Multiple-Quality Cournot Oligopoly and the Role of Market Size

Miao, Zhuang and Long, Ngo Van

Central University of Finance and Economics, McGill University

1 July 2017

Multiple-Quality Cournot Oligopoly and the Role of Market Size

Zhuang Miao and Ngo Van Long

24 July 2018

Multiple-Quality Cournot Oligopoly and the Role of Market Size

Abstract: We model an oligopoly where firms can choose the quality level of their products by incurring set-up costs that generally depend on the quality level. If the set-up cost is independent of product quality, firms may choose to supply both types of quality.We focus on the long run equilibrium where free entry and exit ensure that the profit for each type of firm is zero. Using this framework, we study the implications of an increase in the market size. We show that for the existence of an equilibrium where some firms specialize in the low quality product it is necessary that the set-up cost for the lower quality product, adjusted for quality level, is lower than that for the higher quality product. In the case where the unit variable costs are zero, or they are proportional to quality level (so that unit variable costs, adjusted for quality, are the same), we show that an increase in the market size leads to (i) an increase in the fraction of firms that specialize in the high quality products, (ii) the market shares (both in value terms and in terms of volume of output) of high quality producers increases, and (iii) the prices of both types of product decrease. In the case where higher quality requires higher set up cost (per unit of quality) but lower unit variable cost (per unit of quality), subject to certain bounds on the difference in unit variable costs, we obtain the result that an increase in the market size decreases the number of low quality firms, increases the number of high quality firms, and decreases the prices of both products. In the special case where the set up cost is independent of the quality level, we find that all firms will produce both type of quality levels. In this case, an increase in the market size will reduce the value share of the low quality product, but will leave their volume share unchanged; and the market expansion induces a fall in the relative price of the low quality product, and in the prices of both products in terms of the numeraire good. We carry out an empirical test of a version of the model, where set-up costs now refer to set-up costs to establish an export market, and they vary according to the quality of the product that the firm exports to that market. We show that the data supported the hypothesis that the average qualities are higher for bigger export markets.

JEL classifications: L10, L13, L19

Keywords: Multiproduct firms; Cournot competition; Vertical product differentiation; Cost structure; Market size.

1 Introduction

In many oligopolistic industries, firms supply a variety of products that differ mainly in terms of quality. Some firms are known to specialize in products of high quality while others occupy the lower end of the quality spectrum. The purpose of this paper is to study the role of the market size on the average quality level of the vertically differentiated products that an oligopolistic industry produces.

Trade liberalization is one of major factors that have contributed to the expansion of market size. In the international trade literature, the effect of market size expansion on consumers' welfare has been largely studied using the monopolistic competition framework, where firms produce horizontally differentiated products (Krugman, 1980; Melitz, 2003; Melitz and Trefler, 2012), and trade gains are explained in terms of the lowering of prices and the increase in the number of horizontally differentiated product varieties that the average consumer has access to. The decrease in prices is due to the expansion of the scale of operations of the representative firm, which reduces the firm's average cost. While the monopolistic competition framework is convenient, the CES utility function assumed in this literature (e.g., Melitz, 2003) produces the counterfactual result that the ratio of equilibrium price over the constant unit variable cost is a constant, independent of the market size. This is contrary to the empirical evidence, see, e.g. Edmond et al. (2015). In our paper, we assume instead that firms are oligopolists and study the long run equilibrium of an industry that produces both high and low quality products. The mark-up is not constant in our model.[1]

[1] Long et al. (2011) examine the long run equilibrium in a model where firms are oligopolists with ex-ante cost heterogeneity. Their paper however assumes that the all the firms in the industry produce the same homogenous product. Our paper distinguishes low quality products from high quality products.

Our model is built on Johnson and Myatt (2006), where firms can choose the quality levels from a discrete set $\{S_1, S_2, ..., S_m\}$, and they compete in quantities, taking the inverse demand function for each quality type as given. However, we replace their assumption that all firms incur the same set-up cost (regardless of the quality of the product that firms offer) with a more plausible one: firms that wish to specialize in the lower quality product incur a lower set-up cost than that of firms that produce the high quality product. In addition, while Johnson and Myatt (2006) are mainly concerned with the short run equilibrium, where the number of firms are fixed, the focus of our model is the long run equilibrium, where free entry and exit ensures that profit is zero. Using this framework, we study the implications of an increase in the market size. We show that for the existence of an equilibrium where some firms specialize in the low quality product it is necessary that the set-up cost for the lower quality product, adjusted for quality level, is lower than that for the higher quality product. In the case where the unit variable costs are zero, or they are proportional to quality level (so that unit variable costs, adjusted for quality, are the same), we show that an increase in the market size leads to (i) an increase in the fraction of firms that specialize in the high quality products, (ii) the market shares (both in value terms and in terms of volume of output) of high quality producers increases, and (iii) the prices of both types of product decrease. In the case where higher quality requires higher set up cost (per unit of quality) but lower unit variable cost (per unit of quality), subject to certain bounds on the difference in unit variable costs, we obtain the results that each firm will choose will specialize in only one quality level, and that an increase in the market size decreases the number of low quality firms, increases the number of high quality firms, and decreases the prices of both products. In the special case where the set up cost is independent of quality level, we find that all firms will produce both quality levels. In this case, an increase in the market size will reduce the value shares of low quality products, but will leave their volume share unchanged; and the market expansion induces a fall in the relative price of the low quality product, and in the prices of both products in terms of the numeraire good.

We carry out an empirical test of a version of the model, where set-up costs now refer to set-up costs to establish an export market, and they vary according to the quality of the product that the firm exports to that market. We show that the data supported the hypothesis that the average prices of the products are lower for bigger export markets, and the market share of the high quality product is increasing in the market size.

This paper is related to the theoretical work of Fajgelbaum, Grossman and Helpman (2011) concerning quality differentiation in international trade between the North (the rich countries) and the South (the poor countries). However, while our model assumes oligopoly (i.e. firms choose their strategies and are aware of strategic interactions among them), the model of Fajgelbaum, Grossman and Helpman (2011) assumes monopolistic competition (i.e., there are no strategic interactions among firms). Under oligopoly, each firm knows that the quantities and/or qualities chosen by its rivals depend on their knowledge of the firm's cost and strategy. In contrast, under monopolistic competition, each firm takes as given the market aggregates (such as aggregate expenditure on the products of the industry, and the industry price level), and sets its own price, as if it were a monopolist. In Fajgelbaum, Grossman and Helpman (2011) firms set prices, each assuming that its price has no effects on the industry's price index (this is the standard assumption of the monopolistic competition model). In our model, each firm decides on its output, knowing that its output will affect the industry's output and hence prices. Another difference between our model and Fajgelbaum, Grossman and Helpman (2011) is that we assume that the market is not fully covered, i.e., there are some consumers that do not buy the product of the industry under study: they spend their entire income on the numeraire good which is produced by a perfectly competitive sector. As pointed out by Motta (1993, page 116) and others, the assumption that the market is not fully covered is made so that the inverse demand functions for various quality levels can be derived from the consumers' demand. In contrast, in Fajgelbaum, Grossman and Helpman (2011) there is no need to have inverse demand functions, as firms set prices directly. Another major difference is that in Fajgelbaum, Grossman and Helpman (2011) the marginal rate of substitution between the quality (of the differentiated goods) with the quantity of the numeriare good depends on the level of income. (We will discuss this in more details in the next section).

2 A brief literature review

This section provides a brief review of the literature on oligopoly with vertically differentiated products. There are two canonical approaches regarding the costs of producing higher quality products. The first approach assumes that to produce a higher quality product, a firm must pay a higher fixed cost, while the variable costs are independent of quality level. The fixed costs may be regarded as R&D costs. The second approach assumes that to produce a higher quality product, the firm must incur higher variable cost per unit of output, and there are no fixed costs. This corresponds to situations where production of higher quality goods require the use of more qualified labor or more expensive intermediate inputs. The first approach was adopted by authors such as Shaked and Sutton (1982, 1983, 1984), Bonanno (1986), Ireland (1987), Motta (1993, Part II). Studies using the second approach includes Mussa and Rosen (1987), Gal-Or (1983), Champsaur and Rochet (1989), Motta (1993, Part III), and Johnson and Myatt (2006).

Concerning the mode of competition, most authors assume Bertrand competition with heterogeneous consumers. Gabszewicz and Thisse (1989, 1980) and Shaked and Sutton (1982, 1983) consider a duopoly where one firm produces the high quality product and the other firm produces the low quality product, and the firms compete by setting the prices. Champsaur and Rochet (1989) also restrict attention to a duopoly with Bertrand competition, but allow each firm to be a multi-product firm. (See also Tirole,1988, and Choi and Shin, 1992, for Bertrand competition in the case where the market is fully covered.) A number of authors assume quantity competition (Bonanno, 1986, Gal-Or, 1983, Johnson and Myatt, 2006). Motta (1993) consider both types of competition. Most authors assume that the number of firms are fixed, though Johnson and Myatt (2006) also discuss the long run equilibrium when free entry eliminates excess profit.

Concerning the choice of quality levels, many authors assume that firms can choose any level of quality in a continuum $[S_{\min}, S_{\max}]$. Johnson and Myatt (2006), in contrast, assume firms must choose quality levels from a discrete set, $\{S_1, S_2, ..., S_m\}$, and they use the upgrade approach: each firm can upgrade a low quality product to a higher quality product by incurring an upgrade cost (a variable cost, not a fixed cost). It is as if the firm must produce an additional component to turn a low quality unit into a higher quality unit. The authors make direct assumptions on the consumers' valuation of upgrades. In their model, upgrading involves an increase in the marginal cost, but no increase in fixed costs: whether a firm produces a low quality product, or a high quality product, or both, the fixed cost is the same.

In our paper, we take a more general approach: high quality products may involve both higher fixed costs (e.g. more expensive plants and other overhead costs), as well as higher variable costs, even though we also consider the special case where the fixed costs are independent of product quality.

On the specification of demand, the typical specification is that each consumer buys at most one unit of the product.[2] This seems a reasonable specification for products such as cars, smart phones, computers, etc. It is assumed that consumers are heterogeneous with respect to a taste parameter θ. A consumer of type θ has the net utility function $u_\theta = \theta S - p$ where S is the quality of the product, and p is its price. The parameter θ can also be interpreted as the consumer's marginal rate of substitution between income and quality, so that a higher θ corresponds to a lower marginal utility of income; in other words, a consumer with a higher income would have a higher θ (see Tirole, 1988, p. 96). Indeed, in Gabszewicz and Thisse (1979, 1980), Shaked and Sutton (1982, 1983, 1984), Bonanno (1986), and Ireland (1987), consumers are supposed to be heterogeneous in terms of income.

[2] In a different class of models, consumers are identical and and buy more than one unit, see Sutton (1991) and Motta (1992a, 1992b). In these models, under Cournot competition, firms will choose the same quality.

A simple formulation is that there are only two firms in the industry (Motta, 1993): one firm (say firm 1) produces the high quality product, while the other produces the low quality product. Motta (1993) assumes that the market is not fully covered, i.e., in equilibrium, some set of consumers will choose not to purchase, because their θ is too low relative to the price of either product.[3]

Motta (1993, part I) assumes that fixed cost is quadratic in quality: for any quality level S in the continuum of feasible qualities $[S_{\min,}S_{\max}]$, the associated fixed cost is $F_S = S^2/2$. This implies that the quality-adjusted fixed cost, $f_S \equiv (1/S)F_S$, is $S/2$, and thus the quality-adjusted fixed cost increases in quality level, i.e., if $S_H > S_L$ then $f_{S_H} = S_H/2 > f_{S_L} \equiv S_L/2$. (In our paper, we do not restrict to the quadratic specification). Under this assumption, Motta (1993) considers a two-stage games between two firms. In the first stage, they choose the quality level (and incur the associated fixed cost), and in the second stage, they compete as two Cournot rivals (or alternative, as two Betrand rivals). Motta (1993) finds that when both firms know they will compete as Cournot rivals, their quality differentiation will be relatively small (the ratio of high quality to low quality is about 2) while if they know they will compete as Bertrand rivals, they will choose quality levels that are further apart (the ratio of high quality to low quality is about 5). The intuition behind this result is that since price competition tends to be fiercer than quantity competition (for any given pairs of quality levels), the firms will try to differentiate their products more to reduce rivalry.[4] The result that

[3]As pointed out by Motta (1993), if the market is fully covered, then total demand is independent of the prices, and thus the demand functions cannot be inverted, and hence one cannot consider Cournot competition. Note that if the lowest θ is zero, then it is automatically true that the market is not fully covered.

[4]If the fixed costs are zero, or do not increase too much with quality, and variable costs are zero, and the upper bound $S_{\max}$ is low, so that at $S_{\max}$ the marginal cost of quality is lower than the

Bertrand firms tend to have greater vertical differentiation of quality is robust: if costs of quality improvements are variable costs rather than fixed costs, the same principle applies.[5]

Concerning profits in the case of where the quality-adjusted fixed costs increase with quality, Motta (1993) finds that in the case of duopoly, the sum of profits is higher under Bertrand rivalry.[6] However, this result is reversed if quality improvement involve higher variable costs rather than higher fixed costs. Under either specification of the costs of quality, consumers are always better off under Bertrand competition.

Gal-Or (1983, 1985, 1987) and Motta (1993, Part III) study Cournot competition when quality improvements involve an increase in variable costs. While Gal-Or (1983) assumes in her model that qualities and quantities are simultaneously chosen, Motta (1993, Part III) assumes two stage competition. Motta (1993) assumes that the unit variable cost, adjusted for quality, is increasing in quality level, and finds that the firms will choose to be different: a low quality firm and a high quality firm. This result is consistent with those of Gal-Or (though she uses a slightly different utility function).

Finally, we should mention a related paper with vertical product differentiation which assumes monopolistic competition instead of oligopoly. Its authors, Fajgelbaum, Grossman and Helpman (2011), consider an equilibrium model where firms choose among vertically differentiated product qualities, and horizontally differentiated vari-

marginal revenue of quality, the two Cournot firms will both choose the highest quality level, $S_{\max}$. See Bonanno (1982) and Ireland (1987, pp.71-74).

[5]Under Bertrand competition, Shaked and Sutton (1982) show that if there are neither fixed costs nor variable costs, the two firms will choose two different quality levels, $S_1 = S_{\max}$ and S_2 is higher than $S_{\min}$. In Shaked and Sutton (1984), if fixed costs exist and are strongly increasing in quality level, then $S_1 < S_{\max}$.

[6]This is in sharp constrast to Vives (1985) who showed that Bertrand firms earn lower profits than Cournot firms (under the assumption that product specifications are exogenous).

eties. They assume that in equilibrium, each firm chooses only one quality level q, where q belongs to a finite set $Q \equiv \{q_1, q_2, ..., q_m\}$.[7] For any given q, there is a discrete set of varieties J_q. Each firm that has chosen quality level q must decide which variety in the given set J_q it wants to specialize in. By definition of monopolistic competition, each firm believes that its price does not affect the demand facing any other firm.

Each consumer h has a given income, y^h, and must allocate this income between a perfectly divisible and homogeneous numeraire good and one unit of the differentiated good. This unit can be of any quality $q \in Q$, and can be of any variety $j \in J_q$. The price of the chosen variety, denoted by p_j, is set by firm $j \in J_q$. The consumer pays p_j for the unit of the differentiated good, and thus her expenditure on the numeraire good is $y^h - p_j$. Call z this expenditure on the numeraire good. The utility of the consumer is assumed to be

$$u^h = zq + \varepsilon_j^h \text{ where } j \in J_q$$

where ε_j^h is her idiosyncratic evaluation of the attributes of variety j. Each individual h has a vector

$$\varepsilon^h \equiv \left(\varepsilon_1^h, \varepsilon_2^h, ..., \varepsilon_F^h\right)$$

where F is the number of firms in the industry. The multiplicative term zq indicates that the marginal utility of quality depends on how many units of the homogeneous good she consumes, which of course depends on her income. This formulation implies that a person with a higher income will value quality more.[8] It is assumed that the vectors ε are independently distributed according to a generalized extreme value (GEV)

[7] In the simplest case, the set Q consists of only two quality levels, so that $Q \equiv \{H, L\}$.

[8] This multiplicative formulation makes this paper different from the additive formulation in the industrial organization literature, which follows McFadden (1978) and Berry et al. (1995).

distribution, as in McFadden (1978). For example, if there are two quality levels, H and L, and within each level, there are two product varieties, then the GVE distribution is

$$G(\varepsilon) = e^{-\left(e^{-(\varepsilon_1/\theta_H)}+e^{-(\varepsilon_2/\theta_H)}\right)^{\theta_H}} \times e^{-\left(e^{-(\varepsilon_1'/\theta_L)}+e^{-(\varepsilon_2'/\theta_L)}\right)^{\theta_L}}$$

where $0 < \theta_H < 1$ and $0 < \theta_L < 1$.

More generally,

$$G(\varepsilon) = \exp\left\{-\sum_{q\in Q}\chi_q\right\}$$

where

$$\chi_q \equiv \left[\sum_{j\in J_q}\eta_j^{1/\theta_j}\right]^{\theta_j} \text{ and } \eta_j \equiv e^{-\varepsilon_j},\ j \in J_q$$

It can be shown that under this GEV distribution, the following results hold:

(i) among all consumers who buy a product with quality q, the the percentage who buys variety j in J_q is

$$\rho_{j|q} = \left(e^{-qp_j/\theta_q}\right)\left[\frac{1}{\sum_{i\in J_q} e^{-qp_i/\theta_q}}\right] \tag{1}$$

(ii) among all consumers with income y, the fraction who buy quality q is

$$\rho_q(y) = \left[\sum_{i\in J_q} e^{-(y-p_i)q/\theta_q}\right]^{\theta_q} \times \frac{1}{\sum_{\omega\in Q}\left[\sum_{i\in J_\omega} e^{-(y-p_i)\omega/\theta_\omega}\right]^{\theta_\omega}} \tag{2}$$

(iii) among all consumers with income y, the fraction who choose variety j with quality q is , for $j \in J_q$,

$$\rho_j(y) = \rho_{j|q} \times \rho_q(y) \tag{3}$$

Under monopolistic competition, a firm j that produces quality q will set p_j while assuming that this will have no effect on the term inside the square brackets on the right-hand side of eq. (1), and no effect on any term in equation (2). Fajgelbaum, Grossman and Helpman (2011) assume that θ_q is increasing in q. Under this assumption, and standard assumptions on production costs, they show that richer countries export higher quality goods. This result is consistent with our model, where we show that a country with a bigger market size tends to produce higher average quality and lower average price. However, in our model, the equilibrium mark-up on unit variable cost is not a constant, which is a well documented fact (Edmond et al., 2015).

3 The basic model

We consider an oligopoly with vertically differentiated products. Specifically, for simplicity, we assume there are on two quality levels, denoted by S_L and S_H, where $0 < S_L < S_H$.[9] In this section, we consider the simplest case: we assume for the moment that a firm must either produce a high quality product, or a low quality product, but not both, and they incur different set-up costs: a firm that wishes to produce a high quality product must incur a higher set-up cost. In imposing (in this section) the restriction that each firm is a single-product firm, our basic model is similar to Gabszewicz and Thisse (1989, 1980) and Shaked and Sutton (1982, 1983); however while these authors assume that firms compete as Bertrand rivals, i.e., they set prices, in our model we assume firms are Cournot rivals, i.e., they choose quantities.

In adopting Cournot competition, we follow Johnson and Myatt (2006). However, our model differs from Johnson and Myatt (2006) in two important respects. First, Johnson and Myatt (2006) assume that all firms have the same fixed cost, regardless of the quality they produce. In contrast, in our model, we focus on the more plausible

[9]In this paper, we follow the approach of Johnson and Myatt (2006) in that we do not address the issue of how S_H and S_L are determined. The set of quality levels is discrete and given.

case where the fixed costs depend on the quality level: firms that produce the high quality product must incur a higher fixed cost. The difference in fixed costs play an important role in our model. Our main focus is to determine the equilibrium prices and quantities, and the long-run equilibrium number of firms. Second, Johnson and Myatt (2006) didn't investigate the impacts of the change of market conditions on the equilibrium results while our study analyzes the role of the market size in affecting the equilibrium results.

3.1 Consumers

We assume there is a continuum of heterogeneous consumers. They differ from each other in terms of their intensity of preference for quality, which is represented by a parameter θ, where $0 \leq \theta \leq \overline{\theta}$. Let $G(\theta)$ denote the fraction of consumers whose intensity of preference is smaller than or equal to θ. We assume that $G(0) = 0$, $G(\overline{\theta}) = 1$ and $G'(\theta) > 0$ for all $\theta \in (0, 1)$.

Each consumer buys at most one unit of the good. She must decide whether to buy one unit of the high quality product, or one unit of the low quality product, or she does not buy any. A consumer of type θ places a value θS_H on the consumption of a unit of the high quality product, and a value θS_L on the consumption of a unit of the low quality product. Let P_H (respectively, P_L) denote the market price of the high quality product (respectively, low quality product). Her net utility is $\theta S_H - P_H$ or $\theta S_L - P_L$, depending on which product she buys. If she does not buy either product, her net utility is zero. We assume that $P_H > P_L$. Let us define the following ratios:

$$\theta_L \equiv \frac{P_L}{S_L},\ \theta_H \equiv \frac{P_H}{S_H},\ \theta_I \equiv \frac{P_H - P_L}{S_H - S_L} \tag{4}$$

In what follows, we assume that equilibrium prices are such that $0 < \theta_L < \theta_H < \theta_I < \overline{\theta}$.[10] It is easy to show that

$$\frac{P_H - P_L}{S_H - S_L} > \frac{P_H}{S_H} \iff \frac{P_L}{S_L} < \frac{P_H}{S_H}$$

A consumer whose θ equals θ_L is indifferent between not buying the good and buying one unit of the low quality product at the price P_L. Similarly, a consumer whose θ equals θ_L is indifferent between not buying the good and buying one unit of the high quality product at the price P_H. And a consumer with $\theta = \theta_I$ will be indifferent between the two alternative purchases.

The fraction of the population who purchases the high quality product is $G(\overline{\theta}) - G(\theta_I)$, the fraction who purchases the low quality product is $G(\theta_I) - G(\theta_L)$, and the fraction who does not buy the good is $G(\theta_L) > 0$.

3.2 Producers

We assume that any firm that wants to produce the high quality product must incur an upfront cost (or set-up cost) F_H , and any firm that wants to produce the low quality product must incur F_L, where $0 < F_L < F_H$. These are entry costs to the market. They may correspond to the cost of purchasing equipment, or possibly $R\&D$ costs. In this section, we assume that a firm that incurs F_H can only produce the high quality product, and a firm that incurs F_L can only produce the low quality product. After entry, firms choose their output level and compete as Cournot rivals. The marginal production costs for high and low quality products are C_H and C_L respectively. We assume that the marginal cost of a product is lower than its valuation by the consumer

[10] For example, if $\overline{\theta} = 2$, $S_L = 1, S_H = 2$, $P_L = 1, P_H = 2.2$ then $\theta_L = 1$, $\theta_H = 1.1$, $\theta_I = 1.2$

with the highest θ:

$$C_H < \overline{\theta} S_H \text{ and } C_L < \overline{\theta} S_H$$

We will solve for both the short-run and the long-run equilibrium. In the short-run, the numbers of firms of each types are fixed at n_L and n_H, and there are no entry nor exit: the firms have incurred their set-up costs F_L and F_H, and they make their output decisions, competing as Cournot rivals. We solve for the Cournot equilibrium output of each type of product, and the resulting short-run equilibrium prices P_H and P_L. Gross profits (before subtracting the entry costs) can then be calculated. In the long run, free entry and exit implies that the net profit of each firm is zero. The zero-profit conditions determine the long-run equilibrium number of firms. (As usual in this literature, we ignore the integer problem.)

3.3 Short-run Cournot equilibrium with two types of firms

In the short run, we take n_H and n_L as given. Cournot rivalry means that firms determine their outputs, knowing that the market prices will be determined by the industry outputs of each product. Firms take the inverse demand functions as given. Let us specify the inverse demand functions.

3.3.1 The inverse demand functions

Let N denote the mass of consumers. Given the prices P_L and P_H, we can compute θ_I and θ_L as functions of (P_L, P_H). The number of consumers who demand the high quality product is

$$X_H = N\left[G(\overline{\theta}) - G(\theta_I(P_L, P_H))\right] \tag{5}$$

and the number of consumers who demand the low quality product is

$$X_L = N\left[G(\theta_I) - G(\theta_L(P_L, P_H))\right] \tag{6}$$

From these demand functions, we can compute the inverse demand functions

$$P_H = P_H(X_H, X_L) \tag{7}$$

$$P_L = P_L(X_H, X_L) \tag{8}$$

In order to obtain an explicit solution, let us assume that the distribution of θ is uniform over the interval $\left[0, \overline{\theta}\right]$. Then

$$G(\theta) = \theta/\overline{\theta} \text{ for } \theta \in \left[0, \overline{\theta}\right]$$

Equations (5) and (6) become

$$X_H = N\left[\frac{\overline{\theta}}{\overline{\theta}} - \frac{\theta_I}{\overline{\theta}}\right] = \frac{N}{\overline{\theta}}\left[\overline{\theta} - \frac{P_H - P_L}{S_H - S_L}\right] \tag{9}$$

$$X_L = N\left[\frac{\theta_I}{\overline{\theta}} - \frac{\theta_L}{\overline{\theta}}\right] = \frac{N}{\overline{\theta}}\left[\frac{P_H - P_L}{S_H - S_L} - \frac{P_L}{S_L}\right] \tag{10}$$

These equations yield the inverse demand functions

$$P_H = \left(1 - \frac{X_H}{N}\right)\overline{\theta}S_H - \frac{X_L}{N}\overline{\theta}S_L \tag{11}$$

$$P_L = \left(1 - \frac{X_H}{N} - \frac{X_L}{N}\right)\overline{\theta}S_L \tag{12}$$

Then

$$\frac{\partial P_H}{\partial X_H} = -\frac{\overline{\theta}S_H}{N}, \frac{\partial P_H}{\partial X_L} = -\frac{\overline{\theta}S_L}{N}, \ \frac{\partial P_L}{\partial X_L} = \ \frac{\partial P_L}{\partial X_H} = -\frac{\overline{\theta}S_L}{N} \tag{13}$$

Notice that a unit increase in the output X_H affects P_L the same way as a unit increase in the output X_L. In contrast, the effect of a unit increase in X_H on P_H is stronger that of X_L on P_H.

3.3.2 The output decision and market equilibrium in the short run

Let x_H^i denote the output of the high-quality firm i, and X_H^{-i} denote the sum of outputs of all other high-quality firms. The high-quality firm i chooses x_H^i to maximize its profit, taking X_H^{-i} and X_L as given

$$\max \pi_H^i = x_H^i \left[P_H(X_H^{-i} + x_H^i, X_L) - C_H\right] \tag{14}$$

The first order condition for an interior equilibrium is[11]

$$\left[P_H(X_H^{-i} + x_H^i, X_L) - C_H\right] + x_H^i \frac{\partial P_H(X_H^{-i} + x_H^i, X_L)}{\partial X_H} = 0 \tag{15}$$

Similarly, for any low-quality firm j, the corresponding first order condition is

$$\left[P_L(X_H, X_L^{-j} + x_l^j) - C_L\right] + x_L^j \frac{\partial P_L(X_H, X_L^{-j} + x_L^j)}{\partial X_L} = 0 \tag{16}$$

In a symmetric equilibrium, $x_H^i = (1/n_H)X_H$ and $x_L^j = (1/n_L)X_L$. Substituting into the two first order conditions, we obtain a system of two equations that determines the equilibrium outputs, X_H^* and X_L^*, for given n_L and n_H

$$[P_H(X_H, X_L) - C_H] + \frac{X_H}{n_H} \frac{\partial P_H(X_H, X_L)}{\partial X_H} = 0 \tag{17}$$

$$[P_L(X_H, X_L) - C_L] + \frac{X_L}{n_L} \frac{\partial P_L(X_H, X_L)}{\partial X_L} = 0 \tag{18}$$

Thus the optimal output of the representative high-quality firm and that of the low quality firms are

$$x_H^* = \frac{[P_H(X_H^*, X_L^*) - C_H]}{-\frac{\partial P_H(X_H^*, X_L^*)}{\partial X_H}}, \qquad x_L^* = \frac{[P_L(X_H^*, X_L^*) - C_L]}{-\frac{\partial P_L(X_H^*, X_L^*)}{\partial X_L}} \tag{19}$$

[11]We restrict attention to interior equilibrium outputs for simplicity.

Substituting (19) into the profit function (14) we can expressed the firm's profit in terms of its equilibrium output:

$$\pi_H^* = -\frac{\partial P_H(X_H^*, X_L^*)}{\partial X_H}(x_H^*)^2 = \frac{\overline{\theta} S_H \left(x_H^*\right)^2}{N}, \qquad \pi_L^* = -\frac{\partial P_L(X_H^*, X_L^*)}{\partial X_L}(x_L^*)^2 = \frac{\overline{\theta} S_L \left(x_L^*\right)^2}{N} \tag{20}$$

Substituting into (17) and (18) yields the following system of equations

$$\left(1 - \frac{X_H}{N}\right)\overline{\theta} S_H - \frac{X_L}{N}\overline{\theta} S_L - C_H = \frac{\overline{\theta} S_H}{N}\frac{X_H}{n_H} \tag{21}$$

$$\left(1 - \frac{X_H}{N} - \frac{X_L}{N}\right)\overline{\theta} S_L - C_L = \frac{\overline{\theta} S_L}{N}\frac{X_L}{n_L} \tag{22}$$

Dividing both sides of eq. (21) by $\overline{\theta} S_H/Nn_H$ and both sides of eq. (22) by $\overline{\theta} S_L/Nn_L$, we obtain

$$n_H N(1 - c_H) - n_N X_H - k n_H X_L = X_H \tag{23}$$

$$n_L N(1 - c_L) - n_H X_H - n_L X_L = X_L \tag{24}$$

where

$$c_H \equiv \frac{C_H}{\overline{\theta} S_H} < 1, \quad c_L \equiv \frac{C_L}{\overline{\theta} S_L} < 1 \text{ and } k \equiv \frac{S_L}{S_H} < 1 \tag{25}$$

Solving, we obtain the equilibrium outputs as functions of c_L, c_H, k, n_L, n_H and N

$$X_H^* = n_H N \frac{((1 - c_H)(1 + n_L) - k(1 - c_L)n_L)}{1 + n_H + n_L + (1 - k)n_H n_L} \tag{26}$$

$$X_L^* = n_L N \frac{((1 - c_L)(1 + n_H) - (1 - c_H)n_H)}{1 + n_H + n_L + (1 - k)n_H n_L} \tag{27}$$

Substituting the equilibrium outputs into equations (11) and (12) we obtain the equilibrium prices

$$P_H^* = \left[1 - \frac{1}{N}X_H^* - \frac{k}{N}X_L^*\right]\overline{\theta} S_H$$

$$P_L^* = \left[1 - \frac{1}{N}X_H^* - \frac{1}{N}X_L^*\right]\overline{\theta}S_L$$

Each high-quality firm's equilibrium output is

$$x_H^* = \frac{X_H^*}{n_H} = \frac{N((1-c_H)(1+n_L) - k(1-c_L)n_L)}{1+n_H+n_L+(1-k)n_H n_L}$$

and, using (20), its profit is

$$\pi_H^* = \overline{\theta}S_H N\left(\frac{(1-c_H)(1+n_L) - k(1-c_L)n_L}{1+n_H+n_L+(1-k)n_H n_L}\right)^2 \tag{28}$$

Similarly,

$$\pi_L^* = \overline{\theta}S_L N\left(\frac{(1-c_L)(1+n_H) - (1-c_H)n_H}{1+n_H+n_L+(1-k)n_H n_L}\right)^2 \tag{29}$$

3.4 The long-run free-entry equilibrium

In the long-run equilibrium, free entry and exit ensure that each firm's net profit is zero. The zero-profit conditions determine the number of high-quality producers and low-quality producers. To solve for the equilibrium number of firms n_L and n_H, we equate the profit (before subtracting the fixed cost) for each of type of firm with the corresponding fixed cost:

$$\pi_H^* = F_H$$

$$\pi_L^* = F_L$$

Using the definitions

$$f_H = \frac{F_H}{\overline{\theta}S_H} \text{ and } f_L = \frac{F_L}{\overline{\theta}S_L}$$

the zero profit conditions become

$$\left(\frac{(1-c_H)(1+n_L) - k(1-c_L)n_L}{1+n_H+n_L+(1-k)n_H n_L}\right)^2 = \frac{f_H}{N} \tag{30}$$

$$\left(\frac{(1-c_L)(1+n_H) - (1-c_H)n_H}{1+n_H+n_L+(1-k)n_H n_L}\right)^2 = \frac{f_L}{N} \tag{31}$$

3.4.1 The special case when quality-adjusted marginal costs are the same for both products

In this sub-section, we solve for the long-run equilibrium number of firms of each type, under the assumption that the quality-adjusted marginal costs are identical, $c_H = c_L = c$. Then

$$\frac{(n_L - kn_L + 1)^2}{(n_H + n_L + n_H n_L(1-k) + 1)^2} = \frac{f_H}{(1-c)^2 N} \tag{32}$$

$$\frac{1}{(n_H + n_L + n_H n_L(1-k) + 1)^2} = \frac{f_L}{(1-c)^2 N} \tag{33}$$

Dividing the first equation by the second equation, we get

$$(n_L - kn_L + 1)^2 = \frac{f_H}{f_L} \equiv \beta$$

i.e.

$$n_L(1-k) + 1 = \sqrt{\beta}$$

Thus

$$n_L = \frac{\sqrt{\beta} - 1}{1 - k} \tag{34}$$

Notice that $n_L > 0$ if and only if $f_H > f_L$.

Next, use (33) to get

$$n_H\left(1 + (1-k)n_L\right) + (n_L + 1) = (1-c)\sqrt{\frac{N}{f_L}}$$

$$n_H = \frac{(1-c)(1-k)\sqrt{N} + (k - \sqrt{\beta})\sqrt{f_L}}{(1-k)\sqrt{\beta f_L}} \tag{35}$$

3.4.2 Main results for the basic model

For ease of reference, we state the following three lemmas.

Lemma 1 (The case where marginal production costs are proportional to product quality)

Assume $c_H = c_L = c \geq 0$. In the long run equilibrium (zero profits for both types of firms),

(i) the low quality product is supplied ($n_L > 0$) only if $F_H/S_H > F_L/S_L$, i.e., the set-up cost per unit of quality is increasing in quality level.

(ii) an increase in the market size, N, will increase the number of high-quality firms but leave the number of low-quality firms unchanged.

The total number of firms in the long-run equilibrium is

$$n_L + n_H = \frac{(1-c)(1-k)\sqrt{N} + (k-\sqrt{\beta})\sqrt{f_L} + (\sqrt{\beta}-1)\sqrt{\beta f_H}}{(1-k)\sqrt{\beta f_L}}$$

Clearly the total number of firms increases in the market size. The ratio of number of low-quality firms to the total number of firms is

$$R_L = \frac{\sqrt{\beta f_L}(\sqrt{\beta}-1)}{(1-c)(1-k)\sqrt{N} + (k-\sqrt{\beta})\sqrt{f_L} + (\sqrt{\beta}-1)\sqrt{\beta f_L}}$$

This ratio decreases in N.

The ratio of the number of high-quality firms to the total number of firms is $R_H = 1 - R_L$, and it increases in the market size.

Now consider the quantities sold. From (26) and (27) the total quantity sold is

$$X_H^* + X_L^* = (1-c)N\frac{(n_H + n_L + (1-k)n_H n_L)}{1 + (n_H + n_L + (1-k)n_H n_L)}$$

The ratio of X_L^* to total quantity sold is

$$M_L = \frac{X_L^*}{X_H^* + X_L^*} = \frac{1}{\frac{n_H}{n_L} + 1 + (1-k)n_H}$$

Since n_L does not change with N, and n_H increases in N, we conclude that M_L falls as N increases. Thus we can state:

Lemma 2: *Assume $c_L = c_H = c$, and $f_H > f_L$. As the market size increases, the share of high-quality firms, $n_N/(n_H + n_L)$, increases, and so does their market share in quantity terms, $X_H^*/(X_H^* + X_L^*)$*

What about the market share in value terms?

Lemma 3: *Assume $c_L = c_H = c$ and $f_H > f_L$. The equilibrium prices in the long-run equilibrium are:*

$$P_L = \overline{\theta} S_L \left(\frac{1}{1 + n_H + n_L + (1-k) n_H n_L} \right) = \frac{\overline{\theta} S_L}{(1-c)} \sqrt{\frac{f_L}{N}} \tag{36}$$

$$P_H = \overline{\theta} S_H \left(\frac{1 + (1-k) n_L}{1 + n_H + n_L + (1-k) n_H n_L} \right) = \frac{\overline{\theta} S_H}{(1-c)} \sqrt{\frac{f_H}{N}} \tag{37}$$

They fall as the market size expands. However their ratio, P_H/P_L, is independent of the market size, N:

$$\frac{P_H}{P_L} = \left(\frac{S_H}{S_L} \right) \sqrt{\frac{f_H}{f_L}} > \left(\frac{S_H}{S_L} \right) > 1$$

The market share of low-quality sales is

$$\frac{P_L X_L}{P_H X_H + P_L X_L} = \frac{k n_L}{n_H \left(n_L (1-k) + 1 \right)^2 + k n_L} = \frac{k n_L}{n_H \beta + k n_L} \tag{38}$$

As the market size N increases, n_H increases but n_L is unchanged, and the market share of low-quality sales (in value terms) decreases.

Proof: omitted.

From Lemmas 1 to 3, we can state

Proposition 1 *Assume the marginal cost per unit of quality is constant, i.e., $c_H = c_L = c$, and $f_H > f_L$. In a Cournot oligopoly with free entry and exit, in equilibrium the* *relative price P_L/P_H is independent of the market size. As the market size increases, the prices P_L and P_H both decrease by the same proportion, and the market share (in value terms) of the high quality product increases. Welfare of each type of consumer increases.*

Remark Welfare increases because the prices fall. The fraction of the market served by the oligopoly rises, because $\theta_L \equiv P_L/S_L$ falls. The fraction of the market that is supplied by the high-quality producer rises, because $\theta_I \equiv (P_H - P_L)/(S_H - S_L)$ falls, while P_L/P_H is unchanged. In fact, in the long run equilibrium (with free entry and exit), we have

$$\frac{P_H}{P_L} = \left(\frac{S_H}{S_L}\right)(1 + n_L(1-k)) \text{ (independent of } N\text{)}$$

$$\frac{X_L}{X_H} = \frac{n_L}{n_H + (1-k)n_H n_L} \text{ (falls as } N \text{ rises)}$$

$$\theta_I = \frac{P_H - P_L}{S_H - S_L} = \frac{\left(\frac{1+n_L(1-k)}{k} - 1\right) P_L}{S_H - S_L} \text{ (falls as } N \text{ rises)}$$

4 Commitment to product lines

In the preceding section, we considered a Cournot oligopoly with both high and low quality products, under the assumption that a firm produces either the high quality product, or the low quality product, but not both.

We now relax that single-product-firm assumption, and allow each firm to decide whether to produce both quality levels, or only one. The solution turns out to depend on the relationship among the technological parameters, namely the fixed costs per unit of quality, F_H/S_H and F_L/S_L, and the unit variable costs per unit of quality, C_H/S_H

and C_L/S_L. In what follows, we consider only two cases. In Case 1, we assume that the technology has the following properties: (i) $C_H \geq C_L$, and (ii) higher quality requires higher fixed cost per unit of quality but involves lower unit variable cost per unit of quality, i.e.,

$$\frac{F_H}{S_H} \geq \frac{F_L}{S_L} \text{ and } \frac{C_H}{S_H} \leq \frac{C_L}{S_L} \tag{39}$$

The assumption that $C_H \geq C_L$ and eq. (39) imply $C_L/S_H \leq C_H/S_H \leq C_L/S_L$.

In Case 2, we assume that (a) $F_H = F_L = F > 0$, which implies that $F_H/S_H < F_L/S_L$, and (b) $C_H/S_H > C_L/S_L$. Note that Johnson and Myatt (2006) assume $F_H = F_L = F$ throughout their analysis.

As before, we define

$$f_H = \frac{F_H}{\bar{\theta} S_H},\ f_L = \frac{F_L}{\bar{\theta} S_L},\ c_H = \frac{C_H}{\bar{\theta} S_H},\ c_L = \frac{C_L}{\bar{\theta} S_L}$$

Then in Case 1, $f_H \geq f_L$ and $c_H \leq c_L$, and we say that in this case, the "quality-adjusted fixed cost" increases with quality upgrading, and the "quality-adjusted unit variable cost" decreases with quality upgrading. In Case 2, the reverses hold.

4.1 Case 1: higher quality requires higher fixed cost (per unit of quality) and involves lower variable cost (per unit of quality)

We now consider Case 1, in which the following cost configuration holds:

$$f_H \geq f_L \text{ and } c_H \leq c_L \tag{40}$$

That is, the quality-adjusted fixed cost is higher for the high quality product, and the quality-adjusted variable cost is lower for the high quality product. We assume that any firm j that has invested only F_L is not able to produce the high-quality product. Its output of the low quality product is denoted by x_L^j. In contrast, we assume that any firm i that has invested F_H can produce both quality levels. Its outputs are denoted by x_H^i and x_L^i.

Let us establish an important Lemma:

Lemma 4 *Assume $c_H \leq c_L$. Any firm that has invested F_H and is able to produce both products will find it optimal to specialize in the high quality product.*

Proof: see the appendix.

In what follows, we assume $c_H < c_L$ (because the borderline case where $c_H = c_L$ has been considered in the previous section). Then, due to Lemma 4, all firms that have invested F_H will specialize in the high quality product, and all firms that have invested F_L will specialize in the low quality product.

Given n_L and n_H, the equilibrium outputs are

$$\frac{X_H^*}{N} = \frac{(1-c_H)n_H(1+n_L) - k(1-c_L)n_H n_L}{1+n_H+n_L+(1-k)n_H n_L} = \frac{z_H n_H(1+n_L) - kz_L n_H n_L}{1+n_H+n_L+(1-k)n_H n_L} \tag{41}$$

$$\frac{X_L^*}{N} = \frac{(1-c_L)n_L(1+n_H) - (1-c_H)n_H n_L}{1+n_H+n_L+(1-k)n_H n_L} = \frac{z_L n_L(1+n_H) - z_H n_H n_L}{1+n_H+n_L+(1-k)n_H n_L} \tag{42}$$

where $z_i \equiv 1 - c_i$, and $z_H > z_L$.

Now we must determine the equilibrium number of each type of firm, under free entry. Zero profits imply

$$\left(1 - \frac{X_H^*}{N} - k\frac{X_L^*}{N} - c_H\right)\frac{X_H^*}{N} = \frac{n_H f_H}{N} \equiv n_H g_H \tag{43}$$

$$\left(1 - \frac{X_H^*}{N} - \frac{X_L^*}{N} - c_L\right)\frac{X_L^*}{N} = \frac{n_L f_L}{N} \equiv n_L g_L \tag{44}$$

From (43) we obtain

$$n_H = \frac{z_H - \sqrt{f_H/N} + n_L\left(z_H - kz_L - \sqrt{f_H/N}\right)}{(1 + n_L(1-k))\sqrt{f_H/N}} \tag{45}$$

Using (44) and (45) we obtain

$$\begin{aligned} n_L &= \frac{z_L\sqrt{\beta} + n_H(z_L - z_H)\sqrt{\beta} - z_H}{z_H - kz_L} \\ &= \frac{z_L(\sqrt{\beta} + n_H\sqrt{\beta}) - z_H(1 + n_H\sqrt{\beta})}{z_H - kz_L} \end{aligned} \tag{46}$$

In Diagram 1, Curve 2 depicts equation (46): it has a negative slope. Curve 1 depicts equation (45). Its slope is given by

$$\frac{dn_H}{dn_L} = \frac{k\left[\sqrt{\frac{N}{f_H}}\left(\frac{C_L}{\overline{\theta}S_L} - \frac{C_H}{\overline{\theta}S_H}\right) - 1\right]}{(1 + n_L(1-k))^2}$$

This slope can be positive or negative. Thus we must consider two sub-cases

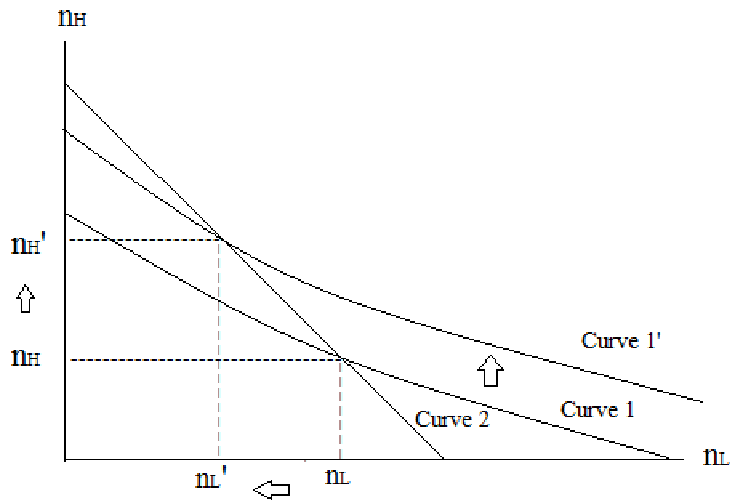
4.1.1 Subcase (i): Curve 1 has a negative slope (the market size is not too large)

The slope of Curve 1 is negative if and only if

$$z_H - z_L \equiv c_L - c_H < \sqrt{\frac{f_H}{N}} \tag{47}$$

i.e., if N is not too big. Assuming that condition (47) is satisfied, we can show that the Curve 1 is strictly convex. Then Curve 1 and Curve 2 intersect at most once in the positive orthant.

Diagram 1. Equilibrium number of the firms with different quality levels, downwards sloping of Curve 1



For a given n_L, an increase in N will shift Curve 1 up.

The vertical intercept of Curve 1 is

$$y_1 = \frac{z_H - \sqrt{g_H}}{\sqrt{g_H}} \tag{48}$$

this intercept is positive if and only if

$$z_H > \sqrt{g_H} \tag{49}$$

and the horizontal intercept is

$$x_1 = \frac{z_H - \sqrt{g_H}}{\sqrt{g_H} - (z_H - kz_L)} \tag{50}$$

Assuming $z_H > \sqrt{g_H}$, the horizontal intercept is positive if and only if

$$z_H - \sqrt{g_H} < kz_L \tag{51}$$

Now, consider Curve 2. Since by assumption $z_H \geq z_L$, at $n_H = 0$, we have $n_L > 0$ only if β is sufficiently large, such that

$$\sqrt{\beta} > z_H - z_L$$

We can rewrite eq (46) as follows:

$$n_H = \frac{\left(z_L\sqrt{\beta} - z_H\right) - n_L(z_H - kz_L)}{(z_H - z_L)\sqrt{\beta}} \tag{52}$$

Curve 2 is a straight-line with negative slope. The vertical intercept of Curve 2 is

$$y_2 = \frac{\left(z_L\sqrt{\beta} - z_H\right)}{(z_H - z_L)\sqrt{\beta}} \tag{53}$$

This intercept is positive if and only if

$$z_L\sqrt{\beta} - z_H > 0 \tag{54}$$

The horizontal intercept is

$$x_2 = \frac{z_L\sqrt{\beta} - z_H}{(z_H - kz_L)} \tag{55}$$

It is positive iff $z_L\sqrt{\beta} - z_H > 0$.

In brief, there exists a unique equilibrium with $n_H > 0$ and $n_L > 0$ (as illustrated by Diagram 1) if we assume the following conditions:

First, we require that $y_2 > y_1$, i.e.

$$0 < c_L - c_H < \sqrt{\frac{f_H}{N}} - \sqrt{\frac{f_L}{N}} \tag{56}$$

Second, we require that $x_1 > x_2$

$$\frac{z_H - \sqrt{g_H}}{\sqrt{g_H} - (z_H - kz_L)} > \frac{z_L\sqrt{\beta} - z_H}{(z_H - kz_L)} > 0 \tag{57}$$

Recall that previously we have also made the following requirements[12]

$$z_H - \sqrt{g_H} > 0 \tag{58}$$

[12]Note that if condition (56) is satisfied, than condition (59) is satisfied.

$$\sqrt{g_H} - (z_H - kz_L) > 0 \tag{59}$$

$$z_L\sqrt{\beta} - z_H > 0, \text{ i.e. } \sqrt{f_H/f_L} > \frac{z_H}{z_L} \equiv \frac{1-c_H}{1-c_L} > 1 \tag{60}$$

and

$$z_H - z_L > 0 \text{ i.e. } c_H < c_L \tag{61}$$

Equations (61) and (59) allow us to re-write (57) as

$$(z_H - kz_L)(z_H - \sqrt{g_H}) - (\sqrt{g_H} - (z_H - kz_L))\left(z_L\sqrt{\beta} - z_H\right) > 0$$

which is equivalent to

$$\sqrt{f_H/N} - k\sqrt{f_L/f_H} < z_H - kz_L \tag{62}$$

i.e.,

$$\frac{\sqrt{f_N}}{\sqrt{N}} < (1-c_H) - k(1-c_L) + k\sqrt{\frac{f_L}{f_H}} \tag{63}$$

Now consider a small increase in N. Curve 2 is not affected by N. An increase in N will shift Curve 1 upwards. The result is that the intersection point of the two curves will move up along Curve 2, implying an increase in the number of high quality firms and a decrease in the number of low quality firms. The ratio X_L^*/X_H^* is, from (41) and (42),

$$\frac{X_L^*}{X_H^*} = \frac{z_L(\frac{1}{n_H}+1) - z_H}{z_H(\frac{1}{n_L}+1) - kz_L}$$

When n_L falls and n_H rises, the denominator gets larger and the numerator gets smaller, implying that in quantity terms, the market share of the low quality product falls.

From the above analysis, we obtain the following Proposition, under the assumption that Curve 1 has a negative slope, i.e.,

$$0 < c_L - c_H < \sqrt{f_H/N} \tag{64}$$

Proposition 2: *Assume $c_H < c_L$, and $c_L - c_H < \sqrt{f_H/N}$. There exists a unique Cournot equilibrium with $n_H^* > 0$ and $n_L^* > 0$ such that each type of firms optimally chooses to specialize in either the high or the low quality product, provided the additional assumptions (i) to (v) below hold. Furthermore, an increase in the market size will increase the number of high-quality firms, decrease the number of low-quality firms, and decrease the market share of the low quality product.*

(i) Large increment in fixed cost for quality upgrade, $F_H/S_H > F_L/S_L$

(ii) The higher quality product has higher unit variable cost, $C_H > C_L$, but lower quality-adjusted unit variable cost, i.e., $C_H/S_H < C_L/S_L$.

(iii) The ratio of quality-adjusted fixed costs, f_H/f_L is sufficiently great relative to the ratio $(1 - c_H)/(1 - c_L)$, i.e. condition (60) holds.

(iv) The market size, N, is not too large:

$$1 - c_H > \sqrt{\frac{f_H}{N}} > (1 - c_H) - \frac{S_L}{S_H}(1 - c_L) \tag{65}$$

(v) Conditions (56) and (63) hold:

$$c_L - c_H < \sqrt{\frac{f_H}{N}} - \sqrt{\frac{f_L}{N}} \tag{66}$$

$$\sqrt{\frac{f_H}{N}} < (1 - c_H) - \frac{S_L}{S_H}(1 - c_L) + k\sqrt{\frac{f_L}{f_H}} \tag{67}$$

4.1.2 Subcase (ii): Curve 1 has a positive slope (The market size is large)

Now, we turn to the case where the slope of Curve 1 is positive. This is depicted in Figure 6. Curve 1 has a non-negative slope if and only if

$$z_H - z_L \equiv c_L - c_H \geq \sqrt{\frac{f_H}{N}} \tag{68}$$

Then, if condition (68) is met, we have

$$z_H - \sqrt{g_H} \geq z_L$$

where $g_H \equiv \frac{f_H}{N}$, and the vertical intercept of Curve 1 is

$$y_1' = \frac{z_H - \sqrt{g_H}}{\sqrt{g_H}} > \frac{z_L}{\sqrt{f_H/N}} > 0$$

If y_1' is equal to or smaller than y_2 (the vertical intercept of Curve 2), then there will be a unique intersection with both $n_L \geq 0$ and $n_H \geq 0$. Thus, if in addition to (68) we assume that

$$y_1' \equiv \frac{z_H - \sqrt{g_H}}{\sqrt{g_H}} \leq \frac{\left(z_L\sqrt{\beta} - z_H\right)}{(z_H - z_L)\sqrt{\beta}} \equiv y_2 \tag{69}$$

then we have an interior Cournot equilibrium. Since we are dealing with the case where $z_H > z_L$, condition (69) is equivalent to

$$\frac{z_H - z_L}{\sqrt{g_H}} \leq \frac{\sqrt{\beta} - 1}{\sqrt{\beta}} \tag{70}$$

The right side of the inequality is less than one, i.e. $\frac{\sqrt{\beta}-1}{\sqrt{\beta}} < 1$. Recall that we have the condition $z_H - \sqrt{g_H} \geq z_L$, which implies that $\frac{z_H - z_L}{\sqrt{g_H}} \geq 1$. Thus the inequality (70) conflicts with condition (68). In this case, when the slope of Curve 1 is non-negative, there are no non-negative solutions for n_H and n_L simultaneously.

4.2 Case 2: The higher quality product requires higher variable cost (per unit of quality), but does not require a larger fixed cost

Now we turn to the opposite case: $F_H = F_L$ and $C_H/S_H > C_L/S_L$. Johnson and Myatt (2006, Section 5, Proposition 9) show that if there are only two possible quality levels with the identical fixed costs $F_H = F_L = F$, in equilibrium no firm restricts itself to selling only the low quality product. Thus, in this case the industry consists of n identical firms, each producing both products. Applying this result to our model, we can solve for the equilibrium output of each product, given that the number of firms is n, and the market size is N. The first order conditions for each firm are

$$\left(1 - \frac{1}{N}X_H\right) - \frac{X_L}{N}k - x_H^i\frac{1}{N} - x_L^i\frac{1}{N}k = \frac{C_H}{\bar{\theta}S_H} \tag{71}$$

$$-x_i^H\frac{1}{N} + \left(1 - \frac{1}{N}X_H - \frac{1}{N}X_L\right) - x_L^i\frac{1}{N} = \frac{C_L}{\bar{\theta}S_L} \tag{72}$$

Subtracting equation (72) from equation (71), we obtain

$$\frac{C_H}{\bar{\theta}S_H} - \frac{C_L}{\bar{\theta}S_L} = \frac{1}{N}(X_L + x_L^i)(1-k) > 0$$

Under symmetry, we have

$$x_L^i = \frac{X_L}{n}$$

Then

$$c_H - c_L = \frac{X_L}{N}\left(1+\frac{1}{n}\right)(1-k)$$

$$\frac{X_{L^*}}{N} = \frac{c_H - c_L}{\left(1+\frac{1}{n}\right)(1-k)} > 0 \tag{73}$$

Re-write eq. (72) as

$$1 - \frac{X_H}{N}\left(1+\frac{1}{n}\right) - \frac{X_L}{N}\left(1+\frac{1}{n}\right) = \frac{C_L}{\overline{\theta} S_L} \tag{74}$$

Thus

$$\frac{X_H}{N}\left(1+\frac{1}{n}\right) = \frac{1-k+(kc_L - c_H)}{1-k}$$

$$\frac{X_H^*}{N} = \frac{(1-k)-(c_H - kc_L)}{\left(1+\frac{1}{n}\right)(1-k)} \tag{75}$$

Thus $X_H^* > 0$ iff

$$c_H - kc_L < 1-k$$

iff

$$k < \frac{1-c_H}{1-c_L} < 1 \tag{76}$$

From (73) and (75), *the ratio of low quality output to high quality output is independent of the market size*, N.

The prices are

$$P_H^* = \left[1 - \frac{1}{N}X_H^* - \frac{k}{N}X_L^*\right]\overline{\theta} S_H \tag{77}$$

$$P_L^* = \left[1 - \frac{1}{N}X_H^* - \frac{1}{N}X_L^*\right]\overline{\theta} S_L \tag{78}$$

Substituting (73) and (75) into (77) and (78) we obtain, after simplification,

$$P_H^* = \left(\frac{nc_H+1}{n+1}\right)\overline{\theta} S_H \text{ and } P_L^* = \left(\frac{nc_L+1}{n+1}\right)\overline{\theta} S_L \tag{79}$$

Then, since this subsection deals with the case where $c_L < c_H$, we obtain

$$\frac{P_L^*}{\bar{\theta} S_L} = \frac{nc_L + 1}{n+1} < \frac{nc_H + 1}{n+1} = \frac{P_H^*}{\bar{\theta} S_H}$$

and

$$\frac{P_L^*}{P_H^*} = \frac{k\left(nc_L + 1\right)}{\left(nc_H + 1\right)} < 1$$

Thus, an increase in n leads to lower prices for both products, and a fall in the ratio P_L^*/P_H^*. The zero profit condition is

$$P_H^* \frac{X_H^*}{N} + P_L^* \frac{X_L^*}{N} - C_H \frac{X_H^*}{N} - C_L \frac{X_L^*}{N} = \frac{nF}{N} \tag{80}$$

This condition is equivalent to

$$\left(\left(\frac{nc_H + 1}{n+1}\right)\bar{\theta} S_H - C_H\right)\left(\frac{1 - k + \left(kc_L - c_H\right)}{\left(1 + \frac{1}{n}\right)\left(1 - k\right)}\right) + \left(\left(\frac{nc_L + 1}{n+1}\right)\bar{\theta} S_L - C_L\right)\left(\frac{c_H - c_L}{\left(1 + \frac{1}{n}\right)\left(1 - k\right)}\right) = \frac{nF}{N}$$

i.e.,

$$\left(\frac{1 - c_H}{n+1}\right)\left(\frac{1 - k + \left(kc_L - c_H\right)}{(n+1)(1-k)}\right) + k\left(\frac{1 - c_L}{n+1}\right)\left(\frac{c_H - c_L}{(n+1)(1-k)}\right) = \frac{F}{N\bar{\theta} S_H}$$

i.e.

$$\frac{\Delta}{\left(1 - k\right)\left(n + 1\right)^2} = \frac{F}{N\bar{\theta} S_H}$$

where

$$\Delta \equiv \left(1 - c_H\right)\left(1 - k + \left(kc_L - c_H\right)\right) + k\left(1 - c_L\right)\left(c_H - c_L\right) > 0$$

Note that $\Delta > 0$ because we have assumed condition (76). The equilibrium number of firms is

$$n^* = \sqrt{\frac{S_H N \bar{\theta} \Delta}{\left(1 - k\right) F}} - 1$$

A doubling of the market size will increase the number of firms, but by a smaller proportion.

Proposition 3: *Assume* $F_L = F_H = F$ *,* $c_H > c_L$*,* $\frac{S_L}{S_H} < \frac{1-c_H}{1-c_L}$*, and the fixed cost is small relative to the market size* N*. Then*

(i) in equilibrium, each will produce both low and high quality products.

(ii) the long-run equilibrium number of firms is uniquely determined. An increase in the market size will lead to a larger number of firms.

(iii) an increase in the market size will reduce P_H*,* P_L*, and also the relative price* P_L/P_H

(iv) an increase in the market size will reduce the value share of the low quality products, $P_L X_L/(P_L X_L + P_H X_H)$*, but the ratio of outputs,* X_L/X_H*, is not affected by the market size.*

4.3 Discussions

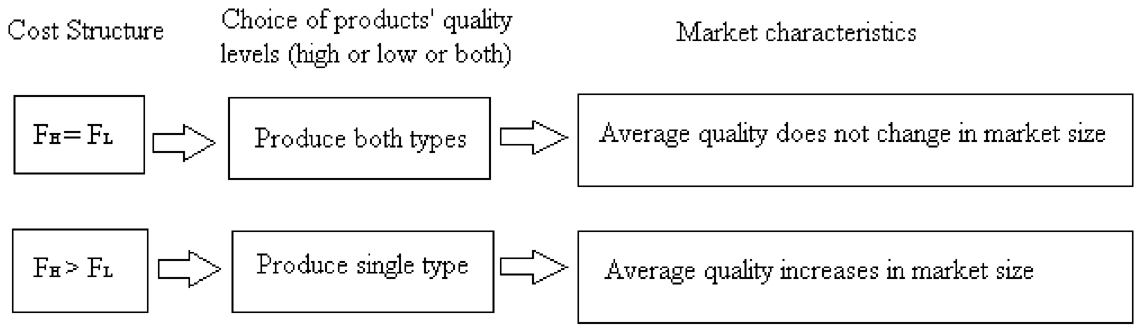
Our results show that, under free entry and exit, in a Cournot oligopoly that produces both high and low quality products, there are two main gains from an expansion of the market size. First, the prices fall, and thus more consumers are served. Second, the market share of the high quality product increases, which implies that on average, consumers have greater access to the higher quality product. We may say that the average quality rises. The mechanism differs, depending on the cost configuration. If the quality-adjusted fixed cost is higher for the high quality product ($f_H > f_L$), and firms must specialize in one of the products, then, in the case where the unit variable costs (adjusted for quality) are the same, $c_H = c_L$, when the market size expands, more firms will enter the high-quality market segment, driving down the price P_H, and the low-quality product price P_L also falls, but the ratio P_L/P_H is independent of the market size.

When $f_H > f_L$ but $c_H \leq c_L$, we show that even though any firm that has incurred F_H is able to produce both products, it will refrain from doing so. This is true even if the firm is making a short run output decision, and even if it is a monopoly (Lemma 4). The reason is that it is not worthwhile for a firm that is capable to produce both goods to produce the low quality product, at a high unit variable cost (adjusted for quality), $c_L \geq c_H$, to compete with its own high quality product. In other words the firm is avoiding the phenomenon called 'cannibalization: producing the low quality good will lower its price, reducing the demand for the high quality product. It follows that the assumption made in section 3 (that the high-quality firms produce only the high-quality product, when $c_L = c_H = c$) is in fact justified. Considering the case where $f_H > f_L$ and $c_H < c_L$, we show that under some additional restrictions, a long run equilibrium exists with two types of firms, each committed to specialize in one type of product. When the market size expands, the effects are an increase the number of high-quality firms, a decrease the number of low-quality firms, and a decrease the market share of the low quality product.

We also consider the case where the fixed costs are the same for all firms regardless of whether they produce the high quality product only, or the low quality product only, or both products. In this case we show that all firms will produce both products, provided that the quality adjusted unit variable cost of the high quality product is higher than that of the low quality one, i.e., $c_H > c_L$. Thus the cannibalization effect is not too strong to discourage the production of the low quality good in this case. We show that an increase in the market size will reduce P_H, P_L, and also reduce the relative price P_L/P_H, as well as the value share of the low quality products, $P_L X_L/(P_L X_L + P_H X_H)$, however the ratio of the two outputs, X_L/X_H, is unaffected.

The following diagram (diagram 3) summarizes the relationships among cost structure, firms' strategies, and market characteristics.

Diagram 3. Cost structure, firms' strategies, and the industrial structure



5 Some empirics

While this paper is primarily theoretical, in this section, we make a modest attempt to bring the theory to the data. One of our theoretical prediction is that the price level is lower for the larger market. Another result is that if the quality-adjusted unit variable cost is higher for the high quality product, then an increase in the size of the market will increase the market share of the high quality product. Using data covering China's exporting firms for the years 2002 to 2006, we test these predictions by exploring: (i) the relationship between the average quality offered to an export market and the size of that market, and (ii) the relationship between the price of a product offered by a single firm and the size of the market. The data sets contains a wide range of variables that are suitable for testing our hypotheses. These include the price of the products supplied by each firm in each export market, the volume of transaction, and the category of the products by the HS6 code. These data are obtained from the China's General Administration of Customs. In addition, we assemble the data that describe the macro characteristics of the destination countries using the databases of the World Bank and the CEPII. These characteristics include population, GDP per capita, GINI coefficient index, geographic distance between any two countries and the import tariffs imposed by the destination countries. The table below describes the features of the data set.

Table 1. Data Description

VARIABLES	Obs.	Mean	Std. Dev.	Min.	Max.
$ln(price)$	17,564,024	0.937	2.027	-10.528	17.910
$ln(quantity)$	17,564,024	7.799	2.813	0	23.213
$ln(price\_average)$	1,206,135	1.339	2.467	-9.861	17.281
$ln(quantity\_average)$	1,206,135	9.276	3.661	0	24.091
$ln(Tariff\_rate)$	654,765	0.0871	0.0928	0	2.785
$ln(populatoin)$	976	15.172	2.318	9.173	20.873
$ln(GDP\,per\,capita)$	893	8.998	1.268	6.362	11.683
$ln(Gini)$	294	3.641	0.257	2.785	4.171
$ln(distance)$	208	4.463	1.496	-0.00487	7.349

In our empirical specifications, we must modify the theory by adding the assumption that exporting to a market requires the firm to incur a set-up cost for that market, and this cost is increasing in the quality level of the products. We suppose that the export set-up cost is country-specific, which may seem a reasonable assumption if tastes vary across countries.[13] Furthermore, since there are no direct statistics on quality levels, we will use the average price, after controlling for the average quantity, as a measure of the average quality. A modest justification of this approach is as follows.

We consider a log form relationship between average private and average quality, based on the inverse demand functions for each type of product:

$$\overline{\ln P_j} = \overline{\ln S_j} + \ln\left[1 - f(\overline{X}_j)\right] + \ln\left(\overline{\theta}_j\right) + \text{constant }_j \tag{81}$$

where $\overline{\ln P_j}$ is the average of the logarithm of the prices of the products in market j, $\overline{X}_j$ is the average quantity of the products[14], $\overline{\ln S_j}$ is the average of the logarithm of the

[13] Di Comite et al. (2014) provide empirical support for this hypothesis.

[14] In the estimation, for simplicity we use the average value (in logarithm) to control this term.

quality of the products, which is expected to be an increasing function of the market size. The Appendix provides some justification for this formulation.

The actual test involves running the following regression model

$$\overline{\ln P_{jkt}} = \beta_1 \ln(size_{kt}) + \beta_2 \ln \overline{X_{jkt}} + Z'_{kt}\gamma + \eta_j + \zeta_t + \varepsilon_{jkt} \tag{82}$$

where $\overline{\ln P_{jkt}}$ is the average of log of price for variety j in country k in year t, $\ln(size_{kt})$ indexes the market size,$\ln \overline{X_{jkt}}$ is the log of average quantity of the product group j exported to market k in year t, Z_{kt} controls other characteristics of the destination markets, η_j is the industry fixed effect, and ζ_t is the time fixed effect. The estimation results (see the first and second columns of Table 2 below) show that the average price in each country-industrial sector is increasing in the market size. [15]

Next, we test the relationship between the price level offered by a single firm to a market and the size of that market. Using equations (36) and (37), we construct the log form relation between the price level offered by firm i to market j as follows:

$$\ln P_{ij} = \ln S_i + \frac{1}{2}\ln f_i - \frac{1}{2}\ln(1-c) - \frac{1}{2}\ln N_j \tag{83}$$

Then the regression model is specified as follows:

$$\ln P_{ijt} = \beta_1 \ln N_{jt} + Z'_{jt}\gamma + \eta_i + \zeta_t + \varepsilon_{ijt} \tag{84}$$

[15] Based on our theoretical settings, we use the population size as the measure of the market size to run our main regressions. We also check the robustness when using the real GDP as the indicator for market size and controlling for all the control variables. The estimation results are consistent with our main regressions. We also run the regressions using the log of average price as our dependent variable, i.e. $\ln \overline{P_{jkt}}$. The estimation results are consistent with our main regressions.

where $\ln P_{ijt}$ is the price level offered by firm i to market j at time t, $\ln N_{jt}$ is the market size of country j at time t, Z_{it} controls the macro characteristics of market j, η_i controls the firm-level fixed effects, and ζ_t controls the time fixed effects. The results (see the third and forth columns of Table 2 below) show that the firm-country-level price is decreasing in the market size.

Table 2. Product price and the market size

Dependent variables	Log of price Country-variety level		Log of price Firm-country-variety level	
ln(populatoin)	0.105***	0.138***	-0.00527***	-0.00499***
	(0.00278)	(0.00409)	(0.00153)	(0.00138)
ln(quantity)	-0.111***	-0.136***		
	(0.00229)	(0.00276)		
ln(distance)	-0.00253	-0.0285***	0.0303***	0.00764***
	(0.00242)	(0.00414)	(0.00177)	(0.00201)
ln(Tariff_rate)	-1.079***	-0.456***	-0.239***	-0.0509***
	(0.0288)	(0.0318)	(0.0218)	(0.0168)
ln(GDP per capita)		0.164***		0.0413***
		(0.00496)		(0.00298)
ln(Gini)		-0.319***		-0.0953***
		(0.0157)		(0.00799)
Observations	638,781	196,939	6,928,948	1,256,626
Adj R-squared	0.887	0.903	0.908	0.931
Variety-time FE	YES	YES		
Firm-variety-time FE			YES	YES

Standard errors are clustered at industrial level
*** p<0.01, ** p<0.05, * p<0.1

6 Concluding remarks

This paper studies the Cournot equilibrium of an oligopoly with multiple product quality under free entry and exit. The paper highlights the dependence of prices and average quality on the market size. Firms must incur a set-up cost. In the first stage of the game, each decides whether it will produce the low quality product or the high quality product. If the set-up cost is independent of product quality, firms may choose to supply both types of quality. We show that for the existence of an equilibrium where some firms specialize in the low quality product it is necessary that the set-up cost for the lower quality product, adjusted for quality level, is lower than that for the higher quality product. In the case where the unit variable costs are zero, or they are proportional to quality level (so that unit variable costs, adjusted for quality, are the same), we show that an increase in the market size leads to (i) an increase in the fraction of firms that specialize in the high quality products, (ii) the market shares (both in value terms and in terms of volume of output) of high quality producers increases, and (iii) the prices of both types of product decrease. In the case where higher quality requires higher set-up cost (per unit of quality) but lower unit variable cost (per unit of quality), subject to certain bounds on the difference in unit variable costs, we obtain the result that an increase in the market size decreases the number of low quality firms, increases the number of high quality firms, and decreases the prices of both products. In the special case where the set-up cost is independent of the quality level, we find that all firms will produce both quality levels. In this case, an increase in the market size will reduce the value share of low quality products, but will leave their volume share unchanged. Both the relative price P_L/P_H and the nominal prices P_H and P_L fall.

We carried out an empirical test of a version of the model, where set-up costs now refer to set-up costs to establish an export market, and they vary according to the quality of the product that the firm exports to that market. We show that the data supported the hypothesis that the private price for a single firm are lower and average qualities of the product are higher for bigger export markets.

Appendix

Proof of Lemma 4

The inverse demand functions are

$$P_H = \left(1 - \frac{1}{N} X_H\right) \overline{\theta} S_H - \frac{X_L}{N} \overline{\theta} S_L$$

$$P_L = \left(1 - \frac{1}{N} X_H - \frac{1}{N} X_L\right) \overline{\theta} S_L$$

Suppose firm i has incurred the fixed cost F_H in stage 1. Then in stage 2, firm i's optimization problem is to choose $x_H^i \geq 0$ and $x_L^i \geq 0$ to maximize

$$\left(P_H - C_H\right) x_H^i + \left(P_L - C_L\right) x_L^i$$

The F.O.C. with respect to x_H^i is

$$\left(P_H - C_H\right) + x_H^i \frac{\partial P_H}{\partial X_H} + x_L^i \frac{\partial P_L}{\partial X_H} \leq 0 \ (= 0 \text{ if } x_H^i > 0)$$

and the F.O.C. with respect to x_L^i is

$$x_H^i \frac{\partial P_H}{\partial X_L} + \left(P_L - C_L\right) + x_L^i \frac{\partial P_L}{\partial X_L} \leq 0 \ \ (= 0 \text{ if } x_L^i > 0)$$

Thus, if both x_H^i and x_L^i are positive, then we must have

$$\left(1 - \frac{1}{N} X_H\right) \overline{\theta} S_H - \frac{X_L}{N} \overline{\theta} S_L - C_H - x_H^i \frac{1}{N} \overline{\theta} S_H - x_L^i \frac{1}{N} \overline{\theta} S_L = 0$$

$$-x_i^H \frac{1}{N} \overline{\theta} S_L + \left(1 - \frac{1}{N} X_H - \frac{1}{N} X_L\right) \overline{\theta} S_L - C_L - x_L^i \frac{1}{N} \overline{\theta} S_L = 0$$

i.e., with $k = S_L/S_H < 1$,

$$\left(1 - \frac{1}{N}X_H\right) - \frac{X_L}{N}k - \frac{C_H}{\overline{\theta} S_H} - x_H^i \frac{1}{N} - x_L^i \frac{1}{N}k = 0$$

$$-x_i^H \frac{1}{N} + \left(1 - \frac{1}{N}X_H - \frac{1}{N}X_L\right) - \frac{C_L}{\overline{\theta} S_L} - x_L^i \frac{1}{N} = 0$$

i.e.,

$$\left(1 - \frac{1}{N}X_H\right) - \frac{X_L}{N}k - x_H^i \frac{1}{N} - x_L^i \frac{1}{N}k = \frac{C_H}{\overline{\theta} S_H} \tag{A.1}$$

$$-x_i^H \frac{1}{N} + \left(1 - \frac{1}{N}X_H - \frac{1}{N}X_L\right) - x_L^i \frac{1}{N} = \frac{C_L}{\overline{\theta} S_L} \tag{A.2}$$

Subtracting (A.2) from (A.1) and using eq. (39) we obtain

$$\frac{C_H}{\overline{\theta} S_H} - \frac{C_L}{\overline{\theta} S_L} = \frac{1}{N}(X_L + x_L^i)(1 - k) > 0$$

which cannot hold, since we have assumed that $\frac{C_H}{\overline{\theta} S_H} - \frac{C_L}{\overline{\theta} S_L} \leq 0$. **Q.E.D.**

Justification for equation (81)

Let us explain how we arrived at equation (81). Using the results in our theoretical part, we have the following inverse demand functions in log forms

$$\ln P_H = \ln S_H + \ln \overline{\theta} + \ln \left(1 - \frac{X_H^*}{N} - \frac{kX_L^*}{N}\right) \tag{A.3}$$

$$\ln P_L = \ln S_L + \ln \overline{\theta} + \ln \left(1 - \frac{X_H^*}{N} - \frac{X_L^*}{N}\right) \tag{A.4}$$

We define the average value of any variable y (be it price, quantity, or quality) by

$$\overline{y} = \frac{n_H}{n_H + n_L} y_H + \frac{n_L}{n_H + n_L} y_L$$

Then

$$\overline{X} = \frac{n_H}{n_H + n_L} X_H^* + \frac{n_L}{n_H + n_L} X_L^*$$

From (A.3) and (A.4)

$$n_H \ln P_H = n_H \ln S_H + n_H \ln \overline{\theta} + n_H \ln \left(1 - \frac{X_H^*}{N} - \frac{kX_L^*}{N}\right)$$

$$n_L \ln P_L = n_L \ln S_L + n_L \ln \overline{\theta} + n_L \ln \left(1 - \frac{X_H^*}{N} - \frac{X_L^*}{N}\right)$$

Adding these two equations, and dividing both sides by $n_H + n_L$, we get

$$\overline{\ln P} = \overline{\ln S} + \ln \overline{\theta} + \frac{n_H}{n_H + n_L} \ln \left(1 - \frac{X_H^*}{N} - \frac{kX_L^*}{N}\right) + \frac{n_L}{n_H + n_L} \ln \left(1 - \frac{X_H^*}{N} - \frac{X_L^*}{N}\right) \tag{A.5}$$

Then eq (A.5) can be rearranged as follows

$$\overline{\ln P} = \overline{\ln S} + \ln \left(\overline{\theta}\right) + \ln \left[1 - \left(\frac{X_H^*}{N} + \frac{X_L^*}{N}\right)\right] + R_H \ln v \tag{A.6}$$

where $R_H \equiv n_H/(n_H + n_L)$ and

$$v \equiv \frac{1 - \frac{X_H^*}{N} - \frac{kX_L^*}{N}}{1 - \frac{X_H^*}{N} - \frac{X_L^*}{N}} > 1$$

Recall that in we have found that there is a monotone increasing (but not linear) relationship between the equilibrium number of firms, $n_H + n_L$, and the market size. Therefore we can approximate the term $\left(\frac{X_H^*}{N} + \frac{X_L^*}{N}\right)$ in eq. (A.6) by some function $f(\overline{X})$.

References

[1] Benasi, C., Chirco, A. and C. Colombo (2006). "Vertical Differentiation and the Distribution of Income." *Bulletin of Economic Research* 58(4): 345-367.

[2] Berry, S., J. Levinsohn, and A. Pakes (1995), "Automobile Prices in Market Equilibrium." *Econometrica* 63(4): 841-90.

[3] Berry, S., and J. Waldfogel (2010). "Product Quality and Market Size" *The Journal of Industrial Economics* 58(1), pp. 1-31.

[4] Bonanno, G. (1986), "Vertical Differentiation with Cournot Competition." *Economic Notes* 15, pp. 68-91.

[5] Champsaur, P. and Rochet, J.-C. (1989), "Multiproduct Duopolists." *Econometrica* 57, pp. 533-557.

[6] Choi, J. C. and Shin, H. S. (1992). A Comment on a Model of Vertical Product Differentiation. *Journal of Industrial Economics,* 40, pp. 229-231.

[7] Di Comite, F., Thisse, J.-F. and Vandenbussche, H. (2014), "Verti-zontal Differentiation in Export Markets." *The Journal of Industrial Economics* 93(1), pp. 50-66.

[8] Edmond, C., V. Midrigan, and D.Y. Xu (2015), "Competition, Mark-ups, and the Gains from International Trade." *American Economic Review*, 105(10), pp 3183-3221.

[9] Fajgelbaum, P., G. M. Grossman, and E. Helpman (2011), "Income Distribution, Product Quality, and International Trade." *Journal of Political Economy*, 119(4): 721-765.

[10] Gabszewicz J. and Thisse, J.-F. (1979), "Price Competition, Quality, and Income Disparities." *Journal of Economic Theory*, 20, pp. 340-359.

[11] Gabszewicz, J. and Thisse, J.-F. (1980), "Entry (and Exit) in a Differentiated Industry," *Journal of Economic Theory*, 22, 327-338.

[12] Gal-Or, E. (1983), "Quality and Quantity Competition." *Bell Journal of Economics*, 14, 590-600.

[13] Gal-Or, E. (1985), "Differentiated Industries without Entry Barriers." *Journal of Economic Theory*, 37, pp. 310-339.

[14] Gal-Or, E. (1987), "Strategic and Non-Strategic Differentiation." *Canadian Journal of Economics* 20, pp. 340-356.

[15] Helpman, E. and Krugman, P. (1985), *Market Structure and Foreign Trade*, MIT Press.

[16] Ireland, N. (1987), *Product Differentiation and Non-Price Competition.* Basil Blackwell.

[17] Jappelli, T. and L. Pistaferri (2010), "Does Consumption Inequality Track Income Inequality in Italy?" *Review of Economic Dynamics* 13(1), 327-338.

[18] Johnson, J. P. and D. P. Myatt (2006), "Multiproduct Cournot Oligopoly." *The Rand Journal of Economics* 13(1), pp. 133-153.

[19] Krueger, D. and F. Perri (2006). Does Income Inequality Lead to Consumption Inequality? Evidence and Theory. *Review of Economic Studies* 73(1), pp 163-193.

[20] Krugman, P. R. (1980), "Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review,* 70 (5), 950-59.

[21] Long, N. V., H. Raff, and F. Stähler (2011), "Innovation and Trade with Heterogeneous Firms." *Journal of International Economics*, Vol 84(2), pp. 149-159.

[22] McFadden, D. (1978), "Modelling the Choice of Residential Location." In *Spatial Interaction Theory and Planning Models*, edited by A. Karlvist, L. Lundqvist, F. Snickars, and J. Weibull, pp 75-96 Amsterdam: North Holland.

[23] Melitz, M. J. (2003), "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity." *Econometrica,* Vol 71(6), pp. 1695-1725.

[24] Melitz, M. J. and D. Trefler (2012), "Gains from Trade when Firms Matter." *Journal of Economic Perspectives,* 26, pp. 91-118.

[25] Motta, M. (1992a), "Sunk Costs and Trade Liberalization." *Economic Journal* 102, pp. 578-587.

[26] Motta, M. (1992b), "Cooperative R&D and Vertical Product Differentiation." *International Journal of Industrial Organization*, 10, pp 643-661.

[27] Motta, M. (1993), "Endogenous Quality Choice: Price vs. Quantity Competition." *Journal of Industrial Economics,* 41 (2), pp. 113-131.

[28] Mussa, M. and Rosen, S. (1982), "Monopoly and Product Quality." *Journal of Economic Theory* 18, 301-317.

[29] Shaked, A. and Sutton, J. (1982), "Relaxing Price Competition through Product Differentiation." *Review of Economic Studies,* 49, pp. 3-14.

[30] Shaked, A. and Sutton, J. (1983), "Natural Oligopolies." *Econometrica* 51, 1469-1483.

[31] Shaked, A. and Sutton, J. (1984), “Natural Oligopolies and International Trade.” In H. Kierzkowski (ed.), *Monopolistic Competition and International Trade* (Clarendon, Oxford.)

[32] Shaked, A. and Sutton, J. (1987), “Product Differentiation and Industrial Structure.” *Journal of Industrial Economics* 36, 131-146.

[33] Singh, N. and Vives, X. (1984),“ Prices and Quality Competition in a Differentiated Duopoly.” *Rand Journal of Economics* 4, pp. 546-554.

[34] Tirole, J., (1988), *The Theory of Industrial Organization.* The MIT Press, Cambridge, MA.

[35] Vives, X. (1985), “On the Efficiency of Cournot and Bertrand Equilibria with Product Differentiation.” *Journal of Economic Theory*, 36, pp. 166-175.

[36] Yurko A. V. “Does Income Inequality Affect Market Outcomes in Vertically Differentiated Market?” *International Journal of Industrial Organization* 29(4), pp. 493-503.