this case, the growth and welfare effects of inflation are similar with the benchmark case, without providing any new and interesting insights. Thus, we do not report the results here, but they are available upon request.

upstream R&D, R&D overinvestment at equilibrium $i = 0$ is sufficient for the Friedman rule to be suboptimal. The welfare is an inverted-U function of inflation with the the welfare-maximizing inflation rate 0.625 larger than the upper bound of inflation rate 0.15. Thus the optimal strategy, in terms of welfare-maximization, is raising the nominal interest rate from $i = 0$ to $i = 0.2140$, corresponding to $\pi = 0.15$, and the associated welfare gain is $\kappa = 0.3106\%$.



(a) (b)

Fig. 8. (a) Inflation and economic growth (b) Inflation and social welfare.

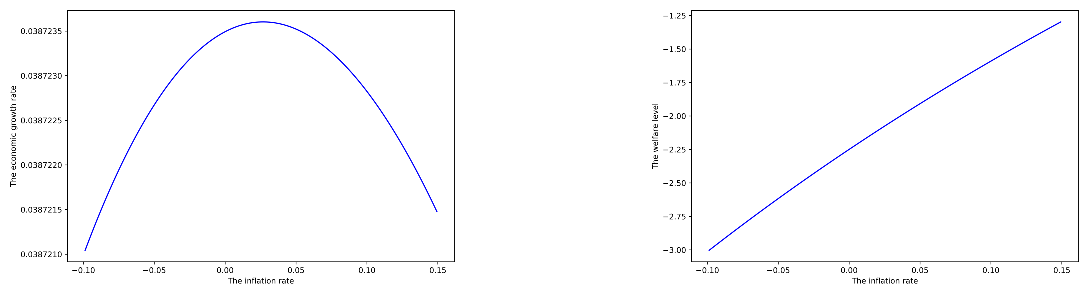
5.3.4 Inverted-U effect of inflation on growth

Lastly, when raising the markup to $\mu = 1.69$,[25] and simultaneously adjusting the two CIA constraint parameters to $\xi_x = 0.01$ and $\xi_y = 1$,[26] we find that the growth effect of aggregate technology is an inverted-U function of inflation. Fig.9a and Fig.9b show the growth and welfare effect of inflation, respectively. The intuition for an inverted-U effect of inflation on growth can be explained as follow. According to Proposition 2, given the support of a large markup as well as a less CIA constraint in upstream R&D sector, an increase in the nominal interest rate i is able to yield a sufficiently positive growth rate of technology, through increasing its R&D labor, that can dominate the negative effect stemming from the more constrained sector (downstream sector) where R&D labor is decreasing in i. Thus, monetary policy is growth-enhancing, before i rises to the threshold value above which the above-mentioned domination is reversed because the growth-enhancing effect from upstream sector becomes weaker while the growth-retarding effect from the downstream sector continues to strengthen. Overall, the inverted-U effect of inflation on the growth rate of aggregate technology emerges. In addition, the growth-maximization inflation rate is found to be around 2.68%, which is line with the empirical estimates of Ghosh and Phillips (1998) (i.e., 2.5%) and López-Villavicencio and Mignon (2011) (i.e., 2.7%).

[25] Comin (2004) shows that the markups charged by innovators should be well above the generally estimated average markup in the economy. Thus it is not unrealistic to consider a higher markup of $\mu = 1.69$.

[26] It is useful to note that according to Proposition 2, either setting any one CIA constraint to zero or choosing any parameter groups with equal CIA constraints will lead to a monotonic effect of i on g. Moreover, given the low relative productivity of downstream R&D productivity, a positive growth effect of inflation is available when the upstream R&D sector features less cash constrained than its downstream counterpart. Intuitively, the growth-enhancing effect of inflation is possible when the labor reallocation acts in the way of from the less productive sector to the more productive one.

Furthermore, the level of welfare is a concave function of inflation, indicating that the Friedman rule is suboptimal. However, the interior solution for the welfare-maximizing inflation rate is tremendously large and exceeds the upper bound $\pi = 0.15$. Therefore, the monetary authority is able to lead the economy to the second-best equilibrium at $i = 0.2488$ (i.e., $\pi = 0.15$), and the largest welfare gains is the gap between $i = 0$ and $i = 0.2488$ yielding $\kappa = 8.1950\%$.



(a) (b)

Fig. 9. (a) Inflation and economic growth (b) Inflation and social welfare.

6 Conclusion

In this study, we analyze the growth and welfare effects of monetary policy in a Schumpeterian growth model in which both vertical sectors engage in R&D activities and CIA constraints are present in R&D investments. We find that a higher nominal interest rate reallocates resources from the more-cash-constrained R&D sector to the less one. In addition to the usual growth-retarding effect of CIA constraints by increasing the cost of R&D, this cross-sector reallocation on R&D labor generates a growth-enhancing effect, which would lead economic growth to have an inverted-U shape on the nominal interest rate. Moreover, we examine the necessary and sufficient conditions for the (sub)optimality of the Friedman rule by relating the underinvestment and overinvestment of R&D in the decentralized equilibrium. We find that this relationship is crucially affected by the presence of CIA constraints, the relative productivity between upstream R&D and downstream R&D, and the strength of markup, given that these factors determine the interaction between the welfare effects brought by the reallocation on different types of labor, and thus the optimal design of monetary policy. Finally, by calibrating this two-R&D-sector model to the US economy, our quantitative results suggest that the growth-maximizing and welfare-maximizing rates of nominal interest are generally zero, i.e., they are given by the Friedman rule. Also, it is found that the welfare effect of monetary policy is quantitatively significant by altering the nominal interest rate from the steady-state value to the optimal value. Therefore, our analysis serves to provide an analytical and quantitative justification that reveals the importance of taking into account R&D investments in an economy with vertical industries when considering the role of monetary policy in economic growth and social welfare.

Appendix A

Proof of Proposition 1

Suppose that a time path of $[i_t]_{t=0}^{\infty}$ is stationary. First, define a transformed variable $\Psi_{y,t} \equiv N_t e_t / v_{y,t}$, and its law of motion is given by

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = n + \frac{\dot{e}_t}{e_t} - \frac{\dot{v}_{y,t}}{v_{y,t}}. \tag{A.1}$$

Similarly, define another transformed variable $\Psi_{x,t} \equiv N_t e_t / v_{x,t}$, and its law of motion is given by

$$\frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}} = n + \frac{\dot{e}_t}{e_t} - \frac{\dot{v}_{x,t}}{v_{x,t}}. \tag{A.2}$$

To derive the law of motion for $v_{y,t}$, substituting (13) into (20) yields

$$\frac{\dot{v}_{y,t}}{v_{y,t}} = r_t + \varphi_y l_{r,t}^y - \left(\frac{\mu - 1}{\mu}\right) \Psi_{y,t}. \tag{A.3}$$

Likewise, combining (17) and (21) yields the law of motion for $v_{x,t}$ such that

$$\frac{\dot{v}_{x,t}}{v_{x,t}} = r_t + \varphi_x l_{r,t}^x - \left(\frac{\mu - 1}{\mu^2}\right) \Psi_{x,t}. \tag{A.4}$$

Plugging (A.3) and (A.4) into (A.1) and (A.2) respectively, along with the Euler equation (6), yields

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \left(\frac{\mu - 1}{\mu}\right) \Psi_{y,t} - \varphi_y l_{r,t}^y - \rho, \tag{A.5}$$

and

$$\frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}} = \left(\frac{\mu - 1}{\mu^2}\right) \Psi_{x,t} - \varphi_x l_{r,t}^x - \rho. \tag{A.6}$$

Also, using the zero-expected-profit condition of upstream R&D (24) yields an expression of $l_{x,t}$ such that

$$l_{x,t} = \left(\frac{1 + \xi_y i}{\mu^2 \varphi_y}\right) \Psi_{y,t}, \tag{A.7}$$

and using (25) to relate $l_{x,t}$ to $\Psi_{x,t}$ yields

$$l_{x,t} = \left(\frac{1 + \xi_x i}{\mu^2 \varphi_x}\right) \Psi_{x,t}. \tag{A.8}$$

Now, $\Psi_{x,t}$ can be expressed as a function of $\Psi_{y,t}$ such that

$$\Psi_{x,t} = \left[\frac{(1 + \xi_y i)\varphi_x}{(1 + \xi_x i)\varphi_y}\right] \Psi_{y,t}. \tag{A.9}$$

Therefore, it is obvious that

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \frac{\dot{\Psi}_{x,t}}{\Psi_{x,t}}. \tag{A.10}$$

Using (A.10) together with (A.5), (A.6) and (A.9), we derive a relationship between $l_{r,t}^y$ and $l_{r,t}^x$ such that

$$l_{r,t}^x = \left(\frac{\varphi_y}{\varphi_x}\right) l_{r,t}^y + \left[\frac{(\mu-1)(1+\xi_y i)\varphi_x}{\mu^2\varphi_y(1+\xi_x i)} - \frac{\mu-1}{\mu}\right] \frac{\Psi_{y,t}}{\varphi_x}. \tag{A.11}$$

Finally, to derive the relationship between $\Psi_{y,t}$ and $l_{r,t}^y$, using the labor-market-clearing condition $l_{x,t} + l_{r,t}^y + l_{r,t}^x = 1$ and substituting (A.7) and (A.11) into it yields

$$\left(1 + \frac{\varphi_y}{\varphi_x}\right) l_{r,t}^y = 1 - \left[\frac{1+\xi_y i}{\varphi_y} + \frac{(\mu-1)(1+\xi_y i)}{\varphi_y \mu^2 (1+\xi_x i)} - \frac{\mu-1}{\mu\varphi_x}\right] \Psi_{y,t}. \tag{A.12}$$

Substituting (A.12) into (A.6) yields an autonomous dynamical equation for $\Psi_{y,t}$ such that

$$\frac{\dot{\Psi}_{y,t}}{\Psi_{y,t}} = \frac{\varphi_x}{\varphi_y + \varphi_x}\left[\frac{1+\xi_y i}{\mu^2}\left(1 + \frac{\mu-1}{1+\xi_x i}\right) + \frac{\mu-1}{\mu}\right]\Psi_{y,t} - \left(\frac{\varphi_x\varphi_y}{\varphi_y+\varphi_x} + \rho\right). \tag{A.13}$$

Given that $\Psi_{y,t}$ is a control variable and the coefficient on $\Psi_{y,t}$ is positive in (A.13), the dynamics of $\Psi_{y,t}$ is characterized by saddle-point stability in this model such that $\Psi_{y,t}$ jumps immediately to its interior steady-state value given by

$$\Psi_y = \frac{\mu^2[\rho(1+\varphi_y/\varphi_x) + \varphi_y]}{(1+\xi_y i)\left[1 + (\mu-1)/(1+\xi_x i)\right] + \mu(\mu-1)}. \tag{A.14}$$

Equations (A.7) and (A.14) imply that when $\Psi_{y,t}$ is stationary, $l_{x,t}$ and $l_{r,t}^y$ must be stationary, which in turn implies that $l_{r,t}^x$ is stationary as well according to (A.11). Hence, l_t must also be stationary.

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