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Return level applied to portfolio analysis

Working Paper

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In this paper, we estimated return levels of a portfolio of two assets using extreme value theory.

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I. S&P 500 and AEX Index

We estimated monthly returns, measured as the log return of the first day of the month to the first day of the next month, of S&P 500 Index and AEX 500 Index from 2008 to 2017, downloaded from Yahoo Finance, for a total 120 observations per index.

Next table summarized main statistical data:

Index	Obs.	Mean	Std. Dev.
S&P 500	120	0.60%	0.0440
AEX	120	0.20%	0.0531

II. Extreme Events

Extreme events occur when a risk takes values from the tail of its distribution (McNeil, 1999).

For a deeper mathematical treatment of the extreme value theory, Coles (2001) is recommended.

II.1. Return Level

The return level R_k^n is the level expected, on average, to be exceeded in one out of *k* periods of length *n*.

The return period is the amount of time expected to wait for particular return level to be exceed; return period is the inverse of the probability of an event, a called "100 years event" has a 1% (1/100) probability of exceed the record level in any one year.

For a Generalized Pareto Distribution, the *k* year return level is defined:

$$R_k \approx \mu + \frac{\tilde{\sigma}}{\tilde{\xi}} ([k * n_y * Pr(X>\mu)]^{\xi} - 1) \text{ for } \tilde{\xi} \neq 0$$

where μ is the defined threshold, $\tilde{\sigma}$, and $\tilde{\xi}$ are the parameters of the Generalized Pareto Distribution, n_y is the number of observations per year, and Pr (X>μ) is equal to number of exceedances of threshold (*Nu*) divided by total number of observations (*N*).

Using the in2extRemes Toolkit developed by Eric Gilleland and Richard Katz, within statistical software R, we conducted the estimation of the Generalized Pareto Distributions.

II.1.1. Left Tail

For the period 2008 – 2017, the AEX Index recorded a total of 51 cases of negatives monthly returns, and a maximum negative monthly return of 22.0%, and the SP&500 Index recorded a total of 43 cases of negatives monthly returns, and a maximum negative monthly return of 18.6%.

For the evaluation period we procced to estimated monthly returns for alternatives portfolios composition, assuming investing **α** (with α= 0%, 5%, 10%, 15%, 20%.... 80%, 85%, 90%, 95% and 100%) in the SP&500 index and investing **(1- α)** in the AEX Index.

In consistency with Suarez (2018) and Suarez (2017) we selected a 3% negative return as threshold (μ).

Next table summarized Generalized Pareto Distribution parameters for the portfolios combinations calculated:

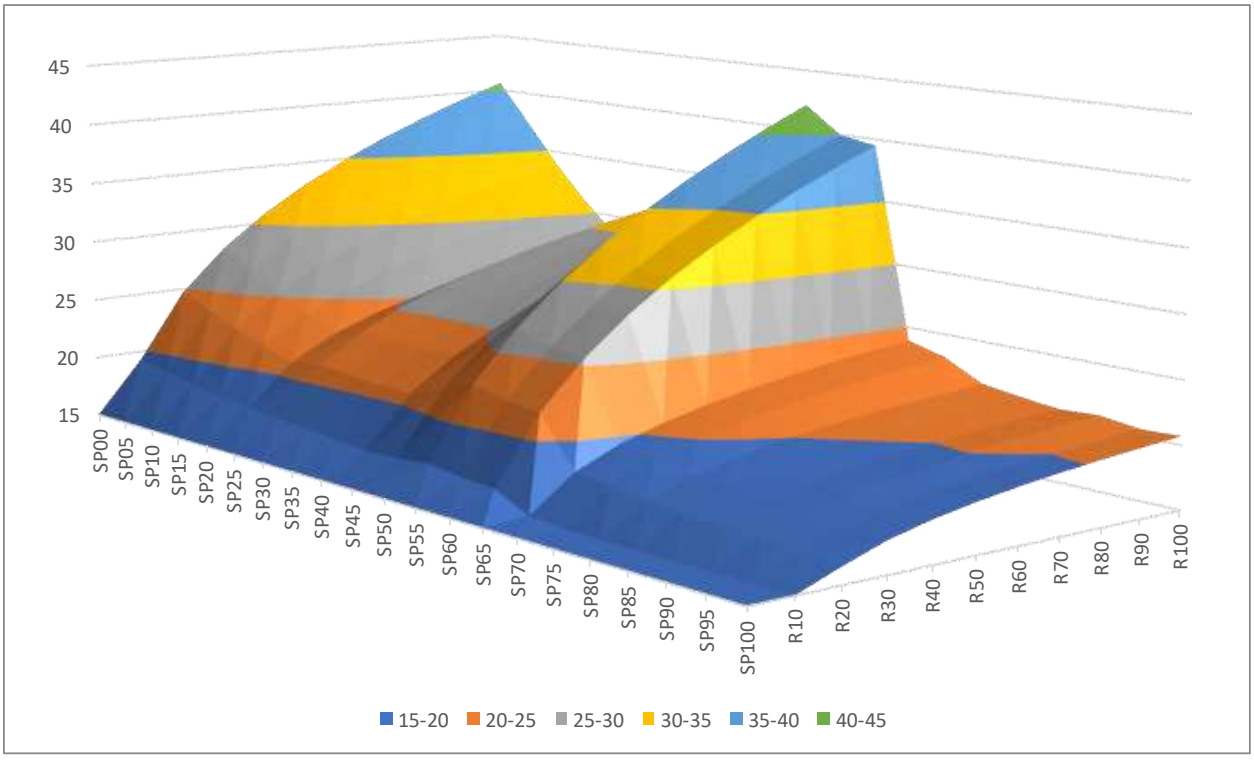
	$SP_{0\%}$	$SP_{5\%}$	$SP_{10\%}$	$SP_{15\%}$	$SP_{20\%}$	$SP_{25\%}$	$SP_{30\%}$	$SP_{35\%}$	$SP_{40\%}$	$SP_{45\%}$	$SP_{50\%}$
$\tilde{\sigma}$	3.7433	4.0185	4.3792	4.8040	5.2941	4.4180	4.2557	4.0785	3.8828	3.2364	2.7252
ξ	0.2112	0.1633	0.1035	0.0381	-0.0316	0.0701	0.0829	0.0998	0.1223	0.2263	0.3237
	$SP_{55\%}$	$SP_{60\%}$	$SP_{65\%}$	$SP_{70\%}$	$SP_{75\%}$	$SP_{80\%}$	$SP_{85\%}$	$SP_{90\%}$	$SP_{95\%}$	$SP_{100\%}$	
$\tilde{\sigma}$	2.7435	2.7071	4.5229	4.5767	5.2030	5.2313	5.2471	4.7583	4.8272	4.4494	
ξ	0.3052	0.3052	-0.0455	-0.0659	-0.1508	-0.1635	-0.1737	-0.1295	-0.1456	-0.1109	

Next table summarized return levels values for the portfolios combinations calculated:

	$SP_{0\%}$	$SP_{5\%}$	$SP_{10\%}$	$SP_{15\%}$	$SP_{20\%}$	$SP_{25\%}$	$SP_{30\%}$	$SP_{35\%}$	$SP_{40\%}$	$SP_{45\%}$	$SP_{50\%}$
R_{10}	19.3	18.8	18.4	18.0	17.6	17.4	17.2	17.0	16.8	16.9	17.1
R_{20}	24.7	23.7	22.7	21.7	20.9	21.3	21.1	20.9	20.8	21.7	22.8
R_{30}	28.2	26.8	25.3	24.0	22.8	23.7	23.4	23.3	23.3	24.8	26.8
R_{40}	30.9	29.1	27.3	25.6	24.2	25.4	25.2	25.1	25.2	27.3	29.9
R_{50}	33.1	31.0	28.8	26.9	25.2	26.7	26.5	26.5	26.7	29.2	32.6
R_{60}	35.0	32.6	30.1	27.9	26.0	27.8	27.7	27.7	27.9	31.0	34.9
R_{70}	36.6	34.0	31.3	28.8	26.7	28.8	28.7	28.7	29.0	32.5	36.9
R_{80}	38.1	35.2	32.2	29.6	27.3	29.6	29.5	29.6	29.9	33.8	38.8
R_{90}	39.4	36.3	33.1	30.3	27.9	30.3	30.3	30.4	30.8	35.0	40.5
R_{100}	40.6	37.3	33.9	30.9	28.4	31.0	31.0	31.1	31.6	36.1	42.1

	$SP_{55\%}$	$SP_{60\%}$	$SP_{65\%}$	$SP_{70\%}$	$SP_{75\%}$	$SP_{80\%}$	$SP_{85\%}$	$SP_{90\%}$	$SP_{95\%}$	$SP_{100\%}$
R_{10}	16.8	16.6	15.2	15.0	15.0	14.9	14.7	14.5	14.4	14.2
R_{20}	22.1	21.9	18.0	17.6	17.2	17.0	16.8	16.6	16.5	16.3
R_{30}	25.8	25.5	19.5	19.1	18.4	18.2	17.9	17.8	17.6	17.5
R_{40}	28.8	28.4	20.6	20.1	19.2	18.9	18.7	18.6	18.4	18.3
R_{50}	31.2	30.8	21.4	20.8	19.8	19.5	19.2	19.2	18.9	18.9
R_{60}	33.3	32.9	22.1	21.4	20.3	20.0	19.7	19.7	19.4	19.4
R_{70}	35.2	34.8	22.6	22.0	20.7	20.3	20.0	20.1	19.8	19.8
R_{80}	37.0	36.5	23.1	22.4	21.1	20.7	20.3	20.4	20.1	20.1
R_{90}	38.5	38.1	23.5	22.8	21.3	20.9	20.6	20.7	20.3	20.4
R_{100}	40.0	39.5	23.9	23.1	21.6	21.2	20.8	21.0	20.6	20.7

Next graph depicts return levels values for the portfolios combinations calculated:



As can be inferred from the graph, any portfolio with an investment of 85% or higher in the S&P 500 Index represent a “zone” with the lower return level values; in other words, there is a lower probability to exceed a particular negative return value in any one year compare with a portfolio that invest more than 15% in the AEX Index.

In fact, the lower return value portfolio (S&P 500 Index: 95%, AEX Index= 5%) at the one hundred years event (R_{100}) is consistent with the Markowtiz minimum variance portfolio solution.

III. CONCLUSION

We have conducted the analysis of the return level values in the case of an investment portfolio of two assets.

Future lines of research could apply estimation of return level using extreme value theory to optimization of portfolios.

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