Modeling and Forecasting Inflation in Zimbabwe: a Generalized Autoregressive Conditionally Heteroskedastic (GARCH) approach

NYONI, THABANI

University of Zimbabwe

22 July 2018

Modeling and Forecasting Inflation in Zimbabwe: a Generalized Autoregressive Conditionally Heteroskedastic (GARCH) approach

Thabani Nyoni

Department of Economics, University of Zimbabwe, Harare, Zimbabwe

Email: nyonithabani35@gmail.com

Abstract

Of uttermost importance is the fact that forecasting macroeconomic variables provides a clear picture of what the state of the economy will be in future (Sultana et al, 2013). Nothing is more important to the conduct of monetary policy than understanding and predicting inflation (Kohn, 2005). Inflation is the scourge of the modern economy and is feared by central bankers globally and forces the execution of unpopular monetary policies. Inflation usually makes some people unfairly rich and impoverishes others and therefore it is an economic pathology that stands in the way of any sustainable economic growth and development. Models that make use of GARCH, as highlighted by Ruzgar & Kale (2007); vary from predicting the spread of toxic gases in the atmosphere to simulating neural activity but Financial Econometrics remains the leading discipline and apparently dominates the research on GARCH. The main objective of this study is to model monthly inflation rate volatility in Zimbabwe over the period July 2009 to July 2018. Our diagnostic tests indicate that our sample has the characteristics of financial time series and therefore, we can employ a GARCH – type model to model and forecast conditional volatility. The results of the study indicate that the estimated model, the AR (1) – GARCH (1, 1) model; is indeed an AR (1) – IGARCH (1, 1) process and is not only appropriate but also the best. Since the study provides evidence of volatility persistence for Zimbabwe's monthly inflation data; monetary authorities ought to take into cognisance the IGARCH behavioral phenomenon of monthly inflation rates in order to design an appropriate monetary policy.

Key Words: ARCH, Forecasting, GARCH, IGARCH, Inflation Rate Volatility, Zimbabwe.

1. Introduction

Although being slightly perceptible, inflation is one of the most difficult to define and bound complex phenomena (Baciu, 2015). Inflation can be defined as the persistent and continuous rise in the general prices of commodities in an economy (Nyoni & Bonga, 2018a). Inflation is the persistent increase in the general price level within the economy which affects the value of the domestic currency (Fatukasi, 2012). Maintenance of price stability continues to be one of the main objectives of monetary policy for most countries in the world today (Nyoni & Bonga, 2018a) and Zimbabwe is not an exception. Sound and productive level of inflation, as noted by Idris & Bakar (2017); is certainly regarded as a repercussion of fiscal prudence and essential criteria for the attainment of a sustainable level of growth and development. Owing to the fact that the effect of monetary policy has a time lag, as already noted by Bokil & Schimmelpfennig (2005); policy makers inevitably require frequent updates to the path of inflation. Policy makers can get prior indication about possible future inflation through inflation forecasting. In this study we use the GARCH approach to model monthly inflation rate volatility in Zimbabwe.

Accomplishment of price stability, in the sense of a low and steady inflation, is key to economic growth and it is one of the objectives of almost every central bank throughout the world (Iqbal & Sial, 2016). Currently, policy makers in Zimbabwe are facing the twin challenge of maintaining price stability while stimulating economic growth. Achieving these two goals simultaneously is by no means easy. Just like other developing nations, Zimbabwe's main macroeconomic targets are generally three – fold, namely; a growth target to support higher employment and poverty alleviation; an inflation target to maintain internal economic stability; and a target for stability of the Balance of Payments (BoP). Therefore, Zimbabwe generally needs a combination of three policy instruments, namely; monetary policy, fiscal policy and policies for managing the BoP; in order to achieve her macroeconomic targets. In attempting to model and forecast monthly inflation rate, this study will give birth to policy prescriptions that are envisaged to assist policy makers in properly coordinating Zimbabwe's macroeconomic targets in a sustainable manner.

A number researchers have analyzed inflation in Zimbabwe, for example; Chhibber *et al* (1989), Dzvanga (1995), Sunde (1997), Makochekanwa (2007), Pindiriri & Nhavira (2011), Pindiriri (2012), Kavila & Roux (2015), Mpofu (2017) and Kavila & Roux (2017) but no study has attempted to employ the GARCH approach to modeling and forecasting inflation in Zimbabwe, hence the need for this study. The rest of the paper is organized as follows: literature review; materials & methods; results: presentation, interpretation & discussion; and conclusion & recommendations; in chronological order.

2. Literature Review

Theoretical Literature Review

The Monetarist Theory of Inflation

The monetarist school of thought, also known as the modern Quantity Theory of Money (QTM); argues that inflation is always and everywhere a monetary phenomenon which comes from rapid expansion in the quantity of money than in the expansion in the quantity of output (Nyoni & Bonga, 2018a). Hinged on the QTM, monetarists believe that the quantity of money is the main determinant of the price level. In their diagnosis of the QTM, monetarists finalize that any change in the quantity of money affects only the price level, leaving the real sector of the economy totally unaffected.

The Keynesian Theory of Inflation

Money creation does not have a direct impact on aggregate demand but rather through interest rates, which themselves have a minimal impact on aggregate demand (Samuelson, 1971). Keynesians strictly disputed the monetarist view that the velocity of circulation of money is stable; instead, arguing that it is rather unstable. The monetarist view on policy rules was criticized by Modigliani (1977), who advocated for the use of stabilization policies dealing with inflation. Keynesians also rejected the monetarist notion that markets are able to adjust freely to disruptions and return to full employment level of output.

The Neo – Keynesian Theory of Inflation

Based in the Keynesian school of thought, the Neo – Keynesian Theory of Inflation states that there are three types of inflation; namely demand pull, cost – push and structural inflation. Demand pull inflation occurs when aggregate demand exceeds available supply. Cost – push inflation occurs due to sudden decrease in aggregate supply. Structural inflation basically occurs as a result of (structural) changes in monetary policy in the country.

Demand Pull Inflation

Demand pull theorists agree that the primary cause of inflation is the persistent increase in the aggregate demand for goods against a relatively fixed supply of goods (Addison & Burton, 1979). The source of inflation emanates from changes to the demand side of the aggregate goods market. An increase in aggregate demand without a corresponding increase in supply, results in an inflationary gap which induces increase in prices (Kavila & Roux, 2017). Inflation is generated by the pressures of excess demand as it approaches and exceeds the full employment level of output (Keynes, 1936; Smithies, 1942). If the economy is operating at full employment output and aggregate demand rises, the output level cannot respond automatically because of full employment constraints. Consequently, the only way to clear the goods market is by raising the money prices for goods. The proponents of demand pull inflation argue that the inflationary gap in the goods market results from a disequilibrium in the money market (Kavila & Roux, 2017). Therefore, inflation is caused by an expansionary monetary policy (Friedman, 1956), where the rate of expansion in the quantity of money is more than the rate of increase in output (Kavila & Roux, 2017).

Cost – push Inflation

Cost push theories of inflation cite non – monetary supply – oriented influences that raise costs and hence prices (Kavila & Roux, 2017). While recognizing the significance of monetary expansion as a source of inflation, proponents of cost push theory view monetary growth as playing the accommodating or passive role (Humphrey, 1977). Cost inflation theories emphasize the increase of production costs instead of excess aggregate demand (Kavila & Roux, 2017).

Structural Inflation

Structuralist models of inflation emphasize on supply – side factors as determinants of inflation (Bernanke, 2005). Structuralism generally argues that there are factors unique to a country's institutional dynamics that define a country's predisposition to inflation. The structuralist ideology attributes inflation to bottlenecks in the economy and such bottlenecks include inelastic supply, monopoly tendencies and labor inflexibility amongst others. Food prices, administered prices, wages and import prices, as noted by Khan & Schimmelpfennig (2006); are also considered as sources of inflation. Structuralists argue that the world is inflexible to such an extent that the price mechanism tends to fail to help the market achieve equilibrium.

Expectations Theory of Inflation

Inflation expectations are perceptions or views of economic agents about future inflation trends (Mohanty, 2012). Inflation expectations are a basic building block in today's macroeconomic theory and monetary policy (Gali, 2008; Sims, 2009). The ability of central banks to achieve price stability is greatly influenced by economic agents' inflation expectations and in this regard, the management of the perceptions is a critical component in the enhancement of price stability (Bomfin & Rodenbusch, 2000; Loleyt & Gurov, 2010).

Expectations can be formed either adaptively or rationally. Adaptive expectations exist when economic agents form their expectations based on past and current experience with inflation. Owing to their backward looking approach, adaptive expectations have attracted a lot of criticisms, giving birth to the rational expectations theory. Rational expectations, as noted by Mohanty (2012); are formed when economic agents take into account all the available information about inflation and then factor in the central bank's monetary policy reaction function. The Phillips curve relationship, as already highlighted by Elhers & Steinbach (2007); is now expectations augmented, further signifying the importance of expectations in macroeconomic policy.

As suggested by various theories of inflation (discussed here), indeed causes of inflation differ, of course according to school of thought. While economic thinking on the nature and effect of inflation may differ, we cannot rule out the existence of a consensus on the need to control the inflation process. Unfortunately, in Zimbabwe; monetary authorities have a well – known history of gross macroeconomic mismanagement.

Empirical Literature Review

The table below shows a summary of the reviewed previous studies:

Table 1

Author	Year	Country	Study Period	Method	Key Findings
Nor *et al*	2007	Malaysia	January 1980 – December 2004	✓ GARCH ✓ GARCH – MEAN ✓ EGARCH ✓ EGARCH – MEAN	The EGARCH model gives better estimates of sub – periods volatility.
Saleem	2008	Pakistan	January 1990 – May 2007	✓ VAR ✓ ARCH ✓ GARCH ✓ EGARCH	Inflation is volatile in nature The time effect model is significant
Sek & Har	2012	Korea, Philippines, Thailand	Korea: September 1985 – June 2010 Philippines: January 1985 – June 2010 Thailand: January 1987 – June 2010	✓ GARCH (1, 1)	There is lower volatility of inflation and also persistence of inflation declines
Osarumwense & Waziri	2013	Nigeria	January 1995 – December 2011	GARCH (1, 1)+ARMA (1, 0)	There would not be any major high volatility persistence in Nigeria
Benedict	2013	Ghana	January 1965 – December 2012	Various ARCH – family type models	EGARCH (2, 1) model is superior in performance
Awogbemi *et al*	2015	Nigeria	January 1997 – December 2007	✓ ARCH ✓ GARCH	Volatility seems to persist in all commodity items studied.
Moroke & Luthuli	2015	South Africa	2002 – 2014 (Quarterly)	GARCH type models	AR (1) - IGARCH model suggested a high degree of persistence in the conditional volatility of the series. AR (1) - EGARCH model was found to be more robust in forecasting volatility effects than the AR (1) – GJR – GARCH (2, 1) models
Uwilingiyimana *et al*	2015	Kenya	January 2000 – December 2014	✓ ARIMA (1, 1, 12) ✓ GARCH (1, 2)	ARIMA (1, 1, 12) – GARCH (1, 2) model is the best model for forecasting inflation in Kenya.
Fwaga *et al*	2017	Kenya	January 1990 – December 2015	✓ EGARCH ✓ GARCH	The EGARCH (1, 1) model is the best model for forecasting Kenyan inflation data.
Banerjee	2017	41 countries	January 1958 – February 2016	GARCH (1, 1)	In the long – run, conditional volatility of inflation is 3.5 times greater in developing countries compared to advanced countries.

As shown in table 1 above, no relevant study has been done in the context of Zimbabwe so far. Hence, this study is the first of its kind.

3. Materials & Methods

Model Building & Estimation Procedures

Autoregressive Conditionally Heteroskedastic (ARCH) model

A typical structural model can be expressed as follows:

$$\mathbf{w_t=\theta_0+\theta_1\zeta_{1t}+\theta_2\zeta_{2t}+\theta_3\zeta_{3t}+\theta_4\zeta_{4t}+\varepsilon_t} \quad \ldots\ldots \quad \mathbf{[1]}$$

where θ_0 is the model constant, $\theta_1 - \theta_4$ are estimation parameters, ζ_{1t} - ζ_{4t} are explanatory variables, w_t is the dependent variable and ε_t is the white noise error term.

or more compactly as:

$$\mathbf{w_t=Z\theta+\varepsilon_t} \quad \ldots\ldots \quad \mathbf{[2]}$$

with $\varepsilon_t \approx N\ (0, \sigma_t^2)$

where Z and θ are vectors of explanatory variables and estimation parameters respectively.

As usual, homoskedasticity is assumed; that is:

$$\mathbf{var\ (\varepsilon_t)= \sigma_t^2} \quad \ldots\ldots \quad \mathbf{[3]}$$

If equation [3] above is not true, then that would be known as heteroskedasticity and this would imply that standard error estimates are misleading. In financial time series, however; it is rare that the variance of the errors will be constant over time, and therefore it is reasonable to take into account a model that does not assume that the variance is constant and yet describes how the variance of the errors evolves; and such a model is an ARCH model. To explain the mechanics behind the ARCH model, we begin by operationally defining the conditional variance of a random variable, ε_t:

$$\mathbf{\sigma_t^2=var(\varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots)=E[\varepsilon_t-E(\varepsilon_t)^2|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots]} \quad \ldots\ldots \quad \mathbf{[4]}$$

assuming that:

$$\mathbf{E(\varepsilon_t)=0} \quad \ldots\ldots \quad \mathbf{[5]}$$

such that:

$$\mathbf{\sigma_t^2=var(\varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots)=E[\varepsilon_t^2|\ \varepsilon_{t-2}, \ldots]} \quad \ldots\ldots \quad \mathbf{[6]}$$

Equation [6] above simply avers that the conditional variance of a zero mean normally distributed random variable ε_t is equal to the conditional expected value of the square of ε_t. The equation below:

$$\mathbf{\sigma_t^2=\varphi_0+\varphi_1\varepsilon^2_{t-1}} \quad \ldots\ldots \quad \mathbf{[7]}$$

is known as an ARCH(1) model simply because the conditional variance depends only on one lagged squared error. Equation [7] is not a complete model because so far we have not mentioned anything about the conditional mean; which describes how the dependent variable, w_t;

varies over time. There is no rule of thumb on how to specify the conditional mean equation; therefore it can take apparently any form that the researcher wishes. By combining equations [1] and [7], we can show that a typical full model looks like equations [8] and [9] below:

$$\mathbf{w_t=\theta_0+\theta_1\zeta_{1t}+\theta_2\zeta_{2t}+\theta_3\zeta_{3t}+\theta_4\zeta_{4t}+\varepsilon_t\ ;\ \varepsilon_t\approx N(0,\sigma_t^2)} \quad \textbf{[8]}$$

$$\mathbf{\sigma_t^2=\varphi_0+\varphi_1\varepsilon^2_{t-1}} \quad \textbf{[9]}$$

The model given by equations [8] and [9] can be generalized to a case where the error variance depends on p lags of squared errors, which is operationally defined as an ARCH (p) model:

$$\mathbf{\sigma_t^2=\varphi_0+\varphi_1\varepsilon^2_{t-1}+\ldots+\varphi_p\varepsilon^2_{t-p}} \quad \textbf{[10]}$$

Generalized ARCH (GARCH)[1] model

The equation below:

$$\mathbf{\sigma_t^2=\varphi_0+\varphi_1\varepsilon^2_{t-1}+\phi_1\sigma^2_{t-1}} \quad \textbf{[11]}$$

is the simplest but often very useful case of a GARCH process, the GARCH(1,1) model; where σ_t^2 is the conditional variance, φ_0 is the constant, $\varphi_1\varepsilon^2_{t-1}$ is the information about the volatility during the previous period, and $\phi_1\sigma^2_{t-1}$ is the fitted variance from the model during the previous period. A GARCH model can be expressed as an ARMA process of squared residuals. For instance, given equation [11] above, we know that:

$$\mathbf{E_{t-1}[\varepsilon^2_t]=\sigma_t^2} \quad \textbf{[12]}$$

such that:

$$\mathbf{\varepsilon^2_t=\varphi_0+(\varphi_1+\phi_1)\varepsilon^2_{t-1}+\mu_t-\phi_1\mu^2_{t-1}} \quad \textbf{[13]}$$

which is an ARMA (1, 1) process. Here:

$$\mathbf{\mu_t=\varepsilon^2_t-E_{t-1}[\varepsilon^2_t]} \quad \textbf{[14]}$$

is the white noise error term. Given the ARMA representation of the GARCH model, we can conclude that stationarity of the GARCH (1, 1) model requires:

$$\mathbf{\varphi_1+\phi_1<1} \quad \textbf{[15]}$$

Taking the unconditional expectation of equation [11], we get:

$$\mathbf{\sigma^2=\varphi_0+\varphi_1\sigma^2+\phi_1\sigma^2} \quad \textbf{[16]}$$

so that:

---

[1] GARCH models are better than ARCH models because they are more parsimonious and thus they subsequently deal with the problem of over – fitting.
ARCH and GARCH models have emerged as the most proeminent tools for estimating volatility, because they are adequate to capture the random movement of the financial data series (Anton, 2012). The basic and most widespread model is GARCH (1, 1) (Ruzgar & Kale, 2007). ARCH models were developed by Engle (1982). GARCH models were developed independently by Bollerslev (1986) and Taylor (1986).

$$\sigma^2=\frac{\mathcal{G}}{\mathcal{H}} \quad [17]$$

where $\mathcal{G}=\varphi_0$ and $\mathcal{H}=1-\varphi_1-\phi_1$. For this unconditional variance to exist, then the (inequality) equation [15] must hold water and for it to be positive, then:

$$\mathcal{G}>0 \quad [18]$$

Equation [11] above can be generalized to a case where the current conditional variance is parameterized to depend upon p lags of the squared error and q lags of the conditional variance, which is operationally defined as a GARCH(p, q) model; as shown below:

$$\sigma_t^2=\varphi_0+\varphi_1\varepsilon^2_{t-1}+\ldots+\varphi_p\varepsilon^2_{t-p}+\phi_1\sigma^2_{t-1}+\ldots+\phi_q\sigma^2_{t-q} \quad [19]$$

Equation [19] can also be expressed as follows[2]:

$$\sigma_t^2=\varphi_0+\varphi(\ell)\varepsilon_t^2+\phi(\ell)\sigma_t^2 \quad [20]$$

where $\varphi(\ell)$ and $\phi(\ell)$ denote the AR and MA polynomials respectively, with:

$$\varphi(\ell)=\varphi_1\ell+\ldots+\varphi_p\ell^p \quad [21]$$

and:

$$\phi(\ell)=\phi_1\ell+\ldots+\phi_q\ell^q \quad [22]$$

or as:

$$\sigma_t^2=\varphi_0+\sum_{i=1}^{p}\varphi_i\varepsilon^2_{t-i}+\sum_{j=1}^{q}\phi_j\sigma^2_{t-j} \quad [23]$$

where condition [15], is now generalized as:

$$\sum_{i=1}^{p}\varphi_i+\sum_{j=1}^{q}\phi_j<1 \quad [24]$$

If all the roots of the polynomial:

$$|1-\phi(\ell)|^{-1}=0 \quad [25]$$

lie outside of the unit circle, we get:

$$\sigma_t^2=\varphi_0\,|1-\phi(\ell)|^{-1}+\varphi(\ell)\,|1-\phi(\ell)|^{-1}\varepsilon_t^2 \quad [26]$$

which may be regarded as an ARCH (∞) process, just because the conditional variance linearly depends on all previous squared residuals. The unconditional variance is then given by:

$$\sigma^2\equiv E(\varepsilon_t^2)=\varphi_0/(1-\sum_{i=1}^{p}\varphi_i-\sum_{j=1}^{q}\phi_j) \quad [27]$$

Obviously, the unconditional variance will be infinite if:

$$\varphi_1+\ldots+\varphi_p+\phi_1+\ldots+\phi_q=1 \quad [28]$$

---

[2] where ℓ is the lag operator.

In many financial time series, conditions [15] and [24]; whose implication is basically the same, are close to unity; indicating persistent volatility. In the event that:

$$\varphi_1+\phi_1=1 \quad \text{[29]}$$

or more generally:

$$\sum_{i=1}^{p}\varphi_i+\sum_{j=1}^{q}\phi_j=1 \quad \text{[30]}$$

or equally rewritten:

$$\varphi(\ell)+\phi(\ell)=1 \quad \text{[31]}$$

then the resulting process is not covariance stationary[3] and is technically referred to as an Integrated GARCH or IGARCH, implying that current information remains vital when forecasting the volatility for all horizons. However, Nelson (1990) argues that although GARCH (or IGARCH) model is not covariance stationary, it is strictly stationary or ergodic and the standard asymptotically based inference procedures are generally valid. In the IGARCH model, as noted by Anton (2012); any shock to volatility is permanent and the unconditional variance is infinite.

Model Selection Criteria (Goodness – of – fit Measure)

The model selection criterion used in most studies is usually based either on all or one of the following:

$$AIC=-2\log(L)+2(m) \quad \text{[32]}$$

$$BIC=-2\log(L)+m\log n \quad \text{[33]}$$

$$HQ=-2\log(L)+2m\log(\log n) \quad \text{[34]}$$

$$SIC=-2\log(L)+(m+m\log n) \quad \text{[35]}$$

where AIC is the Akaike Information Criterion, BIC is the Bayesian Information Criterion, HQ is the Hannan – Quinn Criterion, SIC is the Shwarz Information Criterion, n is the sample size, m is the number of parameters in the model and log (L) is the loglikelihood. The optimal model (the one that strikes a balance between goodness of – fit and parsimony) is usually the one that minimizes these criteria. In this study we take into consideration all of the above criteria, except the BIC.

Table 2

Model			AIC	HQC	SIC
i	GARCH (1, 1)	AR (1)	221.6992	228.1762	237.6798
ii	GARCH (2, 1)	AR (1)	211.0228	218.5793	229.6668

[3] Covariance stationarity is the condition that is more frequently assumed in GARCH models. It does require that all first and second moments exist whereas strict stationarity does not. In this one respect, covariance stationarity is a stronger condition (Holton, 1996).

Table 2 above shows that model [ii] is the one with the lowest AIC, HQC and SIC values. In general, the model with the lowest selection criterion values is usually taken as the optimal model. However, in this study we decide (the other way round) to consider model [i]; the reason being that of the need to specify the most *parsimonious* model while also ensuring that the chosen model is the best in terms of both in – sample and out – sample forecasts, as confirmed by forecast evaluation statistics shown in table 3 below. Model [i] is the most parsimonious model and has a better forecast accuracy (as shown in table 3 below), therefore it is preferred. According to Brooks (2008), a GARCH (1, 1) model will be sufficient to capture the volatility clustering in the data, and rarely is any higher order model estimated or even entertained in the academic finance literature. In the same line of thought, Hansen & Lunde (2005) argue that there is no evidence that a GARCH (1, 1) model is outperformed by any other model/s. Model [i] is the appropriate model since it strikes a balance amongst goodness – of – fit, parsimony and forecast performance[4]. Table 3 below also further justifies our decision to choose model [i].

Forecast Evaluation (Forecasting Performance Measure)

The forecast evaluation statistics shown in table 2 below were computed as follows:

$$\mathbf{MSE}=\frac{1}{T}\sum_{t=1}^{T}(r^2{}_t-\sigma^2{}_t)^2 \quad \text{[36]}$$

$$\mathbf{MAE}=\frac{1}{T}\sum_{t=1}^{T}\left|r^2{}_t-\sigma^2{}_t\right| \quad \text{[37]}$$

$$\mathbf{RMSE}=\sqrt{MSE} \quad \text{[38]}$$

where $r^2{}_t$ is used as a substitute for the realized or actual variance, $\sigma^2{}_t$ is the forecasted variance, and T is the number of observations in the simulations of the sample.

Table 3

Model			Mean Squared Error (MSE)[5]	Root Mean Squared Error (RMSE)	Mean Absolute Error (MAE)
i	GARCH (1, 1)	AR (1)	1.2159	1.1027	0.54827
ii	GARCH (2, 1)	AR (1)	1.2556	1.1205	0.55919

Smaller values of the forecast evaluation statistics generally indicate that the forecast is quite good. As shown in table 3 above, model [i], as compared to model [ii]; is the one with the lowest forecast evaluation statistics, hence it is quite better in terms of forecast performance.

Model Specification

The appropriate equations for the mean and variance were specified as follows:

---

[4] In practice a GARCH (1, 1) specification often performs very well (Verbeek, 2004) and thus, as noted by Wang (2009); in empirical applications a GARCH (1, 1) model is widely adopted. In the same line of thought, Kozhan (2010); notes that the most commonly used model is a GARCH (1, 1) model with only three parameters in the conditional variance equation.
Usually a GARCH (1, 1) model with only three parameters in the conditional variance equation is adequate to obtain a good model fit for time series (Zivot, 2008).

[5] The best forecasting value is evaluated from the mean squared error (Dritsaki, 2018).

AR (1)[6] – GARCH (1, 1) model[7], econometrically presented as follows:

$$\mathbf{INF_t=\lambda_0+\lambda_1INF_{1t-1}+\varepsilon_t;\ \varepsilon_t\approx N\ (0,\ \sigma_t^2)} \quad \text{[39]}$$

$$\mathbf{\sigma_t^2=ɗ+\alpha_1\varepsilon^2_{t-1}+\beta_1\sigma^2_{t-1}} \quad \text{[40]}$$

where:

$$\mathbf{ɗ\geq 0}$$

$$\mathbf{\alpha_1\geq 0}$$

$$\mathbf{\beta_1\geq 0} \quad \text{[41]}$$

where equation [39] is the mean equation, equation [40] is the variance equation; λ_0 is the constant (in the mean equation), λ_1 is the estimation parameter (in the mean equation), INF_t is monthly inflation rate at time t, INF_{t-1} is previous period monthly inflation rate; ɗ is the constant (in the variance equation), α_1 & β_1 are estimation parameters (in the variance equation); everything else remains as previous defined above.

Data Collection

All the data used in this study was collected from Zimbabwe Statistics Agency (ZimStats).

Diagnostic Tests

Testing for Stationarity

The ADF[8] test

The monthly inflation rate series was tested for stationarity using the Augmented – Dickey Fuller (ADF) unit root test as follows:

$$\mathbf{\Delta INF_t=b_0+\sum b_j\Delta INF_{t-j}+\beta_t+\gamma INF_{t-1}+\mu_t} \quad \text{[42]}$$

Where Δ is the difference operator, b_0 is the drift parameter, β is the coefficient on a time trend, μ_t is the error term, and INF_t is monthly inflation at time t.

The null hypotheses of no unit root tests are:

H_0: $\beta=\gamma=0$ (if there is a trend [F – test]) and H_0: $\gamma=0$ (if there is no trend [t – test]). If the null hypothesis is rejected, like in this case; the data are stationary and can be used without differencing and if the null hypothesis is accepted, there is a unit root; therefore we ought to difference the data. Since the probability (p) value was found to be equal to 0.0006835, the null hypothesis was rejected at 1% level of significance and therefore the series is stationary.

Testing for ARCH effects

[6] The intrinsic nature of a time series is that successive observations are dependent or correlated (Faisal, 2012) and therefore, we specify an AR (1) process for the mean equation.
[7] Based on the selection criteria in table 1 and forecast evaluation statistics in table 2 below.
[8] The model may be run without t if a time trend is not necessary (Salvatore & Reagle, 2002).

The Lagrange Multiplier (LM) Test

Run the mean equation or any postulated linear regression of the form given [in equation one above]; for example:

$$\mathbf{Y_t = \gamma_0 + \gamma_1\Omega_{1t} + \gamma_2\Omega_{2t} + \gamma_3\Omega_{3t} + \gamma_4\Omega_{4t} + z_t} \quad \text{[43]}$$

Where Y_t is the dependent variable, $\gamma_0 - \gamma_4$ are estimation parameters of the mean equation, Ω_{1t} - Ω_{4t} are explanatory variables and z_t is the disturbance term. Save the residuals, $\hat{z}_t$. Square the residuals and regress them on p own lags to test for ARCH of order p, that is; run the regression:

$$\mathbf{\hat{z}^2_t = \gamma_0 + \gamma_1\hat{z}^2_{t-1} + \ldots + \gamma_p\hat{z}^2_{t-p} + \Upsilon_t} \quad \text{[44]}$$

where Υ_t is an error term. Obtain R^2 from this regression. The test statistic, defined as TR^2 (the number of observations multiplied by the coefficient of multiple correlation); is distributed as a $\chi^2(p)$. The null and alternative hypotheses are:

H_0: $\gamma_1=0$ and $\gamma_2=0$ and $\gamma_3=0$ and … and $\gamma_p=0$

H_1: $\gamma_1\neq0$ or $\gamma_2\neq0$ or $\gamma_3\neq0$ or $\gamma_p\neq0$

In this study, the ARCH / GARCH effects test described above was done and the results are shown below:

Chi – square (2) = 105.663 [0.0000000000000000000000113622]

The results of the test above indicate that there are (G) ARCH effects in the chosen (optimal) model, since the p – value [which is approximately equal to zero] is significant at 1% level of significance and therefore it is appropriate to estimate a GARCH model.

4. Results: Presentation, Interpretation & Discussion

Descriptive Statistics

Table 4

Description	Statistic
Mean	1.4825
Median	0.8
Minimum	-7.7
Maximum	6.1
Standard Deviation	2.2714
Skewness	-0.61431
Excess Kurtosis	1.2829

As shown in the table above, the mean is positive but is not close to zero as anticipated. The difference between the maximum and minimum is 13% and this indicates that there are generally no outliers in the data since this difference is generally small and quite reasonable for our data set. The skewness coefficient is negative as shown, implying that our time series has a long left tail and is non – symmetric. The rule of thumb for kurtosis, according to Nyoni & Bonga (2017h); is that it should be around 3 for normally distributed variables. As shown in the table above, excess kurtosis is equal to 1.2829, implying that our variable (monthly inflation rate) is not normally distributed.

GARCH (1, 1) Model Results[9]

Table 5

Variable	Coefficient	Std. Error	z	p – value
λ_0	0.0431191	0.0535928	0.8046	0.4211
AR (λ_1)	0.980045	0.0231912	42.26	0.0000***
ɗ	0.0371421	0.0188943	1.966	0.0493**
ARCH (α_1)	0.575929	0.109556	5.257	0.000000146***
GARCH (β_1)	0.424071	0.989479	4.286	0.0000182***
$\alpha_1 + \beta_1$	1			

The model tabulated above can be presented as follows:

$$\mathbf{INF_t=0.04+0.98INF_{1t-1}} \quad \textbf{[45]}$$

p: (0.421) (0.000)

S.E (0.054) (0.023)

$$\boldsymbol{\sigma_t^2=0.04+0.58\varepsilon^2_{t-1}+0.48\sigma^2_{t-1}} \quad \textbf{[46]}$$

p: (0.049) (0000) (0000)

S.E: (0.019) (0.11) (0.99)

Interpretation & Discussion of Results

The estimated model is acceptable simply because the necessary and sufficient conditions (specified in [41]) have been met. As theoretically expected, the parameters λ_0 and α_1 are greater than zero (0) and β_1 is positive to ensure that the conditional variance σ_t^2 is non – negative and therefore the positivity constraint of the GARCH model is not violated. Thus the estimated GARCH (1, 1) model seems quite good for explaining the behavior of monthly inflation rate volatility in Zimbabwe. Both α_1 and β_1 are positive and statistically significant at 1% level of significance. α_1 is positive and statistically significant, indicating that strong GARCH effects are apparent. In fact a 1% increase in previous period volatility leads to an approximately 0.58% increase in current volatility of monthly inflation. β_1 is positive & significant and less than one, indicating that the impact of old news on volatility is significant. In fact an increase of 1% in previous period variance will lead to approximately 0.42% increase in current volatility of monthly inflation rate. Since[10]:

$$\boldsymbol{\alpha_1+\beta_1=1} \quad \textbf{[47]}$$

[9] The *, ** and *** means significant at 10%, 5% and 1% levels of significance; respectively.

[10] α_1 & β_1 are as defined previously.

it confirms that the estimated GARCH model is not covariance stationary but rather ergodic or strictly stationary. The estimated model can thus be referred to as an IGARCH (1, 1) process. This implies that persistent variance is not inexistent and volatility shocks have a permanent effect and therefore current information remains critical for the forecasts of the conditional variances for all horizons. Caporale *et al* (2003) noted that the high estimated volatility persistence obtained using GARCH models might be attributed to structural changes in the variance process. Tsay (2002) highlights the fact that the actual cause of persistence deserves a careful investigation. Our results are similar to previous studies such as Moroke & Luthuli (2015) whose AR (1) – IGARCH (1, 1) model suggested a high degree of persistence in the conditional volatility of the inflation series in South Africa.

Forecasting

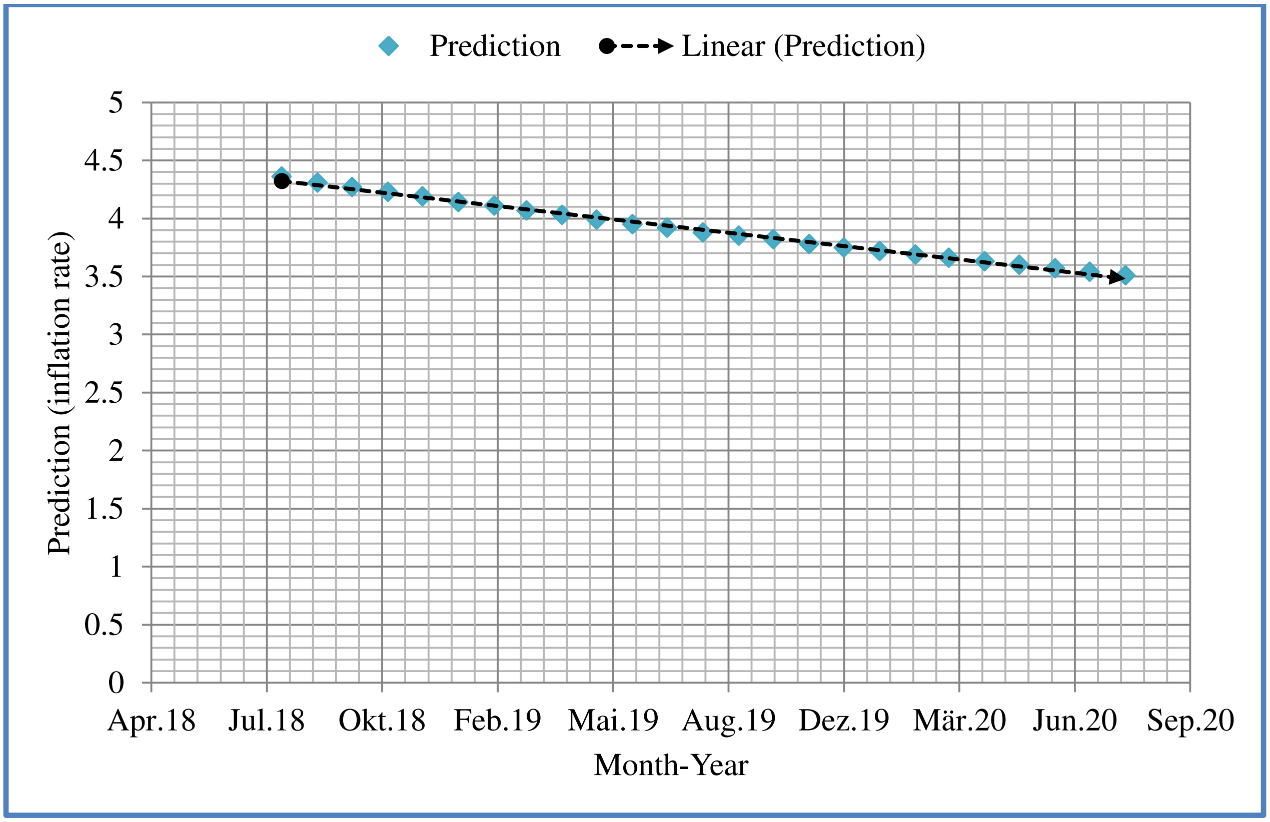
Table 6

Month – Year	Prediction	Std. Error	95% interval
August 2018	4.36	0.381	3.61 – 5.10
September 2018	4.31	0.567	3.2 – 5.42
October 2018	4.27	0.727	2.84 – 5.69
November 2018	4.23	0.874	2.51 – 5.94
December 2018	4.19	1.04	2.20 – 6.17
January 2019	4.14	1.148	1.89 – 6.39
February 2019	4.11	1.278	1.6 – 6.61
March 2019	4.07	1.405	1.31 – 6.82
April 2019	4.03	1.529	1 – 7.03
May 2019	3.99	1.651	0.76 – 7.23
June 2019	3.95	1.77	0.48 – 7.42
July 2019	3.92	1.888	0.22 – 7.62
August 2019	3.88	2.004	-0.04 – 7.81
September 2019	3.85	2.117	-0.3 – 8
October 2019	3.82	2.23	-0.55 – 8.19
November 2019	3.78	2.34	-0.8 – 8.37
December 2019	3.75	2.45	-1.05 – 8.55
January 2020	3.72	2.557	-1.29 – 8.73
February 2020	3.69	2.664	-1.53 – 8.91
March 2020	3.66	2.769	-1.77 – 9.08
April 2020	3.63	2.872	-2 – 9.26
May 2020	3.6	2.975	-2.23 – 9.43
June 2020	3.57	3.076	-2.46 – 9.6
July 2020	3.54	3.176	-2.68 – 9.77
August 2020	3.51	3.275	-2.91 – 9.93

The above table is a summary of the forecasting results of monthly inflation rate over the period August 2018 – August 2020.

Predicted monthly inflation rates over the period August 2018 – August 2020

Figure 1



The graph above shows that monthly inflation rate in Zimbabwe is generally on a downward trend over the next 2 years (or even beyond); as long as economic conditions seldom fluctuate much. The downward trend can be attributed to the adoption of the United States Dollar in February 2009, just after Zimbabwe had reached the climax of the 2008[11] hyper – inflationary era. The graph indicates that by December 2018, inflation will be approximately 4.14% and by June 2019, inflation will be somewhere around 3.95%. This downward spiral is expected to continue into the future, especially given the fact that there are no plans to re – introduce the Zimbabwean dollar. Inflation that ranges from 0% - 10% or simply one digit figure, according to Nyoni & Bonga (2018a); is beneficial for economic growth and this is true because such low inflation rate ensure price stability, which is one of the most crucial objectives of monetary policy. This reasoning is in line with many researchers such as Hasanov (2010) and Marbuah (2010) who assert that inflation that is less than 9% or generally low (single – digit – inflation) is generally beneficial to economic growth. Low or moderate inflation as noted by Hossin (2015); is an indicator of macroeconomic stability and creates an environment conducive for investment. Nyoni & Bonga (2017h) emphasize that lower rates of inflation currently obtaining in Zimbabwe

---

[11] A protracted economic crisis that occurred in Zimbabwe for over a decade (1998 – 2008) culminated in the hyperinflation episode of 2007 – 2008(IMF, 2009; Kamniski & Ng, 2011; Kairiza, 2012; Pindiriri, 2012). This led to the abandonment of the local currency as a medium of exchange (IMF, 2009; Pindiriri, 2012). Informal dollarization of the economy manifested itself during second half of 2008, as economic agents responded to the failure of the local currency to fulfill the basic functions of money (Kavila & Roux, 2017).

are favorable. However, as already warned by Nyoni & Bonga (2018a); policy makers need to be aware of too low inflation (i.e. inflation that is lower than 0% or in the negative territory).

5. Conclusion & Recommendations

Monetary policy is more effective when it is forward looking (Svensson, 2005; Faust & Wright, 2013). Achieving and maintaining price stability will be efficient and effective the better we understand the causes of inflation and the dynamics of how it evolves (Kohn, 2005). The ARCH/GARCH processes have been proved useful in modeling various economic phenomena and have received a great amount of attention in the economic literature (Lee & Kim, 2001). One goal of time series analysis is to forecast the future values of the time series data (Ping *et al*, 2013). Central banks forecast inflation considering all relevant factors (Hanif & Malik, 2015). The Reserve Bank of Zimbabwe (RBZ), being the central bank; can only have some control over future inflation. This is clear testimony to the fact that inflation forecasting is not unimportant in monetary policy making. In this study, we fit an AR (1) – GARCH (1, 1) model which has been shown to be indeed an AR (1) – IGARCH (1, 1) model. The study recommends the recognition of the IGARCH behavioral phenomenon exhibited by monthly inflation rates, if monetary policy design is anything to go by in Zimbabwe. Since the implications of IGARCH models are "too strong", there is need for further research to consider fractionally integrated models such as the Fractionally Integrated GARCH (FIGARCH) model, Fractionally Integrated Exponential GARCH (FIEGARCH) model and the Fractionally Integrated APARCH (FIAPARCH) model amongst others; when analyzing inflation dynamics in Zimbabwe.

REFERENCES[12]

[1] Addison, J. T & Burton, J (1979). The Demise of Demand Pull and Cost Push in Inflation Theory, *Birmingham*.

[2] Anton, S. G (2012). Evaluating the Forecasting Performance of GARCH Models: Evidence from Romania, *Elsevier – Procedia – Social and Behavioral Sciences*, 62 (2012): 1006 – 1010. https://www.researchgate.net/publication/271638409

[3] Awogbemi, C. A., Abayomi, A & Bassey, B. E (2015). Modeling volatility in financial time series: evidence from Nigeria inflation rates, *Mathematical Theory and Modeling*, 5 (13): 28 – 41. http://www.iiste.org/Journals/index.php/MTM/article/view/27724

[4] Baciu, I (2015). Stochastic models for forecasting inflation rate: Empirical evidence from Romania, 7th International Conference on Globalization and Higher Education in Economics and Business Administration – GEBA – 2013, *Elsevier – Procedia*

[12] The **APPENDIX** section is merged as follows: time series plot, residual plot, 95% confidence ellipse & 95% marginal intervals and the [shaded area] 95% confidence interval (forecast). The time series plot and the residual plot indicate volatility clustering of the series. The joint confidence regions (or the sets of confidence ellipse) generally indicate the region in which the realization of two test statistics must lie for us not to reject the null hypothesis. The forecast graph and the confidence ellipse indicate that the accuracy of our forecast is quite reasonable and acceptable since it falls within the 95% confidence interval.

Economics and Finance, 20 (2015): 44 – 52. https://www.sciencedirect.com/science/article/pii/S2212567115000453

[5] Banerjee, S (2017). Empirical Regularities of Inflation Volatility: Evidence from Advanced and Developing Countries, *South Asian Journal of Macroeconomics and Public Finance*, 6 (1): 133 – 156. http://smp.sagepub.com

[6] Benedict, M (2013). Modeling rates of inflation in Ghana: an application of ARCH type models, *University of Ghana*, MPhil Thesis, http://ugspace.ug.edu.gh

[7] Bernanke, B. S (2005). Inflation in Latin America – A New Era? – Remarks at the *Stanford Institute for Economic Policy Research* – Economic Summit, February 11. http://www.federalreserve.gov/boarddocs/speeches/2005/20050211/default.htm

[8] Bokil, M & Schimmelpfennig, A (2005). Three Attempts at Inflation Forecasting in Pakistan, *IMF*, Working Paper No. WP/05/105. https://www.researchgate.net/publication/24046515

[9] Bollerslev, T (1986). Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31: 307 – 327. http://public.econ.duke.edu/~boller/Published_Papers/joe_86.pdf

[10] Bomfin, A & Rodenbusch, V (2000). Opportunistic and Deliberate Disinflation under Imperfect Credibility, *Journal of Money, Credit and Banking*, 32: 707 – 721.

[11] Brooks, C (2008). Introductory Econometrics for Finance, 2nd Edition, *Cambridge University Press*, Cambridge, UK. https://www.amazon.com/Introductory-Econometrics-Finance-second-Brooks/dp/B00BUFKBTE

[12] Caporale, G. M., Pittis, N & Spagnolo, N (2003). IGARCH models and structural breaks, *Applied Economics Letters*, 10: 765 – 768. https://www.researchgate.net/publication/24067795

[13] Chhibber, A. J., Cottani, R., Firuzabadi, F & Walton, M (1989). Inflation, price controls and fiscal adjustment in Zimbabwe, *World Bank*, Working Paper No. WPS 192.

[14] Dritsaki, C (2018). Modeling and Forecasting of British Pound/US Dollar Exchange Rate: An Empirical Analysis, *Springer International Publishing*, https://www.researchgate.net/publication/324568583

[15] Dzwanga, M (1995). The determinants of inflation in Zimbabwe, Economics Working Papers, *University of Zimbabwe*, Harare.

[16] Elhers, N & Steinbach, M. R (2007). The formation of inflation expectations in South Africa, *South Africa Reserve Bank*, Working Paper No. WP/07/06.

[17] Engle, R. F (1982). Autoregressive Conditional Heteroskedasticity with estimates of variance of UK inflation, *Econometrica*, 50: 987 – 1008.

[18] Faisal, F (2012). Forecasting Bangladesh's Inflation Using Time Series ARIMA Models, *World Review of Business Research*, 2 (3): 100 – 117. http://www.wrbrpapers.com/static/documents/May/2012/7.%20Faisal.pdf

[19] Fatukasi, B (2012). Determinants of inflation in Nigeria: An Empirical Analysis, *International Journal of Humanities and Social Science*, 1 (18): 262 – 271.

[20] Faust, J & Wright, J. H (2013). Forecasting Inflation – in Economic Forecasting, 2A (2013), *Elsevier*.

[21] Friedman, M (1956). The Quantity Theory of Money: a restatement – In Friedman, M (Ed), Studies in the Quantity of Money, *University of Chicago Press*, Chicago.

[22] Fwaga S. O., Orwa, G & Athiany, H (2017). Modeling Rates of Inflation in Kenya: An Application of Garch and Egarch models, *Mathematical Theory and Modeling*, 7 (5): 75 – 83. http://r.search.yahoo.com/_ylt=A0geK91VplVbenkADJcPxQt.

[23] Gali, J (2008). Monetary policy, inflation and business cycle: an introduction to the new Keynesian framework, *Princeton University Press*.

[24] Hanif, M. N & Malik, M. J (2015). Evaluating the performance of inflation forecasting models of Pakistan, *SPB Research Bulletin*, 11 (1): 01 – 36. https://mpra.ub.uni-muenchen.de/66843/

[25] Hansen, P. R & Lunde, A (2005). A forecast comparison of volatility models: does anything beat a GARCH (1, 1)? *Journal of Applied Econometrics*, 20: 837 – 889. https://onlinelibrary.wiley.com/doi/full/10.1002/jae.800

[26] Hasanov, F (2010). Relationship between inflation and economic growth in Azerbaijani economy: is there any threshold effect? *Asian Journal of Business and Management Sciences*, 1 (1): 6 – 7. https://www.researchgate.net/publication/228304726

[27] Holton, G (1996). "Contingency Analysis" family of websites including a risk glossary, June 16 2018. http://www.contingencyanalysis.com ; http://www.riskglossary.com

[28] Hossin, M. S (2015). The Relationship Between Inflation and Economic Growth: An Empirical Analysis from 1961 to 2013, *International Journal of Economics, Finance and Management Sciences*, 3 (5): 426 – 434. https://www.researchgate.net/publication/293014504

[29] Humphrey, T. M (1977). On Cost Push Theories in the pre – war Monetary Literature, *Federal Reserve Bank of Richmond – Economic Review*, May/June 1977.

[30] Idris, M & Bakar, R (2017). The relationship between inflation and economic growth in Nigeria: a conceptual approach, *Asian Research Journal of Arts & Social Sciences*, 3 (1): 1 – 15. https://www.researchgate.net/publication/317045613

[31] International Monetary Fund (2009). Country Report No. 09/139, *IMF*, Article IV, Consultation – Staff Report.

[32] Iqbal, S & Sial, M. H (2016). Projections of Inflation Dynamics for Pakistan: GMDH Approach, *Journal of Economics and Political Economy*, 3 (3): 536 – 559. http://kspjournals.org/index.php/JEPE/article/view/904

[33] Kairiza, T (2012). Unbundling Zimbabwe's journey to hyperinflation and official dollarization, GRIPS Policy Information Centre, http://www.grips.ac.jp/r-center/wp-content/uploads/09-12.pdf

[34] Kaminski, B & Ng, F (2011). Zimbabwe's foreign trade performance during the decade of economic turmoil: will exports recover? Prepared for *Zimbabwe's Diagnostic Trade Integration Study in Africa Region* (AFTP 1).

[35] Kavila, W & Roux, L. P (2015). Inflation dynamics in a dollarized economy: the case of Zimbabwe, *Economic Research South Africa (ERSA)*, Working Paper No. 606.

[36] Kavila, W & Roux, P. L (2017). The reaction of inflation to macroeconomic shocks: The case of Zimbabwe (2009 – 2012), *Economic Research South Africa (ERSA)*, ERSA Working Paper No. 707. https://www.researchgate.net/publication/320036434

[37] Keynes, J. M (1936). The General Theory of Employment, Interest and Money, *Cambridge University Press*.

[38] Khan, M. S & Schimmelpfennig, A (2006). Inflation in Pakistan: Money or Wheat? *IMF*, Working Paper No. WP/06/60.

[39] Kohn, D. L (2005). Modeling inflation: A policy maker's perspective, International research forum on monetary policy, Conference Paper, Frankfurt.

[40] Kozhan, R (2010). Financial Econometrics with E – views, *Ventus Publishing*. http://zums.ac.ir/files/research/site/ebooks/finance/financial-econometrics-eviews.pdf

[41] Lee, O & Kim, J (2001). Strict stationarity and functional central limit theorem for ARCH/GARCH models, *Bull. Korean Math. Soc.*, 38 (3): 495 – 504. http://basilo.kaist.ac.kr/mathnet/kms_tex/112227.pdf

[42] Loleyt, A & Gurov, I (2010). The Process of Inflation Expectations Formation, *Bank of Russia*.

[43] Makochekanwa, A (2007). A dynamic inquiry into the causes of hyper – inflation in Zimbabwe, Working Paper Series 2007 – 10, *University of Pretoria*, Pretoria. https://www.up.ac.za/media/shared/61/WP/wp_2007_10.zp39552.pdf

[44] Marbuah, G (2010). On the inflation growth – nexus: testing for optimal inflation for Ghana, *Journal of Monetary and Economic Integration*, 11 (2): 71 – 72. https://www.researchgate.net/publication/230886067

[45] Modigliani, F (1977). The monetarist controversy or should we forsake stabilization policies, American Economic Review.

[46] Mohanty, D (2012). The Importance of Inflation Expectations, *S. P. Jain Institute of Management & Research*, Mumbai.

[47] Moroke, N & Luthuli, A (2015). An Optimal Generalized Autoregressive Conditional Heteroskedasticity Model for Forecasting the South African inflation volatility, *Journal of Economics and Behavioral Studies*, 7 (4): 134 – 149.

[48] Mpofu, R. T (2017). Macroeconomic variables and food price inflation, non – food price inflation and overall inflation: A case of an emerging market, *Risk Governance & Control: Financial Markets & Institutions*, 7 (2): 38 – 48. http://dx.doi.org/10.22495/rgcv7i2art4

[49] Nor, A. H. S. M., Ling, T. Y & Maarof, F (2007). On the relationship between inflation and inflation uncertainty: an application of the GARCH family models, *Sains Malaysiana*, 36 (2): 225 – 232. https://ukm.pure.elsevier.com/en/publications/on-the-relationship-between-inflation-rate-and-inflation-uncertai

[50] Nyoni T & Bonga W. G (2017h). Population Growth in Zimbabwe: A Threat to Economic Development? *DRJ – Journal of Economics and Finance,* 2 (6): 29 – 39. https://www.researchgate.net/publication/318211505

[51] Nyoni, T & Bonga, W. G (2018a). What Determines Economic Growth in Nigeria? *DRJ – Journal of Business and Management*, 1 (1): 37 – 47. https://www.researchgate.net/publication/323068826

[52] Osarumwense, O & Waziri E. I (2013). Modeling monthly inflation rate, using Generalized Autoregressive Conditionally Heteroskedastic (GARCH) models: Evidence from Nigeria, *Australian Journal of Basic and Applied Sciences*, 7 (7): 991 – 998.

[53] Pindiriri, C & Nhavira, J (2011). Modeling Zimbabwe's inflation process, *Journal of Strategic Studies*, 2 (1).

[54] Pindiriri, C (2012). Monetary reforms and inflation dynamics in Zimbabwe, *International Journal of Finance and Economics*, 90 (2012): 207 – 222. https://www.scribd.com/document/119314804/hyperinflation-in-zimbabwe

[55] Ping, P. Y., Miswan, N. H & Ahmad, M. H (2013). Forecasting Malaysian Gold Using GARCH Model, Applied Mathematical Sciences, 7 (58): 2878 – 2884. https://www.researchgate.net/publication/290630444

[56] Ruzgar, B & Kale, T (2007). The use of ARCH and GARCH models for estimating and forecasting volatility, *Universitesi Bilimler Enstitusu Dergisi*, 2 (4): 78 – 109. https://www.researchgate.net/publication/257342835

[57] Saleem, N (2008). Measuring volatility of inflation in Pakistan, *The Lahore Journal of Economics*, 13 (2): 99 – 128. http://www.lahoreschoolofeconomics.edu.pk/EconomicsJournal/Journals/Volume%2013/Issue%202/6%20Nadia_Saleem_(ed_TTC_March_16).pdf

[58] Salvatore, D & Reagle, D (2002). Statistics and Econometrics, 2nd Edition, *McGraw – Hill*, New York.

[59] Samuelson, P. A (1971). Reflections on the Merits and Demerits of Monetarism – in Issues in Fiscal and Monetary Policy: The Eclectic Economist Views the Controversy, Ed, J. J. Diamond, *De Paul University*.

[60] Sek, S. K & Har, W. M (2012). Does inflation targeting work in emerging East – Asian Economies? *Panoeconomicus*, 5 (2012): 599 – 608. http://www.doi.10.2298/PAN1205599S

[61] Sims, C. A (2009). Inflation expectation, uncertainty and monetary policy, *BIS* Working Paper No. 275. https://www.bis.org/publ/work275.pdf

[62] Smithies, A (1942). The Behavior of Money, National Income under Inflationary Conditions, *Quarterly Journal of Economics*, 57 (1): 113 – 128.

[63] Sultana, K., Rahim, A., Moin, N., Aman, S & Ghauri, S. P (2013). Forecasting inflation and economic growth of Pakistan by using two time series models, *International Journal of Business and Economic Research*, 2 (6): 174 – 178. http://www.doi.10.11648/j.ijber.20130206.17

[64] Sunde, T (1997). The dynamic specification of the inflation model in Zimbabwe, Economics Working Papers, *University of Zimbabwe*, Harare.

[65] Svensson, L. E. O (2005). Monetary policy with judgement: forecasting targeting, *International Journal of Central Banking*, 1 (1). http://www.nber.org/papers/w11167

[66] Taylor (1986). Modeling Financial Time Series, *Wiley*, New York.

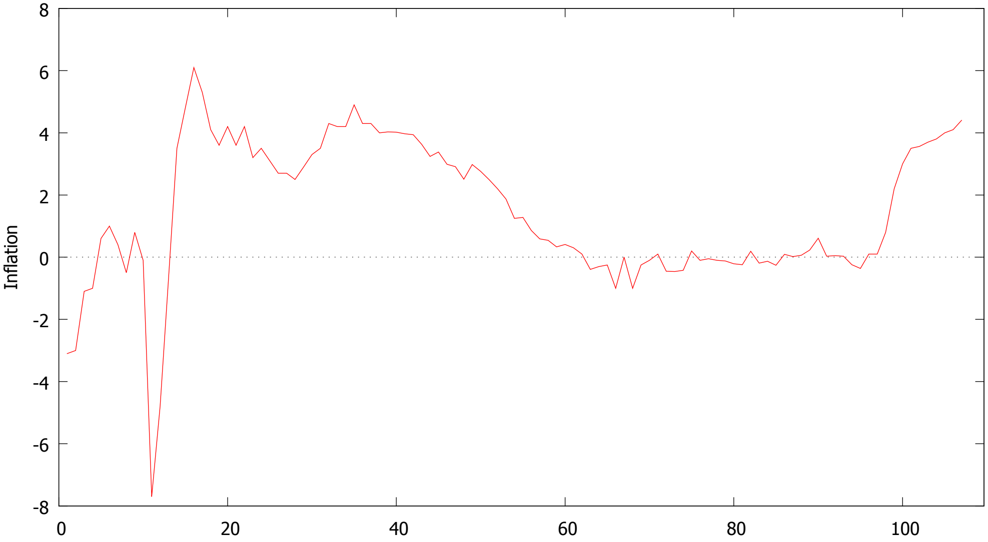
[67] Tsay, R (2002). Analysis of Financial Time Series – Financial Econometrics, *John Wiley & Sons*, New York. https://onlinelibrary.wiley.com/doi/full/10.1002/0471667196.ess3146

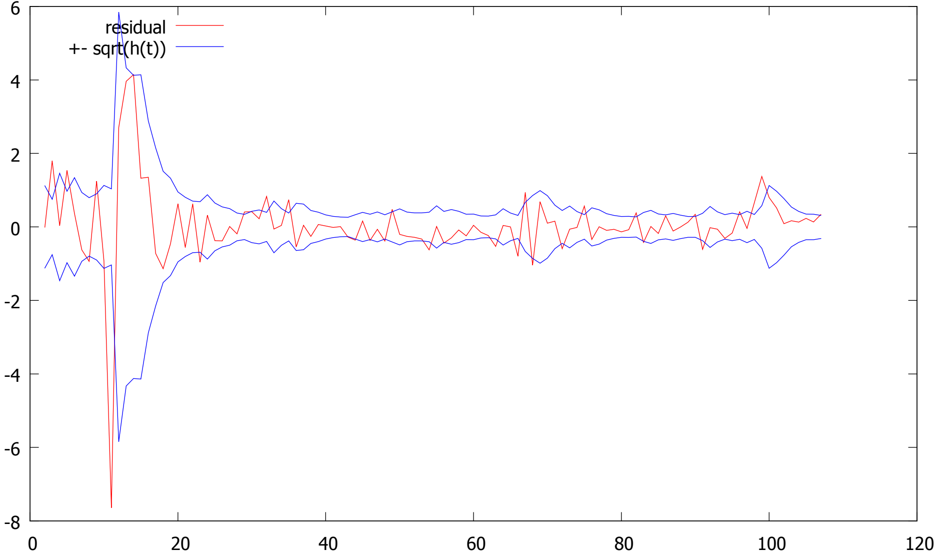
[68] Uwilingiyimana, C., Mungatu, J & Hererimana, J. D (2015). Forecasting inflation in Kenya using ARIMA – GARCH models, *International Journal of Management and Commerce Innovations*, 3 (2): 15 – 27.

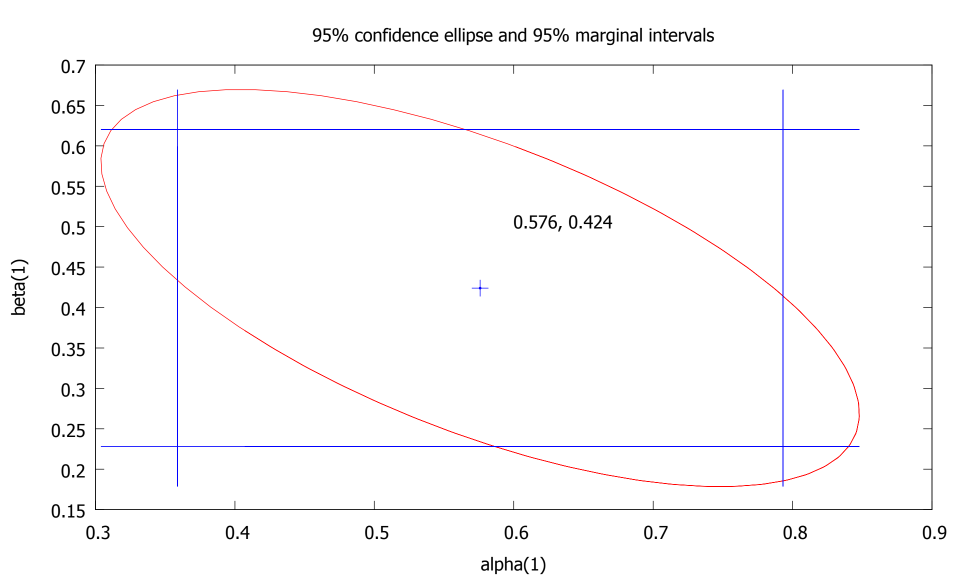
[69] Verbeek, M (2004). A Guide to Modern Econometrics, 2nd Edition, *John Wiley & Sons – Ltd*, England. https://thenigerianprofessionalaccountant.files.wordpress.com/2013/04/modern-econometrics.pdf

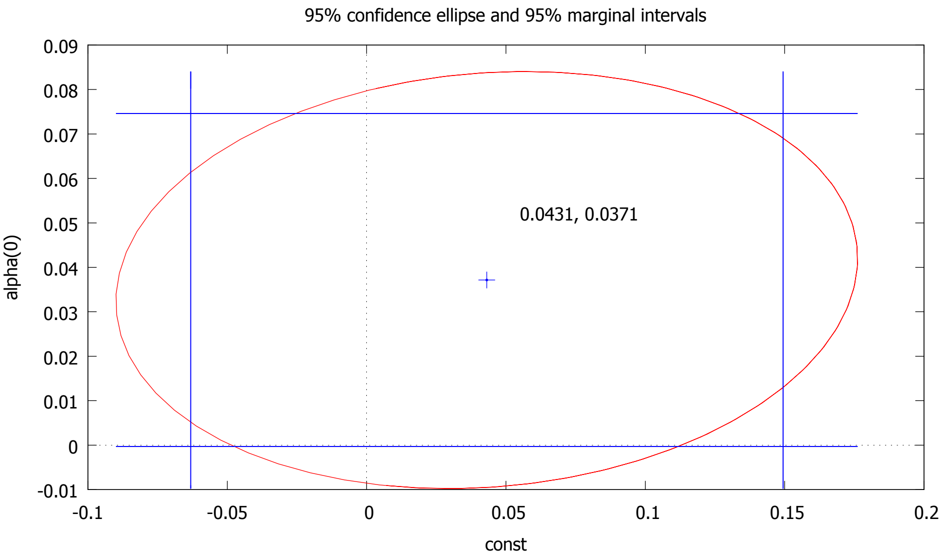
[70] Wang, P (2009). Financial Econometrics, 2nd Edition, *Routledge – Taylor & Francis Group*, London. http://dl4a.org/uploads/pdf/file1.pdf

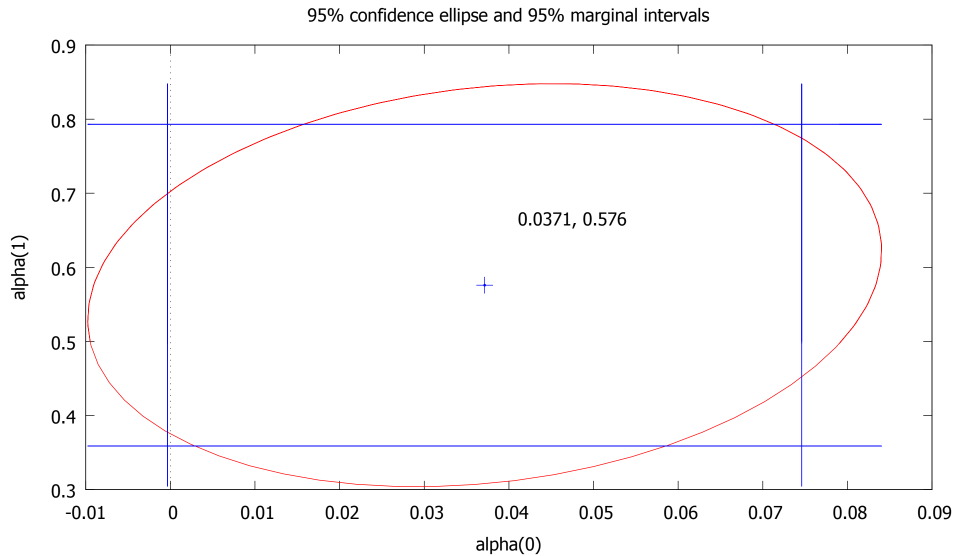
[71] Zivot, E (2008). Practical Issues in the Analysis of Univariate GARCH Models. https://pdfs.semanticscholar.org/686a/b78f58999bcd67ad342c28f9c6a5d32454c1.pdf


Inflation
8
6
4
2
0
-2
-4
-6
-8
0
20
40
60
80
100

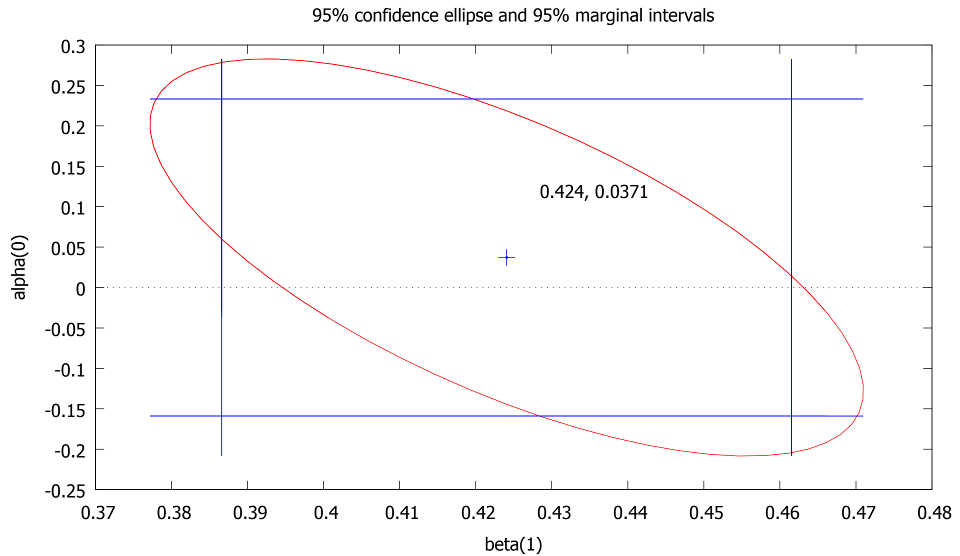

residual
+- sqrt(h(t))
6
4
2
0
-2
-4
-6
-8
0
20
40
60
80
100
120

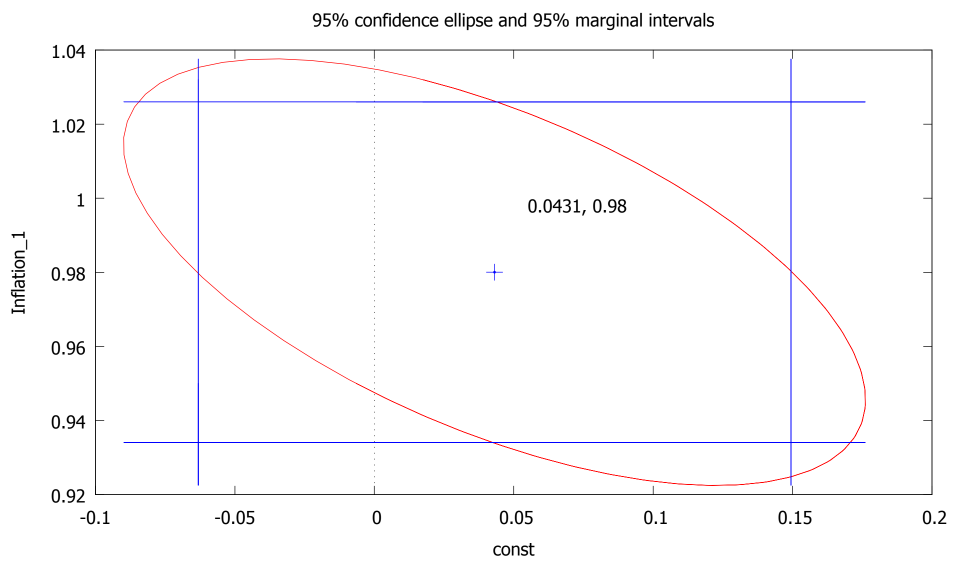

95% confidence ellipse and 95% marginal intervals
0.576, 0.424
beta(1)
alpha(1)
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.3
0.4
0.5
0.6
0.7
0.8
0.9

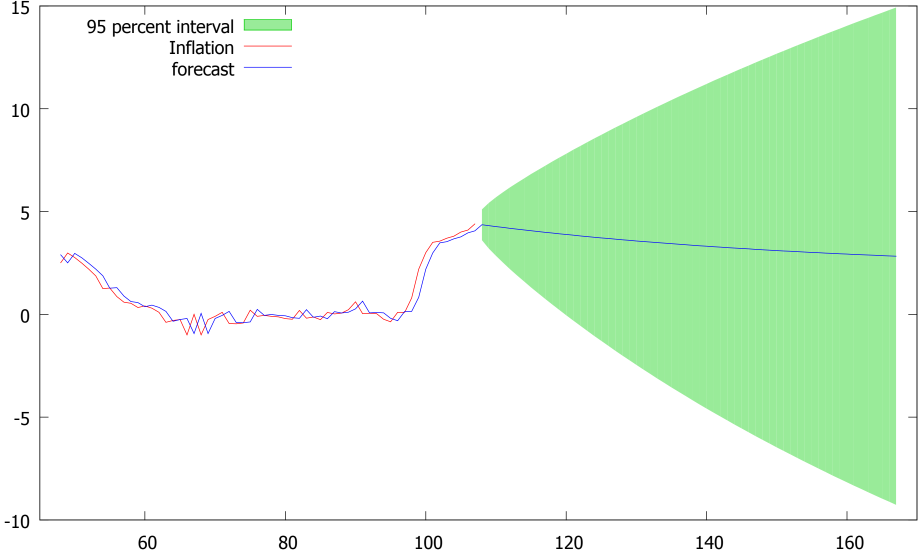

95% confidence ellipse and 95% marginal intervals
0.0431, 0.0371
alpha(0)
const
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.1
-0.05
0
0.05
0.1
0.15
0.2



95% confidence ellipse and 95% marginal intervals
0.0371, 0.576
alpha(1)
alpha(0)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09


95% confidence ellipse and 95% marginal intervals
0.424, 0.0371
alpha(0)
beta(1)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0.37
0.38
0.39
0.4
0.41
0.42
0.43
0.44
0.45
0.46
0.47
0.48


95% confidence ellipse and 95% marginal intervals
0.0431, 0.98
Inflation_1
const
1.04
1.02
1
0.98
0.96
0.94
0.92
-0.1
-0.05
0
0.05
0.1
0.15
0.2


95 percent interval
Inflation
forecast
15
10
5
0
-5
-10
60
80
100
120
140
160