Return and volatility spillovers between South African and Nigerian equity markets

Phume, Maphelane Palesa and Bonga-Bonga, Lumengo

University of Johannesburg, University of Johannesburg

16 May 2018

Return and volatility spillovers between South African and Nigerian equity markets

Maphelane Palesa Phume
University of Johannesburg, School of Economics

Professor Lumengo Bonga-Bonga
University of Johannesburg, School of Economics

Abstract

South Africa and Nigeria are the wo biggest African economies by the size of their economies, translated in their gross domestic product (GDP). Portfolio investors who are interested to invest in the African stock markets should be interested in uncovering whether the two stock exchange markets complement and provide the opportunity for asset diversification or that the two markets are strictly substitutable. It is in that context that this paper evaluates the cross-transmission of returns and volatility shocks between the two countries to infer the extent of interdependence of the two stock exchange markets. Moreover, the paper makes inferences on optimal portfolio weights and hedge ratios when holding assets from the two markets. To that end, estimations based on multivariate GARCH (general autoregressive conditional heteroscedastic) model are used. The results of the empirical analysis suggest evidence of stock market returns and volatility spillovers from South African stock markets to Nigerian stock markets , and not other way around. Moreover, the results suggest that it is ideal to constitute a portfolio and set an optimal hedge ratio by combining assets from the South African and Nigerian stock markets and that investors should engage in dynamic rebalancing of portfolio weights and hedge ratio.

1. Introduction

The South African stock market is the largest in Africa in terms of market capitalisation and it is the most liquid stock market on the continent (World Bank, 2016). In terms of market capitalisation, South Africa accounts for roughly 75 percent of the total African stock market capitalisation (Farid, 2013). With regard to stock market liquidity, South Africa was nearly five percent more liquid than Africa's second largest stock market (Egypt) in 2015 (World Bank, 2016). Due to its comparatively sophisticated infrastructure, its judicial and political institutions in the African region, and its induction as the only African member of the BRICS group in the last decade, South Africa is largely considered to be "the gateway to African investment" (Kahn, 2011). Its financial system is comparable to those of the most developed economies in the world.

Alternatively, Nigeria was the largest African economy in terms of gross domestic product (GDP) in 2015. Additionally, average annual GDP growth in the Nigerian economy was nearly double the South African rate during the 2000-2014 period (World Bank, 2016). A quicker GDP growth is a common comparative analysis factor for speculative investment between the stock markets of different countries (Ritter, 2005; Barra, 2010). Some studies note two positive effects of a higher GDP growth on stock market performance. Firstly, a more rapid GDP growth may lead to higher speculative demand for financial instruments such as stocks, and consequently to higher stock prices (Maduka & Onwuka, 2013). Secondly, a faster GDP growth may imply an accelerated average profit growth for listed companies, which is transmitted to faster average earnings per share growth, and ultimately to higher average stock prices (Barra, 2010; Ritter, 2005).

Given South Africa's robust stock market in terms of liquidity and market capitalisation relative to the Nigerian stock market, one may argue that there are some benefits for investors to take up more positions in the South African stock market relative to the Nigerian stock market. However, each country's economic potential should deter such a trivial decision. It is worth noting that Nigeria's robust real economy in terms of GDP growth and total GDP relative to the South African economy suggests that there are some benefits to financial investment in the Nigerian stock market. Thus, it is the task of empirical analysis to reveal the opportunities for optimal portfolio allocation and hedging when investing in the two African stock markets.

In modern finance, reliable estimates of return and volatility spillovers are essential for optimal portfolio diversification and hedging. For example, Arouri et al. (2011) show that return and volatility transmission across different assets is a crucial element for portfolio selection and design as well as for risk management. Kumar(2012) assesses the first and second moment transmission between gold and Indian industrial sector by making use of a multivariate GARCH model. the author shows that such an assessment is helpful for portfolio selection and diversification benefits. In this regard, it is beneficial for financial investors who intend to invest in the South African and Nigerian stock markets to gain insight into the cross-transmissions of return and volatility spillover between the stock markets of these two African economies.

It is important to note that previous studies apply a plethora of empirical analysis techniques to determine stock market returns and volatility spillovers between multiple African markets (Duncan & Kabundi, 2013; Sugimoto, Matsuki & Yoshida, 2014; King & Botha, 2015). However, very little is known about the bilateral return and volatility spillover effects between the South African and Nigerian stock markets. This paper attempts to fill this gap.

Thus, the paper addresses the following research question. What is the magnitude and significance of bilateral stock market returns and volatility spillovers between the South African and Nigerian stock markets? The objective of the paper is to evaluate the cross-transmission of return and volatility spillovers between South African and Nigerian stock markets.

Optimal portfolio diversification and hedge ratios for risk averse investors depends on reliable estimates of volatility and asset correlation. Return and volatility spillovers highlight the importance of asset diversification in modern finance. In so doing, optimal portfolio weights and hedge ratios for risk averse investors holding positions in South African and Nigerian stock markets can be inferred.

The paper makes use of a model from the multivariate general autoregressive conditional heteroscedastic (MGARCH) family to evaluate the cross-transmissions of returns and volatility spillovers between the South African and Nigerian stock markets. The Quasi Maximum Likelihood Estimation (QMLE) method using the Broyden-Flecther-Goldfarb-Shanno (BFGS) algorithm is employed to determine the results of the proposed MGARCH model.

Estimates from MGARCH models can be used to construct optimal portfolio weights and hedge ratios (Kroner & Ng, 1998). A mean-variance portfolio model is used to evaluate the optimal hedge ratios and portfolio weights of a two-asset portfolio comprising investment positions in South African and Nigerian stock markets. Inputs to the mean-variance portfolio model are generated from the proposed MGARCH model.

The remainder of this paper is as follows. Chapter 2 provides a background of dependence between South African and Nigerian stock markets. Chapter 3 provides a detailed literature review of similar studies in the literature. Chapter 4 provides a detailed description of the research methodology conducted in the paper. Chapter 5 presents and discusses the results of the paper. Chapter 6 concludes the paper.

2. Literature review

Due to their increasing contribution towards global economic growth, growth-oriented investors generally evaluate EMEs for attractive investment opportunities (Balli et al., 2015; Bekaert, 1999; Leke, Lund, Roxburg & van Wamelen, 2010). In addition, a widespread lifting of restrictions on cross-border financial transactions by many emerging economies in the past three decades, coupled with rapid developments in global technology have increased the financial market integration of many EMEs into the global financial investment community (Owusu & Odhiambo, 2012; Tswamuno, Pardee & Wannuva, 2007; Oyovwi & Eshenake, 2013).

However, negative consequences of increased global financial market integration are, inter alia, the increasingly undesirable return and volatility spillover effects across stock markets of different countries. This is particularly the case during periods of financial crises (Sugimoto et al., 2014). Increasing global financial market integration reduces the insulation of the domestic market against global structural shocks, and exposes the domestic market to the risk of financial market contagion (Choudry & Jayasekera, 2014). Notably, the 2007 United States' (US) sub-prime housing market financial crisis expanded to the EU as a sovereign debt crisis in 2008, which ultimately led to declines in stock market prices of both advanced economies and EMEs, i.e. the period widely referred to as the 2007-2009 global financial crisis. It is important to note that literature abounds regarding the transmission of stock market returns and volatility spillover between advanced economies and EMEs (Sugimoto et al., 2014; Piesse & Hearn, 2005; Balli et al., 2015; Duncan & Kabundi, 2013; Öztürk & Volkan, 2015; Korkmaz, Çevik & Atukeren, 2012).

Studies such as Sugimoto et al. (2014), Owusu and Odhiambo (2012), and Farid (2013), find that the degree of financial market integration between South Africa and Nigeria respectively with the EU, may explain the negative effects of the 2007-2009 global financial crisis on these respective African economies.

There is also evidence suggesting that EMEs can be financially integrated with each other (Collins & Biekpe, 2003), thus potentially leading to intra-regional financial market contagion. For example, in 1997, the collapse of the Thai baht led to the spread of currency devaluations and stock market declines throughout the South-East Asia region. A series of events that are commonly referred to as the 1997 Asian contagion. Collins and Biekpe (2003) find that the impacts of the 1997 Asian contagion also spread to the stock markets of some African countries, such as South Africa and Egypt.

It is possible that a consequence of financial market integration is return and volatility spillover between different markets (Leke, Lund, Roxburg & van Wamelen, 2010). Furthermore, financial market integration can expose markets to the risk of financial contagion (Choudry & Jayasekera, 2014), and thus, it is important to highlight some implications of return and volatility spillover effects on the decision-making process of global investors.

Return and volatility spillovers between assets highlight a major cornerstone of modern portfolio theory, namely asset diversification (Masih & Masih, 1999). Following Markowitz (1952), risk averse investors benefit from the diversification of portfolio assets across national borders if–and only if–the asset returns are less than perfectly correlated. A key benefit of asset diversification in the event of less than perfect asset correlation is the minimisation of idiosyncratic risk in the portfolio of assets (Markowitz, 1952). Thus, the optimal allocation of assets in a risk-averse investor's portfolio can be achieved through asset diversification, such that portfolio risk is minimised at every level of the portfolio's returns.

Evaluating the cross-transmission of returns and volatility spillover effects between EMEs, allows for a robust estimation of volatility and asset correlation necessary to achieve an optimal allocation of assets in a portfolio of emerging stock market assets (Sadorsky, 2012). Thus, it is beneficial to evaluate the return and volatility spillover effects of financial market integration during periods of tranquillity, as well as during periods of financial crisis.

There are several studies that have attempted to assess the extent of return and volatility spillovers between assets. These studies have used different approaches. For example, Diebold and Yilmaz (2009) use a vector autoregressive (VAR) model to estimate the return spillover effects between some global stock markets, and they construct a volatility spillover index to estimate their volatility spillover effects. According to Diebold and Yilmaz (2009), volatility spillover can be defined as the future share of variability in one assets' returns, resulting from unexpected changes in the volatility of another asset (Duncan & Kabundi, 2013).

Diebold and Yilmaz (2009) use Cholesky's factorisation to forecast error variance decompositions of a fitted VAR model of volatilities, and they use the generated forecasts to create a volatility spillover index. Diebold and Yilmaz's (2009) framework considers periods of tranquillity and periods of financial crisis for advanced and EME global stock markets. Diebold and Yilmaz (2009) posit that financial market integration can explain the presence of return spillover effects between some global stock markets. Furthermore, volatility spillovers tend to be more pronounced during periods of predetermined financial crises (Diebold & Yilmaz, 2009).

Öztürk and Volkan (2015) use Diebold and Yilmaz's (2009) volatility spillover index approach to account for global structural shock to some EMEs stock markets. Öztürk and Volkan (2015) achieve this by estimating contemporaneous volatility spillovers from some advanced economies to the observed EMEs. Öztürk and Volkan (2015) conclude that accounting for return and volatility spillover effects from advanced economies to EMEs have direct implications for financial hedging and portfolio optimisation, among other things.

However, the use of Cholesky's factorisation to achieve orthogonality of structural shocks, implies that forecasted error variance decompositions depend on the ordering of the variables in the system (Diebold & Yilmaz, 2012). Thus, Diebold and Yilmaz (2012) identify the forecasted error variance decompositions through a generalised impulse response approach, such as Koop, Pesaran, and Potter's (1996) approach, thus removing dependence on the order of the variables in the system.

Sugimoto et al. (2014) use Diebold and Yilmaz's (2012) volatility spillover index approach to estimate volatility spillovers between seven major African stock markets in context of the 2007-2009 global financial crisis. Sugimoto et al. (2014) come to two main conclusions. Firstly, African stock markets are considered to be more likely to be affected by spillovers from global stock markets, than spillovers from commodity and currency markets. Secondly, Sugimoto et al. (2014) found that African markets only experienced the negative effects of the 2008 EU sovereign debt crisis, and not the negative effects of the 2007 US sub-prime housing market financial crisis during the 2007-2009 global financial crisis. Thus, the aggregated spillover effects from the EU are found to outweigh the corresponding effects of the US, even in the wake of the 2007 US financial crisis (Sugimoto et al., 2014).

In another study, Korkmaz et al. (2012) used causality-in-mean and causality-in-variance tests to estimate the inter-regional and intra-regional return and volatility spillover effects between advanced economies and EMEs. Korkmaz et al. (2012) found that both inter-regional and intra-regional return and volatility spillover effects between economies do exist.

Güloğlu, Kaya, and Aydemir (2016) also use a causality-in-mean and causality-in-variance approach to estimate return and volatility spillover effects between Latin American EMEs. Güloğlu et al. (2016) show that although there is some interdependence between the observed Latin American EMEs, the result did not establish the risk of financial market contagion in the region.

Another approach to the estimation of return and volatility spillovers between economies outlined in the literature, is the MGARCH model approach. The volatility of asset returns tends to cluster together (Mandelbrot, 1963). This, suggests that volatility is autocorrelated and time-variant. MGARCH models are widely considered intuitive and effective approaches to modelling the autoregressive and heteroscedastic dynamics of volatility.

Worthington and Higgs (2004) use the Baba, Engle, Kraft, and Kroner (BEKK) MGARCH model developed in Engle and Kroner (1995), to identify the magnitude of inter-regional and intra-regional return and volatility spillovers of advanced economies and EMEs. Worthington and Higgs (2004) specify the conditional mean equation of asset returns as a VAR model to determine return spillovers. Worthington and Higgs (2004) found that the current volatility of emerging markets is best explained by own lagged volatility (volatility persistence) than the cross-transmission of inter-regional and intra-regional volatility spillovers. Furthermore, there is empirical evidence to suggest that inter-regional and intra-regional return spillovers are not homogenous across different economies (Worthington & Higgs, 2004).

Li and Majerowska (2008) use a variation of Kroner and Ng's (1998) Asymmetric BEKK-GARCH (ABEKK-GARCH) model to evaluate the inter-regional and intra-regional return and volatility spillovers of advanced and emerging economies. Li and Majerowska (2008) found that although there is evidence of volatility spillovers from advanced economies to EMEs, the magnitude of inter-regional volatility spillovers is negligible. Thus, financial investors may benefit from adding EME stock market assets into a diversified global stock market portfolio (Li & Majerowska, 2008). Kroner and Ng's (1998) ABEKK-GARCH model is useful because it accounts for asymmetric volatility effects.

Alternatively, Ling, and McAleer (2003) and Sadorsky (2012) propose the use of VAR GARCH models. VAR GARCH models perform more parsimonious evaluations of volatility spillover models than other MGARCH models, such as the BEKK-GARCH and ABEKK-GARCH models (Sadorsky, 2012). In addition, VAR GARCH models can account for dynamic correlations between assets (Sadorsky, 2012), an important feature of financial time series.

Conditional volatility estimates generated from volatility models, can be used to construct optimal hedge ratios and optimal portfolio weights (Sadorsky, 2012; Kroner & Sultan, 1993). Optimal hedge ratios between assets and optimal portfolio weights of assets held in the portfolio, such that the portfolio's expected returns are maximised and the portfolio's variance is minimised, can be determined using the mean-variance portfolio theory (Kroner & Sultan, 1993). Inputs to the mean-variance equation, specified in mean-variance portfolio theory, are derived from the volatility estimates generated from volatility models (Sadorsky, 2012).

3. Methodology

This paper follows the VAR GARCH methodology used in Sadorsky (2012), with an extension accounting for asymmetric volatility effects. The resulting model used in this study is referred to as the Asymmetric VAR GARCH (AVAR GARCH) model. Some advantages of the AVAR GARCH model include the model's ability to estimate volatility spillovers between stock market assets parsimoniously, model dynamic asset correlations, and account for asymmetric volatility effects. Like Sadorsky's (2012) model, the AVAR GARCH model has its foundations in MGARCH models.

The AVAR GARCH model consists of two jointly estimated equations, namely the conditional mean equation and the conditional variance equation. Firstly, the conditional mean equation assesses inter-regional and intra-regional return spillovers between economies. Secondly, the conditional variance equation assesses inter-regional and intra-regional volatility spillovers between economies.

In this study, a simple VARX[1] model is used to specify the conditional mean equation of asset returns. The VARX conditional mean equation in this study allows for estimation of intra-regional and inter-regional return spillovers between economies. Equations (1) and (2) below algebraically represent the VARX conditional mean equation of asset returns used in the study:

[1] The X in VARX indicates a VAR model with exogenous variables

$$r_{j,t} = \emptyset_0 + \sum_{j=1}^{N}\sum_{s=1}^{S} \varphi_j r_{j,t-s} + \sum_{m=1}^{M}\sum_{s=1}^{S} \alpha_m X_{m,t-s} + \sum_{j=1}^{N} \varepsilon_{j,t} \quad (1)$$

$$\varepsilon_{j,t}|\Omega_{t-s} \sim N(0, H_t)$$

$$\text{where } \varepsilon_{j,t} = v_{j,t} H_t^{1/2} \text{ and } v_{j,t} \sim N(0,1) \quad (2)$$

In equation (1), $r_{j,t}$ is a *N x 1* vector of asset returns, where *N* is the number of endogenous variables in the system. $\emptyset_0$ is an *N x 1* intercept vector and φ_j is a *N x N* coefficient matrix of historical asset returns. The off-diagonal elements of φ_j represent the cross dynamic returns between the endogenous asset returns, while the diagonal elements of φ_j represent the effects of own lagged returns on each endogenous asset. X_m is a *m x 1* matrix of returns of exogenous variables at time ***t****,* and α_m is a *N x m* coefficient matrix of unilateral return spillovers from developed economies to emerging economies, where *m* is the number of exogenous variables in the model. $\varepsilon_{j,t}$ is a *N x 1* vector of serially uncorrelated random residuals at time ***t.*** The vector of serially uncorrelated residuals ($\varepsilon_{j,t}$) given the information set Ω available at time $t - s$, where $t \neq s$ is distributed with a mean of zero and an *N x N* time-dependent variance-covariance matrix of H_t. Theoretically, the variance-covariance matrix H_t is restricted to be positive definite (Tsay, 2005). $\varepsilon_{j,t}$ is a function of a normally distributed deterministic component $v_{j,t}$ and time-varying variance-covariance matrix H_t.

Equation 2 shows that the serially uncorrelated innovations of asset returns ($\varepsilon_{j,t}$) are normally distributed with a constant mean of zero, and a non-constant variance of H_t. Thus, it is necessary to model H_t and to evaluate its subsequent effects on $r_{j,t}$ in equation 1. H_t is a variance-covariance matrix of asset returns, and can be expanded as follows:

$$H_t = \begin{pmatrix} h_{ii,t} & h_{ij,t} \\ h_{ji,t} & h_{jj,t} \end{pmatrix} \quad (3)$$

By definition, H_t in equation (3) is positive definite. The diagonal elements of H_t represent the variance of asset returns, while the off-diagonal elements of H_t represent the covariance of asset returns. The AVAR GARCH model is used to evaluate H_t.

The AVAR GARCH model evaluates H_t in two steps. The first step evaluates inter-regional and intra-regional volatility spillover effects between economies. H_t is defined using a multivariate GARCH model with AR terms. The multivariate GARCH model with AR terms used to evaluate H_t in equation 4 is represented algebraically below:

$$H_t = C + \sum_{j=1}^{N}\sum_{p=1}^{P} A_j \varepsilon_{j,t-p} + \sum_{j=1}^{N}\sum_{l=1}^{L} B_j H_{l,t-l} + \sum_{m=1}^{M}\sum_{q=1}^{Q} \omega_m \aleph_{m,t-q} \quad (4)$$

Equation (4) above specifies a GARCH process with AR terms to evaluate variances of endogenous assets (Ling & McAleer, 2003; Sadorsky, 2012). j=1,…,N specifies the number of endogenous assets in the system, p=1,…,P specifies the chosen lags of past innovations and l=1,…,L specifies the chosen lags of past volatility. $\boldsymbol{A_j}$ is a coefficient matrix of short run persistence of asset *j* and volatility spillovers between assets *i* and *j,* where *i ≠ j*. Diagonal elements of $\boldsymbol{A}$ capture short-run persistence of *j* and off-diagonal elements of $\boldsymbol{A}$ capture volatility spillovers between assets *i* and *j,* where *i ≠ j*. $\boldsymbol{B_j}$ is a coefficient matrix of persistence in the long run. $\boldsymbol{C}$ is an *N x 1* vector of constants. $\boldsymbol{\omega_m}$ is an *N x m* parameter matrix of volatility spillovers from developed economies to the stock markets of emerging economies, while $\aleph_{\boldsymbol{mt}}$ is an *m x N* matrix of unconditional volatility of developed economies' stock markets. The last term in equation (4) accounts for unilateral volatility spillovers from developed to emerging stock markets.

The estimated results from the AVAR GARCH model can be used to determine optimal hedge ratios and portfolio weights for financial investors in a portfolio of stock market assets. This is achieved in two main steps. In the first step, a portfolio of *N* risky stock market assets is constructed using Markowitz's (1952) mean-variance portfolio theory. Equations (5) and (6) below show algebraic expressions of the mean-variance portfolio theory as introduced in Markowitz (1952):

$$\boldsymbol{r_{portfolio,t}} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \boldsymbol{w_{j,t} r_{j,t}} \text{ where } w_{j,t} = \begin{cases} 0 \; if \; \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \boldsymbol{w_{j,t}} < 0 \\ \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \boldsymbol{w_{j,t}} \; if \; 0 \le \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \boldsymbol{w_{j,t}} \le 1 \\ 1 \; if \; \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \boldsymbol{w_{j,t}} > 1 \end{cases} \quad \textbf{(5)}$$

$$\boldsymbol{\sigma^2_{portfolio,t}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \sum_{\boldsymbol{j=1}}^{\boldsymbol{N}} \boldsymbol{w_{i,t} H_t w_{j,t}} \quad \textbf{(6)}$$

In equation (5), the returns for a portfolio of *N* risky assets at time $\boldsymbol{t}$ ($\boldsymbol{r_{portfolio,t}}$) are simply the weighted average of returns for each risky asset held in the portfolio where w_j is the weight of the portfolio held in asset *j*. In equation (6), the variance of a portfolio of *N* risky assets at time $\boldsymbol{t}$ ($\boldsymbol{\sigma^2_{portfolio,t}}$) is a function of the portfolio weights held in each asset, as well the variance-covariance matrix ($\boldsymbol{H_t}$). Thus, estimates of the $\boldsymbol{H_t}$ generated in the AVAR GARCH model can be used to infer portfolio risk in equation (6).

In the second step, the portfolio constructed using the mean-variance portfolio theory is evaluated to achieve optimal hedge ratios or portfolio weights for risk-averse financial investors. A risk-averse financial investor in a two-asset portfolio achieves an optimal hedge ratio if the risk of holding one long/short position in asset ***i*** can be optimally insured by holding a corresponding $\boldsymbol{w_j}$ short/long position in asset ***j*** (Kroner & Sultan, 1993). Equation (7) below is an algebraic representation of a two-asset portfolio where one long/short position in asset ***i*** is hedged with a short/long position in asset ***j***:

$$\boldsymbol{\sigma^2_{portfolio,t} = h_{ii,t} + w_{j,t}{}^2\, h_{jj,t} + 2h_{ij,t}w_{j,t}} \qquad \textbf{(7)}$$

Using the definition of optimal hedge ratios described above, an optimal hedge of one long position in asset ***i*** with a short position in asset ***j*** of a two-asset portfolio can be derived from equation (8) as follows:

$$\boldsymbol{\frac{\partial \sigma^2_{portfolio,t}}{\partial w_{j,t}} = 0}$$

$$\boldsymbol{2h_{jj,t}w_{j,t} + 2h_{ij,t} = 0}$$

$$\boldsymbol{w_{j,t} = -\frac{h_{ij,t}}{h_{jj,t}}} \qquad \textbf{(8)}$$

where $\boldsymbol{-\frac{h_{ij,t}}{h_{jj,t}}}$ is the proportion of asset ***j*** shorted to hedge the risk of a long position in asset ***i***.

Alternatively, a risk-averse financial investor in a two-asset portfolio achieves optimal portfolio weights if the weights of each asset held in the portfolio simultaneously maximise portfolio returns and minimise portfolio variance (Markowitz, 1952). Equation (9) below represents the algebraic expression of a two-asset portfolio in which portfolio weights can be optimised:

$$\boldsymbol{\sigma^2_{portfolio,t} = w_{i,t}{}^2 h_{ii,t} + w_{j,t}{}^2\, h_{jj,t} + 2h_{ij,t}w_{i,t}w_{j,t}} \quad \text{where } \boldsymbol{w_{i,t} = 1 - w_{j,t}} \qquad \textbf{(9)}$$

To optimise the portfolio weights in equation (9), the portfolio variance is minimised as follows:

$$\boldsymbol{\frac{\partial \sigma^2_{portfolio,t}}{\partial w_{j,t}} = 0}$$

$$\boldsymbol{w_{j,t} = \frac{h_{ii,t} - 2h_{ij,t}}{h_{ii,t} - 2h_{ij,t} + h_{jj,t}}} \text{ where } \boldsymbol{w_{j,t}} = \begin{cases} 0 \ if\ \boldsymbol{w_{j,t}} < 0 \\ \boldsymbol{w_{j,t}}\ if\ 0 \le \boldsymbol{w_{j,t}} \le 1 \\ 1\ if\ \boldsymbol{w_{j,t}} > 1 \end{cases} \qquad \textbf{(10)}$$

and $\boldsymbol{\frac{h_{ii,t} - 2h_{ij,t}}{h_{ii,t} - 2h_{ij,t} + h_{jj,t}}}$ is the optimal portfolio weight of asset ***j*** held in a two-asset portfolio of assets ***i*** and ***j***.

4. Data description

This section of the study provides a detailed description of the data used to estimate the cross-transmission of volatility between South African and Nigerian stock markets in this study. The data for the Nigerian stock market used in the study is the Nigerian All Share Index (NGSEINDX). The NGSEINDX is computed using only ordinary shares, and it tracks the general movement of all equities listed on the Nigerian Stock Exchange (NSE), including those listed on the Alternative Securities Market, regardless of capitalisation (The Nigerian Stock Exchange, 2016). The South African stock market uses the Johannesburg All Share Index (ASI). The Johannesburg ASI tracks the top 99 percent of total market capitalisation of listed companies on the Johannesburg Stock Exchange (JSE) (JSE, 2016).

The data used in this study are weekly share index data at closing prices, and are observed from 28 September 2000 to 8 September 2016. Notably, the observed estimation period in the study (i.e. 21 September 2000 to 8 September 2016) includes the 2007-2008 global financial crisis. Thus, it is important to note the findings in Sugimoto et al. (2014) inter alia, stating that African stock markets like South Africa and Nigeria were more severely affected by the 2008 EU sovereign debt crisis than they were by the 2007 US subprime housing market crisis.

Thus, the presence of inter-regional returns and volatility spillover effects between South African and Nigerian stock markets with the UK are also represented in the model. For simplicity, only unilateral returns, and volatility spillover effects from the UK stock market (FTSE100) are considered in the model. Thus, the FTSE100 is observed as an exogenous variable in the estimation model, to capture inter-regional returns and volatility spillover effects from the UK to South African and Nigerian stock markets.

The use of weekly data rather than daily data in the estimation allows for synchronisation and provides global financial investors with beneficial and practical portfolio rebalancing solutions[2]. Asset returns are computed as $100 \times \ln(\frac{p_t}{p_{t-1}})$, where p_t is the closing index value and time t.

The conditional mean equation as in Equation (1), can be expressed as:

[2] Weekly data is also of a high enough frequency to accurately capture heteroscedasticity and does not require the manipulation of data; e.g. accounting for missing data points like weekends and public holidays in the case of higher frequency data such as daily and intra-daily data (Tsay, 2005).

$$r_{j,t} = \emptyset_0 + \sum_{j=1}^{N}\sum_{s=1}^{S}\varphi_j r_{j,t-s} + \sum_{m=1}^{M}\sum_{s=1}^{S} a_m \; FTSE100 \; + \sum_{j=1}^{N} \varepsilon_{j,t} \qquad \textbf{(11)}$$

$FTSE100$ represents a series of UK stock market continuously compounded returns. The $FTSE100$ variable is treated exogenously to account for unilateral return spillovers from the UK to South African and Nigerian stock markets.

5. Estimation and discussion of results

This section provides the estimated results of the AVAR GARCH and the modern portfolio theory models. The section begins by outlining some descriptive statistics of the data used in the study. The second part of the section presents and discusses the results of the estimated AVAR GARCH model following the methodology described in section 3. The third part of the section presents and discusses inferences for optimal hedge ratios and portfolio weights using equations (8) and (10) respectively. The fourth part of the section summarises the chapter's findings.

5.1. Descriptive statistics

Table 1 below depicts the descriptive statistics of the data described in section 4.

Table 1: Descriptive statistics of observed data

Number of observations for each time series: 833
Weekly data from: 2000:09:28 to 2016:09:08

Series	**Mean**	**Variance**	**Skewness**	**Kurtosis**	**Jarque-Bera statistic**
NSE returns ($r_{1,t}$)	0.16%	10.42%	-0.0439 (0.605326)	3.5861 (0.000)	446.62 (0.000)
JSE returns ($r_{2,t}$)	0.23%	7.22%	0.0308 (0.716760)	3.2112 (0.000)	358.04 (0.000)
FTSE100 returns ($FTSE100$)	0.01%	5.68%	-0.5990 (0.000)	3.2771 (0.000)	422.55 (0.000)

*P-values in parenthesis

Table 1 indicates that average stock market returns are the highest in the South African stock market (0.23%), second highest in the Nigerian stock market (0.16%), and the lowest in the UK stock market (0.01%). This result is consistent with the theory that EMEs offer attractive investment opportunities for growth investors, due to their increasing contribution to global economic growth (Levine, 1996). Table 1 also reveals that Nigerian stock market returns are the most variable, with a variance of 10.42%.

South African stock market returns are the second most variable (7.22%), and the UK stock market returns are the least variable (5.68%) between the three stock markets. This result is consistent with the theory that stock markets with high market capitalisations and more liquidity are less risky than less developed stock markets (Levine, 1996).

It is important to note that the stock market returns of all three countries are not normally distributed. The skewness of the data suggests that UK stock market returns are frequent, positive, and small, with few events of severe losses, while the distribution of South African and Nigerian stock market returns are generally symmetric. However, the individual distributions of the UK, South African, and Nigerian stock market returns are generally heavy in the tails. These results confirm the theory that the stock markets of advanced economies are generally less volatile than the stock markets of EMEs (Ritter, 2005).

Figure 1 below shows the dynamics of the weekly stock market returns for UK, Nigerian, and Nigerian stock markets.

Figure 1: Weekly returns for FTSE100, NSE, and JSE (2000:09:28 to 2016:09:08)

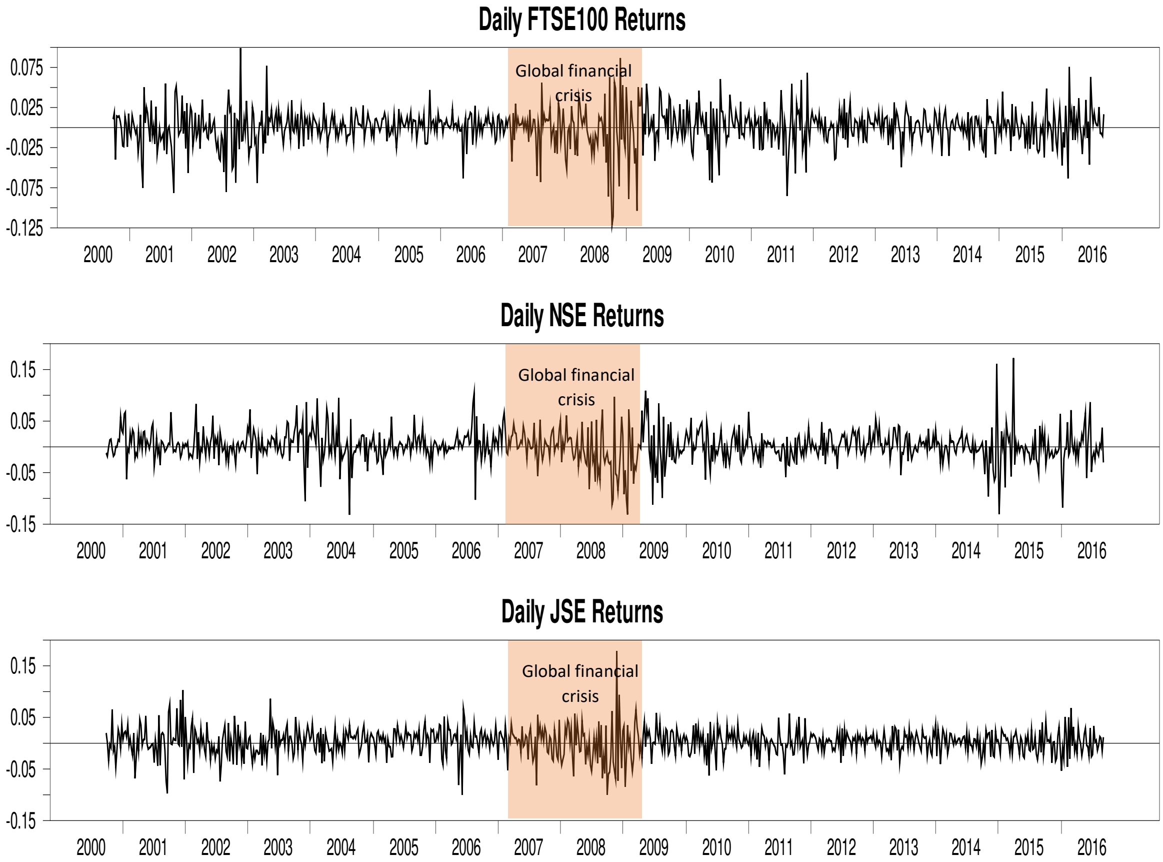


Figure 1 shows that UK, Nigerian, and South African stock market returns are each mean reverting, thus, suggesting the presence of stationarity in the data. Table 2 below shows results from the ADF test for stationarity of UK, Nigerian, and South African stock market returns.

Table 2: Augmented Dickey Fuller test for stationarity in FTSE100, NSE, and JSE returns

Series	ADF T-statistic	ADF p-value
NSE returns (r_1)	-5.51157	0.01
JSE returns (r_2)	-5.86649	0.01
FTSE100 returns ($FTSE100$)	-5.30774	0.01

Note: The alternative hypothesis of the ADF Test is stationarity, tested at lag order 16.

Table 2 indicates that the Nigerian, UK, and South African stock market returns are each stationary at a one percent level of significance. Thus, using the MLS estimation method for the parameters in equations (1) and (11) may not produce spurious regression results.

Figure 2 below shows the autocorrelation functions (ACFs) and PACFs for Nigerian, UK, and South African stock market returns respectively.

Figure 2: ACF and PACF for FTSE100, NSE, and JSE returns

ACF and PACF for weekly JSE returns

Date: 08/22/17 Time: 20:30
Sample: 9/21/2000 9/08/2016
Included observations: 833

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.115	-0.115	10.964	0.001
		2	0.037	0.024	12.115	0.002
		3	-0.033	-0.026	13.025	0.005
		4	-0.045	-0.053	14.694	0.005
		5	0.035	0.027	15.742	0.008
		6	0.047	0.057	17.603	0.007
		7	-0.060	-0.054	20.623	0.004
		8	0.026	0.010	21.178	0.007
		9	0.048	0.064	23.105	0.006
		10	0.006	0.017	23.139	0.010
		11	-0.007	-0.016	23.176	0.017
		12	-0.075	-0.072	27.887	0.006
		13	0.048	0.045	29.809	0.005
		14	0.055	0.063	32.361	0.004
		15	0.037	0.037	33.547	0.004
		16	-0.015	-0.010	33.739	0.006
		17	0.004	0.013	33.752	0.009
		18	-0.003	0.008	33.759	0.013
		19	-0.044	-0.061	35.397	0.013
		20	0.041	0.030	36.839	0.012
		21	-0.034	-0.011	37.834	0.013
		22	0.026	0.015	38.425	0.016
		23	0.005	-0.006	38.448	0.023
		24	-0.025	-0.030	38.967	0.028
		25	0.013	0.019	39.121	0.036
		26	-0.007	-0.001	39.169	0.047
		27	-0.041	-0.042	40.650	0.044
		28	0.007	-0.013	40.691	0.057
		29	-0.044	-0.041	42.362	0.052
		30	0.027	0.016	42.982	0.059
		31	0.039	0.032	44.299	0.057
		32	-0.024	-0.008	44.820	0.066
		33	0.057	0.057	47.598	0.048
		34	-0.039	-0.020	48.935	0.047
		35	0.022	0.014	49.342	0.055
		36	-0.035	-0.037	50.429	0.056

ACF and PACF for weekly NSE returns

Date: 08/22/17 Time: 20:31
Sample: 9/21/2000 9/08/2016
Included observations: 833

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.054	0.054	2.4706	0.116
		2	0.022	0.019	2.8857	0.236
		3	0.019	0.017	3.2016	0.362
		4	0.091	0.089	10.084	0.039
		5	0.082	0.072	15.697	0.008
		6	0.043	0.032	17.256	0.008
		7	-0.038	-0.048	18.486	0.010
		8	0.076	0.070	23.351	0.003
		9	0.027	0.007	23.959	0.004
		10	0.064	0.050	27.449	0.002
		11	-0.022	-0.029	27.860	0.003
		12	0.012	0.006	27.979	0.006
		13	0.054	0.043	30.453	0.004
		14	0.049	0.027	32.509	0.003
		15	-0.011	-0.015	32.608	0.005
		16	0.025	0.019	33.138	0.007
		17	-0.006	-0.013	33.166	0.011
		18	-0.006	-0.030	33.194	0.016
		19	-0.002	-0.005	33.197	0.023
		20	-0.017	-0.020	33.439	0.030
		21	0.026	0.028	34.027	0.036
		22	0.058	0.051	36.883	0.024
		23	-0.025	-0.030	37.415	0.029
		24	0.021	0.022	37.797	0.036
		25	0.049	0.049	39.855	0.030
		26	0.064	0.047	43.410	0.017
		27	-0.027	-0.043	44.022	0.021
		28	-0.047	-0.045	45.890	0.018
		29	-0.013	-0.017	46.027	0.023
		30	-0.013	-0.036	46.175	0.030
		31	0.045	0.048	47.938	0.027
		32	0.036	0.041	49.058	0.027
		33	-0.003	0.011	49.066	0.036
		34	0.031	0.023	49.908	0.038
		35	0.021	0.003	50.294	0.045
		36	0.016	0.005	50.518	0.055

ACF and PACF for weekly FTSE100 returns

Date: 08/22/17 Time: 20:32
Sample: 9/21/2000 9/08/2016
Included observations: 833

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.048	-0.048	1.9406	0.164
		2	-0.021	-0.023	2.2928	0.318
		3	-0.074	-0.076	6.8981	0.075
		4	-0.028	-0.036	7.5445	0.110
		5	0.057	0.051	10.286	0.068
		6	0.006	0.005	10.319	0.112
		7	-0.051	-0.053	12.514	0.085
		8	0.031	0.033	13.307	0.102
		9	-0.033	-0.029	14.245	0.114
		10	0.009	-0.003	14.316	0.159
		11	-0.055	-0.056	16.864	0.112
		12	-0.048	-0.051	18.783	0.094
		13	0.103	0.093	27.816	0.010
		14	0.022	0.023	28.239	0.013
		15	0.006	0.004	28.267	0.020
		16	0.056	0.074	30.934	0.014
		17	0.013	0.038	31.082	0.020
		18	0.027	0.018	31.698	0.024
		19	-0.059	-0.052	34.647	0.015
		20	0.030	0.042	35.403	0.018
		21	0.043	0.039	36.988	0.017
		22	0.056	0.055	39.667	0.012
		23	-0.031	-0.023	40.475	0.014
		24	-0.041	-0.020	41.913	0.013
		25	-0.006	0.017	41.944	0.018
		26	0.064	0.047	45.482	0.010
		27	-0.035	-0.035	46.565	0.011
		28	-0.031	-0.028	47.394	0.012
		29	-0.019	-0.012	47.692	0.016
		30	-0.011	-0.032	47.800	0.021
		31	0.073	0.053	52.383	0.010
		32	0.007	0.024	52.424	0.013
		33	-0.020	-0.013	52.787	0.016
		34	-0.016	-0.023	53.002	0.020
		35	0.010	0.003	53.081	0.026
		36	0.010	-0.003	53.175	0.032

Figure 2 suggests the presence of some autocorrelation in JSE, NSE, and FTSE100 weekly returns over the period 2000-2016. Thus, it is appropriate to use a VAR model to estimate the conditional mean of the JSE and NSE's stock returns. The VAR model specification allows for autocorrelations and cross-autocorrelations in the JSE and NSE's weekly returns. Lagged FTSE100 returns are included in the conditional mean model specification to control for return spillovers from developed economies to South African and Nigerian stock markets. For simplicity, the effects of lagged FTSE100 returns on JSE and NSE returns are assumed to be exogenous.

5.2. Estimated return and volatility spillovers between the South African and Nigerian stock markets

The Box Jenkins (1976) approach is used to determine the appropriate lag length of asset returns in conditional mean equation (1) and volatility in conditional variance equation (4). For the sake of reducing the sacrifice for degrees of freedom, the appropriate lag lengths of asset returns, and volatility in the conditional mean and conditional variance equations respectively is assumed to be one. Table 3 below shows the estimated results for equations (1) to (4) in section 3.

Table 3: Jointly estimated conditional mean equation and conditional variance equation

	Parameter	Coefficient	Standard Error	T-Statistic	Probability-Value
Conditional Mean Equation					
	φ_{11}	0.11611543	0.02710	4.28324000	0.0000184
	φ_{12}	0.03218211	0.00968	3.32214000	0.0008932
	$\emptyset_{01}$	0.32229732	0.12994	2.48034000	0.0131255
	α_{11}	0.04267099	0.04178	1.02116000	0.3071775
	φ_{21}	0.01842969	0.06810	0.27059000	0.7867040
	φ_{22}	-0.04650007	0.07534	-0.61720000	0.5371022
	$\emptyset_{02}$	0.21209494	0.07425	2.85615000	0.0042881
	α_{21}	0.77338524	0.13031	5.93496000	0.0000000
Conditional Variance Equation					
	c_{11}	0.80541074	0.02352	34.23679000	0.0000000
	c_{21}	0.03616129	0.19714	0.18342000	0.8544652
	a_{11}	0.28590155	0.00113	251.9930500	0.0000000
	a_{12}	0.26281278	0.01110	23.66328000	0.0000000
	a_{21}	-0.04450880	0.04603	-0.96680000	0.3336443
	a_{22}	0.09352216	0.03134	2.98386000	0.0028463
	b_{11}	0.72845754	0.01504	48.41645000	0.0000000
	b_{12}	-2.48284401	0.09114	-27.24174000	0.0000000
	b_{21}	0.52655282	0.69160	0.76135000	0.4464468
	b_{22}	0.82800525	0.26699	3.10121000	0.0019272
	$\vartheta_{3,11}$	-0.11950913	0.00132	-90.39433000	0.0000000
	$\vartheta_{3,22}$	0.01835514	0.17598	0.10430000	0.9169310
	$\aleph_1$	0.04636390	0.03769	1.22998000	0.2187039
	$\aleph_2$	0.02171521	0.07842	0.27689000	0.7818674
	ϑ_1	0.01730035	0.00358	4.82751000	0.0000013
	ϑ_2	0.85201071	0.10748	7.92672000	0.0000000

The results reported in Table 3 suggests evidence of dynamic stock market correlations between South African (ϑ_2) and Nigerian (ϑ_2) stock markets. Furthermore, the data suggests evidence of asymmetric volatility in Nigerian stock markets ($\vartheta_{3,11}$) and no evidence of asymmetric volatility in South African ($\vartheta_{3,22}$) stock markets

The results reported in Table 3 also show that return spillovers from the UK to the South African stock market (α_{21}) are highly significant and large, while return spillovers from the UK to Nigerian stock markets (α_{11}) are statistically insignificant. Large return spillovers from the UK to South African stock market may be an indication of strong economic and financial links between South Africa and the UK. Although historical social, economic, and financial data suggest some social, economic, and financial links between Nigeria and the UK, stock market illiquidity and underdevelopment in Nigeria may have resulted in some informational inefficiency in the Nigerian stock market's incorporation of changes in the stock market returns of developed economies (Maduka & Onwuka, 2013).

Moreover, Table 3 above shows that volatility spillovers from the UK to Nigerian stock markets ($\aleph_1$) and volatility spillovers from the UK to South African stock markets ($\aleph_2$) are statistically insignificant. Statistically insignificant volatility spillovers from the UK to South African and Nigerian stock markets suggests that any volatility spillovers from developed economies to the South African and Nigerian stock markets may have already been reflected in stock market returns specified in the conditional mean equation.

The results reported in Table 3 also suggest the presence of statistically significant and positive unidirectional return spillovers between the South African and Nigerian stock markets. Return spillovers from the South African stock market to the Nigerian stock market (φ_{12}) are statistically significant and positive, while return spillovers from the Nigerian to the South African stock market (φ_{21}) are statistically insignificant. This result may suggest that South African stock market returns are more affected by changes in emerging stock markets with high market capitalisation than those with relatively low market capitalisation, such as Nigeria. This result is consistent with Piesse and Hearn (2005), who state that changes in more developed stock markets are likely to affect the returns of less developed stock markets.

The parameters of the conditional variance equation in Table 3 also indicate the presence of unidirectional volatility spillovers between South African and Nigerian stock markets. The volatility spillovers from Nigerian stock markets to South African stock markets (a_{21}) are statistically insignificant, while those from the South African to Nigerian stock markets (a_{12}) are statistically significant and positive. Theoretically, this may suggest a significant impact of structural shocks in a more developed South African stock market on a relatively less developed Nigerian stock market. Thus, the South African stock market is seemingly unaffected by volatility shocks to the Nigerian stock market in the short-run, and volatility shocks in the more developed South African stock market directly affect the volatility of Nigeria's stock market. Similar results are suggested in the long-run. Long-run persistence of volatility shocks from Nigeria to South Africa (b_{21}) is also statistically insignificant. Interestingly, there is some statistically significant negative long-run persistence of volatility shocks from South Africa's stock market to Nigeria's stock market (b_{12}). This result could indicate a possible decoupling of South African and Nigerian stock markets in the long-run, resulting from initial positive volatility shocks in South African stock markets.

5.3. Opportunities for portfolio optimisation and optimal hedge ratios

Although not accurate, given the presence of heteroscedasticity between the NSE and JSE returns, Table 4 nonetheless shows the unconditional correlation matrix ($\bar{\boldsymbol{Q}}$) of the NSE and JSE's returns

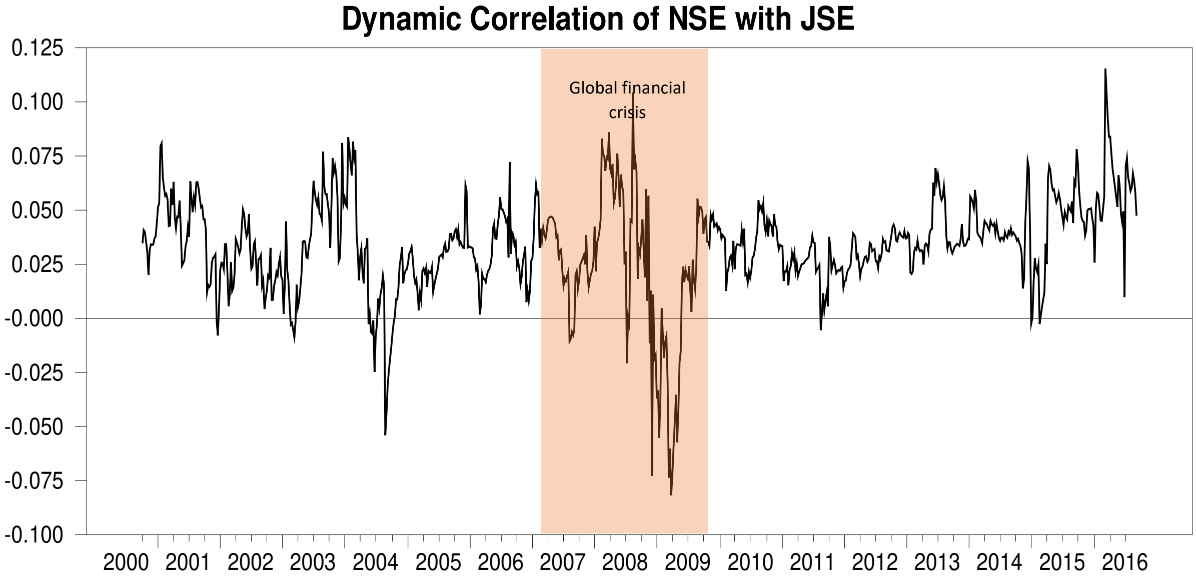
Table 4: Unconditional correlation matrix ($\bar{\boldsymbol{Q}}$) of NSE and JSE continuously compounded returns

	NSE	JSE
NSE	1.000000	0.03431
JSE	0.03431	1.000000

The results in Table 4 show a positive correlation between the South African and Nigerian stock market returns that is close to zero. The low correlation between the JSE and NSE suggests some benefits to diversification and hedging for global financial investors in the stock markets of Africa's two largest economies.

Figure 3 below displays the dynamic conditional correlation between the South African and Nigerian stock market returns obtained from the estimated conditional variance equation in Table 3.

Figure 3: Weekly conditional correlation of NSE with JSE: 2000-2016



It is evident from Figure 3 that the correlation between South African and Nigerian stock market returns increased sharply in the middle of the global financial crisis (mid-2008) then fell in the beginning of 2009, and ultimately became negative towards the end of the global financial crisis. The significant decline in the correlation between South African and Nigerian stock markets towards the end of the global financial crisis suggests some decoupling between the South African and Nigerian stock markets. This outcome suggests the possibility of some meaningful portfolio diversification and hedging opportunities for global financial investors that participate in the two stock markets.

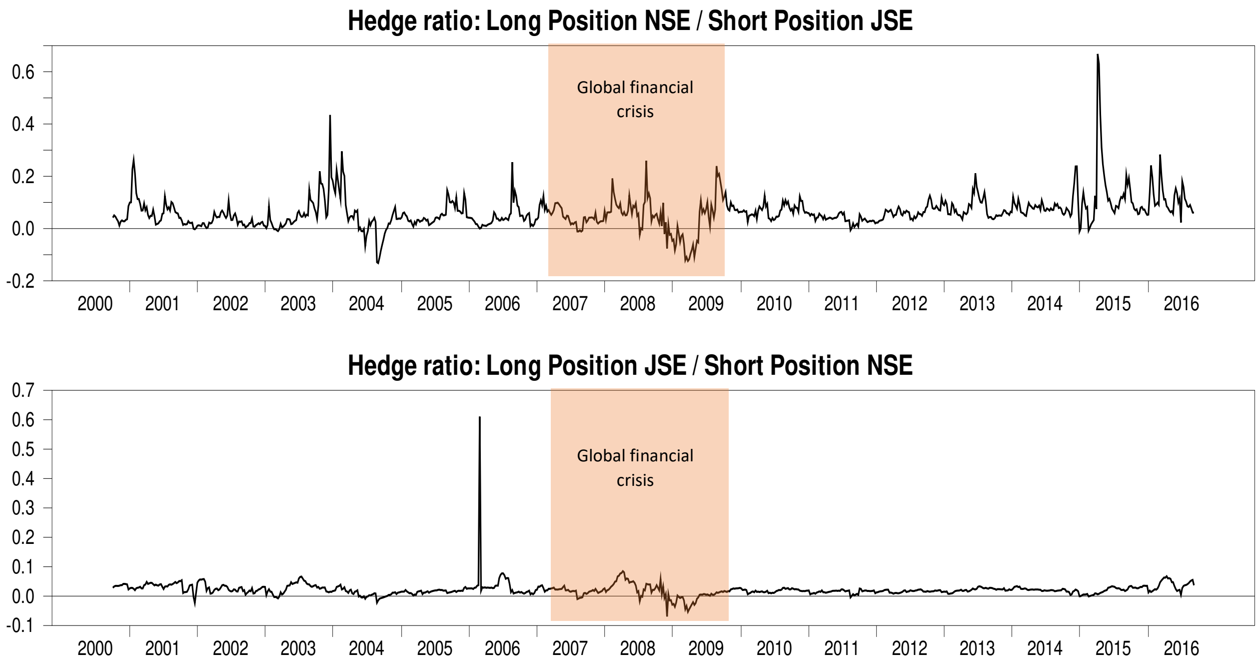
Regarding the hedging ratio, Table 5 summarises the hedge ratio statistics for positions in the South African and Nigerian stock markets over the observed period. Table 5 suggests the relatively inexpensive cost of hedging a long position in the South African stock market with a short position in the Nigerian stock market. Over the observed period, a R1 long position in the South African stock market can be hedged with a R0.02 short position in the Nigerian stock market on average. Whereas, a R1 long position in the Nigerian stock market can be hedged with a R0.06 short position in the South African stock market.

Table 5: Hedge ratio long position/short position summary statistics

Long Position/Short Position	Mean	Standard Deviation	Minimum	Maximum
NSE/JSE	0.06	0.06	-0.13	0.67
JSE/NSE	0.02	0.03	-0.07	0.61

While Table 5 reports the average optimal hedge ratio for an investor that holds positions in the two stock exchange markets, Figure 4 below shows the dynamic optimal hedge ratios for the South African and Nigerian stock markets during the observed period.

Figure 4: Dynamic weekly optimal hedge ratios for positions in the NSE and JSE: 2000-2016



In Figure 4 above, the cost of hedging a long position in the South African stock market with a short position in the Nigerian stock market is relatively inexpensive. As mentioned earlier in this study, the South African stock market is more liquid and developed than the Nigerian stock market. This suggests that risk-averse financial investors consider South African stock markets to be riskier than the Nigerian stock markets (Levine, 1996). Thus, the result in Figure 4 may further suggest that risk-averse investors in the Nigerian and South African stock markets require less compensation for holding less risky South African stock market assets in comparison to holding riskier Nigerian stock market assets (Wu, 2001). However, this differs to the beginning of 2006, when a R1 long position in the South African stock market could be hedged with a more than R0.60 short position in the Nigerian stock market.

As expected, the cost of hedging South African with Nigerian stock markets during the middle of the 2007-2009 global financial crisis (mid-2008) increased, as did the correlation of many stock market assets during that period. A possible decoupling of

South African and Nigerian stock markets at the tail-end of the 2007-2009 global financial crisis suggested by the statistically significant negative long-run persistence of volatility shocks from South African stock markets to Nigerian stock markets illustrated as (b_{12}) in Table 3, may explain the corresponding declining cost of hedging South African with Nigerian stock markets.

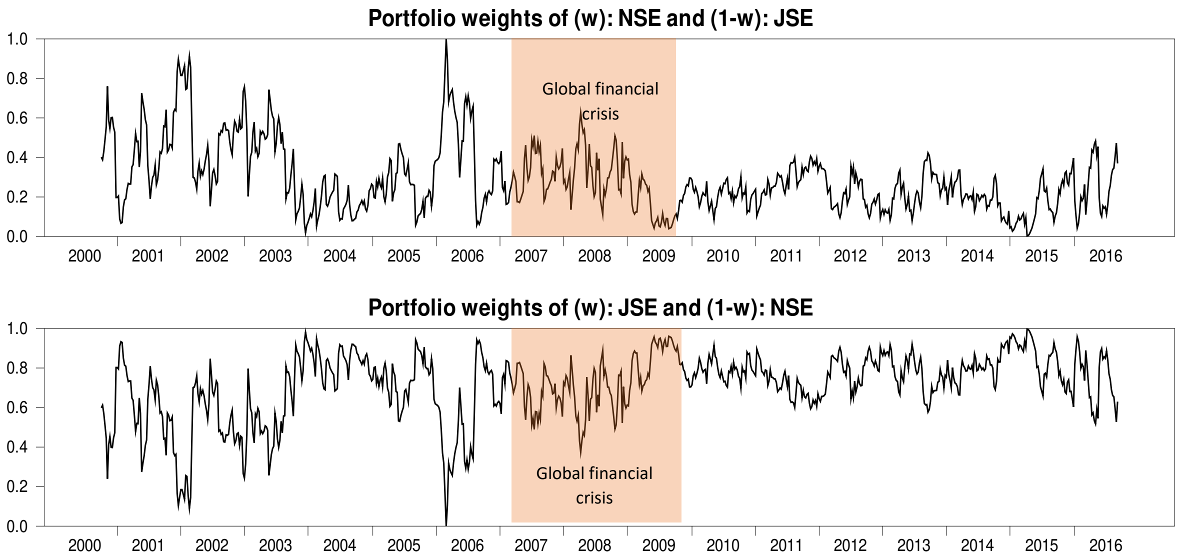
Like dynamic hedge ratios, dynamic portfolio weights provide global financial investors with optimal portfolio rebalancing solutions. Table 6 below shows the portfolio weight summary statistics for a two-asset portfolio of South African and Nigerian stock markets in the observed period. Table 6 suggests that an optimal two-asset portfolio, on average, be held by global financial investors seeking to benefit from portfolio diversification between South African and Nigerian stock markets, and should consist of 29 percent in the Nigerian stock market and 71 percent in the South African stock market.

Table 6: Portfolio weights summary statistics

(w)/(1-w)	Mean	Standard Deviation	Minimum	Maximum
NSE/JSE	0.29	0.17	0.00	1.00

However, the dynamic nature of the correlations between the South African and Nigerian stock markets, arguably justifies an analysis of dynamic portfolio weights in a two-asset portfolio of South African and Nigerian stock market assets. Figure 5 below shows the dynamic portfolio weights that should be held in Nigerian and South African stock markets for global financial investors seeking optimal portfolio diversification opportunities in the stock markets of Africa’s two largest economies.

Figure 5: Dynamic weekly optimal portfolio weights for a two-asset portfolio of assets NSE and JSE: 2000-2016



The first diagram in Figure 5 shows the dynamic portfolio weights of Nigerian stock market assets held in a two-asset portfolio of South African and Nigerian stock market assets, whereas the second diagram in Figure 5 shows the dynamic portfolio weights of South African stock market assets held in a two-asset portfolio of South African and Nigerian stock market assets.

The data in Figure 5 suggests that investors should have held less South African stock market assets and more Nigerian stock market assets during the global financial crisis of 2007-2009 in general. However, towards the end of the crisis, the data suggests holding less Nigerian stock market assets and more South African stock market assets in general. Similar results suggesting greater portfolio weightings in Nigerian stock market assets than South African stock market assets, such as at the beginning of the years 2003 and 2006 respectively, can be highlighted.

6. Conclusion

The objective of this study was to evaluate the cross-transmission of return and volatility spillovers between South African and Nigerian stock markets, thus, inferring the optimal hedging ratio and portfolio weights for risk-averse financial investors seeking investment opportunities in the stock markets of Africa's two largest economies in terms of GDP. The transmission of stock market returns and volatility between any two markets is the presence of financial market integration, including inter alia, social, economic, and financial links between countries, and has previously been identified as one of the explanations for the returns and volatility spillovers between different economies. Historical data suggests that South Africa and Nigeria share some social, economic, and financial links, and therefore, it is arguably justifiable to evaluate the cross-transmission of volatility between South African and Nigerian stock markets

It is important to note that historic events like the 2007-2009 global financial crisis, inter alia, have suggested some co-movement between poor stock market performance in advanced economies like the UK, with poor stock market performance in South Africa and Nigeria. Coincidently, historical data also suggests that South Africa and Nigeria share social, economic, and financial links with the UK. In addition, historical data also suggests that the economic and social links shared by South Africa and Nigeria with the UK are arguably the strongest relative to all other international economies. Thus, it is justifiable to control for stock market spillovers from the UK to South African and Nigerian stock markets.

In achieving the objective of the study, the magnitude and significance of returns and volatility spillovers between South African and Nigerian stock markets are estimated, such that returns and volatility spillover from advanced economies like the UK to South African and Nigerian stock markets are controlled for. To this end, the study makes use of an AVAR GARCH model with conditional mean and conditional variance. The AVAR GARCH model accounts for asymmetric volatility effects in stock markets, as well as dynamic conditional correlations between different stock markets.

The empirical results of the estimated AVAR GARCH model suggest that return and volatility spillovers from South African stock markets to Nigerian stock markets are unilateral, such that stock market returns and volatility spillovers from South Africa to Nigeria are highly statistically significant, positive, and large, whereas stock market returns and volatility spillovers from Nigeria to South Africa are statistically insignificant. These results are seemingly consistent with previous studies suggesting a significant influence of South African stock market on the stock markets of other major African countries (Kahn, 2011; Piesse & Hearn, 2005)). Furthermore, the empirical results of the AVAR GARCH model suggest that Nigerian stock markets are subject to asymmetric volatility effects, whereas South African stock markets are not.

The study also suggests some benefits to portfolio diversification between South African and Nigerian stock markets, and that the correlation between South African and Nigerian stock markets is time-varying. Following Sadorsky (2012), the results of asset correlation and volatility from the estimated AVAR GARCH model are used to make inferences on portfolio optimisation regarding holding South African and Nigerian stock market assets according to the Modern Portfolio Theory.

The study infers that, on average, investors should hold a larger proportion of South African stock market assets relative to Nigerian stock market assets in a portfolio. However, this is not consistent for all time periods.

The study also infers that the cost of hedging a long position in the South African stock market with a short position the Nigerian stock market is relatively inexpensive. On average, a R1 long position in the South African stock market can be hedged with a R0.02 short position in the Nigerian stock market, whereas on average, a R1 long position in the Nigerian stock market can be hedged with a R0.06 short position in the South African stock market. However, this is not consistent for all time periods.

Portfolio optimisation and optimal hedge ratios rely on reliable estimates, volatility, and asset correlation (Sadorsky, 2012). Thus, it can be argued that the presence of dynamic correlations between South African and Nigerian stock markets, justify a dynamic rebalancing of portfolio weights and hedging to achieve optimal portfolio diversification between the stock markets of the two African countries.

References

Arouri, M.E.H.; Jouini, J. and Nguyen, D.K. (2011). Volatility spillovers between oil prices and stock retunrs: implication for portfolio management. Journal of International Money and Finance, 30(7): 1387-1407

Baba, Y., Engle, R.F., Kraft, D.F. & Kroner, K.F. (1990). *Multivariate SimultaneousGeneralized ARCH.* Department of Economics, University of California at San Diego. Working Paper.

Balli, F., Balli, H.O., Louis, R.J. & Vo, T.K. (2015). The transmission of market shocks and directional linkages: Evidence from emerging economies. *International Review of Financial Analysis*, *42*:349-357.

Barra, M.S.C.I. (2010). Is there a link between GDP growth and stock returns? *Research Bulletin.*

Bekaert, G. (1999). Emerging Stock Markets and Market Integration. *NBER Reporter, 2000*:8-11.

Collins, D., & Biekpe, N. (2003). Contagion and interdependence in African stock markets. *South African Journal of Economics*, *71*(1):181-194.

Diebold, F.X. & Yilmaz, K. (2009). Measuring financial asset return and volatility spillovers, with application to global stock markets. *The Economic Journal, 119*(534):158-171.

Diebold, F.X. & Yilmaz, K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting*, *28*(1):57-66.

Duncan, A.S. & Kabundi, A. (2013). Domestic and foreign sources of volatility spillover to South African asset classes. *Economic Modelling*, *31*:566-573.

Engle, R.F. & Kroner, K.F. (1995). Multivariate simultaneous generalized ARCH. *Econometric theory*, *11*(01)122-150.

Farid, S. (2013). Financial integration in African emerging markets. *African Economic Conference,* October 28-30, 2013*,* 1-18.

Güloğlu, B., Kaya, P. & Aydemir, R. (2016). Volatility transmission among Latin American stock markets under structural breaks. *Physica A: Statistical Mechanics and its Applications*, *462*:330-340.

Habibullah, M.S. & Eng, Y.K. (2006). Does financial development cause economic growth? A panel data dynamic analysis for the Asian developing countries. *Journal of the Asia Pacific Economy*, *11*(4):377-393.

Johannesburg Stock Exchange. (2016). *Grow my wealth*: *What is the All Share Index*. Available at: https://www.jse.co.za/grow-my-wealth/what-is-the-jse-all-share-index Accessed on May 10, 2016.

Kahn, M.J. (2011). The BRICS and South Africa as the gateway to Africa. *Journal of the Southern African Institute of Mining and Metallurgy*, *111*(7):493-496.

King, D., & Botha, F. (2015). Modelling stock return volatility dynamics in selected African markets. *Economic Modelling*, *45*, 50-73.

Koop, G., Pesaran, M.H. & Potter, S.M. (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics*, *74*(1):119-147.

Korkmaz, T., Çevik, E.I. & Atukeren, E. (2012). Return and volatility spillovers among CIVETS stock markets. *Emerging Markets Review*, *13*(2):230-252.

Kroner, K.F. & Ng, V.K. (1998). Modeling asymmetric comovements of asset returns. *Review of Financial Studies*, *11*(4)817-844.

Kroner, K.F. & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. *Journal of Financial and Quantitative Analysis*, *28*(04):535-551.

Leke, A., Lund, S., Roxburgh, C. & van Wamelen, A. (2010). What's driving Africa's growth. Available from: http://www.mckinsey.com/global-themes/middle-east-and-africa/whats-driving-africas-growth Accessed on: June 23, 2016.

Levine, R. (1996). Stock markets: a spur to economic growth. *Finance and Development*, *33*:7-10.

Li, H., & Majerowska, E. (2008). Testing stock market linkages for Poland and Hungary: A multivariate GARCH approach. *Research in International Business and Finance*, *22*(3), 247-266.

Ling, S. & McAleer, M. (2003). Asymptotic theory for a vector ARMA-GARCH model. *Econometric Theory*, *19*(02):280-310.

London Stock Exchange. (2016). *London Stock Exchange Group Ticker Plant Product Guide*. Available at: http://www.londonstockexchange.com/products-and-services/millennium-exchange/documentation/gtp001-issue11.pdf Accessed on May 10, 2016.

Maduka, A.C. & Onwuka, K.O. (2013). Financial market structure and economic growth: evidence from Nigeria data. *Asian Economic and Financial Review*, *3*(1):75.

Mandelbrot, B. (1963). New methods in statistical economics. *Journal of political economy*, *71*(5), 421-440.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*(1):77-91.

Masih, A. M., & Masih, R. (1999). Are Asian stock market fluctuations due mainly to intra-regional contagion effects? Evidence based on Asian emerging stock markets. *Pacific-Basin Finance Journal*, *7*(3), 251-282.

Nigerian Stock Exchange. (2016). *Market Indices*, *All Share Index*. Available from: http://www.nse.com.ng/market-data/indices Accessed on May 10, 2016.

Owusu, E.L. & Odhiambo, N.M. (2012). The role of financial liberalization in Nigeria: a review of key policy issues. *Economics, Management, and Financial Markets*, *7*(3):89-107.

Oyovwi, O.D. & Eshenake, S.J. (2013). Financial openness and economic growth in Nigeria: A vector error correction approach. *African Research Review*, *7*(4):79-92.

Öztürk, S.S. & Volkan, E. (2015). Intraindustry Volatility Spillovers in the MENA Region. *Emerging Markets Finance and Trade*, *51*(6):1163-1174.

Piesse, J. & Hearn, B. (2005). Regional integration of stock markets in sub-Saharan Africa. *South African Journal of Economics*, *73*(1):36-52.

Ritter, J.R. (2005). Economic growth and stock returns. *Pacific-Basin Finance Journal*, *13*(5):489-503.

Sadorsky, P. (2012). Correlations and volatility spillovers between oil prices and the stock prices of clean energy and technology companies. *Energy Economics*, *34*(1):248-255.

South African Department of Trade and Industry. (2016). *Research and Statistics, Trade Statistics*. Available at http://tradestats.thedti.gov.za/TableViewer/tableView.aspx?ReportId=574 Accessed on May 10, 2016.

Sugimoto, K., Matsuki, T. & Yoshida, Y. (2014). The global financial crisis: An analysis of the spillover effects on African stock markets. *Emerging Markets Review*, *21*:201-233.

Tsay, R.S. (2005). Analysis of financial time series (Vol. 543). University of Chicago; John Wiley & Sons.

Tswamuno, D.T., Pardee, S. & Wunnava, P.V. (2007). Financial liberalization and economic growth: Lessons from the South African experience. *International Journal of Applied Economics*, *4*(2):75-89.

World Bank. (2016). World Development Indicators Data. Available at: http://databank.worldbank.org/data/reports.aspx?source=world-development-indicators Accessed on May 10, 2016.

Worthington, A., & Higgs, H. (2004). Transmission of equity returns and volatility in Asian developed and emerging markets: a multivariate GARCH analysis. *International Journal of Finance & Economics*, *9*(1), 71-80.

Wu, G. (2001). The determinants of asymmetric volatility. *Review of Financial Studies*, *14*(3):837-859.