



Munich Personal RePEc Archive

# **How Should A Government Finance for Pension Benefit?**

Masaya Yasuoka

Kwansei Gakuin University

20 June 2018

Online at <https://mpra.ub.uni-muenchen.de/87483/>

MPRA Paper No. 87483, posted 24 June 2018 16:24 UTC

# How Should A Government Finance for Pension Benefit?\*

Masaya Yasuoka<sup>†</sup>

June 20, 2018

## Abstract

Based on Ono (2010), this short note presents consideration of the consumption tax and examines how tax reform to maintain the neutrality of pension benefit affects income growth rate and the employment rate. A decrease in the contribution rate of workers with an increase in consumption tax raises employment, but the effect on income growth is ambiguous. A decrease in the contribution rate of firms with an increase in consumption tax decreases the employment and facilitates income growth.

**JEL Classification:** H51, H21, J14

**Keywords:** Aging society, Elderly care services, Tax system, Endogenous growth

---

\*Research for this paper was supported financially by JSPS KAKENHI No. 17K03791. Nevertheless, any remaining error is the author's responsibility.

<sup>†</sup>Corresponding to: School of Economics, Kwansai Gakuin University, 1-155 Uegahara Ichiban-Cho, Nishinomiya, Hyogo 662-8501 Japan, Tel.:+81-798-54-6993, E-mail: yasuoka@kwansai.ac.jp

# 1 Introduction

This paper presents examination of how a government should collect revenue for pension benefits. As shown in Table.1, some differences exist in burden sharing among workers, firms, and governments in some OECD countries. In Japan, the contribution rate of pensions continues to increase and the national government burden sharing of basic pensions was raised from one-third to one-half in 2009. Because of this increased national government burden, the consumption tax rate in Japan was raised from 5% to 8% in 2014. However, this pension reform does not change the level of pension benefits. Why does the government consider a consumption tax? The answer derives from Japan's characteristic aging society with fewer children. In this society, the total revenue to provide the pension benefit decreases or the burden of the workers per capita must continue increasing because of a decrease in the number of workers. This is not a sustainable pension system. However, with the consumption tax, not only workers but also older people pay a consumption tax. Thereby, the government can collect sufficient revenues to provide pension benefits.

[Insert Table. 1 around here.]

Based on the Ono (2010) model setting, this short note presents examination of the effect of an increase in consumption tax instead of a decrease in the contribution rate of the workers or firms to maintain a pension benefit, on the employment rate and the income growth rate. Ono (2010) describes consideration of the contribution rate of the workers and the firms and examines how the means to finance pension benefits affect the employment rate and the income growth rate in the overlapping generations model with unemployment caused by labor unions. Results of this short paper show that a decrease in the contribution rate of workers with an increase in consumption tax raises employment. However, the effect on the income growth is ambiguous. If the government decreases the contribution rate of the firms and increases the consumption tax, then employment decreases and income growth increases. These results imply how the government should collect the revenues to fund pension benefits.

Some studies derive how the pension policy affects income growth. Wigger (1999) describes that a pay-as-you-go pension reduces the income growth rate. However, Yoon and Talmain (2001) demonstrate that income growth in a pay-as-you-go pension system is greater than that in the fully funded system.

A decrease in the contribution rate can raise the income growth rate. Then the pension benefit can be pulled up, as derived by Fanti and Gori (2010). Lin and Tian (2003) report that the consumption tax to finance pension benefits reduces the capital stock per capita in a model with an endogenous labor supply and population growth. Although no report of the relevant literature explains how the consumption tax affects the employment rate in the case of the neutrality of pension benefit, this tax reform should be examined because the government in Japan reformed the pension system as described above. The remainder of this paper consists of the following. Section 2 explains the model settings. Section 3 derives the equilibrium. Section 4 presents examination of the effect of pension reform on the employment and the income growth. The final section concludes this paper.

## 2 Model

This model economy includes agents of four types: households, firms, government and labor union. This section explains the model settings.

### 2.1 Household

Individuals live in two periods: young and old periods. My paper presents consideration of an overlapping generations model. A young generation and old generation exist in each period. The household utility function  $U_t$  is assumed by the following log utility function as

$$U_t = \alpha \ln c_{1t} + (1 - \alpha) \ln c_{2t+1}, \quad 0 < \alpha < 1. \quad (1)$$

In that equation,  $c_{1t}$  and  $c_{2t+1}$  respectively denote consumption by young and old people.  $t$  signifies the period. In the young period, younger people work inelastically to gain wage income, which is allocated into the consumption in the young period and to savings that must be consumed during the old period. However, this paper presents consideration of unemployment: some young people can work; others cannot work because of a lack of available jobs. The government imposes a tax burden to provide pension benefits for older people and the benefit for unemployment. Then, the household's lifetime budget constraint of worker is shown below.

$$(1 + \tau_c)c_{1t} + \frac{(1 + \tau_c)c_{2t+1}}{1 + r_{t+1}} = (1 - \tau_l - \sigma)w_t + \frac{P_{t+1}}{1 + r_{t+1}}. \quad (2)$$

In that equation,  $1 + r_{t+1}$  and  $w_t$  respectively denote an interest rate and a wage rate.  $\tau_l$  ( $0 < \tau_l < 1$ ) denotes the labor income tax rate (contribution rate of workers) to finance for pension benefit  $P_{t+1}$ .  $\sigma$  ( $0 < \sigma < 1$ ) denotes the tax rate for unemployment benefit.  $\tau_c$  ( $0 < \tau_c$ ) denotes the consumption tax rate for pension benefit.

If households are unemployed, then they can obtain unemployment benefit  $u_t$ . Income taxation  $\tau_l$ ,  $\sigma$  for pension benefits and unemployment are exempted. Then, the budget constraint of the unemployed household is shown as

$$(1 + \tau_c)c_{1t} + \frac{(1 + \tau_c)c_{2t+1}}{1 + r_{t+1}} = u_t + \frac{\eta P_{t+1}}{1 + r_{t+1}}, \quad 0 < \eta < 1. \quad (3)$$

An unemployed household is assumed to obtain a pension benefit during the old period that is less than  $P_{t+1}$  because of  $0 < \eta < 1$ . This pension setting is the same as that described by Ono (2010).<sup>1</sup>

With (1) and (2), the optimal allocations of workers,  $c_{1t}^w$  and  $c_{2t+1}^w$  are derived as

$$c_{1t}^w = \frac{\alpha}{1 + \tau_c} \left( (1 - \tau_l - \sigma)w_t + \frac{P_{t+1}}{1 + r_{t+1}} \right), \quad (4)$$

$$c_{2t+1}^w = \frac{(1 - \alpha)(1 + r_{t+1})}{1 + \tau_c} \left( (1 - \tau_l - \sigma)w_t + \frac{P_{t+1}}{1 + r_{t+1}} \right). \quad (5)$$

With (1) and (3), the optimal allocations of unemployment,  $c_{1t}^u$  and  $c_{2t+1}^u$  are derived as

$$c_{1t}^u = \frac{\alpha}{1 + \tau_c} \left( u_t + \frac{\eta P_{t+1}}{1 + r_{t+1}} \right), \quad (6)$$

$$c_{2t+1}^u = \frac{(1 - \alpha)(1 + r_{t+1})}{1 + \tau_c} \left( u_t + \frac{\eta P_{t+1}}{1 + r_{t+1}} \right). \quad (7)$$

## 2.2 Firm

Firms produce final goods with capital stock and labor input in a perfectly competitive market. The product function is assumed as

$$Y_t = K_t^\theta (A_t L_t)^{1-\theta}, \quad 0 < \theta < 1. \quad (8)$$

Therein,  $Y_t$  denotes the final goods.  $K_t$  and  $L_t$  denote the capital stock and the labor input in  $t$  period.

$A_t$  denotes the labor productivity, which is assumed as  $A_t \equiv a \frac{K_t}{L_t}$  ( $0 < a$ ).<sup>2</sup> As shown by Ono (2010), this

paper assumes that the government levies a wage-based tax burden for the firms at the rate (contribution

<sup>1</sup>In Japan, employed households are exempted from paying the pension premium. However, the pension benefit that one can obtain in the old period is less than the pension benefit of the household that is not unemployed in the young period.

<sup>2</sup>Romer (1986) sets the endogenous growth model with externality of physical capital. Grossman and Yanagawa (1993) specify the externality of physical capital such as  $A_t = a \frac{K_t}{L_t}$ .

rate of the firms) of  $\tau_f$  ( $0 < \tau_f < 1$ ) because of the pension benefit for older people. Then, the firm's profit  $\pi_t$  is given as

$$\pi_t = K_t^\theta (A_t L_t)^{1-\theta} - (1 + \tau_f) w_t L_t - (1 + r_t) K_t. \quad (9)$$

Given  $A_t$  and maximizing the firm's profit (9) in a competitive market, demand for the physical capital stock and labor input is

$$w_t = \frac{(1 - \theta) A_t^{1-\theta} K_t^\theta L_t^{-\theta}}{1 + \tau_f}, \quad (10)$$

$$1 + r_t = \theta K_t^{\theta-1} (A_t L_t)^{1-\theta}. \quad (11)$$

Presumably, the physical capital stock is fully depreciated in a single period.

### 2.3 Government

The government provides not only the pension benefit for elderly people but also benefits to compensate for unemployment. The pension benefit for older people is financed by taxation for workers and firms. Then, assuming a balanced budget, the budget constraint for the pension benefit for older people is

$$P_{t+1} L_t + \eta P_{t+1} (N - L_t) = (\tau_l + \tau_f) w_{t+1} L_{t+1} + \tau_c (L_t c_{1t}^w + (N - L_t) c_{1t}^u + L_{t-1} c_{2t}^w + (N - L_{t-1}) c_{2t}^u), \quad (12)$$

where  $N$  denotes the total population of younger people, which is assumed to be constant over time. Furthermore,  $L_t$  and  $N - L_t$  respectively denote the population size of workers and of unemployment. If the unemployment benefit is given by the balanced budget, then the following equation is obtained:

$$(N - L_t) u_t = \sigma w_t L_t. \quad (13)$$

### 2.4 Labor Union

This model includes a labor union. The labor union cares not only about the household lifetime income of workers, but also their unemployment. The labor union chooses wage rate  $w_t$  to maximize the following function as considered by Ono (2010) subject to the labor demand function (10).<sup>3</sup>

$$V_t = L_t \left( (1 - \tau_l - \sigma) w_t + \frac{P_{t+1}}{1 + r_{t+1}} \right) + (N - L_t) \left( u_t + \frac{\eta P_{t+1}}{1 + r_{t+1}} \right). \quad (14)$$

---

<sup>3</sup>Some studies have examined the labor union to bring about unemployment. Ono (2010) sets the model by which the labor union cares about the lifetime income of a household with employed and unemployed members. Corneo and Marquardt (2000) consider a Nash negotiation solution within the wage rate and unemployment rate. Daveri and Tabellini (2000) assume the objective function of a labor union that includes only the income in the younger period. Then, because no pension benefit is considered, the effect of pension benefit caused by the policy does not exist.

The labor union maximizes the objective function  $V_t$  subject to the labor demand (10). Then, substituting (10) into (14), the wage rate  $w_t$  to maximize  $V_t$  is derived as

$$w_t = \frac{u_t - \frac{(1-\eta)P_{t+1}}{1+r_{t+1}}}{(1-\theta)(1-\tau_l-\sigma)}. \quad (15)$$

Considering (10) and (15), the population of workers  $L_t$  is determined. Then, the population of unemployed workers  $N - L_t$  is obtainable as well.

### 3 Equilibrium

Considering  $A \equiv a \frac{K_t}{L_t}$ , the wage rate and interest rate are derived as

$$w_t = \frac{(1-\theta)a^{1-\theta} K_t}{1+\tau_f L_t}, \quad (16)$$

$$1+r_t = \theta a^{1-\theta}. \quad (17)$$

The interest rate is constant over time. The aggregate output is derived as  $Y_t = a^{1-\theta} K_t$ . Then, an increase in capital stock represents the income growth. Aggregate household's saving is given as  $S_t \equiv L_t s_t^w + (N - L_t) s_t^u$ , where  $s_t^w \equiv (1-\tau_l-\sigma)w_t - (1+\tau_c)c_{1t}^w$  and  $s_t^u \equiv u_t - (1+\tau_c)c_{1t}^u$  respectively represent the savings of workers and unemployed people. Considering (4) and (6),  $s_t^c$  and  $s_t^u$  are shown as

$$\begin{aligned} s_t^w &= (1-\alpha)(1-\tau_l-\sigma)w_t - \frac{\alpha P_{t+1}}{1+r_{t+1}}, \\ s_t^u &= (1-\alpha)u_t - \frac{\alpha \eta P_{t+1}}{1+r_{t+1}}. \end{aligned} \quad (18)$$

Then, aggregate saving  $S_t$  and the capital market clearing condition  $K_{t+1} = S_t$  are reduced to

$$\frac{K_{t+1}}{K_t} \equiv 1+g = \frac{(1-\theta)a^{1-\theta}(1-\alpha)(1-\tau_l)}{1+\tau_f} - \frac{\alpha N}{1+r} \frac{P_{t+1}}{K_t} ((1-\eta)l_t + \eta), \quad (19)$$

where  $l_t \equiv \frac{L_t}{N}$  and  $1-l_t$  respectively denote the employment rate and the unemployment rate. For given  $K_t$ , the capital stock  $K_{t+1}$  and the income growth rate  $1+g$  are determined as (19). Substituting (16) into (15), the employment rate  $l_t$  is given to support the following equation.

$$\frac{(1-\theta)(1-\tau_l-\sigma)}{l_t} = \frac{\sigma}{1-l_t} - \frac{(1+\tau_f)(1-\eta)P_{t+1}}{(1-\theta)a^{1-\theta}(1+r)K_t} \quad (20)$$

In that equation,  $P_{t+1}$  depends on employment rate  $l_t$ . For given  $K_t$ , the employment rate  $l_t$  is determined. Defining  $L$  and  $R$  as the left-hand-side and right-hand-side of (20), respectively, we can consider the following figure; we obtain the unique  $l_t$ .

[Insert Fig.1 around here.]

Without a consumption tax, the income growth rate is derived as the simple form of

$$1 + g = \frac{\frac{(1-\theta)(1-\alpha)a^{1-\theta}(1-\tau_l)}{1+\tau_f}}{1 + \frac{\alpha(1-\theta)a^{1-\theta}(\tau_l+\tau_f)}{(1+r)(1+\tau_f)}}. \quad (21)$$

Without a consumption tax,

$$\frac{(1-\theta)(1-\tau_l-\sigma)}{\frac{l_t}{1-l_t}} + \frac{(1-\eta)(\tau_l+\tau_f)(1+g)}{1+r} \frac{1}{\frac{l_t}{1-l_t} + \eta} = \sigma \quad (22)$$

can be obtained.  $l_t$  is given to satisfy this equation. We find unique  $l_t$  between 0 and 1.

## 4 Pension Reform

This paper presents examination of the introduction of consumption tax to finance the pension benefit to support the pension benefit level. The pension reform considered in this paper raises the consumption tax  $\tau_c$  and reduces the labor income tax rate  $\tau_l$  or the contribution rate for firm  $\tau_f$ .

### 4.1 Decrease in the labor income tax rate

This subsection presents consideration of the pension reform to introduce consumption tax  $\tau_c$  and to decrease the labor income tax rate  $\tau_l$ : the contribution rate of workers. From total differentiation of (20) with respect to  $\tau_l$  and  $l_t$  at the approximation of  $\tau_c = 0$  for given  $K_t$ , the sign of  $\frac{dl_t}{d\tau_l}$  is expressed as

$$\frac{dl_t}{d\tau_l} = -\frac{1-\theta}{\frac{\sigma l_t}{(1-l_t)^2} + \frac{(1-\theta)(1-\tau_l-\sigma)}{l_t}} < 0. \quad (23)$$

This negative sign signifies that a decrease in  $\tau_l$  with an increase in  $\tau_c$  raises the employment rate  $l_t$  and decreases the unemployment rate for any  $K_t$ . How does this tax reform affect the income growth rate  $1+g$ ? Total differentiation of (19) with respect to  $\tau_l$ ,  $\tau_c$ ,  $l_t$  and  $g$  at the approximation of  $\tau_c = 0$ ,  $\frac{dg}{d\tau_l}$  are derived as

$$\frac{dg}{d\tau_l} = -\frac{(1-\theta)a^{1-\theta}(1-\alpha)}{1+\tau_f} - \frac{\alpha(\tau_l+\tau_f)(1-\theta)(1-\eta)(1+g)}{\theta(1+\tau_f)((1-\eta)l_t+\eta)} \frac{dl_t}{d\tau_l}. \quad (24)$$

The sign of  $\frac{dg}{d\tau_l}$  is ambiguous. The following proposition can be established.

**Proposition 1** A pension reform that decreases the labor income tax rate and increases the consumption tax rate to maintain a pension benefit raises the employment rate. However, the effect on income growth rate is ambiguous.



A decrease in the labor tax rate  $\tau_l$  raises the workers' disposable income  $(1 - \tau_l - \sigma)w_t$ , which is pulled up for given  $w_t$ . Then, the labor union allows a decrease in wage rate  $w_t$  because the worker's disposable income rises. Then, a decrease in the wage rate raises the labor demand shown by (16). Therefore, the employment rate rises.

The effect of pension reform on the income growth is ambiguous because both a positive effect and the negative effect exist. A decrease in  $\tau_l$  raises household saving, which raises income growth. However, this pension reform raises the employment rate and reduces aggregate household saving because the aggregate pension benefit in the old period increases.

## 4.2 Decrease in Contribution Rate of the Firm

This subsection presents an examination of how a decrease in the contribution rate of the firm with an increase in consumption tax does not change the pension benefit. Total differentiation of (20) with respect to  $l_t$  and  $\tau_f$  at the approximation of  $\tau_c = 0$ , the sing of  $\frac{dl_t}{d\tau_f}$  is derived as

$$\frac{dl_t}{d\tau_f} = \frac{\frac{(1-\eta)(1+g)(\tau_f+\tau_l)}{(1+\tau_f)(l_t+\eta(1-l_t))}}{\frac{(1-\theta)(1-\tau_l-\sigma)}{l_t^2} + \frac{\sigma}{(1-l_t)^2}} > 0. \quad (25)$$

A decrease in  $\tau_f$  reduces employment. From total differentiation of (19) with respect to  $\tau_f$ , one can derive  $\tau_c$ ,  $l_t$ , and  $g$  at the approximation of  $\tau_c = 0$ ,  $\frac{dg}{d\tau_f}$  as

$$\frac{dg}{d\tau_f} = -\frac{(1-\theta)a^{1-\theta}(1-\alpha)(1-\tau_l)}{(1+\tau_f)^2} - \frac{\alpha(\tau_l+\tau_f)(1-\theta)(1-\eta)(1+g)}{\theta(1+\tau_f)((1-\eta)l_t+\eta)} \frac{dl_t}{d\tau_f} < 0. \quad (26)$$

Then, the following proposition is established.

**Proposition 2** A decrease in the contribution rate of firms with an increase in consumption tax reduces the employment rate and raises the income growth rate.

As shown by (16), a decrease in the contribution rate of firms  $\tau_f$  raises the wage rate. Then, the labor demand decreases. Considering (19), the effect on income growth is explainable. A decrease in  $\tau_f$  directly raises income growth as shown by the first term of (19). However, a decrease in  $l_t$  with a decrease in  $\tau_f$  reduces the aggregate pension  $((1-\eta)l_t+\eta)P_{t+1}$  and raises the income growth, as shown by the second term of (19).

## 5 Conclusions

This short note presented examination of how the government should use the consumption tax or not in pension reform. As described herein, a decrease in the contribution rate of firms with an increase in consumption tax raises unemployment even if income growth increases. However, a decrease in the contribution rate of workers with an increase in consumption tax reduces unemployment. Nevertheless, income growth is not always pulled up. Although the consumption tax is considered to be caused by a decrease in intergenerational inequality in an aging society with fewer children, this policy might bring about new problems such as a decrease in income growth or an increase in unemployment.

## References

- [1] Corneo G. and Marquardt M. 2000. Public Pensions, Unemployment Insurance, and Growth. *Journal of Public Economics* 75, 293-311.
- [2] Daveri F. and Tabellini G. 2000. Unemployment, Growth and Taxation in Industrial Countries. *Economic Policy* 15, 47-104.
- [3] Fanti L. and Gori L. 2010. Increasing PAYG Pension Benefits and Reducing Contribution Rates. *Economics Letters* 107, 81-84.
- [4] Grossman G. M. and Yanagawa N. 1993. Asset Bubbles and Endogenous Growth. *Journal of Monetary Economics* 31(1), 3-19.
- [5] Lin S. and Tian X. 2003. Population Growth and Social Security Financing. *Journal of Population Economics* 16, 91-110.
- [6] Ono T. 2010. Growth and Unemployment in An OLG Economy with Public Pensions. *Journal of Population Economics* 23, 737-767.
- [7] Romer P. M. 1986. Increasing Returns and Long-run Growth. *Journal of Political Economy* 94(5), 1002-1037.
- [8] Wigger B.U. 1999. Pay-As-You-Go Financed Public Pensions in A Model of Endogenous Growth and Fertility. *Journal of Population Economics* 12, 625-640.
- [9] Yoon Y. and Yalmain G. 2001. Endogenous Fertility, Endogenous Growth and Public Pension System: Should We Switch from A Pay-As-You-Go to A Fully Funded System? *Manchester School* 69(5), 586-605.

## Appendix

### Derivation of (15)

Substituting (10) into (14) and calculating  $\frac{dV_t}{dw_t}$ , we obtain (15) as

$$\begin{aligned}\frac{dV_t}{dw_t} &= \left( \frac{(1-\theta)A_t^{1-\theta}}{1+\tau_f} \right)^{\frac{1}{\theta}} K_t \left( (1-\theta)(1-\tau_l-\sigma)w_t^{-\frac{1}{\theta}} - \frac{1}{\theta} \frac{P_{t+1}}{1+r_{t+1}} w_t^{-\frac{1}{\theta}-1} \right) \\ &+ \frac{1}{\theta} \left( \frac{(1-\theta)A_t^{1-\theta}}{1+\tau_f} \right)^{\frac{1}{\theta}} K_t w_t^{-\frac{1}{\theta}-1} \left( u_t + \frac{\eta P_{t+1}}{1+r_{t+1}} \right) = 0, \\ &(\theta-1)(1-\tau_l-\sigma)w_t^{-\frac{1}{\theta}} - \frac{P_{t+1}}{1+r_{t+1}} w_t^{-\frac{1}{\theta}-1} + w_t^{-\frac{1}{\theta}-1} \left( u_t + \frac{\eta P_{t+1}}{1+r_{t+1}} \right) = 0, \\ &-(1-\theta)(1-\tau_l-\sigma)w_t = \frac{P_{t+1}}{1+r_{t+1}} - u_t - \frac{\eta P_{t+1}}{1+r_{t+1}}.\end{aligned}$$

### Derivation of (23)

From total differentiation of (20) with respect to  $l_t$ ,  $\tau_l$  and  $P_{t+1}$ , we obtain the following equation,

$$-\frac{1-\theta}{l_t} d\tau_l - \frac{(1-\theta)(1-\tau_l-\sigma)}{l_t^2} dl_t = \frac{\sigma}{(1-l_t)^2} dl_t - \frac{(1+\tau_f)(1-\eta)N}{(1-\theta)a^{1-\theta}(1+r)K_t} dP_{t+1}.$$

Given for  $K_t$  and noting that  $dP_{t+1} = 0$  because this paper presents consideration of that the level of pension benefit  $P_{t+1}$  does not change, we obtain (23).

### Derivation of (24)

From total differentiation of (19) with respect to  $g$ ,  $l_t$ ,  $\tau_l$  and  $P_{t+1}$ , we obtain

$$dg = -\frac{\alpha}{1+r} \frac{NP_{t+1}}{K_t} (1-\eta) dl_t - \frac{(1-\theta)a^{1-\theta}(1-\alpha)}{1+\tau_f} d\tau_l - \frac{\alpha N}{1+r} \frac{(1-\eta)l_t + \eta}{K_t} dP_{t+1}.$$

Noting that  $dP_{t+1} = 0$  and  $\frac{NP_{t+1}}{K_t} = \frac{(\tau_l+\tau_f)(1-\theta)a^{1-\theta}(1+g)}{(1+\tau_f)((1-\eta)l_t+\eta)}$ , we obtain (24).

### Derivation of (25)

From total differentiation of (20) with respect to  $l_t$ ,  $\tau_f$  and  $P_{t+1}$ , we obtain

$$-\frac{(1-\theta)(1-\tau_l-\sigma)}{l_t^2} dl_t = \frac{\sigma}{(1-l_t)^2} dl_t - \frac{(1-\eta)P_{t+1}N}{(1-\theta)a^{1-\theta}(1+r)K_t} d\tau_f - \frac{(1+\tau_f)(1-\eta)N}{(1-\theta)a^{1-\theta}(1+r)K_t} dP_{t+1}.$$

Noting that  $dP_{t+1} = 0$ , we obtain (25).

## Derivation of (26)

From total differentiation of (19) with respect to  $g$ ,  $l_t$ ,  $\tau_f$  and  $P_{t+1}$ ,

$$dg = -\frac{(1-\theta)a^{1-\theta}(1-\alpha)(1-\tau_l)}{(1+\tau_f)^2}d\tau_f - \frac{\alpha N}{1+r} \frac{(1-\eta)l_t + \eta}{K_t} dP_{t+1} - \frac{\alpha N}{1+r} \frac{P_{t+1}(1-\eta)}{K_t} dl_t.$$

Noting that  $dP_{t+1} = 0$ , we obtain (26).

	Japan	U.S.A.	U.K.	Germany	France	Sweden
Contribution Rate	18.3%	12.4%	25.8%	18.7%	17.45%	17.21%
(Worker)	9.15%	6.2%	12.0%	9.35%	7.15%	7.0%
(Firm)	9.15%	6.2%	13.8%	9.35%	10.3%	10.21%
Government Burden Share	*	0%	0%	27.3%	36.5%	**

Table 1 Pension Systems in Some OECD Countries.

\*: Japan government burden share is half of the basic pension. \*\*: Sweden government burden is only for guaranteed pensions. (Data: Ministry of Health, Labour and Welfare)

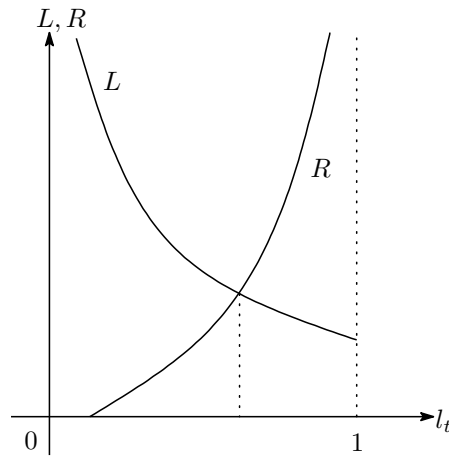


Figure 1 Unique Solution of  $l_t$ .