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Abstract

By introducing Romer (1990)'s endogenous technological change into the multisector growth model developed by Kongsamut, Rebelo and Xie (2001), the paper shows that human capital speeds up economic growth and structural changes in the multisector economy qualitatively and quantitatively.

Keywords: Human Capital; Endogenous Growth; Structural Change.

JEL Classification Numbers: O14; O41.

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1 Introduction

Kongsamut, Rebelo and Xie (2001, KRX) develops a three-sector nonbalanced growth model to explain the Kuznets facts which roughly refers to the massive reallocation of labor from agriculture into manufacturing and services. They argue that the differences in the income elasticity of demand for the different goods are the determining factors for structural changes and the exogenous technological progress is the driving force. However, if there is no technological change, i.e., the exogenous growth rate of technology equals zero, then structural changes never occur. Hence endogenizing the process of technological change is the key to deal with this kind of awkwardness for the demand-side channel[1] to explain structural changes. Moreover, combining structural change with endogenous growth in a unified growth model is helpful for policy and empirical analysis.

By introducing Romer (1990)'s endogenous technological change into the multisector growth model pioneered by KRX, the paper shows that the stock of human capital determines the rate of economic growth endogenously and enforces structural changes in the economy. Specifically, the larger the stock of human capital, the quicker the innovation and knowledge accumulation in the research sector, and the higher the economic growth rate. Meanwhile, since the income elasticity of demand in the agricultural sector is less than manufacturing and services sectors, along with eonomic growth, the employment and production shares of the manufacturing and services sectors increase gradually, and the two shares of agriculture decrease. That is, the industrial structure transforms gradually. Furthermore, endogenous economic growth determines the speeds of expansion or shrink of different sectors. The higher the endogenous growth rate, the quicker the shrink of the agriculture sector and the expansions of manufacturing and services sectors.

The rest of the paper is organized as follows. Section 2 presents our three-sector structural change model with endogenous technological changes. Section 3 describes the monopolistic competitive equilibrium, examines the generalized balanced growth path (GBGP) and derives the endogenous growth rate. Section 4 examines how endogenous economic growth enforces structural changes. Section 5 concludes.

[1] The supply-side channel to explain the phenomena of structural changes is discussed by Baumol (1967), Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008). Comin, Lashkari and Mestieri (2017) combines both two channel to explain structural changes and finds that income effects account for over 80% of the observed pattern of structural change.

2 The Model

2.1 Production

The production side of the economy consists of three sectors: a final-goods sector, an intermediate-goods sector, and a research sector. The final-goods sector is made up of three subsectors: agriculture, manufacturing and services. In each final-goods subsector, perfectly competitive firms produce a homogeneous final good using labor, human capital and all kinds of intermediate goods. Each subsector utilizes the constant return to scale Cobb-Douglas production function with different technological parameters and factor income shares. Specifically,

$$A_t = B_A \left(b_t^A H_Y(t)\right)^\alpha \left(N_t^A\right)^\beta \int_{i=0}^{T_t} \left(\phi_{it}^A x_{it}\right)^{1-\alpha-\beta} di, \tag{1}$$

$$M_t + \dot{K}_t + \delta K_t = B_M \left(b_t^M H_Y(t)\right)^\alpha \left(N_t^M\right)^\beta \int_{i=0}^{T_t} \left(\phi_{it}^M x_{it}\right)^{1-\alpha-\beta} di, \tag{2}$$

$$S_t = B_S \left(b_t^S H_Y(t)\right)^\alpha \left(N_t^S\right)^\beta \int_{i=0}^{T_t} \left(\phi_{it}^S x_{it}\right)^{1-\alpha-\beta} di, \tag{3}$$

where $B_i > 0, i \in \{A, M, S\}$ are three technological parameters; α, β, $(1-\alpha-\beta) \in (0,1)$; $H_Y(t)$ is the total amount of human capital used in the final-goods sector, b_t^A, b_t^M, and b_t^S are the shares used in these three subsectors, $b_t^A + b_t^M + b_t^S = 1$; all of the labor force are employed in the final-goods sector and normalized to one, N_t^A, N_t^M, and N_t^S stand for the shares/amounts employed in the three subsectors, $N_t^A + N_t^M + N_t^S = 1$; each subsector utilizes all kinds of intermediate-goods in its production, ϕ_{it}^A, ϕ_{it}^M, and ϕ_{it}^S stand for the shares of demand for the intermediate-good i, $\phi_{it}^A + \phi_{it}^M + \phi_{it}^S = 1$, $i \in [0, T_t]$; the outputs of agriculture (A_t) and services (S_t) can be used for consumption, and the output of manufacturing can be consumed (M_t) or invested $\left(\dot{K}_t + \delta K_t\right)$, then equations (1), (2), and (3) are also market-clearing conditions for the three subsectors; the knowledge stock (T_t) of the economy is determined endogenously by the amout $(H_A(t))$ of human capital utilized in the knowledge sector and the current knowledge stock (T_t); finally, the prices of the products of the three final-goods are positive constants, P_A, P_M, and P_S.

The profit-maximization problem of each subsector will be then discussed. By taking the price of its product P_i, the wage rates of the labor force and human capital w_{Lt}, w_{Ht}, and the prices of all intermediate-goods $\{p_{jt}, j \in [0, T_t]\}$ as given, subsector $i \in \{A, M, S\}$ solves the problem,

$$\max_{\left\{N_t^i, b_t^i, \phi_{jt}^i, j \in [0.T_t]\right\}} \pi_i(t) = \left\{ \begin{array}{c} P_i B_i \left(b_t^i H_Y(t)\right)^\alpha \left(N_t^i\right)^\beta \int_{j=0}^{T_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj \\ -w_{Lt} N_t^i - w_{Ht} b_t^i H_Y(t) - \int_{j=0}^{T_t} p_{jt} \phi_{jt}^i x_{jt} dj \end{array} \right\}.$$

The FOCs with respect to N_t^i, b_t^i, and ϕ_{jt}^i are as follows

$$P_i B_i \left(b_t^i H_Y(t)\right)^{\alpha} \beta \left(N_t^i\right)^{\beta-1} \int_{j=0}^{T_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj = w_{Lt}, \tag{4}$$

$$\alpha P_i B_i \left(b_t^i H_Y(t)\right)^{\alpha-1} \left(N_t^i\right)^{\beta} \int_{j=0}^{T_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj = w_{Ht}, \tag{5}$$

$$P_i B_i \left(b_t^i H_Y(t)\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta} = p_{jt}. \tag{6}$$

The first two optimality conditions show that the marginal product values of labor force and human capital equal the wage rates of them in each subsector i. The third one displays that the equality between marginal revenue and marginal cost for any intermediate good j in each subsector i, which are also the (inverse) demand functions for any intermediate good j in each subsector i.

Knowledges are designs for the intermediate goods. Being created and granted a patent, a piece of knowledge can be used to produce a kind of intermediate good. The types of these intermediate goods are determined by the knowledge stock created by the research sector. Hence the amount of the types of intermediate-goods is essentially the knowledge stock T_t. Any kind of intermediate good is produced by a single monopolistic firm. The decision process of any monopolistic firm can be separated into two steps: first, he pays the price P_{jTt} to buy the patent for producing intermediate good j in the competitive patents market, which is the sunk cost for the monopolistic firm. Since the patents market is competitive, the price of new design j is the present value of the profits flow $\{\pi_{j\tau}\}_{\tau=t}^{\infty}$ generated by monopolistic firm j, $P_{jTt} = \int_{\tau=t}^{\infty} \exp\left(-\int_{s=t}^{\tau} r(s)\, ds\right) \pi_{j\tau} d\tau$. Second, in the monopoly pricing problem, monopolistic firm j rents capital (as variable costs) and produces intermediate goods j to satisfy the demand of the final-goods sector for its products. It is assumed that the unit cost for any intermediate good is the same $\eta\,(>0)$ units of capital. Then the profit-maximization problem of any monopolistic firm j can be summerized as: $\pi_{jt} = \max_{p_{jt}, x_{jt}} p_{jt}x_{jt} - r\eta x_{jt}$.Substituting the demand functions for the intermediate good j (equation (6)) into π_{jt}, we can derive the following necessary conditions:

$$r(t)\,\eta = P_i B_i \left(b_t^i H_Y(t)\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta)^2 \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta}, i \in \{A, M, S\}. \tag{7}$$

Combining (6) and (7) yields us

$$p_{jt} = \frac{1}{1-\alpha-\beta} r_t \eta \equiv p_t. \tag{8}$$

Note that $r_t\eta$ is the marginal cost for producing another unit of intermediate good, and $\frac{1}{1-\alpha-\beta}\,(>1)$ is the mark-up over marginal cost. In order to earn the monopoly profit, all monopolistic firms price their products over their marginal costs. Moreover, all monopolistic firms set the same monopoly price, which shows that the pricing behaviors display some symmetry.

It is known from equations (6), (7), and (8) that at optimum, $\left\{\phi_{jt}^{i} x_{jt} : i \in \{A, M, S\}\right\}$ do not depend on j, that is, the optimal demand for each intermediate good is the same among the three subsectors. Since $\phi_{jt}^{A} x_{jt} + \phi_{jt}^{M} x_{jt} + \phi_{jt}^{S} x_{jt} = x_{jt}$ does not depend on j, i.e., $x_{jt} = x_t$, $\left\{\phi_{jt}^{i} : i \in \{A, M, S\}\right\}$ do not depend on j either. That is, these three subsectors in final-goods sector use the same share of each intermediate good, namely,

$$\phi_{jt}^{A} = \phi_{t}^{A}, \phi_{jt}^{M} = \phi_{t}^{M}, \phi_{jt}^{S} = \phi_{t}^{S}. \tag{9}$$

Furthermore, each monopoly firm earns the same monopoly profit, namely,

$$\pi_{jt} = (\alpha + \beta) p_t x_t = \pi_t, j \in [0, T_t]. \tag{10}$$

Combining equation (4) and (5) leads to the first efficiency condition in production:

$$\frac{b_t^A}{N_t^A} = \frac{b_t^M}{N_t^M} = \frac{b_t^S}{N_t^S} = 1, \tag{11}$$

which displays that at optimum each subsector in the final-goods sector utilizes the same weight for both labor and human capital. Combining equation (4) and (6) leads to the second efficiency condition in production:

$$\frac{\phi_t^A}{N_t^A} = \frac{\phi_t^M}{N_t^M} = \frac{\phi_t^S}{N_t^S} = 1, \tag{12}$$

which shows that at optimum each subsector in the final-goods sector uses the same weight for both all intermediate goods and labor. Due to (11) and (12) , we have the following

Proposition 1 *The optimal weights of labor, human capital and all intermediate goods employed in each subsector of the final-goods sector are equal, namely,*

$$N_t^A = b_t^A = \phi_t^A, N_t^M = b_t^M = \phi_t^M, N_t^S = b_t^S = \phi_t^S. \tag{13}$$

Substituting (13) into (6), we have the following

Proposition 2 *At optimum the price and technological parameters of these three subsectors in the final-goods sector satisfy the second efficiency condition:*

$$P_A B_A = P_M B_M = P_S B_S. \tag{14}$$

The research sector uses human capital $H_A(t)$ and the existing stock of knowledge T_t to produce new knowledge, with the following knowledge production function

$$\dot{T}_t = \delta H_A(t) T_t, \tag{15}$$

where $\delta (> 0)$ is the productivity parameter. The knowledge production function shows that devoting more human capital to research leads to a higher production rate of new designs, and the larger the total stock of designs, the higher the productivity of the researchers employed in the research sector will be. Due to its partially excludability and nonrivalry of consumtion, the production of knowledge cannot be determined by the private maximizing behavior. The evolution of knowledge however follows the trajectory described by equation (15). In the paper, knowledge refers to designs for new intermediate goods. Then the accumulation of knowledge represents the increase of the types of intermediate goods.

Since human capital is freely mobile, no arbitrage requires it has the same rate of return among the research sector and three subsectors in the final-goods sector, namely,

$$P_T(t)\,\delta T_t = P_i B_i \alpha H_Y(t)^{\alpha-1} T_t x_t^{1-\alpha-\beta}, i \in \{A, M, S\}. \tag{16}$$

2.2 Optimal Consumption and Savings

The representative consumer makes production and capital accumulation decisions in order to maximize the discounted utility of consumption stream for three final goods, namely,

$$\max_{\{A_t, M_t, S_t, K_{t+1}\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{1-\sigma} - 1}{1-\sigma} dt \right\}, \tag{17}$$

subject to the flow budget constraint (FBC):

$$P_A A_t + P_M \left(\dot{K}_t + \delta K_t + M_t\right) + P_A A_t = \sum_{i \in \{A,M,S\}} P_i B_i \left(b_t^i H_Y(t)\right)^{\alpha} \left(N_t^i\right)^{\beta} \int_{j=0}^{T_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj, \tag{18}$$

where $\rho \in (0,1)$ is the subjective time preference rate; $\sigma \in (0,+\infty)$ is the constant coefficient of relative risk aversion; $\overline{A}\,(>0)$ is the level of subsistence consumption, $\overline{M}\,(>0)$ and $\overline{S}\,(>0)$ represent home production of manufacturing and services; $u, v, w \in (0,1)$ stand for the relative utility weights of consumption for agriculture, manufacturing and servies, satisfying $u+v+w=1$.

By utilizing the capital market clearing condition ($K_t = \int_{j=0}^{T_t} \eta x_{jt} dj$) and monopolistic pricing formula (8), substituting (10), (13), and (14) into (18), we obtain the new flow budge constraint:

$$\dot{K}_t = B_M H_Y(t)^{\alpha} \eta^{\alpha+\beta-1} K_t^{1-\alpha-\beta} T_t^{\alpha+\beta} - \delta K_t - \frac{B_M}{B_A} A_t - \frac{B_M}{B_S} S_t - M_t. \tag{19}$$

The necessary conditions for optimality can be summerized as the following nonlinear system of differential equations:

$$\frac{u}{v}\frac{M_t+\overline{M}}{A_t-\overline{A}}=\frac{B_M}{B_A}, \tag{20}$$

$$\frac{w}{v}\frac{M_t+\overline{M}}{S_t+\overline{S}}=\frac{B_M}{B_S}, \tag{21}$$

$$\frac{\dot{\overline{M_t+\overline{M}}}}{M_t+\overline{M}}\left(=\frac{\dot{\overline{A_t-\overline{A}}}}{A_t-\overline{A}}=\frac{\dot{\overline{S_t+\overline{S}}}}{S_t+\overline{S}}\right)=\frac{1}{\sigma}\left[(1-\alpha-\beta)\,\eta^{\alpha+\beta-1}B_M H_Y\,(t)^{\alpha}\left(\frac{K_t}{T_t}\right)^{\alpha+\beta}-\delta-\rho\right], \tag{22}$$

together with (19). Equations (20) and (21) are the optimal allocation conditions for capital and labor among agricuture, manufacturing and services. Equation (22) is consumption Euler equation, which is essentially the intertemporal optimization condition between consumption and savings. The flow budget constraint or social resource constraint (19) shows that optimal allocations of the final goods between consumption and investment in each period.

3 Monopolistic Competitive Equilibrium and GBGP

Since all intermediate-good firms are monopoly firms, the decentralized equilibrium of the multi-sector economy is a monopolistic competitive equilibrium, which can be stated formally by

Theorem 1 *A monopolistic competitive equibrium of the multi-sector economy is composed of equilibrium price sequences* $\left\{P_A, P_M, P_S, (p_{jt}, P_{jTt})_{j\in[0,T_t]}, w_{Ht}, w_{Lt}, r_t\right\}$ *and allocation sequences* $\left\{A_t, M_t, S_t, H_{Yt}, H_{At}, N_t^A, N_t^M, N_t^S, \phi_t^A, \phi_t^M, \phi_t^S, (x_{jt})_{j\in[0,T_t]}\right\}$*, satisfying: (1) The representative consumer consumes and accumulates physical capital to maximize the objective function (17), subject to the FBC (18)or (19); (2) In the final-goods sector, given its production technology, each subsector chooses labor, human capital and all of the intermediate goods to maximize its profits; (3) Given the demand for its products, any intermediate-good monopoly firm* $j\in[0,T_t]$ *chooses monopoly price* p_{jt} *to maximize its monopoly profit* π_{jt}*; (4) The research sector uses human capital* $H_A(t)$ *and the existing knowledge stock* $A(t)$ *to develop new knowledge with the technology (15); (5) The markets for three final goods clear, i.e., equations (1), (2), and (3) hold; (6) Labor market clears, i.e.,* $N_t^A+N_t^M+N_t^S=1$*; (7) The market for human captial clears, i.e.,* $\left(b_t^A+b_t^M+b_t^S\right)H_{Yt}+H_{At}=H_{Yt}+H_{At}=H$*; (8) Capital market clears, i.e.,* $K_t=\int_{j=0}^{T_t}\eta x_{jt}dj$*; (9) Any patent market clears.*

In order to examine the stationary equilibrium and the corresponding generalized balanced growth path (GBGP), we introduce the following

Definition *A generalized balanced growth path (GBGP) is a trajectory along which the real interest rate is constant* r^**.*

The trajectory of the real interest rate is given by

$$r_t = (1-\alpha-\beta)\,\eta^{\alpha+\beta-1} B_M H_Y(t)^{\alpha}\left(\frac{K_t}{T_t}\right)^{\alpha+\beta} - \delta. \tag{23}$$

Setting $r_t = r^*$ and using (22) and (23), we have

$$\frac{\dot{\overline{M_t+\overline{M}}}}{M_t+\overline{M}} = \frac{\dot{\overline{A_t-\overline{A}}}}{A_t-\overline{A}} = \frac{\dot{\overline{S_t+\overline{S}}}}{S_t+\overline{S}} = \frac{1}{\sigma}\left(r^*-\rho\right) \equiv g^*, \tag{24}$$

where $g^* \equiv \frac{1}{\sigma}\left(r^*-\rho\right)$ is defined as the growth rate of $A_t - \overline{A}$, $M_t + \overline{M}$, and $S_t + \overline{S}$ on GBGP. Since (K_t/T_t) is constant on GBGP, $K_t = \int_{j=0}^{T_t} \eta x_{jt} dj$ and $x_{jt} = x_t$, we know that $x_t = K_t/(\eta T_t)$ is constant, i.e., $x_t = x^*$. From the market-clearing condition of human capital and (23), we know that human capital employed both in the final-goods and research sectors are constant on GBGP, i.e., $H_{Yt} = H_Y^*$, $H_{At} = H_A^*$. On GBGP, we know that $P_T^* = \pi^*/r^*$, and (16) as $P_T^* = P_M B_M (\alpha/\delta) H_Y^{*\alpha-1} x^{*1-\alpha-\beta}$. Combining the two equations with (7), (8), and (10) yields

$$H_Y^* = \frac{\alpha r^*}{\delta(\alpha+\beta)(1-\alpha-\beta)}, H_A^* = H - \frac{\alpha r^*}{\delta(\alpha+\beta)(1-\alpha-\beta)}. \tag{25}$$

From (15), we know that both knowledge stock and physical capital grow at the same rate, namely,

$$g_T^* = g_K^* = g^* = \delta H - \frac{\alpha r^*}{(\alpha+\beta)(1-\alpha-\beta)}. \tag{26}$$

Since all endogenous variables grow at the same rate on GBGP, we have

$$\frac{1}{\sigma}\left(r^*-\rho\right) = \delta H - \frac{\alpha r^*}{(\alpha+\beta)(1-\alpha-\beta)}. \tag{27}$$

Solving (27) and (26) for r^* and g^* gives us

$$r^* = \frac{\delta H\sigma + \rho}{\sigma\Lambda + 1}, \tag{28}$$

$$g^* = \frac{\delta H - \Lambda\rho}{\sigma\Lambda + 1}, \tag{29}$$

where $\Lambda \equiv \alpha/(\alpha+\beta)(1-\alpha-\beta)$. Equation (29) shows that the rate of economic growth depends on the total stock of human capital, time discount rate, and technological parameters of the research and final-goods sectors. The larger the total stock of human capital, or the smaller the time discount rate or the larger the elasticity of intertemporal substitution ($EIS = 1/\sigma$), the higher the endogenous rate of economic growth.

Proposition 3 *The higher the total stock of human capital in the economy, the more human capital employed in the research sector, the faster knowledge accumulates. Hence the rate of economic growth is higher. Namely,*

$$\frac{dr^*}{dH} = \frac{\delta\sigma}{\sigma\Lambda+1} > 0, \frac{dH_A^*}{dH} = \frac{1}{\sigma\Lambda+1} > 0, \frac{dg_T^*}{dH} = \frac{dg^*}{dH} = \frac{\delta}{\sigma\Lambda+1} > 0.$$

Proposition 4 *The larger the EIS, the lower the equilibrium real interest rate, the more human capital employed in the research sector, the higher the rate of economic growth.*

$$\frac{dg^*}{d\sigma} = \frac{\delta H - \rho\Lambda}{(\sigma\Lambda + 1)^2} > 0, \frac{dH_A^*}{d\sigma} = -\frac{\alpha}{\delta\Lambda}\frac{\delta H - \rho\Lambda}{(\sigma\Lambda + 1)^2} > 0, \frac{dg^*}{d\sigma} = -\frac{\Lambda(\delta H - \rho\Lambda)}{(\sigma\Lambda + 1)^2} < 0.$$

With the larger EIS, consumers are likely to consume less and save more. Hence the physical capital accumulates more rapidly, which then lowers the equilibrium interest rate. Due to $P_T^* = \pi^*/r^*$ and $w_H = P_T\delta T$, the equilibrium prices of patents rises and equilibrium earnings of human capital employed in the research sector increase. More human capitals transfer to the research sector, which raises the speeds of knowledge accumulation and economic growth.

4 Endogenous Growth and Structural Changes

On GBGP, using (1), (2), (3), (22), (24), and the following knife-edge condition

$$\frac{\overline{A}}{B_A} = \frac{\overline{M}}{B_M} + \frac{\overline{S}}{B_S}, \tag{30}$$

we obtain the dynamic equations for the employment shares of the three final goods

$$\dot{N}_t^A = -g^*\frac{\overline{A}}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} T_t} = -\frac{\delta H - \Lambda\rho}{\sigma\Lambda + 1}\frac{\overline{A}}{B_A H_Y^{*\alpha}\eta^{\alpha+\beta-1}k^{*1-\alpha-\beta}T_t} < 0, \tag{31}$$

$$\dot{N}_t^M = -g^*\frac{\overline{M}}{B_M H_Y^{*\alpha} x^{*1-\alpha-\beta} T_t} = \frac{\delta H - \Lambda\rho}{\sigma\Lambda + 1}\frac{\overline{A}}{B_M H_Y^{*\alpha}\eta^{\alpha+\beta-1}k^{*1-\alpha-\beta}T_t} > 0, \tag{32}$$

$$\dot{N}_t^S = -g^*\frac{\overline{S}}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta} T_t} = \frac{\delta H - \Lambda\rho}{\sigma\Lambda + 1}\frac{\overline{S}}{B_S H_Y^{*\alpha}\eta^{\alpha+\beta-1}k^{*1-\alpha-\beta}T_t} > 0. \tag{33}$$

Theorem 2 *A generalized balanced growth path (GBGP) with a constant equilibrium interest rate (24) and a constant equilibrium endogenous growth rate (29) exists whenever the knife-edge condition (30) holds. On the GBGP, as is implied by equations (31), (32), and (33), the employment share declines in agriculture, rises in both manufacturing and services.*

Theorem 2 shows that on GBGP if technology changes and hence the economy grows endogenously, because the demand elasticity of income for agriculture is less than one and the demand elasticities for both manufacturing and services are larger than one, along with the growth of the economy, even though all the three final goods expand, the speeds of expansion for both manufacturing and services are larger than the one for agriculture, then the employment (and production value) share of agriculture decreases gradually, the ones for manufacturing and services increase correspondingly. Labor forces in the economy transfer from agriculture to both manufacturing and services, which displays that the industry structure upgrades gradually. Furthermore, an increase of the stock of human capital in the economy raises the equilibrium growth rate and hence enforces structural changes.

Corollary 1 *Economic growth accelerates structural changes. Namely, If the equilibrium growth rate* g^* *is larger, even though the signs of* $\dot{N}_t^i, i \in \{A, M, S\}$ *are unchanged, their absolute values become larger. However, if there exist no innovations and growths, structural changes never emerge.*

Corollary 1 shows us that if the equilibrium rate of economic growth is higher, then agriculture shrinks more quickly, manufacturing and services expand more quickly. However, if there exist no innovations and growth, namey, $g^* = 0$, even though the demand elasticities among three final goods are different, the industrial structure will not change, i.e., $\dot{N}_t^i = 0, i \in \{A, M, S\}$. Therefore, economic growth is the fundamental driving force of structural changes.

In order to investigate how human capital promotes economic growth and structural change quantitatively, we do some numerica analysis. Let $\overline{A} = 1$, $A_0 = 2$, $B_A = 4$, $\eta = 1$, $\alpha = 0.3$, $\beta = 0.3$, $T_0 = 1$, $\overline{S} = 0.5$, $S_0 = 0.5$, $B_S = 2.5$. The stock of human capital in the economy is taken to be 0.2, 0.4, and 0.8. The associated equilibrium growth rates are calculated as 0.02, 0.05, and 0.11, respectively. Figures 1, 2 and 3 display the evolution of the employment shares of three final goods. It is shown that with higher level of human capital, the employment share of agriculture decreases more quickly, whereas the employment shares of manufacturing and services increase more quickly. The higher level of human capital speeds up economic growth and hence structural changes. (Insert Figures here.)

Due to (32) and (33), we have the following

Corollary 2 *If the model parameters satisfy* $\frac{\overline{M}}{B_M} > \frac{\overline{S}}{B_S}$, *then the expansion of manufacturing is quicker than services; if the model parameters satisfy* $\frac{\overline{M}}{B_M} < \frac{\overline{S}}{B_S}$, *then the expansion of manufacturing is slower than services; If* $\frac{\overline{M}}{B_M} = \frac{\overline{S}}{B_S}$, *then the expansion speeds of both manufacturing and services are the same.*

5 Conclusion

By introducing endogenous technological change into the multisector growth model pioneered by KRX, the paper shows that human capital speeds up economic growth and enforces structural changes in the economy. It is shown that the larger the stock of human capital, the quicker the innovation and knowledge accumulation in the research sector, and the higher the economic growth rate. Since the income elasticity of demand in the agricultural sector is less than manufacturing and services sectors, along with eonomic growth, the employment and production shares of the manufacturing and services sectors increase gradually, and the two shares of agriculture decrease. Furthermore, endogenous economic growth determines the speed of expansions or shrinks of different sectors. The higher the endogenous growth rate, the quicker the shrink of the agriculture sector and the expansions of manufacturing and services sectors.

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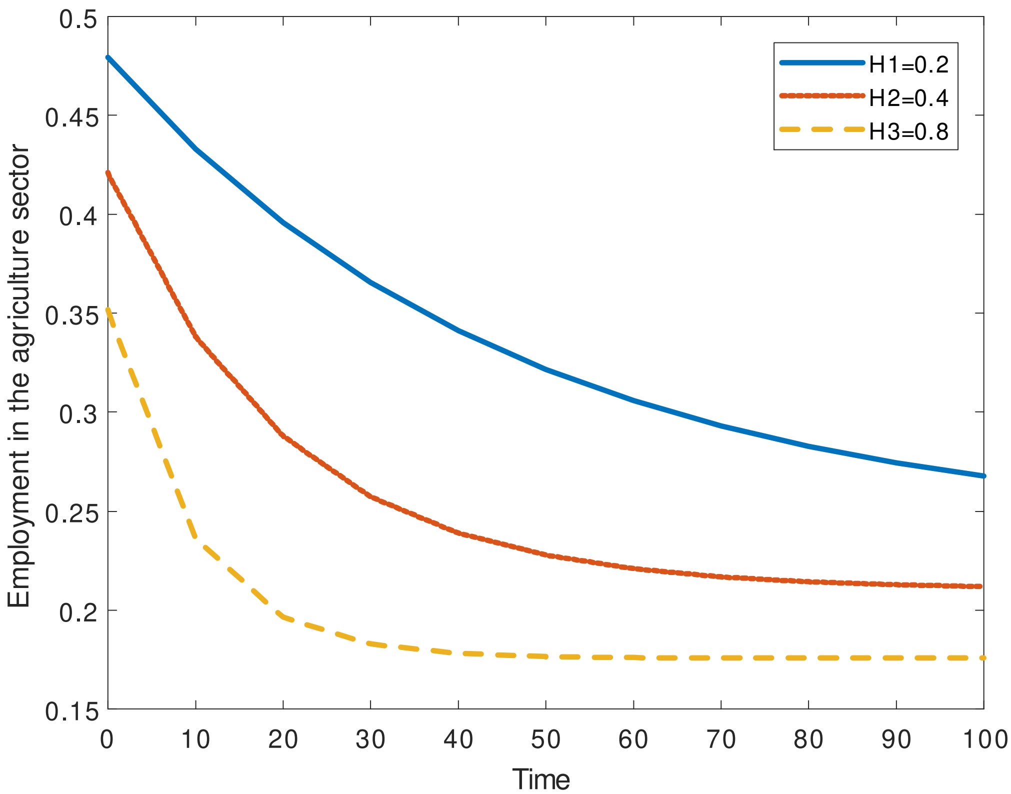

Figure 1: Human capital and employment in the agriculture sector

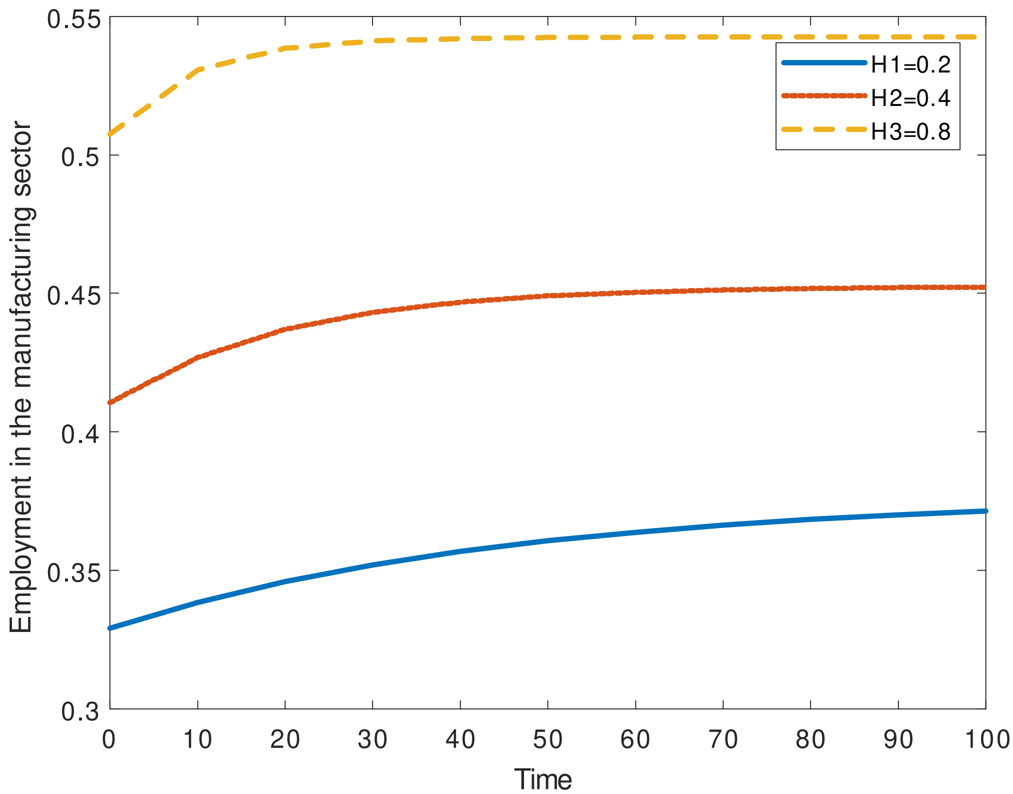

Figure 2: Human capital and employment in the manufacturing sector

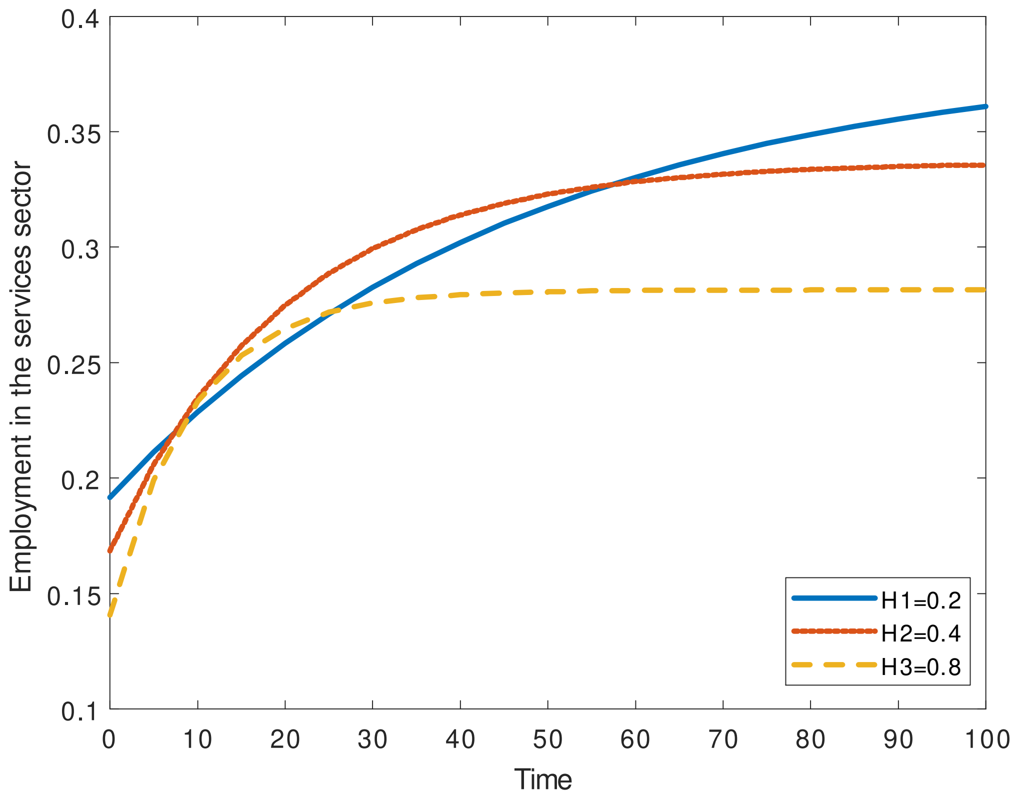

Figure 3: Human capital and employment in the services sector