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Abstract

This paper considers a model of competition in prices and price complexity levels, which accommodates concave and convex confusion technologies. In symmetric equilibrium, the probability of using high complexity increases in the number of firms. In the limit, as the number of competitors goes to infinity, firms use high complexity almost surely but the impact on consumer welfare depends on the characteristics of the confusion technology. Specifically, industry profits converge to the highest level with concave confusion technologies and to the lowest level when with convex confusion technologies. An improvement in consumer sophistication increases consumer welfare but does not reduce market complexity.

Keywords: price complexity, confusion technology, oligopoly markets

JEL classification: D03, D43, L13

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1 Introduction

In many retail markets, firms commonly use technical or involved language in their price disclosures, partitioned prices, or differentiated price formats, which make it harder for consumers to identify the best offer. Price complexity is prevalent in markets for financial and banking products and energy retail. Although, it may be regarded as a benign by-product of intrinsic product complexity, recent research has raised concerns about its strategic use to soften price competition. In nearly homogeneous product markets, price complexity has been associated with consumer ignorance, inertia, lack of sophistication, dispersion in prices and price formats, positive mark-ups, and unintended responses to increased competitive pressure, such as lower transparency for consumers.

Discussing the challenges of consumer financial regulation, Campbell (2016) points out that "financial ignorance is pervasive and unsurprising given the complexity of modern financial products". The 2015 UK Competition and Market Authority investigation of the retail banking market found that price complexity may prevent consumers from receiving good value and identifying the best deals. A 2007 EC study of EU mortgage credit markets and the 2011 report by the UK Independent Commission on Banking echo these concerns.[1] There is mounting evidence that firms use price complexity to exploit consumer bounded rationality.

Strategic price complexity draws on consumers' behavioral biases and on differences in their sophistication levels. To assess and compare complex prices, consumers need to spend more time and/or effort, and some are more likely to make mistakes or suboptimal choices due to confusion. Strategic firms may deliberately increase price complexity to create consumer confusion and soften competition in nearly homogeneous product markets.[2] Research in psychology and economics has shown that consumers' decisions are sensitive to the way in which different alternatives are presented and to the difficulty of the choice environment.

A recent theoretical literature explores the role of price and choice complexity in homogeneous product markets, see Spiegler (2016) for a synthesis. These models send a consistent message that complexity and equilibrium obfuscation increase in response to intensified competition, weakening its positive effects, and identify the possibility that an increase in the number of firms harms consumer welfare.

[1] For instance, Woodward and Hall (2012) show that, in US mortgage markets, deals with the arrangement fee rolled into the interest rate are better than those that quote these fees separately. For a recent discussion of related empirical work, see Campbell (2016).

[2] Both experimental economics and marketing research show that more fragmented multi-part tariffs can limit price comparability, create confusion, and lead to suboptimal consumer choices. See Kalaycı and Potters (2011), Kalaycı (2015), and the review by Greenleaf et al. (2015).

This paper develops a richer modelling framework to accommodate more general confusion technologies, and address some key questions. Is the degree of complexity a good indicator of market performance? Does an increase in consumer sophistication increase market transparency? Under what conditions does an increase in the number of competitors harm consumer welfare? What type of tests can be used to assess the impact of competition on welfare? Robust answers to these questions are crucial to the policy relevance of extant findings.

This analysis focuses on homogeneous product oligopoly markets where firms compete in price and price complexity. Like in Carlin (2009), complexity makes it more difficult to assess prices and compare offers, and prevents some consumers from identifying the best deal. Given firms' complexity choices, some consumers are 'informed' or 'experts', while others are 'confused'. The experts are able to assess all prices and purchase the lowest-price product. The confused buy from a randomly selected firm (or make random errors). The shares of experts and confused consumers depend on firms' price complexity choices. A unilateral incremental increase in a firm's price complexity increases the share of confused as it raises the difficulty of assessing price offers.

A distinctive feature of our extended model is that it allows a firm's price complexity to have a non-trivial impact on the difficulty of assessing a competitor's offer and can accommodate a wider gamut of consumer behaviors. The increase in the share of confused triggered by an increase in a firm's price complexity may be either reduced or magnified by the complexity of rivals' price offers. Specifically, this analysis considers confusion technologies which are either convex or concave in firms' aggregate price complexity choice. One interpretation of this model is that an increase in complexity raises the cost of gathering market-wide information and dissuades more consumers from assessing offers.[3] Then, convexity or concavity in confusion can be related to convexity or concavity of the cost of gathering market-wide price information.

As an illustration, consider a firm which increases the complexity of its price offer by including more technical terms or using more sophisticated jargon. In a situation where there is learning by doing, the effect of an incremental increase in complexity would be smaller if rivals' aggregate complexity were higher. Consumers may get better at deciphering technical language the more they are exposed to it, and so there would be some reciprocal cancellation of firms' complexity levels. On the other hand, in a situation where consumers are more

[3] Alternatively, it may cause more random consumer mistakes, or it may make more consumers uphold their default-options.

likely to be demoralized or make mistakes as the informational load increases, the effect of an incremental increase in complexity may be larger when rivals' aggregate complexity is higher.

In this setting, price complexity underlies consumer heterogeneity and in symmetric equilibrium firms choose prices randomly from a closed interval, according to a continuous distribution function.[4] Moreover, there is a positive relationship between prices and complexity levels: when a firm sets a relatively low price, it benefits from more transparency, while when it sets a relatively high price, it benefits from more confusion. Despite product homogeneity, prices are strictly above marginal cost and expected profits are strictly positive.

A duopoly analysis introduces some preliminary results, showing that an increase in the convexity of the confusion technology (or a reduction in its concavity) leads to an increase in transparency, a decrease in the lowest price associated with low complexity, a (weak) increase in the lowest price associated with high complexity, a decrease in average prices and expected industry profits, and an increase in expected consumer surplus. Intuitively, a deviation to low complexity and a lower price is relatively more profitable when the confusion technology is less concave or more convex, as a larger decrease in rivals' confusion effectiveness triggers a larger increase in the share of experts.

An increase in consumer sophistication decreases average prices and expected profits at both firm and industry level. However, it has no impact on the frequency of using low complexity. From a consumer policy perspective, programmes that raise consumer awareness or understanding of the market (e.g., financial literacy programmes) boost consumer surplus but do not reduce the overall complexity of the market. This suggests that market transparency is not a good indicator of the effectiveness of consumer awareness initiatives or, more generally, of how well a market performs.

The analysis of oligopoly markets indicates that an increase in the number of firms leads to an increase in each firm's probability of using high complexity and so to lower market transparency. In relatively concentrated markets, expected industry profit is not monotonic in the number of firms. In highly fragmented markets, if the confusion technology is concave, expected share of confused and industry profits are bounded away from zero and converge to the highest possible level as the number of competitors goes to infinity. In contrast, if the confusion technology is convex, expected number of confused consumers and industry profits converge to zero. These findings suggest that standard competition policy may backfire in relatively concentrated markets and be undesirable in fragmented markets where the confusion

[4] This framework is related to the broad literature on price dispersion; see Baye, Morgan, and Scholten (2006) for a survey.

technology is concave in the complexity level.

Drawing on the interpretation that concavity in confusion relates to the concavity of the information gathering cost, our findings suggest that consumer learning by doing may not improve market performance in highly competitive environments. Indeed it may coexist with poor outcomes for consumers, as it is compatible with highly effective confusion technologies. On the other hand, highly fragmented markets with convex confusion technologies may perform well. More generally, when competitive pressure is high, although firms use the high complexity almost surely, the impact of complexity on market performance and an effective policy approach depend crucially on the confusion technology characteristics.

This analysis is related to those in Piccione and Spiegler (2012) and Chioveanu and Zhou (2013), when price complexity is the main source of confusion, and especially to Carlin (2009).[5] The latter analyzes the impact of complexity on market transparency in a model where firms choose complexity levels from a closed interval. Carlin assumes that the confusion technology is linear in the aggregate price complexity. Our analysis complements his work by considering second order effects in confusion and showing that these have policy implications.

Using the results in Dasgupta and Maskin (1986), Carlin (2009) proves the existence of a symmetric mixed strategy equilibrium, where firms only use the highest and lowest complexity levels. Drawing on this result, our analysis also revisits the setting with no second order effects to characterize firms' pricing strategies and profits in the unique symmetric equilibrium. Our findings indicate that the convergence properties of expected industry profits in a model with no second order effects are a limiting case, qualitatively consistent with the concave confusion technology case.

Piccione and Spiegler (2012) explore a general comparability structure in a duopoly model where consumers enter the market with a default bias, and identify a necessary and sufficient condition for firms to earn max-min profits in equilibrium. In their setting, consumers are initially attached to one firm and can only make price comparisons when formats are compatible. The model presented here can also be interpreted as one where default-biased consumers are randomly shared across firms and make market-wide price comparisons with a probability which depends on the overall market complexity. Chioveanu and Zhou (2013) analyze an oligopoly framework where firms can choose one of two price presentation formats. We show that an increase in the number of firms induces firms to rely more on frame complexity and

[5] More broadly, this article contributes to the literature on behavioral industrial economics, in particular, to work that explores the interaction between strategic firms and boundedly rational consumers. See Spiegler (2011), Grubb (2015), and Heidhues and Koszegi (2018) for related discussions and surveys of recent work. Armstrong (2008) provides a thorough discussion of policy issues.

may harm consumer welfare. In these analyses, the emphasis is on the price comparability structure and how it is affected by price presentation.[6]

A different branch of literature explores strategic obfuscation in sequential search models with fully rational consumers. In Ellison and Wolitzky (2012), consumers' search costs are strictly convex in shopping time. They argue that strict convexity of the disutility of time is a realistic assumption because 'disutility would be convex in a standard time allocation model with decreasing returns to leisure'. But, they also note that the consideration of cost concavity might be a more suitable if there is consumer learning by doing.[7]

Next section formulates the model, introduces a taxonomy of confusion technologies, and discusses its microfoundations. Section 3 presents some preliminary results, while section 4 illustrates the equilibrium derivation and basic findings in a duopoly model. Section 5 characterizes the unique symmetric mixed-strategy equilibrium in the general oligopoly model. Section 6 focuses on comparative statics and convergence results, and discusses consumer protection and competition policy implications. All proofs missing from the text are relegated to the appendices.

2 Model

Consider a market for a homogeneous product with $n \geq 2$ identical firms. Marginal costs of production are constant and normalized to zero. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay at most $v = 1$.[8] Firms simultaneously and noncooperatively choose prices, p_i for $i \in N = \{1, 2, ...n\}$, and the complexity of their prices. There are two possible complexity levels and each firm can choose just one of them, $k_i \in \{\underline{k}, \bar{k}\}$, where $\bar{k} > \underline{k}$. The profile of firms' price complexity choices is $\mathbf{k}$, an n-vector whose i-th component is k_i. Complexity is related to the way in which firms convey price information and can be adjusted at the same time as prices, which is reflected by the timing.[9]

A reduced-form model which may accommodate different interpretations is introduced first where complexity increases the difficulty of assessing firms' price offers and is a source of

[6] Closer to Carlin (2009), the current analysis considers a two stage process where complexity focuses consumers' attention and may dissuade them completely from making price comparisons.

[7] See also Wilson (2010) and Taylor (2017) for related analyses of obfuscation.

[8] The normalizations of marginal costs and reservation values are made for expositional ease and without loss of generality.

[9] Price complexity in this setting is a form of 'price framing'. Alternatively, firms may engage in 'product framing' - a form of spurious product differentiation, e.g., involving decisions on product size or packaging - which cannot be adjusted as frequently as prices and is better captured by a sequential move framework. See the related discussion in Chioveanu and Zhou (2013).

consumer confusion.[10] Based on firms complexity choices, there are two types of consumers, experts (or informed) and confused (or uninformed). The experts purchase the cheapest product that provides positive surplus, while the confused buy at random, so that they choose a particular firm with probability $1/n$. The model is also consistent with a default-bias interpretation, whereby each consumer is initially assigned to one firm and the firms have equal bases of consumers ex-ante. In this case, confused consumers uphold their default option, while the experts select the cheapest alternative.

The firms' price complexity levels determine the share of experts in the market, which is assumed to be a symmetric function. Due to symmetry, for a given $\mathbf{k}$, the fractions of experts and confused are solely determined by $n-m$, the count of $\bar{k}$ in $\mathbf{k}$ (or alternatively, by m, the count of $\underline{k}$ in $\mathbf{k}$) where $m \in \{0, 1, ...n\}$. Hence, the share of experts can be written as a function,

$$\mu : \mathbb{N} \rightarrow [0, 1] \ .$$

This analysis considers markets, like those for financial or banking products, where there are always some expert consumers, although it allows all consumers to be confused if all firms use $\bar{k}$. Specifically, the share of experts when $n-m$ firms use $\bar{k}$ is $\mu(n-m) \geq 0$, with strict inequality for $m \geq 1$. Denote $\mu(n) \equiv \mu_{\text{min}}$ and, for simplicity, let $\mu(0) = 1$.[11]

By making it more difficult to assess firms' offers, an increase in one firm's price complexity increases the fraction of confused in the market, so

$$1 - \mu(n-m) < 1 - \mu(n-m+1) \ . \tag{1}$$

To capture the impact of a rival's price complexity on the difficulty of evaluating a firm's offer, let

$$\mu(n-m-1) - \mu(n-m) = \frac{\mu(n-m) - \mu(n-m+1)}{\beta - 1} \ , \tag{2}$$

for all m and $\beta > 1$. This formulation parametrizes the rate of change of the incremental share of experts, which is summarized by $(\beta - 1)$, and provides a tractable way to explore more general confusion technologies. If $\beta > 2$ ($\beta < 2$), a unilateral incremental increase in complexity is more (less) effective when the rivals' complexity is higher (lower), i.e. firms' complexity levels reinforce (offset) each other in creating confusion. Specifically, depending on the value of $\beta \neq 2$, (2) provides a taxonomy of confusion technologies.

[10] The main analysis adopts a microfoundation of consumer confusion rooted in the cost of processing information, which is presented at the end of this section.

[11] In a market where $\mu(0) \in (0, 1)$, possibly due to the intrinsic characteristics of the product, firms cannot completely eliminate price complexity. So, even when all firms use the simplest price structure available, some consumers get confused. However, this does not change the results qualitatively.

Case 1 ***Concavity.*** *When $1 < \beta < 2$, the confusion technology $(1-\mu)$ is concave in the number of complex prices in the market. Formally, (1) and (2) imply that*

$$(1-\mu(n-m)) - (1-\mu(n-m-1)) > (1-\mu(n-m+1)) - (1-\mu(n-m)) \ .$$

so that a firm's incremental increase in price complexity is more effective in creating confusion when rivals' price complexity is lower (in the sense that fewer competitors employ $\bar{k}$).

Case 2 ***Convexity***. *When $\beta > 2$, the confusion technology $(1-\mu)$ is convex in the number of complex prices in the market. Formally, (1) and (2) imply that*

$$(1-\mu(n-m)) - (1-\mu(n-m-1)) < (1-\mu(n-m+1)) - (1-\mu(n-m)) \ .$$

so that a firm's incremental increase in price complexity is more effective when rivals' price complexity is higher.

The LHS differences in Cases 1 and 2 capture the increase in the share of confused triggered by a unilateral increase in complexity when $n-m-1$ other firms use $\bar{k}$ (and m rivals use $\underline{k}$), while the RHS differences capture the corresponding increase when $n-m$ other firms use $\bar{k}$ (and m rivals use $\underline{k}$).

Expression (2) implies that

$$1-\mu(1) = \frac{\mu(n-m-1)-\mu(n-m)}{(\beta-1)^{n-1-m}} = \frac{\mu(n-1)-\mu_{\min}}{(\beta-1)^{n-1}} \ .$$

Then it follows that

$$1-\mu(n-m) = \frac{(1-\mu_{\min})[(\beta-1)^{n-m}-1]}{(\beta-1)^{n}-1} \quad \text{for} \quad \beta \neq 2 \ . \tag{3}$$

The share of confused for given $n-m$ can be expressed in terms of the primitives of the model: the market structure (n), the curvature (β), and the degree of consumer sophistication as measured by the highest possible share of confused consumers $(1-\mu_{\min})$.[12]

Case 3 ***No Second Order Effects.*** *When $\beta = 2$, the price complexity of one firm does not affect the effectiveness of an increase in a rival's complexity. Formally, (1) and (2) imply that*

$$(1-\mu(n-m)) - (1-\mu(n-m-1)) = (1-\mu(n-m+1)) - (1-\mu(n-m)) \ .$$

This case is closely related to Carlin (2009) and (3) is replaced by

$$1-\mu(n-m) = \frac{(1-\mu_{\min})(n-m)}{n} \quad \text{for} \quad \beta = 2 \ . \tag{4}$$

The main analysis focuses on $\beta \neq 2$, but a characterization of the equilibrium for $\beta = 2$ is included at the end of section 5 and the policy discussion in section 6 covers all cases.

[12] It can shown that if $\mu(0) \in (0,1)$, the share of confused when $n-m$ firms exactly use $\bar{k}$ is equal to the expression in (3) plus a constant equal to $(1-\mu(0))$.

As the share of experts is assumed to be a symmetric function of firms' price complexity levels, in the setting above it is expressed as a function of the count of $\bar{k}$ in $\mathbf{k}$. Alternatively, the share of experts could be written as a function of the profile of firms' price complexity choices, $\varphi(\mathbf{k})$. Then, Case 1 corresponds to a situation where there is substitutability in confusion, that is, the confusion technology $(1-\varphi)$ satisfies strictly decreasing differences, while Case 2 corresponds to a situation where there is complementarity in confusion, that is, the confusion technology $(1-\varphi)$ satisfies strictly increasing differences.[13] A more detailed description of this alternative formulation is relegated to the online appendix.

2.1 A Microfoundation of Consumer Confusion

The reduced form model can be related to an environment where consumers have limited time availability and incur a time cost to assess firms' offers, so that they use a two-step approach. Depending on their time availability and the overall market complexity, they first decide if to collect information on prices or not. The consumers can directly observe the overall market complexity: for instance, they may receive information amalgamated by the media or social networks or transmitted via word-of-mouth. However, they do not observe individual firms' complexity levels unless they decide to collect information. Complexity focuses consumers' attention to this particular aspect and may dissuade them from gathering price information. When a consumer decides to gather information, they buy the lowest-price product. When a consumer decides not to collect price information, they select a product randomly or uphold their default-option.

Consumers have time availability t, which is a random variable, and incur a time cost to assess firms' offers and identify the best deal. Let t be distributed on an interval $[t_L, t_H]$ according to a continuous probability distribution function $G(t)$, with $G'(t) > 0$.[14] Let $\gamma(n-m)$ be consumers' cost of gathering information where $n-m$ is the count of $\bar{k}$ in $\mathbf{k}$ and measures market-wide complexity. Suppose that it is more costly for consumers to assess price information when more firms use $\bar{k}$, that is, $\gamma(n-m) < \gamma(n-m+1)$.[15] Then, if $t > \gamma(n-m)$, consumers gather information on all price offers and choose the best deal. This happens with probability $1 - G(\gamma(n-m))$. If $t < \gamma(n-m)$, consumers choose a random product or uphold their default option. This happens with probability $G(\gamma(n-m))$.

[13] Here substitutability and complementarity are not strategic (as they do not relate to firms' profits). See Vives, 1999, Ch. 2 for general discussion.

[14] Alternatively, consumers may be heterogeneous and $G(t)$ gives the distribution of time availability in the population.

[15] Even if consumers restrict their consideration sets, in a more complex environment it would be harder to decide which firms to include.

The resulting share of confused is

$$1 - \mu(n-m) = G(\gamma(n-m)) \ .$$

As $G' > 0$, $G(\tau(n-m)) < G(\tau(n-m+1))$ then $1-\mu(n-m) < 1-\mu(n-m+1)$, as in the reduced-form model. Moreover, (2) requires that

$$(G \circ \gamma)(n-m) - (G \circ \gamma)(n-m-1) = \frac{(G \circ \gamma)(n-m+1) - (G \circ \gamma)(n-m)}{\beta - 1} \ . \qquad (5)$$

In the examples below, consumers' time availability is a draw from a uniform distribution.

Example 1 *Let $G \sim U[a,b]$ for $b > a \geq 0$ so that $G(t) = (t-a)/\ (b-a)$, $(1-\mu(n-m)) = (\gamma(n-m)-a)\ /(b-a)$, and $\gamma(n-m) = \{c\,[(\beta-1)^{n-m}-1]\,/\ (\beta-2)\} + a$. It can be checked that (5) holds. For $\beta \in (1,2)$ and $\beta > 2$, $\gamma(n-m)$ identifies a concave and, respectively, convex cost sequence.*

Using (2) and (5), consistency requires that in this example

$$c = \frac{(1-\mu_{\min})(b-a)(\beta-2)}{(\beta-1)^n - 1} \ .$$

Example 2 *Let $G \sim U[a,b]$ for $b > a \geq 0$ so that $G(t) = (t-a)/\ (b-a)$ and $\gamma(n-m) = c(n-m)+a$. It can be checked that (5) implies that $\beta = 2$.*

Like in the previous example, consistency with the reduced form model requires that

$$c = \frac{(1-\mu_{\min})(b-a)}{n} \ .$$

In Example 1, concavity of the confusion technology (Case 1) captures learning-by-doing in the market. In contrast, convexity of the confusion technology (Case 2) captures an environment where there is information processing overload: consumers are increasingly likely to give up as aggregate complexity increases.

Our reduced form model is closely related to the frameworks in Piccione and Spiegler (2012) and Chioveanu and Zhou (2013). However, these papers analyze the impact of price complexity (and, more generally, price framing) on the comparability structure. In contrast, the microfoundation presented above focuses on markets where consumers use a two-stage decision making process and where strategic price complexity directs consumer attention on this particular feature of the market. Here, complexity makes gains from comparisons or switching less obvious and may discourage information gathering or reinforce inertia.

Below alternative conceptual foundations for Cases 1 and 2 are discussed.

Complexity as a Source of Errors. Suppose that the presence of complex prices increases the probability that consumers make a random choice error. Consider a market where exactly

$n-m$ firms are using the highest complexity ($\bar{k}$), with $m \leq n-1$. Suppose that the probability that consumers make a random error (which may be due to upholding a default-option) because firm i uses $\bar{k}$ is given by $\rho_{n-m} \in (0,1)$ and is identical but independent of the probability that they get confused because firm j uses $\bar{k}$.[16] Then, the overall probability that consumers get confused when facing $n-m$ complex prices is $1-(1-\rho_{n-m})^{n-m}$, which implies that $\mu(n-m) = (1-\rho_{n-m})^{n-m}$. Using (3), consistency requires

$$1-(1-\rho_{n-m})^{n-m} = \frac{(1-\mu_{\min})[(\beta-1)^{n-m}-1]}{(\beta-1)^{n}-1} \,. \tag{6}$$

Example 3 ***Concavity of Confusion Technology.*** *Suppose that $\rho_{n-m} = \rho$ for all m. Then $\mu(n-m) = (1-\rho)^{n-m}$ satisfies (1) and the confusion technology is concave, i.e. Case 1 applies.*

In this case, the effectiveness of confusion (as given by ρ) is constant, but the pool of consumers that a firm can confuse by increasing its price complexity from $\underline{k}$ to $\bar{k}$ is decreasing in the number of competitors which use complex prices. For example, when only one rival uses $\bar{k}$, the 'target' group of a firm is $(1-\rho)$, whereas when two rivals use $\bar{k}$, the 'target' group shrinks to $(1-\rho)^2$. Intuitively, the confusion technology is concave also when ρ_{n-m} is decreasing in $(n-m)$ as then, when more rivals use complex prices, a firm not only faces a smaller target group but also is less effective.

The discussion above shows that a necessary condition for the confusion technology to be convex is $\rho_{n-m} > \rho_{n-(m+1)}$, i.e., the probability that consumers make a random error when any firm i uses $\bar{k}$ must increase in the number of complex prices. This may be the case, for instance, if consumers are more likely to make an error when the choice environment is more difficult. The following numerical example illustrates this case.

Example 4 ***Convexity of Confusion Technology.*** *Let $n = 4$, $\beta = 3$, and $(1-\mu_{\min}) = 0.5$. Then, using (6), $\rho_1 = 0.0125$ ($\mu(1) = 0.9875$), $\rho_2 = 0.0250$ ($\mu(2) = 0.95$), $\rho_3 = 0.0573$ ($\mu(3) = 0.8375$), and $\rho_4 = 0.1591$. It is then easy to check that Case 2 applies.*

3 Preliminary Findings

Given competitors' price and price complexity choices $(\mathbf{p}_{-i}, \mathbf{k}_{-i})$, firm i's ex post profit can be written as

$$\pi_i(p_i, k_i, \mathbf{p}_{-i}, \mathbf{k}_{-i}) = p_i \cdot \left[\frac{(1-\mu(n-m))}{n} + q_i(p_i, \mathbf{p}_{-i})\mu(n-m)\right] \,,$$

[16] This interpretation can be related to an irrational version of the quantal response equilibrium (QRE), where players make random errors when choosing a strategy, even when they know which is the best deal, and where each firm is equally likely to be selected.

where firm i's share of informed consumers, $q_i(p_i, \mathbf{p}_{-i})$, is given below.

$$q_i(p_i, \mathbf{p}_{-i}) = \begin{cases} 1 & \text{if } p_i < \min\{p_j, 1\} \text{ for } \forall\, j \in N,\, j \neq i \;; \\ \dfrac{1}{|H|} & \text{if } p_i = p_j \leq 1,\, \forall\, j \in H \subseteq N, \text{ and } p_l > p_i,\, \forall\, l \in N \setminus H,\, j, l \neq i \;; \\ 0 & \text{if } p_i > \min\{p_j, 1\} \text{ for } j \in N,\, j \neq i \;. \end{cases}$$

The first term in square brackets in π_i is firm i's share of confused consumers, while the second term is firm i's share of experts. All experts buy from firm i if its price is strictly lower than rivals' prices. If firm i and other firms tie at the lowest price, then they share equally the experts. If at least one firm $j \neq i$ offers a price p_j lower than p_i, then firm i does not sell to experts.

Although confused consumers are unable to accurately compare prices, they do not pay more than their valuation. If firms charge prices above $v = 1$, the consumers would realize at checkout (or after purchase) and decline the purchase (or return the product free of cost). Therefore, in this setting, firms have no incentives to charge prices in excess of consumers' willingness to pay. Moreover, confused consumers do not infer prices from price complexity levels. So, even if high complexity were consistently associated with higher prices, consumers would not understand this relationship. If consumers participate infrequently in the market, they may not have a chance to learn about this relationship. For instance, some consumers only take few mortgages in their lifetime, or change their gas and electricity providers infrequently.

Lemma 1 *There is no equilibrium where all firms use pure price complexity strategies.*

Proof. (a) Suppose all firms choose $\underline{k}$ for sure (that is, there are $\mu(0) = 1$ experts). Then, firms would compete á la Bertrand and make zero profit. But, if a firm unilaterally deviates to $\bar{k}$ and a positive price (no greater than one), it makes strictly positive profit. A contradiction.

(b) Suppose exactly $n - m$ firms use $\bar{k}$ (for $m \geq 1$). (a) implies that $m < n$. Then there are $\mu(n - m) < 1$ experts and $1 - \mu(n - m)$ confused consumers, and the results in Varian (1980) apply. The unique candidate equilibrium dictates mixed strategy pricing according to a c.d.f. on $[p_0, 1]$ and each firm's expected profit is $(1 - \mu(n - m))/n = p_0\,[1 - (1 - \mu(n - m))(n - 1)/n]$. But, if one of the $n - m$ firms choosing $\bar{k}$ deviates to $(p_0, \underline{k})$ it makes profit $p_0\,[1 - \;(1 - \mu(n - m - 1))(n - 1)/n] > p_0\,[1 - \;(1 - \mu(n - m))(n - 1)/n]$.

(c) Suppose all firms choose $\bar{k}$ for sure (that is, there are $\mu(n) = \mu_{\text{min}}$ experts). If $\mu_{\text{min}} > 0$, the argument in (b) applies unchanged. If $\mu_{\text{min}} = 0$, then in the unique candidate equilibrium $p_i = 1$ and $\pi_i = 1/n$ for all i. But then if a firm unilaterally deviates to $\underline{k}$ and price

$p_i = 1 - \varepsilon$, it obtains profit $(1-\varepsilon)\ [(1-\mu(n-1))/n + \mu(n-1)] > 1/n$ for $\varepsilon < \mu(n-1)/[1/(n-1) + \mu(n-1)]$. ■

Lemma 1 implies that in any candidate equilibrium at least one firm randomizes its price complexity. Therefore, with positive probability firms face both expert and confused consumers. The conflict between the incentives to fully exploit confused consumers and to vigorously compete for the experts rules out equilibria involving pure price strategies. The proof of the following result is standard and therefore omitted.[17]

Lemma 2 *There is no equilibrium where all firms use pure pricing strategies.*

Lemmas 1 and 2 show that in any equilibrium there must be dispersion in both price complexities and prices. A firm's strategy space is $[0,1] \times \{\underline{k}, \bar{k}\}$. Denote by $\xi_i \equiv \xi_i(p_i, k_i)$ firm i's mixed strategy over price and price complexity. This analysis focuses on symmetric mixed strategies, where $F(p)$ is the marginal c.d.f. of firms' random prices, defined on an interval $S \subseteq [0,1]$, and $\lambda(p)$ and $1-\lambda(p)$ are the probabilities that price $p \in S$ is associated with complexity level $\underline{k}$ and $\bar{k}$, respectively. Then, the overall probability of using $\underline{k}$ is $\int_{p \in S} \lambda(p) dF(p) \in (0,1)$.

The following result presents properties of firms' pricing strategies.

Lemma 3 *In symmetric equilibrium, (i) the support of the pricing distribution (S) is a connected interval; (ii) if* $\mu_{\min} > 0$ *the pricing c.d.f. is continuous everywhere, while if* $\mu_{\min} = 0$*, it is continuous everywhere except possibly at the upper bound of S; and (iii)* $\inf S = p_0 > 0$ *and* $\sup S = 1$.

Next section builds on these findings to characterize the symmetric Nash equilibrium. Section 4 illustrates the approach in a simple duopoly framework and presents some preliminary comparative statics results. Section 5 analyses a general oligopoly model, while section 6 discusses competition and consumer protection policy implications, by combining analytical results and numerical simulations.

4 Duopoly Analysis

Consider a duopoly market and suppose firm $j \neq i$ follows the mixed strategy ξ_j presented in section 3. At a given price p_j, firm j uses the lowest complexity $\underline{k}$ with probability $\lambda(p_j)$ and the highest complexity with probability $1 - \lambda(p_j)$. Firm j's price is a random draw from its price c.d.f. Then, firm i's profit at price $p \geq p_0 = \min S > 0$ and complexity $\underline{k}$ is given by

$$\begin{aligned} \pi_i\left(p, \underline{k}\right) &= p \int_p^1 \left[\lambda(p_j)\mu(0) + (1-\lambda(p_j))\mu(1)\right] dF(p_j) + \\ & \frac{p}{2} \int_{p_0}^1 \left[\lambda(p_j)(1-\mu(0)) + (1-\lambda(p_j))(1-\mu(1))\right] dF(p_j)\ . \end{aligned} \tag{7}$$

[17] See, for instance, the analyses in Varian (1980) and Rosenthal (1980).

For a given realization of rival's price p_j, the first square bracket gives the expected share of experts. If firm j also uses $\underline{k}$, which happens with probability $\lambda(p_j)$, the share of experts is $\mu(0)$; if, instead, it uses $\bar{k}$, which happens with probability $1-\lambda(p_j)$, the share of experts is $\mu(1)$. Similarly, for a given p_j, the second square bracket gives the expected share of confused consumers. As firm j's price p_j is a random draw, these expected shares are integrated over the relevant realizations of p_j. Firm i serves experts whenever the rival's price is higher ($p_j > p$), whereas it serves half of the confused consumers for all p_j's.

Firm i's profit at price p and complexity $\bar{k}$ is

$$\begin{aligned}\pi_i\left(p,\bar{k}\right) &= p\int_p^1 \left[\lambda(p_j)\mu(1)+(1-\lambda(p_j))\mu(2)\right]dF(p_j)+ \\ &\quad \frac{p}{2}\int_{p_0}^1 \left[\lambda(p_j)(1-\mu(1))+(1-\lambda(p_j))(1-\mu(2))\right]dF(p_j)\ . \end{aligned} \tag{8}$$

The terms in square brackets present the expected shares of experts and, respectively, confused consumers for a given realization of rival's price p_j. However, in this case, if firm j chooses the lowest complexity $\underline{k}$ - which happens with probability $\lambda(p_j)$ - there are $\mu(1)$ experts and $(1-\mu(1))$ confused. If firm j chooses the high complexity $\bar{k}$ - which happens with probability $1-\lambda(p_j)$ - then there are $\mu(2)=\mu_{\min}$ experts and $(1-\mu(2))=(1-\mu_{\min})$ confused. Firm j's price p_j is a random draw from its pricing c.d.f. The share of experts is integrated over all the price realizations where firm i serves these consumers (i.e., for all $p_j > p$). Firm i serves half of the confused consumers for all p_j's.

Substituting $\mu(0)=1$, and using (3), (7) and (8), the incremental profitability of an increase in complexity is

$$\pi_i\left(p,\bar{k}\right)-\pi_i\left(p,\underline{k}\right)=\frac{p(1-\mu(1))}{2}\left(-\int_p^1 \Lambda(p_j)dF(p_j)+\int_{p_0}^p \Lambda(p_j)dF(p_j)\right)\ , \tag{9}$$

where $\Lambda(p_j)\equiv[1+(1-\lambda(p_j))(\beta-2))]>0$. Evaluating (9) at $p=p_0$ and $p=1$ gives, respectively,

$$\frac{p_0(1-\mu(1))}{2}\left(-\int_{p_0}^1 \Lambda(p_j)dF(p_j)\right)<0 \text{ and } \frac{(1-\mu(1))}{2}\left(\int_{p_0}^1 \Lambda(p_j)dF(p_j)\right)>0\ .$$

(9) is strictly increasing in p. The first term in brackets (which is negative) decreases as p increases (the integration range is smaller), whereas the second term in brackets (which is positive) increases as p increases (the integration range is wider). To maximize its expected profit, firm i chooses

$$k(p)=\begin{cases}\underline{k} \text{ if } p<\hat{p}\ , \\ \bar{k} \text{ if } p>\hat{p}\ , \\ k,\ \forall k\in\{\underline{k},\bar{k}\} \text{ if } p=\hat{p}\ ,\end{cases} \tag{10}$$

where the threshold price $\hat{p}$ follows from equating (9) to zero. So, in symmetric equilibrium, prices below a cut-off level $\hat{p} > p_0$ are associated with complexity level $\underline{k}$, while price above $\hat{p}$, are associated with $\bar{k}$, and each firm assigns probability $\lambda = \int_{p_0}^{1} \lambda(p)dF(p) = F(\hat{p}) \in (0,1)$ to complexity level $\underline{k}$ and $1-\lambda$ to complexity level $\bar{k}$. A formal proof of this result is presented in the next section for the general oligopoly model.

Suppose now that firm $j \neq i$ uses the price complexity strategy identified above and that $F(p)$ satisfies Lemma 3. Then, firm i's expected profits at $p = p_0$ and when $p \to \hat{p}$ are

$$\pi_i\left(p_0, \underline{k}\right) = p_0 \left[\lambda + \frac{(1-\lambda)(1+\mu(1))}{2}\right] \quad \text{and} \quad \lim_{p\to\hat{p}} \pi_i\left(p, \underline{k}\right) = \frac{\hat{p}}{2}(1-\lambda)(1+\mu(1)) \ ,$$

whereas firm i's expected profits at $p = \hat{p}$ and $p = 1$ are

$$\pi_i\left(\hat{p}, \bar{k}\right) = \frac{\hat{p}}{2}\left[1 - \lambda\mu(1) + (1-\lambda)\mu_{\min}\right] \quad \text{and} \quad \pi_i\left(1, \bar{k}\right) = \frac{1}{2}\left[1 - \lambda\mu(1) - (1-\lambda)\mu_{\min}\right] \ .$$

The expected profits for an arbitrary price p are presented in the appendix. Constant profit conditions imply that in duopoly equilibrium the probability of using $\bar{k}$ and expected individual profit are given by

$$1 - \lambda = \frac{1}{\beta} \in (0,1) \quad \text{and} \quad \pi = \frac{\left(1-\mu_{\min}\right)(2\beta - 1)}{2\beta^2} \ , \tag{11}$$

while the boundary prices p_0 and $\hat{p}$ are

$$p_0 = \frac{(1-\mu_{\min})(2\beta-1)}{2\beta^2 - (1-\mu_{\min})} \in (0,1) \quad \text{and} \quad \hat{p} = \frac{(1-\mu_{\min})(2\beta-1)}{2\beta - (1-\mu_{\min})} \in (0,1] \ . \tag{12}$$

Note that $\hat{p} = 1$ iff $\mu_{\min} = 0$.

Finally, $F(p)$ is implicitly determined by $\pi(\underline{k}, p) = \pi$ and $\pi(\bar{k}, p) = \pi$, and presented in the appendix. For $\mu_{\min} = 0$, the price distribution has a mass point at the upper bound of its support as $(1 - F(1)) = 1/\beta$.

Proposition 1 *In the unique symmetric duopoly equilibrium, firms choose prices randomly according to the c.d.f. $F(p)$ presented in (A1). Prices on $[p_0, \hat{p})$ are associated with the low price complexity $(\underline{k})$, while prices on $[\hat{p}, 1]$ are associated with the high price complexity $\left(\bar{k}\right)$, with p_0 and $\hat{p}$ defined in (12). Each firm chooses $\underline{k}$ with probability $\lambda = F(\hat{p})$ and $\bar{k}$ with probability $1-\lambda$. The probability λ is presented in (11), together with a firm's expected equilibrium profit, π.*

For $\mu_{\min} > 0$, symmetric equilibrium price c.d.f. is continuous everywhere. In contrast, for $\mu_{\min} = 0$, $\hat{p} = 1$ and the equilibrium price distribution is continuous for all $p < 1$, but has an atom at the upper bound as $lim_{p\nearrow 1} F\left(p\right) = (\beta - 1)/\beta < F(1) = 1$. There is dispersion in both prices and price complexity levels, a positive relationship between prices and complexity, and firms' strategies and market outcomes depend on both β and $\mu_{\min}$.

β is related to the curvature of the confusion technology. When $\beta \in (1,2)$, an increase in β translates into a reduction in the concavity of the confusion technology: a firm's attempt to confuse consumers is thwarted to a lesser extent by rival's complexity choices. When $\beta > 2$, an increase in β corresponds to an increase in the convexity of the confusion technology, and so a firm's attempt to confuse is magnified to a higher extent by competitor's complexity choice. $\mu_{\min}$ is the share of expert consumers when both firms use the high price complexity ($\bar{k}$) and measures consumer sophistication, with a higher value corresponding to higher rationality or sophistication.

Figure 1 illustrates symmetric equilibrium price c.d.f.s for given $\mu_{\min}$ and two values of β. Solid lines correspond to prices associated with the low complexity $\underline{k}$, while dashed lines correspond to $\bar{k}$.

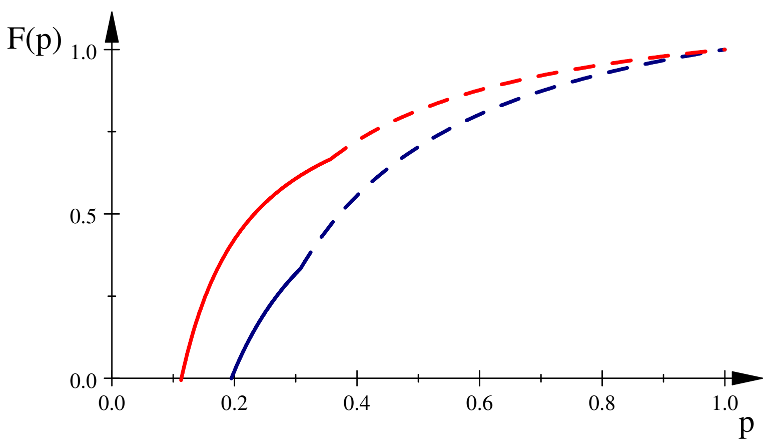

Figure 1: Symmetric Equilibrium Pricing Distributions when $\mu_{\min} = .6$. In bue for $\beta = 3/2$, where $p_0 = 0.195$, $\hat{p} = 0.307$, and $\pi = 0.178$. In red for $\beta = 3$, where $p_0 = 0.113$, $\hat{p} = 0.357$ and $\pi = 0.111$.

Corollary 1 *In the symmetric duopoly equilibrium, the probability of using the low price complexity (λ) and expected share of confused increase in β, the lower bound of the pricing c.d.f. (p_0) decreases in β, the cut-off price ($\hat{p}$) increases in β for $\mu_{\min} > 0$ and is independent of β for $\mu_{\min} = 0$, expected individual profit (π), expected industry profit (2π), and the average price decrease in β.*

An increase in β increases market transparency as firms use $\underline{k}$ more frequently, and it boosts expected consumer surplus, as it decreases the average price. However, it also leads to more price dispersion (as it lowers p_0). As total welfare is constant (and normalized to one), a decrease in industry profit corresponds to an equal increase in consumer surplus. As β increases, a firm's incentive to use the high complexity is lower as it operates in an environment where strategic complexity is more effective (or, at least, less ineffective). Figure 2 illustrates how the probability of using $\underline{k}$ and industry profit vary with β, for given $\mu_{\min}$.

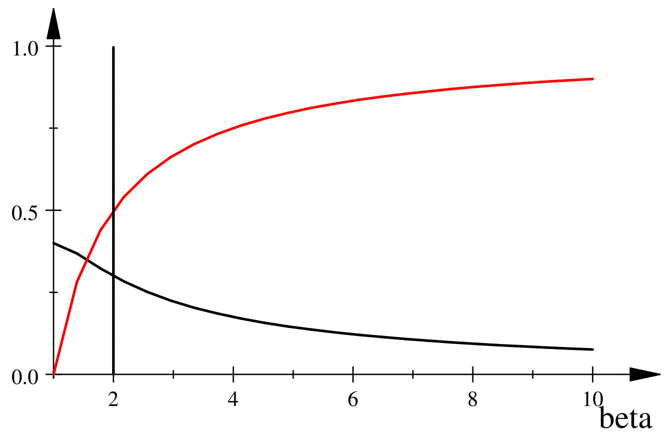

Figure 2: Probability of using $\underline{k}$ (in red) and industry profit (in black) as functions of β for $n = 2$ and $\mu_{\min} = .6$.

While average prices decrease in β, it is less obvious how the average price conditional on low complexity varies with β. For instance, in Figure 1, an increase in β from 3/2 to 3 triggers an increase in the cumulative probability at each price p which was assigned positive density for $\beta = 3/2$ and also a decrease in p_0. These two effects indicate that the average price for $\beta = 3$ is lower than that for $\beta = 3/2$. However, the change in the average price conditional on low complexity is affected by an additional effect which partly offsets these two, as the cut-off price $\hat{p}$ increases in β. Using (A1), the price distribution conditional on the price being below the cut-off value $\hat{p}$ and the corresponding conditional expected price are presented in the appendix. It can be checked that the expected price conditional on low complexity is not generally monotonic in β. For instance, for $\mu_{\min} = 0.6$, it increases for low values of β and then decreases, while for $\mu_{\min} = 0$, it monotonically decreases in β.[18]

Corollary 2 *In symmetric duopoly equilibrium, the probability of using the low price complexity (λ) is independent of $\mu_{\min}$, while the lower bound of the pricing c.d.f. (p_0), the cut-off price ($\hat{p}$), expected individual profit (π), expected industry profit (2π), expected share of confused, the average price, and the average price conditional on low complexity, all decrease in $\mu_{\min}$.*

Consumer policies which increase the degree of consumer sophistication, for instance, financial literacy programmes, decrease average prices and expected share of confused consumers, and increase expected consumer surplus. However, they increase price dispersion (by

[18] When $\mu_{\min} = 0.6$, the expected price conditional on low complexity is 0.25115 for $\beta = 1.05$, 0.25149 for $\beta = 1.15$, and 0.24292 for $\beta = 1.5$.

decreasing the lower bound of the pricing support, p_0) and do not affect market transparency as reflected by firms' probability of choosing low complexity. These results suggest that market transparency and price dispersion may not be good indicators of the success of consumer education programmes or of market performance.

5 Oligopoly Equilibrium

This section characterizes the symmetric equilibrium in oligopoly markets. It mainly focuses on a framework where the confusion technology is either concave or convex in firms' price complexity levels (where either $\beta < 2$ or $\beta > 2$), but also includes a variant without second order effects ($\beta = 2$).

Consider an arbitrary firm $i \in N$. Suppose that firm i's competitors follow the mixed strategy presented in section 2. So the competitors choose prices randomly from a distribution function $F(p)$, which satisfies Lemma 3, the probability that a (random) price p is associated with the low complexity $\underline{k}$ is given by $\lambda(p)$, and the probability that p is associated with high complexity $\bar{k}$ is $1 - \lambda(p)$. Although firm i's competitors choose prices from the same c.d.f., their price draws may be different and so the probabilities of using $\underline{k}$ may also be different across these firms.

Let $\wp(N_{-i})$ be the power set of $N_{-i} = N \setminus \{i\}$ and $M_m = \{M \subset \wp(N_{-i}) \mid |M| = m\}$. $\wp(N_{-i})$ is the set of all subsets of N_{-i} and M_m is the set of all subsets of $\wp(N_{-i})$ of cardinality m. Consider competitors' ex post price profile $\mathbf{p}_{-i} \in S^{n-1}$ and let p_j and p_l ($j,\, l \neq i$) stand for elements of $\mathbf{p}_{-i}$. Denote by

$$\mathsf{P}^m_{n-1} = \sum_{M \subset M_m} \left[\prod_{j \in M} \lambda(p_j) \prod_{k \in N_{-i} \setminus M} (1 - \lambda(p_l)) \right] , \tag{13}$$

the overall probability that out of $n-1$ rivals exactly $m \geq 0$ use $\underline{k}$ and $n-m-1$ use $\bar{k}$).

Example 5 *Let $n = 3$, $N_{-i} = \{j, l\}$, and $\mathbf{p}_{-i} = (p_j, p_l)$. Using (13), $\mathsf{P}^2_2 = \lambda(p_j)\lambda(p_l)$, $\mathsf{P}^1_2 = [\lambda(p_j)(1 - \lambda(p_l)) + (1 - \lambda(p_j))\lambda(p_l)]$, and $\mathsf{P}^0_2 = (1 - \lambda(p_j))(1 - \lambda(p_l))$. In this case, $\wp(N_{-i}) = \{\varnothing, \{j\}, \{l\}, \{j, l\}\}$, $M_2 = \{j, l\}$, $M_1 = \{\{j\}, \{l\}\}$, and $M_0 = \{\varnothing\}$.*

Using (13), firm i's expected profit at price $p \geq p_0 = \min S > 0$ and complexity $\underline{k}$ is given by

$$\begin{aligned} \pi_i(p, \underline{k}) \;&=\; p \int_p^1 \ldots \int_p^1 \left[\sum_{m=0}^{n-1} \mathsf{P}^m_{n-1} \mu(n-m-1) \right] dF(p_j) \ldots dF(p_l) + \\ &\frac{p}{n} \int_{p_0}^1 \ldots \int_{p_0}^1 \left[\sum_{m=0}^{n-1} \mathsf{P}^m_{n-1} (1 - \mu(n-m-1)) \right] dF(p_j) \ldots dF(p_l) \; . \end{aligned}$$

The first term in $\pi_i(p, \underline{k})$ corresponds to firm i's profit on the expert consumer segment. When out of $n-1$ rivals exactly $m \geq 0$ use $\underline{k}$ and $n-m-1$ use $\bar{k}$, given that firm i also employs $\underline{k}$, there are $\mu(n-m-1)$ experts. The first term in square brackets sums up over all values of m. Firm i serves experts only if its price is lower than rivals' prices, which is reflected when integrating over $p_j > p$ for all $j \neq i$. The second term is firm i's profit on the confused consumer segment. Firm i serves a share $1/n$ of these consumers. The second term in square brackets sums the corresponding share of confused $(1-\mu(n-m-1))$ over all values of m. Firm i serves its share of confused regardless of rivals' prices, which is reflected by integrating over $p_j > p_0$ for all $j \neq i$.

Using (13), firm i's expected profit at price p and complexity $\bar{k}$ is

$$\begin{aligned}\pi_i\left(p, \bar{k}\right) &= p\int_p^1 ... \int_p^1 \left[\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m \mu(n-m)\right] dF(p_j)...dF(p_l) + \\ &\quad \frac{p}{n}\int_{p_0}^1 ... \int_{p_0}^1 \left[\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m (1-\mu(n-m))\right] dF(p_j)...dF(p_l)\ .\end{aligned}$$

The expression for $\pi_i\left(p, \bar{k}\right)$ is analogous to the one for $\pi_i(p, \underline{k})$, with the difference that, when exactly $m \geq 0$ out of $n-1$ rivals use the lowest complexity $\underline{k}$, as firm i employs $\bar{k}$, there are $\mu(n-m)$ experts and $(1-\mu(n-m))$ confused.

The incremental profitability of an increase in complexity $(\pi_i\left(p, \bar{k}\right) - \pi_i(p, \underline{k}))$ is given by

$$\begin{aligned}&-\frac{(n-1)\,p}{n}\int_p^1 ... \int_p^1 \left[\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m (\mu(n-m-1)-\mu(n-m))\right] dF(p_j)...dF(p_k) \\ &+\frac{p}{n}\int_{p_0}^p ... \int_{p_0}^p \left[\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m (\mu(n-m-1)-\mu(n-m))\right] dF(p_j)...dF(p_k)\ .\end{aligned}$$

This expression can be simplified using the following result which draws on (3).

Lemma 4 *Suppose $n \geq 2$ and $m \leq n-1$. Let $\mathbf{p}_{-i}$ be the ex post price profile of firm i's competitors and $\Lambda(p_j) \equiv [1 + (1-\lambda(p_j))(\beta-2)]$, where p_j is an element of $\mathbf{p}_{-i}$. Then,*

$$\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m (\mu(n-m-1)-\mu(n-m)) = (1-\mu(1)) \prod_{j\neq i} \Lambda(p_j)\ .$$

By (4), the incremental profitability of an increase in complexity becomes

$$\pi_i\left(p, \bar{k}\right) - \pi_i(p, \underline{k}) = p(1-\mu(1))\left(-\frac{n-1}{n}\prod_{j\neq i}\int_p^1 \Lambda(p_j)dF(p_j) + \frac{1}{n}\prod_{j\neq i}\int_{p_0}^p \Lambda(p_j)dF(p_j)\right)\ . \tag{14}$$

This expression generalizes the incremental profitability of an increase in complexity presented in section 4, and similar reasoning leads to the next result.

Proposition 2 *In symmetric mixed-strategy oligopoly equilibrium, a firm's complexity level depends only on its price. The firms choose prices according to a c.d.f. $F(p)$ with support $S = [p_0, 1]$. If $p < \hat{p}$, firms choose complexity $\underline{k}$, if $p > \hat{p}$, firms choose complexity $\bar{k}$, and if $p = \hat{p}$, firms are indifferent between the two complexity levels.*

When a firm chooses relatively low prices, it benefits from more market transparency, as it is more likely to serve the experts. In contrast, when a firm chooses relatively high prices, it relies more on confused consumers and benefits from higher complexity. As a result, in equilibrium there is a positive relationship between prices and price complexity levels and a firm's complexity choice is determined by its price draw. Each firm assigns probability $\lambda = F(\hat{p}) \in (0, 1)$ to complexity level $\underline{k}$ and $1 - \lambda$ to complexity level $\bar{k}$.

The proof of the next result uses the approach illustrated in section 4 and is relegated to the appendix. Drawing on Proposition 2, a firm's expected profit for a price $p \in [p_0, \hat{p})$ (which is associated with low complexity, $\underline{k}$) and for a price $p \in [\hat{p}, 1]$ (which is associated with high complexity, $\bar{k}$), and mixed strategy equilibrium constant profit conditions are also presented there. Recall that $\mu(n) = \mu_{\min}$.

Proposition 3 *For $n \geq 2$ and $\beta \neq 2$, there is a unique symmetric equilibrium, where firms choose prices randomly according to a c.d.f. $F(p)$ defined on $[p_0, 1]$ where $p_0 > 0$. Each firm chooses complexity $\bar{k}$ with probability $1 - \lambda = 1 - F(\hat{p})$, where*

$$1 - \lambda = \frac{n^{-1/(n-1)}}{1 + (\beta - 2)(1 - n^{-1/(n-1)})} \in (0, 1) \ , \tag{15}$$

and complexity $\underline{k}$ with probability λ. Each firm's expected profit is

$$\pi = (1 - \mu_{\min}) \frac{\left[n(1 - \lambda)^{n-1}(\beta - 1)^n - 1\right]}{n[(\beta - 1)^n - 1]} \ . \tag{16}$$

If $\mu_{\min} > 0$, then $\hat{p} < 1$ and $F(p)$ is continuous everywhere. If $\mu_{\min} = 0$, then $\hat{p} = 1$ and $F(p)$ is continuous for $p \in [p_0, \hat{p})$, but has an atom at $\hat{p}$.

In homogeneous product markets, where firms compete by simultaneously choosing prices and price complexity levels, there is dispersion in both dimensions in equilibrium. These findings are consistent with observed patters in markets for financial and banking products, or markets for gas and electricity. Firms make strictly positive profits and charge prices in excess of marginal cost. The firms' equilibrium strategies and market outcomes depend on market structure (as captured by n), the curvature of the confusion technology (β), and the degree of consumer sophistication ($\mu_{\min}$).

A Model without Second Order Effects

So far, the analysis has focused on $\beta \neq 2$. Below, a variant of the model where $\beta = 2$ (and so there are no second order effects) is discussed. The preliminary results in Lemmas 1-3 carry over unchanged. The $\beta = 2$ case is closely related to Carlin (2009), who analyses a market where "an individual firm's complexity choice may add difficulty to the overall task of becoming informed, [but] does not magnify the effect of other firms' complexity choices on the cost of becoming informed". In Carlin's model the firms choose complexity levels from a closed interval $[\underline{k}, \bar{k}]$, but in symmetric mixed-strategy equilibrium they randomize only between the extreme values of this interval. His analysis proves the existence of a cut-off mixed strategy equilibrium. The next result fully characterizes the unique symmetric equilibrium of a variant with only two price complexity levels ($k_i \in \{\underline{k}, \bar{k}\}$). The proof is relegated to the online appendix.

Proposition 4 *For $n \geq 2$ and $\beta = 2$, there is a unique symmetric equilibrium, where firms choose prices randomly according to a c.d.f. $F(p)$ defined on $[p_0, 1]$ where $p_0 > 0$. Each firm chooses complexity $\bar{k}$ with probability $1 - \lambda = 1 - F(\hat{p})$, given by*

$$1 - \lambda = n^{-\frac{1}{n-1}} \in (0, 1) \ , \tag{17}$$

and complexity $\underline{k}$ with probability λ. Each firm's equilibrium profit π is given by

$$\pi = (1 - \mu_{\min}) \frac{1 - n^{-\frac{1}{n-1}} + n^{\frac{n-2}{n-1}}}{n^2} \ . \tag{18}$$

If $\mu_{\min} > 0$, then $\hat{p} < 1$ and $F(p)$ is continuous everywhere. If $\mu_{\min} = 0$, then $\hat{p} = 1$ and $F(p)$ is continuous for $p \in [p_0, \hat{p})$ but has an atom at $\hat{p}$.

6 Discussion and Policy Implications

This section explores equilibrium comparative statics and convergence results, and discusses their implications for competition and consumer protection policy. It builds on Propositions 3 and 4, and focuses on the impact of changes in (i) competitive pressure as measured by the number of firms (n); (ii) degree of consumer sophistication as measured by the lowest share of experts ($\mu_{\min}$); and (iii) the concavity or convexity of the confusion technology as measured by the value of β.

Corollary 3 *In the symmetric mixed-strategy equilibrium, the probability that each firm assigns to the lowest complexity level $\underline{k}$ (λ) decreases in the number of firms (n), is independent of the degree of consumer sophistication ($\mu_{\min}$), and increases in the degree of concavity/convexity β. Moreover, $\lim_{n\to\infty} \lambda = 0$.*

In more fragmented markets, to offset the impact of increased competitive pressure, firms rely more on consumer confusion and so choose high price complexity more frequently. In the limit, as the number of competitors goes to infinity, firms use high price complexity almost surely. Hence, in settings where firms compete by choosing both prices and price complexity, an increase in the number of firms leads to an increase in the overall complexity of the market. Like in the duopoly framework, the frequency with which firms use low complexity is independent of consumers' degree of sophistication. As a result, initiatives which improve consumer awareness do not lead to reductions in the overall market complexity.

For a given market structure, a lower degree of concavity or higher degree of convexity in confusion technology leads to a decrease in market complexity. When $\beta \in (1,2)$, higher complexity of rivals' offers partly offsets the increase in confusion resulting from a firm's incremental complexity. However, as β increases, this offsetting effect gets smaller and, as a firm's incentives to choose high complexity are reduced, λ increases. When $\beta > 2$, higher rival complexity magnifies the increase in confusion resulting from the incremental complexity of a firm's offer. This creates a free-riding effect and an increase in β further weakens a firm's incentives to use high complexity. Figure 3 provides an illustration.

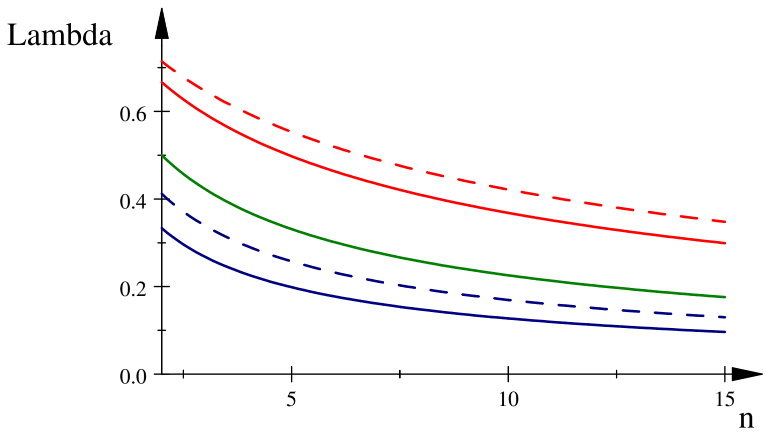

Figure 3: Equilibrium probability of using $\underline{k}$ (λ) as a function of n. From bottom to top for β equal to 1.5, 1.7, 2, 3, and 3.5.

The equilibrium expected share of confused consumers is

$$E\left[1-\mu\right] \equiv \sum_{m=0}^{n} C_n^m \lambda^m (1-\lambda)^{n-m} (1-\mu(n-m)) \ .$$

For a given number of firms, $E[1-\mu]$ aggregates the shares of confused consumers in the sequence $(1-\mu(n-m))_{m=0}^{n}$, weighting each share by the overall probability that m firms choose low complexity and $n-m$ firms choose high complexity. Using (3) for $\beta \neq 2$ and (4) for $\beta = 2$, equilibrium expected share of confused becomes

$$E\left[1-\mu\right] = \begin{cases} (1-\mu_{\min}) \dfrac{\left[1+(1-\lambda)(\beta-2)\right]^n - 1}{(\beta-1)^n - 1} & \text{for} \quad \beta \neq 2 \\ (1-\mu_{\min})(1-\lambda) \quad \text{for} \quad \beta = 2 \end{cases} . \tag{19}$$

This analysis focuses on the impact of convexity or concavity of the confusion technology on market outcomes. For this reason, the comparative statics and convergence results discussed below assume that the lower bound of the share of experts in the market is independent of the number of firms, i.e. $\mu(n) = \mu_{\min}$ is a constant. This is equivalent to assuming that $(1-\mu_{\min})$ - the maximal share of confused consumers - is fixed regardless of the market structure. By fixing the maximal share of confused, when the number of firms is increased, each of them is made smaller with respect to the market, as each firm's maximal base of confused is then fixed at the level $(1-\mu_{\min})/\ n$.[19]

The convergence properties of the expected share of confused consumers in fragmented markets depend on whether the confusion technology is concave or convex in firms' complexity choices.

Corollary 4 *In the symmetric mixed-strategy equilibrium, the expected share of confused consumers in (19) decreases in* $\mu_{\min}$ *and*

$$\lim_{n\to\infty} E\left[1-\mu\right] = \begin{cases} (1-\mu_{\min}) \ \ for \ \ \beta \in (1,2] \\ 0 \ \ for \ \ \beta > 2 \end{cases}.$$

This corollary formalizes Carlin's intuition that the convergence results for the expected number of confused depend on the properties of the confusion technology. If the confusion technology is convex in firms' complexity levels, the expected share of confused consumers converges to zero. For expositional ease, this analysis assumes that $\mu(0) = 1$, that is, when all firms use low complexity, there are no confused consumers. If $\mu(0) < 1$, so that there is some confusion even when all firms use low complexity, then in highly fragmented markets where $\beta > 2$, the expected share of confused consumers would converge to the minimal level $(1-\mu(0))$ instead; see footnote 2. In contrast, if the confusion technology is concave in firms' complexity levels ($\beta \in (1,2)$), the expected share of confused is bounded away from zero and converges to $(1-\mu_{\min})$.

The concavity or convexity of the confusion technology in this model is captured by the sequence of shares $(1-\mu(n-m))_{m=0}^{n}$. A simple illustration is provided below.

Example 6 *Let* $\mu_{\min} = 0$ *and consider (3). If* $\beta = 3$*, then,* $1-\mu(n-m) = (2^{n-m}-1)/(2^n-1)$ *and* $\lim_{n\to\infty}(1-\mu(n-m)) = 2^{-m}$*. If* $\beta = 3/2$*, then* $1-\mu(n-m) = (0.5^{n-m}-1)/(0.5^n-1)$ *and* $\lim_{n\to\infty}(1-\mu(n-m)) = 1$*.*

Intuitively, in fragmented markets, convexity requires a relatively ineffective confusion technology and so, although firms use almost surely high complexity (i.e., despite the lack of market transparency), the expected share of confused converges to zero. In contrast, concavity requires a relatively effective confusion technology. If the confusion technology is concave, in highly fragmented markets, lack of transparency is aligned with a high expected share of

[19] For a discussion of comparative statics in large oligopoly markets, see Chapter 5.2.5. in Vives (2001).

confused consumers. This is also the case when there are no second order effects in confusion, so the results for $\beta = 2$ are qualitatively similar to those that obtain under concavity.

Beyond the convergence results presented in Corollary 4, numerical simulations indicate that the expected number of confused consumers increases monotonically in n for $\beta \in (1, 2]$. For $\beta = 2$, this monotonicity result can be derived analytically using Corollary 3. For all $\beta > 2$, $E[1 - \mu]$ decreases in n at least when n is large enough. For $\beta \geq 3$, its value at $n = 2$ is larger than the value at $n = 3$, and then $E[1 - \mu]$ decreases monotonically in n for $n \geq 3$.[20] However, for $\beta \in (2, 2.8)$ it peaks at some value $n_0(\beta) \geq 3$. Numerical examples also indicate that the expected share of confused consumers strictly decreases in β for given n and $\mu_{\min}$.

Example 7 *Let $\mu_{\min} = 0$. Consider $\beta = 3/2$, 2, or 3. The expected share of confused as a function of the number of firms is illustrated in Figure 4 and presented in the appendix.*

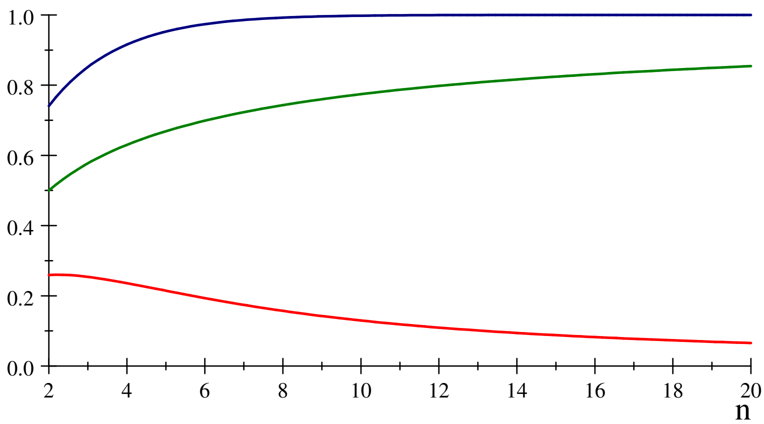

Figure 4: Expected share of confused consumers in equilibrium for $\mu(n) = 0$. In red for $\beta = 3$, in green for $\beta = 2$, and in blue for $\beta = 3/2$.

As discussed, the analysis holds the minimal share of experts fixed, so market fragmentation does not have a direct impact on consumer sophistication. Suppose instead that $\mu_{\min} = \mu(n)$ is a decreasing function of n. Then, the more fragmented the market, the lower the minimal share of experts is. This could be the case in a market where there is choice overload and consumers are more likely to make random choices when they face more options. Using (19), it is easy to see that the qualitative results in Corollary 4 carry over unchanged. For $\beta > 2$, $(1 - \mu(n))$ is bounded above by 1, so that in the limit, the expected share of confused still goes to zero. For $\beta \in (1, 2]$, the expected share of confused converges to $\lim_{n\to\infty}(1 - \mu(n)) > (1 - \mu(2)) > 0$. Now, suppose that $\mu(n)$ is an increasing function of n, that is, the more fragmented the market, the higher the minimal share of experts is.

[20] For $\beta \in [3, 3.15)$, the expected number of confused is an inverted U function of n, but reaches a maximum at some $n_0(\beta) \in (2, 3)$.

Arguably, this is a less realistic case as it requires market structure to have a positive impact on consumer sophistication. For $\beta > 2$, $(1 - \mu(n))$ is bounded below by zero, so the expected share of confused still converges to zero. However, for $\beta \in (1, 2]$, the expected share of confused may converge to zero as almost all consumers are sophisticated in nearly competitive markets.[21]

Corollary 5 *In symmetric mixed-strategy equilibrium, expected industry profit ($\pi_n = n\pi$) decreases in $\mu_{\min}$, and is strictly larger than expected share of confused. Furthermore,*

$$\lim_{n\to\infty} \pi_n = \begin{cases} (1 - \mu_{\min}) & \textit{for} \quad \beta \in (1, 2] \\ 0 & \textit{for} \quad \beta > 2 \end{cases} .$$

An increase in the level of consumer sophistication has a positive effect on consumer surplus (as total welfare is normalized to one). Combined with the findings in Corollary 3, this implies that an improvement in consumer awareness boosts consumer welfare, but does not reduce overall market complexity. Hence, the insight from the duopoly model that market transparency is not a good indicator of market performance carries over to more fragmented environments.

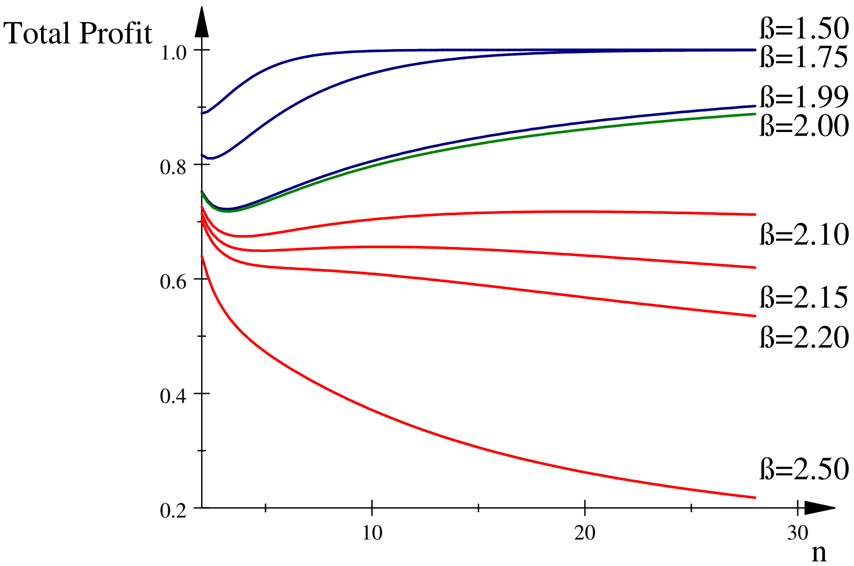

Figure 5: Equilibrium industry profit (π_n) as a function of n for $\mu_{\min} = 0$ and various values of β.

In nearly perfectly competitive markets, when the confusion technology is concave, or when it is linear (i.e., there are no second order effects), expected industry profit is bounded away from zero. As total profit is larger than the expected share of confused, this result is

[21] See also the examples in Carlin, 2009 (section 4, p. 284). In the first one, like in this analysis, $\mu(n)$ is independent of n, while in the second, $\mu(n)$ increases with n.

closely related to Corollary 4. On the other hand, when the confusion technology is convex in complexity levels, expected profit converges to zero as the number of firms goes to infinity. In this case, although firms choose high complexity almost surely, confusion technology is relatively inefficient and fragmented markets may be highly competitive.

In general, industry profit and implicitly consumer surplus are not monotonic in the number of firms. An increase in competitive pressure gives firms stronger incentives to undercut and attract the experts, but also to use more frequently high complexity ($\bar{k}$), which is associated with relatively higher prices. Numerical simulations also suggest that industry profit decreases in β for given $\mu_{\min}$ and n. Figure 4 provides an illustration for $\mu_{\min} = 0$. Numerical simulations provide further insights. Table 1 presents equilibrium outcomes for various values of n and β, letting $\mu_{\min} = 0.6$.

Table 1 ($\mu_{\min} = 0.6$)

β	n	λ	π	π_n	p_0	$\hat{p}$	$E[1-\mu]$
3/2	3	0.268	0.122	0.365	0.149	0.275	0.852
3/2	4	0.227	0.094	0.377	0.125	0.254	0.916
3/2	10	0.127	0.04	0.4	0.062	0.185	0.998
2	3	0.423	0.096	0.287	0.107	0.324	0.578
2	4	0.370	0.072	0.289	0.084	0.325	0.63
2	10	0.226	0.032	0.320	0.043	0.347	0.774
3	3	0.594	0.056	0.169	0.058	0.363	0.254
3	4	0.540	0.034	0.14	0.036	0.374	0.236
3	10	0.368	0.006	0.064	0.007	0.399	0.13

In these examples, p_0 decreases in n for given β and in β for given n. $\hat{p}$ decreases in n for

$\beta = 3/2$, it increases in n for $\beta = 2$ and $\beta = 3$, and it increases in β for given n. Expected industry profit and expected share of confused increase in n for $\beta = 3/2$ and $\beta = 2$, decrease in n for $\beta = 3$, and decreases in β for given n.

7 Conclusions

This paper develops a richer framework for the analysis of competition in homogeneous product markets where sellers compete by choosing both prices and the complexity of their price structures. Price complexity increases the cost of gathering information about prices and identifying the best deal, and is a source of consumer confusion. A distinctive feature of the proposed model is that it accommodates more general confusion technologies, which may be either concave or convex in firms' complexity levels. Our results suggest that the characteristics of the confusion technology have an impact on market outcomes and implications for the design and assessment of competition and consumer protection policy. Furthermore, the characteristics of confusion technology can be related to consumers' cost of gathering market-wide information and to a wider range of underlying consumer behavior.

Despite product homogeneity, in markets with strategic price complexity, there is equilibrium dispersion in both prices and price complexity levels, there is a positive relationship between prices and complexity, and firms make strictly positive profits. In relatively concentrated markets, an increase in the number of competitors triggers an increase in market complexity but has ambiguous effects on consumer surplus. In nearly perfectly competitive markets, although firms use almost surely high price complexity, if the confusion technology is convex, expected industry profits converge to zero, whereas, if the technology is concave, expected industry profits converge to the highest level. An improvement in consumer sophistication boosts consumer welfare but does not reduce the overall complexity of the market. Therefore, interventions which increase consumer awareness are beneficial, but their effectiveness cannot be measured by the degree of market transparency they achieve.

8 Appendix

8.1 Preliminary Results

Proof of Lemma 3. (i) Let $\widetilde{S}$ be the convex hull of S. Suppose there is a gap $G \subset \widetilde{S}$. Let $A = \{p \in S \mid p \leq \inf G\}$, $p_a = \max A$ and $p' = \sup G$. Clearly, $p_a \in S$ and $F(p_a) = F(p')$. But then $\pi_i(p', k, \xi_j) > \pi_i(p_a, k, \xi_j)$. A contradiction.
(ii) (a) Suppose there is a mass point at some $p' \in S$ with $0 < p' < \max S$. Then, there is a positive probability of a tie at this price. (a1) If $\lambda(p') > 0$, the expected share of experts at this price is strictly positive when all firms tie. Then, firm i is better off deviating to $p' - \epsilon$ as there is a discrete increase in market share and only a marginal decrease in price. This deviation applies also at $p' = \max S$, so that firms cannot have a mass point at the upper bound of S. (a2) If $\lambda(p') = 0$, the share of experts at this price is $\mu_{\min}$. If $\mu_{\min} > 0$, the argument in (a1) applies unchanged. If $\mu_{\min} = 0$, in the event of a tie at p', all consumers are confused and firm i is better off moving mass from price p' to price $p = \max S$. Note that this argument does not rule out a mass point at the upper bound of S when $\mu_{\min} = 0$. (c)

Suppose there is a mass point at $p' = 0$. In this case, all firms make zero profits. If $\lambda(p') < 1$, then there are some confused consumers when firms tie at this price and firm i is better off deviating to $p = 1$ and making positive profits on its share of confused. If $\lambda(p') = 0$, all consumers are experts when firms tie, but then firm i is better off deviating to $p = 1$ and $k_i = \bar{k}$, where it makes positive profit.

(iii) It follows from (i) that in symmetric equilibrium S is a bounded interval. Suppose $p_h = \sup S < 1$. If $\mu_{\min} > 0$, by (ii) firms pricing c.d.f.s are continuous everywhere, so a firm charging $p \to p_h$ sells only to its share of confused consumers and it is clearly better off deviating to a higher price $p = 1$. If $\mu_{\min} = 0$, there may be a positive probability of a tie at p_h but, as all consumers are confused when all firms tie, deviating to $p = 1$ is profitable. A contradiction. As $p_h = 1$, firms' expected profit in the mixed strategy equilibrium will be strictly positive, so it must be that $p_0 = \min S > 0$. ■

8.2 Duopoly Analysis

Expected Profits

Using (10), we re-write a firm's expected profits. If firm i chooses complexity $\underline{k}$ and charges a price $p \in [p_0, \hat{p})$, as $\mu(0) = 1$, (7) becomes

$$\pi_i(p, \underline{k}) = p\left[(\lambda - F(p)) + (1-\lambda)\mu(1) + \frac{(1-\lambda)(1-\mu(1))}{2}\right].$$

If instead firm i chooses $\bar{k}$ and charges $p \in [\hat{p}, 1]$, expression (8) becomes

$$\pi_i(p, \bar{k}) = p\left[(1-F(p))\mu(2) + \frac{\lambda(1-\mu(1))}{2} + \frac{(1-\lambda)(1-\mu(2))}{2}\right].$$

Cumulative Price Distributions

The cumulative price distribution is presented below.

$$1 - F(p) = \begin{cases} \dfrac{(1-\mu_{\min})}{2\beta^2}\left(\dfrac{2\beta-1}{p}+1\right) & \text{for } p \in [p_0, \hat{p}), \\ \dfrac{(1-\mu_{\min})(2\beta-1)}{2\beta^2\mu_{\min}}\left(\dfrac{1}{p}-1\right) & \text{for } p \in [\hat{p}, 1], \text{ if } \mu_{\min} > 0. \end{cases} \quad \text{(A1)}$$

Proof of Proposition 1. It is easy to see that λ, p_0, $\hat{p}$, and $F(p)$ as presented in (11), (12), and (A1) are all well defined.

Consider a unilateral deviation to $(p < p_0, \underline{k})$. The deviator's market share is equal to that corresponding to $p = p_0$ but, as the price is lower, this deviation is not profitable.

Consider a unilateral deviation to $(p > \hat{p}, \underline{k})$. This results in deviation profit

$$\pi^d(p,\underline{k}) = p\left[(1-F(p))\mu(1) + \frac{\lambda(1-\mu(0))}{2} + \frac{(1-\lambda)(1-\mu(1))}{2}\right] <$$
$$\pi(p,\bar{k}) = p\left[(1-F(p))\mu(2) + \frac{\lambda(1-\mu(1))}{2} + \frac{(1-\lambda)(1-\mu(2))}{2}\right].$$

The inequality follows as, for $p > \hat{p}$, $\frac{\left(\pi^d(p,\underline{k}) - \pi(p,\bar{k})\right)}{p} =$

$$\left[(1-F(p))(\mu(1)-\mu(2)) - \frac{\lambda(\mu(0)-\mu(1))}{2} - \frac{(1-\lambda)(\mu(1)-\mu(2))}{2}\right] =$$
$$(\mu(1)-\mu(2))\left[(1-F(p)) - \frac{\lambda}{2(\beta-1)} - \frac{(1-\lambda)}{2}\right] <$$
$$(\mu(1)-\mu(2))\left[(1-F(\hat{p})) - \frac{\lambda}{2(\beta-1)} - \frac{(1-\lambda)}{2}\right] = 0.$$

Consider a unilateral deviation to $\left(p < \hat{p}, \bar{k}\right)$. This results in deviation profit

$$\pi^d(p,\bar{k}) = p\left[(1-\lambda)\mu(2) + (\lambda - F(p))\mu(1) + \frac{(1-\lambda)(1-\mu(2))}{2} + \frac{\lambda(1-\mu(1))}{2}\right] <$$
$$\pi(p,\underline{k}) = p\left[(1-\lambda)\mu(1) + (\lambda - F(p)) + \frac{(1-\lambda)(1-\mu(1))}{2}\right].$$

The inequality follows as, for $p < \hat{p}$, $\frac{\left(\pi^d(p,\bar{k}) - \pi(p,\underline{k})\right)}{p} =$

$$-(1-\lambda)(\mu(1)-\mu(2)) - (\lambda - F(p))(1-\mu(1)) + \frac{(1-\lambda)(\mu(1)-\mu(2))}{2} + \frac{\lambda(1-\mu(1))}{2} =$$
$$(\mu(1)-\mu(2))\frac{-2(\lambda-F(p)) - (\beta-1)(1-\lambda) + \lambda}{2(\beta-1)} <$$
$$(\mu(1)-\mu(2))\frac{-(\beta-1)(1-\lambda) + \lambda}{2(\beta-1)} = 0. \blacksquare$$

Equilibrium Expected Share of Confused Consumers.

$$\sum_{m=0}^{2} C_2^m \lambda^m (1-\lambda)^{2-m}(1-\mu(2-m)) = \frac{(1-\mu_{\min})(3\beta-2)}{\beta^3}\ . \tag{A2}$$

For $\mu_{\min} < 1$ is strictly lower than expected industry profits.

Proof of Corollary 1. Using (11), $d\lambda/d\beta = 1/\beta^2 > 0$.

Differentiating (A2) w.r.t. β gives $-2(1-\mu_{\min})(\beta-1)/\beta^4 < 0$.

Using (12), $dp_0/d\beta = -2(1-\mu_{\min})[1-\mu_{\min} + 2\beta(\beta-1)]/\ [2\beta^2 - (1-\mu_{\min})]^2 < 0$.

Using (12), $d\hat{p}/d\beta = 2(1-\mu_{\min})\mu_{\min}/\ [2\beta - (1-\mu_{\min})]^2$. So, for $\mu_{\min} > 0$, $d\hat{p}/d\beta > 0$ and $\mu_{\min} = 0$, $d\hat{p}/d\beta = 0$.

Using (11), $d\pi/d\beta = (1/2)(d\pi_2/d\beta) = -(1-\mu_{\min})(\beta-1)/\ \beta^3 < 0$.

Using (A1), $E(p) = \int_{p_0}^{\hat{p}} (1-F(p))\,dp + \int_{\hat{p}}^{1} (1-F(p))\,dp + p_0$. By Leibniz's rule,

$dE(p)/d\beta = (1-F(\hat{p}))\,(d\hat{p}/d\beta) - (1-F(p_0))\,dp_0/d\beta +$

$\int_{p_0}^{\hat{p}} (-dF(p)/d\beta)\,dp - (1-F(\hat{p}))\,(d\hat{p}/d\beta) + \int_{\hat{p}}^{1} (-dF(p)/d\beta)\,dp + dp_0/d\beta =$

$\int_{p_0}^{\hat{p}} (-dF(p)/d\beta)\, dp + \int_{\hat{p}}^{1} (-dF(p)/d\beta)\, dp$.

But, for $p \in [p_0, \hat{p})$, using (A1), $dF(p)/d\beta = (1-\mu_{\min})(\beta - 1 + p)/\ \beta^3 p > 0$. For $p \in [\hat{p}, 1]$ and $\mu_{\min} > 0$, $dF(p)/d\beta = (1-\mu_{\min})(\beta-1)(1-p)/\ \beta^3 \mu_{\min} p > 0$. Then it follows that $dE(p)/d\beta < 0$. For $\mu_{\min} = 0$, $\hat{p} = 1$ and the second integral in $E(p)$ is a constant, so $dE(p)/d\beta = \int_{p_0}^{\hat{p}} (-dF(p)/d\beta)\, dp < 0$. ■

Conditional Price Distribution. The price distribution conditional on the price being below the cut-off value $\hat{p}$ and the corresponding conditional expected price are given by

$$F(p \mid p < \hat{p}) = F(p)/F(\hat{p}) = \frac{2\beta^2 - (1-\mu_{\min})}{2\beta(\beta-1)} - \frac{(1-\mu_{\min})(2\beta-1)}{2\beta(\beta-1)p} ;$$
$$E(p \mid p < \hat{p}) = \frac{(1-\mu_{\min})(2\beta-1)}{2\beta(\beta-1)} \ln \frac{2\beta^2 - (1-\mu_{\min})}{2\beta - (1-\mu_{\min})} . \tag{A3}$$

Proof of Corollary 2. Using (11), $d\lambda/d\mu_{\min} = 0$.

Using (12), $dp_0/d\mu_{\min} = -2(2\beta-1)\beta^2/\ [2\beta^2 - (1-\mu_{\min})]^2 < 0$.

Using (12), $d\hat{p}/d\mu_{\min} = -2(2\beta-1)\beta/\ [2\beta - (1-\mu_{\min})]^2 < 0$.

Using (11), $d\pi/d\mu_{\min} = -(2\beta-1)/\ 2\beta^2 < 0$.

Differentiating (A2) w.r.t. $\mu_{\min}$ gives $-(3\beta-2)/\beta^3 < 0$.

Using (A1), $E(p) = \int_{p_0}^{\hat{p}} (1 - F(p))\, dp + \int_{\hat{p}}^{1} (1-F(p))\, dp + p_0$. Then, by Leibniz's rule, $\int_{p_0}^{\hat{p}} (-dF(p)/d\mu_{\min})\, dp + \int_{\hat{p}}^{1} (-dF(p)/d\mu_{\min})\, dp$.

But, for $p \in [p_0, \hat{p})$, using (A1), $dF(p)/d\mu_{\min} = [(2\beta-1)/p + 1]/\ 2\beta^2 > 0$. For $p \in [\hat{p}, 1]$ and $\mu_{\min} > 0$, $dF(p)/d\mu_{\min} = (2\beta-1)(1-p)/\ 2\beta^2 \mu_{\min} p > 0$. It follows that $dE(p)/d\mu_{\min} < 0$. For $\mu_{\min} = 0$, $\hat{p} = 1$ and the second integral in $E(p)$ is a constant, so $dE(p)/d\mu_{\min} = \int_{p_0}^{\hat{p}} (-dF(p)/d\mu_{\min})\, dp < 0$.

Using (A3), it follows that

$$dE(p \mid p < \hat{p})/\ d\mu_{\min} =$$
$$-\left\{ \frac{(1-\mu_{\min})(2\beta-1)}{[2\beta - (1-\mu_{\min})]\left[2\beta^2 - (1-\mu_{\min})\right]} + \frac{2\beta-1}{2\beta(\beta-1)} \ln \frac{2\beta^2 - (1-\mu_{\min})}{2\beta - (1-\mu_{\min})} \right\} < 0. \ ■$$

8.3 Oligopoly Analysis

Proof of Lemma 4. By (3) and (13),

$$\Phi = \sum_{m=0}^{n-1} \mathsf{P}_{n-1}^{m} (\mu(n-m-1) - \mu(n-m)) = (1-\mu(1)) \sum_{m=0}^{n-1} \mathsf{P}_{n-1}^{m} (\beta-1)^{n-m-1} =$$
$$(1-\mu(1)) \sum_{m=0}^{n-1} \left\{ \sum_{M \subset M_m} \left[\prod_{j \in M} \lambda(p_j) \prod_{k \in N_{-i} \setminus M} (1 - \lambda(p_k)) \right] (\beta-1)^{n-m-1} \right\}.$$

For a given m, $|N_{-i} \setminus M| = n - m - 1$, so we can re-write Φ as

$(1-\mu(1))\sum_{m=0}^{n-1}\left\{\sum_{M\subset M_m}\prod_{j\in M}\lambda(p_j)\prod_{k\in N_{-i}\setminus M}[(1-\lambda(p_k))(\beta-1)]\right\}.$

As we first sum over all values of m (for $0\leq m\leq n-1$), and then over all subsets of cardinality m in $\wp(N_{-i})$, we are effectively summing up over all the subsets M in $\wp(N_{-i})$. Then, Φ becomes

$(1-\mu(1))\sum_{M\subset\wp(N_{-i})}\prod_{j\in M}\lambda(p_j)\prod_{k\in N_{-i}\setminus M}[(1-\lambda(p_k))(\beta-1)]\,.$

Consider rival l's price p_l. Each term in the sum above contains either $\lambda(p_l)$ or $(1-\lambda(p_l))$. In fact, for each term in the summation which includes $\lambda(p_l)$, there is a 'pair' which includes $(1-\lambda(p_l))$ and all other multipliers are the same. Formally, take some set $M^l\subset\wp(N_{-i})$ such that $l\in M^l$. Then, $\exists M^{-l}$ such that $M^l=M^{-l}\cup\{l\}$. Pairing all such subsets, we can re-write the sum above by factoring out the term $[\lambda(p_l)+(1-\lambda(p_l))(\beta-1)]$. So,

$$\Phi=(1-\mu(1))\left[\lambda(p_l)+(1-\lambda(p_l))(\beta-1)\right]\sum_{M\subset\wp(N_{-i,-l})}\prod_{j\in M}\lambda(p_j)\prod_{k\in N_{-i}\setminus M}[(1-\lambda(p_k))(\beta-1)]\ ,$$

where $\wp(N_{-i,-l})$ is the power set of $N\setminus\{i,l\}\equiv N_{-i,-l}$.

By iteration,

$$\Phi=(1-\mu(1))\prod_{j\neq i}[\lambda(p_j)+(1-\lambda(p_j))(\beta-1)]=(1-\mu(1))\prod_{j\neq i}[1+(1-\lambda(p_j))(\beta-2)]\ .$$

■

Proof of Proposition 2. Using (14), let

$$\Delta(p)\equiv-\frac{n-1}{n}\prod_{j\neq i}\int_p^1\Lambda(p_j)dF(p_j)+\frac{1}{n}\prod_{j\neq i}\int_{p_0}^p\Lambda(p_j)dF(p_j)=\frac{\pi_i(p,k)-\pi_i(p,k)}{p(1-\mu(1))}\ .$$

As $\Lambda(p_j)>0$ (see Lemma 4), it is easy to check that $\Delta(p_0)<0$ and $\Delta(1)>0$. Furthermore, $\Delta(p)$ is strictly increasing in p. As changes in p only affect the integration ranges in $\Delta(p)$, $\dfrac{d\Delta(p)}{dp}=[1+(1-\lambda(p))(\beta-2))]\left(\dfrac{n-1}{n}\Omega_1+\dfrac{1}{n}\Omega_2\right)$ where

$\Omega_1=\sum_{j\neq i}\left\{\prod_{k\neq i,k\neq j}\int_p^1[1+(1-\lambda(p_{j_k}))(\beta-2))]dF(p_{j_k})\right\}>0$ and

$\Omega_2=\sum_{j\neq i}\left\{\prod_{k\neq i,k\neq j}\int_{p_0}^p[1+(1-\lambda(p_{j_k}))(\beta-2))]dF(p_{j_k})\right\}>0.$

So, to maximize its expected profit a firm i chooses according to (10), where the threshold price $\hat{p}\in(p_0,1]$ satisfies $\Delta(\hat{p})=0$. ■

Proof of Proposition 3.

Step 1: Expected Profits. At price $p\in[p_0,\hat{p})$, which is associated with $\underline{k}$, firm i's expected profit is

$$\pi\left(p,\underline{k}\right)=$$
$$p\left\{\sum_{m=0}^{n-1}C_{n-1}^{m}(1-\lambda)^{n-1-m}\left[(\lambda-F\left(p\right))^{m}\mu(n-m-1)+\frac{1}{n}\lambda^{m}(1-\mu(n-m-1))\right]\right\}.$$

When $n-1-m$ competitors choose $\bar{k}$ (and so price above $\hat{p}\geq p$) and m competitors choose $\underline{k}$ (which happens with probability $\lambda^{m}(1-\lambda)^{n-1-m}$) firm i serves $1/n$ of the $(1-\mu(n-m-1))$ confused. It also serves $\mu(n-m-1)$ experts whenever all m firms choosing $\underline{k}$ offer prices higher than p (which happens with probability $(\lambda-F\left(p\right))^{m}(1-\lambda)^{n-1-m}$). This gives the term in square brackets. The term in curly brackets considers all possible combinations of $n-1$ taken m and gives firm i's market share at price p.

Then, firm i's expected profits at $p=p_0$ and when $p\rightarrow\hat{p}$ are,

$$\pi\left(p_0,\underline{k}\right)=p_0\left\{\sum_{m=0}^{n-1}C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}[\mu(n-m-1)+\frac{1}{n}(1-\mu(n-m-1))]\right\}, \quad \text{(A4)}$$
$$\lim_{p\rightarrow\hat{p}}\pi\left(\hat{p},\underline{k}\right)=\hat{p}\left[(1-\lambda)^{n-1}\varphi(1)+\frac{1}{n}\sum_{m=0}^{n-1}C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(1-\mu(n-m-1))\right]. \quad \text{(A5)}$$

At price $p\in[\hat{p},1]$, which is associated with $\bar{k}$, firm i's expected profit is

$$\pi\left(p,\bar{k}\right)=p\left[(1-F(p))^{n-1}\mu(n)+\sum_{m=0}^{n-1}C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\frac{(1-\mu(n-m))}{n}\right].$$

When m competitors choose $\underline{k}$ (and so offer prices below $\hat{p}<p$) and $n-1-m$ competitors choose $\bar{k}$, firm i serves a share $1/n$ of the $(1-\mu(n-m))$ confused consumers. The second term in square brackets considers all possible combinations of $n-1$ taken m. Firm i serves expert consumers only if all rivals choose high complexity (that is, if $n-m=n$) and if it offers the lowest price, as reflected by the first term in square brackets.

Then, firm i's expected profit at $p=\hat{p}$ is

$$\pi\left(\hat{p},\bar{k}\right)=\hat{p}\left[(1-\lambda)^{n-1}\mu(n)+\sum_{m=0}^{n-1}C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\frac{(1-\mu(n-m))}{n}\right].$$

Step 2: Lambda. The equilibrium probability of choosing $\bar{k}$ presented in (15) follows from,

$\lim_{p\rightarrow\hat{p}}\pi\left(\hat{p},\underline{k}\right)=\pi\left(\hat{p},\bar{k}\right)\Leftrightarrow$

$(1-\lambda)^{n-1}(\mu(n-1)-\mu(n))=\frac{1}{n}\sum_{m=0}^{n-1}C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(\mu(n-m-1)-\mu(n-m))\Leftrightarrow$

$(\mu(n-1)-\mu(n))(1-\lambda)^{n-1}=\frac{1-\mu(1)}{n}\left(\sum_{m=0}^{n-1}C_{n-1}^{m}\lambda^{m}\left[(1-\lambda)(\beta-1)\right]^{n-1-m}\right)\Leftrightarrow$

$(1-\mu(1))\left[(1-\lambda)(\beta-1)\right]^{n-1} = \dfrac{1-\mu(1)}{n}[1+(1-\lambda)(\beta-2)]^{n-1}.$

Step 3: Equilibrium Profit. (16) obtains by evaluating the expected profit at $p = 1$ and using (3).

$\pi\left(1,\bar{k}\right) = \sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\dfrac{(1-\mu(n-m))}{n} =$

$\dfrac{(1-\mu(n))}{n[(\beta-1)^{n}-1]}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}[(\beta-1)^{n-m}-1] =$

$\dfrac{(1-\mu(n))}{n[(\beta-1)^{n}-1]}\left[(\beta-1)\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(\beta-1)^{n-1-m}-1\right] =$

$\dfrac{(1-\mu(n))}{n[(\beta-1)^{n}-1]}\left\{(\beta-1)\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}\left[(1-\lambda)(\beta-1)\right]^{n-1-m}-1\right\} =$

$\dfrac{(1-\mu(n))}{n[(\beta-1)^{n}-1]}\left\{(\beta-1)[1+(1-\lambda)(\beta-2)]^{n-1}-1\right\}.$

Then Step 2 implies that $[1+(1-\lambda)(\beta-2)]^{n-1} = n\left[(1-\lambda)(\beta-1)\right]^{n-1}$ and (16) follows.

Step 4: Equilibrium Boundary Prices and Pricing Distribution Functions.

► p_0 and $\hat{p}$ are defined by $\pi(p_0,\underline{k}) = \pi$ and $\lim_{p\to\hat{p}}\pi\left(\hat{p},\underline{k}\right) = \pi$, where the LHS terms are presented in (A4) and (A5), respectively, the RHS term is given by (16), and λ follows from (15).

Firm i's market share at p_0 - the term in curly brackets in (A4) - is larger than its market share when $p \to \hat{p}$ - the term in square brackets in (A5) because $(1-\lambda)^{n-1}\mu(n-1) < \sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\mu(n-m-1)$. This implies that $p_0 < \hat{p}$ as the equilibrium constant profit condition requires $\pi\left(p_0,\underline{k}\right) = \lim_{p\to\hat{p}}\pi\left(\hat{p},\underline{k}\right)$.

Moreover, $\hat{p} \leq 1$, with equality iff $\mu(n) = 0$. Using (A5) and (16), $\hat{p} \leq 1$ requires that

$\dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(1-\mu(n-m)) \leq$

$(1-\lambda)^{n-1}\mu(n-1) + \frac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(1-\mu(n-m-1)) \Leftrightarrow$

$\dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(\mu(n-m-1)-\mu(n-m)) \leq (1-\lambda)^{n-1}\mu(n-1) \Leftrightarrow$

$\dfrac{1}{n}\left[\lambda+(1-\lambda)(\beta-1)\right]^{n-1} - \left[(1-\lambda)(\beta-1)\right]^{n-1} \leq (1-\lambda)^{n-1}\dfrac{\mu(n)}{1-\mu(1)}.$

(15) implies that the LHS is equal to zero. The RHS is nonnegative and may be equal to zero only if $\mu(n) = 0$. So $\hat{p} < 1$ if $\mu(n) > 0$ and $\hat{p} = 1$ if $\mu(n) = 0$.

Below, we focus on the pricing c.d.f.s. Note that π is defined in (16) and independent of p.

► For $p \in [p_0, \hat{p})$, $F(p)$ is implicitly determined by $\pi(\underline{k}, p) = \pi$.

Differentiating w.r.t. p both sides gives

$-F'\left(p\right)\sum_{m=1}^{n-1} C_{n-1}^{m}\left[m(\lambda-F\left(p\right))^{m-1}(1-\lambda)^{n-1-m}\mu(n-m-1)\right] = -\dfrac{\pi}{p^{2}}.$

In this interval, $F\left(p\right) < F\left(\hat{p}\right) = \lambda$ and the summation is strictly positive. Then, $F(p)$ is strictly increasing in p.

For $p \in [\hat{p}, 1]$, $F(p)$ is implicitly determined by $\pi(\bar{k}, p) = \pi \Leftrightarrow$

$(1 - F(p))^{n-1}\mu(n) + \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \dfrac{(1 - \mu(n-m))}{n} = \dfrac{\pi}{p}$.

It is easy to see that $F(p)$ is strictly increasing on this interval.

Furthermore, $F(p)$ is continuous at $\hat{p}$, and satisfies $F(p_0) = 0$ and $F(1) = 1$.

Step 5: No Profitable Unilateral Deviations.

Consider a unilateral deviation to $(p < p_0, \underline{k})$. The deviator's market share is equal to that corresponding to $p = p_0$ but, as the price is lower, this deviation is not profitable.

Consider a unilateral deviation to $(p > \hat{p}, \underline{k})$. This results in deviation profit

$\pi^d(p, \underline{k}) = p\left[(1 - F(p))^{n-1}\mu(n-1) + \dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}\lambda^m(1 - \mu(n-m-1))\right]$.

But $\pi^d(p, \underline{k}) < \pi\left(p, \bar{k}\right) \Leftrightarrow$

$(1 - F(p))^{n-1}\mu(n-1) + \dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}\lambda^m(1 - \mu(n-m-1)) <$

$(1 - F(p))^{n-1}\mu(n)) + \dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^m(1-\lambda)^{n-1-m}(1 - \mu(n-m)) \Leftrightarrow$

$(1 - F(p))^{n-1}(\mu(n-1) - \mu(n)) < \dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^m(1-\lambda)^{n-1-m}(\mu(n-m-1) - \mu(n-m)) \Leftrightarrow$

$(1 - F(p))^{n-1}(\beta - 1)^{n-1} < \dfrac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^m\left[(1-\lambda)(\beta-1)\right]^{n-1-m} \Leftrightarrow$

$(1 - F(p))^{n-1}(\beta - 1)^{n-1} < \dfrac{1}{n}[\lambda + (1-\lambda)(\beta-1)]^{n-1}$.

The last inequality follows from the fact that, as $p > \hat{p}$,

$(1 - F(p))^{n-1}(\beta - 1)^{n-1} < (1 - F(\hat{p}))^{n-1}(\beta - 1)^{n-1} = (1-\lambda)^{n-1}(\beta-1)^{n-1}$.

It is then easy to check using (15) that $(1-\lambda)^{n-1}(\beta-1)^{n-1} = \dfrac{[\lambda + (1-\lambda)(\beta-1)]^{n-1}}{n}$.

Consider a unilateral deviation to $\left(p < \hat{p}, \bar{k}\right)$. This results in deviation profit.

$\pi^d\left(p, \bar{k}\right) = p\left\{\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}[(\lambda - F(p))^m\mu(n-m) + \lambda^m\dfrac{(1-\mu(n-m))}{n}]\right\}$.

But, $\pi^d\left(p, \bar{k}\right) < \pi\left(p, \underline{k}\right)$ as

$\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}[(\lambda - F(p))^m\mu(n-m) + \lambda^m\dfrac{(1-\mu(n-m))}{n}] <$

$\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}\left[(\lambda - F(p))^m\mu(n-m-1) + \lambda^m\dfrac{(1-\mu(n-m-1))}{n}\right] \Leftrightarrow$

$\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}(\mu(n-m-1) - \mu(n-m))[-(\lambda - F(p))^m + \dfrac{\lambda^m}{n}] < 0 \Leftrightarrow$

$\sum_{m=0}^{n-1} C_{n-1}^{m}\left[(1-\lambda)(\beta-1)\right]^{n-1-m}[-(\lambda - F(p))^m + \dfrac{\lambda^m}{n}] < 0 \Leftrightarrow$

$-\sum_{m=0}^{n-1} C_{n-1}^{m}\left[(1-\lambda)(\beta-1)\right]^{n-1-m}(\lambda - F(p))^m +$

$\frac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\left[(1-\lambda)(\beta-1)\right]^{n-1-m}\lambda^{m} < 0 \Leftrightarrow$

$-\left[(1-\lambda)(\beta-1)+\lambda-F(p)\right]^{n-1}+\frac{1}{n}\left[(1-\lambda)(\beta-1)+\lambda\right]^{n-1} < 0.$

The last inequality follows as for $p < \hat{p}$,

$-\left[(1-\lambda)(\beta-1)+\lambda-F(p)\right]^{n-1} < -\left[(1-\lambda)(\beta-1)\right]^{n-1}.$

But, using (15), $-\left[(1-\lambda)(\beta-1)\right]^{n-1}+\frac{1}{n}\left[(1-\lambda)(\beta-1)+\lambda\right]^{n-1}=0.$ ■

8.4 Discussion and Policy Implications

Proof of Corollary 3.

For $\beta \neq 2$, λ is defined in (15) and for $\beta = 2$ it is defined in (17). It is easy to see that when evaluated at $\beta = 2$, expression (15) reduces to (17). Therefore, this proof uses (15) to cover both cases.

Re-write (15) as $\lambda(\tilde{n}) = 1 - \frac{\tilde{n}}{\beta-1-(\beta-2)\tilde{n}}$ where $\tilde{n} = n^{-1/(n-1)} \in (0,1)$,

with $\frac{d\tilde{n}}{dn} = \frac{n^{-n/(n-1)}(1-n+n\log n)}{(n-1)^2} > 0.$

Using the chain rule, $\frac{d\lambda}{dn} = \frac{d\lambda}{d\tilde{n}}\frac{d\tilde{n}}{dn} < 0$ as $\frac{d\lambda}{d\tilde{n}} = -\frac{\beta-1}{\left[\beta-1-(\beta-2)\tilde{n}\right]^2} < 0.$

$\lim_{n\to\infty}\tilde{n} = \exp\left[\lim_{n\to\infty}\ln\left(n^{-1/(n-1)}\right)\right] =$

$\exp\left[\lim_{n\to\infty}\left(-\frac{\ln n}{n-1}\right)\right] =$

$\exp\left[-\lim_{n\to\infty}\left(\frac{\ln n}{n-1}\right)\right]$. By L'Hôpital's rule, $\lim_{n\to\infty}\left(\frac{\ln n}{n-1}\right) = \lim_{n\to\infty}\frac{1}{n} = 0$. So, $\lim_{n\to\infty}\tilde{n} = 1$ and $\lim_{n\to\infty}\lambda = 0$.

Moreover, $\frac{d\lambda}{d\mu(n)} = 0$ and $\frac{d\lambda}{d\beta} = \frac{\tilde{n}(1-\tilde{n})}{\left[\beta-1-(\beta-2)\tilde{n}\right]^2} > 0.$ ■

Proof of Corollary 4.

▶ Suppose $\beta \in (1,2)$. $\lim_{n\to\infty} E[1-\mu] = \lim_{n\to\infty}\frac{(1-\mu_{\min})\left\{1-\left[1+(1-\lambda)(\beta-2)\right]^n\right\}}{1-(\beta-1)^n}$.

In this case, $(\beta-1) < 1$ and $1+(1-\lambda)(\beta-2) < 1$.

As $\lim_{n\to\infty}[1-(\beta-1)^n] = 0$ and $\lim_{n\to\infty}\left\{1-\left[1+(1-\lambda)(\beta-2)\right]^n\right\} = 1$, it follows that $\lim_{n\to\infty} E[1-\mu] = (1-\mu_{\min})$.

▶ Suppose $\beta > 2$. It is convenient to write

$\lim_{n\to\infty} E[1-\mu] = \lim_{n\to\infty}\frac{(1-\mu_{\min})\left\{\left[(\beta-1)-\lambda(\beta-2)\right]^n-1\right\}}{(\beta-1)^n-1} =$

$(1-\mu_{\min})\frac{\lim_{n\to\infty}\left\{\left[(\beta-1)-\lambda(\beta-2)\right]^n-1\right\}}{\lim_{n\to\infty}\left[(\beta-1)^n-1\right]}.$

$\lim_{n\to\infty} \{[(\beta-1)-\lambda(\beta-2)]^n - 1\} = \lim_{n\to\infty} [(\beta-1)^n - 1] = \infty$,

as $(\beta-1) > (\beta-1) - \lambda(\beta-2) = 1 + (1-\lambda)(\beta-2) > 1$.

However, the first inequality implies that the denominator converges faster. It follows that $\lim_{n\to\infty} E[1-\mu] = 0$.

► Suppose $\beta = 2$. By Corollary 3, $\lim_{n\to\infty}(1-\lambda) = 1$.

Then, $\lim_{n\to\infty}(1-\mu(n))(1-\lambda) = (1-\mu_{\min})$.

In all cases, the impact of $\mu_{\min}$ is straightforward. ■

Example 7: Expected share of confused as a function of n.

$$E\left[1-\mu\right] = \begin{cases} \dfrac{\left[1-n^{-1/(n-1)}/(1+n^{-1/(n-1)})\right]^n - 1}{(1/2)^n - 1} & \text{for} \quad \beta = 3/2 \\ n^{-1/(n-1)} \quad \text{for} \quad \beta = 2 \\ \dfrac{\left[2/(2-n^{-1/(n-1)})\right]^n - 1}{2^n - 1} & \text{for} \quad \beta = 3 \end{cases}.$$

Proof of Corollary 5.

Expected industry profit in equilibrium is

$$\pi_n = \begin{cases} (1-\mu_{\min})\dfrac{n(1-\lambda)^{n-1}(\beta-1)^n - 1}{(\beta-1)^n - 1} & \text{for} \quad \beta \neq 2 \\ (1-\mu_{\min})\dfrac{\lambda + n(1-\lambda)}{n} & \text{for} \quad \beta = 2 \end{cases}.$$

► Consider $\beta \neq 2$. Using (16), industry profit can be written as

$$\pi_n = \frac{(1-\mu_{\min})\left\{\dfrac{(\beta-1)^n}{\left[1+(\beta-2)(1-n^{-1/(n-1)})\right]^{n-1}} - 1\right\}}{(\beta-1)^n - 1} =$$

$$\frac{(1-\mu_{\min})\left[\beta-1-(\beta-2)n^{-1/(n-1)}\right]\left[\dfrac{\beta-1}{\beta-1-(\beta-2)n^{-1/(n-1)}}\right]^n - 1}{(\beta-1)^n - 1} =$$

$$\frac{(1-\mu_{\min})\left[\beta-1-(\beta-2)n^{-1/(n-1)}\right][\lambda+(1-\lambda)(\beta-1)]^n - 1}{(\beta-1)^n - 1}.$$

As $\lim_{n\to\infty}\left[\beta-1-(\beta-2)n^{-1/(n-1)}\right] = 1$, it follows that

$\lim_{n\to\infty} \pi_n = \lim_{n\to\infty} \dfrac{(1-\mu_{\min})[\lambda+(1-\lambda)(\beta-1)]^n - 1}{(\beta-1)^n - 1}$.

But, the RHS is $\lim_{n\to\infty} E[1-\mu]$. Using Corollary 4, the results follow.

► Consider $\beta = 2$, using (17), $\lim_{n\to\infty} \pi_n = \lim_{n\to\infty}(1-\mu_{\min})[\dfrac{\lambda}{n} + (1-\lambda)]$. Using Corollary 3, the result follows. ■

Proof of Corollary 5.

▶ Consider $\beta \neq 2$.

Using the expressions in Corollaries 4 and 5,

$$\pi_n - E[1-\mu] = (1-\mu_{\min}) \left\{ \frac{n(1-\lambda)^{n-1}(\beta-1)^n - 1}{(\beta-1)^n - 1} - \frac{[1+(1-\lambda)(\beta-2)]^n - 1}{(\beta-1)^n - 1} \right\} =$$

$$(1-\mu_{\min}) \frac{n(1-\lambda)^{n-1}(\beta-1)^n - [1+(1-\lambda)(\beta-2)]^n}{(\beta-1)^n - 1} =$$

$$(1-\mu_{\min}) \frac{(\beta-1)[1+(1-\lambda)(\beta-2)]^{n-1} - [1+(1-\lambda)(\beta-2)]^n}{(\beta-1)^n - 1},$$

where the last equality uses step 3 in the proof of Proposition 3.

Then, $\pi_n - E[1-\mu] = (1-\mu_{\min}) \dfrac{[1+(1-\lambda)(\beta-2)]^{n-1}(\beta-2)(2-\lambda)}{(\beta-1)^n - 1} > 0.$

▶ Consider $\beta = 2$.

$$\pi_n - E[1-\mu] = (1-\mu_{\min}) \left[\frac{\lambda + n(1-\lambda)}{n} - (1-\lambda) \right] = \frac{(1-\mu_{\min})\lambda}{n} > 0. \ \blacksquare$$

Online Appendix

Substitutability vs. Complementarity in Confusion

Recall that the profile of firms' price complexity choices is $\mathbf{k}$, an n vector whose i-th component is k_i. The vector $\mathbf{k}_{-i}$ is obtained from $\mathbf{k}$ by omitting the i-th component, so $(k_i, \mathbf{k}_{-i}) = \mathbf{k}$. Denote by $\mathbf{k}_m$ an $n-1$ vector with m components equal to $\underline{k}$ and the remaining $n-1-m$ components equal to $\bar{k}$, for $0 \leq m \leq n-1$.

The firms' price complexity levels determine the fraction of experts in the market, which is given by the following function

$$\varphi \colon \{\underline{k}, \bar{k}\}^n \to [0,1] \ .$$

An increase in one firm's price complexity level lowers the comparability of competing offers and therefore the fraction of experts in the market, so $\varphi(\underline{k}, \mathbf{k}_{-i}) > \varphi(\bar{k}, \mathbf{k}_{-i})$ for all $i \in N$. For expositional simplicity and without loss of generality, let $\varphi(\underline{k}, \mathbf{k}_{n-1}) = 1$. To capture the impact of a rival's price complexity on the difficulty of evaluating a firm's offer let

$$\varphi(\underline{k}, \mathbf{k}_m) - \varphi(\bar{k}, \mathbf{k}_m) = \frac{(\varphi(\underline{k}, \mathbf{k}_{m-1}) - \varphi(\bar{k}, \mathbf{k}_{m-1}))}{\beta - 1} \ ,$$

for $\beta > 1$. Using $\varphi(\underline{k}, \mathbf{k}_{n-1}) = 1$, (2) implies that

$$1 - \varphi(\bar{k}, \mathbf{k}_{n-1}) = \frac{(\varphi(\underline{k}, \mathbf{k}_m) - \varphi(\bar{k}, \mathbf{k}_m))}{(\beta-1)^{n-1-m}} = \frac{(\varphi(\underline{k}, \mathbf{k}_0) - \varphi(\bar{k}, \mathbf{k}_0))}{(\beta-1)^{n-1}} \ .$$

As $\varphi(\bar{k}, \mathbf{k}_m) = \varphi(\underline{k}, \mathbf{k}_{m-1})$, using (3),

$$\begin{aligned}\varphi(\bar{k}, \mathbf{k}_m) &= \frac{\varphi(\bar{k}, \mathbf{k}_0) \sum_{k=0}^{n-1-m} (\beta-1)^k + \sum_{k=n-m}^{n-1} (\beta-1)^k}{\sum_{k=0}^{n-1} (\beta-1)^k} \\ &= 1 - \frac{(1-\varphi(\bar{k}, \mathbf{k}_0))[(\beta-1)^{n-m}-1]}{(\beta-1)^n - 1} \ .\end{aligned}$$

Case 1. Substitutability in Confusion. *When $1 < \beta < 2$, the confusion technology, $(1-\varphi)$, satisfies strictly decreasing differences.*

Formally, for $\beta \in (1,2)$,

$$\left(1-\varphi(\bar{k}, k_m)\right) - \left(1-\varphi(\underline{k}, \mathbf{k}_m)\right) > \left(1-\varphi(\bar{k}, k_{m-1})\right) - \left(1-\varphi(\underline{k}, \mathbf{k}_{m-1})\right) \ .$$

so that a firm's incremental increase in price complexity is more effective in creating confusion when rivals' price complexity is lower (in the sense that fewer competitors employ $\bar{k}$).

Case 2. Complementarity in Confusion. *When $\beta > 2$, the confusion technology, $(1-\varphi)$, satisfies strictly increasing differences.*

For $\beta > 2$,

$$\left(1-\varphi(\bar{k}, k_m)\right) - \left(1-\varphi(\underline{k}, \mathbf{k}_m)\right) < \left(1-\varphi(\bar{k}, k_{m-1})\right) - \left(1-\varphi(\underline{k}, \mathbf{k}_{m-1})\right) \ .$$

so that a firm's incremental increase in price complexity is more effective when rivals' price complexity is higher.

Case 3. No Second Order Effects in Confusion. *When $\beta = 2$, the price complexity of one firm does not affect the effectiveness of an increase in a rival's complexity.*

$$\left(1-\varphi(\bar{k}, k_m)\right) - \left(1-\varphi(\underline{k}, \mathbf{k}_m)\right) = \left(1-\varphi(\bar{k}, k_{m-1})\right) - \left(1-\varphi(\underline{k}, \mathbf{k}_{m-1})\right) \ .$$

so that a firm's incremental increase in price complexity is independent of rivals' price complexity levels.

Proof of Proposition 4. First note that the incremental profitability of an increase in complexity in this case is given by $\pi_i\left(p, \bar{k}\right) - \pi_i\left(p, \underline{k}\right) =$

$p \int_p^1 ... \int_p^1 \left[\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m \left(\mu(n-m) - \mu(n-m-1)\right)\right] dF(p_j)...dF(p_k) +$

$\frac{p}{n} \int_{p_0}^1 ... \int_{p_0}^1 \left[\sum_{m=0}^{n-1} \mathsf{P}_{n-1}^m (\mu(n-m-1) - \mu(n-m))\right] dF(p_j)...dF(p_k) =$

$p\left(1-\mu(1)\right)\left[-(1-F(p))^{n-1} + \frac{1}{n}\right]$.

But, at $p = p_0$, the term above is negative, while at $p = 1$, it is positive. As $F(p)$ is strictly increasing in p, $\pi\left(p, \bar{k}\right) - \pi_i\left(p, \underline{k}\right) < 0$ for $p < \hat{p}$ and firms choose $\underline{k}$, while $\pi\left(p, \bar{k}\right) - \pi_i\left(p, \underline{k}\right) > 0$

for $p > \hat{p}$ and firms choose $\bar{k}$. The cut-off price $\hat{p}$ solves $(1 - F(\hat{p}))^{n-1} = 1/n$.

Step 1 in the proof of Proposition 3 carries over unchanged. Then, the constant profit condition requires that

$\pi(\hat{p}, \underline{k}) = \lim_{p \to \hat{p}} \pi(\hat{p}, \bar{k}) \Leftrightarrow$

$(1-\lambda)^{n-1}(\mu(n-1) - \mu(n)) = \dfrac{1}{n} \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} (\mu(n-m-1) - \mu(n-m)) \Leftrightarrow$

$(1-\lambda)^{n-1} = \dfrac{1}{n} \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \Leftrightarrow$

$(1-\lambda)^{n-1} = \dfrac{1}{n}$ and (17) follows.

Equilibrium profit (as a function of λ) obtains by evaluating the expected profit at $p = 1$ and using (4).

$\pi(1, \bar{k}) = \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \dfrac{(1-\mu(n-m))}{n} =$

$(1-\mu(n)) \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \dfrac{(n-m)}{n^2} =$

$(1-\mu(n)) \dfrac{\lambda + n(1-\lambda)}{n^2}$ and (18) follows.

Below, we identify the equilibrium boundary prices and pricing c.d.f.s.

$\pi(p_0, \underline{k}) = p_0 \left\{ \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \left[\mu(n-m-1) + \dfrac{1}{n}(1-\mu(n-m-1)) \right] \right\} =$

$p_0 \left\{ \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \left[1 - \dfrac{n-1}{n}(1-\mu(n-m-1)) \right] \right\} =$

$p_0 \left\{ 1 - (1-\mu(n)) \dfrac{(n-1)^2(1-\lambda)}{n^2} \right\}.$

Then, using $\pi(p_0, \underline{k}) = \pi$, where the RHS is defined in (18), it follows that

$p_0 = \dfrac{(1-\mu(n))[\lambda + n(1-\lambda)]}{n^2 - (1-\mu(n))(n-1)^2(1-\lambda)}.$

$\pi(\hat{p}, \bar{k}) = \hat{p} \left[(1-\lambda)^{n-1}\mu(n) + \sum_{m=0}^{n-1} C_{n-1}^{m} \lambda^m (1-\lambda)^{n-1-m} \dfrac{(1-\mu(n-m))}{n} \right] =$

$\hat{p} \left[(1-\lambda)^{n-1}\mu(n) + (1-\mu(n)) \dfrac{\lambda + n(1-\lambda)}{n^2} \right].$

Then, using $\pi(\hat{p}, \bar{k}) = \pi$, where the RHS is defined in (18), it follows that

$\hat{p} = \dfrac{(1-\mu(n))[\lambda + n(1-\lambda)]}{n - (1-\mu(n))(n-1)\lambda}.$

It is easy to check that $\hat{p} \leq 1$, with equality if and only if $\mu(n) = 0$, and that $p_0 < \hat{p}$.

$F(p)$ is implicitly determined by $\pi(\underline{k}, p) = \pi$ and $\pi(\bar{k}, p) = \pi$.

For $p \in [p_0, \hat{p})$, $F(p)$ solves

$\sum_{m=0}^{n-1} C_{n-1}^{m} \left[(\lambda - F(p))^m (1-\lambda)^{n-1-m} \mu(n-m-1) + \lambda^m (1-\lambda)^{n-1-m} \dfrac{(1-\mu(n-m-1))}{n} \right] =$

$\dfrac{\pi}{p} \Leftrightarrow$

$\sum_{m=0}^{n-1} C_{n-1}^{m}(\lambda - F(p))^{m}(1-\lambda)^{n-1-m}\mu(n-m-1) +$
$\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\frac{(1-\mu(n-m-1))}{n} = \frac{\pi}{p} \Leftrightarrow$
$\frac{1}{n}(1-F(p))^{n-2}\left\{(1-\lambda)[1+\mu(n)(n-1)] + (\lambda - F(p))n\right\} +$
$\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\frac{(1-\mu(n-m-1))}{n} = \frac{\pi}{p}.$

It is straightforward to see that both the LHS and the RHS above are decreasing in $F(p)$. So $F(p)$ is strictly increasing in p in this interval.

For $p \in [\hat{p}, 1]$, if $\mu(n) > 0$, $F(p)$ solves
$(1-F(p))^{n-1}\mu(n) + \sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}\frac{(1-\mu(n-m))}{n} = \frac{\pi}{p} \Leftrightarrow$
$(1-F(p))^{n-1}\mu(n) + \frac{(1-\mu(n))}{n^2}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(n-m) = \frac{\pi}{p}$
$(1-F(p))^{n-1}\mu(n) + (1-\mu(n))\frac{\lambda + n(1-\lambda)}{n^2} = \frac{\pi}{p}$
$(1-F(p))^{n-1}\mu(n) = \pi\left(\frac{1}{p} - 1\right).$

It is easy to see that $F(p)$ is strictly increasing in this interval.

No Profitable Unilateral Deviations.

Consider a unilateral deviation to $(p < p_0, \underline{k})$. The deviator's market share is equal to that corresponding to $p = p_0$ but, as the price is lower, this deviation is not profitable.

Consider a unilateral deviation to $(p > \hat{p}, \underline{k})$. This results in deviation profit
$\pi^{d}(p, \underline{k}) = p\left[(1-F(p))^{n-1}\mu(n-1) + \frac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}\lambda^{m}(1-\mu(n-m-1))\right].$
But $\pi^{d}(p, \underline{k}) < \pi\left(p, \bar{k}\right) \Leftrightarrow$
$(1-F(p))^{n-1}(\mu(n-1) - \mu(n)) < \frac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m}(\mu(n-m-1) - \mu(n-m)) \Leftrightarrow$
$(1-F(p))^{n-1} < \frac{1}{n}\sum_{m=0}^{n-1} C_{n-1}^{m}\lambda^{m}(1-\lambda)^{n-1-m} \Leftrightarrow (1-F(p))^{n-1} < \frac{1}{n}$ as $(1-F(p))^{n-1} < (1-\lambda)^{n-1}$.

Consider a unilateral deviation to $\left(p < \hat{p}, \bar{k}\right)$. This results in deviation profit.
$\pi^{d}\left(p, \bar{k}\right) = p\left\{\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}[(\lambda - F(p))^{m}\mu(n-m) + \frac{\lambda^{m}(1-\mu(n-m))}{n}]\right\}.$
But, $\pi^{d}\left(p, \bar{k}\right) < \pi(p, \underline{k})$ as
$\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}(\mu(n-m-1) - \mu(n-m))[-(\lambda - F(p))^{m} + \frac{\lambda^{m}}{n}] < 0 \Leftrightarrow$
$\sum_{m=0}^{n-1} C_{n-1}^{m}(1-\lambda)^{n-1-m}[-(\lambda - F(p))^{m} + \frac{\lambda^{m}}{n}] < 0 \Leftrightarrow -(1-F(p)^{n-1} + \frac{1}{n} < 0$ as $(1-F(p)^{n-1} > (1-\lambda)^{n-1}$. ■

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