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February 2018

OPTIMAL FINANCIAL CONTRACTS WITH UNOBSERVABLE INVESTMENTS

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Abstract. Motivated by the informational 'opacity' that often characterizes small firms, this article studies a security design problem in which outside investors are unable to observe entrepreneurs' investments decisions and their firms' net-worth, both before and after contracts are signed. The investment size affects the probability of higher profit realizations and depends both on the amount of the entrepreneur's initial capital (or type) and on the firm's access to outside funds. The interconnectedness of these three different forms of asymmetric information implies as many risks in the design problem: a possible adverse selection on entrepreneurs' types; moral hazards both on investments and on the release of information related to the firm's income/profits. Our approach and model is an extension of the classical, reduced-form one used in the moral hazard literature. The results we present establish that outside finance should take the form of a debt contract, whose terms are: a firm's capitalization requirement, a fund size, a payment schedule, a verification/auditing rule. Optimal contracts form a menu; those for higher capitalized firms are of larger size, carry a lower interest rate and lower expected costs of auditing. The payment schedule is the one of standard debt contracts; however, due to the role played by capital, the 'bankruptcy' region is narrower than prescribed in previous studies and decreasing in the firm's initial net-worth.
(*JEL Classification Numbers*: D820, D860, G110, G320.)
(*Keywords*: Security design; asymmetric information; moral hazard; investment decisions; debt contracts, collateral.)

Contents

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I acknowledge financial support from the Italian Ministry of University and Research (PRIN 2015). I thank Martin Meier, Enrico Minelli and Paolo Siconolfi for very helpful discussions and comments. I also thank the participant of the Colloquia on Games and Contracts with Asymmetric Information and of FEBS 2018 conference. All remaining omissions and errors are my own responsibility.

1. Introduction

Data show that the contribution of small and medium companies (SMEs) to the European economy is considerable. Across EU28 in 2015, SMEs accounted for two thirds of the employment and slightly less than three fifths of the value added in the non-financial business sector. This relative importance of SMEs extends to all high-income OECD countries, to most of high-income non-OECD ones, to Est-Asia (particularly to China) and the Pacific (see Kushnir, Mirmulstain and Ramalho, 2010).

Despite their relevance, SMEs often find themselves at a disadvantage with respect to large firms in accessing to finance, both in terms of higher costs and lower opportunities; something that is frequently explained by referring to perhaps the most common of SMEs' characteristics: 'informational opacity.' The 'opacity' of small firms and their consequent difficulties in accessing to finance, are largely associated to three factors (*e.g.* see the discussion in Beger and Udell, 1998; and the survey by Abdulsaleh and Worthington, 2013): a very simplified governance; softer regulatory requirements on accounting and transparency (information disclosure); a low capitalization.

Indeed, the vast majority of small companies are family owned or are simple partnerships, characterized by an elementary owner-manager structure. Moreover, especially for young firms, owners do often use personal assets to either directly finance investments or to access credit. This makes harder for outside investors to distinguish the financial situation of a firm from the personal one of its owners. Information acquisition is then worsen by a consolidated tendency of regulators to reduce the administrative burden for SMEs, which in many situations translates into weak information requirements, especially on their financial status.[1] Finally, agency costs are exacerbated by a diffused undercapitalization of small companies, which for technologically oriented firms is worsen by a tendency to invest in assets that are difficult to evaluate and largely unsuited to be used as collateral: 'soft' and intangible capital.

Motivated by such evidence, in this paper we study the optimal financial structure and decisions of firms, whose risk-neutral entrepreneurs need to raise outside finance in a context in which several agency problems combine. In particular, our security design problem is one in which intermediaries (or outside investors) *i)* are unable to observe the firms' initial financial status (or net-worth), *ii)* neither observe nor can verify how their funds are invested, *iii)* can only contract on final returns (firms' cash flows) that are subject to a costly verification.

[1]The requirement of an annual balance sheet and a profit/loss account has been reaffirmed, even recently, in the EU Responsible Business Package (see European Parliament, Directive 2013/34/EU), also for limited-liability companies. In addition, depending on their dimension (small or micro firms), the Directive, allows EU Members to introduce further significative simplifications at national level, especially in terms of a consistent reduction of financial information. Similarly, in the US, small companies are not required to release financial information on 10K forms.

The principal-agent model we propose builds on and extends the classical *moral hazard* one due to Innes (1990).[2] It follows Innes' in the timing of the agents' actions and in some of the main restrictions on contracts. Instead, it differs from Innes' in several, relevant aspects. First, a real investment decision replaces an 'effort' choice. This implies that, in our context, an agent decision is subject to budget-feasibility, while in Innes' it is not, as actions only produce a subjective welfare cost. Second, in line with this interpretation, we assume that agents can divert outside funds from productive investment and hide them outside their firms. Third, to represent the idea that it is difficult to distinguish the financial situation of the firm from the one of its owner-manager, we assume that firms' capital (net-worth) is unobservable by the intermediaries. Thus, even if (as in Innes') the realization of a real investment project is observable, because correctly reported in the profit-loss statement of the company, the firm's capital may remain private information. Finally, in the last part of the paper we extend the model to the case in which the realization of a real investment project is observable only at a cost, capturing auditing costs.

Given the presence of *moral hazard* and, possibly, of *adverse selection*, the space of contracts we consider is multidimensional, consisting not only of a (final) payment schedule, as in Innes', but also of a loan size, of an initial 'down-payment', and of an auditing (or verification) rule. The down-payment can assume different forms, for example, configuring a participation fee (*e.g.* in the spirit of franchising), a security deposit (*e.g.* bank credit credit lines), a collateral requirement (*e.g.* in secured credit card and bank credit lines) and a minimum capital requirement (*e.g.* a minimum capital requirement prescribed by corporate law to limited liability companies, corporations, etc.). Alternative forms of this instrument have various impact on agents' incentives and decisions. A third contractual dimension is the loan size that may, possibly, be used to induce a dose of '*credit rationing*,' when the demand of outside funding exceeds that size.[3] Finally, when we consider the case of costly state verification on final returns on firms' activities, a forth contractual dimension is the set of states in which the agent commits to certify profits.

Our main result is an optimal menu of contracts with the following characteristics:

1.i) A payment schedule typical of *standard debt contracts* (SDCs) if either profit realizations are subject to costly verification or if they are verifiable at no cost and a monotonicity condition is imposed;[4]

1.ii) An initial down-payment that takes the form of a maximal capital participation by the agent when SDCs are optimal, and of a full collateralization, otherwise;

1.iii) A loan size that induces no credit 'rationing' or 'restriction' to the agent;

[2] Innes (1990) uses the reduced-form approach pioneered by Mirrlees (1976) and Hölmstrom (1979).

[3] Some path-breaking articles on the role of: *a)* collateral are Stiglitz & Weiss (1981) and Chan & Thakor (1987); *b)* security (or bond) deposit is Lewis & Sappington (2000); *c)* franchise fee is Mathewson & Winter (1984). Stigliz & Weiss cit. is also a classic of credit rationing.

[4] SDCs are defined similarly to Gale & Helwig (1985); loosely speaking, as financial contracts whose (final) payment schedule dictates a fixed (non-contingent) payment $z > 0$ for high enough, profit realizations and a 'default-payment' or an equity-type payment (realized profits + collateral) otherwise.

1.iv) An interest rate schedule that is monotonically decreasing in the firm's initial capital and in its debt size, up to the point that financial opportunities sustain the first-best (*i.e.* the level of investment that would be optimal in the absence of asymmetric information). A verification region that is also monotonically decreasing in the same variables.

Moreover, the optimal menu of contracts supports entrepreneurial policies with the following characteristics.

2.i) A unique optimal financial structure is derived for each firm; entrepreneurs' most preferred source of finance is internal capital, followed by outside debt: if initial capital is sufficient to fund the first-best level of individual investment, the entrepreneur does not demand outside finance; otherwise, he agrees/decides to endow the firm with the maximal (secured) level of capital and raises outside funding to finance the desired investment project, whose size is below the first-best (*i.e.* there is underinvestment);

2.ii) Entrepreneurs who subscribe a debt contract, use all the available funds to invest in the firm, implying that investments are sensitive to cash flows and that less capitalized firms are more exposed to agency costs and underinvestment.

The fact that the optimal menu is composed of SDCs is a robust result in our analysis, in the sense that it holds for any arbitrarily small verification cost. This essentially strengthen Innes (1990), who exogenously restricts the space of contracts to one with non-decreasing payment schedules. The underline idea on the optimality of SDCs in the presence of moral hazard is by Jensen and Meckling (1976): debt strengthens incentives to invest relative to equity by redistributing most of the agent's financial cost from 'good' to 'bad' states ('bankruptcy' states). However, in our analysis the shape of the payment schedule differs with respect to the traditional one in the literature, essentially due to the role played by initial capital, under limited-liability and costly state verification. It turns out that a fixed payment is optimal also in a portion of the verification region, which are financed using realized profits and a partial liquidation of secured capital (*i.e.* down-payment), and calibrated so as to satisfy limited-liability and the ex-post, incentive compatibility of the verification rule. We interpret this region as one in which the firm faces a 'liquidity' shortage. Instead, for 'extremely low' cash-flow realizations, capital is completely liquidated; something that we interpret as a 'bankruptcy' event (*e.g.* Gale and Hellwig, 1985).

Optimal contracts prescribe a full down-payment (in 1.ii). This can essentially be explained based on two considerations. First, the down-payment, either in the form of collateral or of capital participation, allows to mitigate moral hazard by increasing the power of the incentive scheme. In fact, in order to avoid the payment loss, occurring at low profit realizations, each agent tends to increase real investment; this, in turn, allows creditors to offer a loan contract with a lower interest rate, thereby sustaining incentives. Thus, even if initial capital were common knowledge, optimal contracts would still prescribe full down-payment. The incentive effects of capital are consistent with the findings of some contributions in the literature on banking and credit markets (*e.g*, Chan and Thakor, 1987), which however –to my knowledge- do not approach the problem as a general one of security design.[5] They are also consistent with the literature on financial intermediation and monitoring in which an increase in initial capital tends to reduce the cost of access to outside finance, by weakening liquidity-constraints and reducing monitoring cots, for example, in Gale and Hellwig (1985), and in Hölmstrong and Tirole (1993). In absence of a down-payment, our analysis says that entrepreneurs with low capitalized firms would claim to have high capital in order to access to contracts with a lower interest rate and a larger size (by 1.iv).[6]

The fact that firms with higher net-worth have an easier and cheeper access to credit (in 1.iv), it is also in line with the empirical evidence. Data show that debt contracts of larger size are typically issued at lower interest rates and have lower default rates; something that is referred as an evidence that private companies of smaller size, typically, have worse credit conditions.

Strictly speaking, optimal contracts entail no credit 'rationing' (in 1.iii); although, all firms subscribing a debt contract will underinvest with respect to the first-best (in 2.i).[7] The fact that underinvestment is linked to insufficient internal capital and to cash flow is in line with a consolidated empirical literature, started by Fazzari, Hubbard, and Petersen (1988).[8] More recently, this evidence has been reinforced, for example, by Borisova and Brown (2013, 2015).

The uniqueness of financial decisions in 2.i), unsurprisingly, contrasts with the traditional neoclassical theory of the firm and invalidates Modigliani-Miller theorem. Moreover, the implied firm's financial structure is in line with a modern version of the *pecking order theory*: when a firm experiences an imbalance of internal cash-flow and real investment opportunities, it resorts to external finance raising the debt-to-equity ratio. In other words, rather than having in mind a static optimal debt-equity ratio, everything else equal, our entrepreneurs adjust their financial structure to the initial net-worth position and investment opportunities.[9] In this paper, debt is preferred to equity (*i.e.* to issue a participation to the firm's profits) as debt produces a more powerful incentive scheme, supporting higher

[5]For example, in Chan & Thakor. cit. the mechanism design problem restricts the space of contracts to that of bank loans. Moreover, they assume that agents/borrowers have unlimited collateral that can be used to finance an investment of exogenously fixed amount.

[6]This view is not consistent with some contributions in the same literature, which highlight the adverse selection effects caused by higher collateral values (see, for example, Stiglitz & Weiss, cit.). However, in Stiglitz & Weiss cit. collateral is added to contracts designed with respect to the interest rate alone; thus, again contracts are not defined as a solution of a general mechanism design problem.

[7]Unlike in this paper, it is sometimes the case in this literature that credit-rationing is identified with underinvestment (*e.g.* Gale and Hellwig, 1985).

[8]See also, Hoshi, Kashyap, & Sharfstein (1991), Gertler & Gilchrist (1994).

[9]Although the goal of this paper is not matching empirical facts, the implied financial structure is coherent with (at least) part of the empirical evidence; see, for example, Shyam-Sunder & Myers (1999) and, more recently, Borisova & Brown (2013, 2015). For an opposite view, see for example Frank & Goyal (2003) .

investments and firm's expected profits.[10] Thus, although entrepreneurs would rather finance investments through internal funds, a contract based on a combination of a SDC and a minimum capital requirement can reproduce an efficient equity-based compensation scheme; a scheme that reduces agency costs and boosts firm investments (in 2.ii).

Related literature on optimal security design. To our knowledge, the only paper that has investigated a two-period, security-design problem which has paired moral hazard with asymmetric information on agents' initial capital, is Lewis and Sappington (2000).[11] Yet, their focus and model differ substantially from ours. First, the problem they analyze is one of optimal delegation in which a principal, the owner of a project, sicks to select one or more agents for its implementation; agents' effort and initial wealth are unobservable to the principal. Our problem, instead, is one in which a regulator (principal) designs financial contracts that maximize the surplus generated by entrepreneurs (agents) investments subject to the participation of a financier (principal) conditional to the specified information asymmetries.[12] This distinction is not purely semantic: the introduction of incentive-compatibility constraints makes the underline optimal design problems non-concave; hence, the solution attained from its primal and dual formulations may not just consist in a different sharing rule of social surplus. Second, the model considered in Lewis and Sappington is binomial; a project either success or fail. This restriction, in our perspective, severely limits the possibility to distinguish between a SDC and more complex forms of non-linear financial agreements. Third, the space of contracts we consider encompasses the one in Lewis and Sappington's. We argue later that the type of down-payments they consider (the 'up front bond' payment in their language) is feasible for our designer but not optimal. This is again due to the fact that the possibility of capital confiscation has the effect of increasing the incentive to invest in the firm (*i.e.* impairing moral hazard), instead of just working as a screening (or signaling) device; something that, again, is in line with the literature on banking and credit markets under asymmetric information (see, for example, Chan and Thakor, 1987). Finally, our analysis extends to the case of costly state verification on the outcomes of the agent's activity; something that is more in line with the idea of an 'opacity' on the financial status of the firm.

A few other contributions study two-period models of optimal financial contracts and firm investments in the presence of a two-dimensional moral hazard. In the spirit of Jensen and Meckling (1976), Biasis and Casamatta (1999) and Hellwig (2009) consider the case in which entrepreneurs must exert costly unobservable effort to improve the distribution of cash flows (in the sense of first-order stochastic dominance), and they must abstain to choose projects which are too risky (in the sense of second-order stochastic dominance) to

[10]In more traditional models of pecking order theory (*e.g.*, Myers & Majluf, 1984), equity is the most disfavor mean of outside finance because it produces the worst form of adverse selection.
[11]See also Lewis & Sappington (2001) and the references therein.
[12]Apart from Innes (1990), our formulation is common to many other contributions in the literature on optimal security design (see, for example, problem (2) in Gale & Hellwig, 1985). It is mostly natural under perfect competition on financial markets.

avoid a fall of expected revenues. The prevalence of risk-shifting justifies contracts that have an equity component; while, a debt contract is optimal when costly effort prevails. Povel and Raith (2004), do also consider a contracting problem in which firms revenues depend on an unobservable decision of the entrepreneur, but it focuses on costly revenues verification. This setup is closer to ours, but differs in two relevant aspects. First, unlike ours, it does not consider that agents might have unobservable initial capital; which might affect firm's revenues in a way that goes beyond a scale effect. Second, as for the other two latest contributions, the security design problem follows the state-space approach, and studies optimal contracting by separately analyzing different entrepreneurial choices, concerning the investment scale (a continuous variable) and either effort or project riskiness (binary variables). Our analysis, instead, follows the reduced-form approach and replaces a binary 'effort' choice with an continuous investment choice.[13] The implied decision model is one in which investment, unlike effort, is restricted by a budget constraint and is sensitive to both internal and external financing opportunities. Moreover, as for R&D expenditure, investments affect the distribution of the revenues of an innovation project.[14] In this respect, our model is close, in spirit, to the benchmark of neoclassical growth models of R&D and innovation;[15] a benchmark that seams to fit better certain technologies used for SMEs intensely involved in R&D.

Organization. The paper unfolds as follows. Section 2 presents the model and some preliminary results. Section 3 defines the optimal mechanism-design problem and states our first, main result, theorem 1, referring to the case in which firms' profits are observable and verifiable. The case of costly state verification of profits is analyzed in section 4, where theorem 2 is established. Most proofs are collected in the Appendix.

2. Basic structure and preliminary results

The model is one with a single consumption good, two time periods, date 0 and 1, and uncertainty over the second period. Agents are risk neutral entrepreneurs and intermediaries. Each entrepreneur is the single-owner of a firm and his type is determined by the firm's first period, endowment of capital (net-worth) a in $A := [\underline{a}, \bar{a}]$. Initial net-worth is the consequence of an unrepresented past and may reflect the return of the entrepreneur's past production and financial decisions, as well as personal wealth injected by the entrepreneur into the firm.

[13]The reduced-form approach was pioneered by Mirrlees (1974), Hölmstrom (1979) and followed by Innes (1990), among others.
[14]In this paper as well as in Povel and Raith (2004), the main results are established under the assumption that, respectively, investments and effort are such that distributions satisfy the monotone likelihood ratio property, in the sense of Milgrom (1981). This is know to imply first-order stochastic dominance.
[15]See, for example, the simple representation in Greenwald and Stiglitz (1990), and the more elaborated models of Shumpeterian growth started by Aghion and Howitt (1992, 2005); see also Aghion, Dewatripont, & Rey (1999).

In date 0, all agents are uncertain on the outcome of each firm's production activity, which is represented by the realization of a production project yielding (gross) profits π in $\Pi := [0, \infty)$ in date 1. Although, profit opportunities Π are equal across firms, their likelihood depends on each entrepreneur's investment choice, $x \geq 0$. We, respectively, let $g(\pi|x)$ and $G(\pi|x)$ denote the (conditional) density and the distribution functions of profits when the investment is x. g and G are assumed to be common knowledge.[16]

The representation of the state space Π can also be used to identify specific 'innovation states,' for example, referred to a technological improvement or to an improvement in the firm's organization. We define an *innovation* state π_s, as the threshold-profit such that an increase of x raises the probability of each and every $\pi \geq \pi_s$ and reduces that of all $\pi < \pi_s$. According to this definition, $1 - G(\pi_s|x)$ measures the probability that an entrepreneur, investing x, successfully innovates.[17]

Throughout the paper, the following assumptions will be maintained, unless differently specified.

Assumption 1. *The distributions g and G satisfy the following properties.*

(1) *$g(\pi|x)$ and $G(\pi|x)$ are twice continuously differentiable functions of x.*
(2) *Innovation requires investment, $G(\pi_s|0) = 1$;*
(3) *Entrepreneur's investment increases the likelihood of higher profits; for all x,*

$$\frac{\partial}{\partial \pi}\left(\frac{g_x(\pi|x)}{g(\pi|x)}\right)$$

is positive on Π and strictly positive for almost all (a.a.) $\pi \geq \pi_s$;

Property (3) is a *monotone likelihood ratio condition* (MLR) (Milgrom, 1981) and implies that the class of distributions considered satisfies *first-order stochastic dominance* over firms' outcomes. By (3), investment increase the likelihood of innovation, $G_x(\pi|x) \leq 0$, for all $\pi > 0$ (holding with strict inequality at a.a. $\pi \leq \pi_s$).

An additional, technical assumption is the convexity of the distribution function:

Assumption 2. *For all $(x, z) > 0$, $\int_z^\infty g(\pi|x)d\pi = 1 - G(z|x)$ is strictly convex in x.*

This assumption prescribes a form of stochastic diminishing return to scale in investment, $G_{xx}(z|x) \leq 0$ for all $(x, z) > 0$. It is common in the moral hazard literature, as it makes possible to exploit the 'first-order approach' (see Rogerson, 1985). Conditional distributions

[16] This modeling choice of profits is the reduced-form, due to Mirrlees (1974) (see also Hölmstrom, 1979) and it has proved to give simpler characterizations of the optimum than those found using the state-space approach. The state-space approach would write firm's profit as the outcome of investment/effort x and a random variable $\widetilde{\theta}$, say, $\pi = \widetilde{\theta} f(x)$. Given a distribution of $\widetilde{\theta}$, one can deduce the distribution of π, using the relationship $\pi = \widetilde{\theta} f(x)$. Though, the reverse relationship –from the reduced-form to the state-space approach- is not always true. Necessary and sufficient conditions on the distribution functions to achieve the equivalence of the two approaches have been, recently, established in Poblete & Spulberg (2012).

[17] The identification of one, or more, of such innovation-states is only for expositional reasons and can be removed without altering any of our results.

satisfying the two assumptions and, particularly, MLR and convexity, can be found in the class of bivariate exponential, often used in applications.[18]

Financial contracts and intermediaries. Contracts are exchanged in date 0, before entrepreneurs implement their investment decisions. Outside investors are risk neutral intermediaries who participate to contracts provided they do not expect to make losses. We shall make precise that contracts are written contingent to the information that is verifiable. In our context, this consists in profit realizations, for which verification may occur at a cost, and in the level of the firm's initial capital, when this is 'secured' or certified.

A financial contract l in $\mathcal{L}$ is characterized by a tuple, $(B_l, \boldsymbol{P}_l, \alpha_l, \Pi_l^v)$, where: $B_l \geq 0$ is the amount of funds (or loan size); α_l is the fraction of the up-front payment (or down-payment) of capital required to an agent to enter the contract; $\boldsymbol{P}_l$ is the payment schedule, a function of the firm's verifiable net-worth, depending on the firm's (verifiable) profits π and the down-payment $\alpha_l a_l$; if profits are verifiable at a cost, $\Pi_l^v \subset \Pi$ denotes the verification region.

Up-front payments are secured in a bank deposit, which pays a fixed interest rate R; moreover, unless some form of terminal confiscation is prescribed by the contract, they remain a firm's property. Payments are restricted so that $\boldsymbol{P}_l$ satisfies two properties: *(a)* it is piecewise smooth and right differentiable on Π; *(b)* it satisfies *limited liability*, for all π in Π and l in $\mathcal{L}$,

$$\text{(LL)} \qquad 0 \leq \boldsymbol{P}_l \leq \pi + \alpha_l a_l R$$

The upper bound in (LL) is often motivated by legal provisions, or by the ability of the entrepreneurs (especially in small business) to hide funds and other sources of income. Accordingly, capital confiscation may occur either unconditionally or conditionally to some future states. In the first case, we have contracts such as those prescribing a participation (or franchise) fee; in the second case we have collateralized loans, for which confiscation occurs, for example, in 'bankruptcy states' (states with 'particularly' low profit realizations). The lower-bound, $0 \leq \boldsymbol{P}_l$ establishes that creditors' liability is limited by the loan offered to the firm (*i.e.* in the worse possible scenario, a creditor looses the amount of the loan). This excludes contracts in which the financier buys equity rights from the entrepreneur, which she may eventually waive, transfer back or loose.[19]

Intermediaries raise funds by collecting deposits at the riskless rate $R > 0$ and their participation condition is: for all contracts l in $\mathcal{L}$, and investment $x > 0$,

$$\text{(IP)} \qquad \int \boldsymbol{P}_l g(\pi|x) d\pi - RB_l \geq 0$$

[18]For example, $1 - G(z|x) = a(bcx + 1)\exp[-za(bcx + 1)]$, $(a, b, c) > 0$, $z > 0$, known as Arnold & Strauss (1988) model.
[19]An equity contract of this type and size κ_l imposes a LL on the side of the financier, $-\kappa_l \leq \min_\pi \boldsymbol{P}_l$.

Entrepreneurs. For every contract l, a *project* of an entrepreneur of type a, is characterized by a pair of financial and real investments (a', x) which solves,

$$(\mathcal{F}(l,a)) \qquad \max_{(a',x)\in\mathbb{R}^2_+} \mathbb{E}[V(a',x;\pi,l,a)|x] :=$$

$$= \left\{ \int [\pi - \boldsymbol{P}_l] g(\pi|x) d\pi + \alpha_l a R + R a' : a' + x \leq B_l + (1-\alpha_l) a \right\}$$

We shall denote the project of an entrepreneur a, associated to contract l,

$$a'(l,a), \;\; x(l,a)$$

Implicit in this formulation is that firms cannot monitor other firms and that putting money into a safe deposit is the only feasible financial investment. Also, in line with the finance literature, we interpret the entrepreneur's decision to set $a' = 0$ as one in which he commits to maximal *equity financing* of a risky investment x.

Some additional definitions and preliminary results. We say that a financial contract l in $\mathcal{L}$ is a *standard debt contract* (SDC), in the sense of Gale & Hellwig (1985), if there exists a state z_l in Π such that, for all $\pi > z_l$, some constant payment z_l is required; while, for all states $\pi \leq z_l$, a 'maximum payment' is required to the firm; here, by (LL), the latest is bounded above by the verifiable net-worth (*i.e.* initial down-payment and verifiable realized profits). When $\pi \leq z_l$ and the payment prescribes a full appropriation of the firm's net-worth, we have the equivalent of what are usually called 'bankruptcy states.' Instead, a financial contract l in $\mathcal{L}$ is a *live-or-die* (LDC) in the sense of Innes (1990), if it is like a SDC except that the prescribed payment at all $\pi > z_l$ is equal to zero.

Moreover, we say that a financial contract l in $\mathcal{L}$ is *monotone* (M) if the payment schedule is non-decreasing. Monotonicity has been explained as a way to prevent situations in which profits are strategically 'manipulated'. For example, if payments violate (M) as in a LDC, an agent who can ex-post access to (hidden) borrowing can fictitiously increase profits so as to escape bankruptcy and the consequent loss of collateral; this, of course, is easier to accomplish if profits are unverifiable.[20] We shall argue below that monotonicity is a robust property in theory, in the sense that it is obtained endogenously in a large class of mechanisms (see our section 4).

Lemma 1 (Investment)**.** *Assume that G is a continuous distribution. Then, for every entrepreneur of type a in A, contract l in $\mathcal{L}$ and investment x, such that (IP) holds, a solution to $\mathcal{F}(l,a)$ exists. Moreover, under assumption 2, the solution is unique if contracts are either SDCs or LDCs.*

Proof. The first part is the result of the objective function being continuous and bounded on a compact domain of the choice variables. By (IP) and (LL), $RB_l \leq \mathbb{E}[\boldsymbol{P}_l|x] \leq \mathbb{E}[\pi|x] <$

[20]See, also the discussion in Innes (1990), p.50.

∞.[21] Individual budget constraint implies that $0 \leq a' \leq a + B - x \leq a + B < \infty$. Thus,

$$\mathbb{E}[V(a', x; \pi, t, l)|x] \leq \mathbb{E}[\pi|x] + Ra' < \infty$$

Hence, by continuity, the objective is bounded from above. It is also bounded from below by the return from inaction, Ra. Therefore, existence of a solution follows from Weierstrass Theorem. Uniqueness holds for both SDCs and LDCs under assumption 2 and LL. ■

Under assumption 1(1) and 2 we can characterize a project (a', x) in $(a'(l, a), x(l, a))$, as the pair of continuous functions that satisfy the following condition: whenever $a' = B_l + (1 - \alpha_l)a - x > 0$,

$$\frac{\partial}{\partial x}\mathbb{E}[V(a', x; \pi, l, a)|x)] = 0 \tag{x}$$

Let $\mathbb{E}\left[V(a', x; \pi, R, a)|x)\right]$ denote the expected profits of a typical entrepreneur who can save at the risk-free rate R. As we argue later, this contract is the optimal one in the absence of information asymmetries, hence it is hereafter addressed as the *first-best* contract (labeled as, $l = R$). The following lemma defines the first-best level of investment.

Lemma 2 (Investment efficiency). *Under assumption 1(1) and 2, for all a in A, there exists a unique efficient (first-best) investment* x^o*,*

$$\frac{\partial}{\partial x}\mathbb{E}\left[V(a', x; \pi, R, a)|x)\right]_{x=x^o} = 0$$

Finally, to focus on situations in which outside financing is demanded by a non-negligible portion of entrepreneurs, we assume that a 'large' fraction of them would not be able to implement their projects without borrowing (*i.e.* choosing to subscribe the null contract $l = 0$). This is summarized in the following assumption.

Assumption 3. *There is an open interval* A' *of* A *such that, for all* a *in* A'*, at a solution* (a', x) *to* $\mathcal{F}(0, a)$*,*

$$\frac{\partial}{\partial x}\mathbb{E}\left[V(a', x; \pi, l, a)|x)\right]_{l=0} > 0$$

Throughout the rest of the paper we shall redefine the type space A as equal to A'.

3. Optimal financial contracts when profit realizations are observable

Suppose that profit realizations are observable and verifiable; thus, we simplify $\mathcal{L}$ by dropping the verification region Π^v. We define and characterize optimal contracts and analyze entrepreneurial policy decisions, assuming that: *i)* before contracting, each agent type sends a message concerning his type a and project (a', x); *ii)* once a contract l is signed, each agent chooses the project to implement; *iii)* profit realizations are publicly observed after projects have been finalized.

Contracts and entrepreneurial policies are defined based on a revelation mechanism, which establishes a sharing-rule over the firm's net-worth.

[21] Hereafter, expectations are all computed with respect to the densities g.

Definition 1 (Mechanism $\mathcal{M}$)**.** *A **mechanism** in $\mathcal{M}$ is:*

i) *a set of messages $M \subset A \times \mathbb{R}^2_+$, of typical element $m = (a, a', x)$, that each entrepreneur can send to the principal,*

ii) *a profile of outcome functions $M \to \mathcal{L} \times \mathbb{R}^2_+$ whose values identify a feasible contract $l(m)$ in $\mathcal{L}$ and an actual implementation of the the project, $(a'(m), x(m))$; where a contract $l(m)$ in $\mathcal{L}$ identifies a triplet $(B_{l(m)}, \boldsymbol{P}_{l(m)}, \alpha_{l(m)})$ defined above.*

*$\mathcal{M}$ is a **direct-mechanism** if for each type $\widetilde{a}$ in A, the message sent is truthful, $m \equiv (\widetilde{a}, \widetilde{a}', \widetilde{x})$, $l(m) \equiv \widetilde{a}$ and the announced project is implemented, $(a'(m), x(m)) \equiv (a'(\widetilde{a}), x(\widetilde{a}))$.*

A solution to our security design problem is the outcome of an optimal, direct mechanism, defined as follows.

Definition 2 (Optimal mechanism)**.** *An **optimal mechanism** is a direct-mechanism in $\mathcal{M}$ such that, for every entrepreneur of type a, each value of the outcome functions (l, a', x) solves problem $\mathcal{P}$:*

$$\max_{\{\bar{l}, (\bar{a}', \bar{x})\} \in (\mathcal{L}, \mathbb{R}^2_+)} \mathbb{E}[V(\bar{a}', \bar{x}; \pi, \bar{l}, a)|x] \quad s.t.$$

$$\text{(CC)} \qquad (a', x) \in \arg\max_{\{\bar{a}', \bar{x}\}} \mathbb{E}[V(\bar{a}', \bar{x}; \pi, a, \bar{l})|\bar{x}]$$

$$\text{(IC)} \qquad l \in \arg\max_{\{\bar{l}\}} \mathbb{E}[V(\bar{a}', \bar{x}; \pi, a, \bar{l})|\bar{x}]$$

$$\text{(IP)} \qquad \mathbb{E}[\boldsymbol{P}_l|x] - RB_l \geq 0$$

(CC) and (IC), respectively, denote a *commitment* and an *incentive-compatibility* constraint. The first ensures that the agent does not deviate from the announced project (a', x); the second, that the agent spontaneously subscribes the contract designed for his type.

Observe that, as an outcome of the optimal mechanism, we determine both the *optimal menu of contracts* offered to all entrepreneurs and the corresponding *optimal entrepreneurial policy.*[22]

Theorem 1. *Let assumptions 1 through 3 hold. The menu of optimal contracts in $\mathcal{L}$ and optimal entrepreneurial policies is the outcome of a direct mechanism in $\mathcal{M}$ such that, for all a in A,*

(1) *entrepreneur a subscribes a financial contract l ($\equiv a$) that takes one of the following two forms, depending on whether or not monotonicity of the payment schedule is assumed:*

[22] Restricting to direct mechanism is without loss of generality and follows from a standard application of the *Revelation Principle.*

(i) *If monotonicity (M) is assumed, it takes the form of a SDC with payment schedule,*

$$(P^*) \qquad P_a(\pi) = \begin{cases} z_a, & \text{for all } \pi > z_a - aR, \\ \pi + aR, & \text{otherwise,} \end{cases}$$

for some $z_a > aR$, it requires maximal capital participation of the agent ($\alpha_a a = a$) if (CC) binds and no need of capital participation otherwise; or,

(ii) *If (M) is not assumed, contracts have the form of LDCs with 'full collateral' if (CC) binds and no need of a collateral otherwise;*

(2) *the contract brakes even (*i.e. *(IP) holds with equality);*

(3) *entrepreneur a chooses an inefficient investment project, $0 < x(a) < x^o$, whenever (CC) binds;*

(4) *entrepreneur a does not resort to hidden saving ($a'(l, a) = 0$).*

(5) *the optimal menu of contracts is characterized by an investment schedule $x(a)$ and an interest rate schedule z_a/B_a which are respectively monotonically increasing and decreasing for almost all a in $\{A : x(a) < x^o\}$ and are constant otherwise.*

For a clearer understanding of the effects produced by different sources of information asymmetries it is useful to, first, deal with the case in which types are common knowledge (*i.e.* the case of 'pure moral hazard') and then to introduce asymmetric information on entrepreneurs' types in A. This will be, respectively, done in the next two subsections in which proposition 1 and 2 will be established. The proof of theorem 1 follows directly from these propositions.

3.1. **Pure moral hazard.** When a type a is verifiable at no cost, before contracting, the message space M is reduced to projects (a', x) and $\mathcal{P}$ is redefined by dropping (IC). For notational simplicity, in the rest of this section, we shall omit indices of contracts and agent types, (l, a).

Proposition 1. *The menu of optimal contracts in $\mathcal{L}$ and optimal entrepreneurial policies is such that, for each entrepreneur a, properties (1)-(5) in the theorem hold.*

Proof of proposition 1: see the Appendix

The case of pure moral hazard is the closest to Innes (1990). As in that article, the mechanism is linear in the payment function, something that explains why a solution takes either the form of a SDC or of a LDC, depending on whether or not one assumes monotonicity (M). What is new with respect to Innes' is the presence of initial capital and the fact that this affects the demand of outside funding required to implement the firm's project (*i.e.* Innes' 'effort' is subject to a budget constraint). The presence of initial capital allows to weaken the limited-liability constraint on the side of the firm and, correspondingly, to increase the punishment scheme in bankruptcy states. In other words, similarly to collateral, the firm's

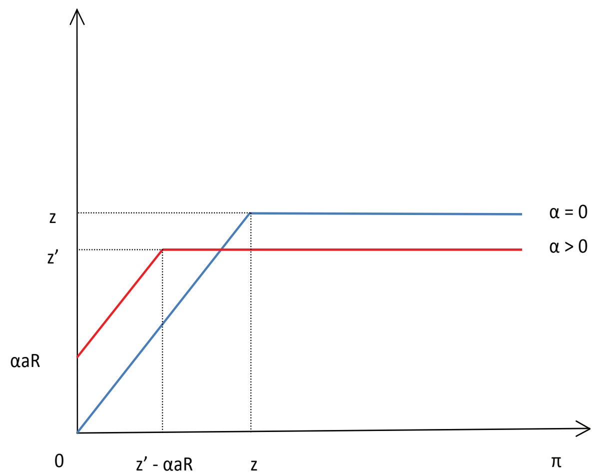

FIGURE 1. Contracts with and without down-payment

capitalization reduces moral hazard. What we actually establish in the proposition is that, thanks to capital, the optimal contract can now be offered at a lower interest rate; this is clearly illustrated in figure 1, where z decreases with α. In turn, a decrease in the interest rate boosts expected profits and real investments. We show that the latest effects are such that the debt size can be adjusted to sustain investments, without violating intermediaries' participation. This reasoning can be used to argue that there is a unique financial structure of the firm that is a mix of capital and debt, with (secured) capital being equal to the maximum wealth available to the entrepreneur in date 0.

A first implication of the introduction of a down-payment is on the definition of the 'bankruptcy' region. In Innes cit. and in the rest of the literature, this is identified with those states $\pi \leq z$, which do not allow the firm to payback their debt through cash flows. Here, because of the presence of a down-payment, there is an intermediate region $(z' - aR, z]$, of 'liquidity' shortage, in which the contract keeps prescribing a fixed payment z that the firm pays integrating its insufficient cash flow with a partial liquidation of secured assets (its contractual down-payment). Only for 'extremely low' states, in $[0, z' - aR]$, the fim goes 'bankrupt' and suffers a total seizure of secured capital.

An additional implication of the use of capital is that, along the menu of optimal contracts, defined with respect to a in A, the interest rate z/B is monotonically decreasing and real investment, x, is monotonically increasing up to the first-best scale x^o. In other words, by increasing x, the entrepreneur escapes liquidity problems and bankruptcy, states in which the firm looses its profits and some of its capital.

The effects of capital we have just highlighted can also be used to explain why, at an optimal contract, hidden savings a' is zero; that is, the entrepreneur chooses full internal-equity participation. Reasoning by contradiction, we argue in the proof that, if at an optimal SDC, (B, z, α), the entrepreneur had to choose $x > 0$ and $a' > 0$, there would be an alternative contract with a higher capital requirement and a lower cost $\widetilde{z} < z$ such that, if subscribed by the agent, it would finance a project characterized by a higher real investment $\widetilde{x} > x$ and a lower saving $\widetilde{a}' < a$; something that would produce a contradiction.

3.2. **Private information on firms' initial capital.** So far, we have considered the case in which the regulator/principal observes entrepreneurs' initial capital (*i.e.* their types). In this section, we relax this assumption and prove that the optimal mechanism delivers the same menu of contracts, in $\mathcal{L}$; that is, $\mathcal{L}^*$ is a menu of incentive-compatible contracts (*i.e.* contracts satisfying (IC) in $\mathcal{M}$). This menu induces all entrepreneurs to truthfully reveal their types and subscribe the contracts which are designed to support their optimal projects. For expositional reasons, we assume that financial contracts are exclusive; in the sense that every entrepreneur can subscribe a single financial contract. Later, in remark 3.1, we explain why –at the optimal menu- entrepreneurs do actually prefer to subscribe a single contract than to shop for a portfolio of different contracts.

Consider contracts that require the entrepreneur who sends a message a to dispose a down-payment of a. Clearly, under full down-payment, an entrepreneur of type $\widetilde{a}$ can only send messages with $a \leq \widetilde{a}$, which rules out the possibility that he can access to (hence deviate to) SDCs designed for higher capitalized firms. Then, the following is true.

Proposition 2. *Under private information on types in A, a.a. contracts l in $\mathcal{L}^*$ are incentive-compatible.*

Proof. Incetive-compatibility prescribes that every agent $\widetilde{a}$ in A prefers to follow the behavior prescribed by the mechanism $\widetilde{a} \in \mathcal{L}^*$ with an expected payoff $\nu(\widehat{a}) \equiv \nu(\widetilde{a}, \widetilde{a})$, instead of taking an alternative mechanism $\ell \neq \widetilde{a} \in \mathcal{L}^*$ and use it to implement an individually optimal project $(a'(\ell, \widetilde{a}), x(\ell, \widetilde{a}))$.[23] Since for a type $\widetilde{a}$ the only feasible messages are those with $a \leq \widetilde{a}$, consider these deviations.

First, we show that for all $\ell \leq \widetilde{a}$, $x(\ell, \widetilde{a}) = x(\ell)$. Indeed, $x(\ell)$ is feasible for type $\widetilde{a} > \ell$, who can claims to have a firm with capital $a_l < \widetilde{a}$ and implement $x(\ell) = B_\ell < B_\ell + \widetilde{a} - a_\ell$. This implies that $x(\ell, \widetilde{a}) \geq x(\ell)$. Next, because types have the same objectives and type ℓ is not rationed at the optimal contract $\ell \in \mathcal{L}^*$, $x(\ell)$ is also individually optimal for $\widetilde{a}$, implying that $a'(\ell, \widetilde{a}) = \widetilde{a} - a_\ell$; namely, $x(\ell, \widetilde{a}) \leq x(\ell)$.

Second, recall that for a.a. ℓ in $\mathcal{L}^*$, $a'(\ell) = 0$, $x(\ell) = B_\ell$, and (IP) holds with equality on $\mathcal{L}^*$.

[23] Later, we use the notation $(a'(\ell), x(\ell)) := (a'(\ell, \ell), x(\ell, \ell))$.

Next, we are going to exploit these two last properties of $\mathcal{L}^*$ to verify that such menu of contracts is incentive-compatible. For all $a \leq \widetilde{a}$ and entrepreneur $\widetilde{a}$,

$$\begin{aligned}
\nu(\widetilde{a},\widetilde{a}) - \nu(a,\widetilde{a}) &= \int [\pi - \boldsymbol{P}_{\widetilde{a}}] g(\pi|x)(\widetilde{a}) d\pi + R\widetilde{a} - \int [\pi - \boldsymbol{P}_a] g(\pi|x)(a,\widetilde{a}) d\pi - R[a + a'(a,\widetilde{a})] \\
&= \int [\pi - \boldsymbol{P}_{\widetilde{a}}] g(\pi|x)(\widetilde{a}) d\pi + R\widetilde{a} - \int [\pi - \boldsymbol{P}_a] g(\pi|x)(a) d\pi - R[a + \widetilde{a} - a] \\
&= \int \pi [g(\pi|x)(\widetilde{a}) - g(\pi|x)(a)] d\pi - RB_{\widetilde{a}} + RB_a \\
&= \int \pi [g(\pi|x)(\widetilde{a}) - g(\pi|x)(a)] d\pi - R[x(\widetilde{a}) - x(a)]
\end{aligned}$$

This, by the Fundamental Theorem of Calculus, for all $a \leq \widetilde{a}$, can be written as,

$$\int_a^{\widetilde{a}} \left[\left(\int \pi g_x(\pi|x)(\ell) d\pi - R \right) \frac{dx}{da}(\ell) \right] d\ell$$

For the latest to be positive, it suffices that for all $a \leq \ell \leq \widetilde{a}$, the argument in the square brackets is positive. Indeed, this is true since, first, by (CC'),

$$\int \pi g_x(\pi|x)(\ell) d\pi - R = \int \boldsymbol{P}_\ell(\pi) g_x(\pi|x)(\ell) d\pi \geq 0$$

with strict inequality if $x(\ell) < x^o$; second, by proposition 1 (and particularly, lemma 7 in the appendix), for all $\ell \leq \bar{a}$, $(dx/da)_\ell \geq 0$, holding with strict inequality for a.a. contracts ℓ for which $x(\ell) < x^o$ (occurring for those a with a binding (CC)). Therefore, $\nu(\widetilde{a},\widetilde{a}) - \nu(a,\widetilde{a}) > 0$ if $x(\ell) < x^o$ and is zero otherwise. ■

It is interesting to remark that the introduction of private information on the initial firms' capital does not alter the original incentive scheme provided by the optimal mechanism in the pure moral hazard case (proposition 1). Thus, contracts retain the original properties and deliver an interest rate schedule, $(z_a/B_a)_{a\in\mathcal{L}^*}$, that is decreasing in the initial firms' capital a. This, again, proves that higher capitalized firms have better financial opportunities. Capital participation is used here, both as an optimal incentive devise (preventing moral hazard) and as an efficient screening (or signaling) devise.[24] The capital down payment increases incentives to invest, by relaxing the limited liability, and preserves incentive-compatibility; something that is achieved by eliminating the possibility that agents lie on their type, pretend to have higher capital, and access more favorable contracts.

Remark 3.1 (Exclusivity)**.** *In the context of our proposition 2, the optimal menu of contracts is such that no entrepreneur has an incentive to subscribe multiple contracts. This follows immediately from the fact that the interest rate is decreasing in the loan size and in the capital down-payment. Thus, an entrepreneur with capital a, who needs B to finance*

[24] The optimal mechanism could be equivalently implemented as the outcome of a game in which either the firms decide on the initial capital contribution or the intermediaries compete in offering contracts that contain an initial capital requirement.

a project, can obtain higher expected profits by choosing a single contract with full down-payment a, than by a portfolio of loans (each of which is designed for a lower capitalized firm) of total size $\bar{B} \geq B$ and total down-payment $\bar{a} \leq a$.

4. Optimal financial contracts with costly state verification

We now consider the case in which firms' informational 'opacity' also concerns profits. Assume that monitoring profits is feasible, albeit costly. In the spirit of Townsend (1979), we assume that in date-1 states π are *verifiable* by the firm, paying an 'auditing cost.' We assume that, for all contracts l in $\mathcal{L}$, this cost is a constant fraction λ of the payment due P_l. We redefine contracts by adding to $(B, P, \alpha)_l$ a *verification region* Π_l^v in Π. The verification region is part of the contract; determined in date-0, it indicates the states in which a firm is supposed to pay to be audit.

Consequently, the definition of the mechanism is modified in two elements. First, the message space includes date-1 messages on whether or not the firm is in a verification state (according to the contract). Second, the definition of the outcome function is adapted such that *feasibility* also requires a *consistence* (or commitment) condition, which ensures that date-1 messages are truthful: the entrepreneur, upon observing any π in Π_l^v, will proceed to the firm auditing; and will otherwise not audit, for states π in $\Pi_l^o = \Pi \setminus \Pi_l^v$.

As in Townsend (1979) (see his lemma 2.1), for some constant $c_l > 0$, consistence is achieved if and only if payment schedules satisfy the following two restrictions; namely:

(c) (i) for all π in $\Pi_l^o \equiv \Pi \setminus \Pi_l^v$, $P_l(\pi) = c_l$;

(ii) for all π in Π_l^v, $P_l(\pi)(1+\lambda) \leq c_l$.

These conditions are intuitively clear. If the agent is in a non-verification state he can always claim the lowest possible level of profit in Π_l^o, hence this level will actually pin down the consistent interest rate in this region. In contrast, if π is in the verification region Π_l^v, he will actually proceed to pay the verification cost if this will lead to a payment cost $(1+\lambda)P_l(\pi)$ that does not exceed c_l, the total payment due under no verification.

For completeness, we provide a formal definition of the mechanism.

Definition 3 (Mechanism $\mathcal{M}'$)**.** *A **mechanism** in $\mathcal{M}'$ is:*

- ***i)*** *a set of date-0 messages $M_0 \subset A \times \mathbb{R}_+^2$, of typical element $m_0 = (a, a', x)$, and date-1 messages m_1 in $M_1 \subset \Pi$, which an entrepreneur can send to the principal, $M \equiv M_0 \times M_1$, $m \equiv (m_0, m_1) \in M$;*
- ***ii)*** *a profile of outcome functions $M \to \mathcal{L} \times \mathbb{R}_+^2$ whose values identify a feasible contract $l(m)$ in $\mathcal{L}$ and an actual implementation of the the project, $(a'(m), x(m))$; where a contract $l(m)$ in $\mathcal{L}$ identifies a tuple $(B_{l(m)}, \boldsymbol{P}_{l(m)}, \alpha_{l(m)}, c_{l(m)})$ and a verification region $\Pi_{l(m)}^v$.*

A solution to our security design problem is the outcome of an optimal, direct mechanism:

Definition 4 (Optimal mechanism)**.** *For all* $0 \leq \lambda < 1$*, an* ***optimal mechanism*** *is a direct-mechanism in* $\mathcal{M}'$ *such that, for every entrepreneur of type a in A, each value of the outcome functions solves problem* $\mathcal{P}'$*:* $(a', x, B, \boldsymbol{P}, B, \alpha, c)$ *and the region* Π^o *(*$\Pi^v \equiv \Pi - \Pi^o$*), maximize,*

$$\int_{\Pi^o} [\pi - c] g(\pi|x) d\pi \int_{\Pi^v} [\pi - P(\pi)(1+\lambda)] g(\pi|x) d\pi + Ra'$$

subject to,

$$0 \leq a' = B - x + (1-\alpha)a, \quad 0 \leq \alpha \leq 1, \quad 0 \leq B,$$

$$\text{(CC')} \qquad \int_{\Pi^o} [\pi - c] g_x(\pi|x) d\pi + \int_{\Pi^v} [\pi - P(\pi)(1+\lambda)] g_x(\pi|x) d\pi - R = 0$$

$$\text{(IP')} \qquad c \int_{\Pi^o} g(\pi|x) d\pi + \int_{\Pi^v} P(\pi) g(\pi|x) d\pi - RB \geq 0$$

$$\text{(c)} \qquad P_l(\pi) = c_l, \quad \textit{for all } \pi \textit{ in } \Pi_l^o \equiv \Pi \setminus \Pi_l^v$$

$$P_l(\pi)(1+\lambda) \leq c_l, \quad \textit{otherwise}$$

$$(LL_v) \qquad 0 \leq (1+\lambda)P(\pi) \leq \pi + \alpha a R, \quad \textit{for all } \pi \textit{ in } \Pi^v$$

$$(LL_o) \qquad 0 \leq P(\pi) \leq \pi + \alpha a R, \quad \textit{otherwise.}$$

Under costly state verification the outcome of a direct, optimal menu is characterized in the following theorem.

Theorem 2. *Let assumptions 1 through 3 hold. In the presence of costly state verification on date-1 profits, the menu of optimal contracts takes the form of a SDC with a payment schedule,*

$$P_a(\pi) = \begin{cases} z_a, & \textit{for all } \pi \in \Pi_a^o; \\ \frac{z_a}{1+\lambda}, & \textit{for all } \pi \in \Pi_a^{vH} \equiv \{\pi' \in \Pi_a^v : \pi' \in [z_a - aR, z_a]\}; \\ \frac{\pi + aR}{1+\lambda}, & \textit{for all } \pi \in \Pi_a^v \setminus \Pi_a^{vH}. \end{cases}$$

where, $\Pi_a^v = [0, z_a]$*,* $\Pi_a^o = \Pi \setminus \Pi_a^v$*. All the other properties of the contracts are identical to those numbered* (2) *through* (5) *in theorem 1.*

Proof of theorem 2: see the Appendix

Observe that the payment schedule, and the whole result in this theorem coincides with theorem 1, if we let the verification cost λ go to zero and we introduce monotonicity of the payment schedule (M). This can be easily explained observing the similarity of the constraint (M) with the consistence requirement (c). Indeed, consistence imposes a 'monotonicity' of the payment schedule, $(1+\lambda)P_a(\pi) \leq P_a(z_a)$, for all states π in Π_a^v. A monotonicity restriction is not implied by (c) in the verification region, in which the asymmetric information on realized profits vanishes. Yet, optimality prescribes the maximal feasible punishment in such region, yielding monotonicity of the optimal payment schedule in Π_a^v too. Therefore, after taking into account the verification costs, so as to avoid a failure of consistence/monotonicity, the optimal payment schedule in the two theorems are the same:

defining P_λ as the payment inclusive of the auditing cost, when payed, one obtains a graph of the payment schedule identical to the one in figure 1 above.

Next observe that, since theorem 2 holds for all $\lambda > 0$, we can use this result to claim that SDCs with a monotone payment schedule are the only robust, optimal menu one should really focus on and that, even theoretically, LDCs can be thought as atypical.

5. Conclusions

This article studies a security design problem in which outside investors are unable to observe entrepreneurs' investments decisions and their firms' net-worth, both before and after contracts are signed. The interconnectedness of these three different forms of asymmetric information implies as many risks in the design problem: a possible adverse selection on entrepreneurs' types (initial capital) and moral hazards, both on investments and on the release of final information regarding firm's income/profits. Since investments are unobservable and the exact consistency of firms' net-worth is unverifiable, contracts can only be written contingently on future verified profits and on an initial down-payment, in the form of secured capital.

Our approach to the problem builds on the classical, reduced-form one used in the moral hazard literature. Investment decisions, similarly to effort, affect the firms success probability, increasing the likelihood of higher returns from production. However, differently from effort and similarly to R&D expenditure, investment size is budget constraint; it depends on both the amount of the entrepreneur's initial capital (past firm's cash flow) and the firm's access to outside funds.

The results we present establish that outside finance should take the form of a debt contract, whose terms are: a firm's capital requirement, a fund size, an interest rate, a verification rule. Optimal contracts form a menu. Contracts with higher capital prescriptions offer more funds at a lower interest rate and have a smaller verification region. State verification occurs when profit realizations are 'low' and it is accompanied by a gradual confiscation of capital; a seizure of the entire capital occurs only when profits drop below a 'bankruptcy' threshold. At optimal contracts, firms privilege internal to external funds. In particular, only entrepreneurs who have insufficient capital decide to borrow. These entrepreneurs are exactly those who underinvest in their firm. Their credit conditions are tighter the lower is their size (capital), the higher is their desired leverage. These properties of optimal contracts are robust to the introduction of private information on entrepreneurs initial net-worth. The presence of down-payments grants incentive-compatibility, by preventing agents to lie on their capital and subscribe contracts designed for higher capitalized firms. Finally, our findings hold regardless contracts are assumed to be exclusive. The fact that the interest rate schedule is decreasing in the down-payment, represents a disincentive to contracting with multiple intermediaries.

We have motivated our interest in this security-design problem also referring to the 'opacity' that typically characterizes the financial situation and decisions of small firms, representing a particularly large fraction of the non-financial sector, in most developed countries. These firms heavily rely on bank credit. They raise outside funds in the form of standard credit lines, provided by a single or very few banks. The interest rate conditions on credit lines are typically decreasing in the loan size, which is highly positively correlated with firms' dimension (in general, capital); that is, highly capitalized firms are less leveraged and borrow at a lower interest rate. The strong dependency of SMEs to the banking sector and to traditional forms of credit lines, has often be considered a major cause of inefficiency and underinvestment. Instead, to some extent, our analysis, along with other results in the literature on optimal financial contracts, question this conclusion. A proper combination of standard debt contracts and collateral or minimum capital requirement may constitute an efficient (in sense of *second-best*) corporate finance, particularly, in a context in which R&D type of investments are the engine of high productivity and production revenues. It is so, in the presence of this predominant form of moral hazard, essentially because debt-contracts provide entrepreneurs with a reward scheme that is equivalent to the best equity-base one, compatible with incentives. This is confirmed also in the presence of adverse selection, due to the positive effect of down-payments in separating firms types, by their different levels of initial capital. Finally, it extends to situations in which the outcome of firm activities is verifiable only at a cost. In particular, this latter aspect highlights the fact that, even if the weak information requirements adopted by regulators are undisputed, a minimum dose of auditing may have two positive effects. First, it can efficiently prevent firms from hiding profits from their (tax and non-tax) creditors. Second, it may provide an incentive to strengthen firms' capital; something that it has often proved to be essential to survive negative shocks, both of cyclical and idiosyncratic nature.

Appendix

Proof of Proposition 1. We denote by $\mathcal{L}^*$ the menu of optimal contracts.

To prove this result, consider the Lagrangian of problem $\mathcal{P}$, in which (M) is dropped, budget-balance is used to eliminate a' and (CC) is substituted with,

$$\text{(CC')} \qquad \frac{\partial}{\partial x}\mathbb{E}[V|x] \equiv \int [\pi - \boldsymbol{P}] g_x(\pi|x) d\pi - R \geq 0$$

Also, restrict to no hidden-borrowing, assume $a' \geq 0$, and denote the candidate multipliers by $(\psi, \gamma, \mu, \eta, \theta, \xi, \zeta)$,

$$\max_{x,P,B,\alpha} \int [\pi - \boldsymbol{P}] g(\pi|x) d\pi + \alpha a R + (1+\psi) R\left[(1-\alpha)a + B - x\right] + \gamma \left[\int \boldsymbol{P} g(\pi|x) d\pi - RB\right] +$$
$$+ \mu \left[\int [\pi - \boldsymbol{P}] g_x(\pi|x) d\pi - R\right] + \int \eta(\pi)[\pi + \alpha a R - \boldsymbol{P}] g(\pi|x) d\pi +$$
$$+ \int \theta(\pi) \boldsymbol{P} g(\pi|x) d\pi + \xi B R + \zeta_0 \alpha a R + \zeta_1 (1-\alpha) a R$$

Necessary conditions for optimality are:

$$
\begin{aligned}
\text{(f1)} \qquad & x: \ \frac{\partial}{\partial x}\mathbb{E}[V|x] + \gamma \int \boldsymbol{P} g_x(\pi|x) d\pi + \mu \frac{\partial^2}{\partial x^2}\mathbb{E}[V|x] - \psi R + \\
& + \int \eta(\pi)[\pi + \alpha a R - \boldsymbol{P}] g_x(\pi|x) d\pi + \int \theta(\pi) \boldsymbol{P} g_x(\pi|x) d\pi = 0, \\
& \psi: \ \psi \geq 0, \ \psi\left[(1-\alpha)a + B - x\right] = 0
\end{aligned}
$$

$$
\begin{aligned}
\text{(f2)} \qquad & \boldsymbol{P}: \ g(\pi|x)\left\{-1 + \gamma - \mu \frac{g_x(\pi|x)}{g(\pi|x)} - [\eta(\pi) - \theta(\pi)]\right\} = 0, \\
& \eta, \theta: \ \eta(\pi) \geq 0, \ \eta(\pi)[\pi + \alpha a R - P(\pi)] = 0, \quad \theta(\pi) \geq 0, \ \theta(\pi) P(\pi) = 0, \quad \textit{for all } \pi
\end{aligned}
$$

$$
\text{(f3)} \quad B, \xi: \ 1 + \psi - \gamma + \xi = 0, \ \xi \geq 0, \ \xi B = 0
$$

$$
\text{(f4)} \quad \alpha, \zeta: \ 1 - (1+\psi) + \int \eta(\pi) g(\pi|x) d\pi = \zeta_1 - \zeta_0, \ \zeta_0, \zeta_1 \geq 0, \ \zeta_0 \alpha = 0, \ \zeta_1(1-\alpha) = 0.
$$

Like in Innes (1990), risk-neutrality implies that the problem is linear in P. Let $\varphi(\pi, x, \cdot) := -1 + \gamma - \mu \frac{g_x(\pi|x)}{g(\pi|x)}$; then, for any given α, the optimal payment schedule satisfies the following,

$$
\begin{aligned}
\varphi(\pi, x, \cdot) = -\theta(\pi) < 0 \quad & \textit{only if} \quad P_\lambda = 0, \quad \eta(\pi) = 0, \\
\varphi(\pi, x, \cdot) = 0 \quad & \textit{only if} \quad P_\lambda \in [0, \pi + \alpha a R\}], \quad \eta(\pi) = 0 = \theta(\pi) \\
\varphi(\pi, x, \cdot) = \eta(\pi) > 0 \quad & \textit{only if} \quad P_\lambda = \pi + \alpha a R, \quad \theta(\pi) = 0
\end{aligned}
$$

The proof of proposition 1 follows from the application of lemma 3 trough 8 below.

Lemma 3. *In the context of theorem 1, for any non-trivial contract in $\mathcal{L}^*$ with a non-binding commitment constraint (CC'), properties (1)-(3) in the theorem hold, with the investment level equal to x^o. $a' \geq 0$ is non-binding.*

Proof. We aim at showing that, at a solution, $\mu = 0$ implies $\gamma > 0$ (precisely, $\gamma = 1$) and $\psi = 0$. First, since a first-best entails a non-contingent payment R, we claim that, w.l.o.g., $B = x - a > 0$ and $\alpha = 0$. This, by complementary slackness, implies $\xi = 0$. $(f3)$ implies $\gamma \geq 1 + \psi \geq 1$. Next, we show that $\gamma \leq 1$. By contradiction, suppose $\gamma > 1$, then $P(\pi) = \pi + \alpha a R$ binding at all π occurring with positive probability; otherwise, $\eta(\pi) = 0$ would imply $\varphi(\pi, x, \cdot) = -1 + \gamma + \theta(\pi) > 0$, contradicting $(f2)$. However, $P(\pi) = \pi + \alpha a R$ at all π occurring with positive probability violates (CC'). Indeed, if it were true, we would have $\frac{\partial}{\partial x}\mathbb{E}[V|x] = -R < 0$. Therefore, we conclude that $\mu = 0$ implies $\gamma = 1$.
By complementary slackness, $\gamma = 1 > 0$ implies that (IP) holds with equality.
$\psi = \mu = 0$ and $\gamma = 1$ in $(f1)$ imply that,

$$
0 = \frac{\partial}{\partial x}\mathbb{E}[V|x] + \int \boldsymbol{P} g_x(\pi|x) d\pi = \int \pi g_x(\pi|x) d\pi - R
$$

where the rhs is zero at $x = x^o$. Then, (CC) holds by Assumption 3.
Clearly, in this case, there is some indeterminacy of the mechanism: a mechanism with

payment R, size $B = x^o - a$, $\alpha = 0$ and $a' = 0$ is equivalent to one with -same uncontingent payment- loan size $B' = a' + B + \alpha a$, $0 < \alpha \leq 1$ and $a' \neq 0$; where it should be noted that, at a solution, any value of α is interior: $\gamma = 1$ and $\xi = 0$, by $(f3)$, imply $\psi = 0$, and $(f4)$ yields $\zeta_1 = \zeta_0 = 0$.

■

Lemma 4. *In the context of theorem 1, for any non-trivial contract in $\mathcal{L}^*$ with a binding commitment constraint (CC'), properties (1)-(3) in the theorem hold and $a' \geq 0$ is non-binding.*

Proof. We proceed in steps.

Step 1: A solution satisfies (IP) with equality.
$(f3)$ implies $\gamma = 1 + \psi > 0$. By complementary slackness (IP) binds.

Step 2: A solution satisfies (CC') with equality.
From $\mu > 0$ and complementary slackness, $0 = \frac{\partial}{\partial x}\mathbb{E}[V|x]$.

Step 3: A solution is a LDC.
$\mu > 0$ and MLR imply that $\varphi(\pi, x, \cdot)$ is decreasing in π, strictly decreasing for almost all $\pi \geq \pi_s$. Moreover $\varphi(\pi, x, \cdot) \geq 0$ at some π occurring with positive probability, otherwise $P = 0$ would be chosen violating (IP). Moreover, we know that, generically, a solution satisfies $\varphi(\pi, x, \cdot) > 0$ at some π. Also, (CC') implies that we cannot have $P = \pi + \alpha a R$ at all π and thus, at a solution, it must be that $\varphi(\pi, x, \cdot) \leq 0$ at some π. By continuity and the Intermediate Value Thm., we conclude that there exists a $z \geq \pi_s > 0$ such that $\varphi(\pi, x, \cdot) = 0$. The fact that $z \geq \pi_s$ follows from Step 1, $\gamma \geq 1$. Moreover, z is unique by strict monotonicity of g_x/g on $[\pi_s, \infty)$. The optimal payment schedule is typical of a LDC, in the sense of Innes (1990),

$$\boldsymbol{P} = \begin{cases} 0, & \pi > z \\ \pi + \alpha a R & otherwise \end{cases}$$

Step 4: We now show that, because at a solution $\int \boldsymbol{P} g_x(\pi|x)d\pi > 0$, the solution is inefficient, $0 < x < x^o$, and satisfies (CC) with a non-binding constraint $a' \geq 0$.
First, because (CC') binds ($\mu > 0$), $0 = \frac{\partial}{\partial x}\mathbb{E}V[\pi|x]$ is a necessary optimality condition for the agent at a non-binding constraint $a' \geq 0$; implying $\psi = 0$. Moreover,

$$(o) \qquad \begin{aligned} 0 &= \frac{\partial}{\partial x}\mathbb{E}[V|x] \\ &= \frac{\partial}{\partial x}\left[\int [\pi - \boldsymbol{P}]g(\pi|x)d\pi] + \alpha a R + R(B + a(1-\alpha) - x)\right] \\ &= \frac{\partial}{\partial x}\int [\pi - Rx]g(\pi|x)d\pi - \int \boldsymbol{P} g_x(\pi|x)d\pi \end{aligned}$$

Using, $0 = \frac{\partial}{\partial x}\mathbb{E}[V|x]$, $\gamma = 1, \psi = 0$, $(f1)$ reads,

$$0 < -\mu\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x] = \int \boldsymbol{P} g_x(\pi|x)d\pi + \int \eta(\pi)[\pi + \alpha aR - \boldsymbol{P}]g_x(\pi|x)d\pi + \int \theta(\pi)\boldsymbol{P} g_x(\pi|x)d\pi$$

with the latest inequality following from assumption 2 and $\mu > 0$. By complementary slackness in (f2), the last two terms are zero, yielding $\int \boldsymbol{P} g_x(\pi|x)d\pi > 0$. The latest can be used in (o) to find,

$$(*) \qquad \frac{\partial}{\partial x}\int [\pi - Rx]g(\pi|x)d\pi > 0$$

As (CC') holds with equality, a solution x satisfies necessary and sufficient conditions for individual optimality (CC), with a non-binding constraint $a' \geq 0$. Finally, by assumption 2, $\int [\pi - R]g(\pi|x)d\pi$ is strictly concave in x; hence, $(*)$ implies $x < x^o$. $0 < x$ follows from assumption 3.

Step 5: Full collateralized LDC contracts.
For any non-trivial contract, we have seen that, $\xi = 0 = \psi$, $\gamma = 1$, irrespectively of $\mu \geq 0$. Hence, (f4) reduces to,

$$\text{(f4')} \qquad \int \eta(\pi)g(\pi|x)d\pi = \zeta_1 - \zeta_0 \geq 0$$

where the inequality follows from $\nu \geq 0$. This implies an optimal prescription: $\alpha = 1$ if $lhs > 0$ (*i.e.* if $\eta(\pi) > 0$ at some π) and $\alpha \in [0, 1]$ if $lhs = 0$. Next, integrating (f2) on Π,

$$\int [\eta(\pi) - \theta(\pi)]g(\pi|x)d\pi = \gamma - 1 = 0$$

Since, by (f2), $\theta(\pi) \geq 0$ holds strictly at some π, occurring with positive probability, we conclude that a solution of $\mathcal{P}$ yields $\zeta_1 > 0 = \zeta_0$ and, by complementary slackness, it delivers $\alpha = 1$. ■

Lemma 5. *In the context of theorem 1, restrict contracts to have a monotone payment schedule (M). Then, any non-trivial solution to $\mathcal{P}$ is a SDC such that properties (1)-(3) stated in the theorem hold.*

Proof. The proof reiterates the ones given for lemma 3 and 4. The only difference is for the latest lemma, when it comes to the definition of the payment schedule $P(\cdot)$. Obviously, LCD violates (M) at all $\pi > z$. The best financial contract is therefore one that distribute the payments so as to impose the lowest possible burden on 'high' states, in which the marginal effect of investment is increasingly larger, and otherwise impose the highest costs on 'low' states, without violating (M) and (LL). This is achieved by letting z be such that $\varphi(z, \cdot) = 0$, as above, and to start confiscating capital at all $\pi \leq z$, so as to satisfy (M). To this end, and only for expositional convenience, we momentarily redefine contracts by adding to the original elements (B, P, α) a *confiscation function*, $\phi : \Pi \to [0, 1]$, which specifies the fraction of capital that is confiscated in each state π. Then, (LL) prescribes $P(z) \leq z + \phi(z)\alpha aR$. We let $\phi(z) = 0$ such that, in state $\pi = z$ the firm makes profit z and pays $P(z) = z$. Next, for all $\pi < z$, we define,

$$\phi(\pi) = \min\left\{1, \frac{z-\pi}{\alpha a R}\right\}$$

As the second term in brackets is decreasing in π, from 0 (when $\pi = z$) to values higher than 1 (when $\pi \searrow 0$, provided that $z/\alpha a R > 1$), we identify two sub-regions in $[0, z]$: $(z-\alpha a R, z]$ in which there is partial confiscation, $\phi < 1$, and $[0, z-\alpha a R]$ of full 'bankruptcy', with $\phi = 1$. Using the definition of ϕ into $\boldsymbol{P}$, the optimal payment schedule in (P^*) is,

$$(P^*) \qquad P(\pi; z) = \begin{cases} z, & \text{for all } \pi > z - \alpha a R; \\ \pi + aR, & \text{otherwise.} \end{cases}$$

It should now be clear that the confiscation function was just an intuitive, technical devise, and that we do not need to keep track of it hereafter.

■

The following establishes property (4) in the theorem.

Lemma 6. *In the context of theorem 1, any optimal financial contract l in $\mathcal{L}^*$ entails no hidden savings.*

Proof. By contradiction, suppose that an optimal SDC, (B, z), entails $a' > 0$. Let us marginally reduce B and increase x: $dx = -\epsilon dB > 0$, $\epsilon > 0$ arbitrarily small. This is feasible if $da' = dB - dx = dB(1+\epsilon)$, and if it satisfies (IP). (IP) holds if,

$$\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)\frac{dz}{dB} + \left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\frac{dx}{dB} - R = 0$$

where, under (M), $\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x] := \int_z^\infty g(\pi|x)d\pi > 0$.[25] Or, equivalently,

$$(\star) \qquad \frac{dz}{dB} = \frac{R + \left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\epsilon}{\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]}$$

whose rhs, evaluated at an optimal contract, is positive. Next, totally differentiating the agent's objective,

$$d\mathbb{E}[V|x] = -\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)dz + \left(\frac{\partial}{\partial x}\mathbb{E}[V|x]\right)dx + RdB$$

which, evaluated at the optimal contract (entailing $\frac{\partial}{\partial x}\mathbb{E}[V|x] = 0$), reduces to,

$$\frac{d\mathbb{E}[V|x]}{dB} = -\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)\frac{dz}{dB} + R$$

[25] Notice that $\frac{d}{dz}D_x\mathbb{E}[\boldsymbol{P}|x] := \int_z^\infty g_x d\pi > 0$, because $g_x > 0$ for a.a. $\pi \geq z \geq \pi_s$, by assumption 1.

Using $(\star)$,

$$\begin{aligned}\frac{d\mathbb{E}[V|x]}{dB} &= -\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)\left[\frac{R+\left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\epsilon}{\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]}\right]+R \\ &= -\left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\epsilon < 0\end{aligned}$$

for any $\epsilon > 0$. It remains to check that (CC') continues to hold, for at least some $\epsilon > 0$. At the initial, optimal contract, (CC') holds if,

$$-\left(\frac{\partial}{\partial z}D_x\mathbb{E}[\boldsymbol{P}|x]\right)dz + \left(\frac{\partial^2}{\partial x^2}E[V|x]\right)dx \geq 0$$

where, under (M) (*i.e.* for collateralized SDCs), $\frac{\partial}{\partial z}D_x\mathbb{E}[\boldsymbol{P}|x] := \int_z^\infty g_x(\pi|x)d\pi$. Using this to determine ϵ:

$$\epsilon \equiv -\frac{dx}{dB} \geq \left(\frac{\frac{\partial}{\partial z}D_x\mathbb{E}[\boldsymbol{P}|x]}{\frac{\partial^2}{\partial x^2}E[V|x]}\right)\frac{dz}{dB}$$

By $(\star)$, $dz/dB > 0$ implying that the right hand side of the latter expression is negative, when evaluated at the initial optimal contract. Therefore, (CC') holds for all $\epsilon > 0$. To sum up, a marginal reduction of the loan size, accompanied with a feasible adjustment of z (decreasing) and x (increasing) improves the agent's welfare; which provides the desired contradiction. As the initial contract delivering $a' > 0$, was arbitrary, this concludes our proof. It is straightforward to reiterate the argument for the case of a LDC. ■

Lemma 7. *The menu of optimal, monotone SDCs is characterized by an investment schedule $x(a)$ and a loan threshold $z(a)$ which are, respectively, monotonically increasing and decreasing for almost all a in $\{A : x(a) < x^o\}$ and constant otherwise.*

Proof. We essentially verify that the Implicit Function Theorem holds and that it delivers the desired monotonicity properties of $(z, x)(a)$. From (CC') holding with equality at almost all (a.a.) a, we find,

$$(+)\qquad \frac{dx}{da}(a) = \left(\frac{\frac{\partial}{\partial z}\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi}{\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x(a)]}\right)\frac{dz}{da}(a) = -\mu(a)\left(\frac{\frac{\partial}{\partial z}\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi}{\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi}\right)\frac{dz}{da}(a)$$

For any feasible change dz/da, (+) measures the slope of the optimal investment schedule on A. This is zero in the case of no-moral hazard (*i.e.* when the commitment constraint is non-binding, $\mu(a) = 0$), corresponding to efficient investment x^o, for all firms, independently of theirs initial capital. The numerator of (+) is, $(\int_z^\infty g_x(z|x)d\pi)(dz/da)$, evaluated at (a). This implies that -given the capital requirements- it is individually optimal to increase real investments for entrepreneurs with higher capitalized firms if and only if they face a decreasing loan threshold z. Next, we prove that indeed, at any optimal, monotone SDC a, $dz/da < 0$.

Recall that (IP) holds with equality at a.a. contracts in $\mathcal{L}^*$. Hence, the total differential (with respect to a, z, x, B) must be equal to zero almost everywhere (a.e.). Moreover, since for a.a. a, the individual budget constraint balances at zero saving, $\frac{dB}{da} = \frac{dx}{da}$. Hence, passing to SDCs and using $g := g(\pi|x(a))$, for brevity,

$$\begin{aligned}
0 &= \frac{\partial}{\partial a}\left(\int \boldsymbol{P}_a g d\pi\right) da + \frac{\partial}{\partial z}\left(\int \boldsymbol{P}_a g d\pi\right) dz + \frac{\partial}{\partial x}\left(\int \boldsymbol{P}_a g d\pi\right) dx - RdB \\
&= Rda + \frac{\partial}{\partial z}\left(\int_0^z \pi g d\pi + z\int_z^\infty g d\pi\right) dz + \left(\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi - R\right) dx \\
&= R + [1 - G(z|x(a))]\frac{dz}{da} + \left(\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi - R\right)\frac{dx}{da}
\end{aligned}$$

Next, using (+) into the latest expression and rearranging terms,

$$\frac{dz}{da}(a) = \frac{-R}{1 - G(z|x(a)) + \left(\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi - R\right)\left(\frac{\int_z^\infty g_x(z|x(a))d\pi}{\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x(a)]}\right)}$$

The right hand side of this expression is negative iff the denominator is strictly positive, which is true at a.a. optimal, monotone SDCs by lemma 8, below. ■

Lemma 8. *At any monotone SDC a in $\mathcal{L}^*$,*

$$(oo) \qquad R - \int \boldsymbol{P}_a g_x(\pi|x(a))d\pi > \left[\frac{1 - G(z|x(a))}{\int_z^\infty g_x(z|x(a))d\pi}\right]\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x(a)]$$

Proof. By contradiction, suppose that, at an optimal contract (oo) fails to hold. We are going to prove that by, proportionally, increasing B and x and lowering z, we can increase the value of the objective without violating (IP) and the commitment constraint (CC'). Indeed, let $dB = dx > 0$ (and $da' = 0$), be an arbitrarily small change. Since (IP) holds with equality at a.a. a in $\mathcal{L}^*$, differentiating and evaluating at the optimal contract, we find,

$$(o*) \qquad \frac{dz}{dx} = \frac{R - \int \boldsymbol{P} g_x(\pi|x)d\pi}{1 - G(z|x)}$$

which is negative when (oo) fails. We now show that our feasible adjustment dz/dx does not violate incentives. For (CC') to hold,

$$-\left(\frac{\partial}{\partial z}\int \boldsymbol{P} g_x(\pi|x)d\pi\right) dz + \left(\frac{\partial^2}{\partial x^2}E[V|x]\right) dx \geq 0$$

Yielding,

$$\frac{dz}{dx} \leq \frac{\frac{\partial^2}{\partial x^2}E[V|x]}{\int_z^\infty g_x(z|x)d\pi} < 0$$

This does not contradict $(o*)$ iff,

$$\frac{R - \int \boldsymbol{P} g_x(\pi|x)d\pi}{1 - G(z|x)} \leq \frac{\frac{\partial^2}{\partial x^2}E[V|x]}{\int_z^\infty g_x(z|x)d\pi}$$

that we can rewrite as,

$$R - \int \boldsymbol{P} g_x(\pi|x)d\pi \leq \left[\frac{1-G(z|x)}{\int_z^\infty g_x(z|x)d\pi}\right]\frac{\partial^2}{\partial x^2}E[V|x]$$

This holds because we started out assuming (oo) does not. Finally, it is immediate to check that an increase of x, B and a decrease of z, raise the value of the objective, delivering the desired contradiction. ■

§§§

Proof of theorem 2. We define and solve an auxiliary, simpler problem $\mathcal{P}''$ and show that its solution can be used to pin down a solution to $\mathcal{P}'$. To this end, let the payment schedule including the auditing cost be defined as, $P_\lambda(\pi)$ (equal to $(1+\lambda)P(\pi)$ for al π in Π^v).
$\mathcal{P}''$ consists in finding (x, c, P, B, α) maximizing,

$$\int [\pi - P_\lambda] g(\pi|x)d\pi + Ra'$$

subject to, $0 \leq a' = B - x + (1-\alpha)a,\ \ 0 \leq \alpha \leq 1,\ \ 0 \leq B,$

(CC") $$\int [\pi - P_\lambda] g_x(\pi|x)d\pi - R = 0$$

(IP") $$\int \frac{P_\lambda}{1+\lambda} g(\pi|x)d\pi - RB \geq 0$$

the consistence requirement, at all π,

(c) $$P_\lambda \leq c$$

for all π,

(LL) $$0 \leq P_\lambda \leq \pi + \alpha a R$$

First order conditions are computed. Consider the one related to $\boldsymbol{P}_\lambda$, for the case in which (CC') is binding ($i.e.$ $\mu > 0$).

(f2') $$\boldsymbol{P}_\lambda:\quad g(\pi|x)\left\{-1 + \frac{\gamma}{1+\lambda} - \mu\frac{g_x(\pi|x)}{g(\pi|x)} - [\eta(\pi) - \theta(\pi)] - \delta(\pi)\right\} = 0,$$

$$\eta, \theta, \delta:\ \eta(\pi) \geq 0,\ \eta(\pi)[\pi + \alpha a R - P_\lambda] = 0,\quad \theta(\pi) \geq 0,\ \theta(\pi)P_\lambda = 0,$$
$$\delta(\pi) \geq 0,\ \delta(\pi)[P_\lambda - c] = 0,\quad \textit{for all } \pi.$$

Let $\varphi(\pi, x, \cdot) := -1 + \gamma - \mu\frac{g_x(\pi|x)}{g(\pi|x)}$; then, for any given α, and $c \geq \alpha a R$, the optimal payment schedule satisfies the following,

$$\varphi(\pi, x, \cdot) = -\theta(\pi) < 0 \quad \textit{only if} \quad P_\lambda = 0,\quad \eta(\pi) = \delta(\pi) = 0,$$
$$\varphi(\pi, x, \cdot) = 0 \quad \textit{only if} \quad P_\lambda \in [0, \min\{c, \pi + \alpha a R\}],\quad \delta(\pi) = \eta(\pi) = 0 = \theta(\pi)$$
$$\varphi(\pi, x, \cdot) = \delta(\pi) + \eta(\pi) > 0 \quad \textit{only if} \quad P_\lambda = \min\{c, \pi + \alpha a R\},\quad \theta(\pi) = 0$$

As for proposition 1, MLR implies that an optimal payment schedule is one with the lowest payment at high realization of profits and the highest payment at the lowest realizations.

A solution is one in which c can take any value greater than αaR, only if $\gamma = (1+\lambda)^{-1}$, which implies that (IP") holds with equality. Let $c \equiv z$ be such that $\varphi(z, x, \cdot) = 0$. The existence and uniqueness of such z follows from the same arguments used above.

Next, define the subregions $\Pi^v = [0, z]$ and $\Pi^v = \Pi \setminus \Pi^o$ and restrict $P(\pi)$ to satisfy consistence. As in Townsend (1979) (see his lemma 2.1), for $z > 0$, consistence is achieved if and only if payment schedules satisfy the following two restrictions in (c) above; namely:

(i) $P(\pi) = z$, *for all* π *in* $\Pi^o \equiv \Pi \setminus \Pi^v$;

(ii) $P(\pi)(1+\lambda) \leq z$, *otherwise*.

Indeed, for all $\pi \in \Pi^o$, one can at most impose to the creditor a payment equal to the lowest level of profit in Π^o, z; here, we claim that in the verification region it is optimal to have no capital confiscation (Π^o does not contain bankruptcy states). At $\pi = z \in \Pi^v$, consistence requires that $z = P(z)(1+\lambda)$. As we did in the proof of lemma 5, we redefine contracts by adding to (B, P, α) a *confiscation function*, $\phi : \Pi \to [0, 1]$. As before, $\phi(z) = 0$, so that $P(z) = P_\lambda(z) = z$ if the firm chooses not to audit. Otherwise, by taking the highest possible confiscation value,

$$\phi(\pi) = \min\left\{1, \frac{z-\pi}{\alpha aR}\right\}$$

Using this definition of ϕ, we derive the optimal payment schedule,

$$(P^{**}) \qquad P(\pi) = \begin{cases} z, & \text{for all } \pi \in \Pi^o; \\ \frac{z}{1+\lambda}, & \text{for all } \pi \in \Pi^{vH} \equiv \{\pi' \in \Pi^v : \pi' \in (z - \alpha aR, z]\}; \\ \frac{\pi + aR}{1+\lambda}, & \text{for all } \pi \in \Pi^v \setminus \Pi^{vH}. \end{cases}$$

Therefore, for any z and corresponding verification region, this payment schedule imposes the maximal confiscation of initial capital (satisfying both LL and consistence) in the verification region.

Next, using the subregions Π^v, Π^o to rewrite the objective, one obtains the objective of the original problem $\mathcal{P}'$ and, substituting a solution of $\mathcal{P}''$ into (IP"), one obtains,

$$z \int_{\Pi^o} g(\pi|x)d\pi + \int_{\Pi^v} P(\pi)g(\pi|x)d\pi - RB = 0$$

which is (IP'). Thus, we can conclude that a solution of $\mathcal{P}''$ is also a solution of the original problem, $\mathcal{P}'$.

The rest of the proof goes through identically as for theorem 1.

■

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