Why do employers not pay less than advertised? Directed search and the Diamond paradox

Poeschel, Friedrich

OECD

2 May 2018

Why do employers not pay less than advertised? Directed search and the Diamond paradox*

Friedrich Poeschel
OECD

May 2018

Abstract

When employers advertise wages to attract applicants, it is not clear why employers do not renege on these advertisements ex post. Existing models typically assume that employers are somehow committed to their advertisements. This paper provides an economic explanation that aligns with the empirical evidence. Workers' expectations are fixed by advertisements, and they interpret reneging as a deviation by the employer from mutually beneficial cooperation. By consequence, the worker will exert less effort during the employment relationship if the employer reneges on the advertisement. To avoid this, sufficiently patient employers choose not to renege, and commitment to advertisements arises endogenously.

JEL Classification Numbers: J64, D83, J31
Key words: directed search, advertising, Diamond paradox

*The views expressed in this paper are those of the author and do not necessarily reflect those of the OECD or of the governments of its member countries. This paper is based on chapter 2 of my doctoral thesis at the University of Oxford. A previous version of this paper was circulated under the title "Directed Search, Efficiency Wages and the Diamond Paradox". I would like to thank my supervisor Godfrey Keller for many helpful discussions and suggestions. Very helpful feedback was provided by Jim Albrecht, Melvyn Coles, Leo Kaas, Pan Liu, James Malcomson, Andrew Rhodes, Margaret Stevens, Emily Troscianko, and Randy Wright. I have also received useful comments from audiences at Humboldt-Universität Berlin, the University of Oxford, Nuffield College, Beijing Normal University, the University of Vienna, Ecole Polytechnique, the University of Mainz, University of Technology Braunschweig, RWI Essen and the 2013 annual conference of the Verein für Socialpolitik. Financial support from the Economic and Social Research Council (№ PTA-031-2004-00250) is gratefully acknowledged. All errors are my own.
Correspondence: friedrich.poeschel@gmail.com

"If your wage is cut a bit, you'd start to misbehave in your job, but you wouldn't leave."
C. A. Pissarides[1]

1 Introduction

In practice, it seems easy: an employer advertises a job, specifying the wage. Job seekers apply and are invited to interviews. The employer selects an applicant who is then hired at the specified wage. While this description fits millions of hirings every year, the process is not fully understood by labour economists: why do employers not pay a lower wage than they had advertised? After all, they could. Employers are not legally bound by their advertisement, they can say that the advertised wage was meant only for the ideal candidate. And employers have an incentive to pay less in order to save on wage costs.

Of course, many models in labour economics include a hiring process with advertisements. Directed search models (also referred to as models of competitive search) have become the standard theoretical approach in this context. They capture a trade-off employers face: a high wage advertisement attracts many applicants but is costly when it has to be paid, while a low wage advertisement may be cheaper but might attract fewer applicants. However, directed search models do not explain why employers cannot have the best of both worlds by advertising a high wage and then paying a low wage. Instead of an economic explanation, they almost always invoke a commitment assumption, which says that employers are somehow unable to renege on the wages (or contracts or mechanisms) they advertised. In their survey of this literature, Rogerson, Shimer and Wright (2005) find "it is a strong assumption to say that agents commit to the posted terms of trade" and consider it "a potential disadvantage" for this entire class of models.[2]

It would be wrong to dismiss the problem as a theoretical detail. Not only does the commitment assumption take the place of an economic explanation. Commitment is also crucial for the functioning of directed search models, so that their results hinge on a strong assumption. To see this, note first that rational job seekers will ignore advertised wages that are unrelated to the wages employers actually pay. Advertisements therefore become meaningless, and job seekers might just as well apply randomly. When they then attend an interview without a clue about the wage, they might find the wage so low that they regret coming to the interview. Suppose they incur some costs from attending interviews, such as travel costs. By the time of the interview, costs are sunk, and employers do not need to reimburse them. To minimise wage costs, employers only pay as much as job seekers can obtain elsewhere by returning to job search.

Directed search without commitment therefore leads to one robust theoretical outcome: every employer pays job seekers the expected value of job search, so that job seekers are not better off than before the interview. On the contrary, they have incurred

[1] On April 9th, 2008 in Birmingham.

[2] See Rogerson et al. (2005), p. 976.

search costs they are not reimbursed for. Forward-looking workers thus decide not to attend interviews at all, and the labour market breaks down. This surprising outcome is known as the *Diamond paradox*, following Peter Diamond's (1971) seminal contribution. It is frequently encountered in search models, and some remedies the literature has produced will be reviewed below. In most directed search models, however, the commitment assumption is all that protects them from degenerating into Diamond's outcome (see Tudón Maldonado, 2016).

This paper. This paper proposes an intuitive economic explanation why employers do not pay workers less than advertised. The employer's profit from the employment relationship will depend on the effort exerted by the worker. If the employer reneges on the advertisement, the worker responds by exerting little effort. Such "work to rule" or "white strike" can leave the employer worse off than if the advertised wage was paid. The employer therefore prefers to pay the advertised wage. Although the advertisements are cheap talk, they turn out to be accurate.

In order to formalise this reasoning, the model in this paper extends a directed search model beyond the point in time when employer and job seeker match. The post-match interaction during the employment relationship is modelled as a repeated game with an infinite horizon. In each period, the worker chooses effort and the employer pays a wage. In this set-up, mutually beneficial behaviour can be sustained as much as mutually detrimental behaviour. The worker does not know in advance which situation will prevail but would not want to exert high effort only to be paid a low wage. Looking for clues, the worker bases expectations on the advertised wage. If the employer tries to renege on the advertisement, this will be treated as a deviation by the worker, leading to the mutually detrimental situation. Employers then optimally choose not to renege: the model generates their commitment to advertisements endogenously.

Directed search models are therefore not necessarily threatened by the Diamond paradox, which is the theoretical contribution of this paper. To examine whether the reasoning also aligns with employers' and workers' behaviour in practice, the paper surveys the available evidence. In particular, some large-scale empirical studies recently examined the reasons why employers appear to refrain in general from cutting wages. Among several competing theories, the results support primarily the view that employers fear the effects of wage cuts on worker morale and productivity. In light of these findings, the explanation for employers' commitment proposed in this paper appears plausible and may represent the main motivation for employers' behaviour in practice.

Related literature. The contributions most closely related to this paper all take issue with the assumption of commitment by employers. Coles (2001) introduces a version of the wage-posting model by Burdett and Mortensen (1998) without commitment. Employers in the latter model cannot advertise high wages to attract more applicants but can pay high wages in order to also recruit workers who are already employed elsewhere and demand higher wages. However, once an employed job seeker has left the old job, the new employer could reduce the wage. Coles (2001) argues this does not happen because higher wages also retain workers - the additional wage costs can be recouped from saving on the costs of staff turnover. Endogenous commitment arises because employers fear that workers leave, while in this paper, employers fear that they work with low productivity.

Mortensen (2003) also acknowledges the problem of commitment. He suggests that employers will not renege because they fear for their reputation among job seekers, so that they would have trouble recruiting in the future. While this reasoning is yet to be spelled out in a model, it requires that sufficiently many job seekers learn how a given employer has treated previous applicants. It is not clear how this can happen in practice. After all, employees have little to gain from publicly complaining about past treatment, and firms whose reputation is tarnished can change name. The empirical studies surveyed in this paper also consider reputation concerns but find little evidence for them. The reasoning in Coles (2001) receives more empirical support, but the available evidence appears to support most the explanation proposed in this paper.

Masters (2010) presents a directed search model in which employers can use a legal minimum wage or co-operation with unions as commitment devices: applicants are thereby reassured that the actual wage will not fall below a certain floor. This leaves at least two questions. How can employers commit to advertisements of higher wages, and why do employers not pay less than advertised if they operate in labour market segments without unions or minimum wages?

A contribution by Menzio (2007) is closely related to this paper because it obtains a similar result in a different search context. In his model, employers can attract more applicants by advertising some wage-relevant information, much like in a directed search model. In contrast to most directed search models, employers and job seekers bargain over the wage in the interview. Although employers are not committed to their advertisements, those who advertise more positive information end up paying higher wages because they face tougher bargaining behaviour from job seekers: more positive advertisements make job seekers more demanding. A key parallel to this paper is that the effect of advertisements on job seekers' expectations creates a link between advertisements and actual wages. An important difference is that commitment is not needed for search models with bargaining to function - the Diamond paradox only complicates the matter when employers choose wages unilaterally.

Recently, Kim and Kircher (2015) questioned whether commitment is really needed for the functioning of directed search models. Their results imply that commitment is not necessary whenever employers award jobs to workers through a first-price auction. While auction-like situations seem to sometimes occur in labour markets, they have likely remained rare in comparison to wages that are simply set by employers. Then the question remains why employers do not pay less than advertised when they can set wages.

Introducing advertisements that employers do not renege on is one of several ways to avoid the Diamond paradox in search models. Other approaches in the literature typically assume imperfectly informed employers or somehow ensure that employers compete directly for workers despite search frictions. Burdett and Mortensen (1998) is a case in point, as their model relies heavily on the assumption that employers do not know the employment status of job seekers visiting them. Carrillo-Tudela (2009) surveys empirical evidence against this assumption.

Direct competition for workers has been introduced by allowing for multiple offers to workers as in Burdett and Judd (1983). However, then the question arises again why employers should not renege once the worker has come to them and has foregone the competing offer from another employer - unless the other employer remains a "contact" whose offer may be recalled at any time, as in Carrillo-Tudela, Menzio and Smith (2011).

The paper is organised as follows. Section 2 develops a directed search model without commitment, demonstrates the Diamond paradox in this setting, but then extends the model by employment relationships. Section 3 shows how consequences for worker effort lead employers not to renege on their advertisements, so that the Diamond paradox no longer results. This section also discusses how the paper's reasoning goes beyond efficiency wages. Evidence on both pre-match and post-match behaviour as modelled in this paper is surveyed in Section 4 before section 5 concludes.

2 Model

The model adapts Diamond's (1971) set-up to labour markets by introducing wage advertisements and repeated interactions between worker and employer during the employment relationship. Directed search as in Montgomery (1991) and Burdett et al. (2001) is used to account for the role of wage advertisements in agents' pre-match behaviour. The post-match interaction during the employment relationship is set in the game-theoretic environment of Malcomson and McLeod (1989). While directed search is typically set in a static framework, a unified treatment of pre- and post-match behaviour here requires a dynamic setting throughout.

Time is discrete with an infinite horizon. The only agents are a large number of identical employers who can employ at most one worker and a large number of identical workers endowed with an indivisible unit of labour. Agents can be in three states: in search for a match, in an interview, and in an employment relationship. Unmatched agents can transition into an employment relationship via an interview, and they can leave an interview or an employment relationship to search again. Positive payoffs can be earned only in matches. Employers and workers discount future payoffs with discount factors $\beta_F \in (0,1)$ and $\beta_W \in (0,1)$, respectively.

2.1 Pre-match behaviour

A marketplace exists for employers and workers to search and to match, while matched agents leave the marketplace. To obtain a steady state environment, it is assumed that one employer enters the marketplace for each employer who leaves, and likewise for workers. However, a steady state is not needed for the qualitative results in this paper. In order to have a clear notion of the sequence of events, any period t is divided into subperiods t^0, t^1, and t^2. During a pre-match period 0, unmatched employers and workers flow into the marketplace in subperiod 0^0. Including those who have remained unmatched in the marketplace, a total of m employers seek to recruit among a total of n unemployed workers, and both m and n are large numbers.

Wage advertisements In subperiod 0^1, each employer in the marketplace publishes a job advertisement at negligible cost. The advertisement specifies a wage w_t per period of work and the work input or 'effort' that is expected. This information allows workers to derive which benefit the vacancy might hold for them: their benefit b_t per period is defined as the wage net of their disutility of work $d(s_t)$ (so that $b_t = w_t - d(s_t)$), where $s_t \in \mathbb{R}_+$ is the effort exerted by the worker. Let $\tilde{b}$ denote the benefit implicitly advertised by the employer. Having observed the advertisements, workers decide which employer to apply to in subperiod 0^2. Workers prefer jobs with a high benefit, but they are aware that other workers are equally attracted to these jobs, so that the probability of obtaining a job with a high benefit is low. A job with a low benefit attracts few applicants, but the probability of obtaining it is accordingly high.

As the situation is the same for each of the identical workers, each will apply with the same probability σ to a given employer advertising $\tilde{b}$. With probability $(1-\sigma)^n$, all workers apply elsewhere, so that this employer receives at least one application with probability $1-(1-\sigma)^n$. Among several (identical) applicants, the employer selects one applicant at random. Then the probability ρ that a given applicant is selected for an interview next period is $1-(1-\sigma)^n$ divided by the expected number of applicants:

$$\rho = \frac{1-(1-\sigma)^n}{n\sigma} \tag{1}$$

Due to the competition for workers, the employer will only have applicants if the expected utility of applying to this employer is at least $\underline{\Phi}$, a level that is offered to applicants elsewhere in the marketplace. This expected utility depends on the advertised benefit and the chances of obtaining the job. With $\Phi(b_t)$ denoting the value to the worker of being in a match that provides the per-period benefit b_t, workers will only be interested in the employer's advertisement if visiting this employer promises at least as much as is offered elsewhere in the marketplace:

$$\rho\beta_W\Phi(\tilde{b}) - k_W \geq \beta_W\underline{\Phi} - k_W \quad \text{or} \quad \rho\Phi(\tilde{b}) \geq \underline{\Phi} \tag{2}$$

where k_W is an out-of-pocket search cost that a worker incurs from attending an interview. Using Λ to denote the value to the employer of searching for a worker and Θ to denote the value of hosting an interview, the employer's situation is described by:

$$\Lambda = (1-\sigma)^n \beta_F \Lambda + [1-(1-\sigma)^n]\beta_F \Theta \tag{3}$$

In words, the employer will host an interview next period if there is at least one applicant, and will otherwise search. The employer chooses $\tilde{b}$ to maximise Λ subject to equation (2).

Workers' search Having observed all employers' advertisements in subperiod 0^1, each worker applies to one employer in subperiod 0^2. With probability ρ, a given worker is invited to an interview in subperiod 1^0. Let Ω be the value to the worker of being in an interview. Then the value Y to the worker of searching in the marketplace is

$$Y = (1-\rho)\beta_W Y + \rho\beta_W \max\left[\Omega - k_W, Y\right] \tag{4}$$

Equation (4) says that the worker is not invited to an interview with probability $1-\rho$; when invited, however, the worker can choose between going to the interview, which carries the value $\beta_W(\Omega - k_W)$, and not going, which carries the value $\beta_W Y$.

Interviews If at least one worker does apply to the employer in subperiod 0^2, the employer will host an interview for a randomly selected applicant in the following subperiod 1^0. By coming to the interview, the worker incurs the search cost $k_W > 0$. In the interview, the employer makes a take-it-or-leave-it wage offer $\hat{w}_t$ under perfect information. This structure corresponds to Diamond's (1971) setting where sellers unilaterally set prices and buyers only learn the price after coming to the seller at some search cost.

If the worker rejects the wage offer, she will return to the marketplace. In this case, subperiods 1^1 and 1^2 will be like subperiods 0^1 and 0^2, for both the employer and the worker. It is not possible for the worker to recall the employer's offer ex post. If the worker accepts the offer in the interview, worker and employer match and exit the marketplace. In this case, subperiods 1^1 and 1^2 will be part of an employment relationship (see below). An interview period can thus turn into either a match or into another period on the marketplace, depending on whether or not the worker accepts. The worker's situation in the interview can thus be summarised as

$$\Omega = \max[\Phi(b_t), Y] \tag{5}$$

where b_t is determined by the employer's wage offer $\hat{w}_t$. That is, the wage offer will be rejected if the worker prefers further search to a match with this benefit b_t. Let Π denote the value to the employer of being in a match. Then the interview has the value $\Theta = \Pi$ to the employer if the worker accepts, but it has the value $\Theta = \Lambda$ if the worker rejects.

Diamond paradox It is quickly demonstrated that the Diamond paradox arises in this model as in Diamond's (1971) model. Noting that $\rho\beta_W Y$ cancels on the right-hand side of equation (4), it can be rearranged into

$$(6) \qquad Y = \frac{\rho\beta_W}{1-\beta_W} \max\left[\Omega - k_W - Y, 0\right]$$

This formulation shows that a worker who is invited to the interview will only go if $\Omega - Y > k_W$, so that any gain from the interview is at least worth the cost of going there. However, consider the employer's choice of $\hat{w}_t$ in the interview. To minimise wage costs, the employer offers a wage such that the benefit $b_t = \hat{w}_t - d(s)$ leaves the worker indifferent between accepting and rejecting, so that $\Phi(b_t) = Y$. This benefit (or a little bit more to tip the balance) is all the employer needs to offer for the worker to accept in the interview. It follows from equation (5) that $\Omega = Y$, and the worker does not recoup the cost of going to the interview. For a forward-looking worker, it would therefore be irrational to go to the interview, and equation (6) accordingly simplifies to $Y = 0$: since the worker does not go, a match never happens and no benefit is ever received.

This argument generalises to any worker and any period. As it does not depend on the value of ρ, even a single worker without competition from others (i.e. $\rho = 1$) would obtain $Y = 0$. The result is complete market failure: as the search costs would never be recouped, no worker engages in search, and no match is ever observed. This result of an inactive market as the unique equilibrium when there are search costs is known as the Diamond paradox. Its root cause are thus costs that are incurred ex ante and become sunk costs by the time a worker visits an employer, so that the employer has no reason to reimburse the worker for these costs.

In this context, wage advertisements might be a remedy. If employers advertised wages to workers before the interview, this might build a bridge over the point when search costs become sunk costs: workers would only visit employers whose advertised wage offer suggests that they will at least be reimbursed for their search costs. However, this critically depends on advertisements being truthful, so that no employer reneges on the advertised wage offer once a worker has come along.

Yet it is still the case that the worker's costs are sunk by the time the interview takes place, and it is not clear why an employer should then not renege on the advertisement and set $\Phi(b_t) = Y$ after all. While the employer would like to induce worker participation through advertisements, the employer's optimal behaviour is time-inconsistent: once workers participate the employer reneges. Aware of this inconsistency, workers then do not trust advertisements and do not participate. Hence advertisements per se are not enough to avoid the Diamond paradox, which is why nearly all models of directed search make the assumption that employers are committed to advertisements, i.e. somehow unable to renege on them. This assumption is not made in this paper.

2.2 Post-match behaviour

From directed search models, the model in this paper differs primarily by including agents' behaviour after they match: the employment relationship is modelled as repeated and dynamic productive interaction between worker and employer, rather than as a one-off exchange transaction. Production technology is identical across employers and uses labour as the only input. The worker produces perfectly measurable but not verifiable output $p(s_t)$ per period during the match, where $p(\cdot)$ is a strictly concave function and $p(0) = 0$. The worker's disutility from work $d(\cdot)$ is a strictly convex function and $d(0) = 0$.

Stage game As noted before, if the worker accepts the wage offer $\hat{w}_t$ in the interview in subperiod 1^0, worker and employer match and leave the marketplace. In subperiod 1^1, the worker then chooses effort s_t and produces for the employer. The employer pays the actual wage w_t only in subperiod 1^2. This structure constitutes an extensive game in which agents move sequentially and their actions combine to an outcome $(\hat{w}_t, s_t, w_t) \in \mathbb{R}^3_+$. Letting the price for output be 1, so that $p(s_t)$ is also the monetary value of output, the per-period payoff of the employer is $p(s_t) - w_t$ and that of the worker is $b_t = w_t - d(s_t)$.

Time structure During a further period of the employment relationship, the actions and their timing are exactly as in the interview period, except that the worker does not incur any search cost. Keeping in mind that agents transition from a period on the marketplace to an interview period only if the invited worker comes to the interview, the time structure can be summarised as in Figure 1.

Repeated game Unless it breaks up, the match is an infinite repetition of the stage game. The histories in subperiods t^0, t^1, and t^2 of a match period are

$$\begin{aligned} h(t^0) &= \{\tilde{b}, (\hat{w}_1, s_1, w_1), (\hat{w}_2, s_2, w_2), \ldots, (\hat{w}_{t-1}, s_{t-1}, w_{t-1})\} \\ h(t^1) &= h(t^0) \cup \{\hat{w}_t\} \quad \text{and} \quad h(t^2) = h(t^1) \cup \{s_t\} \end{aligned}$$

with the initial history given by the employer's advertisement, $h(1^0) = \{\tilde{b}\}$. Match break-up is endogenous: a rejection of $\hat{w}_t$ by the worker terminates the match, and both the worker and the employer can decide at the end of each full period whether to terminate the match. The match will continue next period only if neither side chooses to terminate it. If the match is terminated, both the worker and the employer return to search on the marketplace, which carries values Y and Λ, respectively. The value $\Pi(s_t)$ to the employer of being in a match can thus be expressed as

$$\Pi(s_t) = p(s_t) - w_t + \alpha\beta_F\Pi(s_{t+1}) + (1-\alpha)\beta_F\Lambda \tag{7}$$

where α is an indicator that equals one if the match continues and otherwise equals zero. Similarly, the value $\Phi(b_t)$ to the worker of being in a match can be expressed as

$$\Phi(b_t) = w_t - d(s_t) + \alpha\beta_W\Phi(b_{t+1}) + (1-\alpha)\beta_W Y \tag{8}$$

The employer's preferences in the infinitely repeated game are defined on the set of infinite sequences of stage game outcomes and satisfy weak separability. Concretely, they are given by δ-discounting, and equivalently for the worker's preferences. Finally, the level of effort that maximises the surplus of the worker's production is denoted

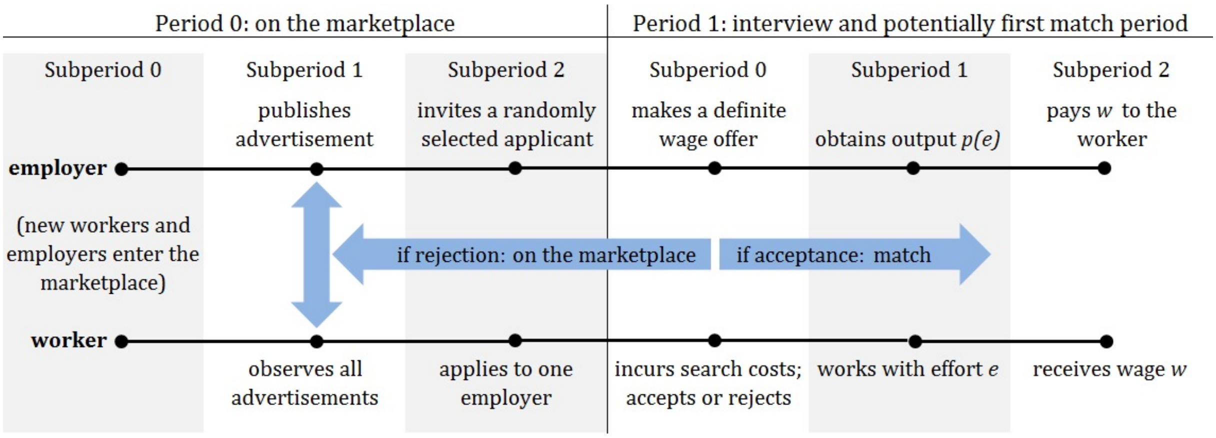
$$\bar{s} = \arg\max_{s_t} p(s_t) - d(s_t) \tag{9}$$

To ensure the existence of gains from trade between worker and employer, it is assumed

$$\frac{p(\bar{s}) - d(\bar{s})}{1-\beta_W} > k_W \tag{10}$$

which says that the worker's search costs can be recouped by the present discounted value of match production over the infinite horizon.

Figure 1: Time structure of pre-match and post-match behaviour



3 Equilibrium

This section identifies an equilibrium in which workers do go to interviews, match with employers, and are paid the same wages as employers advertised. Crucially, truthful advertisements do not reflect a mere assumption that employers are committed to their advertisements, but result endogenously from employers' optimal behaviour. In a nutshell, this result arises from the interdependence of pre- and post-match behaviour: if the employer reneged on the advertisement, co-operation during the employment relationship would be undermined, with adverse effects on profits. This leads employers to only advertise wages they are prepared to pay. Since workers can therefore trust advertisements, they come to interviews as soon as advertisements promise sufficiently high wages.

The section proceeds by stating the equilibrium, whose parts are then proven one at a time through a series of lemmas. In a way that will be clarified by Lemma 4, the equilibrium depends on employers being sufficiently patient, so that β_F is high enough. As behaviour in this equilibrium is time-invariant, time subscripts are dropped. Let $\mu(b = \tilde{b}|\tilde{b})$ denote the worker's subjective probability of obtaining benefit $b = \tilde{b}$ in the employment relationship when $\tilde{b}$ has been advertised by the employer. Since such beliefs are involved, the equilibrium of the model is a *perfect Bayesian equilibrium* (PBE).

Proposition 1 (Endogenous commitment). *For β_F large enough, a PBE exists in which*

(i) each employer chooses the same advertisement $\tilde{b}$ that at least reimburses workers for their search costs

(ii) workers trust all advertisements: $\mu(b = \tilde{b}|\tilde{b}) = 1$

(iii) the SPE profile $(\hat{w}, s, w) = (\tilde{b} + d(\bar{s}), \bar{s}, \tilde{b} + d(\bar{s}))$ is played in every match period, so that matches do not break up

(iv) workers engage in search and match at the first opportunity.

The equilibrium comprises the main results of the model: in contrast to the Diamond paradox, workers here engage in search (part *iv*) because each employer advertises at least the lowest benefit that still reimburses workers for their search costs (part *i*); as advertisements are truthful (part *iii*), workers are right to trust them (part *ii*).

3.1 Pre-match equilibrium

For the proposed equilibrium, the values that various states carry for worker and employer can be determined. That workers go to interviews in equilibrium implies $\Omega - k_W \geq Y$ in equation (4). Solving for Y, the value of search to the worker, one finds

$$Y = \frac{\rho}{\beta_W^{-1} - (1-\rho)} \left[\Phi(b) - k_W\right] \tag{11}$$

where $\Phi(b)$ replaces Ω because every interview in equilibrium turns into a match. Since matches in the proposed equilibrium do not break up, $\alpha = 1$ in equation (8), so that the value to the worker of being in a match is

$$\Phi(b) = \frac{w - d}{1 - \beta_W} \tag{12}$$

With $\alpha = 1$ in equation (7), the value to the employer of being in a match is found as

$$\Pi(s) = \frac{p(s) - w}{1 - \beta_F} \tag{13}$$

As every interview in the proposed equilibrium becomes a match, $\Theta = \Pi(s)$. Then one can solve equation (3) for Λ:

$$\Lambda = \frac{1 - (1 - \sigma)^n}{\beta_F^{-1} - (1 - \sigma)^n} \frac{p(s) - w}{1 - \beta_F} \tag{14}$$

Recall that Λ is the value of search to the employer, i.e. the value at the time when the employer chooses an advertisement. In the proposed equilibrium, the employer does not renege on the advertised wage. By choosing the wage for the advertisement, the employer therefore actually determines w in equation (14). The employer seeks to maximise Λ by setting w as low as possible, but also needs to attract applicants in competition with other employers. Formally, the employer's optimisation problem is to maximise Λ subject to the constraint in equation (2). Using equation (12) to solve the constraint for w gives

$$w = \frac{1 - \beta_W}{\rho} \underline{\Phi} + d(s) \tag{15}$$

Substituting for w in equation (14), the employer's optimisation problem becomes

$$\max \Lambda = \frac{1 - (1 - \sigma)^n}{\beta_F^{-1} - (1 - \sigma)^n} \frac{1}{1 - \beta_F} \left[p(s) - \frac{1 - \beta_W}{\rho} \underline{\Phi} - d(s) \right] \tag{16}$$

This expression indirectly depends on w because σ, the probability that a given worker applies to this employer, depends on the wage advertisement. Hence one can maximise Λ with respect to σ, noting from equation (1) that ρ is a function of σ.

Lemma 1 (Employers' choice of advertisement). *All employers advertise the same*

$$\tilde{b} = \max \left[\frac{(1 - \beta_F) \frac{n}{m} (p(s) - d(s))}{\left[1 - \left(1 - \frac{1}{m}\right)^n\right] \left[\left(1 - \frac{1}{m}\right)^{1-n} - \beta_F \left(1 + \frac{n-1}{m}\right)\right]}, (1 - \beta_W) k_W \right] \tag{17}$$

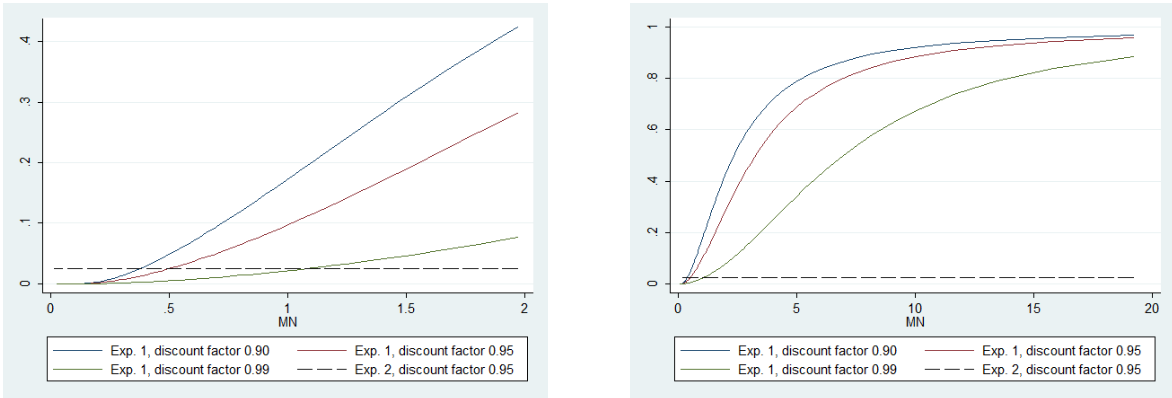
Proof. See appendix. □

Employers choose an advertisement $\tilde{b}$ that offers workers a share of the surplus $p(s) - d(s)$ (the first expression in equation (17)) unless the minimum that workers need to be offered for them to incur the search costs is larger (the second expression). In the derivation of these expressions, σ is replaced by $\frac{1}{m}$ since, given that all employers advertise the same $\tilde{b}$, workers apply to one employer at random. By consequence, both expressions in equation (17) depend on n and m.

Figure 2 shows how the first expression varies with *market tightness*, defined as the ratio of employers to workers $\frac{m}{n}$, for various values of β_F, β_W and k_W. The first expression continuously increases with market tightness: greater competition among employers for workers drives up their choice of advertisement $\tilde{b}$. For high values of market tightness, the advertised share of $p(s) - d(s)$ approaches one, so that workers are offered almost all the surplus from the match. The second expression does not depend on market tightness, so that the first expression exceeds the second expression for sufficiently high market tightness. That is, the market has to be tight enough for employers to offer workers more than the minimum that would just reimburse their search costs.

Figure 2: Dependence of employers' advertisement $\tilde{b}$ on market tightness

(a) For $p(s) - d(s) = 1$, $k_W = 0.5$ and $\frac{m}{n} \leq 2$ (b) For $p(s) - d(s) = 1$, $k_W = 0.5$ and $\frac{m}{n} \leq 20$



To formally characterise the behaviour of the first expression in equation (17), consider a large labour market. Whenever n and m are large, $\left(1 - \frac{1}{m}\right)^n$ is well approximated by $e^{-\frac{n}{m}}$ and $\frac{n-1}{m} \approx \frac{n}{m}$. Then the expression becomes

$$(18) \qquad \frac{(1-\beta_F)\frac{n}{m}(p(s)-d(s))}{\left[1-e^{-\frac{n}{m}}\right]\left[e^{\frac{n}{m}} - \beta_F\left(1+\frac{n}{m}\right)\right]}$$

By L'Hôpital's rule, the limits as $\frac{n}{m} \to \infty$ or $\frac{n}{m} \to 0$ in this expression are the same as for a ratio of the numerator's and denominator's derivatives with respect to $\frac{n}{m}$, denoted H:

$$(19) \qquad H = \frac{(1-\beta_F)[p(s)-d(s)]}{e^{\frac{n}{m}} - \beta_F\left(\frac{n}{m}e^{-\frac{n}{m}} + 1\right)}$$

The limits are therefore found as

$$(20) \qquad \lim_{\frac{n}{m}\to\infty} H \to 0 \quad \text{and} \quad \lim_{\frac{n}{m}\to 0} H \to p(s) - d(s)$$

Since $\frac{n}{m}$ is the inverse of market tightness, workers are offered almost the entire match surplus when market tightness becomes very high, but would be offered nothing when market tightness is very low if it were not for the second expression in equation (17).

Lemma 1 proves part (i) of Proposition 1: when employers choose their advertisements, they take workers' search costs and competing advertisements from other employers into account. However, whether employers pay the wages they advertised is an entirely different question. Without commitment by employers, workers need good reasons to believe that employers will not renege on their advertisements, otherwise it is not rational for them to visit employers. In this paper, post-match behaviour endogenously leads employers to pay the advertised wages (see below). The effect of this endogenous post-match behaviour on workers' pre-match behaviour is the same as that of a commitment assumption: workers trust employers' advertisements and do visit them.

Lemma 2 (Workers' participation conditional on advertisements). *Given the post-match behaviour specified in part (iii) of Proposition 1, workers trust employers' advertisements, engage in search and match at the first opportunity.*

Proof. See appendix. $\square$

Lemma 2 proves parts (ii) and (iv) of Proposition 1, taking the other parts as given. The next section turns to part (iii) of the Proposition.

3.2 Post-match equilibrium

Equilibrium in the repeated interaction during the employment relationship has a simple game-theoretic structure: employer and worker can behave in a mutually beneficial way, but they can also harm each other. Both situations can be supported as subgame perfect equilibria (SPE), where the mutually beneficial SPE is maintained by the threat to move to the mutually detrimental SPE. As the advertisement has in effect defined what the mutually beneficial SPE entails, employers will have to pay the advertised wage if they want to stay in the mutually beneficial SPE. Lemma 3 begins this reasoning with the mutually detrimental situation:

Lemma 3 (Non-cooperation). *Provided employers are sufficiently patient, the repeated game between employer and worker in an employment relationship has a SPE in which the worker only exerts effort $\underline{s} < \bar{s}$ in every period t that leaves the employer indifferent between continuing the match and deviating by not paying the wage and returning to search. The employer pays the wage*

$$\underline{w} = \frac{\rho}{\beta_W^{-1} - (1-\rho)} \left[\tilde{b} - (1-\beta_W)k_W\right] + d(\underline{s}) \tag{21}$$

in every period t, also leaving the worker indifferent between continuing the match and returning to search. The choice of $\hat{w}$ is indeterminate, and employers will be sufficiently patient if $\beta_F \geq \underline{w}/[\Pi(\underline{s}) - \Lambda]$. The worker's payoff is given by $\Phi(\underline{w} - d(\underline{s})) = Y$ and the employer's payoff by $\Pi(\underline{s}) = \underline{w}/\beta_F\Lambda$.

Proof. See appendix. □

The worker's behaviour in the mutually detrimental situation can be interpreted as "white strike", only working the minimum needed to keep the job. The next lemma characterises behaviour in the mutually beneficial SPE and confirms that payoffs for the worker and the employer are strictly higher than in the mutually detrimental SPE. By consequence, the threat of moving to the mutually detrimental SPE can prevent deviations from the mutually beneficial situation. Lemma 4 thereby proves part (iii) of Proposition 1:

Lemma 4 (Cooperation). *Provided employers are sufficiently patient such that*

$$\beta_F \geq \max\left[\frac{\tilde{b} + d(\bar{s}) - \underline{w}}{\Pi(\bar{s}) - \Pi(\underline{s})}, \frac{\underline{w}}{\Pi(\underline{s}) - \Lambda}\right] \tag{22}$$

the repeated game between employer and worker in an employment relationship also has a SPE in which the worker exerts effort $\bar{s}$ and the employer pays the wage $\tilde{b} + d(\bar{s}) > \underline{w}$ so that the profile $(\hat{w}, s, w) = (\tilde{b} + d(\bar{s}), \bar{s}, \tilde{b} + d(\bar{s}))$ is played in every match period. The worker therefore obtains the advertised benefit $\tilde{b}$. This SPE is sustained by the threat of moving to the SPE in Lemma 3 or leaving the match following a deviation: as

$$\Phi(\tilde{b}) > \Phi(\underline{w} - d(\underline{s})) = Y \quad \textit{and} \quad \Pi(\bar{s}) > \Pi(\underline{s}) > \Lambda$$

neither worker nor employer have an incentive to deviate and matches do not break up.

Proof. See appendix. □

Lemma 4 completes the proof of Proposition 1. Crucially, it is optimal for employers in the SPE in Lemma 4 to pay the advertised wage in order to avoid "white strike" or match break-up. Instead, this SPE elicits the surplus-maximising effort level $\bar{s}$ from the worker. Similar mutually beneficial SPE may also be sustained for other effort levels, as raising effort above $\underline{s}$ already increases the surplus. However, the SPE in Lemma 4 entails the highest payoff to the employer given that the advertised benefit $\tilde{b}$ has to be paid. Comparing payoffs for the employer in different SPE simplifies to comparing the per-period payoff $p(s) - w = p(s) - b - d(s)$. Given $b = \tilde{b}$, consider the first expression for $\tilde{b}$ in equation (17). Let λ denote the inverse of its denominator, which is a constant. Then the employer's maximisation problem can be written as

$$\max_s p(s) - \lambda(1 - \beta_F)\frac{n}{m}\left[p(s) - d(s)\right] - d(s)$$

The first-order condition yields

$$\left[1 - \lambda(1 - \beta_F)\frac{n}{m}\right]\left[\frac{\delta p(s)}{\delta s} - \frac{\delta d(s)}{\delta s}\right] = 0$$

As this leads to the same condition on effort as in equation (9), $s = \bar{s}$ is optimal for the employer. With regards to the second expression for $\tilde{b}$ in equation (17), it suffices to note that it is a constant, so that maximisation of $p(s) - b - d(s)$ immediately leads to the same condition as in equation (9). Therefore, the employer chooses the SPE in Lemma 4 among all mutually beneficial SPE by demanding $s = \bar{s}$ from the worker.

3.3 Discussion

Much of the reasoning in this section is not new at all. As one of the first, Solow (1979) formulated the theory that employers avoid wage cuts because they lead to bad morale and low productivity. More generally, a positive link between high wages and high worker effort, known as efficiency wages, has been the focus of an entire literature. That the logic of efficiency wages might not only sustain high wages but also make employers deliver on promises of high wages is, however, a novel approach taken in this paper. This extension merely requires that workers react very similarly to a broken promise of a high wage as to a wage cut.

In this paper's directed search model, job advertisements therefore take on a second role, compared to other directed search models. They are not only a tool for employers to attract candidates, but also set a benchmark against which the employer's behaviour can be evaluated. If the wage actually paid falls short of the benchmark, the worker's goodwill or trust towards the employer may be lost, an implicit contract may be violated, or an arrangement of gift exchange may be undermined.

Whatever the intuition, the benchmark matters as long as candidates arrive expecting the advertised wage and treat a different wage as a deviation from cooperative behaviour. This simple link is at the core of the model in this paper, and it is the only link between pre- and post-match behaviour: by fixing workers' beliefs, the advertisements predetermine the mutually beneficial SPE in the employment relationship. While the advertisements themselves are cheap talk, reneging on them amounts to deviating from the SPE. This interplay of advertisements and efficiency wages produces endogenous commitment in this paper's directed search model and resolves the Diamond paradox.

This solution to the Diamond paradox does not rely on relaxing Diamond's (1971) assumption of perfect information. Imperfect information is at the core of several well-known solutions, including those that involve some form of efficiency wages. In Burdett and Mortensen (1998), employers pay higher wages than necessary to unemployed job seekers only because they cannot distinguish them from employed job seekers. In Shapiro and Stiglitz (1984), employers cannot always observe the worker's effort and therefore pay higher wages to discourage shirking.

Using the model in this paper, the role of this assumption in Shapiro and Stiglitz (1984) is quickly demonstrated. A worker who shirks by choosing $s = 0$ is only detected with probability $q < 1$. Whenever shirking is detected, the match breaks up, but as long as it is not detected, the match continues and the worker saves $d(\bar{s})$. For the worker not to shirk, the payoff from working needs to outweigh the payoff from shirking:

$$b + \beta_W \Phi(b) \geq (1-q)\left[d(\bar{s}) + b + \beta_W \Phi(b)\right] + q\beta_W Y \tag{23}$$

With $b + \beta_W \Phi(b) = \Phi(b)$ and substituting for Y from equation (11), this becomes:

$$\Phi(b) \geq \frac{1}{1-\eta}\left[\frac{1-q}{q} d(\bar{s}) - \eta k_W\right] \tag{24}$$

where $\eta = \beta_W \rho / [\beta_W^{-1} - (1-\rho)]$. The search costs k_W are covered by this payoff $\Phi(b)$ if

$$\frac{1-q}{q} d(\bar{s}) \geq k_W$$

This may hold for q low enough, but cannot hold for $q = 1$. Efficiency wages in this paper are modelled differently and do not require an assumption of imperfect information.

4 Evidence

This section surveys a wealth of empirical evidence and concludes that every main element of the model in this paper is relevant in practice, or even most common. In particular, several recent studies confirm that employers refrain from cutting wages because they fear the effects on workers' productivity (see Section 4.2). This result will carry over to paying less than advertised if workers use the advertisement as a benchmark, as recent evidence from natural and laboratory experiments suggests (see Section 4.3).

4.1 On pre-match behaviour

The basic setting considered in this paper – employers advertise non-negotiable wage offers and then select job seekers in interviews – applies to many hiring processes in practice, as suggested by three recent studies. Hall and Krueger (2012) present survey results from a representative sample of 1,400 workers, collected in the United States in 2008. Brenčič (2012) draws on administrative data from public and private job matching services. These comprise data on 106,000 vacancies registered with the Employment Service of Slovenia in 2001, on 4,400 vacancies registered with the Lancashire Careers Service in the United Kingdom between 1988 and 1992, and on 140,000 vacancies advertised on Monster.com in the United States in 2006. Brenzel, Gartner and Schnabel (2014) dispose of a random sample of 9,300 employers in Germany who were included in the 2011 wave of the annual German Job Vacancy Survey.

Hall and Krueger (2012) find that 32% of workers had known their wage exactly before the first interview, and more than 80% either knew it exactly or had "a pretty good idea". According to Brenčič (2012), 25% of the vacancies in the sample from the United States included wage information, 30% of the vacancies in the sample from Slovenia, and 86% in the sample from the United Kingdom (which were targeted at young job seekers). Employers made a take-it-or-leave-it offer to 63% of workers surveyed in Hall and Krueger (2012). Such posted wages were more frequent for workers with lower educational attainment, as well as in the public sector. Among the workers who faced posted wages, more than 40% exactly knew the wage in advance. Brenzel et al. (2014) similarly find that 62% of employers in Germany had posted the wage for their most recently recruited worker. Posted wages were used less for highly educated job seekers (44%), for already employed job seekers (54%), and in cases when it was difficult to fill the job opening (55%). Posted wages were used more often when the employer was subject to a collective bargaining agreement (73%), and for job seekers who were unqualified (70%), unemployed (68%) or under 25 years old (68%).

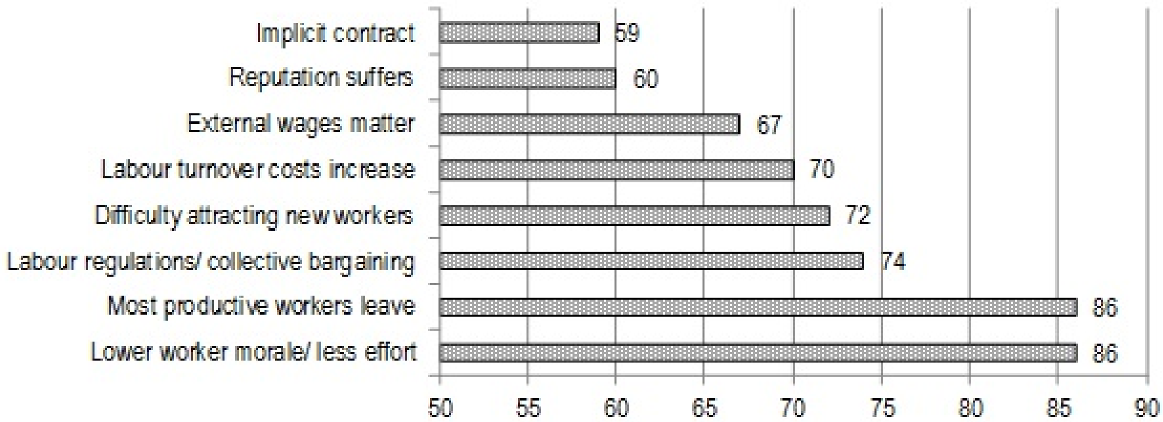
4.2 On post-match behaviour

The model in this paper allows matched workers to react to employers' behaviour by lowering their work effort. Some of the first evidence was presented by Kaufman (1984) and by Blinder and Choi (1990) based on small surveys among managers with a responsibility for workers' wages. Most of them did express fears that a wage cut would lead to bad morale among workers and lower effort.

Since then, employers' views on wage cuts have been surveyed on a larger scale. From more than 300 interviews in the Northeast of the United States, Bewley (1998) concludes that employers primarily avoid cutting wages because the adverse effects on morale would reduce productivity while staff turnover would rise. Surveyed employers therefore rejected wage cuts, expecting that the costs from these consequences would outweigh the savings on wage payments. From a survey of close to 200 large firms in the United States, Campbell and Kamlani (1997) conclude that employers fear lower effort especially among blue-collar workers and higher turnover especially among white-collar workers. Based on a representative survey of 900 wage setters in Swedish firms, Agell and Bennmarker (2007) report that 49% of employers expected that workers would reduce their effort levels following a wage cut, and another 30% considered this reaction possible.

A particularly large survey was carried out in 2007/08 by European central banks, collecting responses from 17 000 firms in 17 countries of the European Union. The survey used a standardised questionnaire with detailed questions on wage setting, and respondents could choose from a range of prepared answers to indicate reasons for their practices in wage setting. Results on firms' reasons to avoid wage cuts (available for 14 countries) are presented by du Caju et al. (2015) and reproduced in Figure 3. With little variation across countries, firm sizes and worker characteristics, two leading reasons emerged: adverse effects on morale and effort, and fears that some workers might leave. Taken together, the survey evidence suggests that effects on morale and productivity are one of two main reasons (alongside rising turnover) why employers avoid wage cuts. Both Agell and Bennmarker (2007) and du Caju et al. (2015) can confirm that wage cuts remain very rare, occurring in only about 3% of the surveyed firms.

Figure 3: Employers' reasons for avoiding wage cuts



Share considering the reason "relevant" or "very relevant", in percentages. Source: du Caju et al. (2015)

In the context of this paper, it is important to verify that wage cuts are rare also for newly hired workers. Galuscak et al. (2012) focus on the wages of newly hired workers in the large survey carried out by European central banks (in this case, data are available for eight countries). They find that only 13% of surveyed firms pay lower wages to new hires than to existing staff when labour market conditions allow. The main reason given for not paying new hires a lower wage is again the adverse effect on their effort (cited by 36% of surveyed firms), followed by concerns about fairness or the firm's reputation (33%) and collective bargaining agreements (28%). These results are in line with reports in Bewley (1998) that, in order not to antagonise new hires, they are paid according to the same standards as existing staff.

It is a different question whether employers are right to fear lower productivity following wage cuts. From detailed survey data on how workers in the United States use their time at work, Burda et al. (2016) find that workers with lower wages spend a greater share of their time at work not working. The effects of outright wage cuts have hardly been studied. Lee and Rupp (2007) investigate the effect of cuts in the wages of pilots working for major U.S. airlines, implemented between 2000 and 2005 when several airlines filed for bankruptcy. For pilots at some airlines, they find a significant reduction in performance, measured by statistics on flight delays. For pilots whose airline was near bankrupt, however, no change in performance was observed, and even the observed effects appeared short-lived. These findings may reflect the specific situation of highly-paid pilots in an industry threatened by bankruptcy: experimental evidence in Henning-Schmidt et al. (2010) suggests that workers only reduce effort when their employer derives undue profits from the wage cut.

Other case studies provide more indirect evidence. Mas (2008) documents significant deteriorations in product quality during disputes over pay between workers and employers in a manufacturing firm, which resulted in sizeable financial losses. Based on an employer-employee survey covering 123 firms in Japan, Kawaguchi and Ohtake (2007) document that morale was significantly lower among workers who experienced decreasing nominal wages. Smith (2015) uses the British Household Panel Survey to establish a causal link between wage cuts and bad morale. In experiments, the link between wage cuts and performance has been documented comparatively often. For example, a field experiment conducted by Kube et al. (2013) produced evidence of large and persistent effects of wage cuts on productivity: workers' average output, measured by the number of books entered into a database, fell by more than 20%.

4.3 On the link between pre-match and post-match behaviour

This section argues that workers' productivity can also be affected when *expected* wages are cut. If this is the case, then the wage expectations created by employers' advertisements (pre-match behaviour) will constitute a link to workers' effort (post-match behaviour). This link is needed for the main theoretical result of this paper.

As workers' expectations are not directly observable, a link with effort is difficult to establish. The most relevant evidence is provided by Mas (2006) who studies indicators of police performance in New Jersey after collective bargaining over police wages. The pay rise demanded by the police union might create expectations, which are violated when the pay rise ultimately found through arbitration turns out to be significantly lower. Mas (2006) finds that police performance, measured e.g. by reported crimes and arrest rates, sharply declined when arbitrators ruled against the police union. The larger the difference between demanded pay rise and actual pay rise, the larger was the decline in police performance. Changes in police performance were therefore not fully explained by actual pay rises alone - also the demanded pay rise was empirically relevant. This strongly suggests that violated wage expectations mattered when the police chose how much effort to exert in their work.

Mas (2006) interpreted his findings as evidence for reference points, in this case created by the demanded pay rises. Related evidence on expectations as reference points has been obtained in several experiments. Ericson and Fuster (2011) document that subjects who expect to receive an item with a higher probability value it more strongly in monetary terms and are less likely to trade it. They conclude that expectations create an endowment effect. Further results by Heffetz and List (2014) underline the role of induced changes in expectations. In the context of this paper, their results suggest that wage expectations induced by an advertisement can create a strong endowment effect, especially when it contrasts with the ultimate wage offer. Consequences for the choice of effort have also been documented: in experiments that require subjects to exert measurable effort - counting zeros in large tables - Abeler et al. (2011) find that significantly less effort is provided when a lower wage payment is expected afterwards.

The literature on organizational behaviour has discussed related findings under the notion of implicit contracts, i.e. the behaviour that employers and workers expect from each other. The mutual expectations are partly based on information received during the recruitment phase (see for example Rousseau and Greller, 1994), which notably includes job advertisements. Several empirical studies such as Turnley and Feldman (2000) and Wanous et al. (1992) found that, when expectations of new workers are not met, this has adverse effects on their morale and their effort. As a result, they appeared to neglect not only their job duties but in particular exhibited less organizational citizenship behaviour beyond their immediate job duties. Such reactions may be interpreted as "work to rule" or "white strike". Robinson and Morrison (2000) document particularly strong reactions from workers in cases when the employer was perceived to purposefully renege.

Together, the evidence presented on pre-match and post-match behaviour as well as on the link between them implies that a wide range of employers should be concerned about lower effort levels if they renege on advertised wages, which can explain why they normally do not renege.

5 Conclusions

This paper has addressed an issue in directed search models: why do employers not pay less than they advertised? The simple solution offered here is that employers could but prefer not to because they fear adverse effects on workers' morale and productivity. As shown in the model of this paper, attempts to renege on advertised wages can easily reduce the employer's profit even though search frictions prevent the worker from leaving: the worker can choose to exert only as much effort as is needed to keep the job.

Essentially, employer's dependence on the worker's effort during an employment relationship shapes the employer's behaviour before employment begins, during the matching process. Directed search models have thus far not captured this mechanism because they only model the matching process. To capture the mechanism in this paper, a standard directed search model was extended by a game-theoretic model of the employment relationship and placed in a dynamic setting. In this framework, it arises endogenously that employers do not renege on their advertisements.

Thus far, it has been most common in directed search models to assume that employers are somehow committed to their advertisements. The findings in this paper imply that the results of these models should still hold when the assumption is relaxed - presumably, employment relationships follow the matching process in each of these directed search models, so that commitment can arise endogenously. Without exogenous or endogenous commitment, directed search models suffer from the Diamond paradox: they collapse to a state in which workers do not engage in search. The explanation for endogenous commitment provided in this paper therefore also offers an economic rationale why this outcome is normally avoided in practice.

Finally, the mechanism at the core of this paper's reasoning - that wage decreases reduce workers' morale and productivity - has been identified as leading reason for wage rigidity by three decades of empirical research, and has likewise been confirmed by recent experimental findings. Several contributions in the literature suggest that this mechanism is not limited to decreases of actual wages but also applies to decreases of promised wages as in this paper.

A Appendix: Omitted proofs

Proof of Lemma 1. The objective is to maximise Λ by taking the derivative of the right-hand side of equation (16) with respect to σ. For reasons of presentation, consider first the derivative of $\frac{1-(1-\sigma)^n}{\beta_F^{-1}-(1-\sigma)^n}$. Using the quotient rule, this derivative is found as

$$\frac{n(1-\sigma)^{n-1}\left[\frac{1}{\beta_F}-(1-\sigma)^n\right]-[1-(1-\sigma)^n]n(1-\sigma)^{n-1}}{\left[\frac{1}{\beta_F}-(1-\sigma)^n\right]^2}=n(1-\sigma)^{n-1}\frac{\frac{1}{\beta_F}-1}{\left[\frac{1}{\beta_F}-(1-\sigma)^n\right]^2}$$

Next rewrite the remainder of the right-hand side of equation (16) as follows to make the dependence on σ explicit:

$$\frac{1}{1-\beta_F}\left[p(s)-d(s)-\frac{1-\beta_W}{\frac{1-(1-\sigma)^n}{n\sigma}}\underline{\Phi}\right]$$

The derivative of this part is, by the quotient rule,

$$\left[-\frac{1-\beta_W}{1-\beta_F}\underline{\Phi}\right]\left[\frac{n(1-(1-\sigma)^n)-n^2\sigma(1-\sigma)^{n-1}}{[1-(1-\sigma)^n]^2}\right]$$

With the derivatives of the parts, one can put together the first-order condition for the entire right-hand side of equation (16) according to the product rule:

$$\begin{aligned}
&n(1-\sigma)^{n-1}\frac{\frac{1}{\beta_F}-1}{\left[\frac{1}{\beta_F}-(1-\sigma)^n\right]^2}\frac{1}{1-\beta_F}\left[p(s)-d(s)-\frac{1-\beta_W}{\rho}\underline{\Phi}\right] \quad + \\
&\frac{1-(1-\sigma)^n}{\frac{1}{\beta_F}-(1-\sigma)^n}\left[-\frac{1-\beta_W}{1-\beta_F}\underline{\Phi}\right]\left[\frac{n\left[1-(1-\sigma)^n\right]-n\left[n\sigma(1-\sigma)^{n-1}\right]}{[1-(1-\sigma)^n]^2}\right] \quad = \quad 0 \qquad (25)
\end{aligned}$$

Note that, with the definition of ρ in equation (1), the last term can be rewritten as follows:

$$\frac{n\left[1-(1-\sigma)^n\right]-n\left[n\sigma(1-\sigma)^{n-1}\right]}{\left[1-(1-\sigma)^n\right]^2}=\frac{n}{1-(1-\sigma)^n}\left[1-\frac{1}{\rho}\left(1-\sigma\right)^{n-1}\right]$$

Using this equivalence and dividing equation (25) by n and $1-\beta_W$ before multiplying it by $1-\beta_F$ and $\frac{1}{\beta_F}-(1-\sigma)^n$ leads to

$$(1-\sigma)^{n-1}\frac{\frac{1}{\beta_F}-1}{\frac{1}{\beta_F}-(1-\sigma)^n}\left[\frac{p(s)-d(s)}{1-\beta_W}-\frac{\underline{\Phi}}{\rho}\right]-\underline{\Phi}\left[1-\frac{1}{\rho}\left(1-\sigma\right)^{n-1}\right]=0$$

Now call $\frac{\frac{1}{\beta_F}-1}{\frac{1}{\beta_F}-(1-\sigma)^n}=x$ and divide by $(1-\sigma)^{n-1}$ to obtain

$$\begin{aligned}
& x\left[\frac{p(s)-d(s)}{1-\beta_W}-\frac{\underline{\Phi}}{\rho}\right]-\underline{\Phi}\left[\left(1-\sigma\right)^{1-n}-\frac{1}{\rho}\right]=0\\
\text{or}\quad & x\left[\frac{p(s)-d(s)}{1-\beta_W}\right]=\underline{\Phi}\left[\frac{1}{\rho}(x-1)+\left(1-\sigma\right)^{1-n}\right]\\
\text{so that}\quad & \underline{\Phi}=\frac{x}{\frac{1}{\rho}(x-1)+\left(1-\sigma\right)^{1-n}}\left[\frac{p(s)-d(s)}{1-\beta_W}\right]
\end{aligned}$$

The constraint in equation (15) requires

$$w-d(s)=\frac{1-\beta_W}{\rho}\underline{\Phi}=\frac{x}{x-1+\rho(1-\sigma)^{1-n}}\left(p(s)-d(s)\right)$$

using the result obtained for $\underline{\Phi}$. After substituting for x according to its definition and multiplying by $\frac{1}{\beta_F}-(1-\sigma)^n$ and then by β_F, one obtains

$$\begin{aligned}
w-d(s) &= \frac{1-\beta_F}{1-\beta_F+\left[\rho(1-\sigma)^{1-n}-1\right]\left[1-\beta_F(1-\sigma)^n\right]}\left(p(s)-d(s)\right)\\
&= \frac{1-\beta_F}{\rho\left[(1-\sigma)^{1-n}-\beta_F(1-\sigma)\right]-\beta_F\left[1-(1-\sigma)^n\right]}\left(p(s)-d(s)\right)\\
&= \frac{1-\beta_F}{\rho\left[(1-\sigma)^{1-n}-\beta_F\left[1+(n-1)\sigma\right]\right]}\left(p(s)-d(s)\right) \qquad (26)
\end{aligned}$$

where the last step uses that $1-(1-\sigma)^n=n\sigma\rho$ by the definition of ρ. As $w-d(s)$ is the worker's per-period benefit, the expression obtained is the share of the surplus $p(s)-d(s)$ that employers advertise as benefit to workers. Note that the expression is the same for every employer: in equilibrium, all employers choose the same combination of σ and $w-d(s)$, so that all offer the same $\underline{\Phi}$ to workers. Workers therefore apply to one employer at random, and each employer has a chance of $\sigma=1/m$ of receiving an application from a given worker. Substituting first for ρ and then for σ in equation (26), one obtains

$$\tilde{b} = \frac{(1-\beta_F)\frac{n}{m}}{\left[1-\left(1-\frac{1}{m}\right)^n\right]\left[\left(1-\frac{1}{m}\right)^{1-n} - \beta_F\left(1+\frac{n-1}{m}\right)\right]}\left(p(s)-d(s)\right) \tag{27}$$

as given in equation (17). However, this $\tilde{b}$ may be too low to reimburse workers for their search costs k_W. The minimum per-period benefit employers have to advertise to attract workers in expectation just reimburses them for their search costs, so that

$$\frac{\tilde{b}}{1-\beta_W} = k_W$$

which gives the other expression in equation (17). Hence $\tilde{b}$ is the maximum of these two expressions. □

Proof of Lemma 2. Taking part (iii) of Proposition 1 as given means that employers do offer workers who visit them what they advertised. Workers therefore trust the advertisements. Employers' advertisements are specified in Lemma 1, and offer workers a per-period benefit of at least $(1-\beta_W)k_W$. The value to the worker of receiving this per-period benefit in a match that does not break up is $\Phi = k_W > 0$. In general, the value to the worker of being in the interview is $\Omega = \max[\Phi, Y]$, as the worker can choose to accept the match or to continue searching. To prove that $\Phi \geq Y$ here, suppose to the contrary that $Y > \Phi$. Then the worker would always decide against the match and prefer to continue searching. As the worker then never obtains any positive payoff, it follows that $Y \leq 0$. However, this yields a contradiction because $Y > \Phi$ is impossible if $Y \leq 0$ and $\Phi > 0$. Hence $\Phi \geq Y$ and, by equation (5), $\Omega = \Phi$: the worker always accepts the match, which proves that the worker matches at the first opportunity. As workers are homogeneous, employers also wish to match at the first opportunity. Next substituting $\Omega = \Phi = k_W$ in equation (4), one obtains

$$Y = (1-\rho)\beta_W Y + \rho\beta_W \max[0, Y] \tag{28}$$

Using the same argument as before, $Y > 0$ is impossible, as the worker would then never go to an interview, which implies $Y = 0$. Hence $Y \leq 0$ so that workers prefer attending interviews at least weakly, and they thus do engage in search. □

Proof of Lemma 3. The proof considers non-cooperation as a single deviation from the equilibrium in Proposition 1 and proceeds by backward induction. In subperiod t^2 of a match, the worker has already exerted effort. The employer can take advantage of the situation by not paying any wage. With $w = 0$, the per-period payoff for the worker equals $-d(s) \leq 0$, so that $\Phi(0 - d(s)) \leq 0$. As the worker can instead obtain $\beta_W Y \geq 0$ by returning to search, not paying any wage will end the match. The lowest wage that retains the worker in the match, denoted $\underline{w}$, makes the worker indifferent between another match period and returning to search: $\beta_W \Phi(\underline{w} - d(s)) = \beta_W Y$. The employer prefers paying $\underline{w}$ to not paying any wage whenever

$$p(s) - \underline{w} + \beta_F \Pi(s) \geq p(s) + \beta_F \Lambda \quad \text{or} \quad \beta_F \left[\Pi(s) - \Lambda\right] \geq \underline{w} \tag{29}$$

which will hold if employers are sufficiently patient, i.e. $\beta_F \geq \underline{w}/[\Pi(s) - \Lambda]$. Equation (29) also gives the employer's payoff as specified in Lemma 3, for a general s.

The level of $\underline{w}$ follows from the worker's indifference. In the equilibrium in Proposition 1, employers leave workers with benefit $\tilde{b}$ in every period of the match, so that equation (11) can be written as

$$Y = \frac{\rho}{\beta_W^{-1} - (1 - \rho)} \left[\frac{\tilde{b}}{1 - \beta_W} - k_W\right] \tag{30}$$

Since the match does not break up if the employer pays $\underline{w}$ in every period, $\Phi(\underline{w} - d(s)) = [\underline{w} - d(s)]/[1 - \beta_W]$ in analogy to equation (12). Indifference between $\beta_W \Phi(\underline{w} - d(s))$ and $\beta_W Y$ therefore requires:

$$\underline{w} = (1 - \beta_W)Y + d(s) \tag{31}$$

Substituting for Y from equation (30) gives the expression in Lemma 3 for a general s.

By backward induction, the worker can anticipate the employer's behaviour and choose the level of s accordingly. If the worker sets $s = 0$, the employer cannot obtain a positive per-period payoff $p(s) - w$. To avoid a loss, the employer will not pay any wage and instead obtain $\beta_F \Lambda > 0$ by returning to search. Anticipating that the employer is prepared to pay a wage $\underline{w}$ so that the worker does not gain from match break-up, the worker also seeks to avoid match break-up by choosing the lowest effort that retains the employer in the match, denoted $\underline{s}$. The worker's optimisation problem is therefore to maximise the benefit while ensuring that equation (29) just holds:

$$\max_s \underline{w} - d(s) \quad \text{subject to} \quad \underline{w} = \beta_F \left[\Pi(s) - \Lambda\right]$$

The constraint can be rewritten as

$$\underline{w} = \frac{\beta_F}{1 - \beta_F} \left[p(s) - (1 - \beta_W)Y - d(s)\right] - \beta_F \Lambda$$

using equation (31). After substituting for $\underline{w}$ in $\max_s \underline{w} - d(s)$ and noting that Y and Λ

are given equilibrium values, the first-order condition requires for $\underline{s}$ that

$$\frac{\delta p(s)}{\delta s} = \frac{1}{\beta_F}\frac{\delta d(s)}{\delta s} \tag{32}$$

By contrast, equation (9) requires for $\bar{s}$ that

$$\frac{\delta p(s)}{\delta s} = \frac{\delta d(s)}{\delta s} \tag{33}$$

Since $1/\beta_F > 1$, the left-hand side of equation (32) exceeds that of equation (33). By the strict concavity of $p(\cdot)$, the equality in equation (32) must occur at a lower level of s than the equality in equation (33). Therefore, $\underline{s} < \bar{s}$.

The choices $\underline{s}$ and $\underline{w}$ form a Nash equilibrium in the match period because unilateral deviations are not profitable. If the employer deviates to a lower wage than $\underline{w}$, the match will break up; however, effort level $\underline{s}$ was chosen such that the employer does not gain from match break-up. If the employer deviates to a higher wage, the match continues but the employer's per-period payoff decreases. Similarly, if the worker deviates to a lower effort level than $\underline{s}$, the match will break up but $\underline{w}$ was chosen such that the worker does not gain from match break-up. If the worker deviates to a higher effort level, the employer will adjust the wage as given in Lemma 3 to reflect the increase in $d(s)$, but the worker would still be left with the same benefit $w - d(s)$.

Since match break-up is avoided, this Nash equilibrium is repeated in every match period, and the infinite repetition of a Nash equilibrium is by definition a SPE of the repeated game. All this is independent of the employer's choice of $\hat{w}$ in t^0, so that $\hat{w}$ may take any value in $\mathbb{R}_+$. $\square$

Proof of Lemma 4. Let players use *grim trigger strategies* as follows. In each period t^0, the employer offers $\hat{w} = \tilde{b} + d(\bar{s})$ and demands $s = \bar{s}$ if there has been no earlier deviation by either side. In each period t^1, the worker chooses $s = \bar{s}$ unless there was a deviation. In each period t^2 the employer pays $\tilde{b} + d(\bar{s})$ unless there was a deviation.

If there was a deviation by either side, players either switch to the SPE in lemma 3 or leave the match and return to search. If the employer deviated by offering or paying some wage $w \geq \underline{w}$ or the worker deviated to some effort $s \geq \underline{s}$, players switch to the SPE in lemma 3: the worker instead chooses $s = \underline{s}$, the employer chooses $w = \underline{w}$ (with $\hat{w}$ indeterminate), and likewise in every subsequent period. As noted before, any SPE must also be a NE, and therefore the threat of switching is credible. In case a deviation involves offering or paying a lower wage than $\underline{w}$ or lower effort than $\underline{s}$, worker and employer react by not working and not paying, respectively, and the match breaks up.

The threat of these actions can sustain a SPE with $s = \bar{s}$ and $w = \tilde{b} + d(\bar{s})$ if both players prefer not to deviate. The worker prefers not to deviate to $s \geq \underline{s}$ because

$\Phi(b) = b/(1-\beta_W)$ would decrease: slightly rewriting equation (21) gives the benefit in the mutually detrimental SPE as

$$\underline{w} - d(\underline{s}) = \frac{\rho}{\beta_W^{-1} - (1-\rho)} \left[\tilde{b} - (1-\delta)k_W\right] < \tilde{b} \tag{34}$$

where the inequality holds because $\tilde{b} - (1-\delta)k_W < \tilde{b}$, while $\rho/(\beta_W^{-1} - (1-\rho))$ is the probability of having an interview (see equation (11), for example) and is therefore smaller than 1. Rearranging the inequality in equation (34) gives $\tilde{b} + d(\underline{s}) > \underline{w}$ which implies $\tilde{b} + d(\bar{s}) > \underline{w}$ as claimed in Lemma 4 because $d(\underline{s}) < d(\bar{s})$ follows from $\underline{s} < \bar{s}$ (see Lemma 3). By consequence, $\Phi(\tilde{b}) > \Phi(\underline{w} - d(\underline{s}))$. As the worker is indifferent between staying and leaving in the SPE in Lemma 3, also $\Phi(\tilde{b}) > Y$ so that the worker does not prefer to deviate to some effort $s < \underline{s}$ either.

Similarly to equation (29), the employer prefers not to deviate whenever the non-deviation payoff at least matches the payoffs from paying $\underline{w}$ and thereby moving to the SPE in Lemma 3 and from paying nothing and thereby ending the match:

$$p(\bar{s}) - \tilde{b} - d(\bar{s}) + \beta_F \Pi(\bar{s}) \geq \max\left[p(\bar{s}) - \underline{w} + \beta_F \Pi(\underline{s}), p(\bar{s}) + \beta_F \Lambda\right] \tag{35}$$

The first expression on the right-hand side exceeds the second whenever

$$\beta_F[\Pi(\underline{s}) - \Lambda] \geq \underline{w}$$

which is the same condition as in equation (29). Therefore the second expression can be ignored provided $\beta_F \geq \underline{w}/[\Pi(\underline{s}) - \Lambda]$. Then equation (35) leads to:

$$\beta_F \geq \frac{\tilde{b} + d(\bar{s}) - \underline{w}}{\Pi(\bar{s}) - \Pi(\underline{s})} \tag{36}$$

Lemma 4 requires β_F to meet both of these two thresholds. Which one is higher depends on parameter values that determine $\tilde{b}$ and the worker?s benefit included in $\underline{w}$. Finally, $\Pi(\bar{s}) > \Pi(\underline{s})$ if

$$p(\bar{s}) - \tilde{b} - d(\bar{s}) + \beta_F \Pi(\bar{s}) > p(\underline{s}) - \underline{w} + \beta_F \Pi(\underline{s})$$

which must hold whenever equation (35) holds, as it only differs by $p(\bar{s}) > p(\underline{s})$ on the right-hand side. $\Pi(\bar{s}) > \Lambda$ follows from $\Pi(\underline{s}) > \Lambda$ in Lemma 3. Hence both players do not deviate if the employer is sufficiently patient, and matches do not break up. $\square$

References

[1] Abeler, Johannes, Falk, Armin, Goette, Lorenz and Huffman, David (2011): "Reference Points and Effort Provision," *American Economic Review* 101, 470-492

[2] Agell, Jonas and Bennmarker, Helge (2007): "Wage Incentives and Wage Rigidity: A Representative View from Within," *Labour Economics* 14, 347-369

[3] Bewley, Truman F. (1998): "Why Not Cut Pay?," *European Economic Review* 42, 459-490

[4] Blinder, Alan S. and Choi, Don H. (1990): "A Shred of Evidence on Theories of Wage Stickiness," *The Quarterly Journal of Economics* 105, 1003-1015

[5] Brenčič, Vera (2012): "Wage Posting: Evidence from Job Ads," *Canadian Journal of Economics* 45, 1529-1559

[6] Brenzel, Hanna, Gartner, Hermann and Schnabel, Claus (2014): "Wage Bargaining or Wage Posting? Evidence from the Employers' Side," *Labour Economics* 29, 41-48

[7] Burda, Michael C., Genadek, Katie R. and Hamermesh, Daniel S. (2016): "Not Working at Work: Loafing, Unemployment and Labor Productivity," NBER Working Paper 21923

[8] Burdett, Kenneth and Mortensen, Dale T. (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review* 39, 257-273

[9] Burdett, Kenneth and Judd, Kenneth L. (1983): "Equilibrium Price Dispersion," *Econometrica* 51, 955-969

[10] Burdett, Kenneth, Shi, Shouyong, and Wright, Randall (2001): "Pricing and Matching with Frictions," *Journal of Political Economy* 109, 1060-1085

[11] Campbell III, Carl M., and Kamlani, Kunal S. (1997): "The Reasons for Wage Rigidity: Evidence from a Survey of Firms," *Quarterly Journal of Economics* 112, 759-789

[12] Carrillo-Tudela, Carlos (2009): "An Equilibrium Search Model When Firms Observe Workers' Employment Status," *International Economic Review* 50, 485-506

[13] Carrillo-Tudela, Carlos, Menzio, Guido, and Smith, Eric (2011): "Job Search With Bidder Memories," *International Economic Review* 52, 639-655

[14] Coles, Melvyn G. (2001): "Equilibrium Wage Dispersion, Firm Size, and Growth," *Review of Economic Dynamics* 4, 159-187

[15] Diamond, Peter A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory* 3, 156-168

[16] Du Caju, Philip, Kosma, Theodora, Lawless, Martina, Messina, Julián and Rõõm, Tairi (2015): "Why Firms Avoid Cutting Wages: Survey Evidence From European Firms," *Industrial and Labor Relations Review* 68, 862-888

[17] Ericson, Keith M. M., and Fuster, Andreas (2011): "Expectations as En-

dowments: Evidence on Reference-Dependent Preferences From Exchange and Valuation Experiments," *Quarterly Journal of Economics* 126, 1879-1907

[18] Galuscak, Kamil, Keeney, Mary, Nicolitsas, Daphne, Smets, Frank, Strzelecki, Pawel and Vodopivec, Matija (2012): "The Determination of Wages of Newly Hired Employees: Survey Evidence on Internal Versus External Factors," *Labour Economics* 19, 802-812

[19] Hall, Robert E. and Krueger, Alan B. (2012): "Evidence on the Incidence of Wage Posting, Wage Bargaining, and On-the-Job Search," *American Economic Journal: Macroeconomics* 4, 56-67

[20] Heffetz, Ori and List, John A. (2014): "Is the endowment effect an expectations effect?," *Journal of the European Economic Association* 12, 13961422

[21] Hennig-Schmidt, Heike, Sadrieh, Abdolkarim and Rockenbach, Bettina (2010): "In Search of Workers' Real Effort Reciprocity a Field and a Laboratory Experiment," *Journal of the European Economic Association* 8, 817-837

[22] Kaufman, Roger T. (1984): "On Wage Stickiness in Britain's Competitive Sector," *British Journal of Industrial Relations* 22, 101-112

[23] Kawaguchi, Daiji, and Ohtake, Fumio (2007): "Testing the Morale Theory of Nominal Wage Rigidity," *Industrial and Labor Relations Review* 61, 59-74

[24] Kim, Kyungmin and Kircher, Philipp (2015): "Efficient Competition Through Cheap Talk: The Case of Competing Auctions," *Econometrica* 83, 1849-1875

[25] Kube, Sebastian, Marchal, Michel Andr and Puppe, Clemens (2013): "Do wage cuts damage work morale? Evidence from a natural field experiment," *Journal of the European Economic Association* 11, 853-870

[26] Lee, Darin, and Rupp, Nicholas G. (2007): "Retracting a Gift: How Does Employee Effort Respond to Wage Reductions?," *Journal of Labor Economics* 25, 725-761

[27] MacLeod, W. Bentley and Malcomson, James M. (1989): "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment," *Econometrica* 57, 447-480

[28] Mas, Alexandre (2008): "Labour Unrest and the Quality of Production: Evidence From the Construction Equipment Resale Market," *The Review of Economic Studies* 75, 229-258

[29] Mas, Alexandre (2006): "Pay, Reference Points, and Police Performance," *Quarterly Journal of Economics* 121, 783-821

[30] Masters, Adrian M. (2010): "A Theory of Minimum Wage Compliance (or Recognition of Unions)," *Labour Economics* 17, 215-223

[31] Menzio, Guido (2007): "A Theory of Partially Directed Search," *Journal of Political Economy* 115, 748-769

[32] Montgomery, James D. (1991): "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics* 106, 163-179

[33] Mortensen, Dale T. (2003): *Wage Dispersion: Why are Similar Workers Paid Differently?,* Cambridge: MIT Press

[34] Robinson, Sandra L. and Wolfe Morrison, Elizabeth (2000): "The Development of Psychological Contract Breach and Violation: A Longitudinal Study," *Journal of Organizational Behaviour* 21, 525-546

[35] Rogerson, Richard, Shimer, Robert and Wright, Randall (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature* 43, 959-988

[36] Rousseau, Denise M., and Greller, Martin M. (1994): "Human Resource Practices: Administrative Contract Makers," *Human Resource Management* 33, 385-401

[37] Shapiro, Carl and Stiglitz, Joseph E. (1984): "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review* 74, 433-444

[38] Smith, Jennifer C. (2015): "Pay Growth, Fairness, and Job Satisfaction: Implications for Nominal and Real Wage Rigidity," *Scandinavian Journal of Economics* 117, 852-877

[39] Solow, Robert M. (1979): "Another Possible Source of Wage Stickiness," *Journal of Macroeconomics* 1, 79-82

[40] Tudón Maldonado, José F. (2016): "Price Dispersion With Ex Ante Homogeneity," University of Chicago mimeo

[41] Turnley, William H. and Feldman, Daniel C. (2000): "Re-Examining the Effects of Psychological Contract Violations: Unmet Expectations and Job Dissatisfaction as Mediators," *Journal of Organizational Behaviour* 21, 25-42

[42] Wanous, John P., Poland, Timothy D., Premack, Stephen L. and Davis, K. Shannon (1992): "The Effects of Met Expectations on Newcomer Attitudes and Behaviors: A Review and Meta-Analysis," *Journal of Applied Psychology* 77, 288-297