Inventory Holding and a Mixed Duopoly with a Foreign Joint-Stock Firm

Ohnishi, Kazuhiro

29 April 2018

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Kazuhiro Ohnishi*

Institute for Basic Economic Science, Japan

Abstract

This paper investigates a mixed duopoly model in which there is a state-owned firm competing with a foreign joint-stock firm. The following situation is considered. In the first period, each firm non-cooperatively decides how many it sells in the current market. In addition, each firm can hold inventory for the second-period market. By holding large inventory, a firm may be able to commit to large sales in the next period. In the second period, each firm non-cooperatively chooses its second-period output. At the end of the second period, each firm sells its first-period inventory and its second-period output and holds no inventory. The paper traces out the firms' reaction functions in the mixed duopoly model.

Keywords: Inventory holding, state-owned firm, foreign joint-stock firm, reaction curves

JEL classification: C72, D43, F23, L30

* Email: ohnishi@e.people.or.jp

1. Introduction

Rotemberg and Saloner (1989) examine a two-period model in which inventory is used by a duopoly to deter deviations from an implicitly collusive arrangement, and establish that higher inventory allows firms to punish cheaters more strongly and can thus help to maintain collusion. Matsumura (1999) presents a Cournot duopoly model with finitely repeated competition, and establishes that two-period competition is insufficient to make private firms collusive. These studies investigate private duopoly models with inventory as a strategic device. Ohnishi (2011) presents a mixed duopoly model in which a welfare-maximizing public firm and a profit-maximizing private firm can use inventory investment as a strategic device, and demonstrates that the equilibrium coincides with the Stackelberg solution where the private firm is the leader.

The analysis of mixed oligopoly models that incorporate state-owned public firms has been performed by many researchers (e.g., see Delbono and Rossini, 1992; Nett, 1994; Willner, 1994; Fjell and Pal, 1996; George and La Manna, 1996; White, 1996; Mujumdar and Pal, 1998; Pal, 1998; Pal and White, 1998; Poyago-Theotoky, 1998; Nishimori and Ogawa, 2002; Bárcena-Ruiz and Garzón, 2003; Ohnishi, 2006, 2009; Bárcena-Ruiz, 2007; Fernández-Ruiz, 2009; Heywood and Ye, 2010; Wang and Lee, 2010; Pal and Saha, 2014; Cracau, 2015). However, these studies consider mixed market models in which state-owned firms compete with capitalist or labor-managed firms, and do not include joint-stock firms.

Only few studies consider joint-stock firms. For example, Meade (1972) shows the differences in incentives, short-run adjustment and so forth among entrepreneurial, co-operative and joint-stock firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a homogeneous final good with inputs of capital and labor, and examines the behavior of profit-maximizing, labor-managed and joint-stock firms. Ohnishi (2010b) presents the equilibrium solution of a quantity-setting model comprising a joint-stock firm and a profit-maximizing capitalist firm, and shows that introducing lifetime employment into the model of quantity-setting duopoly is beneficial only for the joint-stock firm. In addition, Ohnishi (2015b) investigates a mixed duopoly model where a joint-stock firm and a state-owned firm are allowed to offer lifetime employment as a strategic commitment, and presents the equilibrium solution of the mixed duopoly model.

We consider a two-period mixed market model in which a state-owned firm and a foreign joint-stock firm can hold inventory as a strategic device. The game runs as follows. In period one, each firm non-cooperatively decides how many it sells in the current market. In addition, each firm non-cooperatively decides the level of inventory it holds for the second-period market. In period two, each firm non-cooperatively chooses its output. At the end of period two, each firm sells its first-period inventory and its second-period output. We trace out the firms' reaction functions in the mixed duopoly model.

The balance of this paper is organized as follows. In the second section, we describe the model. The third section characterizes best replies for firms in the model. The fourth section presents the results of this study. The final section concludes the paper.

2. The model

We consider a mixed duopoly model in which there is a domestic state-owned firm (firm D) competing with a foreign joint-stock firm (firm F). In the balance of this paper, subscripts D and F refer to firms D and F, respectively, and superscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when i and j are used to refer to firms in an expression, they should be understood to denote D and F with $i \neq j$. The duopolists produce perfectly substitutable goods. The price of each period is determined by $P(S^t)$, where $S^t = s_{\mathrm{D}}^t + s_{\mathrm{F}}^t$ is the aggregate sales of each period. We assume that $P' < 0$ and $P'' \leq 0$.

The two periods of the game are as follows. In the first period, each firm non-cooperatively and simultaneously decides its first-period production $q_i^1 \in [0, \infty)$ and its first-period sales $s_i^1 \in [0, q_i^1]$. Firm *i*'s inventory I_i^1 becomes $q_i^1 - s_i^1$. In the second period, each firm non-cooperatively and simultaneously decides its second-period production $q_i^2 \in [0, \infty)$. At the end of the second period, each firm sells $s_i^2 = I_i^1 + q_i^2$ and holds no inventory. For notational simplicity, we consider the game without discounting.

Since $\sum_{t=1}^2 q_i^t = \sum_{t=1}^2 s_i^t$, economic welfare is

$$W = \sum_{t=1}^{2} \left[\int_0^{S^t} P(x)dx - m_{\mathrm{D}} q_{\mathrm{D}}^t - P(S^t) q_{\mathrm{F}}^t \right] = \sum_{t=1}^{2} \left[\int_0^{S^t} P(x)dx - m_{\mathrm{D}} s_{\mathrm{D}}^t - P(S^t) s_{\mathrm{F}}^t \right] \qquad (1)$$

where $m_{\mathrm{D}} \in (0, \infty)$ denotes firm D's constant marginal cost. Firm D seeks to maximize (1).

The demand and cost conditions that firms face remain unchanged over time. We define

$$w^t \equiv \int_0^{S^t} P(x)dx - m_{\mathrm{D}} s_{\mathrm{D}}^t - P(S^t) s_{\mathrm{F}}^t \tag{2}$$

In addition, since $\sum_{t=1}^{2} q_{\mathrm{F}}^t = \sum_{t=1}^{2} s_{\mathrm{F}}^t$, firm F's profit per capital is

$$\Phi_{\mathrm{F}} = \sum_{t=1}^{2} \left[\frac{P(S^t) s_{\mathrm{F}}^t - m_{\mathrm{F}} q_{\mathrm{F}}^t - f_{\mathrm{F}}^t}{k_{\mathrm{F}}(s_{\mathrm{F}}^t)} \right] = \sum_{t=1}^{2} \left[\frac{P(S^t) s_{\mathrm{F}}^t - m_{\mathrm{F}} s_{\mathrm{F}}^t - f_{\mathrm{F}}^t}{k_{\mathrm{F}}(s_{\mathrm{F}}^t)} \right] \tag{3}$$

where $m_{\mathrm{F}} \in (0, \infty)$ denotes firm F's constant marginal cost, $f_{\mathrm{F}} \in (0, \infty)$ is firm F's fixed cost, and $k_{\mathrm{F}}(s_{\mathrm{F}}^t)$ is firm F's capital input function. Firm F aims to maximize (3).

We assume that $k_{\mathrm{F}}(s_{\mathrm{F}}^t)$ is the function of s_{F}^t with $k_{\mathrm{F}}' > 0$ and $k_{\mathrm{F}}'' > 0$. This assumption means that the marginal quantity of capital used is increasing.

We also assume that firm D is less efficient than firm F, i.e. $m_{\mathrm{D}} > m_{\mathrm{F}}$. This assumption is justified in Gunderson (1979) and Nett (1993, 1994), and is often used in literature studying mixed oligopoly markets (e.g., see George and La Manna, 1996; Mujumdar and Pal, 1998; Pal, 1998; Nishimori and Ogawa, 2002; Matsumura, 2003; Ohnishi, 2006, 2015a; Fernández-Ruiz, 2009). If firm D is equally or more efficient than firm F, then firm D chooses q_{D}^t and s_{D}^t such that price equals marginal cost. Therefore, firm F has no incentive to operate in the market, and firm D acts as a monopoly.

We define

$$\phi_{\mathrm{F}}^t \equiv \frac{P(S^t) s_{\mathrm{F}}^t - m_{\mathrm{F}} s_{\mathrm{F}}^t - f_{\mathrm{F}}^t}{k_{\mathrm{F}}(s_{\mathrm{F}}^t)} \tag{4}$$

The solution concept of this model is subgame perfection. In the next section, we will give supplementary explanations of the model.

3. Supplementary explanations

First, we derive firm D's reaction functions from (2). In the first period, since there is no inventory available, firm D's reaction function is defined by

$$R_{\mathrm{D}}^{1}(s_{\mathrm{D}}^{1}) = \arg\max_{\{s_{\mathrm{D}}^{1} \geq 0\}} \left[\int_{0}^{S^{1}} P(x)dx - m_{\mathrm{D}} s_{\mathrm{D}}^{1} - P(S^{1}) s_{\mathrm{F}}^{1} \right] \tag{5}$$

In the second period, firm D's reaction function without inventory is defined by

$$R_{\mathrm{D}}^{2}(s_{\mathrm{D}}^{2}) = \arg\max_{\{s_{\mathrm{D}}^{2} \geq 0\}} \left[\int_{0}^{S^{2}} P(x)dx - m_{\mathrm{D}} s_{\mathrm{D}}^{2} - P(S^{2}) s_{\mathrm{F}}^{2} \right] \tag{6}$$

and therefore its best response is shown by

$$\overline{R}_{\mathrm{D}}^{2}(s_{\mathrm{F}}^{2}) = \begin{cases} R_{\mathrm{D}}^{2}(s_{\mathrm{F}}^{2}) & \text{if } s_{\mathrm{D}}^{2} > I_{\mathrm{D}}^{1} \\ I_{\mathrm{D}}^{1} & \text{if } s_{\mathrm{D}}^{2} = I_{\mathrm{D}}^{1} \end{cases} \tag{7}$$

Firm D maximizes economic welfare with respect to s_{D}^{t}, given s_{F}^{t}. The equilibrium solution needs to satisfy the following conditions: When inventory is zero, the first-order condition for firm D is

$$P - m_{\mathrm{D}} - P' s_{\mathrm{F}}^{t} = 0 \tag{8}$$

and the second-order condition is

$$P' - P'' s_{\mathrm{F}}^{t} < 0 \tag{9}$$

Therefore, we obtain

$$R_{\mathrm{D}}^{t\,\prime}(s_{\mathrm{F}}^{t}) = \frac{P'' s_{\mathrm{F}}^{t}}{P' - P'' s_{\mathrm{F}}^{t}} \tag{10}$$

In the first period, firm D's reaction function is upward sloping. In the second period, firm

D's best response also slopes upward for $s_{\mathrm{D}}^2 > I_{\mathrm{D}}^1$. This indicates that firm D treats s_{D}^t as strategic complements.[1]

Next, we derive firm F's reaction functions from (4). In period one, since there is no inventory available, firm F's reaction function is defined by

$$R_{\mathrm{F}}^1(s_{\mathrm{D}}^1) = \arg\max_{\{s_{\mathrm{F}}^1 \geq 0\}} \left[\frac{P(S^1)s_{\mathrm{F}}^1 - m_{\mathrm{F}}s_{\mathrm{F}}^1 - f_{\mathrm{F}}^1}{k_{\mathrm{F}}(s_{\mathrm{F}}^1)} \right] \tag{11}$$

In period two, firm F's reaction function without inventory is defined by

$$R_{\mathrm{F}}^2(s_{\mathrm{D}}^2) = \arg\max_{\{s_{\mathrm{F}}^2 \geq 0\}} \left[\frac{P(S^2)s_{\mathrm{F}}^2 - m_{\mathrm{F}}s_{\mathrm{F}}^2 - f_{\mathrm{F}}^2}{k_{\mathrm{F}}(s_{\mathrm{F}}^2)} \right] \tag{12}$$

and therefore its best response is shown by

$$\overline{R}_{\mathrm{F}}^2(s_{\mathrm{D}}^2) = \begin{cases} R_{\mathrm{F}}^2(s_{\mathrm{D}}^2) & \text{if } s_{\mathrm{F}}^2 > I_{\mathrm{F}}^1 \\ I_{\mathrm{F}}^1 & \text{if } s_{\mathrm{F}}^2 = I_{\mathrm{F}}^1 \end{cases} \tag{13}$$

Firm F maximizes profit per capital with respect to s_{F}^t, given s_{D}^t. When inventory is zero, the first-order condition for firm F is

$$(P's_{\mathrm{F}}^t + P - m_{\mathrm{F}})k_{\mathrm{F}} - (Ps_{\mathrm{F}}^t - m_{\mathrm{F}}s_{\mathrm{F}}^t - f_{\mathrm{F}}^t)k_{\mathrm{F}}' = 0 \tag{14}$$

and the second-order condition is

$$(P''s_{\mathrm{F}}^t + 2P')k_{\mathrm{F}} - (Ps_{\mathrm{F}}^t - m_{\mathrm{F}}s_{\mathrm{F}}^t - f_{\mathrm{F}}^t)k_{\mathrm{F}}'' < 0 \tag{15}$$

In addition, we obtain

$$R_{\mathrm{F}}^t{}'(s_{\mathrm{D}}^t) = -\frac{P''s_{\mathrm{F}}^t k_{\mathrm{F}} + P'(k_{\mathrm{F}} - s_{\mathrm{F}}^t k_{\mathrm{F}}')}{(P''s_{\mathrm{F}}^t + 2P')k_{\mathrm{F}} - (Ps_{\mathrm{F}}^t - m_{\mathrm{F}}s_{\mathrm{F}}^t - f_{\mathrm{F}}^t)k_{\mathrm{F}}''} \tag{16}$$

[1] The concept of strategic complements is due to Bulow, Geanakoplos, and Klemperer (1985).

Since $k''_{\mathrm{F}} > 0$, $k_{\mathrm{F}} - s^t_{\mathrm{F}} k'_{\mathrm{F}} < 0$, so that $P'' s^t_{\mathrm{F}} k_{\mathrm{F}} + P'(k_{\mathrm{F}} - s^t_{\mathrm{F}} k'_{\mathrm{F}})$ is positive. This means that firm F treats s^t_{F} as strategic complements.

4. Results

In this section, we trace out the firms' reaction curves in the model described in section 2. There is no inventory available in the first period, and further s^1_i does not affect s^2_i and s^2_j. Firm D's and firm F's reaction curves in the second period is decided by the level of I^1_i, and therefore we consider the second period.

We illustrate both firms' reaction curves by using Figures 1-12. For explanations, the figures are drawn simply. We discuss the following four cases.

Case 1: Neither firm holds inventory.

Case 2: Only firm D holds inventory.

Case 3: Only firm F holds inventory.

Case 4: Each firm holds inventory.

We discuss these cases in orders.

Case 1

Both firms' reaction curves are drawn in Figure 1, where R_i^2 denotes firm *i*'s second-period reaction curve with no inventory. Both firms' reaction curves are upward sloping. The equilibrium is decided in Cournot fashion, i.e., the intersection of firm D's and firm F's second-period reaction curves gives us the equilibrium of the game. The reaction curves cross twice as depicted in Figure 1. Only point N is a stable Cournot equilibrium, since in point N' firm F's second-period reaction curve crosses firm D's from above. In the remainder of this paper, we delete the superscript 2 for brevity's sake.

Case 2

This is the case in which only firm D holds inventory, and this case is illustrated in Figures 2-4. We consider the change of firm D's best response curve, which is drawn in Figure 2. We suppose that firm D maintains the inventory level of I_{D}^{1B} in period two. By holding inventory, firm D's best response changes to (7). Firm D's inventory holding creates a kink in its reaction curve at the level of I_{D}^{1B}. Therefore, firm D's reaction curve becomes the kinked bold broken lines.

From Figure 2, we see that the inventory level of I_{D}^{1B} changes the solution of the game. The intersection of the reaction curves is an equilibrium solution in the second period. That is, if firm D holds I_{D}^{1B}, then the solution occurs at A. We see that economic welfare is higher at A than at N', and A is a stable solution.

We consider the situation drawn in Figure 3. If firm D maintains the inventory level of

I_{D}^{1C} in the second period, then its inventory holding creates a kink in its reaction curve at the level of I_{D}^{1C}. The intersection of the reaction curves is an equilibrium solution in period two. That is, if firm D holds I_{D}^{1C}, then the solution becomes N'. It is easy to see that N' is an unstable equilibrium solution. If firm D maintains the inventory level of I_{D}^{1C}, then there is no stable solution.

We consider the situation drawn in Figure 4. If firm D holds I_{D}^{1E} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{D}^{1E}. In this figure, the reaction curves cross at two points. Both D and N are stable solutions. We see that economic welfare is higher at N than at D.

Case 3

This case is illustrated in Figures 5-7. Firstly, we consider the situation in Figure 5. We suppose that the firm F holds I_{F}^{1G} in the second period. By holding inventory, firm F's best response changes to (13). Firm F's inventory holding creates a kink in its reaction curve at the level of I_{F}^{1G}. Therefore, firm F's reaction curve becomes the kinked bold lines.

From Figure 5, we see that the inventory level of I_{F}^{1G} changes the solution of the game. If firm F holds I_{F}^{1G}, then the solution is at F. However, we see that firm F's profit per capacity is lower at F than at N'.

Secondly, we consider the situation drawn in Figure 6. If firm F maintains the inventory level of I_{F}^{1H} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{F}^{1H}. The intersection of the reaction curves is an equilibrium solution in period two. That is, if firm F maintains the inventory level of I_{F}^{1H}, then the solution becomes N'. In this figure, there is no stable solution.

Thirdly, we consider the situation in Figure 7. If firm F holds I_{F}^{1J} in period two, then its inventory holding creates a kink in its reaction curve at the level of I_{F}^{1J}. The best response curves cross at multiple points as in Figure 7.

Case 4

This case is illustrated in Figures 8-12. In this case, both firms use inventory holding as a strategic commitment device. Firstly, we consider the situation in Figure 8. Suppose that firm D maintains the inventory level of I_{D}^{1B} in period two. By holding inventory, firm D's best response changes to (7). Firm D's inventory holding creates a kink in its reaction curve at the level of I_{D}^{1B}. Therefore, firm D's reaction curve becomes the kinked bold broken lines. In addition, suppose that firm F holds I_{F}^{1G} in period two. By holding inventory, firm F's best response changes to (13). Firm F's inventory holding creates a kink in its reaction curve at the level of I_{F}^{1G}. Therefore, firm F's reaction curve becomes the kinked bold lines.

The solution is decided in Cournot fashion. From Figure 8, we see that inventory holding by each firm changes the solution of the game. Figure 8 indicates that there are three stable solutions.

Secondly, we consider the situation in Figure 9. If firm D maintains the inventory level of

I_{D}^{1C}, then its best response curve becomes the bold broken lines, and if firm F holds I_{F}^{1J}, then its best response curve becomes the bold lines. In Figure 9, both firms' reaction curves do not cross each other. That is, the obvious outcome is that there is no stable solution.

Thirdly, we consider the situation in Figure 10. When firm D holds I_{D}^{1E} in the second period, its quantity best response curve is kinked at the level of I_{D}^{1E}. In addition, inventory holding by firm F kinks its quantity best response curve. Therefore, firm D's best response is depicted as the thick broken lines, while firm F's best response curve is the thick lines. The firms' best response curves cross twice as in Figure 10. It is obvious that both N and D are stable solutions.

Fourthly, we consider the situation drawn in Figure 11. Inventory holding by each firm kinks its quantity best response curve. If firms D and F hold I_{D}^{1W} and I_{F}^{1T} respectively, then firm D's best response is depicted as the thick broken lines, and firm F's best response curve is the thick lines. In this figure, the firms' best response curves cross at multiple points. It is obvious that both N and M are stable solutions.

Fifthly, we consider the case drawn in Figure 12. We suppose that firms D and F hold I_{D}^{1W} and I_{F}^{1J}, respectively. Therefore, firm D's best response is depicted as the thick broken lines, while firm F's best response curve is the thick lines. The firms' best response curves cross four times as in Figure 12. We see that all these points are stable solutions.

5. Conclusion

We have considered a two-period mixed duopoly model in which there is a state-owned firm competing with a foreign joint-stock firm. Each firm is allowed to hold inventory as a strategic device. As a result, we have shown that there are multiple stable Cournot solutions in the international mixed duopoly model.

In the near future, we will extend our analysis by considering a mixed oligopoly model where a state-owned firm competes with both domestic and foreign joint-stock firms.

References

Artz, B., Heywood, J. S., & McGinty, M. (2009). The merger paradox in a mixed oligopoly. *Research in Economics, 63,* 1-10.

Bárcena-Ruiz, J. C. (2007). Endogenous timing in a mixed duopoly: Price competition. *Journal of Economics, 91,* 263-272.

Bárcena-Ruiz, J. C., & Garzón, M. B. (2003). Mixed duopoly, merger and multiproduct firms. *Journal of Economics / Zeitschrift für Nationalökonomie, 80,* 27-42.

Barcena-Ruiz, J. C., & Garzón, M. B. (2007). Capacity choice in a mixed duopoly under price competition. *Economics Bulletin, 12,* 1-7.

Bös, D. (1984). Income taxation, public sector pricing and redistribution. *Scandinavian Journal of Economics, 86,* 166-183.

Bös, D. (1986). *Public enterprise economics.* Amsterdam: North-Holland.

Bulow, J. I., Geanakoplos, J. D., & Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy, 93,* 488-511.

Chao, C. C., & Yu, E. S. H. (2006). Partial privatization, foreign competition, and optimal tariff. *Review of International Economics, 14,* 87-92.

Cracau, D. (2015). The effect of strategic firm objectives on competition. In: Ohnishi, K. (Ed.), *Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 1* (pp. 170-181). Sharjah, UAE: Bentham Science Publishers.

Cremer, H., & Cremer, J. (1992). Duopoly with employee-controlled and profit-maximizing firms: Bertrand vs Cournot competition. *Journal of Comparative Economics, 16,* 241-258.

Cremer, H., Marchand, M., & Thisse, J.-F. (1989). The public firm as an instrument for regulating an oligopolistic market. *Oxford Economic Papers, 41,* 283-301.

Cremer, H., Marchand, M., & Thisse, J.-F. (1991). Mixed oligopoly with differentiated products. *International Journal of Industrial Organization, 9,* 43-53.

Delbono, F., & Rossini, G. (1992). Competition policy vs horizontal merger with public, entrepreneurial, and labor-managed firms. *Journal of Comparative Economics, 16,* 226-240.

Delbono, F., & Scarpa, C. (1995). Upward-sloping reaction functions under quantity competition in mixed oligopolies. *Bulletin of Economic Research, 47,* 341-346.

Fernández-Ruiz, J. (2009). Managerial delegation in a mixed duopoly with a foreign competitor. *Economics Bulletin, 29,* 90-99.

Fjell, K., & Heywood, J. S. (2002). Public Stackelberg leadership in a mixed oligopoly with foreign firms. *Australian Economic Papers, 41,* 267-281.

Fjell, K., & Pal, D. (1996). A mixed oligopoly in the presence of foreign private firms. *Canadian Journal of Economics, 29,* 737-743.

George, K., & La Manna, M. (1996). Mixed duopoly, inefficiency, and public ownership. *Review of Industrial Organization, 11,* 853-860.

Gunderson, M. (1979). Earnings differentials between the public and private sectors. *Canadian Journal of Economics, 12,* 228-242.

Hey, J. D. (1981). A unified theory of the behaviour of profit-maximising, labour-managed and joint-stock firms operating under uncertainty. *Economic Journal, 91,* 364-374.

Heywood, J. S., & Ye, G. (2010). Optimal privatization in a mixed duopoly with consistent conjectures. *Journal of Economics, 101,* 231-246.

Kaplan, A. D. H., Dirlam, J. B., & Lanzillotti R. F. (1958). *Pricing in big business: A case approach.* Washington, D. C.: The Brookings Institution.

Lambertini, L., & Rossini, G. (1998). Capital commitment and Cournot competition with labour managed and profit-maximising firms. *Australian Economic Papers, 37,* 14-21.

Lanzillotti, R. F. (1958). Pricing objectives in large companies. *American Economic Review, 48,* 921-940.

Lin, M, H., & Ogawa, H. (2005). Cost reducing incentives in a mixed duopoly market. *Economics Bulletin, 12,* 1-6.

Lu, Y. (2007). Endogenous timing in a mixed oligopoly consisting of a single public firm and foreign competitors. *Economics Bulletin, 12,* 1-7

Lu, Y., & Poddar, S. (2007). Firm ownership, product differentiation and welfare. *The Manchester School, 75,* 210-217.

Lu, Y., & Poddar, S. (2009). Endogenous timing in a mixed duopoly and private duopoly – 'capacity-then-quantity' game: The linear demand case. *Australian Economic Papers, 48,* 138-150.

Matsumura, T. (1999). Cournot duopoly with multi-period competition: Inventory as a coordination device. *Australian Economic Papers, 38,* 189-202.

Matsumura, T. (2003). Stackelberg mixed duopoly with a foreign competitor. *Bulletin of Economic Research, 55,* 275-287.

Meade, J. E. (1972). The theory of labour-managed firms and of profit sharing. *Economic Journal, 82,* 402-428.

Merrill, W., & Schneider, N. (1966). Government firms in oligopoly industries: A short-run analysis. *Quarterly Journal of Economics, 80,* 400-412.

Mujumdar, S., & Pal, D. (1998). Effects of indirect taxation in a mixed oligopoly. *Economics Letters, 58,* 199-204.

Nett, L. (1991). R&D innovation in a mixed oligopolistic market. (Discussion Paper A-357). Bonn, Germany: University of Bonn.

Nett, L. (1993). Mixed oligopoly with homogeneous goods. *Annals of Public and Cooperative Economics, 64,* 367-393.

Nett, L. (1994). Why private firms are more innovative than public firms. *European Journal of Political Economy, 10,* 639-653.

Nishimori, A., & Ogawa, H. (2002). Public monopoly, mixed oligopoly and productive efficiency. *Australian Economic Papers, 41,* 185-190.

Ohnishi, K. (2006). A mixed duopoly with a lifetime employment contract as a strategic commitment. *FinanzArchiv, 62,* 108-123.

Ohnishi, K. (2008). Strategic commitment and international mixed competition with domestic state-owned and foreign labor-managed firms. *FinanzArchiv, 64,* 458-472.

Ohnishi, K. (2009). Capacity investment and mixed duopoly with state-owned and labor-managed firms. *Annals of Economics and Finance, 10,* 49-64.

Ohnishi, K. (2010a). Inventories and mixed duopoly with state-owned and labor-managed firms. *iBusiness, 2,* 116-122.

Ohnishi, K. (2010b). Lifetime employment contract and quantity competition with profit-maximizing and joint-stock firms. *Journal of Institutional and Theoretical Economics, 166,* 462-478.

Ohnishi, K. (2011). A quantity-setting mixed duopoly with inventory investment as a coordination device. *Annals of Economics and Finance, 12,* 111-121.

Ohnishi, K. (2014). Sequential mixed competition with a foreign joint-stock firm.

International Journal of Social Sciences and Management Studies, 1, 38-52.

Ohnishi, K. (2015a). Inventory holding and international mixed duopoly. *Hellenic Open Business Administration Journal, 1,* 73-89.

Ohnishi, K. (2015b). Lifetime employment and a three-stage model with state-owned and joint-stock firms. In: Ohnishi, K. (Ed.), *Firms' Strategic Decisions: Theoretical and Empirical Findings, Volume 1* (pp. 155-169). Sharjah, UAE: Bentham Science Publishers.

Pal, D. (1998). Endogenous timing in a mixed oligopoly. *Economics Letters, 61,* 181-185.

Pal, D., & Saha, B. (2014). Mixed duopoly and environment. *Journal of Public Economic Theory, 16,* 96-118.

Pal, D., & White, M. D. (1998). Mixed oligopoly, privatization, and strategic trade policy. *Southern Economic Journal, 65,* 264-281.

Rotemberg, J. J., & Saloner, G. (1989). The cyclical behavior of strategic inventories. *Quarterly Journal of Economics, 104,* 73-97.

Poyago-Theotoky, J. (1998). R&D competition in a mixed duopoly under uncertainty and easy imitation. *Journal of Comparative Economics, 26,* 415-428.

Poyago-Theotoky, J. (2001). Mixed oligopoly, subsidization and the order of firms' moves: An irrelevance result. *Economics Bulletin, 12,* 1-5.

Roy chowdhury, P. (2009). Mixed oligopoly with distortions: First best with budget-balance and the irrelevance principle. *Economics Bulletin, 29,* 1885-1900.

Saha B., & Sensarma, R. (2008). The distributive role of managerial incentives in a mixed duopoly. *Economics Bulletin, 12,* 1-10.

Saloner, G. (1987). Cournot duopoly with two production periods. *Journal of Economic Theory, 42,* 183-187.

Waldman, D. E., & Jensen, E. J. (2007). *Industrial organization: Theory and practice.* Boston, MA: Pearson/Addison Wesley.

Wang, L. F. S., & Lee, J. Y. (2010). Partial privatization, foreign competition, and tariffs ranking. *Economics Bulletin, 30,* 2405-2012.

Wang, L. F. S., & Wang, J. (2009). Environmental taxes in a differentiated mixed duopoly. *Economic Systems, 33,* 389-396.

Willner, J. (1994). Welfare maximization with endogenous average costs. *International Journal of Industrial Organization, 12,* 373-386.

White, M. D. (1996). Mixed oligopoly, privatization and subsidization. *Economics Letters, 53,* 189-195.

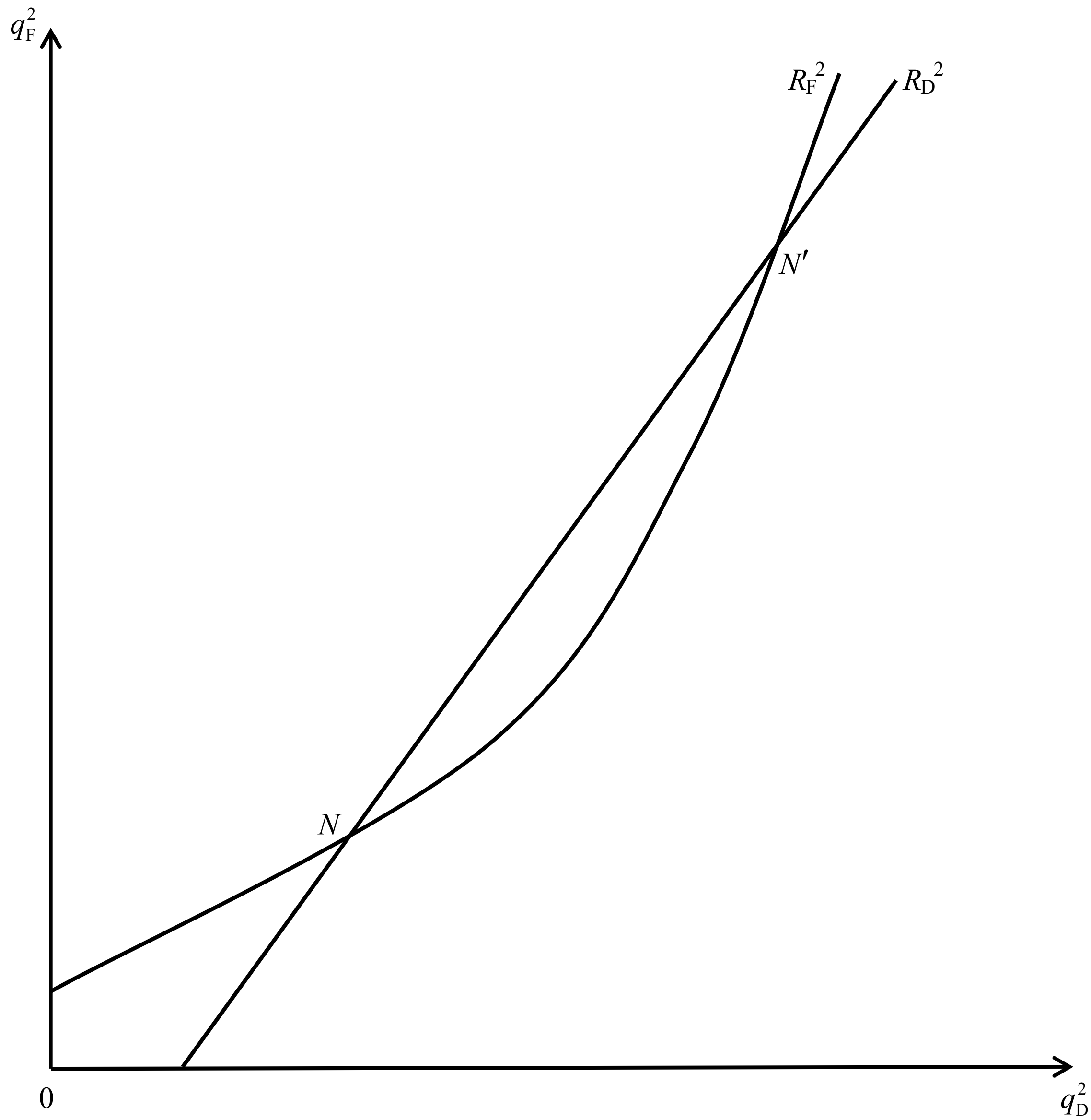

Figure 1: The reaction curves cross twice, but only point N is a stable Cournot equilibrium.

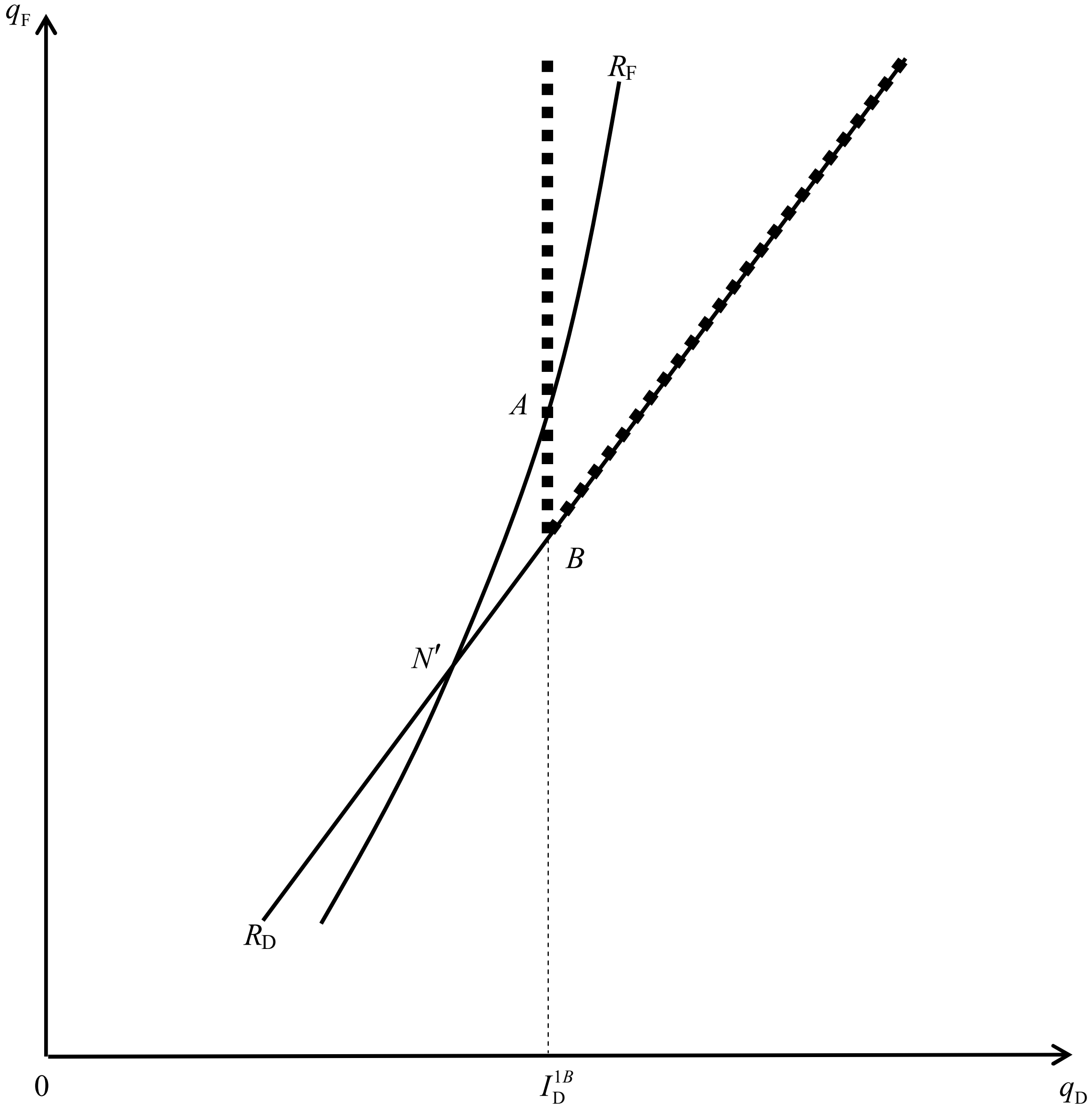

Figure 2: Firm D's best response is kinked at the level of I_D^{1B}.

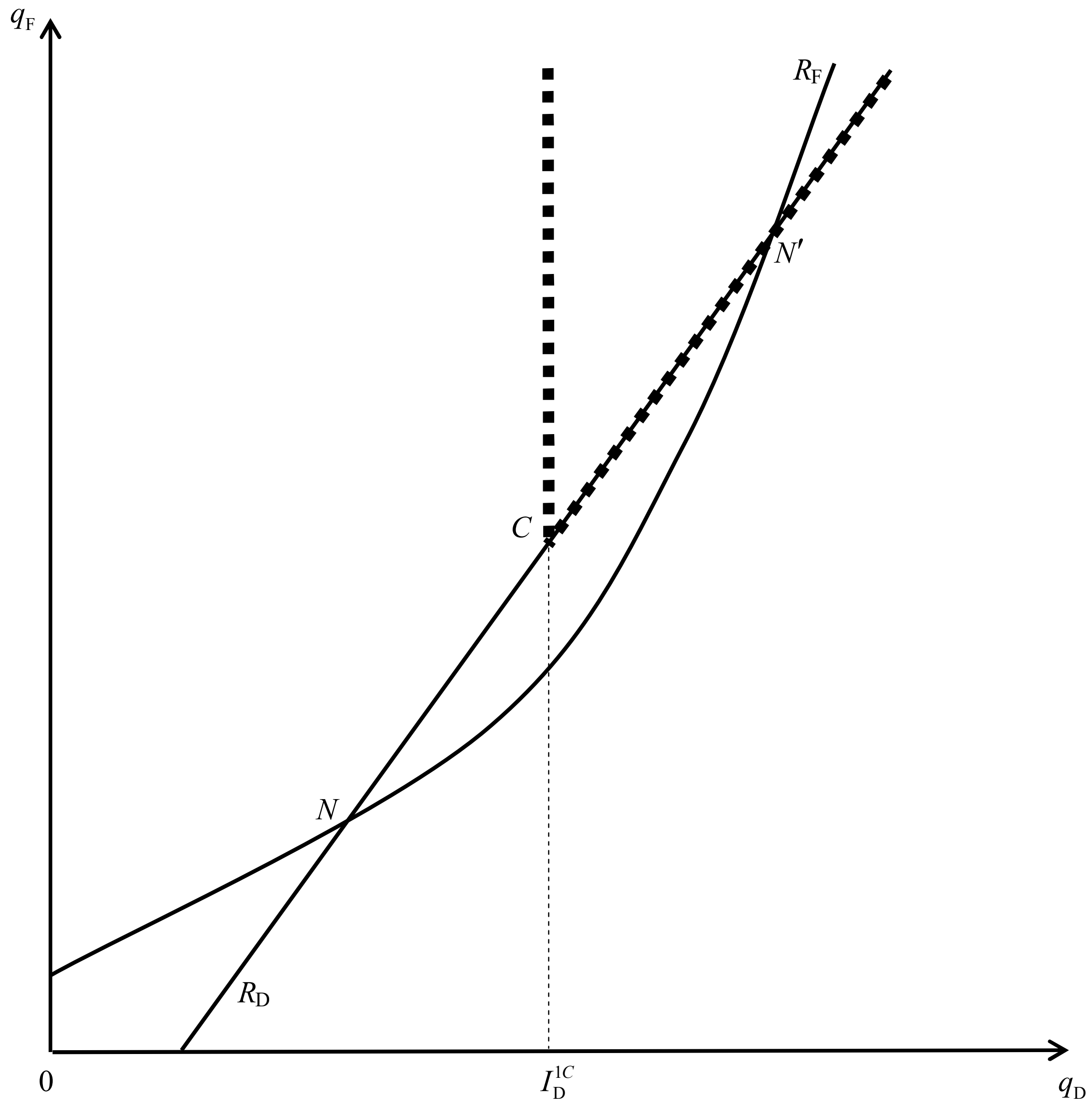

Figure 3: Point N' is not a stable solution.

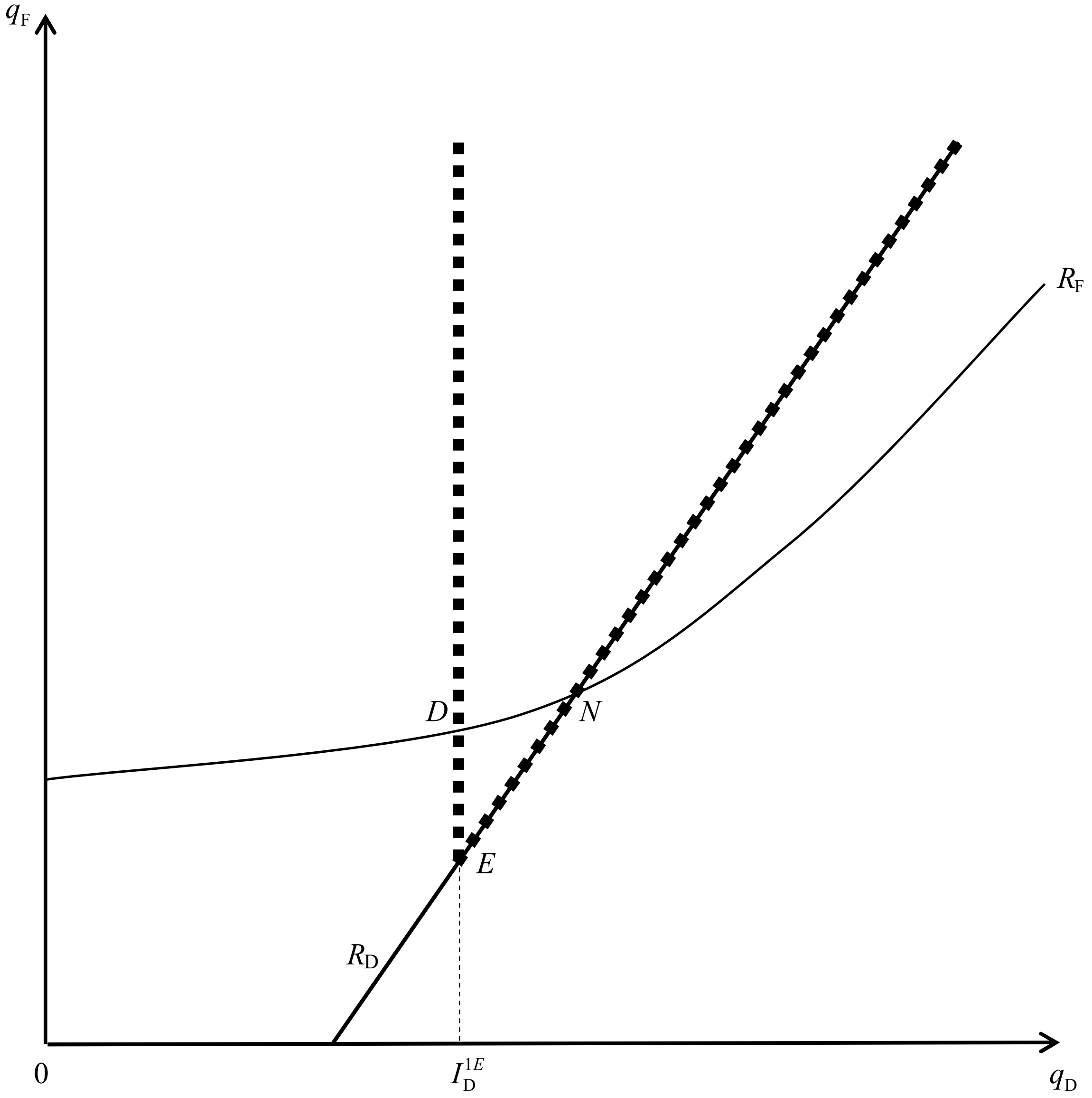

Figure 4: There are two stable solutions.

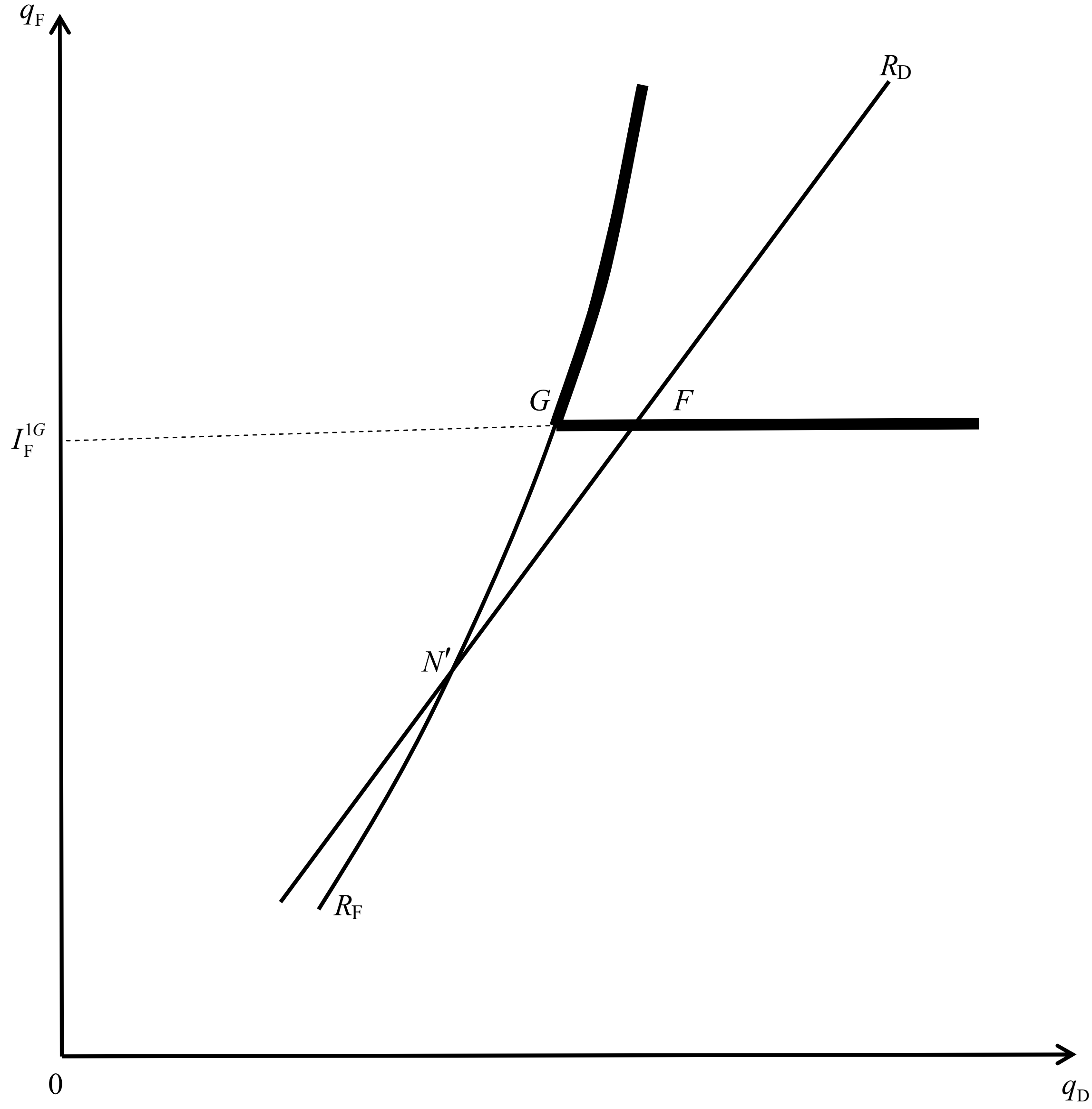

Figure 5: Firm F's best response is kinked at the level of I_F^{1G}.

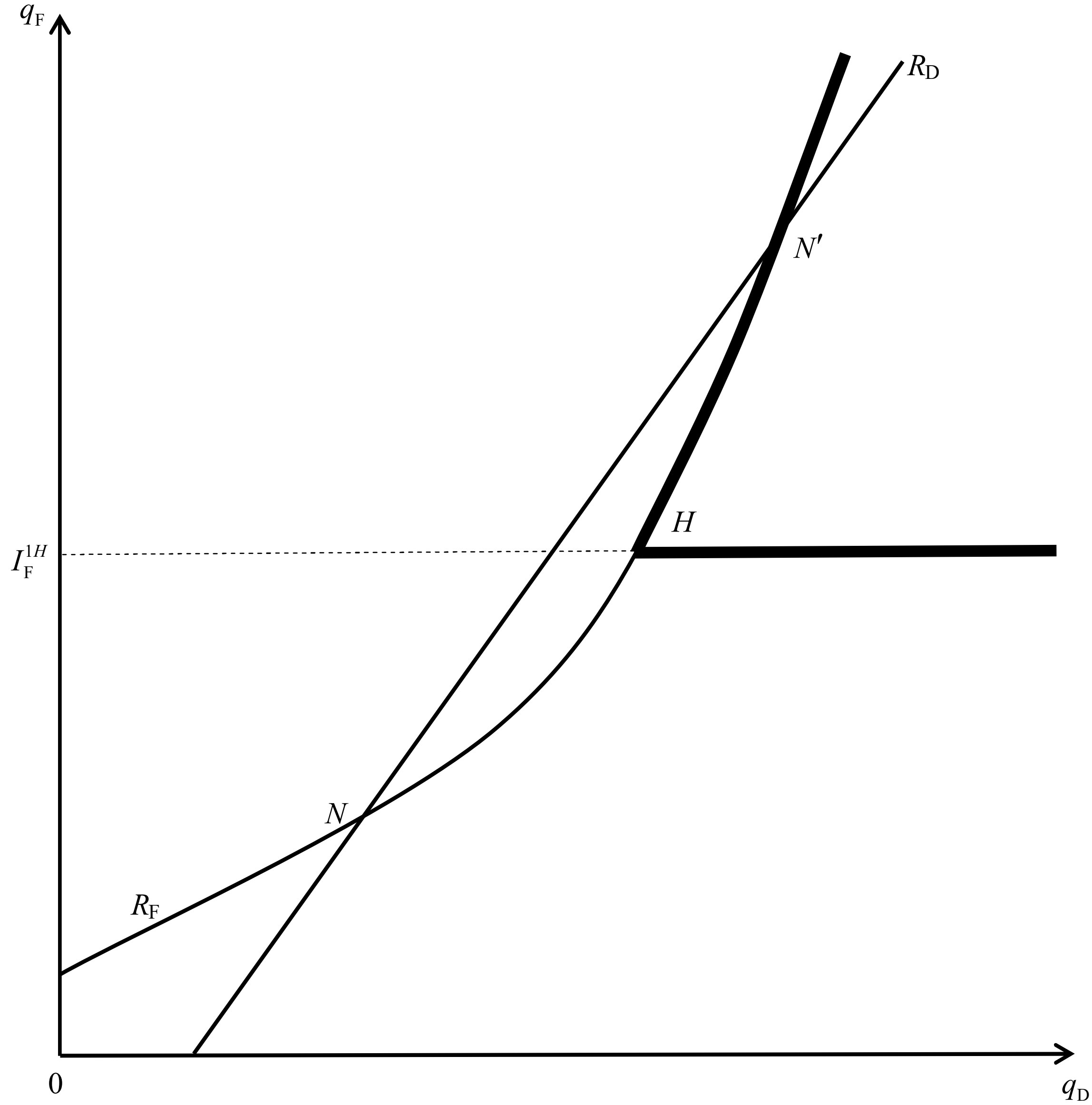

Figure 6: There is no stable solution.

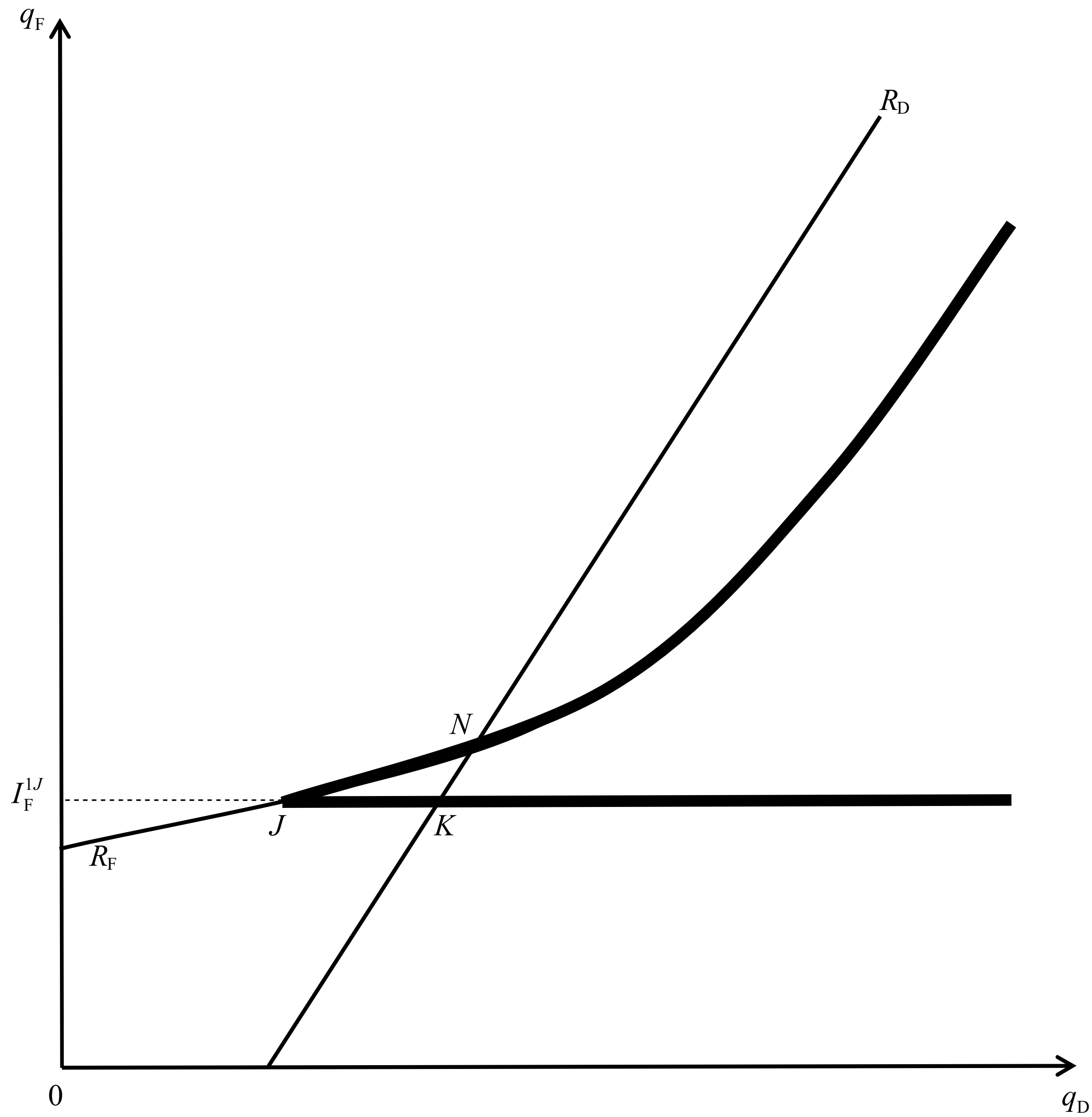

Figure 7: There are two stable solutions.

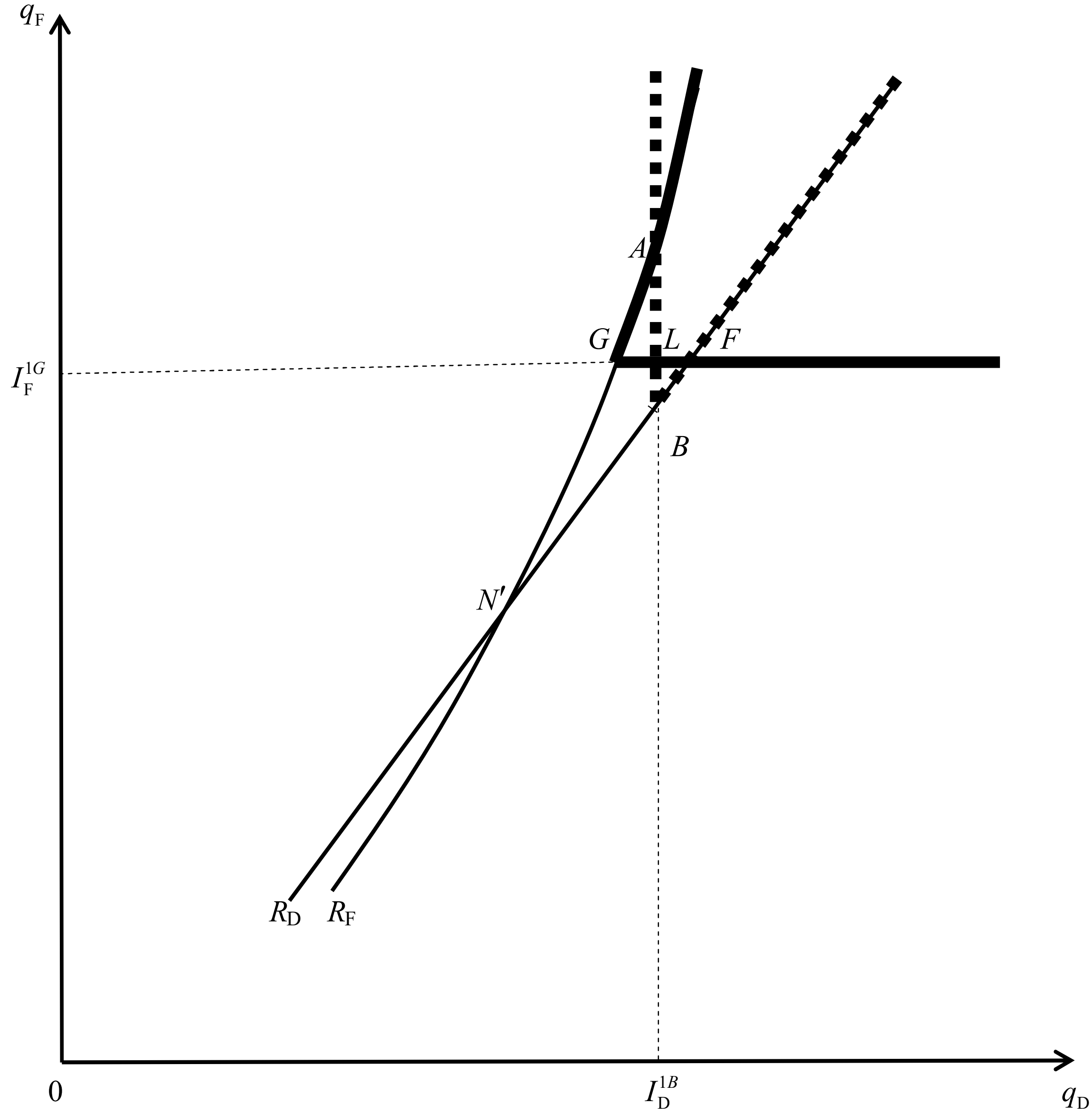

Figure 8: There are three stable solutions.

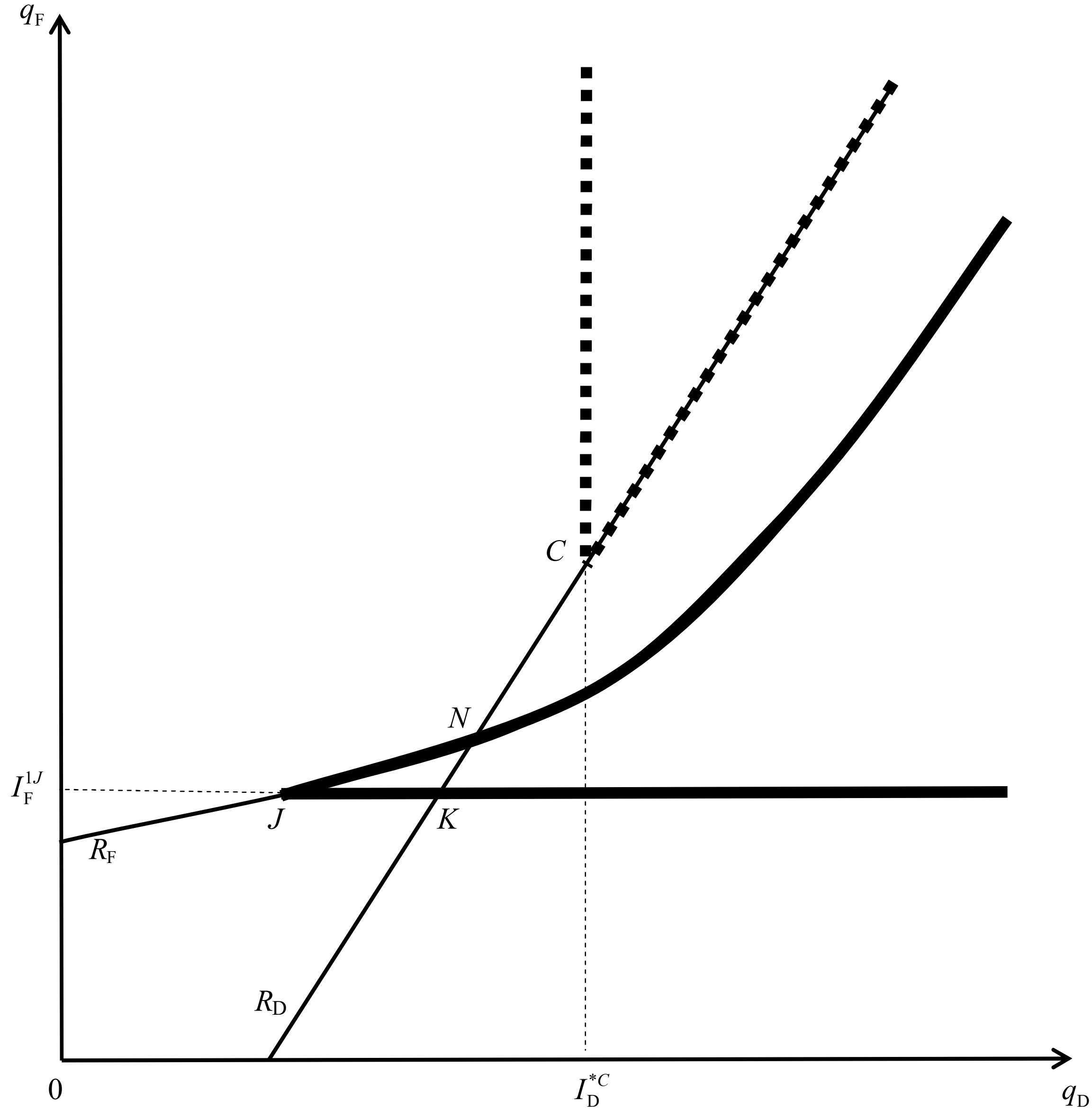

Figure 9: There is no stable solution.

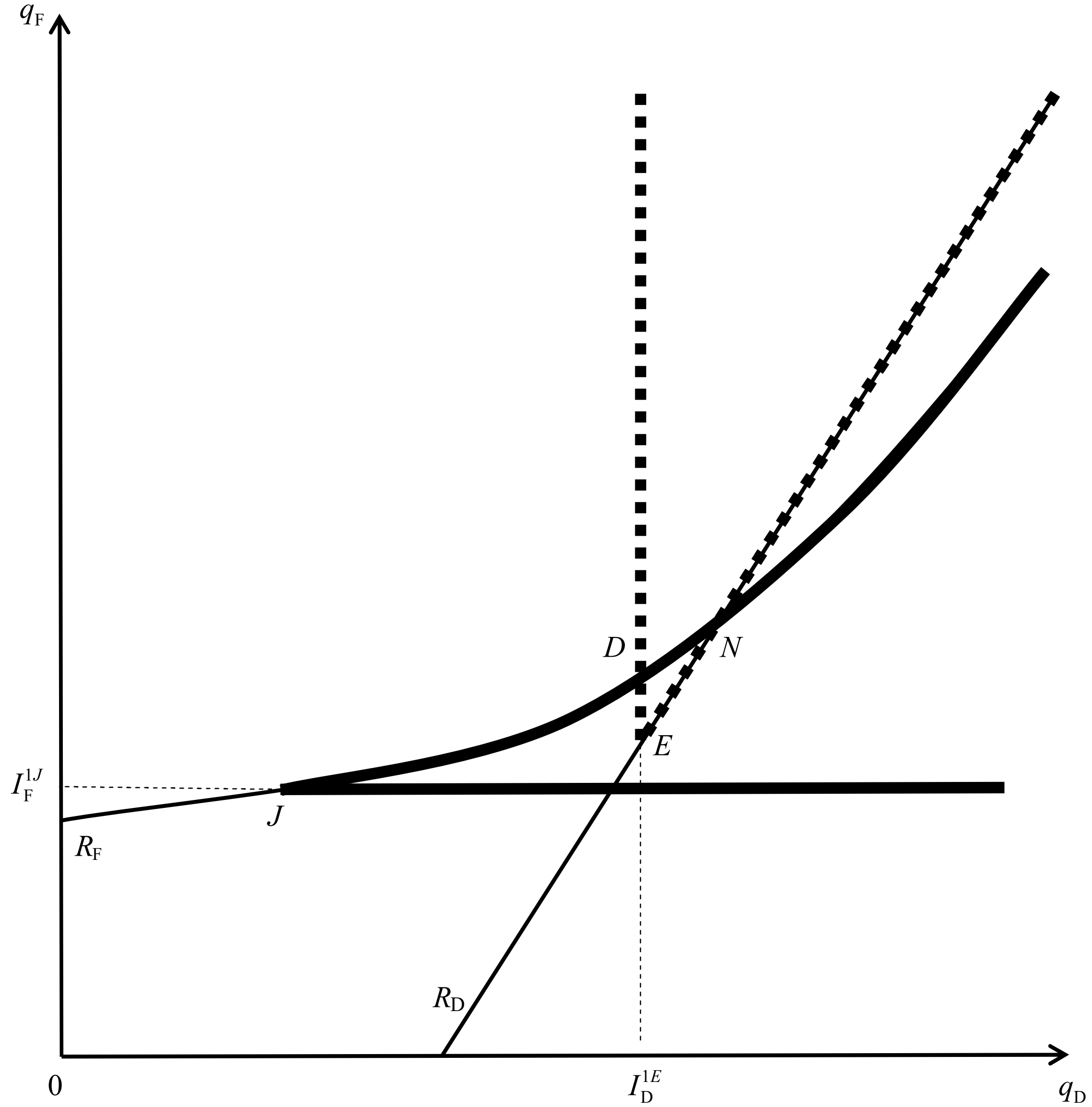

Figure 10: Both *N* and *D* are stable solutions.

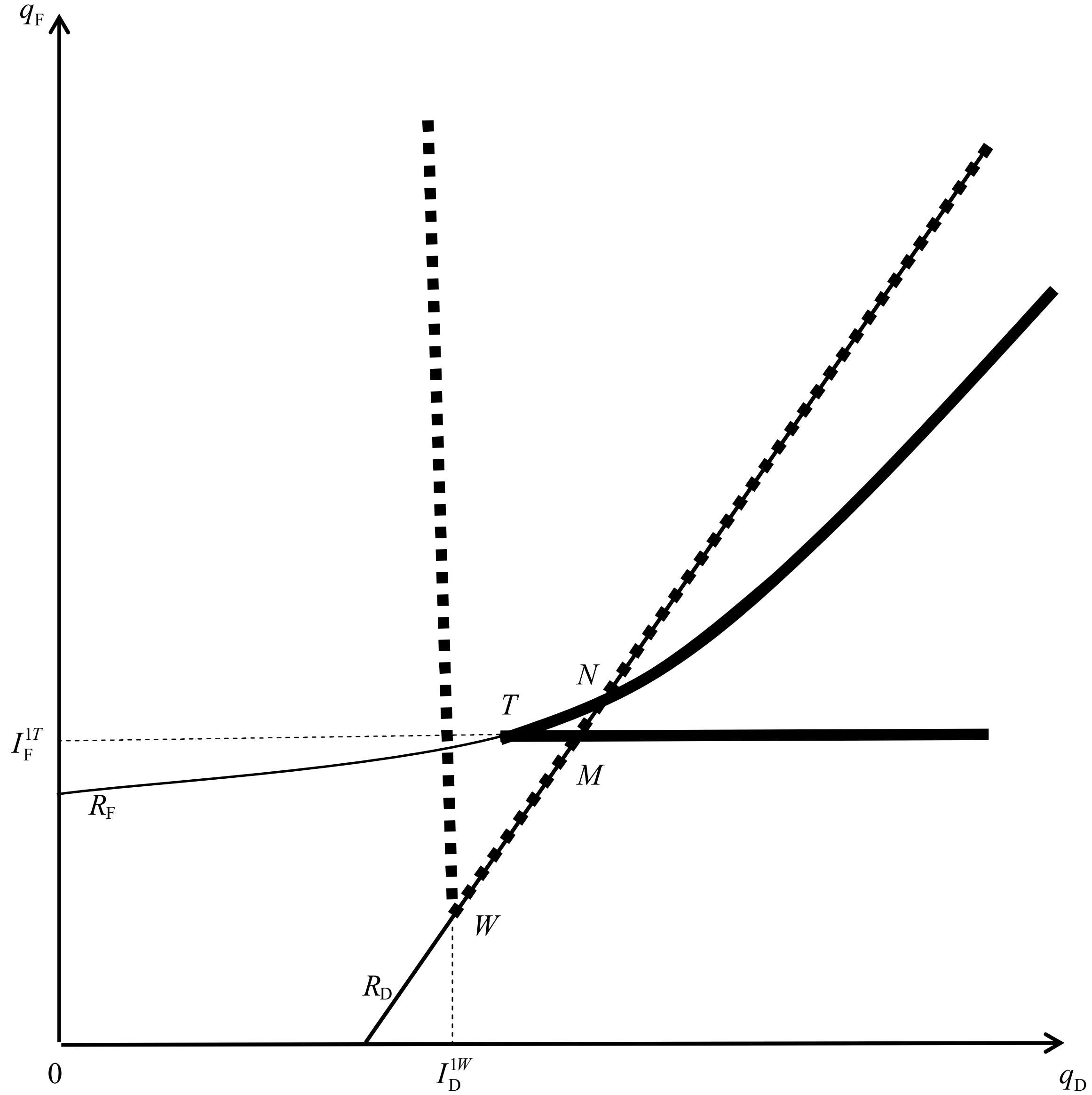

Figure 11: Both N and M are stable solutions.

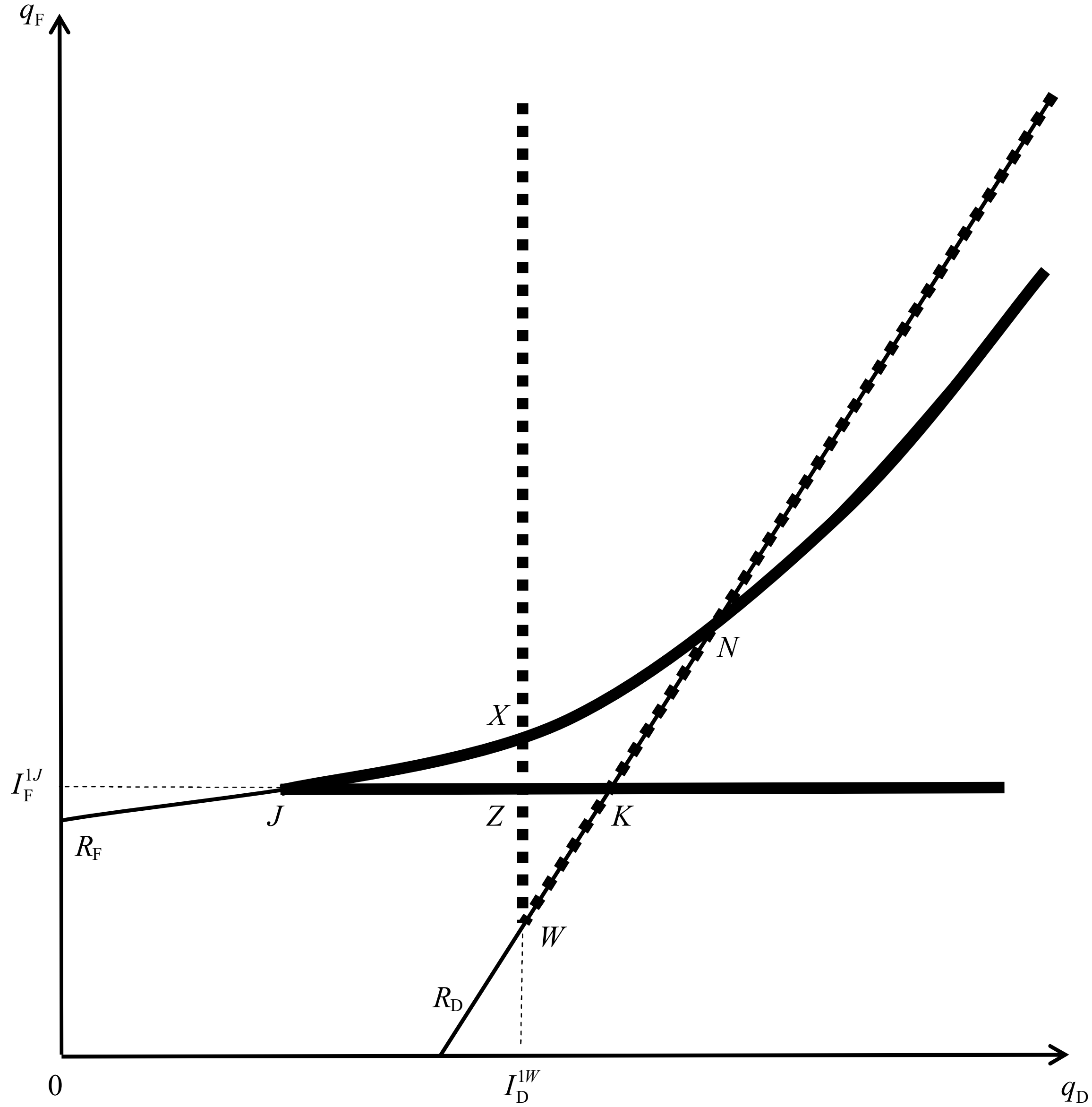

Figure 12: There are four stable solutions.