An Improved LM Curve

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An Improved LM Curve

By MARTIN HIERMEYER*

Over the last decades, the LM curve has largely disappeared from research and, to some extent, also from teaching. Because of its well-known weaknesses, the LM curve is now frequently replaced with an interest rate rule. The paper suggests an improved LM curve which gets rid of the weaknesses of the LM curve but, unlike an interest rate rule, retains the LM curve’s strength of including money, and even expands upon this strength.

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I. Introduction

The LM curve is given by the following equation.

\[(LM) \quad M = L(i, Y), \quad \text{with } L'(i) < 0 \text{ and } L'(Y) > 0\]

The variables are:

- M: Money
- i: Interest rate
- Y: Output

For decades, the LM curve was routinely used to close a two-equation system consisting of an IS curve, and a price setting, or aggregate supply, curve. Today, this system is instead usually closed by an interest rate rule (Friedman 2003).

Unlike the LM curve, an interest rate rule does not include money. Thus, if there is any money at all in the resulting models, money is introduced separately, for example by including money in the utility function of a household, by making money a prerequisite for transactions, or by introducing some form of public finance budget accounting (Walsh 2017).

Following Clarida et al. (1999), the trend away from the LM curve is particularly pronounced in research. In teaching, the LM curve has held its ground better as many undergraduate textbooks still choose to present the
IS-LM-AS model (e.g. Abel, Bernanke and Croushore 2017, Blanchard 2017, Dwivedi 2015, Froyen 2013, Heijdra 2017 or Mankiw 2016).

The resulting disconnect between teaching and research is often lamented (e.g. Carlin and Soskice 2005) and has led to the development of undergraduate-level models that replace the LM curve with an interest rate rule. Examples include the IS-MP-IA model of Romer (2000), the IS-PC-MR model of Carlin and Soskice (2005), or the models of Allsopp and Vines (2000), Taylor (2000), Walsh (2002) and Bofinger et al. (2006).

However, many textbook authors do not seem to be convinced by those models. For example, Mankiw (2006) gives detailed reasons for rejecting the IS-PC-MR model in favor of the IS-LM-AS model – despite the LM curve’s well-known weaknesses.

This paper suggests an improved LM curve – both for teaching and research – which gets rid of the weaknesses of the LM curve, retains the LM curve’s strength of including money, and expands upon this strength.

II. Weaknesses Of The LM Curve

The LM curve has four well-known weaknesses.

Weakness 1. The LM curve assumes that the central bank targets money M. This is considered a weakness since today most central banks target an interest rate such as the federal funds rate rather than money (e.g. Goodhart 2009).

Weakness 2: The LM curve’s money measure M is undefined. Is it high-powered money? M1 money? Some other money measure? There is no obvious answer as “M” exhibits characteristics of several money measures (e.g. Romer 2000).

Weakness 3. The LM curve is unclear about its interest rate i. Some confusion is related to the confusion about money M. After all, since the interest rate i is supposed to clear the market for money M, this again raises the question: Which money M? The confusion is heightened by the fact that the LM curve is usually combined with an IS curve that shares its interest rate i. In a simple form, such an IS curve is given by the following equation.
\((IS)\quad Y = C + I(i) + G, \quad \text{with } I'(i) < 0\)

The variables, if not already defined, are:

- **C**: Consumption spending
- **G**: Government spending
- **I**: Investment spending

Apparently, the interest rate \(i\) thus clears two markets: The IS equation’s “goods market” and the LM equation's money market – despite the fact, that the two markets have different interest rates (e.g. Romer 2000).

Weakness 4. The LM curve cannot explain unconventional monetary policy. It includes only one money measure \(M\), which the central bank already targets in conventional monetary policy. Thus, no distinction can be made between conventional and unconventional monetary policy.

**III. An Improved LM Curve**

The improved LM curve is based on two accounting identities and four plausible assumptions.

1. \(\text{HPM} \equiv \text{CHP} + \text{ER} + \text{RR}\)
2. \(\text{RR} \equiv \text{rrD}, \quad \text{with } 0 < \text{rr} \leq 1\)
3. \(\text{HPM} = \text{HPM}(\text{ffr}), \quad \text{with } \text{HPM}'(\text{ffr}) < 0\)
4. \(\text{CHP} = \text{CHP}(i), \quad \text{with } \text{CHP}'(i) < 0\)
5. \(\text{ER} = \text{ER}(i), \quad \text{with } \text{ER}'(i) < 0\)
6. \(\text{D} = \text{D}(Y), \quad \text{with } \text{D}'(Y) > 0\)

The variables, if not already defined, are:

- **HPM**: High-powered money
- **rr**: Reserve ratio
- **CHP**: Currency held by the public
- **D**: Demand deposits
- **ER**: Excess reserves
- **ffr**: Federal funds rate
- **RR**: Required reserves

Equation (1) defines the components of high-powered money.
Equation (2) defines the reserve ratio.

Equation (3) assumes that demand for high-powered money decreases with the federal funds rate. This is plausible as the federal funds rate is the interest rate on high-powered money.

Equation (4) assumes that the demand for currency held by the public decreases with the interest rate i. This is plausible as the interest rate i reflects the opportunity cost of holding currency instead of paying the currency into a bank savings account.

Equation (5) assumes that excess reserves decrease with the interest rate i. This is plausible as the interest rate i reflects the opportunity cost of holding excess reserves instead of making loans.

Equation (6) assumes that demand for demand deposits increases with output. This is plausible. Output implies transactions. Transactions imply M1 money, as payment with M1 money (currency, check, direct debit or bank wire transfer) is generally the only accepted method of payment. M1 money finally implies currency or demand deposits as M1 money is defined as follows:

\[
(7) \quad \text{M}1 \equiv \text{CHP} + \text{D}
\]

Combining equations (1) through (6) yields the improved LM equation.

\[
(I-LM) \quad \text{HPM}(\text{ffr}) = \text{CHP}(i) + \text{ER}(i) + \text{rrD}(Y),
\]

with \( \text{HPM}(\text{ffr})'(i) = \text{CHP}'(i) + \text{ER}'(i) < 0 \)

and \( \text{HPM}(\text{ffr})'(Y) = \text{rrD}'(Y) > 0 \)

Thus, the I-LM curve replaces the LM curve’s endogenous variable “M” with five endogenous variables: With high-powered money HPM, with the federal funds rate ffr, with currency held by the public CHP, with excess reserves ER, with the reserve ratio rr and with demand deposits D. For \( L'(Y) = \text{rrD}'(Y) \) and \( L'(i) = \text{CHP}'(i) + \text{ER}'(i) \), both curves have the same slope (Figure 1).
IV. Understanding The Improved LM Curve

To fully understand the I-LM curve, consider both a rightward shift in the I-LM curve and a movement along the I-LM curve.

A. Rightward Shift In The Improved LM Curve

Let’s consider the case of a one-percentage point reduction in the federal funds rate \( \text{ffr} \). For \( HPM'(\text{ffr}) = -2 \), \( \text{rr} = 0.1 \) and \( D'(Y) = 1 \), this shifts the I-LM curve to the right by $20 in line with equation (8).

\[
\frac{dY}{dffr} = \frac{HPM'(\text{ffr})}{\text{rr}D'(Y)} < 0
\]

Table 1 shows what happens step-by-step, assuming arbitrary initial values for the involved variables.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency held by the public (CHP)</td>
<td>$60</td>
<td>$60</td>
<td>$60</td>
</tr>
<tr>
<td>Excess reserves (ER)</td>
<td>$31</td>
<td>$33</td>
<td>$31</td>
</tr>
<tr>
<td>Required reserves (RR)</td>
<td>$9</td>
<td>$9</td>
<td>$11</td>
</tr>
<tr>
<td>High-powered money (HPM)</td>
<td>$100</td>
<td>$102</td>
<td>$102</td>
</tr>
<tr>
<td>Additional loans</td>
<td>$300</td>
<td>$300</td>
<td>$300</td>
</tr>
<tr>
<td>Demand deposits (D)</td>
<td>$90</td>
<td>$90</td>
<td>$110</td>
</tr>
<tr>
<td>M1 money (M1)</td>
<td>$150</td>
<td>$150</td>
<td>$170</td>
</tr>
<tr>
<td>Output (Y)</td>
<td>$50</td>
<td>$50</td>
<td>$70</td>
</tr>
</tbody>
</table>

Column (1) shows the economy before the Fed reduces the federal funds rate.

In column (2), the Fed now reduces the federal funds target rate by one percentage point. To meet the lower federal funds target rate, the Fed’s
New York traders lend $2 more in high-powered money to banks (given HPM'(ffr)= -2). Banks initially add the additional high-powered money to excess reserves. In line with that, excess reserves and high-powered money increase by $2 to $33 and $102, respectively.

In column (3), banks use the additional $2 of excess reserves to make additional loans totaling $20 (given rr = 0.1). When banks make such loans, they credit the demand deposits account of the borrower with a demand deposit of the size of the loan (so that the borrower can use the money). This is the reason why demand deposits increase by $20 from $90 to $110. Required reserves increase by $2 from $9 to $11. M1 money, the sum of currency held by the public and demand deposits, also increases by $20. Output increases by $20 in line with demand deposits as the borrowers spend all the borrowed M1 money (given D'(Y) = 1).

Since a $2 reduction in excess reserves made demand deposits increases by $20, the process is also known as the “money multiplier”.

### B. Movement Along The Improved LM Curve

Let’s consider the case of a one-percentage point increase in the interest rate i. For CHP'(i) = -1, ER'(i) = -1, rr = 0.1 and D'(Y) = 1, this increases output by $20 in line with equation (9).

\[
(9) \quad dY/di = -(CHP'(i) + ER'(i))/\text{rr}D'(Y) > 0
\]

Table 2 shows what happens step-by-step, assuming arbitrary initial values for the involved variables.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency held by the public (CHP)</td>
<td>$60</td>
<td>$59</td>
<td>$59</td>
</tr>
<tr>
<td>Excess reserves (ER)</td>
<td>$31</td>
<td>$32</td>
<td>$30</td>
</tr>
<tr>
<td>Required reserves (RR)</td>
<td>$9</td>
<td>$9</td>
<td>$11</td>
</tr>
<tr>
<td>High-powered money (HPM)</td>
<td>[HPM = CHP + ER + RR]</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td>Additional loans</td>
<td>$300</td>
<td>$300</td>
<td>$320</td>
</tr>
<tr>
<td>Demand deposits (D)</td>
<td>[D = RR/rr]</td>
<td>$90</td>
<td>$90</td>
</tr>
<tr>
<td>Savings deposits</td>
<td>$200</td>
<td>$201</td>
<td>$201</td>
</tr>
<tr>
<td>M1 money (M1)</td>
<td>[M1 = CHP + D]</td>
<td>$150</td>
<td>$150</td>
</tr>
<tr>
<td>Output (Y)</td>
<td>$50</td>
<td>$50</td>
<td>$70</td>
</tr>
</tbody>
</table>

Column (1) shows the economy before the reduction in the interest rate.
In column (2), the lower interest rate now makes the public pay $1 of currency into their savings accounts (given CHP'(i) = -1) as the higher interest rate increases the opportunity cost of holding currency. Excess reserves now stand at $32. This is $2 higher than the desired level (after all, given ER'(i) = -1, banks now desire $1 lower excess reserves than in column (1).

In column (3), banks use those $2 of excess reserves to make additional loans totaling $20 (rr = 0.1). As in Table 1, in turn loans, demand deposits, M1 money and output increase by $20.

Since a $2 reduction in excess reserves made demand deposits increases by $20, the process is also known as the “money multiplier”.

V. Advantages Of The Improved LM Curve

The I-LM curve has four advantages vis-à-vis the LM curve and, as we will see, also vis-à-vis models that replace the LM curve with an interest rate rule.

A. The Fed Targets The Federal Funds Rate By Adjusting High-Powered Money So That The Target Is Met

The LM curve is frequently criticized for its assumption that the central bank targets money. This is considered a weakness since most central banks today target an interest rate such as the federal funds rate rather than money.

In contrast, the I-LM curve assumes that the Fed targets the federal funds rate. Even better, the Fed does so by adjusting high-powered money so that the target rate is met.

With that, the I-LM curve goes not only beyond the LM curve. It also goes beyond models that replace the LM curve with an interest rate rule. After all, as Friedman (2003) points out, these models leave open the underlying question how the central bank manages to fix the chosen interest rate in the first place as they do not include high-powered money.
B. The I-LM Curve Is Clear About Its Money Measures

The LM curve is unclear about its money measure “M” which might be high-powered money, M1 money or some altogether different aggregate.

In contrast, the I-LM curve is clear about its money measures as it explicitly includes the following ones: high-powered money, currency held by the public, excess reserves, required reserves and demand deposits. By extension, it thus also includes M1 money, the sum of currency held by the public and demand deposits.

With that, the I-LM curve is not only clearer about money than the LM curve but also “clearer” about money than models that replace the LM curve with an interest rate rule. After all, those models generally do not include any variable labeled money at all (Friedman 2003).

C. The I-LM Curve Is Clear About Its Interest Rates

The LM curve is unclear about its interest rate i. This interest rate apparently clears two very different markets at once, with one being the market for “money” (in whatever definition).

In contrast, the I-LM curve is clear, distinguishing between the federal funds rate ffr on the one hand and the interest rate i on the other hand.

The federal funds rate ffr operates within the I-LM curve. It clears the market for high powered money by matching the supply of high-powered money with the demand for high-powered money.

The interest rate i operates within the “IS-I-LM model” which combines equation (IS) with equation (I-LM). The interest rate i clears the market for credit by matching the I-LM curve’s additional lending with the IS curve’s additional borrowing.

After all, a rightward shift in the I-LM curve reflects additional lending equal to the increase in output (see Table 1). Likewise, a rightward movement along the I-LM curve reflects additional lending equal to the increase in output (see Table 2).

This fits neatly to the IS curve where shifts in the IS curve and movements along the IS curve reflect additional borrowing equal to the increase
in output. For example, if the IS curve shifts to the right by $20, this reflects a $20 increase in credit-financed consumption, investment or government spending. And if a lower interest rate $i$ increases output by $20, this reflects a $20 increase in investment spending.

With that, the I-LM curve goes not only beyond the LM curve. It also goes beyond models that replace the LM curve with an interest rate rule. After all, there is no credit or banking in these models (Friedman 2003).

Note that the federal funds rate $ffr$ and the interest rate $i$ may or may not co-move. For example, in a fiscal expansion, the Fed holds the federal funds rate steady but the interest rate $i$ increases due to the additional government borrowing which the additional government spending brings (see Figure 2, left-hand side). In contrast, in a monetary expansion, both the federal funds rate and the interest rate $i$ move in the same direction. The federal funds rate declines as the Fed reduces it from $ffr_1$ to $ffr_2$, and the interest rate $i$ declines as banks push additional loans into the credit market (see Figure 2, right-hand side).

**Figure 2. Fiscal Expansion (Left) and Monetary Expansion (Right) in the IS-I-LM Model.**

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**D. The I-LM Curve Can Explain Unconventional Monetary Policy**

The LM curve cannot explain unconventional monetary policy. It includes only one money measure $M$, which the central bank already targets in conventional monetary policy. Thus, no distinction can be made between conventional and unconventional monetary policy.

In contrast, the I-LM curve can explain unconventional monetary policy. If the Fed wants to, it can increase equation (I-LM)’s high-powered money directly, bypassing the federal funds rate. It can do so by purchasing assets
such as mortgage-backed securities, bank debt or Treasury notes from banks with high-powered money (“quantitative easing”).

The Fed may do so at the zero lower bound, where bank demand for high-powered money is so low that the federal funds rate has dropped to zero. The low demand for high-powered money may be due to banks being unwilling to lend, say because of strict capital requirements, and/or due to firms being unwilling to borrow, say because of general pessimism. In the first case, ER(i) → ∞ holds and the I-LM curve is horizontal. In the second case, I′(i) = 0 holds and the IS curve is vertical.

Given that the federal funds rate is already zero, the additional supply of high-powered money will not lower the federal funds rate any further. In equation (I-LM), high-powered money increases while the federal funds rate remains steady at zero.

The IS-I-LM model shows that such a direct increase in high-powered will, however, have little or no effect on aggregate demand if the I-LM curve is nearly horizontal, or horizontal (ER(i) → ∞), and/or if the IS curve is nearly vertical, or vertical (I′(i) = 0). In this case, the additional high-powered money will largely, or entirely, end up in currency held by the public and in excess reserves.

And indeed, while quantitative easing increased US high-powered money by about 370% from 2008 to 2015, aggregate demand increased only modestly while excess reserves exploded by about 1300% (Federal Reserve 2018).

Of course, even in the case of a horizontal I-LM curve and a vertical IS curve, policy is not per se powerless. The central bank may use high-powered money to create demand deposits for the government at banks, thus shifting the I-LM curve to the right. The government may then spend this money, thus shifting the IS curve to the right. This is the very mechanism through which all past hyperinflations came about.

A tiny dose of such a potential hyperinflation is currently discussed as a possible way to reflate low-inflation economies. One catchword is “heli-
copter money”. In helicopter money, a central bank lends or gifts households newly created demand deposits, hoping to increase their consumption spending. Alternatively, the central bank might also finance some government spending directly (“people’s quantitative easing”).

With that, the I-LM curve goes not only beyond the LM curve. It also goes beyond models that replace the LM curve with an interest rate rule. After all, these models focus on conventional monetary policy which can be described by an interest rate rule and do not include any high-powered money.

VI. Conclusion

The improved LM curve gets rid of the LM curve’s four weaknesses, retains its strength (the inclusion of money), and expands upon this strength (section V and Table 3 below).

<table>
<thead>
<tr>
<th>TABLE 3—LM CURVE VS. INTEREST RATE RULE VS. IMPROVED LM CURVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM Curve</td>
</tr>
<tr>
<td>Assumes that the central bank targets an interest rate?</td>
</tr>
<tr>
<td>Is clear about its money measures (if any)?</td>
</tr>
<tr>
<td>Is clear about its interest rate(s)?</td>
</tr>
<tr>
<td>Can explain unconventional monetary policy?</td>
</tr>
<tr>
<td>Includes money?</td>
</tr>
<tr>
<td>Shows how the central bank targets an interest rate?</td>
</tr>
<tr>
<td>Includes credit and banking?</td>
</tr>
</tbody>
</table>

With that, the I-LM curve is an useful alternative to the LM curve and to interest rate rules – both for teaching and research.

REFERENCES


Hicks, John R. 1937. “Mr. Keynes and the 'Classics': A Suggested Interpretation”. In Econometrica 5 (2). 147-159.

