Financial incentives for open source development: the case of Blockchain

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Abstract

Unlike traditional open-source projects, developers of open-source blockchain-based projects can reap large financial rewards thanks to a modern form of *seignorage*. I study to what extent this novel form of financing generates incentives to innovate. I consider a developer working on an open-source blockchain-based software that can be used only in conjunction with a specific crypto-token (itself a piece of open-source software). This token is first sold to investors via an Initial Coin Offering (ICO) and then traded on a frictionless financial market. In all equilibria of the game, in each post-ICO period there is a positive probability that the developer sells all his tokens on the market and, as a consequence, no development occurs. Anticipating this, the developer will hold the ICO only when his own funds are insufficient to sustain the development of the software. The equilibrium of the game is, in general, inefficient.

JEL classification: D25, O31, L17, L26,

Keywords: Blockchain, decentralized ledger technologies, Initial Coin Offering (ICO), seignorage, innovation, incentives, open source.

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1 Introduction

This paper studies a new mechanism to finance innovation: seignorage. Seignorage allows the developer of a blockchain-based open-source software to rip a direct financial benefit (in addition to indirect benefits derived from, for example, career concerns)via the creation of a token—itself a piece of open-source software—that must be used in conjunction with the software.

Historically, seigniorage are profits earned by a government by issuing a currency. As the economic activity within a country increases, the value of the currency used in this country also increases, and with it the profit earned by the government issuing this currency. The same mechanism allows developers of open-source blockchain-based projects to benefit from their work. As an illustration, consider a population of agents who wishes to exchange either a good or a service, but are prevented from doing so for lack of the required infrastructure. If this exchange can occur in electronic form, then the missing infrastructure may be a *protocol*, that is, the technical specifications governing the communication between machines. A developer may create the missing protocol, and profit from his innovation by simultaneously creating a *token* and establishing that all exchanges occurring using the protocol must use this token.[1] The developer owns the initial stock of tokens. It follows that, if the developer can credibly commit to a specific supply of tokens and the protocol is successful, then there will be a positive demand for tokens, a positive price for tokens, and positive profits earned by the developer.

Bolockchain enables this mechanism in three ways (see the next subsection for additional details on blockchain). It allows the developer to commit to a specific supply of tokens. Absent this commitment, because the marginal cost of creating electronic tokens is zero, the only possible equilibrium price for tokens is zero, leading to zero profits for the entrepreneur. Using blockchain technology, instead, the rules determining whether (and how) the supply of tokens increases over time can be fully specified initially within the software. If the software is open source, this commitment is credible because anybody can verify the software's source code.[2]

[1] Prices could be expressed in fiat currency (that is, in some numeraire). The important thing is that they need to be settled using the token.

[2] Conversely, it is not possible to finance the development of a closed-source protocol via seignorage, because it is not possible to verify the rules determining the supply of tokens. Of course, it is possible to finance the development of a closed source protocol via a set of fees/prices (see Section 5.1 for a discussion).

Furthermore, blockchain can be used to specify that only a given token can be used to transact using the protocol.[3] Finally, blockchain may be used to create the protocol.

This paper is the first to study the ability of seignorage to generate incentives for innovation. I build a model in which, in every period, a developer exerts effort and invests in the development of a protocol. Initially, the developer owns the entire stock of tokens, and can sell some to investors via an Initial Coin Offering (ICO), modeled as an auction. Subsequently, in every period he can sell or buy tokens on a frictionless market for tokens, in which both users of the protocol and investors are active. The developer can use the proceedings of the sale of tokens to either invest in the development of the protocol or to consume.

The main insight is that, if investors are price takers, then in any post-ICO period there is an anti-coordination problem. If investors expect the developer to develop the software in the future, this expectation should be priced into the token's current price. But if this is the case, then the developer is strictly better off by selling all his tokens, which allows him to "cash in" on the future development without doing any. On the other hand, if investors expect no development to occur, the price of the token will be low. The developer should hold on as many tokens as possible, exert effort and invest in the development of the protocol so to increase the future price of the token. In every post-ICO period, therefore, the equilibrium is in mixed strategy: the price of the token is such that the developer is indifferent between selling all his tokens (and therefore stop developing the protocol) or keeping a strictly positive amount of tokens (and therefore continuing the development of the protocol). The developer randomizes between these two options, in a way that leaves investors indifferent between purchasing tokens in any given period.

The equilibrium at ICO is instead in pure strategies. The important point is that, if the ICO is an auction, then the share of the total supply of tokens sold by the developer is announced initially. Because the incentives to exert effort and invest in the development of the software increase with the share of tokens held by the developer, investors can anticipate the amount of development that will occur in the period following the ICO, which will be reflected in the price of the token at ICO.

In addition, both at ICO and post-ICO there may be a coordination problem.

[3] Of course, it is always possible to modify the source code to accept a different token, therefore creating a "fork": a new protocol, with its own development, incompatible with the initial protocol.

Because of a cash constraint, in every period the developer cannot invest in the development of the software more than his assets. It follows that the developer may sell some of his tokens, as a way to accumulate assets and finance the future development of the software. The number of tokens that the developer needs to sell in order to finance future investments depends on the current price for tokens, therefore generating a coordination problem. If the price is high, the developer needs to sell few tokens and his incentives to invest and develop the software in the future are high. This, in turn, justifies the high price for tokens today. If instead the price today is low, in order to finance future development the developer needs to sell many tokens. But then his incentives to develop the software will be low, which justifies the fact that the price is low today. Therefore at ICO there could be multiple pure-strategy Nash equilibria, while post ICO there could be multiple mixed-strategy Nash equilibria.[4]

When choosing whether and when to hold an ICO the developer is therefore facing a tradeoff. If he holds an ICO, in every subsequent periods with positive probability he will sell all his tokens and not develop the software. Postponing the ICO therefore prevents the creation of a market for tokens and works as a commitment device, because the developer will hold all his tokens for sure and set the corresponding level of effort and investment. On the other hand, if the developer does not sell tokens at ICO he may lack the funds to invest in the development of the protocol. As a consequence, the developer never wants to hold an ICO if his own assets are sufficient to finance the optimum level of investment in the development of the protocol, but may hold the ICO as soon as his own funds are not sufficient to achieve the optimal level of investment.

The equilibrium of the game is, in general, not efficient. The first source of inefficiency is that, as already discussed, the developer may need to sell some tokens to finance the investment in the development of the protocol, but doing so implies that in every subsequent period he will develop the protocol with probability less then one. But even assuming that the developer has sufficient funds to invest optimally, there is a second, more subtle, source of inefficiency. The developer's level of

[4] Clearly, if there are network effects, then there is an additional coordination problem: for given sequence of effort and investment by the developer, there is a coordination problem among users, possibly leading to the existence of a "high adoption" and "low adoption" equilibria. The novelty here is that fixing one of the adoption equilibria, there are multiple equilibrium sequence of effort and investment arising from a coordination problem between investors and the developer.

effort and investment are set so to maximize the value of his stock of tokens. This value depends on the volume of the transaction occurring using the protocol during a given period of time.[5] In the first best, instead, effort and investment should be set so to maximize the *present discounted value* of the surplus generated by the protocol. That is, the fact that the protocol will be used and generate surplus over multiple periods is completely disregarded by the developer who, therefore, will set an inefficient level of effort and investment.

1.1 Blockchain-based protocols

The key premise of this paper is that blockchain can be the technological foundation of various other protocols. To illustrate this fact, it is useful to make an analogy between Blockchain and the *Internet Protocol Suite*.

The Internet protocol suite (commonly known as TCP/IP) was developed in the late '60 and early '70 to allow for the decentralized *transmission* of data, that is, transmission of data via a network of computers in which no node is, individually, essential to the well functioning of the network. It was financed by the Defense Advanced Research Projects Agency (DARPA), with the goal of increasing military communication resilience by moving from a hub-and-spoke model of communication to a complete (or mesh) network model of communication (see Figure 1).[6] The Internet Protocol Suite is the technological foundation of a second set of protocols, also called *application layer protocols*. Those protocols make use of TCP/IP to handle specific types of data in specific context: HTTP for accessing web pages; SMTP, POP, and IMAP for sending and receiving emails; FTP for sending receiving files; and so on.

Blockchain further expands the possible operations that can be performed by a network of computer in which no node is essential. Like TCP/IP, it allows for the decentralised transmission of data, but also permits the decentralised storage, verification and manipulation of data.[7] Blockchain is also similar to TCP/IP in that

[5] This will result from an application of the equation of exchange, usually employed to link a country's price level, real GDP, money supply and velocity of money.

[6] See Hafner and Lyon (1998), in particular the description of the work of Paul Baran (pp 53-64).

[7] Sometimes a distinction is made between blockchain and *decentralized ledger technologies*, where blockchain refers to a specific way to maintain a decentralized ledger. This distinction is not relevant for the purpose of this paper. Another distinction is between "blockchain" meaning the technology, and "the blockchain" meaning a specific application of the blockchain technology,

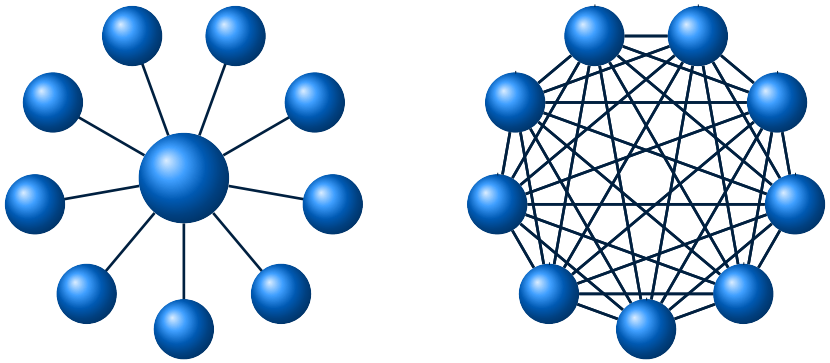


Fig. 1: Hub-and-spoke (left) and mesh (right)

it provides the foundation for a number of other protocols. The most well-known is the Bitcoin protocol: a protocol allowing a network of computers to store data (how many Bitcoin each address owns) and to enforce specific rules regarding how these data can be manipulated (no double spending). The Bitcoin protocol is not only the oldest and most famous application of blockchain technology, but also well illustrates an important point: absent blockchain technology, the same type of data can be maintained only within a traditional organization (typically a bank).

Numerous other open-source blockchain-based protocols currently exist or are being actively developed. In addition to several cryptocurrencies (such as Monero, ZCash, Litecoin), there are protocols for building decentralized computing platforms that can run applications on a network rather than on specific computers (see Ethereum, EOS, Cardano, NEO); protocols for decentralized real-time gross settlement (see Ripple, Stellar); protocols enabling the creation of decentralized marketplaces for storage and hosting of files (see SIA, Filecoin, Storj), for renting in/out CPU cycles (see Golem), for event or concert tickets (see Aventus), for ebooks (see Publica); protocol for generic e-commerce transactions (see Openbazaar); protocols creating fully decentralized prediction markets (see Augur, Gnosis), financial exchanges (see 0xproject), and financial derivatives (see MakerDAO); protocols allowing the existence of fully decentralized organizations (see Aragon) and virtual worlds (see Decentraland); and many more.[8]

usually the Bitcoin blockchain.

[8] To my knowledge, the only well established blockchain projects that are not open source protocols or are not planning to open-source their code are Iota (a protocol that is not fully open source), and Binance Coin (a token that works as a "voucher" to access Binance, a traditional exchange).

1.2 Blockchain and Seignorage

An important difference between the protocols built on TCP/IP and those built on blockchain is the way in which their developers are rewarded. The vast majority of protocols based on TCP/IP are opensource, free to adopt and use. The contributors to these projects are not organized in a single, traditional company, but rather form a loosely-defined group around one (or multiple) project leader and are based on open collaboration (as typical of open source projects). They do not receive immediate, direct financial compensation for their contributions, and are motivated by career concerns (i.e. increase their reputation and reap a financial benefit in the future) and by non-monetary considerations (i.e. the pleasure of sharing, collaborating, contributing to a public good).

Instead, as already discussed in the introduction, the development of blockchain-based protocols can leverage financial incentives via seignorage. This is possible whenever the protocol must be used in conjunction with a token. In case of protocols creating decentralized marketplaces, the token is typically the currency used by the two sides of the market. In blockchain projects without this "marketplace" element, the use of the token can vary. For example, in case of cryptocurrencies such as Bitcoin there are two sides: people who need to exchange bitcoins, and those who use their computers to process these transactions, also called miners. Users of bitcoins "pay" the miners in two ways. One is direct: the sender of bitcoins can pay a fee to process the transaction faster, and this fee is earned by the miner. The second is indirect: the network awards miners new bitcoins for their work. Because of its effect on the price, this increase in the supply of bitcoins amounts to a transfers from the holders of bitcoins to the miners.[9] A similar mechanism is used within decentralized computing platforms such as Ethereum.

The most visible part of seignorage is the Initial Coin Offering (ICO), where the developer sells tokens to investors and users for the first time. The first notable ICO was that of Ethereum in 2014, raising USD 2.3 million in approximately 12 hours. In a recent report PwC estimates that in 2017 there were 552 ICOs raising a total of USD 7 million.[10] The same report notes that the figures for 2018 are likely

[9] See also Huberman, Leshno, and Moallemi (2017). The case of Bitcoin also illustrates another point: that the mechanism by which one side of the market rewards the other may not be a market-clearing price. This aspect, however, will not be relevant here.

[10] See https://cryptovalley.swiss/wp-content/uploads/20180628_PwC-S-CVA-ICO-Report_EN.pdf

to be much larger: just in the first half of 2018 there were 537 ICOs raising more than USD 13 million. Interestingly, some analysts claim that about half of the ICOs launched in 2017 already failed by early 2018.[11]

The less visible part of seignorage is the sale on the open market of tokens that were not sold at ICO. Very few projects disclose whether and to what extent they finance themselves this way.[12] Some projects refer to this practice within blog posts and informal communication.[13] More visible is the practice of rewarding suppliers using tokens. This is often referred to as "bug bounty programs" by which translators, coders, marketers receive tokens for their work.

Despite this difference in visibility, a recent work by Howell, Niessner, and Yermack (2018) show that two sides of seignorage—sale of tokens at ICO and sale of tokens post-ICO—are comparable in terms of number of tokens sold (or expected to be sold). They analyze a sample of around 400 ICOs and find that, on average, only 54 percent of the tokens are sold at ICOs. Interestingly, they also find that only about 1/3 of ICOs include vesting provisions locking up the tokens not sold at ICO (or part of them) for some amount of time.

1.3 Relevant literature.

This paper contributes to the literature on innovation and incentives, in particular to the literature studying the motivation behind contributions to open source software (see the seminal paper by Lerner and Tirole, 2002). With this respect, I show that open source—with its organizational structure and ethos—can coexist with strong financial incentives. Of course, an open question that I do not address here is whether financial rewards will crowd out other motives (see, for example, Benabou and Tirole, 2003), that is, whether the open source ethos will be compromised by the introduction of strong financial incentives.

Closely related is a recent literature building theoretical models of ICOs (see Sockin and Xiong, 2018, Li and Mann, 2018, Catalini and Gans, 2018). The main difference is that, in my model, the developer can sell tokens both at ICO and post ICO. That is, these papers focus on a specific aspect of seignorage, the ICO, while

[11] See http://fortune.com/2018/02/25/cryptocurrency-ico-collapse/.

[12] Ripple is an exception as it announces in advance a schedule for selling parts of its XRP stock, see https://ripple.com/insights/q1-2018-xrp-markets-report/.

[13] For example, see this blog post by the Ethereum foundation https://blog.ethereum.org/2016/01/07/2394/.

my goal is to capture it in its entirety. Of course, the flip side is that these models provide a more realistic and detailed description of how ICOs work, while here I simply assume that an ICO is an auction. A second important difference is that here the quality of the project is endogenous, which allows me to study the incentives for innovation generated by ICOs and seignorage more in general.

There is a small but growing literature studying how blockchain works (see, for example Catalini and Gans, 2016, Huberman, Leshno, and Moallemi, 2017, Dimitri, 2017, Prat and Walter, 2018, Ma, Gans, and Tourky, 2018, Budish, 2018). Within this literature, closely related is Biais, Bisiere, Bouvard, and Casamatta (2018), in which the price of a token and incentives of miners (i.e., the computers that process transactions and therefore constitute the nodes of the bitcoin blockchain) are determined in the equilibrium of a game-theoretic model. Also in my paper prices and incentives are determined in equilibrium, but the interest is in the incentives to develop the software rather than processing transactions. The portion of the model that determines the equilibrium price of the token borrows heavily from Athey, Parashkevov, Sarukkai, and Xia (2017), who propose an equilibrium model of the price of bitcoin in which the demand comes both from users and from investors. The novelty with respect to their paper is that, here, the demand for tokens (originating from both investors and users) is a function of the developer's effort and investment, while the "quality" of the bitcoin protocol is taken as given in their model.

Gans and Halaburda (2015) study platform based digital currencies such as Facebook credits and Amazon coins. These currencies share some similarities with the tokens discussed in the introduction, because they can be used to perform exchanges on a specific platform. They are, however, controlled by their respective platforms, which decide on their supply and the extent to which they can be traded or exchanged. This may explain why, despite some initial concerns,[14] these currencies neither gained wide adoption, nor generated significant profits for the platform issuing them.

A line of literature that is also related is the one studying how the financial market may weaken incentive schemes faced by managers (see, for example, the

[14] See, for example "Could a gigantic nonsovereign like Facebook someday launch a real currency to compete with the dollar, euro, yen and the like?" by Matthew Yglesias on Slate, February 29, 2012 (available at http://www.slate.com/articles/business/cashless_society/2012/02/facebook_credits_how_the_social_network_s_currency_could_compete_with_dollars_and_euros_.html).

seminal work by Diamond and Verrecchia, 1982 and the most recent Bisin, Gottardi, and Rampini, 2008, Acharya and Bisin, 2009). The reason is that, also in my model, the possibility of trading on the financial market reduces the incentives to exert effort and invest. The environment I'm considering here is however different from the one in these papers, because there is no contract between the issuer of the currency (the developer) and those holding the currency (the investors). The developer's incentive problem depends on how the equilibrium price of the token is determined and how this price is affected by the developer's actions.

The remainder of the paper is organized as follows. Section 2 presents a model of seignorage. Section 3 solves for its equilibrium. Section 4 illustrates the first best of the model and compares it to its equilibrium. Section 5 discusses some extensions to the model. Section 6 concludes. Unless otherwise noted, all proofs and mathematical derivations missing from the text are in appendix.

2 The model

The economy is composed of a developer, a large mass of risk-neutral price-taking investors, and a large mass of users. At the beginning of every period $1 \leq t \leq T$, the developer exerts effort e_t and invests i_t into the development of a Blockchain-based protocol, which can be used by users to transact with each other. The development of the protocol lasts T periods, after which the developer exists the game and the protocol continues being used indefinitely. At the beginning of the game, the developer establishes that all transactions using the protocol must be conducted using a specific token, with total supply M, fully owned by the developer.

In period $t_o \leq T$, the developer sells some tokens to investors via an auction. This stage is the ICO (Initial Coin Offering) stage, and its date t_o is chosen by the developer.[15] In each period $t \in \{t_o + 1, ..., T\}$, first the developer exerts effort and invests, then a frictionless market for tokens opens, and then users can use the protocol. In every period after the developer exits (that is, in every $t > T$), first

[15] As we will see, in solving for the equilibrium, the important element of an auction will be that the developer specifies in advance what share of the total stock of tokens will be sold and what share will be kept by the developer. Hence, despite the fact that not all ICOs are auctions (see, for example, the practice of holding *uncapped ICOs* in which the token's price is fixed and the number of tokens sold is determined in equilibrium), the results derived in this paper extend to other types of ICOs as long as these shares are specified in advance.

the market for tokens opens, and then users use the protocol. See Figure 2 for a graphical representation of the timeline.

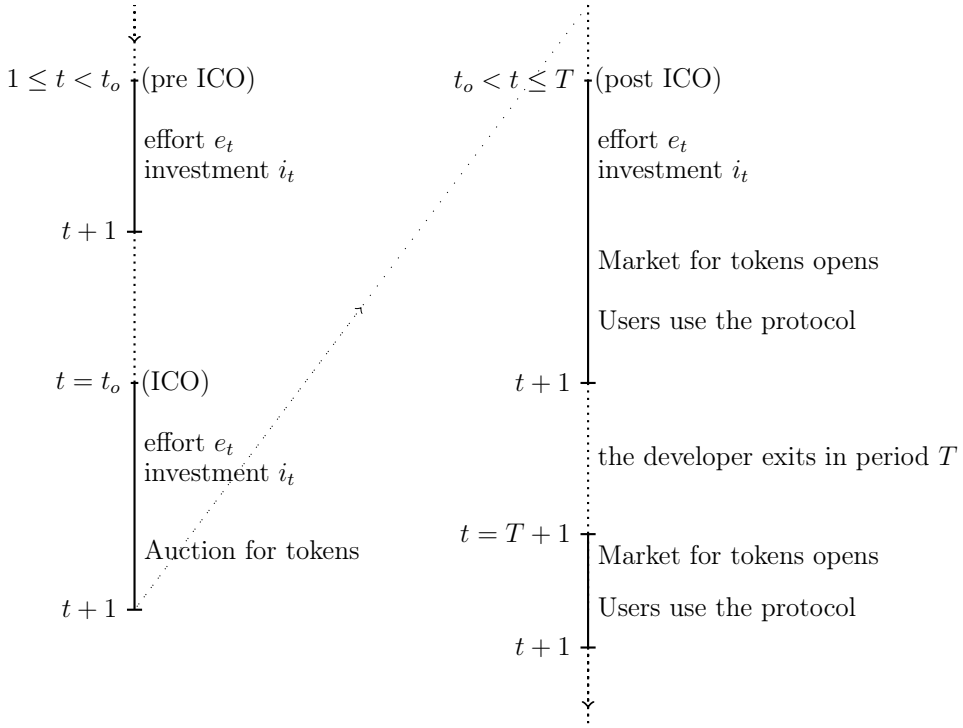


Fig. 2: Timeline

Investors. Investors are risk-neutral profit maximizers with no cash constraints. They can purchase tokens in every period and sell them during any subsequence period. Importantly, when buying or selling tokens on the market they are price takers: their net demand for tokens in period t depends on the sequence of token's prices from period t onward, which they take as given. Investors do not discount the future, and are indifferent between purchasing any amount of tokens in period t whenever $p_t = \bar{p}_t \equiv \max_{s>t} \{E[p_s]\}$, where $\bar{p}_t$ is therefore the largest future expected price. If instead $p_t > \bar{p}$, then the investors' demand for tokens in period t is zero. Finally, if $p_t < \bar{p}$ then the investors' demand for token in period t is not defined.

Users. In every period $t \geq t_o$, there is a market for tokens, on which users can purchase tokens to be used with the protocol. The total value of all exchanges

occurring using the protocol during a given period is the *value of the protocol* and is defined as:

$$V_t = \sum_{s=1}^{t} f(e_s, i_s) \tag{1}$$

where $f(.,.)$ is increasing in both arguments, concave in e_t, with $\lim_{i\to\infty} \left\{ \frac{\partial f(e_t,i_t)}{\partial i_t} \right\} = 0$ for all e_t. For ease of notation, I assume that each user can access the market for tokens only once in every period.[16] This implies that those who use the protocol to purchase goods and services in period t have a demand for tokens in period t equal to $\frac{V_t}{p_t}$. Instead, those who use the protocol to sell goods or services have a supply of tokens in period $t+1$ equal to $\frac{V_t}{p_t}$.[17]

The key assumption is that the value of the protocol is increasing in the sequence of effort and investment. The developer's effort and investment improve the protocol, in the sense of reducing transaction costs, increasing ease of use, increasing security and reliability. As a consequence, more users (both on the selling side and on the buying side) will use the protocol to perform more/larger transactions. Also, although quite general, the above specification abstracts away from a possible coordination problem in the adoption phase of the protocol. That is, because of network externalities, it is possible that for given sequence of effort and investment there are both a "high adoption" equilibrium in which the value of the protocol is high, and a "low adoption" equilibrium in which the value of the protocol is low. With a minimal loss of generality, the reader can interpret V_t as the value of the protocol in one of these equilibria, the one that the developer expects to emerge.[18]

[16] That is, the velocity of the token is 1. Assuming a different velocity will would introduce an additional parameter that is, however, inconsequential with respect to subsequent derivations.

[17] I'm abstracting away from the fact that, from period t point of view, the price in period $t+1$ may be uncertain, which may affect the willingness to trade using the protocol and the value of the protocol. Introducing this additional complication does not change the equilibrium of the game. I will argue later that, because no development occurs past period T, the price of the token is constant from T onward. Furthermore, the value of the protocol from T onward must be larger than the value of the protocol in any previous period. In equilibrium, because of the investors, the price of the token in every period before T is equal (in expected terms) to the price of the token at T, which implies that only the value of the protocol in period T matters in equilibrium. This remains true when uncertainty depresses the value of the protocol in periods before T.

[18] The loss of generality is that either the "high" or the "low" adoption equilibrium may not exist for some levels of efforts and investments, generating a discontinuity in the way effort and investment maps into the value of the protocol.

The developer. Call $Q_t \leq M$ the stock of tokens held by the developer at the beginning of period t, with $Q_1 = M$. Call

$$A_t \equiv a + \sum_{s=1}^{t-1} \left[(Q_s - Q_{s+1}) \cdot p_s - i_s \right] = A_{t-1} - i_{t-1} + p_{t-1}(Q_{t-1} - Q_t)$$

the total resources available to the developer at the beginning of period t, where a are the developer's initial assets (cash) and the rest are resources earned from the sale of tokens in previous periods, net of the investments made. To account for the fact that during periods $t < t_o$ the developer cannot sell tokens, I impose that $p_t \equiv 0$ for all $t < t_o$. Intuitively, in any $t < t_o$ the developer cannot sell tokens but can destroy them, which is equivalent to selling them at price zero. Of course, this will not happen in equilibrium.

In every period, the developers maximizes assets at the end of life A_{T+1} minus the disutility of effort. He faces a per-period feasibility constraint determining the largest investment that can be made:

$$i_t \leq A_t,$$

and a per-period cash constraint determining the maximum amount of tokens that can be purchased by the developer

$$p_t \max \left\{ Q_{t+1} - Q_t, 0 \right\} \leq A_t - i_t.$$

Note that the cash constraint is always tighter than the feasibility constraint, which can therefore be disregarded.

Similarly to investors, also the developer does not discount the future. Hence, his problem can be rewritten in recursive form as, for $t < T$:

$$\begin{aligned} U_t(Q_t, A_t) \equiv \max_{Q_t, e_t, i_t} \Big\{ & -\frac{1}{2} e_t^2 + U_{t+1}(Q_{t+1}, A_t + (Q_t - Q_{t+1}) \cdot p_t - i_t) + \\ & \lambda_t (A_t - i_t - p_t \max \left\{ Q_{t+1} - Q_t, 0 \right\}) \Big\}, \end{aligned}$$

and for $t = T$:

$$U_T(Q_T, A_T) \equiv \max_{e_T, i_T} \left\{ A_T + Q_T \cdot p_T - i_T - \frac{1}{2} e_T^2 + \lambda_T (A_T - i_T) \right\},$$

where λ_t is the Lagrange multiplier associated with the period-t cash constraint. The sequence of effort, investments and Q_t are assumed observable by investors and users at the beginning of each period. The developer understands the price formation mechanism.

3 Solution

3.1 Periods $t \geq T$.

In this section I show that, given the set up of the model and an appropriate equilibrium selection criterion (which I introduce below), the price of the token in period T is strictly increasing in the value of the protocol V_T—and hence in the sequence of effort and investments made by the developer. It is important to keep in mind, however, that the solution to developer's problem will depend exclusively on the fact that p_T is strictly increasing in V_T, while the details of how V_T affects p_T will be relevant only to derive closed-form solutions. That is, the model is robust to different assumptions about what happens from period T onward (for example, regarding the demand and supply of tokens by users or by investors), provided that under these different assumptions p_T is increasing in V_T.

The presence of investors and the fact that no development is possible after period T implies that the price of the token must be constant from period T onward. Investors are therefore indifferent between holding cash and holding the token, which implies that there are multiple equilibria: the price of the token will depend on the stock of tokens held by the investors, who are indifferent between holding any level of tokens.

To break this indeterminacy I impose the following assumption:

Assumption 1. *In equilibrium the stock of tokens held by investors from period $t \geq T$ is $\gamma \cdot M$ for $\gamma \in [0, 1)$.*

That is, out of the many equilibria possible, here I am interested in those in which the demand for tokens by investors is a constant fraction of the stock of tokens M.

The term $\gamma \cdot M$ therefore represents the "speculative" demand for tokens: the demand for tokens driven by the expectation that future investors will also demand $\gamma \cdot M$. Next to this demand, in every period there is a demand and a supply for tokens originating from users. Because the stock of tokens available to users is $(1 - \gamma) \cdot M$, the price for token must solve:

$$p_T = \frac{V_T}{(1-\gamma)M}.$$

The important observation here is that the price at which the developer can sell his tokens in period T is strictly increasing in the value of the protocol V_T, and therefore in the prior sequence of effort and investments.

3.2 The developer's problem.

The fact that the price of the token in period T is increasing in the sequence of effort and investments generates the following tension. Investors are forward looking and are willing to purchase tokens in period $t < T$ at the same price that is expected in period T. But if the developer's future effort is already priced into todays' price, the developer may be better off by selling all his tokens—that is, to benefit from his future effort and investment before exerting any. This section shows that, as a consequence of this tension, in every post-ICO period the equilibrium must be in mixed strategy, in which, in every period with some probability the developer sells all his tokens.

In solving for the developer's problem, the following observation will play a key role. Because investors are price takers, in every $t > t_o$ their demand for tokens depends exclusively on p_t and $\bar{p}_t$ (the largest future price) and not on the quantity of tokens sold by the developer in period t.[19] In particular, if $p_t = \bar{p}_t$, then investors are indifferent between purchasing any amount of tokens. At the same time, the equilibrium price in period t should reflect effort and investments made prior to t. Hence, because the instantaneous demand for tokens by investors is inelastic to the supply of tokens, in every period the developer can sell any amount of tokens at the market price. But because prices react to effort and investment which depend on the stock of tokens held by the developer, the amount sold by the developer in each period will have an effect on future prices.

It is useful to solve the developer's problem by distinguishing two cases. The first is the "rich developer" case, in which the developer's initial assets a are sufficient to cover the optimal level of investment in every period. In this case, the cash constraint is never binding and can be ignored. The second case is that of a "poor developer" in which the cash constraint is binding for at least one period.

3.2.1 Rich developer.

If the cash constraint is never binding, the developer's utility can be written as, for $t \leq T - 1$:

$$\tilde{U}_t(Q_t) \equiv \max_{Q_{t+1}, e_t, i_t} \left\{ (Q_t - Q_{t+1}) \cdot p_t - i_t - \frac{1}{2} e_t^2 + \tilde{U}_{t+1}(Q_{t+1}) \right\},$$

[19] Of course, the equilibrium price will be such that demand equals supply; the point here is simply that in a price-taking environment the demand cannot be a function of the supply.

and for $t = T$:

$$\tilde{U}_T(Q_T) \equiv \max_{e_T, i_T} \left\{ Q_T \cdot p_T - i_T - \frac{1}{2} e_T^2 \right\}.$$

Note that $(Q_t - Q_{t+1}) \cdot p_t - i_t$ is the cash generated in period t, net of investment. Because there is no discounting and the cash constraint is never binding, I can include this cash in period-t utility function (i.e., the period in which it is generated) even if it is consumed in period T.

Consider the last period of the developer's life. The fact that p_T increases in e_T and i_T immediately implies that $\tilde{U}_T(Q_T)$ is strictly convex. The argument is quite standard: if e_T and i_T were fixed, then p_T would be fixed and $\tilde{U}_T(Q_T)$ would be linear in Q_T. However, the optimal e_T and i_T are[20]

$$e^*(Q_T) \equiv \text{argmax}_e \left\{ f(e, i^*(Q_T)) \frac{Q_T}{(1-\gamma)M} - \frac{1}{2} e^2 \right\} \tag{2}$$

$$i^*(Q_T) \equiv \text{argmax}_i \left\{ f(e^*(Q_T), i) \frac{Q_T}{(1-\gamma)M} - i \right\} \tag{3}$$

As long as either $e^*(Q_T)$ or $i^*(Q_T)$ are positive for some $Q_T \leq M$ (an assumption I maintain to avoid trivialities), then optimal effort and investment react to changes in Q_T, which implies that $\tilde{U}_T(Q_T)$ must grow faster than linearly.

Consider now the choice of Q_T in period $T-1$. For given e_{T-1} and i_{T-1}, the developer chooses Q_T so to maximize $p_{T-1}(Q_{T-1} - Q_T) + \tilde{U}_T(Q_T)$, which is strictly convex in Q_T because $\tilde{U}_T(Q_T)$ is strictly convex. It follows that, depending on p_{T-1}, the developer will either sell all his tokens (when p_{T-1} is high), or purchase as many tokens as possible (when p_{T-1} is low), or be indifferent between these two options. The price at which the developer is indifferent is

$$p_{T-1} = \frac{\tilde{U}_T(M)}{M} = \frac{V_{T-1} + f(e^*(M), i^*(M))}{(1-\gamma)M} - \frac{(e^*(M))^2/2 + i^*(M)}{M}, \tag{4}$$

where $\frac{V_{T-1} + f(e^*(M), i^*(M))}{(1-\gamma)M}$ is the period T price in case the developer holds M tokens at the beginning of period T.

[20] With a slight abuse of notation, I ignore the time index when writing optimal effort and optimal investment. I show below that these functions are, in fact, time invariant. Note also that, under the assumptions made on $f(.,.)$ optimal effort and investment must exist. They however may not be unique. In what follows, for ease of exposition I implicitly assume that they are indeed unique, although no result depends on this assumption.

Note, however, that if investors expect the developer to sell all his tokens, they should also expect no effort or investment in period T and therefore p_{T-1} should be low. If instead they expect the developer to set $Q_T = M$, they should expect maximum effort and investments in period T and therefore p_{T-1} should be high. We therefore have an anti-coordination problem, which implies that the unique equilibrium is in mixed strategy: the price will be such that the developer is indifferent, and the developer will randomize between $Q_T = 0$ and $Q_T = M$.

More precisely, if the developer sells all his tokens in period $T-1$, then the price in period T will be $\frac{V_{T-1}}{(1-\gamma)M}$. If instead the developer purchases M tokens in period $T-1$, then $p_T = \frac{V_{T-1}+f(e^*(M),i^*(M))}{(1-\gamma)M}$. Because investors must be indifferent between purchasing in period T or period $T-1$, it must be that

$$p_{T-1} = \frac{V_{T-1}}{(1-\gamma)M} + (1-\alpha_{T-1})\frac{f(e^*(M), i^*(M))}{(1-\gamma)M}$$

where α_{T-1} is the probability that the developer sells all his tokens in period $T-1$, which using (4) can be written as

$$\alpha_{T-1} = (1-\gamma)\,\frac{(e^*(M))^2/2 + i^*(M)}{f(e^*(M), i^*(M))}$$

For intuition, note that $(e^*(M))^2/2 + i^*(M)$ is the cost generated by holding M tokens, coming from the additional effort and investment that the developer will exert in period T. Instead,

$$M \cdot \frac{f(e^*(M), i^*(M))}{(1-\gamma)M},$$

is the benefit of setting $Q_T = M$, coming from the increase in the value of these tokens due to the developer's effort and investment in period T. α_{T-1} is therefore equal to the ratio between cost and benefit of holding M tokens in period T. Note also that, because effort and investment are chosen optimally, the benefit should be at least as large as the cost, and therefore $\alpha_{T-1} \leq 1$.

The following proposition shows that these results generalize to every period in which the market for tokens operates.

Proposition 1 (Equilibrium post-ICO)**.** *In every period $t \in \{t_o + 1, ..., T\}$:*

1. *Optimal effort and investment for given Q_t are $e^*(Q_t)$ and $i^*(Q_t)$, given by* (2) *and* (3)*.*

2. *The developer sells all his tokens (so that $Q_{t+1} = 0$) with probability*

$$\alpha_t = \begin{cases} 1 & \text{if } t = T \\ (1-\gamma)\frac{(e^*(M))^2/2+i^*(M)}{f(e^*(M),i^*(M))} & \text{otherwise} \end{cases} \tag{5}$$

and purchases all tokens (so that $Q_{t+1} = M$) with probability $1 - \alpha_t$.

3. *The price of tokens as a function of past effort and investment is*

$$p_t = \frac{V_t + (1-\alpha_t)(T-t)f(e^*(M), i^*(M))}{(1-\gamma)M}. \tag{6}$$

The proposition is based on the fact that all $\tilde{U}_t(Q_t)$ are strictly convex and, therefore, in every period $t < T$ the equilibrium price must be such that the agent is indifferent between holding all his tokens and selling all his tokens. But this also implies that the agent is indifferent between selling all his tokens in period t or holding M in every period until T. The benefit of exerting effort and of investing in a given period is therefore given by the resulting change in p_T, which is constant over time and given by (2) and (3).

Hence, whenever $Q_t = M$ the value of the protocol increases by $f(e^*(M), i^*(M))$ in period t, while if $Q_t = 0$ the value of the protocol does not change in period t. The probability that $Q_t = 0$ is such that investors are indifferent between holding the token at $t-1$ or at t, and is also constant over time. It follows that the price in period t (equation (6)) reflects past effort and past investment via the term V_t, as well as expected future effort and investment via the term $(1-\alpha_t)(T-t)f(e^*(M), i^*(M))$. This expression can also be interpreted as law of motion of the price, because it implies that, in every period $t \leq T$, the price of token will increase by

$$\frac{(e^*(M))^2/2 + i^*(M)}{M}$$

with probability

$$1 - (1-\gamma)\frac{e^*(M))^2/2 + i^*(M)}{f(e^*(M), i^*(M))}$$

and will decrease by

$$\frac{1}{M}\left(\frac{f(e^*(M), i^*(M))}{1-\gamma} - (e^*(M))^2/2 + i^*(M))\right)$$

otherwise.

Period t_o (the ICO) is characterize by the fact that tokens are sold via an auction. Hence, contrarily to all subsequent periods, in period t_o the price of token depends on the number of tokens sold, which is $M - Q_{t_o}$. Again, in equilibrium investors must be indifferent and therefore, for any number of tokens sold at ICO, it must be that $p_{t_o} = p_{t_o+1}$. Hence, whenever $t_o < T$, the developer's problem at ICO can be written as

$$\begin{aligned}
&\max_{Q_{t_o+1}} \left\{ \tilde{U}_{t_o+1}(Q_{t_o+1}) + (M - Q_{t_o+1}) p_{t_o} \right\} = \\
&\max_{Q_{t_o+1}} \left\{ \max_{e_{t_o+1}, i_{t_o+1}} \left\{ Q_{t_o+1} \cdot p_{t_o+1} - \frac{1}{2} e_{t_o+1}^2 - i_{t_o+1} \right\} + (M - Q_{t_o+1}) p_{t_o+1} \right\} \leq \\
&\max_{Q_{t_o+1}} \left\{ \max_{e_{t_o+1}, i_{t_o+1}} \left\{ Q_{t_o+1} \cdot p_{t_o+1} - \frac{1}{2} e_{t_o+1}^2 - i_{t_o+1} + (M - Q_{t_{o+1}}) p_{t_o+1} \right\} \right\} = \\
&\max_{e_{t_o+1}, i_{t_o+1}} \left\{ M \cdot p_{t_o+1} - \frac{1}{2} e_{t_o+1}^2 - i_{t_o+1} \right\} = \tilde{U}_{t_o+1}(M)
\end{aligned}$$

where the first and the last equality follow from writing $\tilde{U}_{t_{o+1}}(Q_{t_{o+1}})$ explicitly (under the assumption that the developer sells all his tokens in period t_{o+1}). The developer therefore anticipates that the price of tokens will be the same at ICO and in the following period, independently from how many token he sells. The number of token sold, however, determine the equilibrium level of effort and investment in period t_{o+1}. By choosing $Q_{t_{o+1}} = M$, the developer maximizes effort and investments in period t_{o+1}, and therefore the price in period t_{o+1}. If instead $t_o = T$, then the developer sells all his tokens during the ICO, and then exists the game. The following proposition summarizes these observations.

Proposition 2 (Equilibrium at t_o)**.** *If the ICO occurs before T, then the developer does not sell any token at ICO. It follows that $Q_{t_o+1} = M$ with probability 1. Effort and investment in all $t_o + 1$ are $e^*(M)$ and $i^*(M)$ with probability 1. If instead the ICO occurs at period T, then the developer sells all his tokens at ICO.*

Proof. In the text. □

Period $t_o + 1$ is therefore the only period in which the market is open and the developer contributes to the development of the protocol with probability 1.

It is immediate to check that optimal effort and investment between period 1 and t_{o+1} are, again, $e^*(M)$ and $i^*(M)$. In all subsequent periods, instead, the existence of the market for tokens creates a commitment problem: the value of the protocol is

maximized when the developer holds all his tokens until T, but this cannot happen in equilibrium. From period t_{o+2} onward the developer exerts effort and invests with probability less than one, which implies the following proposition:

Proposition 3 (Equilibrium t_o)**.** *The developer holds the ICO either in period T or in period $T-1$.*

Proof. In the text. □

Note that if the ICO is held in period $T-1$ the developer will auction off 0 tokens, and he will sell M tokens on the market in period T. If instead the ICO is in period T the developer sells all his tokens via the auction. Holding the ICO in period $T-1$ or period T, therefore, achieves the same outcome: the developer does not sell any token before period T and sells all his tokens in period T. As a consequence effort and investment are at their optimal level $e^*(M)$ and $i^*(M)$ with probability 1 in every period.

Corollary 1. *The cash constraint is never binding (and hence we are in the "rich developer" case) if and only if $a \geq T \cdot i^*(M)$.*

Proof. Immediate from the above Proposition. □

That is, we are in the "rich developer" case whenever the developer does not need to sell tokens to finance the optimal amount of investment.

Finally, it is easy to check that the developers' utility does not depend on M. From (2) and (3) we know that the equilibrium sequence of investment and effort is also independent from M. By proposition 1, in every post-ICO period the value of all outstanding tokens $p_t M$ is independent from M. The developer's utility is therefore independent from M.

3.2.2 Poor developer

The rich developer case focuses on one side of seignorage: the incentives provided to the developer. It shows that the developer will hold the ICO just before exiting the game, as a way to commit to the optimal level of effort and investment in every period.

There is, however, a second side of seignorage: its ability to channel funds from investors to the developer, to be then used in the development of the protocol. I

now introduce this aspect into the model by assuming that the developer is "poor", in the sense that $a < T \cdot i^*(M)$: the developer cannot invest efficiently in all periods, and the cash constraint could be binding.

To focus on the role of the cash constraint, I assume the following functional form

$$f(e,i) \equiv g(e)\mathbb{1}\{i \geq \bar{i}\}, \tag{A1}$$

where $\mathbb{1}\{\}$ is the indicator function, and $g(e)$ is strictly increasing and strictly concave. Hence, i is an essential input in the development of the protocol, because effort is productive only if $i \geq \bar{i}$. However, investing more that $\bar{i}$ is also not productive. The choice of optimal investment therefore simplifies to the choice between two levels: $\bar{i}$ and 0.

Given this, period-T effort and investment are

$$\hat{e}_T(Q_T, i_T) \equiv \begin{cases} e^*(Q_T) \equiv \text{argmax}_e \left\{ g(e)\frac{Q_T}{(1-\gamma)M} - \frac{1}{2}e^2 \right\} & \text{if } i_T \geq \bar{i} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$\hat{i}_T(Q_T, A_T) \equiv \begin{cases} \bar{i} & \text{if } \bar{i} \leq \max_e \left\{ g(e)\frac{Q_T}{(1-\gamma)M} - \frac{1}{2}e^2 \right\} \text{ and } \bar{i} \leq A_T \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

To avoid trivial equilibria in which there is never any effort or investment, I furthermore assume that

$$\bar{i} < \max_e \left\{ g(e)\frac{1}{1-\gamma} - \frac{1}{2}e^2 \right\} \tag{A2}$$

that is: there is a level of Q_T for which the developer will invest and exert positive effort whenever his assets are sufficient to do so. I call the threshold level $\hat{Q}$, implicitly defined as

$$\hat{Q} \equiv Q : \quad \bar{i} = \max_e \left\{ g(e)\frac{Q}{(1-\gamma)M} - \frac{1}{2}e^2 \right\}. \tag{9}$$

In the remainder of this section, I fully solve for the equilibrium in periods T and $T-1$ depending on whether the ICO happened in period T, $T-1$, or any earlier period. I will only informally discuss the equilibrium in periods before $T-1$. I will, nonetheless, provide a characterization of the optimal timing of the ICO.

Case 1: $t_o = T$. If the ICO occurs in the last period, then optimal effort and investment in period T are given by (7) and (8). The price of a token is therefore:

$$\frac{V_{T-1}}{(1-\gamma)M} + \begin{cases} 0 & \text{if } A_T < \bar{i} \\ \frac{g(e^*(M))}{(1-\gamma)M} & \text{otherwise} \end{cases}$$

In period $T-1$, the choice of optimal investment affects A_T and the period-T optimal effort and investment. This is relevant whenever $\bar{i} \leq A_{T-1} < 2\bar{i}$, that is, whenever assets in period $T-1$ are not sufficient to invest optimally in both period $T-1$ and period T. It is quite immediate to see that, in this case, the final price is always $\frac{V_{T-2}+g(e^*(M))}{(1-\gamma)M}$, independently from whether effort and investment are positive in period $T-1$ or T. The same logic applies to the choice of investment and effort in any earlier period.Define

$$n \equiv \text{argmax}_{k \in \{1,2,\ldots,T\}} \{k \cdot \bar{i} \leq a\} \tag{10}$$

as the number of periods in which the developer can invest efficiently using exclusively his initial assets. The above discussion implies that the developer will invest and exert effort for n periods, and he is indifferent with respect to which ones. The following Proposition summarizes these observations.

Proposition 4 (ICO in period T)**.** *Whenever $t_o = T$, the final value of the protocol is $V_T = n \cdot e^*(M)$.*

Proof. In the text. □

Case 2: $t_o = T-1$. If the ICO occurs in period $T-1$, then the developer can finance some of its period T investment by selling tokens in period $T-1$. Remember that, in equilibrium, the price of tokens at ICO p_{T-1} must be equal to p_T. Hence, for given $M - Q_T$ (i.e., tokens sold at ICO) the price for tokens will be

$$p_T = \frac{V_{T-1}}{(1-\gamma)M} + \begin{cases} 0 & \text{if } A_{T-1} - i_{T-1} + p_T(M - Q_T) < \bar{i} \\ \frac{g(e^*(Q_T))}{(1-\gamma)M} & \text{otherwise} \end{cases} \tag{11}$$

Whenever $A_{T-1} - i_{T-1} < \bar{i}$ (that is, whenever the developer does not have enough own funds to invest in period T), both LHS and RHS of (11) depend on p_T, and therefore for given Q_T there are multiple equilibrium p_T. For intuition, suppose

that the developer announces the sale of $M - Q_T$ tokens at ICO. If investors expect p_T to be low, they will drive down p_{T-1} (the price at ICO), which implies that the level of investment achievable in period T by selling $M - Q_T$ at ICO may be below $\bar{i}$, which justifies the initial expectation. If instead investors expect p_T to be high, in equilibrium p_{T-1} will also be high, which implies that the level of investment achievable in period T by selling $M - Q_T$ tokens at ICO may be above $\bar{i}$, which justifies the initial expectation. This can be interpreted as a coordination problem among investors. For any number of tokens sold by the developer at ICO, investors may coordinate on a "high" equilibrium that leads to high effort and investment in period T, or on a "low" equilibrium leading to "low" (or no) development in period T. Call $p(Q_T)$ the correspondence mapping Q_T to the equilibrium p_T. We therefore have (see also Figure 3):

$$p(Q_T) = \begin{cases} \frac{V_{T-1}}{(1-\gamma)M} & \text{if } \frac{\bar{i}+i_{T-1}-A_{T-1}}{M-Q_T} < \frac{V_{T-1}+g(e^*(Q_T))}{(1-\gamma)M} \\ \frac{V_{T-1}+g(e^*(Q_T))}{(1-\gamma)M} & \text{if } \frac{\bar{i}+i_{T-1}-A_{T-1}}{M-Q_T} > \frac{V_{T-1}}{(1-\gamma)M} \\ \left\{ \frac{V_{T-1}}{(1-\gamma)M}, \frac{V_{T-1}+g(e^*(Q_T))}{(1-\gamma)M} \right\} & \text{otherwise} \end{cases}$$

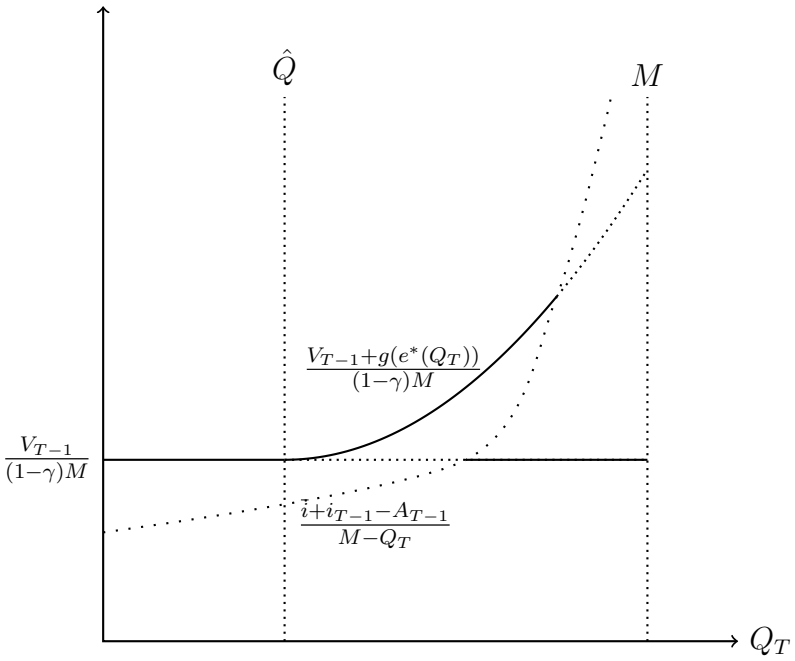


Fig. 3: $p(Q_T)$ whenever $\bar{i} + i_{T-1} > A_{T-1}$.

The choice of Q_T maximizes the period-T utility $U_T(Q_T, A_T)$,. The next lemma shows that $U_T(Q_T, A_T)$ is convex in Q_T, provided that the developer has enough wealth to invest, and provided that he has enough "skin in the game" in the sense of $Q_T > \hat{Q}$.

Lemma 1. *$U_T(Q_T, A_T)$ is strictly convex in Q_T whenever $\bar{i} \leq A_T$ and $Q_T \geq \hat{Q}$, and is otherwise linear in Q_T. $U_T(Q_T, A_T)$ is linearly increasing in A_T with slope 1 (corresponding to the marginal utility of consumption), and has an upward discontinuity at $A_T = \bar{i}$ if and only if $Q_T \geq \hat{Q}$.*

Proof. By the same argument made in the previous case: $U_T(Q_T, A_T)$ is linear in Q_T whenever optimal investment and effort do not change with Q_T, and is strictly convex whenever optimal investment and effort depend on Q_T. Similarly, $U_T(Q_T, A_T)$ is discontinuous in A_T whenever the level of wealth allows for the optimal level of investment. □

Note, however, that

$$A_T = A_{T-1} + (M - Q_T) \cdot p_{T-1} - i_{T-1}$$

where p_{T-1} is the price of tokens at ICO, and must be such that $p_{T-1} \in p(Q_T)$, depending on which equilibrium is expected to emerge in period 3. Hence, the choice of Q_T determines both period T's incentives to exert effort and whether the developer will have enough resources to invest. The next lemma shows that the continuation value is maximized at

$$Q_T^* = M - \frac{\max\{i_{T-1} + \bar{i} - A_{T-1}, 0\}}{p_{T-1}}, \tag{12}$$

which is the largest Q_T such that the developer can invest $\bar{i}$ in period T.

Lemma 2 (Equilibrium in period $T-1$ for $t_o = T-1$)**.** *If $Q_T^* > \hat{Q}$ the developer chooses $Q_T = Q_T^*$; there are positive investment and effort in period T. If instead $Q_T^* \leq \hat{Q}$ then the developer is indifferent between any Q_T, and there are no investment nor effort in period T. When $A_{T-1} - i_{T-1} < \bar{i}$ multiple equilibria are possible and Q_T^* may not be unique. When $A_{T-1} - i_{T-1} \geq \bar{i}$ the equilibrium is unique and $Q_T^* = M$.*

For intuition, remember that the developer has incentives to invest and exert effort in period T only if $Q_T > \hat{Q}$. Whether $Q_T > \hat{Q}$ is attainable depends on the

cash constraint. If this constraint is tight, $Q_T^* \leq \hat{Q}$ and no level of Q_T that allows for positive investment will generate sufficient incentives and hence there will be no development in period T. If instead the cash constraint is sufficiently loose, then $Q_T^* > \hat{Q}$ and for some level of Q_T there will be positive effort and investment in period T.

In this last case, multiple equilibria are possible. That is because the right hand side of (12) may be neither monotonic nor continuous (remember that $p_{T-1} \in p(Q_T)$). That is, even assuming that the investors can solve their coordination problem and therefore $p(Q_T)$ is a function and not a correspondence, there is an additional coordination problem between developer and investors giving rise to multiple equilibrium Q_T^*. Suppose that $A_{T-1} - i_{T-1} < \bar{i}$, so that the developer needs to sell some tokens at ICO in order to finance future development. If the price in period T is expected to be high, so will be price in period $T-1$ and, as a consequence, the developer needs to sell fewer tokens in order to achieve $i_T = \bar{i}$. Because he can hold a large fraction of tokens, future effort will be high, which implies that today's price for token should be large. Similarly, if period-T price is expected to be low, price at ICO will be low, and the developer needs to sell a large fraction of his tokens, which implies that future effort will be low, and so is today's price. If instead $A_{T-1} - i_{T-1} \geq \bar{i}$ then the developer does not need to sell any token to finance his future investment and, as a consequence, in the unique equilibrium $Q_T^* = M$.

Consider now optimal investment and effort in period $T-1$. It is easy to see that optimal effort is again given by (7). The choice of optimal investment, instead, has an inter-temporal element to consider: for given initial assets, the choice of period $T-1$ investment affects the equilibrium at ICO and therefore Q_T^*. This is relevant whenever $A_{T-1} < 2\bar{i}$, in which case the developer may choose not to invest in period $T-1$, so to set $Q_T^* = M$.

It is, however, easy to show that postponing investment is never optimal. Suppose that the developer has sufficient funds to invest only in one period. If the developer invests in period $T-1$, then total utility is

$$\frac{V_{T-2} + g^*(M) + g^*(Q_T^*)}{(1-\gamma)M} M - \frac{1}{2}(e^*(M))^2 - \frac{1}{2}(e^*(g^*(Q_T^*))^2.$$

if instead the developer does not invest in $T-1$, he can set $Q_T = M$ and achieve utility

$$\frac{V_{T-2} + g^*(M)}{(1-\gamma)M} M - (e^*(M))^2.$$

Comparing the above two expressions, it is clear that the developer is better off by using his own funds for investing in period $T-1$, and then financing period-T investment via the sale of tokens at ICO. This reasoning extends to any period prior to the ICO, and therefore implies the following proposition.

Proposition 5 (ICO in period 2)**.** *Whenever $t_o = T-1$, the final value of the protocol is*

$$V_T = \begin{cases} 3g^*(M) & \text{if } a \geq 3\bar{i} \\ ng^*(M) + g^*(Q_T^*) & \text{otherwise} \end{cases}$$

where n is defined in (10) *and Q_T^* is defined in* (12)*.*

Proof. In the text. □

Corollary 2. *The developer prefers to hold the ICO in period $T-1$ than in period T, strictly so when $A_{T-1} < 2\bar{i}$.*

Whenever $A_{T-1} > 2\bar{i}$ the developer has sufficient funds to invest in the last 2 periods. This is the "rich developer" case discussed above, and the developer is indifferent between holding the ICO in period $T-1$ or T, because in either case he will sell all tokens in period T. If instead $A_{T-1} < 2\bar{i}$, then the developer is better off by using his funds to invest in periods $T-1$, raise funds at ICO, and then use these funds to invest in period T.

Case 3: $t_o < T-1$**.** If the ICO occurred in period $t_o < T-1$, then in period $T-1$ there is a market for tokens. Let's start by considering the choice of Q_T, that is, of how many tokens to sell or buy on the market in period $T-1$. For given market price p_{T-1}, the developer's utility as a function of Q_T is:

$$U_T(Q_T, A_{T-1}+(Q_{T-1}-Q_T)\cdot p_{T-1}-i_{T-1})+\lambda_{T-1}(A_{T-1}-i_{T-1}-p_{T-1}\max\{Q_T-Q_{T-1},0\})$$

There are similarities with the previous case (i.e., the case of an ICO in period $T-1$). Also here the choice of Q_T determines the assets available in the following period. As a consequence, the continuation value

$$U_T(Q_T, A_{T-1} + (Q_{T-1} - Q_T) \cdot p_{T-1} - i_{T-1})$$

is strictly convex in Q_T only for

$$\hat{Q} \leq Q_T \leq Q_T^*$$

and is linearly increasing in Q_T otherwise, with a downward discontinuity at Q_T^* (where Q_T^* is defined in (12)).

There are however two important differences with the previous case. The first one is that, here, the developer could have sold some tokens during a previous period, and therefore it is possible that $Q_{T-1} < M$. It follows that the cash constraint in period $T-1$ may be binding. With this respect, note that if the cash constraint in period $T-1$ is binding, then $A_T = 0$ and the cash constraint in period T is binding. Conversely, if the period T cash constraint is binding we have $A_T = \bar{i}$, which implies that the period $T-1$ cash constraint is not binding. Hence, in solving for Q_T, the only constraint that needs to be taken into consideration is the period-T cash constraint.

Second, and most importantly, because investors are price takers, then the market price in period $T-1$ does not depend on Q_T. Only period-T price depends on Q_T, leading to the same type of anti-coordination problem discussed in the "rich developer" case.

Lemma 3 (Equilibrium in period $T-1$ for $t_o < T-1$)**.** *If $Q_T^* \leq \hat{Q}$, then the developer is indifferent between holding any level of Q_T. Effort and investment in period T are zero, so that $p_T = p_{T-1} = \frac{V_{T-1}}{(1-\gamma)M}$.*

If instead $Q_T^* > \hat{Q}$, then, in equilibrium, the developer is indifferent between setting $Q_T = 0$ and setting $Q_T = Q_T^*$. He sets $Q_T = 0$ with probability

$$\alpha_{T-1} = \left(\frac{1}{2}(e^*(Q_T^*)^2 + \bar{i}\right)\left(Q_T^* \cdot \frac{g(e^*(Q_T^*)}{(1-\gamma)M}\right)^{-1}$$

The equilibrium price is

$$p_{T-1} = \frac{V_{T-1} + (1-\alpha_{T-1})g(e^*(Q_T^*))}{(1-\gamma)M}$$

If $A_{T-1} - i_{T-1} \leq \bar{i}$ multiple equilibria are possible, while if $A_{T-1} - i_{T-1} > \bar{i}$ the equilibrium is always unique.

By comparing the above lemma with Lemma 2, we can see the difference between selling tokens in period $T-1$ via an ICO or on the market. The difference is that, whenever $Q^* > \hat{Q}$, if the market for tokens exists the equilibrium is in mixed strategies, while if the tokens are sold via an ICO the equilibrium is in pure strategies.

The reason is that the presence of the market generates the same anti-coordination problem discussed in the previous section. The developer randomizes between selling everything and setting $Q_T = 0$ and holding the maximum number of tokens, which is the minimum between the one at which period-T cash constraint is binding and M.

The other features of the equilibrium are similar. In particular, whenever $A_{T-1} - i_{T-1} \leq \bar{i}$ there could be multiple equilibria. There could be an equilibrium in which p_{T-1} is high, which implies that the developer needs to sell few tokens to finance future investment, and therefore period-T effort is high. Next to this equilibrium, there could be one in which p_{T-1} is low, which implies that the developer needs to sell many tokens to finance future investment, and therefore period-T effort is low. If instead $A_{T-1} - i_{T-1} > \bar{i}$ then the developer does not need to sell any token to achieve $i_T = \bar{i}$, and this coordination problem is absent. In case the market for tokens is open, there are therefore multiple mixed strategy equilibria, each of them corresponding to a different Q_T^* and a different p_{T-1}.

For example, the choice of i_{T-1} affects Q_T^*. Hence, the developer may want to set $i_{T-1} = 0$ even if $A_{T-1} \geq \bar{i}$ and $Q_{T-1} > \hat{Q}$ so to achieve a higher Q_T^*. Not only, but because there are multiple equilibrium Q_T^*, the choice of i_{T-1} may determine what equilibrium emerges in the market for tokens. This difficulty extends to the choice of Q_{T-2}, because Q_{T-2} determines i_{T-1}.

Despite these issues, it is possible to characterize the developer's choice of when to hold an ICO. The reason is that every time the market is open, there is the basic anti-coordination problem discussed earlier and the equilibrium is in mixed strategy. If instead the developer does not hold the ICO and has sufficient funds to invest $\bar{i}$, he will set the optimal level of effort and investments with probability 1. This observation implies the following proposition.

Proposition 6. *In equilibrium $t_o = n$, that is, the developer initially invests using his own funds, and holds the ICO as soon as his funds are below $\bar{i}$.*

Proof. In the text. □

To conclude, note that, also here, the developer's payoff does not depend on M. The reason is that, in each period, the developer's problem depends on M only via the share of M that he holds (see the optimal level of effort (2) and the incentive to set positive investment (9)). Therefore, in each period, the value of the protocol

and the value of all outstanding tokens $p_t M$ depend on the share of tokens held by the developer in each period and not on M. In addition, in all cases analyzed, the equilibrium share of tokens held by the developer in a given period is either zero, M or Q_T^*. It is easy to see that, if $p_t M$ is independent from M, so is Q_T^*/M, and therefore equilibrium the share of tokens held by the developer in each period is also independent from M.

4 First best

In the first best, effort and investment are set so to maximize the present discounted value of the surplus generated by the protocol.[21] Furthermore, the ICO is held immediately so to allow users to use the protocol from the very beginning.

The equilibrium of the game differs from the first best in several ways. As already discussed, in equilibrium the developer will want to hold the ICO only after exhausting his own funds. This is, however, inefficient because users are prevented from using the protocol before the ICO. The equilibrium post ICO is also inefficient because the developer may set zero effort and zero investment even if the social value of his effort and investment is strictly positive.

More interestingly, even assuming that the market for tokens exists so that users can use the protocol and that the developer will set positive effort and investment, there is an additional source of inefficiency. The developer is setting effort and investment so to maximize the value of the protocol in period T, when he will exit the game. A minor observation is that the value of the protocol in a given period (i.e., the value of the transactions that occur using the protocol) is, in general, different from the social surplus generated by the protocol.[22] A more important observation is that, in its objective function, the developer completely disregards the fact that the protocol will generate value over multiple periods, focusing instead exclusively on the period in which he will sell all his tokens and exit the game.

Whether the developer's effort and investment will be above or below their first best level is, however, unclear and depends on γ, which determines the elasticity of the price of token to his effort and investment. If the speculative demand for tokens is sufficiently high, then the developer will exert effort and investment above the

[21] The discount factor should be that of users.

[22] The social surplus depends on the equilibrium utility/profits of users on the buying and selling side of the protocol, as well as on their outside options.

first best. If instead it is low, then the developer may exert effort and invest below the first best.

5 Discussion

5.1 Seignorage vs monopoly pricing

The discussion of the first best is also relevant when comparing seignorage with more standard mechanism such as establishing a set of fees/prices for using the protocol. Profits generated via seignorage depend on the value of the protocol in the moment in which the developer sells his tokens. Under standard monopoly pricing, instead, the monopolist is able, in every period, to capture only a fraction of the value of the protocol (which will depend on the elasticity of supply and demand). But the monopolist is able to earn profits in every period, not only in one period.

Profits under seignorage therefore depend on the value of the protocol in a given period, while profits under standard monopoly pricing will accrue in every period. Which one is larger is, however, ambiguous and depend crucially on γ: the speculative demand for tokens. It is always possible to find a large enough γ such that profits under seignorage are greater than profits under monopoly pricing. For low γ, however, the ranking may reverse.

5.2 Asymmetric information

The results derived above largely extend to a situation in which the developer's productivity is private information. In this case, if the market for token is open, for given price for token there is a threshold productivity above which the developer wants to hold all tokens, below which the developer wants to sell all tokens. The price in every period is equal to the expected price tomorrow, which depends on the developer's expected contribution to the protocol. In every period, if the developer is more productive that the market expectation he will purchase tokens and develop the protocol with probability 1. If the developer is less productive than the market expectation he will sell all tokens and not develop the protocol.[23]

[23] The same argument can be made about wealth. If the developer's wealth is private information and affects the development of the protocol, then a developer who is richer than the market expectation about his wealth will want to purchase all tokens and develop with probability one. Otherwise he will sell all tokens and not develop.

The important observation is that, if at ICO the productivity of the developer is unknown to investors, it will nonetheless be revealed over time. In the moment it is fully revealed, the equilibrium of the game is again the one derived in the previous section. Asymmetry of information therefore implies that developers with above average productivity may contribute to the development of the protocol with probability 1 for some periods. Conversely, developers with below average productivity do not contribute to the protocols initially. After the developer's productivity is revealed, he will contribute with probability less than 1 as in the symmetric information case.

Less obvious is the impact of asymmetric information on the timing of the ICO. A developer of ability greater than the investors' expectation may benefit from anticipating the ICO, because he expects to exert effort and invest in the development of the protocol with probability 1 for few post-ICO periods. But if this is the case, then investors should infer that a developer holding an ICO early is of high ability. This, clearly, cannot be an equilibrium because, now, the high-ability developer does not benefit anymore from anticipating the ICO. The full analysis of this problem is left for future work.

5.3 Multiple, heterogeneous developers

Suppose that there is a population of developers indexed by j, each characterized by a productivity parameter q_t^j (commonly known) so that effort and investment by developer j in period t generates an increase in the value of the protocol equal to $q_t^j f(e_t^j, i_t^j)$. If all developers are "'rich" (that is, the cash constraint is never binding for any developer), in every period t the equilibrium price of the token must be such that the developer with the largest q_{t+1}^i is indifferent between holding all tokens or no tokens.[24] If, furthermore, $\max_j q_t^j$ is constant over time, then the model is formally identical to the one just solved. The only difference is its interpretation: in every period a different developer (the most productive in that period) may purchase tokens and contribute to the development of the protocol.

Contrary to the case considered in the body of the text, now the existence of a

[24] Suppose not. Then the best developer strictly prefers to hold all tokens and exert the maximum level of effort and investment in the following period. But then this developer's contribution to the protocol should already be accounted for in the current price, which implies that this developer strictly prefers to sell all his token, leading to a contradiction.

market for tokens generates an allocative efficiency: the most productive developer works on the project in every period. Of course, as we already saw, this developer contributes to the project only with some probability. It follows that holding an ICO has an additional benefit because it allows the most productive developer to contribute to the project in every period. Absent the ICO, instead, the initial developer will set high effort and investment in every period, but he may not be the most productive developer who could work on the project.

If instead some developer are "poor" (i.e., the cash constraint may be binding), then the most productive developer in a given period may not have enough resources to purchase tokens and/or invest efficiently in the development of the protocol. The developer that, in equilibrium, develops the protocol with positive probability in every period depends partly on productivity and partly on wealth.

6 Conclusion

This paper studies a novel form of financing for open-source software development: seignorage. I show that seignorage is effective at generating incentives and providing financial resources for the development of blockchain-based software. Its effectiveness is, however, limited by the fact that whenever a market for tokens exists, in equilibrium there is a positive probability that the developer will sell all his tokens and that, as a consequence, no development will occur.

Importantly, in the "rich developer" case the developer uses his own resources to finance the investment in the protocol, so that seignorage plays a role exclusively because it generates profits and provides incentives. In the "poor developer" case, seignorage has the additional role of providing resources to be invested into the development of the protocol. The comparison between the two cases shows that the use of seignorage to finance the investment in the protocol is a second-best response to the developer's lack of resource, because the value of the protocol (and the developer's payoff) is always higher in the "rich developer" case. This observation suggests that an external investor (call it a *traditional investor*) could provide capital to the developer so to move from the "poor developer" to the "rich developer" case, and by doing so generate extra surplus. This, however, would depend crucially on the ability of the traditional investor and the developer to contract. Studying the constraints that, in this environment, prevent perfect contracting between a traditional investor and a developer, and establishing whether seignorage and traditional

financing are complements or substitutes is left for future work.

Finally, the model abstracts away from competition, either from other open-source blockchain-based protocols or traditional companies. One interesting aspect of competition is how it changes the timing of the ICO. In particular, if there are "winner takes all" dynamics and network effects, it is conceivable that the developer will want to anticipate the ICO, so to build a sufficiently large user base and prevent the entrance of competitors. But competition is likely to affect the equilibrium of the game in many other ways. The full treatment of this case is also left for future work.

Mathematical appendix

Proof of Proposition 1. In the text I show that if $\tilde{U}_T(Q_T)$ is strictly convex and therefore, in equilibrium, in period $T-1$ the developer is indifferent between selling all his tokens or keeping all his tokens. It follows that I can write

$$\tilde{U}_{T-1}(Q_{T-1}) = \max_{e_{T-1},i_{T-1},e_T,i_T} \left\{ -i_{T-1} - \frac{e_{T-1}^2}{2} - i_T - \frac{e_T^2}{2} + Q_{t-1} \cdot p_T \right\},$$

that is, I can write the utility in period $T-1$ assuming that the developer sells all his tokens in period T. Again, because effort and investment affect p_T, then $\tilde{U}_{T-1}(Q_{T-1})$ is strictly convex and, in equilibrium, in period $T-2$ the developer is indifferent between selling all his tokens or keeping all his tokens. Therefore I can write

$$\tilde{U}_{T-2}(Q_{T-2}) = \max_{e_{T-2},i_{T-2},e_{T-1},i_{T-1},e_T,i_T} \left\{ -i_{T-2} - \frac{e_{T-2}^2}{2} - i_{T-1} - \frac{e_{T-1}^2}{2} - i_T - \frac{e_T^2}{2} + Q_{t-1} \cdot p_T \right\}$$

which is strictly convex. Repeating the same argument implies that all $\tilde{U}_t(Q_t)$ are strictly convex, that in every period the only possible equilibrium is one in which the developer is indifferent between selling all his tokens or purchasing all tokens, and that all $\tilde{U}_t(Q_t)$ can be written as

$$\tilde{U}_t(Q_t) = \max_{e_t,i_t,e_{t+1},i_{t+1},\ldots,e_T,i_T} \left\{ -\sum_s = t^T i_s - \sum_s = t^T \frac{e_s^2}{2} + Q_{t-1} \cdot p_T \right\}.$$

This implies that, in every period, optimal effort and investment are again given by (2) and (3).

Furthermore, for the agent to be indifferent, in every period the price must be $p_t = \frac{\tilde{U}_{t+1}(M)}{M}$. Writing the utility function in period $t+1$ as above, and using optimal effort and investment, we get

$$p_t = \frac{V_t + (T-t) f(e^*(M), i^*(M))}{(1-\gamma)M} - (T-t) \frac{e^*(M)^2/2 + i^*(M)}{M} \tag{13}$$

It follows that if $Q_t = M$, then

$$p_t = \frac{V_{t-1} + (T-t+1) f(e^*(M), i^*(M))}{(1-\gamma)M} - (T-t) \frac{e^*(M)^2/2 + i^*(M)}{M}$$

if instead $Q_t = 0$, then

$$p_t = \frac{V_{t-1} + (T-t) f(e^*(M), i^*(M))}{(1-\gamma)M} - (T-t) \frac{e^*(M)^2/2 + i^*(M)}{M}$$

Call α_{t-1} the probability that in period $t-1$ the developer sells all his tokens. Because investors must be willing to hold tokens between the two periods, it must be that

$$\begin{aligned} p_{t-1} =& \frac{V_{t-1} + (T-t+1)f(e^*(M), i^*(M))}{(1-\gamma)M} - (T-t+1)\frac{e^*(M)^2/2 + i^*(M)}{M} = \\ & \alpha_{t-1}\left(\frac{V_{t-1} + (T-t)f(e^*(M), i^*(M))}{(1-\gamma)M} - (T-t)\frac{e^*(M)^2/2 + i^*(M)}{M}\right) + \\ & (1-\alpha_{t-1})\left(\frac{V_{t-1} + (T-t+1)f(e^*(M), i^*(M))}{(1-\gamma)M} - (T-t)\frac{e^*(M)^2/2 + i^*(M)}{M}\right) \end{aligned}$$

Solving for α_{t-1} yields:

$$\alpha_{t-1} = (1-\gamma)\,\frac{(e^*(M))^2/2 + i^*(M)}{f(e^*(M), i^*(M))}.$$

Finally, the above expression can be used to further simplify (13) and achieve (6)

□

Proof of Lemma 2. As discussed in the text, the choice of Q_T maximizes the continuation value:

$$U_T(Q_T, A_{T-1} + (M - Q_T)\cdot p_T^* - i_{T-1}),$$

where $p_T^* \in p(Q_T)$ depends on which equilibrium is expected to emerge in period T. The important observation is that Q_T determines the assets available in the following period. Therefore, by Lemma 1, the continuation value is strictly convex in Q_T for

$$\hat{Q} \leq Q_T \leq Q_{T-1} - \frac{i_{T-1} + \bar{i} - A_{T-1}}{p_T}$$

and is linearly increasing in Q_T otherwise, with a downward discontinuity at $M - \frac{i_{T-1}+\bar{i}-A_{T-1}}{p_T}$, given by the minimum number of tokens that the developer needs to sell in order to achieve $\bar{i}$ in period T. See Figure 4 for a graphical representation.

Suppose that the "high" equilibrium is expected to emerge, so that $p_T = \max\{p(Q_T)\}$. The discontinuity is at

$$\tilde{Q}'_T \equiv Q_T : \frac{\bar{i} + i_{T-1} - A_{T-1}}{M - Q_T} = \frac{V_{T-1} + g(e^*(Q_T))}{(1-\gamma)M}$$

generating a continuation utility:

$$\frac{V_{T-1} + g(e^*(\min\{\tilde{Q}'_T, M\}))}{(1-\gamma)M}M - \frac{1}{2}(e^*(\min\{\tilde{Q}'_T, M\}))^2$$

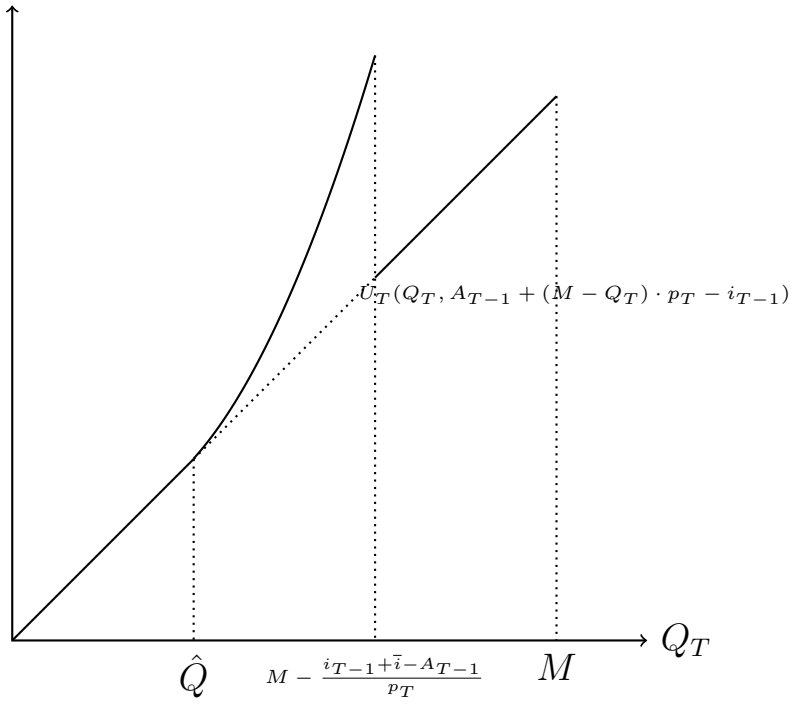


Fig. 4: Continuation value as a function of Q_T.

If the "low" equilibrium is expected to emerge, then the discontinuity is at

$$\tilde{Q}''_T = M - \frac{\left(\bar{i} + i_{T-1} - A_{T-1}\right)(1-\gamma)M}{f(e_{T-2}, i_{T-2}) + f(e_{T-1}, i_{T-1})}$$

generating a continuation utility:

$$\frac{V_{T-1} + g(e^*(\min\{\tilde{Q}''_T, M\}))}{(1-\gamma)M} M - \frac{1}{2}(e^*(\min\{\tilde{Q}''_T, M\}))^2$$

Because period T effort is chosen optimally, it must be that

$$\frac{g(e^*(\min\{\tilde{Q}'_T, M\}))}{(1-\gamma)M} M \geq \frac{1}{2}(e^*(\min\{\tilde{Q}'_T, M\}))^2$$

and

$$\frac{g(e^*(\min\{\tilde{Q}''_T, M\}))}{(1-\gamma)M} M \geq \frac{1}{2}(e^*(\min\{\tilde{Q}''_T, M\}))^2$$

which implies that the two continuation utilities (the one with threshold $\tilde{Q}'_T$ and the one with threshold $\tilde{Q}''_T$) are greater than the continuation utility when the developer

holds $Q_T = M$ and no investment occurs:

$$\frac{f(e_{T-2}, i_{T-2}) + f(e_{T-1}, i_{T-1})}{(1-\gamma)M} M$$

Hence holding either $\tilde{Q}'_T$ or $\hat{Q}''_T$ is preferred to holding the entire stock of tokens M and not investing. The continuation utility is therefore maximized at either $\tilde{Q}'_T$ or $\hat{Q}''_T$, depending on what equilibrium is expected to emerge in period T. □

Proof of Lemma 3. Remember that Q^*_T is the largest possible Q_T such that the period-T constraint is not binding. It follows that, as already discussed, if $Q^*_T \leq \hat{Q}$ then the continuation value is linear in Q_T because there is no Q_T for which the developer will exert effort in period T.

If instead $Q^*_T > \hat{Q}$ then the continuation value is somewhere strictly convex in Q_T for $Q_T \leq Q^*_T$. In this case, there is the same anti-coordination problem discussed for the "rich developer" case and the equilibrium is in mixed strategies. The developer must be indifferent between $Q_T = 0$ and Q^*_T.

The price at which the developer is indifferent is:

$$p_{T-1} = \frac{U_T(Q^*_T, A_{T-1} + (Q_{T-1} - Q^*_T) \cdot p_{T-1} - i_{T-1})}{Q^*_T} = \frac{Q^*_T \left(\frac{V_{T-1} + g(e^*(Q^*_T, \bar{i})}{(1-\gamma)M} \right) - \frac{1}{2}(e^*(Q^*_T, \bar{i})^2 - \bar{i}}{Q^*_T}$$

Furthermore, investors must be indifferent between holding tokens in period T and in period $T-1$, which implies that

$$p_{T-1} = \frac{V_{T-1} + (1 - \alpha_{T-1}) g(e^*(Q^*_T, \bar{i}))}{(1-\gamma)M}$$

where α_{T-1} is the probability that the developer sells all his tokens in period $T-1$. Combining the above two expressions and solving for α_{T-1} yield the expression in the proposition.

For existence and (sometimes) uniqueness of the equilibrium, without loss of generality, assume that whenever $Q^* \leq \hat{Q}$ the agent randomizes between $\max\{Q^*_T, M\}$ and 0. Define Q^*_T as a function of p_{T-1} by:

$$Q(p) \equiv \begin{cases} \min\left\{ Q_{T-1} - \frac{i_{T-1} + \bar{i} - A_{T-1}}{p}, M \right\} & \text{if } Q_{T-1} - \frac{i_{T-1} + \bar{i} - A_{T-1}}{p} > 0 \\ 0 & \text{otherwise,} \end{cases}$$

which is increasing whenever $A_{T-1} - i_{T-1} \leq \bar{i}$ (that is, when the developer needs to sell some tokens in period $T-1$ to invest $i_T = \bar{i}$), and is decreasing otherwise.

Similarly define the equilibrium p_{T-1} as a function of Q_T^* by:

$$p(Q) \equiv \frac{V_{T-1} + (1-\alpha(Q))g(e^*(Q), i^*(Q, A_T))\{i^*(Q) \geq \bar{i}\}}{(1-\gamma)M}$$

where

$$\alpha(Q) \equiv \left(\frac{1}{2}(e^*(Q, i^*(Q, A_T)))^2 + i^*(Q, A_T)\right)\left(Q \cdot \frac{g(e^*(Q, i^*(Q, A_T)))}{(1-\gamma)M}\right)^{-1}$$

The complication here is that, for given Q, A_T is itself a function of $p(Q)$, which implies that $p(Q)$ is a correspondence. The reason is the same discussed in the body of the paper for the case $t_o = T-1$: if the developer needs to sell some tokens to invest in period T, then for given number of tokens sold, period-T investment will be a function of the price at which the developer can sell these tokens. Hence, whenever $A_{T-1} - i_{T-1} < \bar{i}$ (that is, whenever the developer needs to sell some tokens in period $T-1$ to invest $i_T = \bar{i}$), we have

$$p(Q) \equiv \frac{V_{T-1}}{(1-\gamma)M} + \begin{cases} 0 & \text{if either } Q \leq \hat{Q} \text{ or } Q > Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}}{(1-\gamma)M}} \\ \frac{(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M} & \text{if } \hat{Q} \leq Q \leq Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}+(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M}} \end{cases}$$

because

$$Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}}{(1-\gamma)M}} < Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}+(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M}}$$

for all Q, the case $A_{T-1} - i_{T-1} < \bar{i}$ can be split into three subcases:[25]

1. ("high V_{T-1}") Whenever $Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}}{(1-\gamma)M}} > \hat{Q}$ then for some Q we have $p(Q) = \left\{\frac{V_{T-1}}{(1-\gamma)M}, \frac{V_{T-1}+(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M}\right\}$. That is, there are situations in which for given Q_T^*, if p_{T-1} is low the developer will not have enough funds to finance investment in period T, and therefore no development will occur. If instead p_{T-1} is high there is positive probability that the developer will invest and exert effort in period T. Again, this situation can be seen as a coordination problem among investors. For given action taken by the developer in period $T-1$, investors can coordinate on a "high" equilibrium that leads to effort and investment in period T with positive probability, or a "low" equilibrium leading to no development in period T.

[25] The three cases emerge as a function of the three state variables Q_{T-1}, V_{T-1} and $A_{T-1} - i_{T-1}$. For ease of exposition, I describe them solely in terms of V_{T-1}, although for $Q_{T-1} > \hat{Q}$ but sufficiently low, the three cases will indeed emerge as a function of V_{T-1} exclusively.

2. ("low V_{T-1}") Whenever $Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}+(1-\alpha(M))g(e^*(M))}{(1-\gamma)M}} \leq \hat{Q}$, then there is no development in period T and $p(Q) = \frac{V_{T-1}}{(1-\gamma)M}$ for all Q.

3. ("intermediate V_{T-1}") in all other cases, $p(Q)$ is a function, which is equal to $\frac{V_{T-1}}{(1-\gamma)M}$ for $Q \leq \hat{Q}$ and to $\frac{V_{T-1}+(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M}$ otherwise.

Instead, whenever $A_{T-1} - i_{T-1} \geq \bar{i}$ (that is, whenever the developer has enough own funds to invest $i_T = \bar{i}$), then period T investment does not depend on p_{T-1} and therefore

$$p(Q) \equiv \frac{V_{T-1}}{(1-\gamma)M} + \begin{cases} 0 & \text{if either } Q \leq \hat{Q} \\ \frac{(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M} & \text{otherwise} \end{cases}$$

which is a continuous function.

By definition of $\alpha(Q)$, I can write

$$Q \cdot \frac{g(e^*(Q,\bar{i}))}{(1-\gamma)M} - \frac{1}{2}(e^*(Q,\bar{i}))^2 - \bar{i} = (1-\alpha(Q))Q \cdot \frac{g(e^*(Q,\bar{i}))}{(1-\gamma)M}. \tag{14}$$

The LHS of (14) is equal to:

$$\max_e \left\{ Q \cdot \frac{g(e,\bar{i})}{(1-\gamma)M} - \frac{1}{2}e^2 \right\}$$

which is strictly increasing and strictly convex in Q. It follows that the RHS of (14) must also be strictly increasing and strictly convex in Q. This, in turn, implies that $p(Q)$ is strictly increasing whenever Q is such that positive development is expected with some probability in period T, and is constant otherwise.

The equilibrium of the game is a p^* such that $p^* = p(Q(p^*))$ and a $Q^* = Q(p^*)$. Figure 5 represents all possible cases. Whenever both $p(Q)$ and $Q(p)$ are functions, the existence of the equilibrium is readily established. It is enough to note that the range of $p(Q)$ is a closed interval. Call this interval $[a, b]$. The equilibrium is the fixed point of the continuous function $p(Q(p))$ defined over $[a, b]$. Brower's fixed point theorem applies and the fixed point exists.

Whenever $p(Q)$ is a correspondence ($A_{T-1} - i_{T-1} < \bar{i}$, "high V_{T-1}" case) we know that for $\hat{Q} \leq Q \leq Q_{T-1} - \frac{i_{T-1}+\bar{i}-A_{T-1}}{\frac{V_{T-1}+(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M}}$ we have that $\frac{V_{T-1}+(1-\alpha(Q))g(e^*(Q,\bar{i}))}{(1-\gamma)M} \in p(Q)$. Define the threshold value of Q

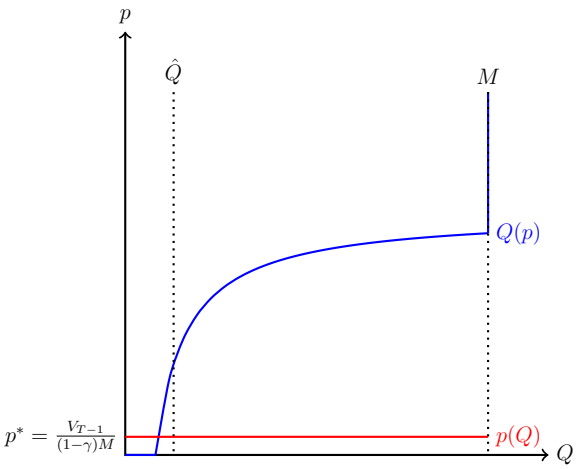
$$\tilde{Q} \equiv Q_{T-1} - \frac{i_{T-1} + \bar{i} - A_{T-1}}{\frac{V_{T-1}+(1-\alpha(\tilde{Q}))g(e^*(\tilde{Q},\bar{i}))}{(1-\gamma)M}}$$

and similarly the corresponding price

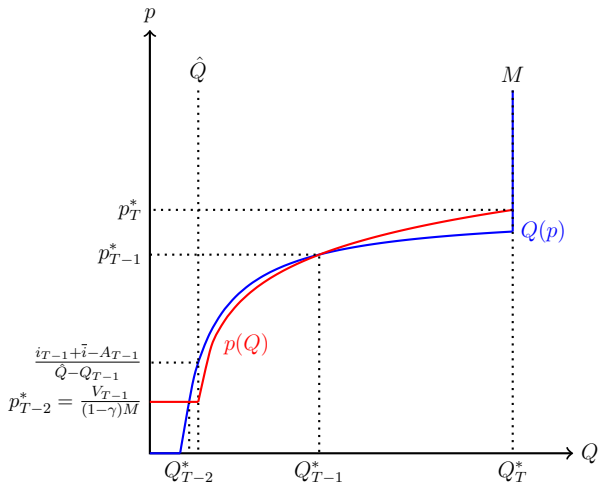
$$\tilde{p} \equiv \frac{V_{T-1} + (1 - \alpha(\tilde{Q}))g(e^*(\tilde{Q}, \bar{i}))}{(1 - \gamma)M} \in p(Q)$$

By definition of $Q(p)$ we have that $\tilde{Q} = Q(\tilde{p})$, which implies that $\{\tilde{Q}, \tilde{p}\}$ is an equilibrium.

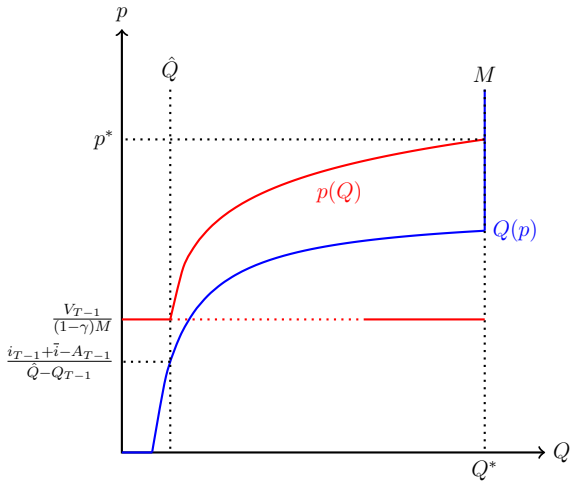
It is quite immediate to see that in case $A_{T-1} - i_{T-1} \geq \bar{i}$ the equilibrium is unique. The equilibrium is unique also in the "low V_{T-1}" case. In all other cases multiple equilibria are possible.



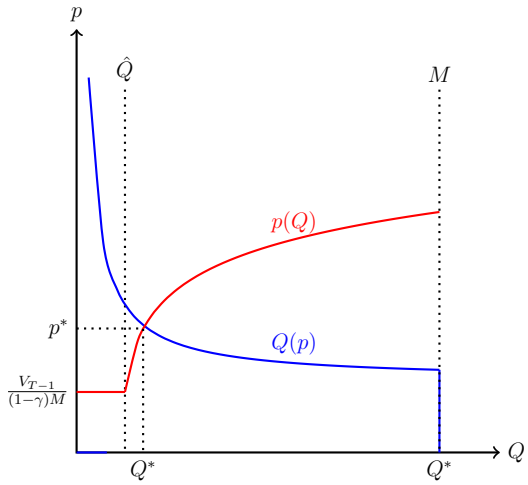
(a) $A_{T-1} - i_{T-1} \leq \bar{i}$; "low V_{T-1}" case.



(b) $A_{T-1} - i_{T-1} \leq \bar{i}$; "intermediate V_{T-1}" case; multiple equilibria



(c) $A_{T-1} - i_{T-1} < \bar{i}$; "high V_{T-1}" case.



(d) $A_{T-1} - i_{T-1} > \bar{i}$

Fig. 5: Equilibrium in $T-1$.

□

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