Stackelberg Mixed Triopoly Games with State-Owned, Labour-Managed and Capitalist Firms

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3 June 2018

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Abstract

This paper investigates three sequential-move games with a capitalist firm, a labour-managed firm and a state-owned firm. The first game is as follows. In stage one, the capitalist firm chooses its output level. In stage two, the other firms choose their output levels simultaneously and independently. In stage three, the market opens and all firms sell their outputs. The structures of the second and third games are nearly identical and differ only in order in which the firms choose output levels in the first two stages. The paper discusses the equilibrium outcomes of the three sequential-move games.

Keywords: Stackelberg games; Capitalist firm; Labour-managed firm; State-owned firm

JEL classification: C72; D21; L30

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1. Introduction

The assumptions of Cournot (simultaneous move) and Stackelberg (sequential move) behaviour are widely used in analyzing oligopoly games. As it is very well known, the profit-maximizing Stackelberg leader can get a profit higher than the follower by committing to a high quantity. Many studies on mixed oligopoly with state-owned and capitalist firms discuss simultaneous-move games. However, only some studies investigate sequential-move games. For example, Poyago-Theotoky (2001) investigates the role that production subsidies play in a quantity-setting mixed market, and shows that the optimal subsidy and equilibrium output levels are identical irrespective of whether the state-owned firm moves simultaneously with the capitalist firms or the state-owned firm acts as a Stackelberg leader. Wang and Mukherjee (2012) compare economic welfare under different numbers of capitalist firms, and show that the entry of capitalist firms increases the state-owned firm's profit, industry profit and economic welfare at the expenses of the consumers. Tao, Zhu and Zou (2013) study and compare economic welfare in a sequential-move mixed duopoly when either the state-own firm or the capitalist firm acts as the leader. They demonstrate that the fact that which firm is the leader affects economic welfare and that whether firms compete in quantity or price also affects the optimal choice of market leader.

Furthermore, some studies on mixed oligopoly comprising labour-managed firms examine sequential-move games. For example, Okuguchi (1993) considers Bertrand and Cournot duopoly games where two labour-managed firms produce differentiated goods, and shows that followership is more advantageous than leadership in both Bertrand-Stackelberg and Cournot-Stackelberg duopoly games. Lambertini (1997) examines an endogenous-timing mixed duopoly model in which a capitalist firm competes with a labour-managed firm, and demonstrates that the quantity-setting game has a unique equilibrium with the capitalist firm in the leader's role and the labour-managed firm in the follower's role. Okuguchi and Serizawa (1998) examine Cournot and Stackelberg duopoly games in which two labour-managed firms coexist with each other, and show that the Cournot firm's output is larger than the Stackelberg follower's output, which in turn is larger than the leader's output.

On the other hand, there are few theoretical researches on sequential-move mixed oligopoly with state-owned and labour-managed firms.

We examine three sequential-move games with a capitalist firm, a labour-managed firm and a state-owned firm. The first game runs as follows. In the first stage, the capitalist firm chooses its output level. In the second stage, the other firms choose their output levels simultaneously and independently. The structures of the second and third games are nearly identical and differ only in order in which the firms choose output levels in the two stages. To the best of the author's knowledge, there is no previous work dealing with such economic situation. The paper discusses the equilibrium outcomes of the three sequential-move games.

The purpose of this study is to show the equilibrium outcomes of three sequential-move games comprising of a capitalist firm, a labour-managed firm and a state-owned firm.

The remainder of this paper proceeds as follows. In the next section, we describe the basic model. The third section derives the reaction functions for all firms. The fourth section discusses each equilibrium outcome of three sequential-move games. The last section concludes the paper. All propositions are proved in the appendix.

2. Basic setting

Let us consider an economy composed of one capitalist firm (firm C), one labour-managed firm (firm L) and one state-owned firm (firm S). Throughout this paper, subscripts C, L and S denote firm C, firm L and firm S, respectively. In addition, when *i*, *j* and *k* are used to refer to firms in an expression, they should be understood to represent C, L and S with $i \neq j \neq k$. There is no possibility of entry or exit. The triopolists produce perfectly substitutable goods. The inverse demand function is $P(Q)$, where $Q = q_\mathrm{C} + q_\mathrm{L} + q_\mathrm{S}$. We assume that $d^2P/dQ^2\,(= \partial^2 P/\partial q_\mathrm{C} = \partial^2 P/\partial q_\mathrm{L} = \partial^2 P/\partial q_\mathrm{S}) \geq 0$ and $dP/dQ + q_i\, d^2P/dQ^2 < 0$. This assumption allows a linear inverse demand function which is adopted in many studies of mixed oligopoly (e.g., see Stewart, 1991; Delbono and Rossini, 1992; Lambertini, 1997; Lambertini and Rossini, 1998; Ohnishi, 2009).

We consider the following timing. In stage one, firm *i* chooses its output level. In stage two, firm *j* and firm *k* choose their output levels simultaneously and independently. In stage three, the market opens and all firms sell their outputs.

Each firm's profit π_i is

$$\pi_i(q_\mathrm{C}, q_\mathrm{L}, q_\mathrm{S}) = P(Q)q_i - w(q_i) - r(q_i) - f, \tag{1}$$

where w represents the labour cost function, r is the capital cost function, and $f > 0$ is the fixed cost. We assume $dw/dq_i > 0$, $d^2w/dq_i^2 > 0$, $dr/dq_i > 0$ and $d^2r/dq_i^2 > 0$.[1] Firm C chooses

[1] We assume that the triopologists face the same cost function and the marginal cost of production is increasing. If the marginal cost of production is constant or decreasing, then firm S will produce where price is equal to marginal cost of production, and will enjoy monopoly. Therefore, this assumption is adopted in many papers studying mixed oligopoly markets (e.g. Harris and Wiens, 1980; Delbono and

$q_C \geq 0$ in order to maximize its own profit.

Economic welfare W is

$$W(q_C, q_L, q_S) = \int_0^Q P(x)dx - w(q_S) - r(q_S) - w(q_C) - r(q_C) - w(q_L) - r(q_L) - 3f. \tag{2}$$

Firm S chooses $q_S \geq 0$ in order to maximize economic welfare.

Firm L's income per worker φ_L is

$$\varphi_L(q_C, q_L, q_S) = \frac{P(Q)q_L - r(q_L) - f}{l(q_L)}, \tag{3}$$

where l represents the labour input function. We assume that $dl/dq_i > 0$ and $d^2l/dq_i^2 > 0$. Firm L sets $q_L \geq 0$ in order to maximize income per worker.

Throughout this study, we adopt subgame perfection as our solution concept. Since the inverse demand function is defined only for non-negative outputs, it is ensured that all outputs obtained in equilibrium are non-negative.

3. Reaction functions

First, we consider firm C's best response, which is defined by

$$R_C(q_L, q_S) = \arg\max_{q_C \geq 0} \left[P(Q)q_C - w(q_C) - r(q_C) - f\right]. \tag{4}$$

Firm C aims to maximize its profit with respect to q_C, given q_L and q_S. The equilibrium outcome needs to satisfy the following conditions: The first-order condition for (4) is

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Rossini, 1992; Delbono and Scarpa, 1995; Poyago-Theotoky, 1998; Bárcena-Ruiz and Garzón, 2003; Ohnishi, 2008; Wang and Wang, 2009).

$$q_{\mathrm{C}} \frac{dP}{dQ} + P - \frac{dw}{dq_{\mathrm{C}}} - \frac{dr}{dq_{\mathrm{C}}} = 0, \tag{5}$$

and the second-order condition for (4) is

$$q_{\mathrm{C}} \frac{d^2 P}{dQ^2} + 2\frac{dP}{dQ} - \frac{d^2 w}{dq_{\mathrm{C}}^2} - \frac{d^2 r}{dq_{\mathrm{C}}^2} < 0. \tag{6}$$

Moreover, we obtain

$$\frac{\partial R_{\mathrm{C}}(q_{\mathrm{L}}, q_{\mathrm{S}})}{\partial q_{\mathrm{L}}} = \frac{\partial R_{\mathrm{C}}(q_{\mathrm{L}}, q_{\mathrm{S}})}{\partial q_{\mathrm{S}}} = -\frac{dP/dQ + q_{\mathrm{C}}\, d^2 P/dQ^2}{2\,dP/dQ + q_{\mathrm{C}}\, d^2 P/dQ^2 - d^2 w/dq_{\mathrm{C}}^2 - d^2 r/dq_{\mathrm{C}}^2}, \tag{7}$$

where $dP/dQ + q_{\mathrm{C}}\, d^2 P/dQ^2 < 0$.

We can now present the following lemma.

Lemma 1: Under quantity competition, firm C's reaction functions are downward sloping.

Second, we consider firm L's best response, which is defined by

$$R_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{S}}) = \arg\max_{q_{\mathrm{L}} \geq 0} \left[\frac{P(Q) q_{\mathrm{L}} - r(q_{\mathrm{L}}) - f}{l(q_{\mathrm{L}})} \right]. \tag{8}$$

Firm L seeks to maximize its income per worker with respect to q_{L}, given q_{C} and q_{S}. The equilibrium solution satisfies the following conditions: The first-order condition for (8) is

$$\left(q_{\mathrm{L}} \frac{dP}{dQ} + P - \frac{dr}{dq_{\mathrm{L}}} \right) l - \left(P q_{\mathrm{L}} - r - f \right) \frac{dl}{dq_{\mathrm{L}}} = 0, \tag{9}$$

and the second-order condition for (8) is

$$\left(q_{\mathrm{L}} \frac{d^2 P}{dQ^2} + 2\frac{dP}{dQ} - \frac{d^2 r}{dq_{\mathrm{L}}^2} \right) l - \left(P q_{\mathrm{L}} - r - f \right) \frac{d^2 l}{dq_{\mathrm{L}}^2} < 0. \tag{10}$$

Moreover, we obtain

$$\frac{\partial R_{\mathrm{L}}(q_{\mathrm{C}},q_{\mathrm{S}})}{\partial q_{\mathrm{C}}}=\frac{\partial R_{\mathrm{L}}(q_{\mathrm{C}},q_{\mathrm{S}})}{\partial q_{\mathrm{S}}}=-\frac{q_{\mathrm{L}} l\, d^2P/dQ^2+(l-q_{\mathrm{L}}\, dl/dq_{\mathrm{L}})\, dP/dQ}{\left(q_{\mathrm{L}}\, d^2P/dQ^2+2\, dP/dQ-d^2r/dq_{\mathrm{L}}^2\right)l-\left(Pq_{\mathrm{L}}-r-f\right)d^2l/dq_{\mathrm{L}}^2}. \quad (11)$$

Since $d^2l/dq_{\mathrm{L}}^2>0$, $l-q_{\mathrm{L}}\, dl/dq_{\mathrm{L}}<0$, and thus $q_{\mathrm{L}} l\, d^2P/dQ^2+\left(l-q_{\mathrm{L}}\, dl/dq_{\mathrm{L}}\right)dP/dQ>0$.

We now state the following lemma.

Lemma 2: Under quantity competition, firm L's reaction functions are upward sloping.

Third, we consider firm S's best response, which is defined by

$$R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}})=\arg\max_{q_{\mathrm{S}}\geq 0}\left[\int_0^Q P(x)dx-w(q_{\mathrm{S}})-r(q_{\mathrm{S}})-w(q_{\mathrm{L}})-r(q_{\mathrm{L}})-w(q_{\mathrm{C}})-r(q_{\mathrm{C}})-3f\right]. \quad (12)$$

Firm S attempts to maximize economic welfare with respect to q_{S}, given q_{C} and q_{L}. The equilibrium needs to satisfy the following conditions: The first-order condition for (12) is

$$P-\frac{dw}{dq_{\mathrm{S}}}-\frac{dr}{dq_{\mathrm{S}}}=0, \quad (13)$$

and the second-order condition for (12) is

$$\frac{dP}{dQ}-\frac{d^2w}{dq_{\mathrm{S}}^2}-\frac{d^2r}{dq_{\mathrm{S}}^2}<0. \quad (14)$$

Moreover, we have

$$\frac{\partial R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}})}{\partial q_{\mathrm{C}}}=\frac{\partial R_{\mathrm{S}}(q_{\mathrm{C}},q_{\mathrm{L}})}{\partial q_{\mathrm{L}}}=-\frac{dP/dQ}{dP/dQ-d^2w/d\,q_{\mathrm{S}}^2-d^2r/d\,q_{\mathrm{S}}^2}. \quad (15)$$

We present the following lemma.

Lemma 3: Under quantity competition, firm S's reaction functions are downward sloping.

Finally, we make the following assumption: $R_{\mathrm{C}}(q_{\mathrm{L}}, q_{\mathrm{S}})$, $R_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{S}})$ and $R_{\mathrm{S}}(q_{\mathrm{C}}, q_{\mathrm{L}})$ intersect at one point. Note that this assumption eliminates the case where only one firm is relatively large or small compared to the other firms and ensures the existence and uniqueness of a Cournot equilibrium.

4. Results

In this section, we will consider the following three games.

Game 1: Firm C is the leader.

Game 2: Firm L is the leader.

Game 3: Firm S is the leader.

We will discuss these games in order.

Game 1

We consider Stackelberg mixed triopoly competition in which firm C is the leader. Firm C chooses q_{C}, and after observing q_{C}, firm L and firm S choose q_{L} and q_{S}, respectively. When firm C is the Stackelberg leader, it chooses q_{C} in order to maximize $\pi_{\mathrm{C}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}))$, and firm L and firm S act as followers and maximize $\varphi_{\mathrm{L}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}})$ and $W(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}})$, respectively.

We now present the following proposition.

Proposition 1: In the equilibrium of Stackelberg mixed triopoly competition with firm C as leader, the following are obtained:

(i) $\pi_C(q_C, R_L(q_C), R_S(q_C)) \geq \pi_C(q_C, q_L, q_S)$;

(ii) $\varphi_L(q_C, R_L(q_C), R_S(q_C) \gtreqqless \varphi_L(q_C, q_L, q_S)$;

(iii) $W(q_C, R_L(q_C), R_S(q_C) \gtreqqless W(q_C, q_L, q_S)$.

Proposition 1 means that Game 1 may or may not be profitable for the followers. We explain the intuition behind this proposition using Figures 1 and 2. In Figure 1, the horizontal axis represents firm C's output, the vertical axis represents firm L's output, R_i is firm *i*'s reaction curve, π_C^* is an iso-profit curve, and φ_L^* is an iso-income-per-worker curve. In Figure 2, the horizontal axis denotes firm C's output, the vertical axis denotes firm S's output, and W^* is an iso-welfare curve. For intuitive explanation, the figures are drawn very simply. As is shown in Figure 1, if firm C is the leader and firm L is the follower, then firm C's optimal output is smaller than the Cournot output. On the other hand, if firm C is the leader and firm S is the follower, then firm C's optimal output is larger than the Cournot output (Figure 2). In consideration of these situations, firm C chooses an output that can achieve the highest level of profit as the leader of the market. Therefore, if firm C decreases its output, firm L's income per worker increases, while if firm C increases its output, firm L's income per worker decreases. On the other hand, if firm C decreases its output, economic welfare also decreases, while if firm C increases its output, economic welfare also increases.

Game 2

We consider Stackelberg competition in which firm L becomes the leader. Firm L chooses q_L, and after observing q_L, firm C and firm S choose q_C and q_S, respectively. Therefore, firm L chooses

q_{L} to maximize $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}}))$, and firm C and firm S act as followers and maximize $\pi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}}))$ and $W(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}}))$, respectively.

We now present the following proposition.

Proposition 2: In the equilibrium of Stackelberg mixed triopoly competition with firm L as leader, the following are obtained:

(i) $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) > \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$;

(ii) $\pi_{\mathrm{C}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) < \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$;

(iii) $W(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) > W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$.

Proposition 2 says that Game 2 can be profitable for the leader and the welfare-maximizing follower. We explain the intuition behind this proposition using Figures 1 and 3. In Figure 3, the horizontal axis denotes firm S's output, and the vertical axis denotes firm L's output. As is shown in Figure 1 (resp. Figure 3), if firm L is the leader and firm C (resp. firm S) is the follower, then firm L's optimal output is larger than the Cournot output (Figures 1 and 3). Firm L chooses an output that can achieve the highest level of income per worker. Therefore, since firm L increases its output, firm C's profit decreases, while economic welfare improves.

Game 3

When firm S is the Stackelberg leader, it chooses q_{S} that maximizes $W(R_{\mathrm{C}}(q_{\mathrm{S}}), R_{\mathrm{L}}(q_{\mathrm{S}}), q_{\mathrm{S}})$, and firm C and firm L maximize $\pi_{\mathrm{C}}(R_{\mathrm{C}}(q_{\mathrm{S}}), R_{\mathrm{L}}(q_{\mathrm{S}}), q_{\mathrm{S}})$ and $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{S}}), R_{\mathrm{L}}(q_{\mathrm{S}}), q_{\mathrm{S}})$, respectively.

We now present the following proposition.

Proposition 3: In the equilibrium of Stackelberg mixed triopoly competition with firm S as leader, the following are obtained:

(i) $W(R_{\mathrm{C}}(q_{\mathrm{S}}), R_{\mathrm{L}}(q_{\mathrm{S}}), q_{\mathrm{S}}) \geq W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$;

(ii) $\pi_{\mathrm{C}}(R_{\mathrm{C}}(q_{\mathrm{S}}), R_{\mathrm{L}}(q_{\mathrm{S}}), q_{\mathrm{S}}) \gtreqless \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$;

(iii) $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{S}}), R_{\mathrm{L}}(q_{\mathrm{S}}), q_{\mathrm{S}}) \gtreqless \varphi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$.

Proposition 3 says that Game 3 may or may not be profitable for the followers. The intuition behind this proposition is as follows. If firm S is the leader and firm C is the follower, then firm S's optimal output is smaller than the Cournot output (Figure 2). On the other hand, if firm S is the leader and firm L is the follower, then firm S's optimal output is larger than the Cournot output (Figure 3). In consideration of these situations, firm S chooses an output that maximizes economic welfare. Therefore, if firm S decreases its output, firm C's profit and firm L's income per worker increase, while if firm S increases its output, firm C's profit and firm L's income per worker decrease.

5. Conclusion

We have discussed the equilibrium outcomes of three sequential-move games comprising a capitalist firm, a labour-managed firm and a state-owned firm. We have found that if the labour-managed firm is the leader, then at equilibrium the labour-managed firm and the state-owned firm can get more than in the Cournot game. However, we cannot have gotten such a clear result for the other games.

Appendix

Proof of Proposition 1

(i) The leader (firm C) chooses q_{C} in order to maximize $\pi_{\mathrm{C}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}))$ and it can choose $q_{\mathrm{C}} = q_{\mathrm{C}}^{N}$. Here the superscript '*N*' denotes the equilibrium outcome of the Cournot-Nash game. Therefore, we obtain $\pi_{\mathrm{C}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}})) = \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$.

When firm C is the leader, it maximizes $\pi_{\mathrm{C}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}))$ with respect to q_{C}. The first-order condition for profit maximization is

$$q_{\mathrm{C}} \frac{dP}{dQ} + P - \frac{dw}{dq_{\mathrm{C}}} - \frac{dr}{dq_{\mathrm{C}}} + q_{\mathrm{C}} \frac{dP}{dQ} \frac{\partial R_{\mathrm{L}}}{\partial q_{\mathrm{C}}} + q_{\mathrm{C}} \frac{dP}{dQ} \frac{\partial R_{\mathrm{S}}}{\partial q_{\mathrm{C}}} = 0. \tag{16}$$

Here $dP/dQ < 0$, $\partial R_{\mathrm{L}}/\partial q_{\mathrm{C}} > 0$ (Lemma 2), and $\partial R_{\mathrm{S}}/\partial q_{\mathrm{C}} < 0$ (Lemma 3). If $q_{\mathrm{C}}\left(dP/dQ\right)\left(\partial R_{\mathrm{L}}/\partial q_{\mathrm{C}}\right) + q_{\mathrm{C}}\left(dP/dQ\right)\left(\partial R_{\mathrm{S}}/\partial q_{\mathrm{C}}\right) < 0$, then $q_{\mathrm{C}}\left(dP/dQ\right) + P - dw/dq_{\mathrm{C}} - dr/dq_{\mathrm{C}} > 0$, and therefore firm C maximizes its profit by choosing $q_{\mathrm{C}} < q_{\mathrm{C}}^{N}$. On the other hand, if $q_{\mathrm{C}}\left(dP/dQ\right)\left(\partial R_{\mathrm{L}}/\partial q_{\mathrm{C}}\right) + q_{\mathrm{C}}\left(dP/dQ\right)\left(\partial R_{\mathrm{S}}/\partial q_{\mathrm{C}}\right) > 0$, then $q_{\mathrm{C}}\left(dP/dQ\right) + P - dw/dq_{\mathrm{C}} - dr/dq_{\mathrm{C}} < 0$, and therefore firm C chooses $q_{\mathrm{C}} > q_{\mathrm{C}}^{N}$.

(ii) If $q_{\mathrm{C}}^{CL} = q_{\mathrm{C}}^{N}$, then $\varphi_{\mathrm{L}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}) = \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Here the superscript '*CL*' denotes the equilibrium outcome of the game where firm C is the leader. Since $\partial \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})/\partial q_{\mathrm{C}} = q_{\mathrm{L}}\, dP/dQ < 0$, if $q_{\mathrm{C}}^{CL} > q_{\mathrm{C}}^{N}$, then $\varphi_{\mathrm{L}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}) < \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$, while if $q_{\mathrm{C}}^{CL} < q_{\mathrm{C}}^{N}$, then $\varphi_{\mathrm{L}}(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}) > \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$.

(iii) If $q_{\mathrm{C}}^{CL} = q_{\mathrm{C}}^{N}$, then $W(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}) = W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Since $\partial W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})/\partial q_{\mathrm{C}} = P - dw/dq_{\mathrm{C}} - dr/dq_{\mathrm{C}} > 0$, if $q_{\mathrm{C}}^{CL} > q_{\mathrm{C}}^{N}$, then $W(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}) > W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$, while if $q_{\mathrm{C}}^{CL} < q_{\mathrm{C}}^{N}$, then $W(q_{\mathrm{C}}, R_{\mathrm{L}}(q_{\mathrm{C}}), R_{\mathrm{S}}(q_{\mathrm{C}}) < W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Q.E.D.

Proof of Proposition 2

(i) Since the leader (firm L) maximizes $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}}))$ and it can choose $q_{\mathrm{L}} = q_{\mathrm{L}}^{N}$, we obtain $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) \geq \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. We show that $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) \neq \varphi_{\mathrm{L}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$ by showing that $q_{\mathrm{L}}^{LL} \neq q_{\mathrm{L}}^{N}$. Here the superscript '*LL*' denotes the equilibrium outcome of the game where firm L is the leader. If firm L is the leader, then it maximizes $\varphi_{\mathrm{L}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}}))$ with respect to q_{L}. The first-order condition is

$$\left(q_{\mathrm{L}}\frac{dP}{dQ} + P - \frac{dr}{dq_{\mathrm{L}}}\right)l - \left(Pq_{\mathrm{L}} - r - f\right)\frac{dl}{dq_{\mathrm{L}}} + q_{\mathrm{L}}\frac{dP}{dQ}\frac{\partial R_{\mathrm{C}}}{\partial q_{\mathrm{L}}} + q_{\mathrm{L}}\frac{dP}{dQ}\frac{\partial R_{\mathrm{S}}}{\partial q_{\mathrm{L}}} = 0. \tag{17}$$

Here $dP/dQ < 0$, $\partial R_{\mathrm{C}}/\partial q_{\mathrm{L}} < 0$ (Lemma 1) and $\partial R_{\mathrm{S}}/\partial q_{\mathrm{L}} < 0$ (Lemma 3). To satisfy (17), $\left[q_{\mathrm{L}}\left(dP/dQ\right) + P - dr/dq_{\mathrm{L}}\right]l - \left(Pq_{\mathrm{L}} - r - f\right)dl/dq_{\mathrm{L}}$ needs to be negative, and therefore $q_{\mathrm{L}}^{LL} > q_{\mathrm{L}}^{N}$.

(ii) When firm L is the Stackelberg leader, then it increases q_{L} (Proposition 2 (i)). Since $\partial \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}}) / \partial q_{\mathrm{L}} = q_{\mathrm{C}}\, dP/dQ < 0$, increasing q_{L} decreases π_{C} given q_{C} and q_{S}, and thus $\pi_{\mathrm{C}}(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) < \pi_{\mathrm{C}}(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$.

(iii) When firm L is the leader, it increases q_{L} (Proposition 2 (i)). Since $\partial W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})/\partial q_{\mathrm{L}} = P - dw/dq_{\mathrm{L}} - dr/dq_{\mathrm{L}} > 0$, increasing q_{L} improves economic welfare given q_{C} and q_{S}, and thus $W(R_{\mathrm{C}}(q_{\mathrm{L}}), q_{\mathrm{L}}, R_{\mathrm{S}}(q_{\mathrm{L}})) > W(q_{\mathrm{C}}, q_{\mathrm{L}}, q_{\mathrm{S}})$. Q.E.D.

Proof of Proposition 3

(i) The leader (firm S) chooses q_S in order to maximize economic welfare $W(R_C(q_S), R_L(q_S), q_S)$ and it can choose $q_S = q_S^N$. Therefore, we obtain $W(R_C(q_S), R_L(q_S), q_S) = W(q_C, q_L, q_S)$.

When firm S is the leader, it maximizes economic welfare $W(R_C(q_S), R_L(q_S), q_S)$ with respect to q_S. The first-order condition for welfare maximization is

$$P - \frac{dw}{dq_S} - \frac{dr}{dq_S} + q_S \frac{dP}{dQ}\frac{\partial R_C}{\partial q_S} + q_S \frac{dP}{dQ}\frac{\partial R_L}{\partial q_S} = 0. \qquad (18)$$

Here $dP/dQ < 0$, $\partial R_C/\partial q_S > 0$ (Lemma 1), and $\partial R_L/\partial q_S > 0$ (Lemma 2). If $q_S(dP/dQ)(\partial R_C/\partial q_S) + q_S(dP/dQ)(\partial R_L/\partial q_S) < 0$, then $P - dw/dq_S - dr/dq_S > 0$, and therefore firm S maximizes economic welfare by choosing $q_S < q_S^N$. On the other hand, if $q_S(dP/dQ)(\partial R_C/\partial q_S) + q_S(dP/dQ)(\partial R_L/\partial q_S) > 0$, then $P - dw/dq_S - dr/dq_S < 0$, and therefore firm S chooses $q_S > q_S^N$.

(ii) If $q_S^{SL} = q_S^N$, then $\pi_C(R_C(q_S), R_L(q_S), q_S) = \pi_C(q_C, q_L, q_S)$. Here the superscript '*SL*' denotes the equilibrium outcome of the game where firm S is the Stackelberg leader. Since $\partial\pi_C(q_C, q_L, q_S)/\partial q_S = q_C\, dP/dQ < 0$, if $q_S^{SL} > q_S^N$, then $\pi_C(R_C(q_S), R_L(q_S), q_S) < \pi_C(q_C, q_L, q_S)$, while if $q_S^{SL} < q_S^N$, then $\pi_C(R_C(q_S), R_L(q_S), q_S) > \pi_C(q_C, q_L, q_S)$.

(iii) If $q_S^{SL} = q_S^N$, then $\varphi_L(R_C(q_S), R_L(q_S), q_S) = \varphi_L(q_C, q_L, q_S)$. Since $\partial\varphi_L(q_C, q_L, q_S)/\partial q_S = q_L\, dP/dQ < 0$, if $q_S^{SL} > q_S^N$, then $\varphi_L(R_C(q_S), R_L(q_S), q_S) < \varphi_L(q_C, q_L, q_S)$, while if $q_S^{SL} < q_S^N$, then $\varphi_L(R_C(q_S), R_L(q_S), q_S) > \varphi_L(q_C, q_L, q_S)$. Q.E.D.

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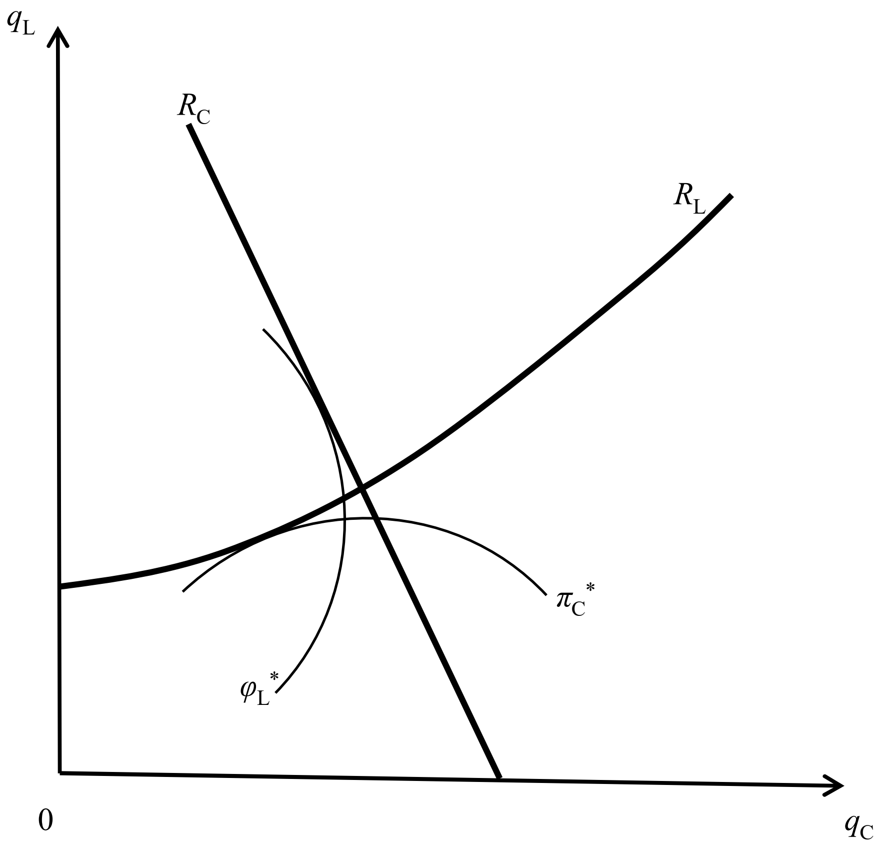

Figure 1: The capitalist firm and the labour-managed firm

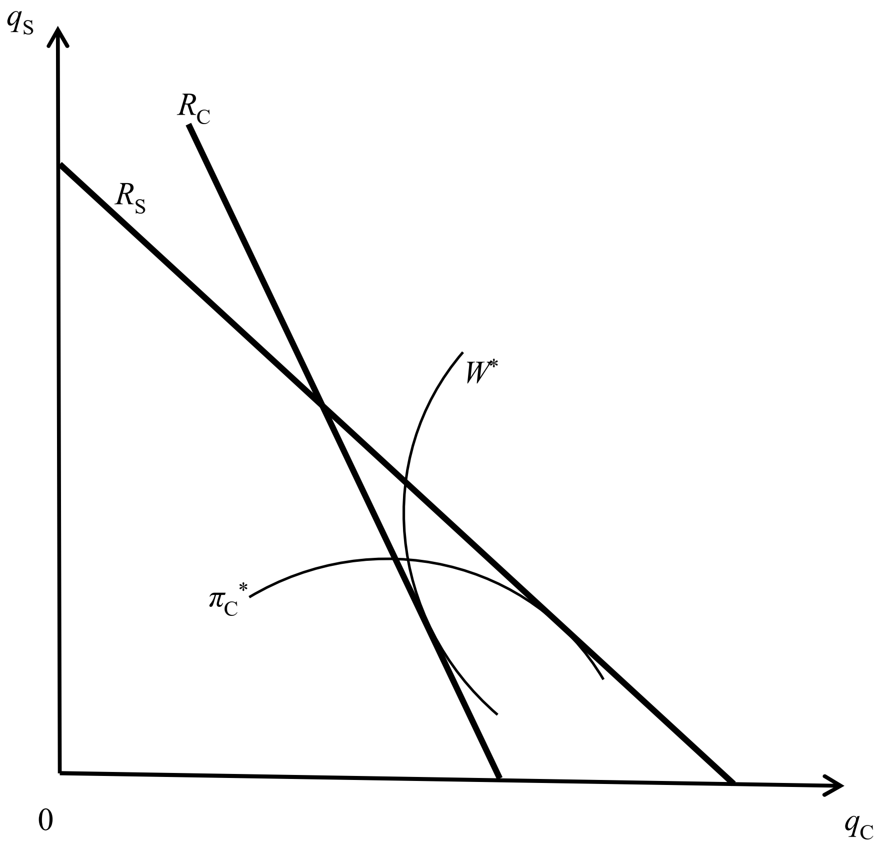

Figure 2: The capitalist firm and the state-owned firm

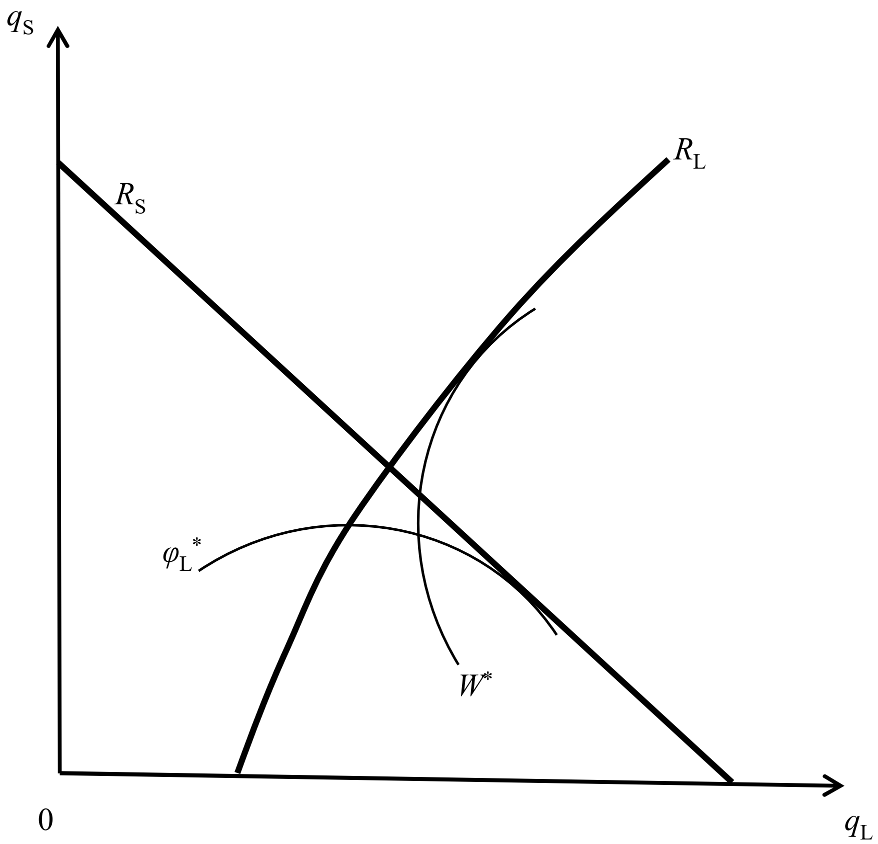

Figure 3: The labour-managed firm and the state-owned firm