Solow-Swan growth model with global capital markets and congestible public goods

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Solow-Swan growth model and the fortunes of the commons

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Abstract. The traditional Solow-Swan growth framework has only one kind privately owned capital. Output saved in a period is transformed into privately owned capital through the saving channel. We add a fiscal channel that taxes output and transforms it into public goods subject to congestion. We show that under the standard conditions the steady state of the economy is determined by the interaction of these two channels and prosperity for a country is only attained if both channels work properly.

Keywords: Economic Growth, Public Goods, Infrastructure, Congestion

JEL Classification: O11, O16, O41, O43

"*An enormous influence on the fate of nations emanates from the economic bleeding which the needs of the state necessitates, and from the use to which the results are put.*"

Schumpeter et al. (1918)

1. Introduction

Why does newly installed infrastructure in developing countries often wither away over time? Why have empirical studies never been able to find a strong impact of foreign aid on long-term economic growth or per-capita income? What kind of reforms will turn a low-income country into a middle-income country? These are the stark questions that still confound policymakers and academics.

The Solow-Swan growth framework[1] remains the most tractable model for understanding economic growth in countries where the evolution of technology is not an endogenously determined process. Its intellectual influence is hard to overstate given its pedagogical value, its relevance in contemporary policy-making and its use in a range of disciplines beyond economics. Yet, the Solow-Swan framework is unable to shed any light on the questions posed above. The objective of the paper is to augment the Solow-Swan growth framework to answer these questions. At the heart of the Solow-Swan growth framework lies the notion that output that is not consumed can be costlessly transformed into new capital that can then be used to produce output in the next period. Exactly what Solow-Swan framework means by its physics-defying notion of capital is important to understand if it is to be augmented to answer the questions we pose.

This paper contends that the received wisdom on Solow-Swan growth framework[2] over the years has stuck too closely to the original formulation. The rather broad category of privately owned capital is unable to satisfactorily accommodate tax-funded publicly owned capital and

[1]The Solow-Swan model of economic growth was developed independently by Robert Solow and Trevor Swan and published simultaneously in Solow (1956) and Swan (1956).

[2]This paper uses the word framework to refer to the way the have been received, which maybe distinct from the ideas in the orignal papers. There is some evidence in Solow (2007) and La Grandville and Solow (2009) that this is an important distinction.

its impact on the economy. The paper uses first principles to argue that it is necessary to augment the Solow-Swan framework by adding the *fiscal channel* that funds public capital — an addition that changes the nature of the model quite dramatically. In doing so, we formulate an approach that incorporates the institutional features associated with the fiscal capacity of a country into the model and show how the convergence properties of the model are contingent on the fiscal capacity. The paper further demonstrates how aspects of fiscal capacity interact with the conventional saving channel in determining the steady state per-capita income level. Consequently, any capital markets reform or foreign aid will have a negligible effect on per-capita income unless it is accompanied by concomitant fiscal reform.

At the heart of Solow-Swan framework is the *saving channel* through which capital accumulation channel takes place and results in the growth of per-capita income. This channel costlessly transforms output that is not consumed into new capital stock. The new capital stock can then either augment capital stock of the economy or compensate for the depreciation of the pre-existing capital stock. Capital stock accumulates or de-accumulates until the economy reaches a *steady-state* where the output that is saved and transformed into capital just about meets the depreciation needs of the pre-existing capital stock and the capital accumulation process comes to a standstill. There are of course specific conditions under which one or more steady states exist and the economy converges or diverges from these steady states.

The Solow-Swan framework was able to show that the economy converges to a unique steady state if the production function of the economy exhibits constant returns to scale and diminishing returns to capital. These properties of the production function and competitive markets for factor inputs create the *tendency of convergence* and homogenisation within the functional sub-units of the modern economy. Given this tendency to homogenise, the distinction between production process at the aggregate level and within its functional sub-unit is inconsequential. Herein lies at least one justification for categorising all variety of capital under the rubric of just one homogenous type of capital.

The received wisdom on Solow-Swam framework suggests that output and capital variables do not represent their actual physicality but an abstract notion of resources. These resources are then represented by nominal assets that can be traded in the capital markets. The homogenisation tendency in the modern economy helps in unifying the myriad physicality of capital and output into one single asset class.[3] Hence, the critical role played by the saving channel in the Solow-Swan framework. There maybe numerous types of capital, but if a single asset class can represent them, then for all practical purposes, they can be represented by a single type of capital. The fact that the abstract notion of capital in economic growth models is inextricably tied with the channel that facilitates its creation is rarely appreciated and requires to be stressed at the risk of inducing ennui in order to create the context for the results of the paper. The fiscal and saving channels capture the dichotomy of the modern economy in terms of the way in which the economic activity can be organised.

1.1. **Public Goods.** Following Samuelson (1954), public goods are evaluated along the dimensions of *excludability* and *rivalry*. Yet, the *calculus of public finances* is the third dimension that is often completely disregarded. Conceptually, all services are excludable at a cost. For instance, a local public road can be made excludable, at a cost that may be considered excessive for the benefits it provides. Rivalry captures the way in which benefits derived from a service varies with the number of people using the service. Public goods are subject to congestion if they provide services that are rivalrous — a category of public good that is associated with what William Forster Lloyd called "the tragedy of the commons" in a pamphlet published in 1833.[4] Hardin (1968) calls this a "rebuttal to the invisible hand" or "tendency to assume that decisions reached individually will, in fact, be the best decisions for an entire society".[5] Herein lies the sharp distinction between congestible public goods provided by the state and private capital intermediated by decentralised capital markets.

[3]Since there is no uncertainty in the framework, its does not matter if the asset is debt or equity.
[4]Hardin (1968) quoting Lloyd (1833)
[5]Hardin (1968) summarising the Adam Smith's influence on contemporary thinking.

Edwards (1990) and Barro and Sala-i Martin (1992) conceptualises public goods congestion as the decline in an individual's ability to use their capital stock to derive benefits from the public good. This definition applies to industrialised countries where capital is abundant and production is capital intensive. In developing countries, capital is scarce and individuals own scant amount of capital. A more appropriate definition of congestion of public goods in developing countries is the decline in individual's ability to directly derive benefits from the public good for some productive use.

In the context of commuting to work, an example of congestion in a high-income country would an individual's declining ability to derive benefits from her car while using a public road due to congestion. An example of congestion in low-income country would be declining benefit from using a publicly owned bus to commute to work. Kremer et al. (2005) find that teachers are less likely to be absent in rural schools in India that are closer to a paved road. Banerjee and Duflo (2006) has similar examples in health care delivery. Consequently, for a low-income country, congestion of public goods like roads, schools, hospital and courts are due to low-stock of public goods. Reasons for which are ultimately due to the low level of economic activity that can be taxed and ploughed back into accumulating public goods. Thompson (1974) argues that even national defense is subject to congestion. Congestion is thus far more widespread that is often considered in economic growth literature and may even apply beyond the context of developing countries.

Samuelson (1954) presented the criteria of excludability and rivalry are discrete innate characteristics of a good, evident at the micro-level. Yet, cost of exclusion and rivalry are a continuum and best understood from a macro general-equilibrium perspective. The cost of exclusion often depends on the set of public goods and institutions available in the society and pattern of rivalry varies over geography and social networks. Whether excluding individuals from a service requires a simple sign or a private militia depends on the legal framework and enforcement capacity of the state and other institutions.

The government's budget constraint determines which public goods are provided. The budget constraint is influenced by the tax revenues derived from economic activity in the economy. Providing public goods that have a propensity to generate tax revenues and loosen its budget constraints will allow it to expand its range of public goods and transition to the optimal mix of public and private capital. The model in this paper is trying to capture the critical role the fiscal and savings channels play in this transition.[6]

1.2. **Economic Growth Literature.** The argument in growth has been about whether the returns to capital are diminishing or not. If they diminish, then growth slows down at the economy approaches steady state. Conversely, if they are not diminishing for instance in an AK types endogenous growth model, then a country can potentially have a perpetual growth. Solow-Swan growth framework had assumed diminishing returns to capital.

Lucas (1988) assumed individuals acquire human capital by spending time in education institutions accumulating skills and Mankiw et al. (1992) assume that people acquire human capital simply by foregoing consumption. In both papers, human capital enters the production function just like the labour-augmenting technological change in the Solow-Swan framework and can provide perpetual growth as human capital of workers keeps increasing. Romer(1986, 1990) modelled knowledge as a non-rivalrous factor input, which was accumulated as a spillover effect of capital investment. Romer's insight was that even though private returns from private capital investment maybe diminishing, if the investment has a spillover effect, then the social returns to capital may be higher and possible non-diminishing.

This led to Rebelo's notion of "*broad capital*" (Rebelo, 1991). Broad capital is a composite variable that includes human capital, ideas, knowledge and organisation capital. Each individual factor input may exhibit diminishing returns but taken together they exhibit non-diminishing returns to scale. The key is the accumulation of all inputs take places through a decentralised market mechanism, i.e., a channel that looks very similar to the saving channel. Thus, their dynamic paths look very similar. Barro (1990) analyses public goods by putting

[6] In analysing public private partnerships, Besley and Ghatak (2001) show that if contracts it is not possible to write complete contracts then the ownership of a public good should lie with a party that values the benefits generated by it relatively more.

it under the hood of broad capital into an endogenous growth model and finds no significant role for fiscal policy.

The range of factor inputs in the production function can be divided into three broad categories — animate, inanimate and abstract. Animate factor inputs are inputs that are inalienable to the humans. In the growth literature, this has been labelled human capital. Inanimate factor inputs are the physical stock of public goods and private capital. The abstract covers ideas, knowledge and beliefs. The economic growth literature has examined capital, human capital and institutions as factors whose accumulation leads to sustained prosperity.

Yet, as we argue below, the important classification is whether it is cost-effective to let the factor accumulation occur through a decentralised market process where the individuals are driven by their personal interest or whether it is more cost-effective to accumulate that factor through collective coordination under the guise of a government, taking into account the inefficiencies associated with any process that involves government. Besley and Ghatak (2005) suggest that given that inherently motivated agents are less incentivised, it is possible to create mission-oriented public bureaucracies, nonprofits and education providers if mission orientation of the organisation is matched with innate mission p preference of the agent. Given the multitude of pathways, it is difficult to establish the empirical link between the role of government plays in providing public goods and how it affects factor accumulation process. In the next section, we present some empirical evidence that looks at this links.

1.3. **Results.** The model sets out the system of autonomous non-linear difference equation system that describe the evolution of the economy. The system has two endogenous variables, the stock of public goods and private capital and it captures the fiscal and saving channel. The main result of the paper is that for an economy with a general production function that exhibits constant returns to scale and diminishing returns to both the stock of congestible public goods and private capital, a unique steady state exists and the economy converges to it. It further shows that both channels are complementary and lethargy in either channel leads results in low per-capita income. This requires concomitantly enhancing the efficacy of the saving and fiscal channel. Needless to say, a country cannot achieve prosperity unless and until is able to manage the "fortunes of the commons".

2. Empirical Evidence

2.1. **Fiscal channel.** Developing fiscal capacity has historically been extremely important for countries that have developed (Besley et al., 2013, Aidt and Jensen, 2009). Besley et al. (2013) show that potentially three types of types of states can emerge in the long run: a *common-interest* state which spends tax revenues on public goods, a *redistributive-state* which spends tax-revenues on transfers, and a *weak-state* with no transfers and a low stock of public goods. Our fiscal channel has two components, the tax rate and the proportion of taxes spent on public goods. It follows from Besley et al. (2013) that developing a "common-interest state" is a pre-requisite for prosperity and a prosperous state can also transition back to a redistributive state. We show in our model that the optimal golden rule tax rate is a function of the proportion of taxes spent on public goods, which in turn depends on the kind of aforementioned state the country has developed.

Fiscal capacity evolves slowly and there are instances where it is effective in spite of its leakages and could potentially transition a weak-state to common-interest state. Engerman and Sokoloff (2007) examine the building of the Erie Canals and other canals by the New York State during the antebellum period considers it a success in spite of the corruption, fraud and mismanagement. The paper suggests that New York state took up the role of constructing the canal largely because the scope of investment was beyond what private firms could manage on their own. The Ghani et al. (2016) find that output levels in the districts in the 0-10 kilometre output levels grew by 49% over the decade after the construction began. Since 0-10 km range contains a third of India's manufacturing base, this represents a substantial boost in India's economic activity. Hsieh and Klenow (2009) has found that there are substantial gaps in marginal products of labour and capital across plants within narrowly defined industries in China and India compared with the United States and attributes this to misallocation of capital. Hsieh and Klenow (2009) are silent on how this marginal product varies is local public goods stock. The pertinent question is whether there exists a larger misallocation of capital between the public goods and private capital.

2.2. **Savings channel.** Besley (1995) present the confounding evidence about the way the poor saving. The savings of the poor are driven by a number of institutional, social and environmental factors. Saving plays numerous roles including the of role of self-insurance in absence of easy access to credit. Consequently, there are *susu* men in Africa who charge a deposit rate, in effect a negative rate for keeping the saving safe. The empirical evidence on poor's saving rate does not chime with the consumption smoothing results derived from either Ramsey-Cass-Koopmans[7] or Diamond (1965) model. Solow-Swan framework with an exogenously given saving rate is thus the appropriate model for our purposes. Dercon (2005) presents evidence from African rural communities that prevalence of covariate risk within the community reduces the lucrative opportunities for saving. During a negative covariate shock like famine or flood, when a lender needs their savings back, the borrower is incapable of paying. Papers like Sapienza (2004) and La Porta et al. (2002) have argued that state control of the banking sector makes it susceptible to elite capture. Conversely, show how the Indian government used nationalisation of the banks Burgess and Pande (2005) to force the nationalised banks to open bank branches in previously unbanked areas. This led to a significant increase in saving rate where new local branches were established and had an overall impact on saving rate in India. Access to formal banking allowed them to diversify their risk portfolio. This is an example of policy that can increase the efficacy of the saving channel. There is no clear consensus on exactly what was responsible for the structural break in India's growth process in the early 1980s.[8] What is clear is that the structural break led to three decades of robust growth in India. Basu and Maertens (2007) suggests that the bank reforms of the 1970s and the subsequent increase in saving rate is possibly responsible for the structural break and subsequent increase in per-capita income.

[7] Ramsey (1928), Cass (1965) and Koopmans et al. (1965)

[8] Wallack (2003), Rodrik and Subramanian (2004), DeLong (2003)

2.3. **Factor accumulation.** A significant strand of the endogenous growth theory stresses that the inalienable human capital the workers possess is one of the key factor inputs that influences the growth trajectory of a country. A range of empirical studies find that public goods are crucial in facilitating the acquisition of human capital. Duflo (2001) establishes a causal link between school building in the 1970s in Indonesia to workers' increased years of education and higher wages in the area where the schools were built. Banerjee and Duflo (2006) and Muralidharan and Prakash (2017) show that transport to school is causally linked to the enrolment rates in school in Bihar (India).

Reinikka and Svensson (2004) find that Uganda spends 20% of its government expenditure on primary education, of which only 13% reaches the school and rest 87% lost to local corruption. Others survey suggest that the situation is similar in other African countries. Reinikka and Svensson (2003) find that an information campaign in the newspaper significantly increased the proportion of funds reaching the schools that had a newspaper outlet in its vicinity. In a similar vein, Besley and Burgess (2002) find that government's in states in India where local language newspaper circulation is high are also more responsiveness when it comes to providing relief for droughts and floods.

Besley and Ghatak (2006) make the useful distinction between market supporting public goods, which give the poor access to markets and market augmenting public goods, which provide the correct level of public good where the market does not. Infrastructure can potentially play both roles by dramatically change the cost of delivering public goods. Das and Hammer (2004) illustrate the problems of sins of omission and sins of commission in the public and private provisions of services. They examine the quality of services provided by doctors in Delhi public and private health care centres. They find that public doctors are better qualified but less diligent whereas private doctors less qualified and ready to please the patients, even if it means over-prescribing the medicines. Grogan and Sadanand (2013) find that electrification increased the propensity of rural women to work outside the home by about 23% in Nicaraguan. Dinkelman (2011) finds that household electrification raises employment by releasing women from home production and enabling microenterprises.

There is a feedback loop between the beliefs people possess and the congestion that results in any economic system. Public goods can potentially shape this feedback loop. For instance in traffic, *de jure* rules and traffic density determine the congestion, which in turn determines the adherence to those rules. Thus, the only *de jure* rules that survive are the ones that correspond to the mixed strategy equilibrium of the system Basu (2010). In the aftermath of the horrific 2006 Mecca stampede, Haase et al. (2016) examined pedestrian crowd flows using computational methods and designed, implemented, and supervised a new set of pathways. Devising pathways also require devising new rules pilgrims are supposed to follow. Devising appropriate *de jure* rule is a resource-intensive process and requires fiscal capacity.

North (1990) maps the role beliefs play in establishing institutions and Williamson (2000) warns that institutions can be classified according to how amenable they are to change. Some institutions are deeply embedded and hardwired into the societies whereas are there are other ones that respond to the government's policy. Designing policy requires attention given that they are easier to destroy than to create. Gneezy and Rustichini (2000) show with an experiment how a badly designed pecuniary fine can easily destroy social institutions and the social institutions does not bounce back once the pecuniary fine is taken away. Policy can thus lead to hysteresis. Lefebvre (1974) gives an account of contemporary space with the advent of modern state and impact accelerated accumulation of capital during had on France during the "Trente Glorieuses" or three decades of prosperity that France witnessed after 1945. What is striking is the significant role transport and communication plays in almost all the empirical results above. In a developing country, these are activities that traditionally fall in the government's domain. The empirical evidence thus points to the importance of the fiscal channel in determining the prosperity of the country.

3. The Environment

The economy's output Y is given by the production function

$$Y = F(\bar{K}, K, AL) \tag{1}$$

where K is the stock of privately owned capital and L is the number of agents in the economy. $\bar{K}$ is the stock of public goods subject to congestion. $\bar{K}$ represents public goods like courts, roads and other infrastructure. Benefits derived from them declines if they are intensively used. Hence, $\bar{K}$ represents public goods subject to congestion, i.e., public goods that are non-excludable but retain some degree of rivalry. A is labour-augmenting technology change that exogenously determined in line with Uzawa (1965). The production function $F(\cdot)$ is described by following assumptions.

Assumption 1. *$F : \mathbb{R}^3_+ \to \mathbb{R}_+$ is homogenous of degree 1.*

Assumption 2. *$F(\bar{K}, K, AL)$ is continuous and twice differentiable in $\bar{K}, K$ and L and satisfies the following conditions: $F_{\bar{K}}, F_K, F_L > 0$, $F_{\bar{K}\bar{K}}, F_{KK}, F_{LL} < 0$ and $F_{\bar{K}K}$,$F_{\bar{K}L}$, $F_{KL} > 0$ for $\bar{K}, K, L \in [0, \infty)$.*

Assumption 3. *$F(0, K, AL) = F(\bar{K}, 0, AL) = F(\bar{K}, K, 0) = 0$ for $\bar{K}, K, L > 0$.*

Assumption 4 (Inada Conditions)**.** *$F(\bar{K}, K, AL)$ satisfies the following conditions: $\lim_{K\to 0} F_{\bar{K}}, F_K, F_L = \infty$ and $\lim_{K\to\infty} F_{\bar{K}}, F_K, F_L = 0$.*

Assumption 1 allows us to transform the production function (1) and write it terms of per effective worker, i.e., : $\frac{Y}{AL} = F\left(\frac{\bar{K}}{AL}, \frac{K}{AL}, 1\right)$ or

$$y = f(\bar{k}, k) \tag{2}$$

where $y = \frac{Y}{AL}$, $\bar{k} = \frac{\bar{K}}{AL}$ and $k = \frac{K}{AL}$ and the properties of $f : \mathbb{R}^2_+ \to \mathbb{R}_+$ follow from the Assumptions 2, 3 and 4. The assumption of monotonicity, strict concavity[9] and differentiability follow from Assumption 1 and 2. There properties are going to be very useful in pinning down the the transitional dynamics of the model.

The production function (2) essentially says that output per effective worker increasing and concave in both private capital per effective worker and public goods per effective worker and the marginal productivity.

[9]The concavity implies that $\frac{f(\bar{k}_t,k_t)}{k}$ and $\frac{f(\bar{k}_t,k_t)}{k}$ are decreasing in $\bar{k}$ and k respectively.

Assumption 5. *$\forall t \in [0, \infty)$, $L_{t+1} = (1+n)L_t$ and $A_{t+1} = (1+g)A_t$.*

Assumption 5 simply implies that L and A grow at constant exogenously determined rates n and A respectively. Private capital K depreciates at a constant rate δ and public goods $\bar{K}$ depreciates at the rate $\bar{\delta}$. Assumption 6 is a restriction on the parameters to ensure global asymptotical stability of the steady state of the growth model.

Assumption 6. *$(\bar{\delta} + n + g) \in (0, 1)$ and $(\delta + n + g) \in (0, 1)$.*

Given Assumption 1, a representative one worker firm suffices for the analysis. The firms are price takers. The one-worker firm maximises its profits $\pi = f(\bar{k}_t, k_t) - (r+\delta)k - w$. The factor markets clear and give us $r + \delta = f'(\bar{k}, k)$ and $w = A\left[f(\bar{k}_t, k_t) - f_k(k, k)k\right]$.

National Income Accounting. τ proportion of the agents' income is taxed by the government, accruing the government tax revenue of τY and leaving agents a disposable income of $(1-\tau)Y$. ς is the public goods investment rate, that is, the government spends ς portion of the tax revenues on public goods and $(1-\varsigma)$ proportion of tax revenues leaks, i.e., it is either redistributed back to the agents through transfers or is lost to corruption. In either case, it simply ends back in the economy as disposable income. Given that the tax revenue lost to corruption ends back in the private hands and becomes part of the disposable income, the final disposable income is $(1-\varsigma\tau)Y$. Thus, from the current output Y, $(1-s)(1-\varsigma\tau)Y$ is consumed, $s(1-\varsigma\tau)Y$ is saved and invested in private capital and $\varsigma\tau Y$ gets invested in public goods in the current period.

Significance of τ, ς and s. It is useful at this point to think about what s, τ and ς parameters of the model signify. τ is simply the marginal rate at which output is taxed in the economy and quite easy to observe. s and ς represent the channels through which output not consumed in the current period is transformed into private capital and public goods respectively. Hence, s and ς are reduced form parameters that represent all the institutions and constraints that affect the private capital and public goods accumulation channel. Consequently, s and ς far more difficult to pin down for empirical purposes.

4. Model

The saving $s(1-\varsigma\tau)Y$ by agents augments the private capital stock after the depreciated private capital stock is replaced. The private capital stock at time t is given by

$$K_{t+1} = s(1-\varsigma\tau)F(\bar{K}_t, K_t, A_tL_t) - \delta K_t. \tag{3}$$

The stock of public goods is augmented by government's tax revenues $\varsigma\tau Y$ after the depreciated public goods stock is replaced. The public goods stock at time t is given by

$$\bar{K}_{t+1} = \varsigma\tau F(\bar{K}_t, K_t, A_tL_t) - \bar{\delta}\bar{K}_t. \tag{4}$$

Using Assumption 5, we can re-write (3) and (4) in terms of per-effective effective worker.

$$k_{t+1} = s(1-\varsigma\tau)\, f(\bar{k}_t, k_t) + (1-\delta-n-g)k_t \tag{5}$$

$$\bar{k}_{t+1} = \varsigma\tau\, f(\bar{k}_t, k_t) + (1-\bar{\delta}-n-g)\bar{k}_t \tag{6}$$

(5) and (6) together are a system of autonomous non-linear difference equation system that describe the evolution of the economy. The dynamic co-evolution of k_{t+1} and $\bar{k}_{t+1}$ is inter-linked through the production function $f(\bar{k}_t, k_t)$ and is independent of time.

The economy is in a steady state if $k_t = k_{t+1} \ldots = k_{t+n}$ and $\bar{k}_t = \bar{k}_{t+1} \ldots = \bar{k}_{t+n}$ for some large integer n. Setting $\bar{k}_{t+1} = \bar{k}_t = \bar{k}$ and $k_{t+1} = k_t = k$ in (5) and (6) gives us Lemma 1.

Lemma 1. *The economy is at a steady state if either $\bar{k} = k = 0$ or the following condition holds:*

$$\begin{aligned} \frac{f(\bar{k}, k)}{k} &= \frac{\delta+n+g}{s(1-\varsigma\tau)} \\ \frac{f(\bar{k}, k)}{\bar{k}} &= \frac{\bar{\delta}+n+g}{\varsigma\tau} \end{aligned} \tag{7}$$

Lemma 1 essentially says that in steady state, the average products of private capital k and public goods $\bar{k}$ are each equal to constants determined by the parameters of the model.

Corollary 1 follows directly from 1 and would help us determine the nature of potential steady states.

Corollary 1. *For $\bar{k}, k > 0$, $\frac{\bar{k}}{k} = \frac{\delta+n+g}{\bar{\delta}+n+g} \cdot \frac{\varsigma\tau}{s(1-\varsigma\tau)}$ always holds in the steady state.*

Corollary 1 says that for all non-zero steady states, the ratio of public goods and private capital in the economy is constant and depends on the relative efficacy of the saving (s) and fiscal channel (ς and τ).

In the rest of the section, we will establish uniqueness and the stability properties of the steady state. As we will see, uniqueness requires very few restrictions on the production function. Conversely, the restrictions required for local and global stability is far more stringent.

Lemma 2. *For $\bar{k} \in (0, \infty)$ and $k \in (0, \infty)$, there exists a unique steady state if the production is homogenous of degree 1 (Assumptions 1) and $\frac{\partial F}{\partial L} > 0$.*

Proof. Given that $F(\bar{K}, K, AL)$ is homogenous of degree 1 and $\frac{\partial F}{\partial L} > 0 \, \forall L \in (0, \infty)$, we can say that for an arbitrary λ,

$$
\begin{aligned}
F(\lambda\bar{k}, \lambda k, 1) &> F(\lambda\bar{k}, \lambda k, \lambda) && \text{if} \quad \lambda \in (0,1) \\
F(\lambda\bar{k}, \lambda k, 1) &< F(\lambda\bar{k}, \lambda k, \lambda) && \text{if} \quad \lambda \in (1,\infty)
\end{aligned}
$$

Using the defintion of $f(\cdot)$ we can write $F\left(\lambda\bar{k}, \lambda k, 1\right) = f(\lambda\bar{k}, \lambda k)$ and $F\left(\lambda\bar{k}, \lambda k, \lambda\right) = \lambda f(\bar{k}, k)$. It follows that

$$
\begin{aligned}
f(\lambda\bar{k}, \lambda k) &> \lambda f(\bar{k}, k) && \text{if } \lambda \in (0,1) \\
f(\lambda\bar{k}, \lambda k) &< \lambda f(\bar{k}, k) && \text{if } \lambda \in (1,\infty)
\end{aligned}
$$

Dividing through by $\bar{k}$ and k respectively and rearranging gives us

$$\frac{f(\lambda\bar{k},\lambda k)}{\lambda\bar{k}} > \frac{f(\bar{k},k)}{\bar{k}} \qquad \frac{f(\lambda\bar{k},\lambda k)}{\lambda k} > \frac{f(\bar{k},k)}{k} \qquad \text{if } \lambda \in (0,1)$$

$$\frac{f(\lambda\bar{k},\lambda k)}{\lambda\bar{k}} < \frac{f(\bar{k},k)}{\bar{k}} \qquad \frac{f(\lambda\bar{k},\lambda k)}{\lambda k} < \frac{f(\bar{k},k)}{k} \qquad \text{if } \lambda \in (1,\infty)$$

Let $\bar{k} = k^\circ$ and $k = k^\circ$ be one set of steady state values that satisfies (7). For uniqueness, we need to show that there cannot be another set that can satisfy (7).

From Lemma 1 we know that in the steady state, the average product of public goods and private capital are a particular constant. If $\lambda \in (0,1)$,

$$\frac{f(\lambda\bar{k}^\circ,\lambda k^\circ)}{\lambda\bar{k}^\circ} > \frac{f(\bar{k}^\circ,k^\circ)}{\bar{k}^\circ} = \frac{\bar{\delta}+n+g}{\varsigma\tau} \qquad \frac{f(\lambda\bar{k}^\circ,\lambda k^\circ)}{\lambda k^\circ} > \frac{f(\bar{k}^\circ,k^\circ)}{k^\circ} = \frac{\delta+n+g}{s(1-\varsigma\tau)}. \tag{8}$$

(8) shows us that if $\lambda \in (0,1)$ the average product of public goods and private capital are higher than the steady state level and it it follows from Assumption 2 that the public and private capital will always less than the steady state level. Conversely, if $\lambda \in (1,\infty)$,

$$\frac{f(\lambda\bar{k}^\circ,\lambda k^\circ)}{\lambda\bar{k}^\circ} < \frac{f(\bar{k}^\circ,k^\circ)}{\bar{k}^\circ} = \frac{\bar{\delta}+n+g}{\varsigma\tau} \qquad \frac{f(\lambda\bar{k}^\circ,\lambda k^\circ)}{\lambda k^\circ} < \frac{f(\bar{k}^\circ,k^\circ)}{k^\circ} = \frac{\delta+n+g}{s(1-\varsigma\tau)} \tag{9}$$

(9) shows us that if $\lambda \in (1,\infty)$ the average product of public goods and private capital are lower than the steady state level and it follows from Assumption 2 that the public and private capital will always greater the steady state level.

This implies that for $\bar{k}, k > 0$, there is a unique steady state associated with $\lambda = 1$ and no other steady states exist for $\lambda \in (0,1)$ and $\lambda \in (1,\infty)$. ■

It follow from Lemma 2 that there is a unique steady state for $\bar{k}, k > 0$. Let $\bar{k}^*$ and k^* respectively be the steady state values of private capital and public goods that satisfies (7).

Local and Global Asymptotically Stability. Defining $g : \mathbb{R}_+^2 \to \mathbb{R}_+$ and $h : \mathbb{R}_+^2 \to \mathbb{R}_+$ allows us to write k_{t+1} and $\bar{k}_{t+1}$ in the following form:

$$k_{t+1} = g(\bar{k}_t, k_t) \equiv s(1 - \varsigma\tau)f(\bar{k}_t, k_t) + (1 - (\delta + n + g))k \tag{10}$$

$$\bar{k}_{t+1} = h(\bar{k}_t, k_t) \equiv \varsigma\tau f(\bar{k}_t, k_t) + (1 - (\bar{\delta} + n + g))\bar{k}. \tag{11}$$

Assumptions 1 and 2 imply that g and h are continuous and twice differentiable. $\frac{\varsigma\tau f(\bar{k}^*, k^*)}{\bar{k}_t} = (\bar{\delta} + n + g)$ and $\frac{s(1-\varsigma\tau)f(\bar{k}^*, k^*)}{k_t} = (\delta + n + g)$ imply that

$$h(\bar{k}^*, k^*) = \bar{k}^* \tag{12}$$

$$g(\bar{k}^*, k^*) = k^* \tag{13}$$

.

Proposition. *The steady state $(\bar{k}^*, k^*)$ of the non-linear difference equation system, $\bar{k}_{t+1} = h(\bar{k}_t, k_t)$ and $k_{t+1} = g(\bar{k}_t, k_t)$, is locally asymptotically stable.*

To establish the local asymptotic stability, we would show below that $h_{\bar{k}}(\bar{k}_t, k_t) \in (0, 1)$ and $g_k(\bar{k}_t, k_t) \in (0, 1)$. That is for a sufficiently small $\varepsilon_1, \varepsilon_2 > 0$, if $|\bar{k}_t - k^*| < \varepsilon_1$ and $|k_t - k^*| < \varepsilon_2$, then $\lim_{t\to\infty} \bar{k}_t = \bar{k}^*$ and $\lim_{t\to\infty} k_t = k^*$.

Proof. It follows from Assumption 1 and 2 that $g_k(\bar{k}_t, k_t)$ and $h_{\bar{k}}(\bar{k}_t, k_t)$ will always exist and will always be positive.

$$g_k(\bar{k}_t, k_t) = s(1 - \varsigma\tau)f_k(\bar{k}_t, k_t) + (1 - (\delta + n + g)) > 0$$

$$h_{\bar{k}}(\bar{k}_t, k_t) = \varsigma\tau f_{\bar{k}}(\bar{k}_t, k_t) + (1 - (\bar{\delta} + n + g)) > 0$$

since $f_k(\bar{k}_t, k_t) > 0$, $f_{\bar{k}}(\bar{k}_t, k_t) > 0$, $s > 0$, $\tau > 0$ and from Assumption 6.

For a strictly concave function differentiable function $f(\bar{k}_t, k_t)$,

$$f(\bar{k}_t, k_t) > f(\bar{k}, 0) + k f_k(\bar{k}, k) = k f_k(\bar{k}, k)$$

$$f(\bar{k}_t, k_t) > f(0, k) + \bar{k} f_{\bar{k}}(\bar{k}, k) = \bar{k}(f_{\bar{k}})(\bar{k}, k)$$

because $f(\bar{k}, 0) = f(0, k) = 0$. $(\delta + n + g) = \frac{s(1-\varsigma\tau)f(\bar{k}^*, k^*)}{k^*} > s(1-\varsigma\tau)f_k(\bar{k}^*, k^*)$ implies that

$$g_k(\bar{k}^*, k^*) = s(1-\varsigma\tau)f_k(\bar{k}^*, k^*) + (1 - (\delta + n + g)) < 1$$

Similarly, $(\bar{\delta} + n + g) = \frac{\varsigma\tau f(\bar{k}^*, k^*)}{\bar{k}^*} > \varsigma\tau f_{\bar{k}}(k^*, \bar{k}^*)$ implies that

$$h_{\bar{k}}(\bar{k}^*, k^*) = \varsigma\tau f_k(\bar{k}^*, k^*) + (1 - (\bar{\delta} + n + g)) < 1$$

Thus, $h_{\bar{k}}(\bar{k}_t, k_t) \in (0, 1)$ and $g_k(\bar{k}_t, k_t) \in (0, 1)$. ■

Proposition. *The steady state of $(\bar{k}^*, k^*)$ of the non-linear difference equation system, $\bar{k}_{t+1} = h(\bar{k}_t, k_t)$ and $k_{t+1} = g(\bar{k}_t, k_t)$, is globally asymptotically stable.*

We would like to show that global stability exists any given pair of $(k_t, \bar{k}_t)$ where $\bar{k}_t \in (0, \bar{k}^*)$ and $k_t \in (0, k^*)$. This entails showing that for any t and $t+1$, $0 < k_t < k_{t+1} < k^*$ and $0 < \bar{k}_t < \bar{k}_{t+1} < \bar{k}^*$.

Proof. We know from Assumption 1 and 2 that $f(\bar{k}_t, k_t)$ is concave in $\bar{k}_t$ and k_t and that $\frac{f(\bar{k}_t, k_t)}{k_t}$ and $\frac{f(\bar{k}_t, k_t)}{\bar{k}_t}$ are decreasing in $\bar{k}_t$ and k_t respectively. Hence, for any $k_t < k^*$, we can write

$$\begin{aligned} \frac{k_{t+1-k_t}}{k_t} &= \frac{s(1-\varsigma\tau)f(\bar{k}_t, k_t)}{k_t} - (\delta + n + g) \\ &> \frac{s(1-\varsigma\tau)f(\bar{k}^*, k^*)}{k^*} - (\delta + n + g) = 0 \end{aligned} \tag{14}$$

For any $\bar{k}_t < \bar{k}^*$, we can write

$$\begin{aligned} \frac{\bar{k}_{t+1} - \bar{k}_t}{\bar{k}_t} &= \frac{\varsigma \tau f(\bar{k}_t, k_t)}{\bar{k}_t} - (\bar{\delta} + n + g) \\ &> \frac{\varsigma \tau f(\bar{k}^*, k^*)}{\bar{k}_t} - (\bar{\delta} + n + g) = 0 \end{aligned} \tag{15}$$

Thus, for any $\bar{k}^* \in (0, \bar{k}^*)$ and $k_t \in (0, k^*)$, we have $\frac{\bar{k}_{t+1} - \bar{k}_t}{\bar{k}_t} > 0$ and $\frac{k_{t+1} - k_t}{k_t} > 0$. This naturally implies that $k_t < ke_{t+1}$ and $\bar{k}_t < \bar{k}_{t+1}$, i.e., private capital and public goods would increase each period if they are not at the steady state level.

Next, we would like to show that for any $\bar{k}_t \in (0, \bar{k}^*)$ and $k_t \in (0, k^*)$, $\bar{k}_{t+1} < \bar{k}^*$ and $k_{t+1} < k^*$. Using (10), (11), (12), (13) and the *Fundamental theorem of Calculus*, we can write

$$\begin{aligned} k_{t+1} - k^* &= g(\bar{k}_t, k_t) - g(\bar{k}^*, k^*) \\ &= \left[g(\bar{k}_t, k_t) - g(\bar{k}^*, k_t)\right] + \left[g(\bar{k}^*, k_t) - g(\bar{k}^*, k^*)\right] \\ &= -\int_{\bar{k}_t}^{\bar{k}^*} g_{\bar{k}}(\bar{k}, k_t) d\bar{k} - \int_{k_t}^{k^*} g_k(\bar{k}^*, k) dk < 0 \end{aligned} \tag{16}$$

We know that $g_{\bar{k}}(\bar{k}_t, k_t) = s f_{\bar{k}}(\bar{k}_t, k_t) > 0$ and $g_k(\bar{k}_t, k_t) \in (0, 1)$ for all $\bar{k}_t, k_t > 0$. Thus, using (14) and (16) it is established that $0 < k_t < k_{t+1} < k^*$. Similarly,

$$\begin{aligned} \bar{k}_{t+1} - \bar{k}^* &= h(\bar{k}_t, k_t) - h(\bar{k}^*, k^*) \\ &= \left[h(\bar{k}_t, k_t) - h(\bar{k}_t, k^*)\right] + \left[h(\bar{k}_t, k^*) - h(\bar{k}^*, k^*)\right] \\ &= -\int_{k_t}^{k^*} h_{\bar{k}}(\bar{k}, k_t) dk - \int_{\bar{k}_t}^{\bar{k}^*} h_k(\bar{k}^*, k) d\bar{k} < 0 \end{aligned} \tag{17}$$

We know that $h_k(\bar{k}_t, k_t) = \tau f_k(\bar{k}_t, k_t) > 0$ and $h_{\bar{k}}(\bar{k}_t, k_t) \in (0, 1)$ for all $\bar{k}_t, k_t > 0$. Thus, using (15) and (17) it is established that $0 < \bar{k}_t < \bar{k}_{t+1} < \bar{k}^*$.

These two arguments together establish that for all $\bar{k}_t \in (0, \bar{k}^*)$ and $k_t \in (0, k^*)$, $\bar{k}_{t+1} \in (0, \bar{k}^*)$ and $k_{t+1} \in (0, k^*)$. Therefore, $\{\bar{k}_t\}_{t=0}^{\infty}$ and $\{k_t\}_{t=0}^{\infty}$ are monotonically increasing and are respectively bounded above by $\bar{k}^*$ and k^*. Moreover, since $(\bar{k}^*, k^*)$ is the unique steady state for $\bar{k}, k > 0$, there exists no $\bar{k}'_t \in (0, \bar{k}^*)$ and $k'_t \in (0, k^*)$ such that $\bar{k}_{t+1} = \bar{k}^* = \bar{k}'$ and $k_{t+1} = k^* = k'$ for any t. Therefore, $\{\bar{k}_t\}_{t=0}^{\infty}$ and $\{k_t\}_{t=0}^{\infty}$ must monotonically converge to $\bar{k}^*$ and k^*. From a similar set of deductive arguments, it follows that for all $\bar{k}_t > \bar{k}^*$ and $k_t > k^*$, $\bar{k}_{t+1} \in (\bar{k}^*, \bar{k}_t)$ and $k_{t+1} \in (k^*, k_t)$ and that $\{\bar{k}_t\}_{t=0}^{\infty}$ and $\{k_t\}_{t=0}^{\infty}$ monotonically converge to $\bar{k}^*$ and k^*. This complete the proof of global stability. ■

5. Cobb-Douglas Steady State

A Cobb-Douglas production function that satisfies the homogenous of degree 1 (Assumption 1) takes the form of $Y = \bar{K}^{\beta} K^{\alpha} (AL)^{1-(\alpha+\beta)}$ where $\alpha, \beta > 0$ and $\alpha + \beta < 1$. In per-capita form, it can be written as

$$y = \bar{k}^{\beta} k^{\alpha} \tag{18}$$

where β and α represent the elasticity of output with respect to public goods and private capital in Cobb-Douglas production function (18).[10]

The system of difference equations for the economy in the Cobb-Douglas case is as follows.

$$k_{t+1} = s(1 - \varsigma\tau)\, \bar{k}^{\beta} k^{\alpha} + (1 - \delta - n - g) k_t \tag{19}$$

$$\bar{k}_{t+1} = \varsigma\tau\, \bar{k}^{\beta} k^{\alpha} + (1 - \bar{\delta} - n - g) \bar{k}_t \tag{20}$$

Figure 1 and 2 below show the results of simulations run on the difference equation system (19) and (20). In Figure 1, the public good stock and private capital start from arbitrary value and are seen to converge to the steady state level.

In Figure 2 the value of public goods and private capital are shocked from their steady state level and then are seen to converge back to the steady state level. Figure 1 and 2

[10]The Euler's theorem allows a rather insightful interpretation of exponents in case of a Cobb-Douglas production function that is homogenous of degree 1. If the factors are paid their marginal product, the exponent of factor is also the output share received by that factor. Thus, share of output hypothetically due to public goods, private capital and labour would be α, β and $1 - (\alpha + \beta)$ respectively. Public goods in this model is funded by exogenously determined proportional tax that is distortionary. Thus, even though public goods are unlikely to receive their marginal product, we can use the exponent to indicate the extent to which the factor contribute to the output.

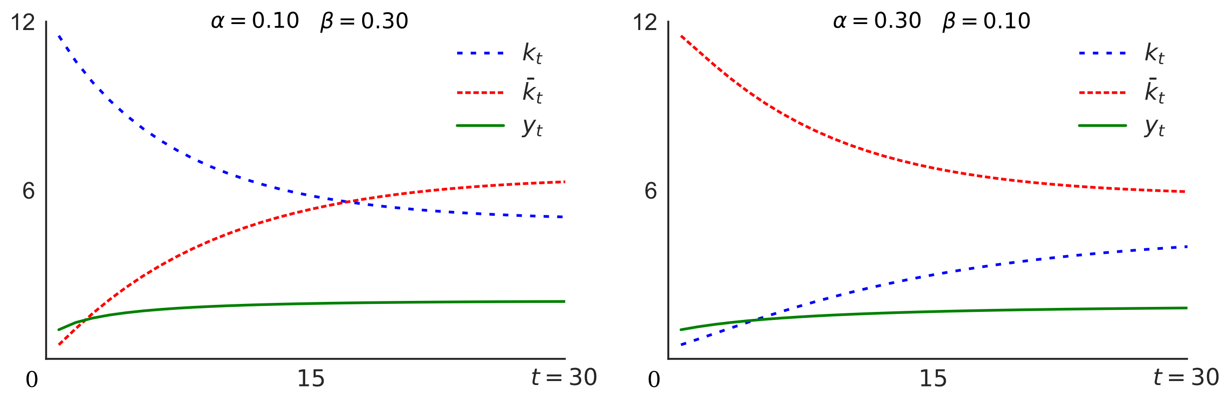

FIGURE 1. Simulation of output and public and private capital over time

show in how the co-evolution of public goods and private capital are interlinked in (19) and (20). In Solow-Swan growth framework either assumes that the two types of capital are pure substitutes or pure complements and always exist in fixed ratio. If they are pure substitutes, investment in public goods has no distinctive role and does not really matter. If they are pure complements, then there exists some mechanism of ensuring that they accumulate in such a way that their proportions remain fixed. Simulations in Figure 1 and 2 show that neither of these two cases can be nested in the economy described by (19) and (20) and its the distinction between public goods and private capital is not superflous.

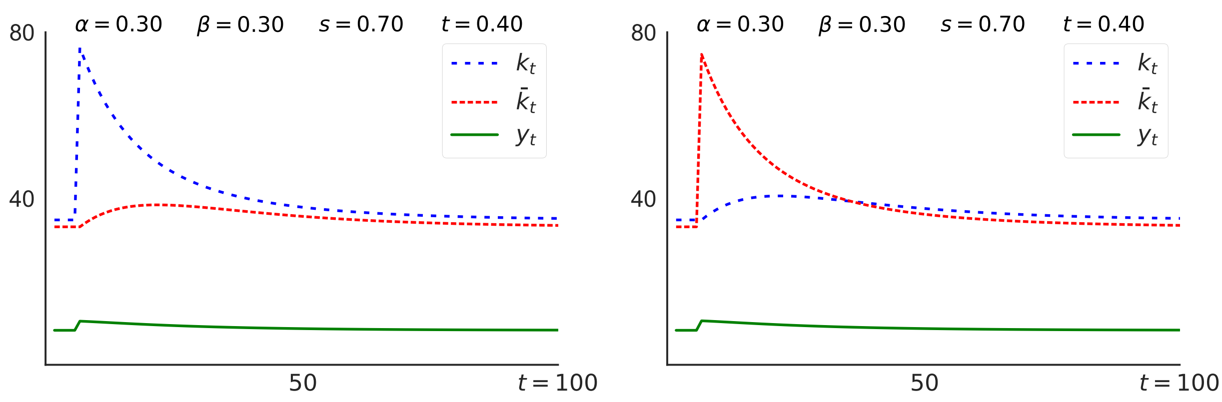

FIGURE 2. The effect of shock increase in public and private capital

Two Channels. The output of the economy depends on stock of both public goods and private capital and their respective values are interlinked in the difference equation system defined by (20) and (19). To gain insight into how this difference equation system works, it is useful to analyse the locus of of all the points where $\bar{k}_{t+1} = \bar{k}_t$ and $k_{t+1} = k_t$ in the $(\bar{k}, k)$ space.

Setting $\bar{k}_{t+1} = \bar{k}_t$ in (20) and gives us (21). This is the $\bar{k}(k)$ locus in Figure 3, the locus of all the points for which $\bar{k}$ remains constant.

$$\bar{k}(k) = \left[\frac{\varsigma\tau}{\bar{\delta} + n + g}\right]^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}} \tag{21}$$

Setting $k_{t+1} = k_t$ in (19) and gives us (22). This gives us the locus $k(\bar{k})$ in Figure 3, the locus of all the points for which k remains constant.

$$k(\bar{k}) = \left[\frac{s(1-\varsigma\tau)}{\delta + n + g}\right]^{\frac{1}{1-\alpha}} \bar{k}^{\frac{\beta}{1-\alpha}} \tag{22}$$

The intersection of the two locus gives us the steady state where both $\bar{k}$ and k remains constant. Starting with any arbitrary $\bar{k}$ and k, the transitional dynamics characterised by (19) and (20) will ensure that the economy will gravitate towards the steady state in Figure 3.

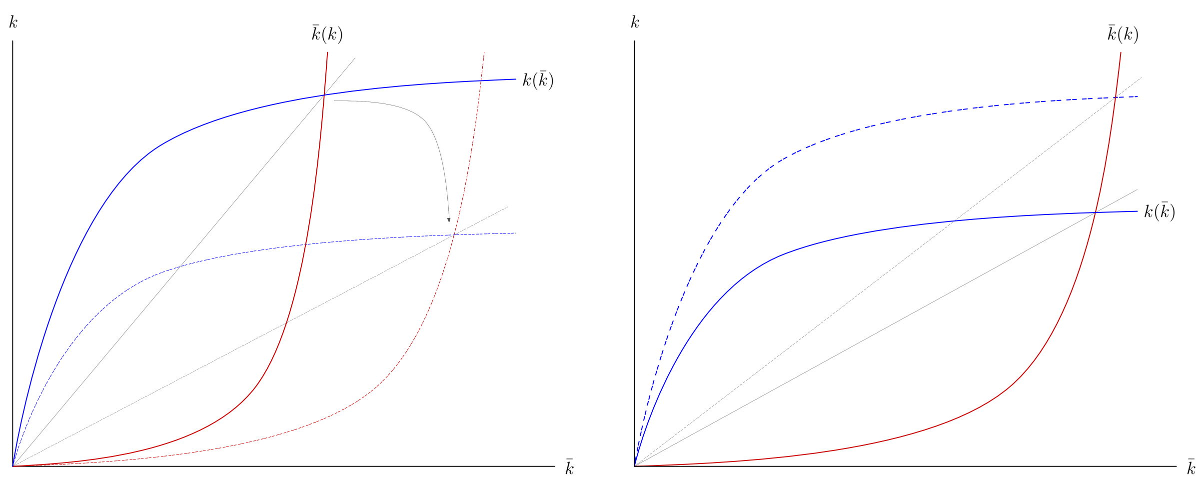

Figure 3. Impact of an increase in $\varsigma\tau$ (left panel) or s (right panel) on $\bar{k}(k)$ and $k(\bar{k})$.

An increase in either ς or τ has an impact on the steady state through two different channels, the *fiscal channel* working through (21) and the *saving channel* working through

(22). For a given k, an increase in $\varsigma\tau$ increases $\bar{k}$ through the *public-goods tax-revenue channel*. For a given $\bar{k}$, an increase in $\varsigma\tau$ reduces the disposable income and thus lowers the k through the *private-capital savings channel*. The left panel of Figure 3 depicts the effect of an increase in $\varsigma\tau$. The $\bar{k}(k)$ locus shifts right and the $k(\bar{k})$ locus shifts down as a result. An increase in $\varsigma\tau$ is likely to lead to a higher $\bar{k}$ and lower k. The impact on output is ambiguous and depends on the values of β and α.

The right panel of Figure 3 depicts the impact of an increase in s. The $k(\bar{k})$ locus shifts up and $\bar{k}(k)$ locus remains where it way, leading to an increase in both k and $\bar{k}$ and a higher output. Similarly, an increase in either n or g shift the $\bar{k}(k)$ towards left and shifts the $k(\bar{k})$ locus down lowering the steady state public goods, private capital and output levels. $\bar{\delta}+n+g$ will only affect the $k(k)$ locus and $\delta+n+g$ will only affect the $\bar{k}(k)$ locus.

5.1. **Steady State in the Cobb-Douglas case.** Lemma 3 characterises the steady state of the economy.

Lemma 3. *For $\bar{k}\in(0,\infty)$ and $k\in(0,\infty)$, the economy has a unique steady $(\bar{k}^*,k^*)$ state where*

$$\begin{aligned}\bar{k}^* &= \left(\frac{s(1-\varsigma\tau)}{\delta+n+g}\right)^{\frac{\alpha}{1-(\alpha+\beta)}}\left(\frac{\varsigma\tau}{\bar{\delta}+n+g}\right)^{\frac{1-\alpha}{1-(\alpha+\beta)}}\\ k^* &= \left(\frac{s(1-\varsigma\tau)}{\delta+n+g}\right)^{\frac{1-\beta}{1-(\alpha+\beta)}}\left(\frac{\varsigma\tau}{\bar{\delta}+n+g}\right)^{\frac{\beta}{1-(\alpha+\beta)}}\end{aligned}\tag{23}$$

The steady state output and consumption per-effective worker is given by

$$y^* = \left(\frac{s(1-\varsigma\tau)}{\delta+n+g}\right)^{\frac{\alpha}{1-(\alpha+\beta)}}\left(\frac{\varsigma\tau}{\bar{\delta}+n+g}\right)^{\frac{\beta}{1-(\alpha+\beta)}}\tag{24}$$

$$c^* = (1-s)\left(\frac{s^{\alpha}\,(1-\varsigma\tau)^{1-\beta}(\varsigma\tau)^{\beta}}{(\delta+n+g)^{\alpha}(\bar{\delta}+n+g)^{\beta}}\right)^{\frac{1}{1-(\alpha+\beta)}}\tag{25}$$

The elasticity of consumption per-effective worker is given by

$$\varepsilon_{c,\tau} = \tau\frac{d\ln c}{d\tau} = \frac{\varsigma\tau}{1-(\alpha+\beta)}\left[\frac{\beta}{\varsigma\tau}-\frac{1-\beta}{1-\varsigma\tau}\right]$$

and is increasing in the range $\tau \in \left(0, \frac{\beta}{\varsigma}\right)$ and decreasing in the range $\tau \in \left(\frac{\beta}{\varsigma}, 1\right)$. The consumption maximising tax rate of $\tau = \frac{\beta}{\varsigma}$ is decreasing in ς implying that any institutional reform process that reduces the leakage or redistribution represented by ς could reduce the tax rate at which the consumption would be maximised.

If β, the elasticity of output with respect to public goods is positive, then (23) and (24) represent the real model of the world. In that case, as $\varsigma\tau \to 0$, not only does $\bar{k}^* \to 0$, it also means that $\bar{k}^* \to 0$. This happens because the accumulation process of public goods and private capital are intricately linked through the nature of the production process. This naturally implies that as $\varsigma\tau \to 0$, we also have $y^* \to 0$. It is important to note that the policy implications that follow are not to simply follow the rhetorical policy of just increasing the tax rate τ. What matters is the $\varsigma\tau$. Thus, the ς needs to increase concomitantly along with τ.

τ thus represents the fiscal capacity of the state in terms of public goods, i.e., the government's ability to collect tax and spend it on public goods. There is a subtle nuance here that needs to be emphasised. If the government spends the resources on a public goods on a project the rate of success is stochastically determined. If the investment succeeds, then the country gets a public good. If the project fails, the resource spent on the project becomes redistribution. Thus, when it comes to investing in stochastic public goods, the ς is determined by the expected value of the public goods and the government cannot ensure a sufficiently high value of ς unless it has mechanism of determining good bets or is good at "picking winners". Increase tax rate with a low rate of "picking winners" will just lead to a low $\varsigma\tau$, which will lead back to low steady state value of $\bar{k}^*, k^*$ and y^*. This is not just a problem of low-income countries, it is a potential problem develop countries can fall into. It is important to take away from this that maxining consumption in the long run requires an effective fiscal channel that includes not just the nominal tax rate but also the institutions that ensure that a sufficiently high proportion of tax revenues are invested in public goods and not entirely lost to redistribution or corruption.

6. Conclusions

In spite of its impeccable spartan elegance, Solow-Swan growth models has not had much to say about development. Given that it takes the saving rate as exogenous has been seen as a flaw. In the case of developing countries, given that we still don't understand exactly how to model saving for the poor, it works as a good workhorse model. It has till now not been able to say anything about how the economic activity in the public and the private domain of the economy interact. The paper has argued that the right way to understand this is to think of channels that transform output into public goods and private capital. We show that prosperity of the country depends on both channels working effectively. The impact of a reform that increases the efficacy of fiscal channel depend also on the state of the saving channel and vice-versa. Capital market reforms and fiscal reforms are often implemented in isolation. It would be judicious to implement them simultaneously.

References

Aidt, T. S. and Jensen, P. S. (2009). Tax structure, size of government, and the extension of the voting franchise in western europe, 1860–1938. *International Tax and Public Finance*, 16(3):362–394.

Banerjee, A. and Duflo, E. (2006). Addressing absence. *Journal of Economic perspectives*, 20(1):117–132.

Barro, R. J. (1990). Government spending in a simple model of endogeneous growth. *Journal of political economy*, 98(5, Part 2):S103–S125.

Barro, R. J. and Sala-i Martin, X. (1992). Public finance in models of economic growth. *The Review of Economic Studies*, 59(4):645–661.

Basu, K. (2010). *Beyond the invisible hand: Groundwork for a new economics*. Princeton University Press.

Basu, K. and Maertens, A. (2007). The pattern and causes of economic growth in india. *Oxford Review of Economic Policy*, 23(2):143–167.

Besley, T. (1995). Savings, credit and insurance. *Handbook of development economics*, 3:2123–2207.

Besley, T. and Burgess, R. (2002). The political economy of government responsiveness: Theory and evidence from india. *The Quarterly Journal of Economics*, 117(4):1415–1451.

Besley, T. and Ghatak, M. (2001). Government versus private ownership of public goods. *The Quarterly Journal of Economics*, 116(4):1343–1372.

Besley, T. and Ghatak, M. (2005). Competition and incentives with motivated agents. *American economic review*, 95(3):616–636.

Besley, T. and Ghatak, M. (2006). Public goods and economic development. *Understanding poverty*, 19:285–303.

Besley, T., Ilzetzki, E., and Persson, T. (2013). Weak states and steady states: The dynamics of fiscal capacity. *American Economic Journal: Macroeconomics*, 5(4):205–35.

Burgess, R. and Pande, R. (2005). Do rural banks matter? evidence from the indian social banking experiment. *American Economic Review*, 95(3):780–795.

Cass, D. (1965). Optimum growth in an aggregative model of capital accumulation. *The Review of economic studies*, 32(3):233–240.

Das, J. and Hammer, J. S. (2004). Strained mercy: Quality of medical care in delhi. *Economic and Political Weekly*, 39(9).

DeLong, J. B. (2003). India since independence: An analytic growth narrative. *In search of prosperity: Analytic narratives on economic growth*, pages 184–204.

Dercon, S. (2005). Risk, insurance, and poverty: a review. *Insurance against poverty*, pages 9–37.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.

Dinkelman, T. (2011). The effects of rural electrification on employment: New evidence from south africa. *American Economic Review*, 101(7):3078–3108.

Duflo, E. (2001). Schooling and labor market consequences of school construction in indonesia: Evidence from an unusual policy experiment. *American economic review*, 91(4):795–813.

Edwards, J. H. (1990). Congestion function specification and the "publicness" of local public goods. *Journal of Urban Economics*, 27(1):80–96.

Engerman, S. L. and Sokoloff, K. L. (2007). Digging the dirt at public expense. *Corruption and Reform*, page 95.

Ghani, E., Goswami, A. G., and Kerr, W. R. (2016). Highway to success: The impact of the golden quadrilateral project for the location and performance of indian manufacturing. *The Economic Journal*, 126(591):317–357.

Gneezy, U. and Rustichini, A. (2000). A fine is a price. *The Journal of Legal Studies*, 29(1):1–17.

Grogan, L. and Sadanand, A. (2013). Rural electrification and employment in poor countries: Evidence from nicaragua. *World Development*, 43:252–265.

Haase, K., Al Abideen, H. Z., Al-Bosta, S., Kasper, M., Koch, M., Müller, S., and Helbing, D. (2016). Improving pilgrim safety during the hajj: an analytical and operational research approach. *Interfaces*, 46(1):74–90.

Hardin, G. (1968). Science. *The tragedy of the commons*, 13(162):1243–1248.

Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing tfp in china and india. *The Quarterly journal of economics*, 124(4):1403–1448.

Koopmans, T. C. et al. (1965). On the concept of optimal economic growth.

Kremer, M., Chaudhury, N., Rogers, F. H., Muralidharan, K., and Hammer, J. (2005). Teacher absence in india: A snapshot. *Journal of the European Economic Association*, 3(2-3):658–667.

La Grandville, O. d. and Solow, R. (2009). Capital-labour substitution and economic growth. *Economic Growth: A Unified Approach*, pages 389–416.

La Porta, R., Lopez-de Silanes, F., and Shleifer, A. (2002). Government ownership of banks. *The Journal of Finance*, 57(1):265–301.

Lefebvre, H. (1974). *La production de l'espace*. Paris: Anthropos.

Lloyd, W. F. (1833). *Two Lectures on the Checks to Population: Delivered Before the University of Oxford, in Michaelmas Term 1832*. JH Parker.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of monetary economics*, 22(1):3–42.

Mankiw, N. G., Romer, D., and Weil, D. N. (1992). A contribution to the empirics of economic growth. *The quarterly journal of economics*, 107(2):407–437.

Muralidharan, K. and Prakash, N. (2017). Cycling to school: increasing secondary school enrollment for girls in india. *American Economic Journal: Applied Economics*, 9(3):321–50.

North, D. (1990). Institutions, economic theory and economic performance. *Institutions, Institutional Change and Economic performance. Nueva York: Cambridge University Press.*

Ramsey, F. P. (1928). A mathematical theory of saving. *The economic journal*, 38(152):543–559.

Rebelo, S. (1991). Long-run policy analysis and long-run growth. *Journal of political Economy*, 99(3):500–521.

Reinikka, R. and Svensson, J. (2003). The power of information: Evidence from a newspaper campaign to reduce capture.

Reinikka, R. and Svensson, J. (2004). Local capture: evidence from a central government transfer program in uganda. *The Quarterly Journal of Economics*, 119(2):679–705.

Rodrik, D. and Subramanian, A. (2004). From" hindu growth" to productivity surge: the mystery of the indian growth transition. Technical report, National Bureau of Economic Research.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of political economy*, 94(5):1002–1037.

Romer, P. M. (1990). Endogenous technological change. *Journal of political Economy*, 98(5, Part 2):S71–S102.

Samuelson, P. A. (1954). The pure theory of public expenditure. *The review of economics and statistics*, pages 387–389.

Sapienza, P. (2004). The effects of government ownership on bank lending. *Journal of financial economics*, 72(2):357–384.

Schumpeter, J. A. et al. (1918). *The crisis of the tax state*. 1991.

Solow, R. M. (1956). A contribution to the theory of economic growth. *The quarterly journal of economics*, 70(1):65–94.

Solow, R. M. (2007). The last 50 years in growth theory and the next 10. *Oxford review of economic policy*, 23(1):3–14.

Swan, T. W. (1956). Economic growth and capital accumulation. *Economic record*, 32(2):334–361.

Thompson, E. A. (1974). Taxation and national defense. *Journal of political economy*, 82(4):755–782.

Uzawa, H. (1965). Optimum technical change in an aggregative model of economic growth. *International economic review*, 6(1):18–31.

Wallack, J. S. (2003). Structural breaks in indian macroeconomic data. *Economic and Political Weekly*, pages 4312–4315.

Williamson, O. E. (2000). The new institutional economics: taking stock, looking ahead. *Journal of economic literature*, 38(3):595–613.