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Instability in the basic New Keynesian model under limited information[1].

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Asbtract.

New solutions to the basic standard New Keynesian model are explored. I extend De Grauwe's model (2012), distinguishing two types of agents and different expectations rules. The central bank fixes the rate of interest. Families and firms determine aggregated demand and supply. Neither of them follows the hypothesis of perfect rational expectations. However, Popper's principle of rationality is applied. From a situation of limited information, even though they learn through rational processes, they are unable to understand their mutual behaviour. Therefore, the expectations in the three equations do not coincide. As a result, the solution does not tend to a single, stationary equilibrium. This conclusion does not depend on the hypothesis of the "animal spirits". Finally, the possibility of a successful learning process is studied. It is considered whether the central bank could learn from the data, finally reaching a stationary optimum equilibrium. The answer is no. The New Keynesian model seems to be basically unstable when agents have limited information. The problem lies in the impossibility to get adequate coordination.

Keywords: Business Cycles, Imperfect Information, Learning, Monetary Policy.

JEL codes: D83, E10, E32, E52.

1. Object.

The basic New Keynesian model has been proposed, among other authors, by Woodford (2001a, 2003), Gali (2008b), De Grauwe (2012). This model is one of the contemporary mainstream macro models. In general, New Keynesian theory puts together two ideas, the structure of the Real Business Cycle theory, with the Keynesian proposal that there are imperfect competition and nominal rigidities (Milani, 2012). Consequently, these models try to give coherence to two different ideas. One is equilibrium and markets clearance. So the business cycle should come from exogenous stochastic shocks. And the other, staggered wages, price contracts and the relevance of monetary policy (De Vroey and Malgrange, 2011, pp. 18-19).

In this paper, I consider De Grauwe`s (2012) model. In order to give reason of business cycles, De Grauwe develops the hypothesis of animal spirits (conceptually similar to the hypothesis of sunspots). The animal spirits hypothesis attempts to explain the economic cycles in GDP and employment on the basis of economic agents that have imperfect information and bounded rationality. Self-fulfilling non-fundamental stochastic shocks to beliefs are proposed to give reason of endogenous economic cycles, for example, Farmer and Guo (1994) and Benhabib and Farmer (1994). First, agents would apply simple rules to predict the future. Then they would check their results: they would compare them with data. The predictions would be modified according to the

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achieved success. Finally, the agents would move through waves of optimism/pessimism.

I analyse the solution of a modified De Grauwe's model (2012). First, only the deterministic model is taken into consideration. Second, two types of agents are distinguished. Third, these agents have got different expectations rules. Fourth, expectations in the formation of interest rates are introduced. Now these rates are not moved by actual output gap, but by the expected output gap. Nor by actual inflation rate, but by the expected inflation (Galí 2008, p.79).

Deterministic model means that this New Keynesian standard model is analysed to study the business cycles that it would generate by itself, without the need to introduce the hypothesis of the animal spirits. Even removing the random disturbances. In De Grauwe's (2012) model, expectations are not formed rationally and there are waves of optimism and pessimism. This assumption does not intervene in this work, which analyses the model and does so based on the two simple ways of predicting that De Grauwe (2012) considers.

There are fundamentalist and extrapolative rules. Therefore, the hypothesis of rational expectations is not used. A fundamentalist rule is one that expects the variable to reach equilibrium at t + 1. An extrapolative rule is one which determines what will happen to the variable in t + 1 from what happened in t - 1.

There are two types of agents, each of which decides some equations of the model. Each type of agent uses a different formation of expectations. They ignore how the other makes decisions, though they try to anticipate these rules. Therefore, an agent believes the other one follows the same structure of decision. All in all, agents only know their own expectations, and they expect everyone else to be using the same rule.

From this initial point, it is studied if it generates a single, stable and stationary equilibrium. Consequently, if the model induces cushioned cycles that lead to that equilibrium over time. Mitra and Bullard (2000): the monetary policy rules should generate models with a determinate rational expectations' equilibrium. Also, when the agents, and crucially the central bank, perform a learning process, and they do not follow rational expectations, their recursive learning process should generate this rational expectations' equilibrium (Mitra and Bullard, 2000).

I consider the existence of learning processes which could modify the conclusions obtained. Both sets of agents can learn: they try to contrast the evolution of variables with what they expected to happen. They learn through econometric estimation, as in Sargent (1993).

2. Basic New Keynesian model and the specification in De Grauwe (2012)

The basic New Keynesian model consists of an aggregate demand equation (IS curve, derived from the Euler equation for consumer optimization), an aggregate supply equation (price setting rule for the monopolistic firms) and a rule for setting the interest rate, usually a formulation of the Taylor rule. E.g. Evans and Honkapohja (2003).

Within this family of models is the De Grauwe's system (2012, pp. 3-4).

$$y_t = a_1 E_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - E_t \pi_{t+1}) + \xi_1 \quad (1)$$
$$\pi_t = b_1 E_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \xi_2 \quad (2)$$
$$r_t = c_1 (\pi_t - \pi^*) + c_2 y_t + c_3 r_{t-1} + \xi_3 \quad (3)$$

Where y_t is the output gap, E_t means expectation, r_t is the nominal interest rate, π is the inflation rate, π^* is the target inflation rate (normalized to 0). ξ is white noise (normally distributed, mean zero, constant standard deviation). Hetzel (2013) analyses the ECB's policy with a similar model. White noise will be excluded from the analysis.

In the New Keynesian model, equations (1) and (2) determine the evolution of the inflation rate, given the development of the output gap. As well as the movement in this output gap based on the change in the real interest rate (Galí, 2008, p.49). The differences between Galí (2008) and De Grauwe (2012), in these equations, are given in De Grauwe's introduction of the lagged variables. In addition, De Grauwe dispenses with the natural interest rate. In Galí (2009), for example, the natural interest rate is subtracted, together with the expectation of future inflation, to the current interest rate (p.3). This type of natural interest is introduced because the output gap appears as the difference between observed and natural. Natural output gap would be the level prevailing if all prices were flexible (Gali, 2008b, p.48) and markets were clearing. Equation (3) contains the fixation of the nominal interest rate by the monetary authority.

The natural interest rate seems to vary (Carlstrom and Fuerst, 2016) and a possible solution is to subtract from the interest rate only the expectation of inflation, given that, in the calibration of the model, the variables appear in terms of variables without trend. In theory, it is feasible. In practice, it requires the employment of the exact procedure to remove the trend. But this choice is theoretical and not testable (Kikut and Muñoz, 1994, p.5). This discussion is not the object of this work. This election does not seem to change the conclusions obtained here.

3. The modified model.

In the model, the following variation has been introduced. In equation (3) the interest rate ceases to depend on π_t and y_t, to become dependent on $E\pi_{t+1}$, Ey_{t+1}. Double expectation is introduced regarding inflation and output gap.

$$r_t = c_1 (E_t\pi_{t+1} - \pi^*) + c_2 E_t y_{t+1} + c_3 r_{t-1} + u_t \qquad (3)'$$

Now the model has three non-predetermined variables: output gap, inflation rate and interest rate[3].

For example, among others, Woodford (2001b); Galí (2008, pp. 79-81); Clarida, Galí, and Gertler (2000) estimate a forward-looking Taylor rule where inflation and output gap expectations are introduced (p.150). Also, Mitra and Bullard (2000, p.8), who consider that expectations are formed in the same way in all equations, that is, all agents share them, and use identical calculation algorithms and the same algorithms to learn and reformulate those expectations (pp. 9 and 24). Likewise, expectations could be formed in four ways (current data, data with delays, expectations regarding the future, expectations regarding the present).

4. Two types of agents.

There are two types of agents. On the one hand, the planning agent or central bank (CB), which establish the monetary policy: it sets the nominal interest rate. On the other hand, the representative agent (RA) that decides the aggregate supply and demand. Therefore, the model differentiates between those who decide monetary policy (CB) and those who develop supply and demand

[3] Following Buiter (1983): a non-predetermined variable is one for which its current value is a function of the expectation we have in the present, about the future values of endogenous and exogenous variables. Then they can respond instantly to changes in expectations. Any modification in expectations generates immediate adjustments in output gap, inflation and interest rate.

decisions (RA). Both can follow different rules for predicting the relevant macroeconomic variables. They do not have rational expectations. Or, at least, not necessarily.

The model can be classified as a 'bottom-up model' (De Grauwe, 2010): all agents have cognitive limitations and limited information. Although they learn from experience.

5. Expectations.

Agents do not form expectations rationally. Where rational expectations mean that subjective probability distribution is equal to objective probability distribution (weak form), e.g. Tesfatsion (2018), p.2. Additionally, agents know all the information known to the modeller (strong form, op. cit., p.4). That is, agents would have every relevant piece of information about expectations, deterministic exogenous variables, properties of the probability distributions, realized values for all endogenous variables and stochastic exogenous variables (p.4).

Evans and Honkapohja (2002) divide the rules for setting interest rates between rules based on fundamentals and rules based on expectations. The rules based on fundamentals are fixed in relation to white noise processes and, sometimes, lagged output gap. The rules based on expectations also suggest a relationship with the expected output gap and inflation rate. It is not exactly the distinction of this work, since the expectations are fixed previously, and they decay into some rule of formation of those predictions. Evans and Honkapohja (1999) distinguish three learning rules. Eductive learning, adaptive learning and rational learning. Eductive is conditional on the continuous application of rational expectations hypothesis (applying the true mathematical conditional expectations), so the solution converges towards rational expectations. Adaptive learning depends on iterative regression procedures. There is no guarantee to reach that rational expectations solution.

I propose a model where agents neither form expectations rationally (in this sense), nor know every relevant factor necessarily. A model where limited information is a central characteristic. The two types of agents try to make the best forecasts, attempting to anticipate the other agents' expectations, but without having an exact knowledge of them.

Popper's principle of rationality applies: "The rationality principle states that each agent acts adequately or appropriately to her situation as she sees it, given her aims" (Frederick, 2013, p. 62, explaining Popper's ideas). In this sense, a rational agent is able to improve his initial theory, or even to act in a way which conflicts with this initial theory, in order to adapt more efficiently to the situation (op. cit., p. 66).

Following De Grauwe (2012), there are two rules to determine the evolution of economic variables. Fundamentalist rule: the agents estimate the steady state value of the output gap (normalized at 0) and they use it to predict the future output gap.

$$E_t^F y_{t+1} = y^* = 0 \qquad (4)$$

In relation to the inflation rate, the announced inflation target is considered to be credible. They believe the objective will be attained.

$$E_t^F \pi_{t+1} = \pi^* \qquad (5)$$

Extrapolative rule. The agents do not know or do not believe in the steady-state output gap. They extrapolate the previous observed output gap into the future. This is a type of adaptive expectations. They predict by inductive experience.

$E_t^e\, y_{t+1} = y_{t-1}$ (6)

$E_t^e\, \pi_{t+1} = \pi_{t-1}$ (7)

The CB hesitates between three policies: applying a fundamentalist rule regarding the output gap and inflation rate, or applying it only to one of those two variables. It may fix the interest rate on the basis of confidence: controlling the inflation rate, the national income can be maintained close to its trend or natural value. Or it may deem the evolution of the output gap to be the only relevant task. The inflation target would be immediately achieved, due to the CB's credibility. I analyse the three possibilities.

The RA builds expectations by the extrapolative rule. There is no confidence that the CB will maintain any fundamentalist objective. They thus take into account the past evolution in order to determine the evolution of aggregate supply and demand. But the interest rate continues to be decided by the CB.

CB and RA are not aware that they do form their expectations differently. They do not know the real rule of formation of expectations, although they understand that it is possible that it does not coincide with the assumptions.

6. Some additional considerations about the scientific literature: the New Keynesian model and the effectiveness of monetary policy.

De Grauwe (2012) states that, when the private sectors of the economy are forward looking, there are two difficulties that the monetary policy of the central bank must face (Evans and Honkapohja, 2002, p.7). First, that the rules of monetary policy and the expectations of agents can generate a disequilibrium. Second, that those rules could lead to the indeterminacy of equilibrium: multiple equilibria with rational expectations. The economy would not reach the optimum. However, these authors conclude that if the interest rate rule is carefully designed, there is no indetermination or disequilibrium.

Galí (2008) analyses the mathematical conditions so that the standard New Keynesian model has a determinate equilibrium. The model proposed here differs from that structure, because it follows De Grauwe (2012), and also because the expectations of equation (3)' may not coincide with the expectations of the other two equations (1) and (2).

Evans and Honkapohja (2003), based on the fact that agents do not have perfect rational expectations, consider whether a stable and optimal equilibrium is attained. That is, if that equilibrium is achieved and if it is the same as what would exist if all the agents had perfect rational expectations. The answer they obtain is the following, if the CB sets rules based on expectations and the expectations of the private agents are correctly considered, that optimal equilibrium is necessarily reached.

There is an open debate about the effectiveness of monetary policy regarding the output gap. Applying a Taylor rule, Galí (2008) comes to a double conclusion, similar to Woodford (2001b). That a policy that is very active to control the output gap reduces the utility of the representative consumer, because it increases the variance of the output gap and inflation (p.83). Especially when the coefficient takes values as high as unity (p. 84). And second, that the monetary authority achieves lower welfare losses when it responds only to movements in the inflation rate, and the less the greater that response. "Hence, and at least in the context of the basic New Keynesian model considered here, a simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy" (p.84). The central bank or planner would not take into account the state of the economic cycle in the national income. In the same way, Mitra and Bullard (2000) recommend monetary policies with little or no reaction to the output gap (p.5). Clarida, Galí and Gertler (2000) consider that the coefficient that multiplies the inflation rate must be higher than 1, otherwise the policy would open the possibility of 'bursts of inflation and output that result from self-fulfulling changes in expectations' (p.178).

De Grauwe (2008, pp. 28-30) considers this possibility. Unlike Galí (2008) and Woodford (2001b), De Grauwe concludes that the intensity of the economic cycle increases. This would be due to the animal spirits hypothesis. Consequently, De Grauwe (2012) defends that a monetary policy that considers only the control of inflation is not an optimal policy ("strict inflation targeting is unlikely to be optimal", p. 69).

Modifying Taylor's equation by introducing expectations regarding the future, together with the fundamentalist formation of these expectations of the CB with respect to the output gap, is equivalent to centering the CB's performance around exclusively the objective of the inflation rate, within an economy that moves through extrapolative movements.

The conclusion reached in this work is: a monetary policy which is active regarding inflation, neutral with respect to the output gap, could produce, in the context of the New Keynesian analysis, practically recurrent economic cycles. It is necessary for the CB to place the output gap also as the objective of its control. Therefore, I obtain a conclusion similar to De Grauwe (2012). On the otrher hand, in a New Keynesian model, a policy considering the control of output gap, but not the evolution of the inflation rate, could produce explosive cycles (without a stable solution).

Finally, monetary policy should be considered in the context of a complex social and economic system, where agents make decisions with limited information. Equilibrium is not assured. Learning is not a feasible solution, since agents may face a serious problem of omitted variables. Agents should reconsider the variables in que equations, but nothing guarantees this will be the result.

7. Calibration and data.

The values used are usually: a_1 = 0.5. a_2=-0.2. b_1= 0.5. b_2=0.05. c_1=1.5. c_2=0.5. c_3=0.5 (De Grauwe, 2012, Galí 2008, among others). Mitra and Bullard (2000) lower b2 to 0.024. Taylor's (1993) initial estimates were 1.5 (c1) y 0.5 (c2). With a constant coefficient of 0.04 and a 2% inflation target.

Mitra and Bullard (2000), in a model without lagged variables use the following calibration in relation to c1 and c2: They must be between 0 and 4. If the coefficient that multiplies the inflation rate is greater than 1, it would be an active rule (p.13). The value of the coefficient of response to inflation should be above 1, so that the original model has a certain solution (Gali, 2008b, p.22). The values of 1.5 and 0.5 (0.5/4 for quarterly values) would be approx. Consistent with the variations observed in the type of the Federal Reserve in the Greenspan period (Gali, 2008b, p.52). However, 'Response to inflation deviations (...) should be 1.5, and the response to output deviations (...) should be 1' (Carlstrom and Fuerst, 2016, p.2).

The introduction of the exchange rate (with a delay) appears in many researchs as a relevant factor (Nelson, 2000). In this paper, it is not important to introduce new variables, given that the object of study is the evolution of the model and its stability, rather than the strict econometric adjustment to data.

Next, I simulate the cyclical behavior of the UK economy. Data taken from the Bank of England[4]. The Hodrick-Prescott filter is applied in such a way that the output gap, the inflation rate and the interest rate equal zero on average (1830-1900). De Grauwe and Ji (2016) also use this HP filter and find a high correlation among the GDP growth rates of the eurozone countries. The behaviour of the three variables is simulated from 1900 on.

8. The solution generated by the model.

When values are given to expectations operators, the general model becomes concrete (decays) and takes specific values. The initial model has non-predetermined variables, but once the expectations in past or present values are specified, the model stops having them. As expectations can be fundamentalist or extrapolative (adaptive), the model lapses into six basic cases.

Determinate solutions are considered in the models: if the solution is unique, stable and stationary, for the three endogenous variables considered, output gap, inflation rate and interest rate. The condition is that all eigenvalues are in the unit circle (eg, Galí, 2008).

Case A, the model contains rational expectations. In consequence, expectations coincide with the actual values.

$$y_t = a_1\, y_t + (1 - a_1)\, y_{t-1} + a_2\, (r_t - \pi_t) + \xi_1 \qquad (1a)$$
$$\pi_t = b_1\, \pi_t + (1 - b_1)\, \pi_{t-1} + b_2\, y_t + \xi_2 \qquad (2a)$$
$$r_t = c_1\, (\pi_t - \pi^*) + c_2\, y_t + c_3\, r_{t-1} + \xi_3 \qquad (3a)$$

Case B. All expectations are resolved in an adaptive way: extrapolative.

$$y_t = y_{t-1} + a_2\, (r_t - \pi_{t-1}) + \xi_1 \qquad (1b)$$
$$\pi_t = \pi_{t-1} + b_2\, y_t + \xi_2 \qquad (2b)$$
$$r_t = c_1\, (\pi_{t-1} - \pi^*) + c_2\, y_{t-1} + c_3\, r_{t-1} + \xi_3 \qquad (3b)$$

Case C. All expectations are fundamentalists.

$$y_t = (1 - a_1)\, y_{t-1} + a_2\, (r_t - \pi^*) + \xi_1 \qquad (1c)$$
$$\pi_t = b_1\, \pi^* + (1 - b_1)\, \pi_{t-1} + b_2\, y_t + \xi_2 \qquad (2c)$$
$$r_t = c_3\, r_{t-1} + \xi_3 \qquad (3c)$$

The following three cases distinguish between types of agents. Private agents (RA) decide on the IS and Phillips curve equations and they do so extrapolatively. They do not believe that the CB manages to maintain the objectives of the monetary policy and prefer to value the past data. The CB decides on the third, setting the interest rate according to different definitions of expectations.

In case D, the CB is fundamentalist, with respect to the inflation rate and output gap.

$$y_t = y_{t-1} + a_2\, (r_t - \pi_{t-1}) + \xi_1 \qquad (1d)$$
$$\pi_t = \pi_{t-1} + b_2\, y_t + \xi_2 \qquad (2d)$$
$$r_t = c_3\, r_{t-1} + \xi_3 \qquad (3d)$$

Case E. The CB is fundamentalist with respect to the output gap.

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4 Three Centuries of Data, http://www.bankofengland.co.uk/research/Pages/onebank/threecenturies.aspx. DD is domestic demand, in constant 1900 prices, from Mitchell (1988). RAT is Bank of England Rate. CPI is consumer prices index, from ONS (O'Donoghue et al (2004)).

$y_t = y_{t-1} + a_2 (r_t - \pi_{t-1}) + \xi_1$ (1e)
$\pi_t = \pi_{t-1} + b_2 y_t + \xi_2$ (2e)
$r_t = c_1 (\pi_t - \pi^*) + c_3 r_{t-1} + \xi_3$ (3e)

Case F. The CB is fundamentalist in relation to the inflation rate.

$y_t = y_{t-1} + a_2 (r_t - \pi_{t-1}) + \xi_1$ (1f)
$\pi_t = \pi_{t-1} + b_2 y_t + \xi_2$ (2f)
$r_t = c_2 y_t + c_3 r_{t-1} + \xi_3$ (3f)

Assuming that the values are normalized to $\pi^* = 0$, for example accepting the trend calculated by HP filter as the interest rate target, model A contains the following solution.

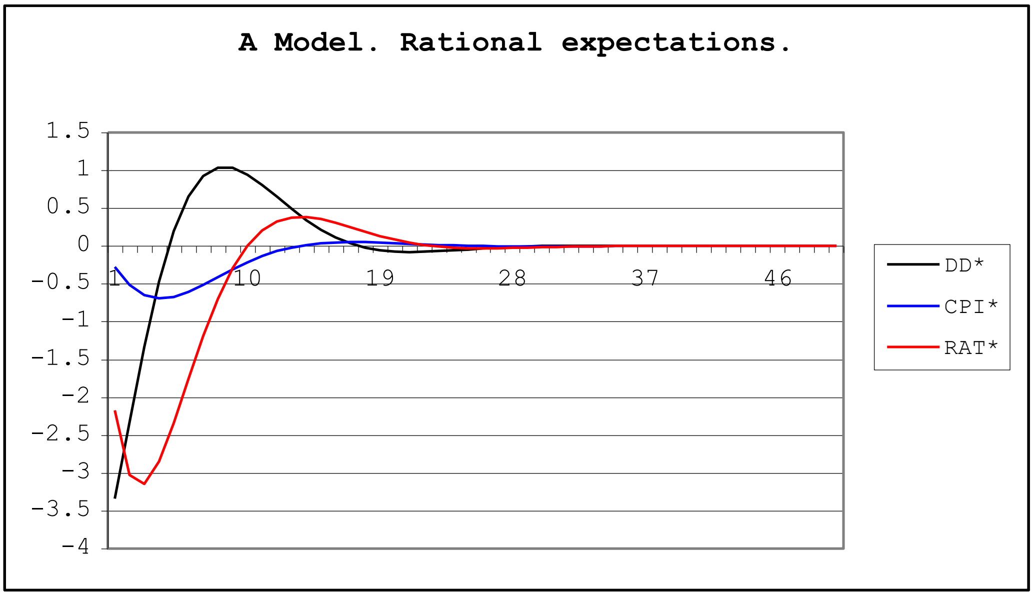
$$\begin{pmatrix} a_1 & a_2 & -a_2 \\ -b_2 & b_1 & 0 \\ -c_2 & -c_1 & 1 \end{pmatrix}\begin{pmatrix} y_t \\ \pi_t \\ r_t \end{pmatrix} = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_3 \end{pmatrix}\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{1t} \\ \varepsilon_{1t} \end{pmatrix} \quad (4)$$

$A\ z_t = B\ z_{t-1} + \xi$ (5)

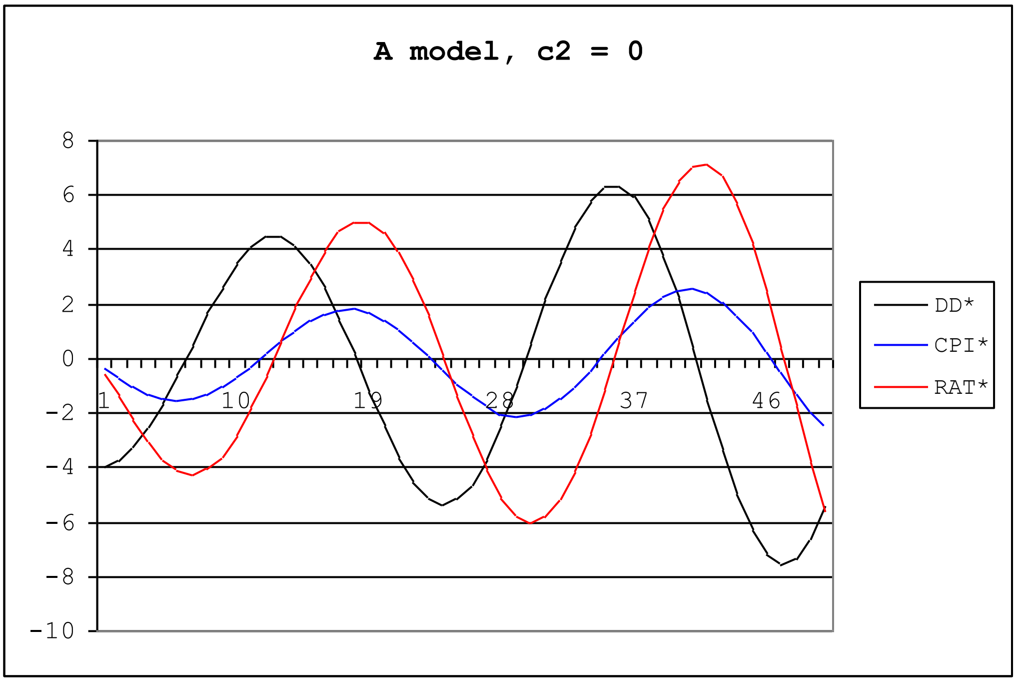
$z_t = A^{-1} B\ z_{t-1} + A^{-1} \xi$ (6)

With the calibration used, models A, B and C have vectors $A^{-1}B$ with eigenvalues less than unity. This generates damped oscillations, and the system tends to a stable and unique equilibrium in $y_t = 0$, $\pi_t = 0$.

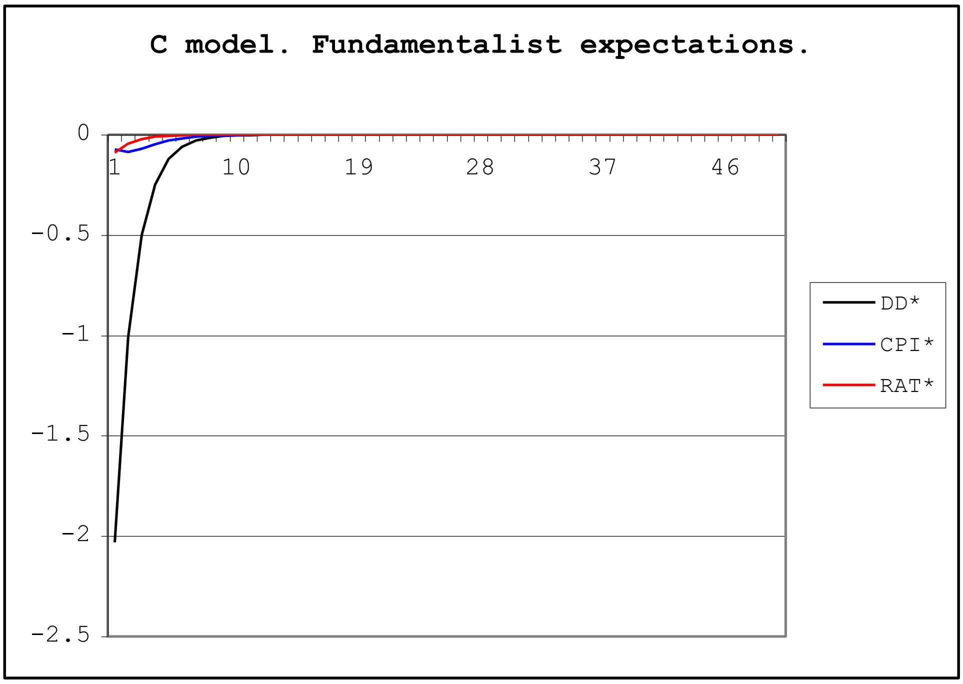
I simulate the values over the next 50 years, from the values in 1900 for the British economy. DD * is the cycle of the output gap once the trend calculated by the HP filter has been subtracted. CPI * is for the inflation rate. RAT * for the nominal interest rate.



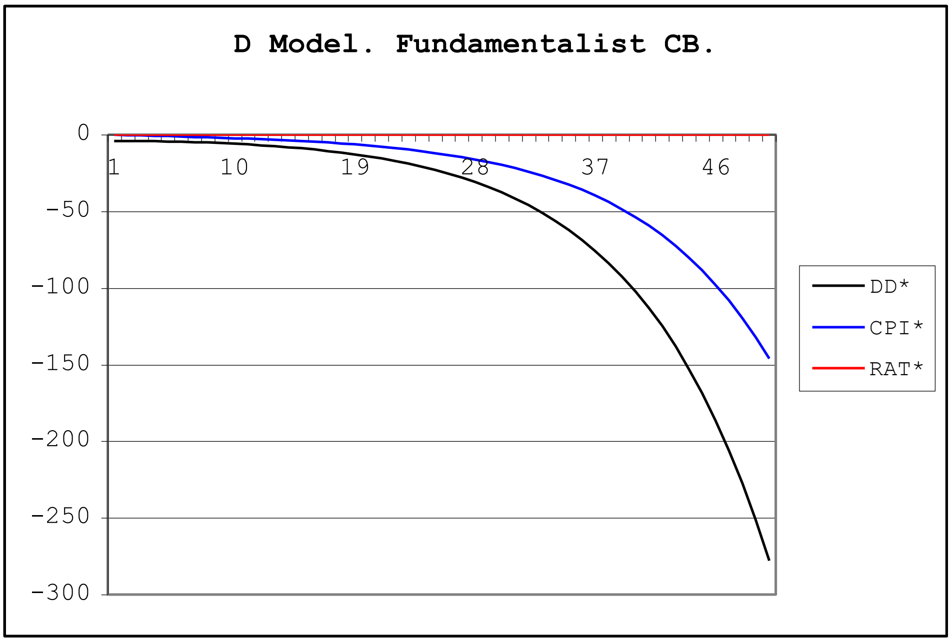
However, if monetary policy ceases to have the simultaneous control over the evolution in the output gap ($c_2 = 0$), then explosive cycles are generated.



Model B follows exactly the same dynamic, with a stationary equilibrium solution, unless monetary policy ignores the evolution of the output gap. Model C shows that if everyone followed fundamentalist expectations, the economy would quickly drift towards stable stationary equilibrium.

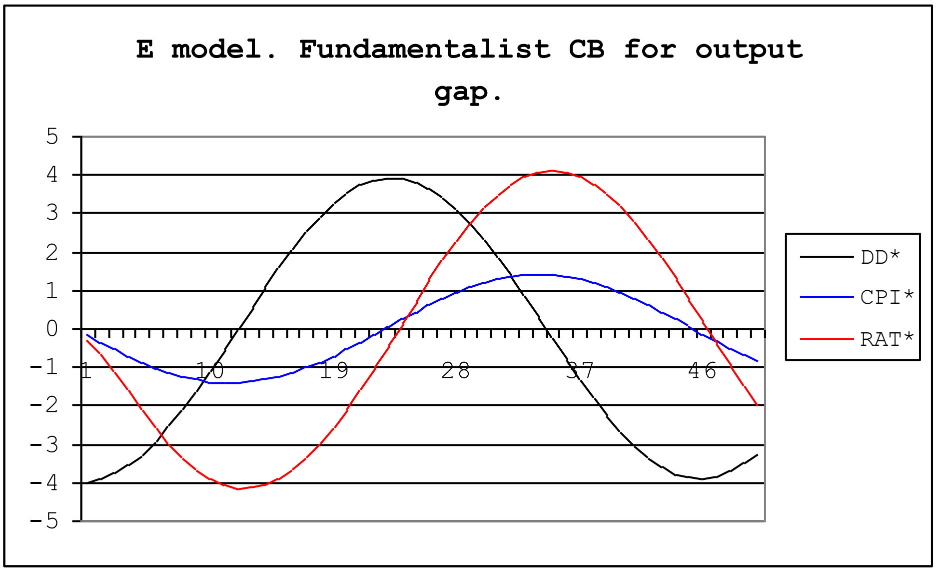


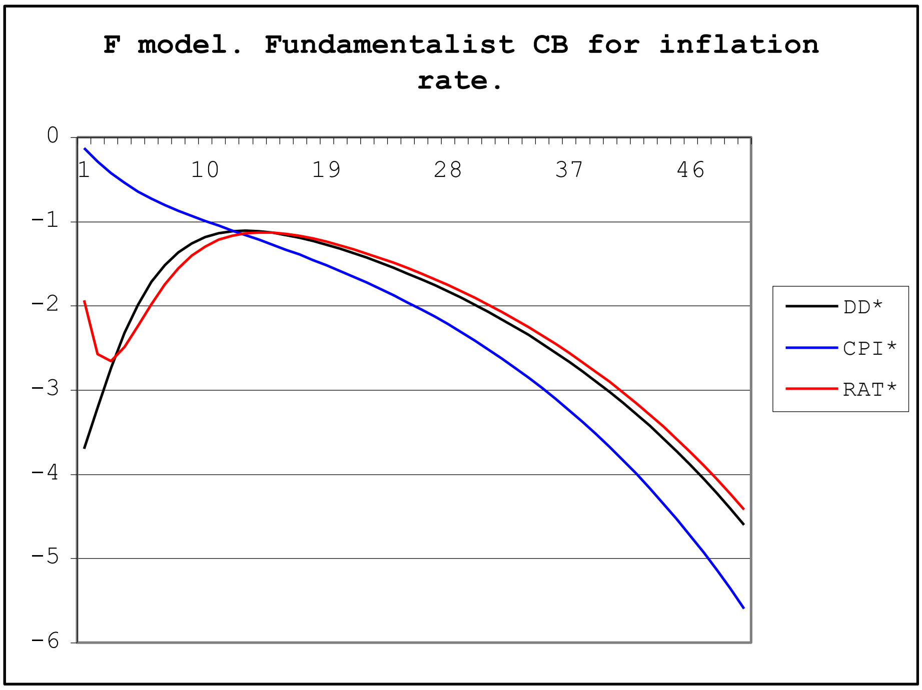
The model D leads to an explosive solution. The system does not tend to a stationary equilibrium.



One of the three eigenvalues is greater than one. The system is explosive when interest rates do not respond neither to the inflation rate, nor to the output gap, and yet private agents have no confidence in that monetary policy and its effectiveness. Therefore, the New Keynesian model discards a policy of constant, autonomous growth of the money supply and prefers an active policy. (Galí considers this type of monetary rule to be non-adequate in front of monetary demand shocks, 2008, pp. 84-85).

Cases E and F. The CB's sole objective is either the inflation rate (E) or the output gap (F).





When the CB pursues only the inflation target (E model), recurring cycles can be produced. Since two of the eigenvalues of the matrix take a value of approximately 1 (0.99971). If the CB establishes an objective of output gap (F model), it can cause values of output gap and rate of inflation that progressively move away from equilibrium. In addition, in the E model, if the CB increases the weighting of the lagged interest rate, softening its monetary policy more, then the matrix of the E model happens to have two eigenvalues above 1. Not so for F model.

Three conclusions. First, the New Keynesian model is stable under rational expectation, under the usual calibration. This is under the assumption that all agents are the same, since all of them know the exact model (De Grauwe, 2008, p. 42). Second, the New Keynesian model must set monetary objectives of inflation rate and output gap. Because it is possible that monetary policy is not credible and economic agents maintain rules such as extrapolative, among other possible. Third, the New Keynesian model can generate unstable situations, with a progressive distancing from an equilibrium situation in output gap and inflation. This is due to the possible lack of coordination among economic agents. This is a similar situation to that considered by Evans and Honkapohja (2006, p. 25): if agents try to forecast through adaptive learning, but the central bank applies a rule based on the presumption of being in the stationary rational equilibrium.

9. The impact of learning. Does learning converge towards the perfectly rational expectations equilibrium?.

I focus on models E and F. The difference of agents and expectations rules may generate recurrent cycles, even explosive ones. A possible solution would be found in learning. The possibility of reaching a single stationary equilibrium through experience must be analysed. If the agents understood that they have different patterns of behaviour, and their expectations followed suit, they would approach the real behavior of the other agents. The end would be the stability under the rational expectations equilibrium: A model.

Evans and Honkapohja (2002) establish a double criterion for valuing learning models. First, if with rational expectations, the system gives a single stationary solution. If so, then the model is determinate. Second, if with adaptive learning, usually by simple regression, this stationary solution would be stable, that is to say reached asymptotically, through this learning process. Solution that would coincide with the result of rational expectations. The learning process is considered by Evans and Honkapohja (2006) as the minimisation of a quadratic loss function. To obtain the minimal values for the deviations of the output gap and the inflation rate, once a discount factor is applied. This type of learning is necessarily based on the knowledge of every relevant factor working in the economic system. If there is just one factor not considered, the minimisation cannot be succesful, since it is trying to get a minimal value of a deviation whose determinants are not known. A completely different approach from the point of view maintained in this work. I consider the possibility of uncertainty over the variables taking place in the system, due to unknown expectations.

The analysis carried out of the New Keynesian model has shown that, if rational expectations are maintained, it is determinate under a usual calibration (A model). When it is assumed that the agents are able to accurately anticipate the expectations of the other agents: their subjective expectation equals objective facts.

This second question is the one posed here for cases E and F, in which the CB, based on a situation of limited information, has decided not to react to the output gap, or to the inflation rate. Not taking into account thar the RA is reacting to the lagged output gap and the lagged inflation rate, CB's model forgets relevant variables and CB tries to know and control the whole economic system with this omission of variables.

Here, the agents know the model's structure. However, they do not know the exact value of the parameters, nor do they know what other agents will do and so they try to predict the reciprocal expectations rules. In addition, they do not have a complete list of variables, because the variables depend on the expectations rules.

In this context, the CB's role in deciding monetary policy is crucial.

Learning is given by econometric methods: the different agents perform an estimation by simple linear regression or by two-stage least squares (2SLS). For example, Marcet y Sargent (1988). The agents could learn in other ways, minimising the differences between the expected and the perceived values. But the most powerful method must be the result of econometric theory. The objective is to know if, applying the best methods, the agents manage to approximate the balance of rational expectations in time.

The agents estimate from their original assumptions. It makes no sense that their estimation is made on factors not considered or from a different structure. This would be an attack on the assumption of limited information. Economic theory must study real agents in actual conditions, not omniscient individuals. Therefore, individuals are rational (Popper's principle): they adapt in the best way, but their knowledge is limited.

In model E, taking into consideration 20 simulated data, the agents reach the following estimate.
The RA estimates:

$$y_t = y_{t-1} - 0.2\,(r_t - \pi_{t-1}) \quad (7)$$
$$\pi_t = \pi_{t-1} + 0.05\,y_t \quad (8)$$
$$r_t = 1.5\,\pi_{t-1} + 0.07389\,y_{t-1} + 0.5\,r_{t-1} \quad (9)$$

Estimation by simple linear regression[5]:

[5] Easyreg program. 2SLS does give very similar results.

Y = DDE	Coefficient	t Student	Y = CPIE	Coefficient	t Student	Y = CPIE	Coefficient	T Student
X(1) = RATE-LAG1[CPIE]	-0.2	-3624974551	X(1) = LAG1(CPIE)	1	11470054239	X(1) = LAG1[CPIE]	1.4926108	366733003
X(2) = LAG1[DDE]	1	2.319E+10	X(2) = DDE	0.05	1419710077	X(2) = LAG1[DDE]	0.0738916	428474123
						X(3) = LAG1[RATE]	0.4926108	357242109
n	20			20			20	
RSS	0			0			0	
R2	1			1			1	
R*2	1			1			1	

This model is called, in the scientific literature, the perceived law of motion (PLM). The E model, as such, is the ALM (actual law of motion). The problem is about minimising the distance between the data and the prediction from the theoretical equations that express what you believe it is happening. This is equivalent to trying to reduce business cycles.

The CB estimates, on the other hand:

$y_t = 0.998980\ y_{t-1} \quad - 0.132125\ r_t$ (10)

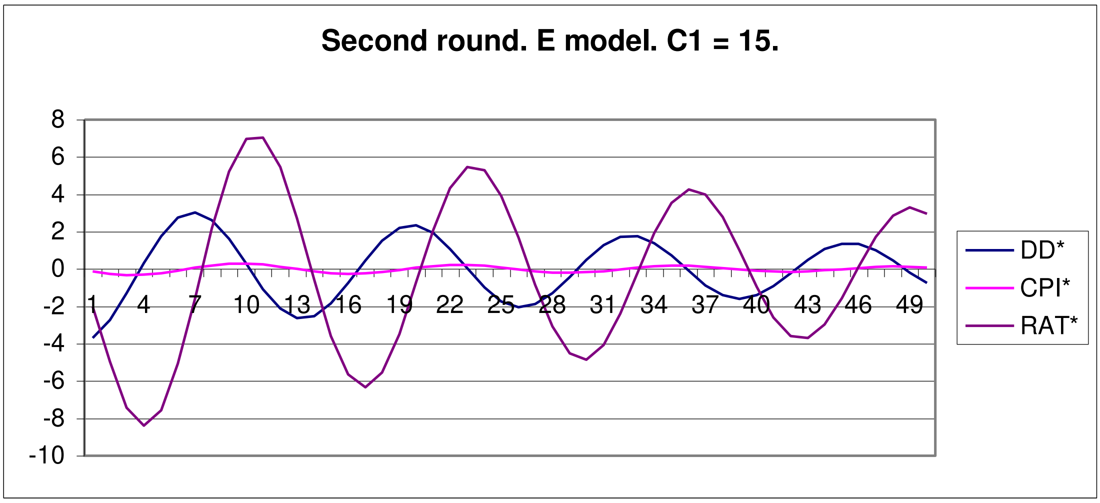
$\pi_t = \pi_{t-1} \quad + 0.05\ y_t$ (11)

$r_t = 1.5\ \pi_t \ + 0.5\ r_{t-1}$ (12)

This is its PLM, with a very significative regression (simple linear regression).

Y = DDE	Coefficient	t Student	Y = CPIE	Coefficient	t Student	Y = CPIE	Coefficient	t Student
X(1) = LAG1(DDE)	0.998980	3397.44	X(1) = LAG1(CPIE)	1	11470054239	X(1) = CPIE	1.5	2853592722
X(2) = RATE	-0.132125	-531.15	X(2) = DDE	0.05	1419710077	X(2) = LAG1[RATE]	0.5	2784056642
N	20			20			20	
RSS	0.00021			0			0	
R2	1			1			1	
R*2	1			1			1	

From the estimation, private agents would continue to be extrapolative, believing that the CB reacts actively against deviations of the inflation rate and very gently against deviations of the output gap with a delay. For its part, the CB has estimated a system of recurrent cycles, so that it would decide a more active monetary policy: it would increase the coefficient c_1 that relates the inflation rate with the interest rate. Because the matrix of its system of equations indicates that the cycles become more cushioned as that value increases. Assuming that c_1 takes a value of 15, for example, there is an improvement, although the agents do not agree about what is happening, and the monetary policy would not be optimal.



It is, of course, possible for the CB to rectify and add an objective for the output gap, until an optimal monetary policy is reached. But nothing guarantees that an agent includes unknown factors.

In F model, the solution from learning is more complex. It would necessarily imply changes in the variables considered, because when the CB tries to estimate its fundamentalist model, it finds that the estimated equations have serious problems: there are estimated coefficients with an incorrect sign.

Y = DDF	Coefficient	t Student
X(1) = LAG1(DDF)	0.6384921	6.631
X(2) = RATF	0.2925965	2.763
N	20	
RSS	0.2045	
R2	0.9694	
R*2	0.9677	

After 20 simulated data, this problem makes clear to the CB that its model and its monetary policy both are erroneous. So finally it would be possibly forced to include the inflation variable, especially with a delay: it offers an optimal adjustment.

This is possible, but not for sure. Economic agents can not find new variables, they can prove, but they can not find them inductively. Then the solution to the problem lies in breaking the assumption of limited knowledge. Imagination can cause new unknown factors to be introduced, but since it is not a matter of data perception or reasoning, the result is uncertain. “Intellectual intuition and imagination are most important, but they are not reliable” (Popper, 1962, p.28).

Assuming that CB’s estimation includes a new variable, the logical question is: Would CB decide to modify your monetary policy?. Evidently yes, because the matrix has an eigenvalue greater than 1, and the solution is explosive. However, controlling only the output gap can not reach equilibrium and must necessarily expand its objectives to include the inflation rate. If, for example, $c_2 = 0.5$ and $c_1 = 1$, then we can reach that equilibrium situation.

In conclusion, in the first case analysed, learning would lead to a better situation. Although not optimal. In the other model, the analysis make agents know they are erroneous. The solution would imply breaching the assumption of limited knowledge, so that agents can spontaneously include relevant variables. There is not a necessary and rational process that leads with certainty to this innovation. Coordination between agents is desirable, but not a compulsory conclusion.

As a matter of fact, the situation in which the agents find themselves is that of omissions of relevant variables (e.g., Clarke, 2005, pp. 342 ss.). This is an unsolvable problem. In fact, to include new variables could not be a solution if there is a continuity of omitted variables. However, "by including additional control variables in our specifications, we could very easily be making the bias on the coefficient of interest worse", and "knowing for sure requires knowing much more than we typically do in practice. In the absence of this kind of omniscience, we need an approach to achieving convincing experimental control that has fewer debilitating side effects" (op. cit., p. 350). Therefore, in this context of rational agents trying to establish the best decisions within the context of limited information available, the problem of omitted variables is very serious, and it can cause strong biases in predictions, and the consequent monetary policies mistakes.

10. Conclusions.

First, heterogeneous agents with uncertain expectations rules must be considered as a possibility to build models and crucially to design possible monetary policies. As De Grauwe (2008) says, the model builder can limit the rules that agents can use to make forecasts by "imposing the condition that forecasts must be consistent with the underlying model" (p, 6). It is compulsory to test the results when this condition is relaxed.

Second, the New Keynesian model can generate explosive cycles when there are different agents forming expectations with different rules, and they have a situation of limited knowledge. That is, when they are not able to coordinate, even if they are rational and they try to learn from experience. Explosive cycles mean disequilibrium.

Third, learning is not a solution, because there is not inductive or deductive procedure to rationally find omitted variables. It is possible, but not for sure. However, omitted variables are a consequence of different expectations rules. There is no certainty that a learning process can lead to determinate, stable and stationary solutions to the model.

Fourth, the absence of determination in a New Keynesian model, in that situation, does not depend neither on random exogenous variables, nor on the formation of self-fulfilling expectations. The lack of coordination among agents, with limited knowledge and different expectations, can cause this situation.

Fifth, the CB must follow active policies to control the inflation rate and the output gap, closely monitoring whether the result of the policies is a volatile situation or not. The reason is the economic agents may not be coordinating their expectations. This result is similar to Mitra and Bullard (2000) who contemplate indeterminate results when expectations are formed on delays.

Sixth, I mentioned the open debate about the effectiveness of monetary policy regarding the output gap. The result obtained in this work is the CB should pursue active policies considering objectives both with respect to the inflation rate and the output gap.

Monetary policy must be aware that it faces an uncertain economic system, with multiple agents pursuing hard-to-know objectives. And, for this reason, the CB should always contemplate the possibility that there are omitted variables, of difficult or impossible knowledge. I leave for successive works if that monetary policy should be one of commitment (rules), or it should resort to discretionary measures when considered necessary.

11. References.

Benhabib, J., & Farmer, R. E. (1994). Indeterminacy and increasing returns. Journal of Economic Theory, 63(1), 19-41.

Buiter, W. H. (1982). Predetermined and non-predetermined variables in rational expectations models. Economics Letters, 10(1-2), 49-54.

Clarida, R., Gali, J., & Gertler, M. (2000). Monetary policy rules and macroeconomic stability: evidence and some theory. The Quarterly journal of economics, 115(1), 147-180.

Clarke, K. A. (2005). The phantom menace: Omitted variable bias in econometric research. Conflict Management and Peace Science, 22(4), 341-352.

De Grauwe, P. (2010). Top-down versus bottom-up macroeconomics. CESifo Economic Studies, 56(4), 465-497.

De Grauwe, P. (2012). Lectures on behavioral macroeconomics. Princeton University Press.

De Grauwe, P., & Ji, Y. (2016). Animal spirits and the international transmission of business cycles.

De Vroey, M., & Malgrange, P. (2011). The History of Macroeconomics from Keynes's General Theory to the present. IRES Discussion Papers, 2011028.

Evans, G. W., & Honkapohja, S. (1999). Learning dynamics. Handbook of macroeconomics, 1, 449-542.

Evans, G. W., & Honkapohja, S. (2003). Adaptive learning and monetary policy design. Journal of Money, Credit, and Banking, 35(6), 1045-1072.

Evans, G. W., & Honkapohja, S. (2006). Monetary policy, expectations and commitment. Scandinavian Journal of Economics, 108(1), 15-38.

Farmer, R. E., & Guo, J. T. (1994). Real business cycles and the animal spirits hypothesis. Journal of Economic Theory, 63(1), 42-72.

Frederick, D. (2013). Popper, rationality and the possibility of social science. THEORIA. Revista de Teoría, Historia y Fundamentos de la Ciencia, 28(1), 61-75.

Gali, J. (2008a). The new Keynesian approach to monetary policy analysis: Lessons and new directions.

Galí, J. (2008b). Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.

Hetzel, R. L. (2013). ECB monetary policy in the recession: A New Keynesian (Old Monetarist) critique (No. 13-07). Federal Reserve Bank of Richmond.

Milani, F. (2012). The modeling of expectations in empirical DSGE models: A survey. In DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments (pp. 3-38). Emerald Group Publishing Limited.

Bullard, J., & Mitra, K. (2000). mDeterminacy, Learnability, and Monetary Policy Inertia, V Federal Reserve Bank of St. Louis Working Paper 2000% 030B.

Carlstrom, C. T., & Fuerst, T. S. (2016). The natural rate of interest in Taylor rules. Economic Commentary, 1.

Marcet, A., & Sargent, T. J. (1988). The fate of systems with" adaptive" expectations. The American Economic Review, 78(2), 168-172.

Muñoz, E., & Kikut, A. (1994). El filtro de Hodrick y Prescott: una técnica para la extracción de la tendencia de una serie. Banco Central de Costa Rica, Departamento de Investigaciones Económicas, DIENT-03-94/R, marzo.

Nelson, E. (2001). UK monetary policy 1972-97: a guide using Taylor rules.

Popper, K. (1963). Conjectures and refutations. The Growth of Scientific Knowledge.

Sargent, T. J. (1993). Bounded rationality in macroeconomics: The Arne Ryde memorial lectures. OUP Catalogue.

Taylor, J. B. (1993, December). Discretion versus policy rules in practice. In Carnegie-Rochester conference series on public policy (Vol. 39, pp. 195-214). North-Holland.

Taylor, J. B. (1999). A historical analysis of monetary policy rules. In Monetary policy rules (pp. 319-348). University of Chicago Press.

Tesfatsion, L. (2018). Introductory notes on rational expectations.

Woodford, M. (2001a). Interest and prices, manuscript, Princeton University. Chart, 1.

Woodford, M. (2001b). The Taylor rule and optimal monetary policy. American Economic Review, 91(2), 232-237.

Woodford, M. (2003). Interest rate and prices.

Woodford, M. (2005). Firm-specific capital and the New-Keynesian Phillips curve (No. w11149). National Bureau of Economic Research.

Woodford, M. (2010). Optimal monetary stabilization policy. In Handbook of Monetary Economics (Vol. 3, pp. 723-828). Elsevier.