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Mihara, H. Reiju

Kagawa University

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H. Reiju Mihara†
Graduate School of Management, Kagawa University
Takamatsu 760-8523, Japan

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Abstract

Consider the problem of allocating k identical, indivisible objects among n agents, where k is less than n. The planner's objective is to give the objects to the top k valuation agents at zero costs to the planner and the agents. Each agent knows her own valuation of the object and whether it is among the top k. Modify the $(k+1)$st-price sealed-bid auction by introducing a small participation fee and the option not to participate in it. This strikingly simple mechanism (modified auction) implements the desired outcome in iteratively weakly undominated strategies. Moreover, no pair of agents can profitably deviate from the equilibrium by coordinating their strategies or bribing each other.

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1 Introduction

I propose the following variant of the second-price (sealed-bid) auction to solve King Solomon's problem. First, the agents need not participate in the auction if they do not want to. Second, they may have to pay a small participation fee—a situation that arises if the number of actual participants exceeds one (or the number of objects to be allocated, more generally). In other words, I modify the second-price auction (Vickrey auction) by introducing an arbitrarily small entry fee and the option not to participate in it. This strikingly simple, intuitively appealing mechanism (modified auction) solves the problem. Most likely, this mechanism, with entry fees, is close to what ordinary people would think of when they learn the notion "second-price auction," hence the title.

When generalized to multiple units of an item, the problem is as follows: k identical, indivisible objects are to be allocated among n agents, where $k < n$. The objective of the "planner" ("auctioneer") is to give the objects at no cost to the k agents with the highest valuations. Each agent knows not only her own valuation of the object but also whether she is among the top k valuation agents.[1]

The $(k+1)$st-price auction (instead of the second-price auction) for k objects, similarly modified, solves this generalized problem (Proposition 1). In other words, this two-stage mechanism implements the desired outcome by iterative elimination of weakly dominated strategies (one round of elimination of weakly dominated strategies, followed by two rounds of elimination of *strictly* dominated ones). In fact, only those top k valuation agents choose to participate in the auction, eliminating the need to actually hold the auction.

The reasoning behind this conclusion is straightforward. In this auction, it is a weakly dominant strategy for each agent to bid her true valuation. The planner can set an entry fee low enough so that the top k valuation agents can profitably obtain the object by paying the $(k+1)$st price and the entry fee. Then the other agents will not enter the auction, since they can only expect to pay the entry fee without getting the object. While the logic is simple, a careful argument specifying *what information is available to which agent* is called for. I state as explicitly as possible the informational assumptions on which each step of the argument is based.

Earlier contributions to King Solomon's problem, such as Glazer and Ma [2] and Moore [3], consider the case of $k = 1$ object and assume that each

[1] There are many situations of this sort, including those mentioned in Glazer and Ma [2, footnote 1]. Consider a temporary loss of data, for example. A certain service (or license) must be provided to eligible claimants before the data is recovered. The number of eligible claimants is known, but the records concerning their identity have been lost. An ineligible person has a very low valuation of the service because to her the service comes with a later punishment.

agent knows the *other agents*' valuations, too. Under this complete information assumption, they construct multi-stage mechanisms that implement the outcome in subgame-perfect equilibrium. Though those mechanisms consist of more than two stages, they have an appealing feature that only one agent moves at each stage.

More recently, assuming that each agent only knows her *own* valuation as well as whether she is one of the top k valuation agents, several authors have constructed ingenious mechanisms that implement the outcome in iteratively weakly undominated strategies. Perry and Reny [5] and Olszewski [4] construct mechanisms for $k = 1$. Bag and Sabourian [1] extend Olszewski's mechanism [4] to any k (they have also investigated the complete information setting). Qin and Yang [6][2] propose an alternative mechanism for any k. Like mine, these mechanisms consist of two stages and lack the feature of only one agent moving at each stage.

As pointed out in most of these papers [2, 3, 5], an auction itself does not solve the problem, since it involves a transfer of money. It is interesting to note that, given this fact, all the authors who deal with the incomplete information settings propose a mechanism, of which a stage game is a modified version of the second-price auction.[3] However, their modifications are fairly sophisticated, not appearing as straightforward as mine. Perry and Reny [5] use a second-price *all-pay* auction with the winner having an *ex post option* to quit. Olszewski [4] uses the second-price auction modified by adding an *extra, non-constant (positive) payment from* the planner. Qin and Yang [6] use a second-price auction with entry fees, but the fees are endogenously determined as an outcome of a game in which *each agent has to guess the other's bid*[4] (for $n = 2$ and $k = 1$). Their mechanism loses the advantage of the second-price auction that each agent need not guess others' bids or valuations.

In Section 4, I compare my mechanism with Olszewski's, which is one of the simplest in the literature. Olszewski's mechanism requires the planner to subsidize the agents out of the equilibrium path.[5] As a result, it is vulnerable to *collusion between agents that bribe each other* to coordinate their strategies. In fact, they can profitably deviate from the equilibrium without even manipulating their bids (Proposition 2). Unlike Olszewski's

[2] I would like to thank Takuma Wakayama for pointing out an earlier version of this paper.

[3] Those dealing with the complete information settings [2, 3] also propose auction-like mechanisms, though the bidding protocols are different from the second-price auction.

[4] Qin and Yang assume that each agent knows the distribution of the other's valuation conditional on her own. I do not assume that the agents know the distributions.

[5] In contrast, if my mechanism fails at the first stage, what comes after the entry fees are collected is just an ordinary second-price auction. So the mechanism is particularly attractive as a compromise solution in situations where solution based on price is not too problematic but has not been used (because of some sort of stigma), such as assignment of parking spaces at some university campus.

(and unlike the second-price auction), my mechanism is not vulnerable to such collusion (Proposition 3).

2 Framework

We consider the problem $\mathcal{P}_n^k$, a multi-unit generalization of King Solomon's problem: k identical, indivisible objects are to be allocated among n agents, where $k < n$. The objective of the planner is to give the objects to the top k valuation agents at zero monetary costs to the planner and the agents. (This defines the choice function to be implemented.)

The framework is as follows: Let $N = \{1, \ldots, n\}$ be the set of agents. Fix a certain number $\delta > 0$, which is known to everyone (i.e., all agents and the planner). Fix a set $Q \subset \mathbb{R}^n$ of possible profiles of valuations of the object such that every profile $(v_1, \ldots, v_n)$ in Q contains at least k nonnegative components. (The valuation by the planner is understood to be zero.) At *Stage* 0, God (Nature) announces a pair (v, H), where $v = (v_1, \ldots, v_n) \in Q$ is a profile of valuations and $H = \{i \in N : v_i \text{ is among the top } k \text{ valuations at } v\}$ is the corresponding set of the k agents with the highest valuations (so $\#H = k$). *While no one needs to know the set Q itself, everyone (including the planner) knows the following condition imposed on Q*: if $i \in H$ and $j \in L := N \setminus H$, then $v_i \geq 0$ and[6]

$$v_i - v_j > \delta. \tag{1}$$

This inequality, which says there is a gap of at least δ between the top k valuations and the other valuations, will serve as a "word of wisdom" that facilitates the construction of a successful mechanism. (Remark 1 discusses a way to relax this assumption.) Every agent i observes $(v_i, H(i))$, where $H(i) = 1$ or 0, depending on $i \in H$ or not.[7] The planner does not observe God's announcement. Each agent i's payoff of obtaining the object and payment $p \in \mathbb{R}$ is $v_i + p$. Each agent's payoff of obtaining no object and payment p is p. Finally, it is common knowledge that the agents and the planner have the knowledge described here.

3 The Solution

The mechanism $\mathcal{M}_n^k$ consists of two stages, Stage 1 followed by Stage 2. (We can regard it as a single-stage mechanism by considering its strategic

[6] This assumption is naturally satisfied if, for example, $n = 3$, $k = 2$ and either (i) $Q = U_1 \times U_2 \times U_3$ for some pairwise disjoint finite sets U_1, U_2, and U_3 in $\mathbb{R}_+$ or (ii) for some disjoint closed intervals U, V in $\mathbb{R}_+$ such that $u \in U$ and $u' \in V$ imply $u > u'$, we have $Q = (U \times U \times V) \cup (U \times V \times U) \cup (V \times U \times U)$.

[7] The implementability result of Bag and Sabourian [1] is valid under this assumption, though they make a stronger assumption that each i observes (v_i, H).

form representation. However, our solution concept—iterated elimination of dominated strategies—seems more appealing if the mechanism is presented in an extensive form.) I describe Stage 2 first.

Stage 2 is the $(k+1)$*st-price sealed-bid auction* for k objects, except that the planner collects the *participation fee* $\delta > 0$ from each agent. There are at least $k+1$ agents participating in the auction and each participating agent i bids $b_i \in \mathbb{R}$. Thus, any bid b_i (including those not corresponding to any profile in Q) is allowed at this stage. Rearrange the *named bids* (b_i, i) according to the dictionary order—first in terms of the value b_i (highest bid first), second in terms of the agent name i (lowest number first). (We need not worry about how ties are broken; we will see below that the k highest bidders are uniquely determined in equilibrium, because of inequality (1).) Let b^{k+1} be the $(k+1)$st bid (i.e., the first component of the $(k+1)$st named bid according to the above order) by the participating agents. The following is what agent i receives, as well as her payoff u_i: footnote (a) if b_i is among the k highest bids (i.e., (b_i, i) is among the first k named bids according to the dictionary order), then i gets the object but pays the $(k+1)$st bid and the participation fee, implying $u_i = v_i - b^{k+1} - \delta$; (b) otherwise, i pays the participation fee, implying $u_i = -\delta$.

Stage 1 is a simultaneous-move game form[8] in which the agents say either "auction" (which means that she is willing to move on to Stage 2 and participate in the auction) or "no (auction)." If at least $k+1$ agents say "auction," then (only) these agents move on to Stage 2; the others get nothing. If less than $k+1$ agents say "auction," then they get the object; the others get nothing. (If no agent says "auction," then no agent gets anything.)

After playing Stage 1, each agent may (partially) observe the result of the first-stage game. Let M_i be an *arbitrary* set of messages that i may receive at the end of Stage 1.[9] Agent i's *strategy* is defined as a function s_i mapping each observation $(v_i, H(i))$ into (m_i, B_i), where $m_i \in \{$"auction", "no"$\}$ is a first-stage action and $B_i\colon M_i \to \mathbb{R}$ is a function assigning a second-stage action (bid) b_i that i will play if she participates in the auction.

Given a mechanism, we say that two strategies s_i and s'_i are *equivalent for* i if for all $s_{-i} := (s_j)_{j \neq i}$, the outcomes corresponding to (s_i, s_{-i}) and to (s'_i, s_{-i}) both give the same assignment (whether she gets the object and how much she needs to pay) to i. For example, given $\mathcal{M}^k_n$, all strategies of the form ("no", B_i) are equivalent for i, giving nothing to i. Let s^*_i be the strategy obtained by setting m_i equal to "auction" if $i \in H$ and "no" if

[8] This stage can be sequential.

[9] I do not make a specific assumption about what information (messages) is available to each agent after Stage 1. This could be any of the following: (i) nothing, (ii) her own choice of "auction" or "no," (iii) whether the number of agents saying "auction" is at least $k+1$, (iv) the number of agents saying "auction", (v) the set of agents saying "auction." We will see that the equilibrium strategy does not use any of these messages.

$i \in L$, and letting $B_i(\cdot) \equiv v_i$.

Proposition 1 *The mechanism $\mathcal{M}_n^k$ solves the problem $\mathcal{P}_n^k$; that is, it implements the choice function by iterative elimination of weakly dominated strategies (one round of elimination of weakly dominated strategies, followed by two rounds of elimination of* strictly *dominated ones). For each agent i, the remaining strategies and s_i^* are all equivalent.*

Proof. First, we begin with an analysis of Stage 2. Since the fees δ are independent of the agents' action, the well-known result for the $(k+1)$st-price auction implies that it is a weakly dominant strategy for each agent to bid her true valuation.[10] *Each agent i thus eliminates in this first round all the strategies* ("auction", B_i) *such that B_i assigns a bid $b_i \neq v_i$ to a message* that does not exclude the possibility that the auction is held.

Next, we consider Stage 1. The agents know that in Stage 2 they will bid their valuations, namely, $b_j = v_j$ for all j, though each agent i observes her own valuation v_i only. Each agent i knows whether $i \in H$ (i.e., $H(i) = 1$) or $i \in L$ (i.e., $H(i) = 0$).

Consider agents $i \in H$ first. We show that *in this second round i eliminates all the strategies* ("no", B_i). If at least k other agents say "auction," then by saying "auction," i can participate in the auction, which is held. In this case, i knows that she will be among the k highest bidders. Her payoff from the auction is $u_i = v_i - b^{k+1} - \delta$, which depends on b^{k+1} yet to be known. She can, however, deduce from (1) that the last value is positive, since she knows that $i \in H$ and $j \in L$ for $v_j = b_j = b^{k+1}$ (the $(k+1)$st bidder's valuation is the $(k+1)$st highest at best, so she cannot be in H by definition). Since saying "no" means a zero payoff, she chooses "auction." If less than k other agents say "auction," then the auction is not held in any case. By saying "auction," she gets the object with a payoff of $v_i \geq 0$ (since $i \in H$). So "auction" is a best response.

Consider agents $i \in L$ next. We show that *in this third round, i eliminates all the remaining strategies* ("auction", B_i). Since i can deduce that the agents $j \in H$ will choose "auction" and bid $b_j = v_j$, she knows that if she says "auction," the auction is held and she gets the payoff of $u_i = -\delta < 0$ (she cannot win the object because she will not be among the k highest bidders). She therefore says "no" and obtains nothing.

At this point in the elimination process, the remaining strategies $s_i = (m_i, B_i)$ are such that m_i is "auction" if $i \in H$ and "no" if $i \in L$ and that B_i assigns v_i to each message not excluding the possibility that the auction is held and i participates in it. Clearly, s_i^* is equivalent to any remaining

[10] The conclusion, say for b_i, can be derived by fixing the bids of the other participating agents and then comparing i's payoffs for bidding her true valuation ($b_i = v_i$) and for bidding something else, for each case: (a) bidding $b_i = v_i$ is among the k highest bids, and (b) otherwise.

strategy for i. Since only those agents in H say "auction" and there are exactly k such agents, the auction will not be held. So each agent in H gets the object and each agent in L gets nothing. ∎

Recall the assumption that the agents and the planner—while they do not necessarily know the set Q of possible profiles of valuations—know that the elements in Q satisfy certain conditions such as inequality (1). Note that this limited knowledge about Q is required only for solving Stage 1. The conclusion for Stage 2 can be obtained without this knowledge (any $\delta \in \mathbb{R}$ will do, implying (1) is irrelevant). (For *constructing* Stage 2, of course, the planner must know δ.)

Remark 1 What if the planner does not know the value of δ in (1) but the agents know it? We come close to solving the problem by modifying Stage 1 as follows: Each agent i announces a nonnegative entry fee δ_i that she would be willing to pay if the auction is held. Like Bag and Sabourian [1, page 47], set an entry fee δ' equal to the smallest *positive* δ_i announced by the agents if at least one agent announces a positive δ_i; set $\delta' = 0$ otherwise. If at least $k+1$ agents announce a positive δ_i, then (only) these agents move on to Stage 2, where the participation fee δ is replaced by δ'; the others get nothing. If less than $k+1$ agents announce a positive δ_i, then they get the object; the others get nothing. In the second round of the elimination process, $i \in H$ can eliminate all the strategies involving $\delta_i = 0$, since they are weakly dominated by a strategy involving $\delta_i = \delta/2$, for example. In the third round, $i \in L$ can eliminate all the strategies involving $\delta_i > 0$ as above. A problem with this argument is that no strategy for $i \in H$ survives: any strategy is weakly dominated by a strategy involving smaller $\delta_i > 0$. To summarize, we can solve the problem, but the solution concept requires a modification. $\|$

4 Discussion

It would be of some interest to compare the mechanism $\mathcal{M}_n^k$ with those in the literature [5, 4, 1, 6] dealing with the incomplete information environments. Of those mechanisms, I focus on Olszewski's [4] since it is simpler than Perry and Reny's [5]. Also, Bag and Sabourian's mechanism [1] for the incomplete information setting is an extension of Olszewski's, not an alternative to it. Qin and Yang's mechanism performs just like mine, if we ignore the complexity of making guesses.

I consider the classical case of Solomon's problem in this section: $n = 2$, $k = 1$, $v_1 > 0$, $v_2 > 0$, and for $i \in H$ (the higher-valuation agent) and $j \in L$ (the lower-valuation agent), $v_i - v_j > \delta > 0$.[11] Note that the planner can

[11] Olszewski constructs another mechanism that solves the problem for $\delta = 0$ (the case

use arbitrarily small δ (because if $\delta > 0$ satisfies the inequality, then so does any positive $\delta' \leq \delta$).

Olszewski's mechanism works as follows: In Stage 1, the two agents say "hers" (corresponding to "auction" in this paper) or "mine" ("no (auction)") simultaneously. If both say "hers," then they *move on to Stage 2*. If only one says "hers," then the agent who says "mine" gets the object. If both say "mine," then both get *nothing*. Stage 2 is a modified second-price auction (modified such that each agent pays the entrance fee δ but receives the other's bid): if $b_i > b_j$, then $u_i = v_i - \delta$ and $u_j = b_i - \delta$.

Table 1 compares the payoffs for the two mechanisms, assuming $b_i > b_j$ and i is the row player.

	"hers"	"mine"
"hers"	$v_i - \delta, b_i - \delta$	$0, v_j$
"mine"	$v_i, 0$	$0, 0$

	"auction"	"no"
"auction"	$v_i - b_j - \delta, -\delta$	$v_i, 0$
"no"	$0, v_j$	$0, 0$

Table 1: Payoffs for Olszewski's mechanism (left) and mine (right).

It is a weakly dominant strategy for each i to play $b_i = v_i$ in Stage 2. The other strategies are eliminated in the first round of elimination of weakly dominated strategies. Olszewski's mechanism requires another round: "hers" is a weakly (but not strictly) dominated strategy for the higher-valuation agent and "mine" is one for the lower-valuation agent (if $i \in H$ and $j \in L$, then $v_j < u_i = u_j = v_i - \delta < v_i$). My mechanism requires two more rounds. But those strategies to be eliminated in the second and the third rounds are *strictly* dominated.

Olszewski's mechanism relies on the availability of transfer from the planner out of the equilibrium path.[12] The reliance on subsidies from outside means that the agents are less likely to find an outsider (planner) who is willing to adopt this mechanism. In contrast, the total amount received by the agents in Stage 2 of my mechanism is negative ($(-b_j - \delta) - \delta = -b_j - 2\delta = -v_j - 2\delta < 0$).

I next consider the possibility (not described by the mechanisms) that the agents could bribe each other to coordinate their strategies. Let $u_i(s)$ be i's payoff from a mechanism, where $s = (s_i, s_j, s_{-ij})$ and $s_{-ij} = (s_k)_{k \notin \{i,j\}}$. We say that a strategy profile $s = (s_i)$ is *stable against bribery involving two agents* if the following condition is violated: there are two agents i, j, their

where the higher valuation and the lower valuation can be arbitrarily close). My mechanism fails to solve such a problem: if $\delta = 0$, the lower valuation agent's strategy "no" is weakly dominated in the second round of elimination if she has a positive valuation. Hence the usual second-price auction (no participation fees) will be held.

[12] The total amount received by the agents in Stage 2 is $-\delta + (b_i - \delta) = b_i - 2\delta$. If we require this value to be non-positive, even if we assume $b_i = v_i$, we have $v_i \leq 2\delta < 2v_i - 2v_j$, implying the inequality $v_i > 2v_j$, not likely in many situations.

strategies s'_i, s'_j, and a bribe $t \in \mathbb{R}$ such that $u'_i := u_i(s'_i, s'_j, s_{-ij}) + t > u_i(s)$ and $u'_j := u_j(s'_i, s'_j, s_{-ij}) - t > u_j(s)$.

Proposition 2 *Suppose that the total amount received by the agents in Stage 2 of Olszewski's mechanism is positive, assuming $i \in H$ bids $b_i = v_i$. Then its equilibrium is not stable against bribery involving two agents, even if the agents bid their true valuations in Stage 2.*

Proof. Consider the strategies such that both agents say "hers" and bid their valuations. The total amount subsidized is $b_i - 2\delta = v_i - 2\delta > 0$ by assumption. Find an $\epsilon > 0$ such that $b_i - 2\delta - \epsilon > 0$. Consider a bribe $\delta + \epsilon$ from $j \in L$ to $i \in H$. Since $b_i > b_j$, the resulting payoffs are: $u'_i = v_i - \delta + \delta + \epsilon = v_i + \epsilon > v_i$; $u'_j = b_i - \delta - \delta - \epsilon = b_i - 2\delta - \epsilon > 0$. ∎

Note that if the agents are not restricted to bidding their true valuations, they can achieve arbitrarily large payoffs,[13] though the availability of subsidies then becomes questionable.

In contrast, my mechanism works better against bribes. I present the result for a more general case of n agents and k objects; it includes the classical case.

Proposition 3 *Suppose that each individual has a positive valuation and is prohibited from submitting a negative bid: $v_i > 0$ and $b_i \geq 0$ for each i. Then the equilibrium of the mechanism $\mathcal{M}_n^k$ is stable against bribery involving two agents.*

Proof. Let s be an equilibrium and suppose it not stable. Then there are agents i, j, strategies s'_i, s'_j, and a bribe t such that $u'_i := u_i(s'_i, s'_j, s_{-ij}) + t > u_i(s)$ and $u'_j := u_j(s'_i, s'_j, s_{-ij}) - t > u_j(s)$. We have

$$u'_i + u'_j = u_i(s'_i, s'_j, s_{-ij}) + u_j(s'_i, s'_j, s_{-ij}) > u_i(s) + u_j(s). \qquad (2)$$

Suppose i, $j \in H$. Then $u_i(s) + u_j(s) = v_i + v_j$. Inequality (2) cannot be satisfied since $u_i(s') \leq v_i$ and $u_j(s') \leq v_j$ for any s'.

Suppose i, $j \in L$. If both says "no," they cannot meet inequality (2). So, suppose that i says "auction," in which case she is worse off (regardless of whether she gets the object), unless she receives a sufficiently large bribe $t > 0$. Then j, who pays the bribe, is worse off (whether she participates in the auction), violating $u'_j > u_j(s)$.

It follows that $i \in H$ and $j \in L$ without loss of generality.

(i) Suppose i says "auction" and j says "no." Then $u'_i = v_i + t > u_i(s) = v_i$ implies $u'_j = 0 - t < 0 = u_j(s)$, a contradiction.

(ii) Suppose i says "no" and j says "auction." Then $u'_i = 0 + t > u_i(s) = v_i$ implies $u'_j = v_j - t < v_j - v_i < -\delta < 0 = u_j(s)$, a contradiction.

[13] For any $\bar{u}_i > 0$ and $\bar{u}_j > 0$, fix a small b_j, find a bribe $t \in \mathbb{R}$ such that $u'_i = v_i - \delta + t > \hat{u}_i$, and find a b_i such that $u'_j = b_i - \delta - t > \hat{u}_j$.

(iii) Suppose i says "no" and j says "no." Then $u'_i + u'_j = 0$ and $u_i(s) + u_j(s) = v_i$, violating (2).

(iv) Suppose i says "auction" and j says "auction." If j gets the object, (2) implies that $u'_i + u'_j = v_j - b^{k+1} - 2\delta > v_i$, where b^{k+1} is the $(k+1)$st highest bid. Then $-b^{k+1} - 2\delta > v_i - v_j > \delta$, implying $-b^{k+1} > 3\delta > 0$, contradicting the assumption that bids are nonnegative. The case where i gets the object is easier. ∎

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A Appendices (Not to be Published)

A.1 Remark on Qin and Yang [6]: More on footnote 4

Qin and Yang [6] also use a second-price auction with entry fees. Their mechanism is more complex than mine in that entry fees are endogenously determined by a game in which each agent has to guess (in the classical case of $n = 2$ and $k = 1$) the other's bid. This increased complexity is partly because they only assume $\delta \geq 0$ in inequality (1), while I focus on the case of $\delta > 0$. They assume that each agent has beliefs about the other agents' valuations conditional on her own—which is very close to assuming a distribution function over the set Q of possible profiles. While one could relax their assumption of expected utility maximization, one cannot discard the use of distribution functions, since one needs it to delete suboptimal guesses (a strategy has to specify an optimal guess even when an agent does not enter an auction). In contrast, I do not assume that the agents know the distribution, not even the set Q of possible profiles.

A.2 Sufficient information for calculating the choice function: More on footnote 7

Bag and Sabourian [1], who also deal with the multi-unit case, assume that each agent knows her own valuation and the *identity* of the top k valuation agents. That is, they assume that each i observes (v_i, H), instead of just $(v_i, H(i))$. Each agent thus has "sufficient information for calculating the choice function" (defined below). One might therefore suspect that their implementability result depends on the assumption that some agents have such information. (One could say that for the purpose of computing the choice function, their setting is in effect that of "complete information.")

It turns out that the informational assumption of Bag and Sabourian [1] can be relaxed. Their implementability result is still valid even when their assumption is relaxed to the assumption that each i observes $(v_i, H(i))$. (This relaxed assumption seems more faithful to the assumption of Perry and Reny, who write "[the highest value agent] may be the only one who knows who the highest value agent is" [5, page 282].) Their result does not depend on the assumption that some agents have sufficient information for calculating the choice function.

Remark 2 The informational setting of the paper is essentially the same as the incomplete information settings of the previous papers [5, 4, 1]. It is incomplete *relative to* the complete information settings of earlier papers [2, 3], in which the agents know other agents' valuations. The reader should note that the word is used in this relative, model-dependent sense. From the point of view of calculating the choice function, the agents here have “more than complete information” since they know who the high valuation agents are (her own valuation is “extra” information).

The notion of “complete information” in the literature is defined with reference to the model. For example, let f be a dictatorial social choice function for which agent 1 is the dictator. Nature announces the agents' preferences at Stage 0. Each agent observes her preference as well as the preference of agent 1. This is usually called an “incomplete information” setting. However, for the purpose of computing f, we could say that each agent has enough (“more than complete”) information for calculating f. $\|$

Let me give a precise notion of sufficient information with reference to the function to be implemented. Let $f\colon E \to X$ be a choice function, where E is a set of environments and X is a set of allocations. Let $\mu_i\colon E \to \Theta_i$ be a function that specifies the information observable to i. (In the framework above, let N_k be the collection of k-agent subsets of N. Then $E \subset Q \times N_k$, $X = N_k$, $f((v,H)) = H$ where outputs are the sets of agents receiving the objects, and $\mu_i((v,H)) = (v_i, H(i))$.) An agent i has *sufficient information for calculating* f if there exists a function $f_i\colon \Theta_i \to X$ such that $f_i \circ \mu_i(e) = f(e)$ for all $e \in E$. (In the framework of this paper, no agent has such information. On the other hand, if μ_i is redefined by $\mu_i((v,H)) = (v_i, H)$ as in Bag and Sabourian [1], then i has such information.)

A.3 More Discussion: Deleted from Section 4

Which mechanism is easier to understand? Different people would have different answers, but my view is that my mechanism is easier. It is just the second-price auction with *constant* participation fees in which participation is voluntary. On the other hand, whatever labels one may use, interpretation of the first-stage actions of Olszewski's mechanism seems to require sufficient understanding of the mechanism. Since people tend to associate no payment with getting no object in an auction, for example, the additional payment of b_i to the lower-valuation agent j can be particularly confusing. If the planner avoids using the label “auction” in order not to confuse them, then even those who have studied the workings of the second-price sealed-bid auction would fail to see the connection. In any case, this is a kind of question that should be answered by empirical methods like opinion surveys and experiments.

Both Olszewski's and my mechanisms are naturally presented as a two-stage mechanism. In both mechanisms, Stage 2 will not be reached in equilibrium. I do not say that the *number of stages* gives a good measure of simplicity, but what is clear is that this measure does not tell which mechanism is simpler. Stage 2 provides a “threat” if it successfully induces the agents to bid their true valuations. *For this stage to work as expected, it should be as straightforward as possible.* We are thus back to the “Which is easier?” question above.