Inventory Holding and a Mixed Duopoly with a Foreign Joint-Stock Firm

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29 April 2018

Online at https://mpra.ub.unimuenchen.de/88223/
MPRA Paper No. 88223, posted 31 July 2018 03:41 UTC
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Abstract
This paper investigates a mixed duopoly model in which there is a state-owned firm competing with a foreign joint-stock firm. The following situation is considered. In the first period, each firm non-cooperatively decides how many it sells in the current market. In addition, each firm can hold inventory for the second-period market. By holding large inventory, a firm may be able to commit to large sales in the next period. In the second period, each firm non-cooperatively chooses its second-period output. At the end of the second period, each firm sells its first-period inventory and its second-period output and holds no inventory. The paper traces out the firms’ reaction functions in the mixed duopoly model.

Keywords: Inventory holding, state-owned firm, foreign joint-stock firm, reaction curves
JEL classification: C72, D43, F23, L30

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1. Introduction

Rotemberg and Saloner (1989) examine a two-period model in which inventory is used by a duopoly to deter deviations from an implicitly collusive arrangement, and establish that higher inventory allows firms to punish cheaters more strongly and can thus help to maintain collusion. Matsumura (1999) presents a Cournot duopoly model with finitely repeated competition, and establishes that two-period competition is insufficient to make private firms collusive. These studies investigate private duopoly models with inventory as a strategic device. Ohnishi (2011) presents a mixed duopoly model in which a welfare-maximizing public firm and a profit-maximizing private firm can use inventory investment as a strategic device, and demonstrates that the equilibrium coincides with the Stackelberg solution where the private firm is the leader.

The analysis of mixed oligopoly models that incorporate state-owned public firms has been performed by many researchers (e.g., see Delbono and Rossini, 1992; Nett, 1994; Willner, 1994; Fjell and Pal, 1996; George and La Manna, 1996; White, 1996; Mujumdar and Pal, 1998; Pal, 1998; Pal and White, 1998; Poyago-Theotoky, 1998; Nishimori and Ogawa, 2002; Bárcena-Ruiz and Garzón, 2003; Ohnishi, 2006, 2009; Bárcena-Ruiz, 2007; Fernández-Ruiz, 2009; Heywood and Ye, 2010; Wang and Lee, 2010; Pal and Saha, 2014; Cracau, 2015). However, these studies consider mixed market models in which state-owned firms compete with capitalist or labor-managed firms, and do not include joint-stock firms.

Only few studies consider joint-stock firms. For example, Meade (1972) shows the
differences in incentives, short-run adjustment and so forth among entrepreneurial, co-operative and joint-stock firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a homogeneous final good with inputs of capital and labor, and examines the behavior of profit-maximizing, labor-managed and joint-stock firms. Ohnishi (2010b) presents the equilibrium solution of a quantity-setting model comprising a joint-stock firm and a profit-maximizing capitalist firm, and shows that introducing lifetime employment into the model of quantity-setting duopoly is beneficial only for the joint-stock firm. In addition, Ohnishi (2015b) investigates a mixed duopoly model where a joint-stock firm and a state-owned firm are allowed to offer lifetime employment as a strategic commitment, and presents the equilibrium solution of the mixed duopoly model.

We consider a two-period mixed market model in which a state-owned firm and a foreign joint-stock firm can hold inventory as a strategic device. The game runs as follows. In period one, each firm non-cooperatively decides how many it sells in the current market. In addition, each firm non-cooperatively decides the level of inventory it holds for the second-period market. In period two, each firm non-cooperatively chooses its output. At the end of period two, each firm sells its first-period inventory and its second-period output. We trace out the firms’ reaction functions in the mixed duopoly model.

The balance of this paper is organized as follows. In the second section, we describe the model. The third section characterizes best replies for firms in the model. The fourth section presents the results of this study. The final section concludes the paper.
2. The model

We consider a mixed duopoly model in which there is a domestic state-owned firm (firm D) competing with a foreign joint-stock firm (firm F). In the balance of this paper, subscripts D and F refer to firms D and F, respectively, and superscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when \( i \) and \( j \) are used to refer to firms in an expression, they should be understood to denote D and F with \( i \neq j \). The duopolists produce perfectly substitutable goods. The price of each period is determined by \( P(S') \), where \( S' = s^t_D + s^t_F \) is the aggregate sales of each period. We assume that \( P' < 0 \) and \( P'' \leq 0 \).

The two periods of the game are as follows. In the first period, each firm non-cooperatively and simultaneously decides its first-period production \( q^t_i \in [0, \infty) \) and its first-period sales \( s^t_i \in [0, q^t_i] \). Firm \( i \)'s inventory \( I^t_i \) becomes \( q^t_i - s^t_i \). In the second period, each firm non-cooperatively and simultaneously decides its second-period production \( q^t_i \in [0, \infty) \). At the end of the second period, each firm sells \( s^t_i = I^t_i + q^t_i \) and holds no inventory. For notational simplicity, we consider the game without discounting.

Since \( \sum_{i=1}^2 q^t_i = \sum_{i=1}^2 s^t_i \), economic welfare is

\[
W = \sum_{r=1}^2 \left[ \int_0^{S^t_r} P(x)dx - m_D q^t_D - P(S')q^t_D \right] = \sum_{r=1}^2 \left[ \int_0^{S^t_r} P(x)dx - m_D s^t_D - P(S')s^t_F \right]
\]

(1)

where \( m_D \in (0, \infty) \) denotes firm D’s constant marginal cost. Firm D seeks to maximize (1).
The demand and cost conditions that firms face remain unchanged over time. We define
\[ w' = \int_0^s P(x)dx - m_D s_D' - P(S') s'_F \]  
(2)

In addition, since \( \sum_{i=1}^2 q'_i = \sum_{i=1}^2 s'_i \), firm F’s profit per capital is
\[ \Phi_F = \sum_{i=1}^2 \left[ \frac{P(S') s'_i - m_F q'_i - f'_i}{k_F(s'_F)} \right] = \sum_{i=1}^2 \left[ \frac{P(S') s'_i - m_F s'_i - f'_i}{k_F(s'_F)} \right] \]  
(3)

where \( m_F \in (0, \infty) \) denotes firm F’s constant marginal cost, \( f_F \in (0, \infty) \) is firm F’s fixed cost, and \( k_F(s'_F) \) is firm F’s capital input function. Firm F aims to maximize (3).

We assume that \( k_F(s'_F) \) is the function of \( s'_F \) with \( k'_F > 0 \) and \( k''_F > 0 \). This assumption means that the marginal quantity of capital used is increasing.

We also assume that firm D is less efficient than firm F, i.e. \( m_D > m_F \). This assumption is justified in Gunderson (1979) and Nett (1993, 1994), and is often used in literature studying mixed oligopoly markets (e.g., see George and La Manna, 1996; Mujumdar and Pal, 1998; Pal, 1998; Nishimori and Ogawa, 2002; Matsumura, 2003; Ohnishi, 2006, 2015a; Fernández-Ruiz, 2009). If firm D is equally or more efficient than firm F, then firm D chooses \( q'_D \) and \( s'_D \) such that price equals marginal cost. Therefore, firm F has no incentive to operate in the market, and firm D acts as a monopoly.

We define
\[ \phi'_F = \frac{P(S') s'_F - m_F s'_F - f'_F}{k_F(s'_F)} \]  
(4)

The solution concept of this model is subgame perfection. In the next section, we will give supplementary explanations of the model.
3. Supplementary explanations

First, we derive firm D’s reaction functions from (2). In the first period, since there is no inventory available, firm D’s reaction function is defined by

$$R_D^1(s_D^1) = \arg \max_{(s_D^1 \geq 0)} \left[ \int_0^{s_D^1} P(x)dx - m_D s_D^1 - P(S^1)s_D^1 \right]$$  \hspace{1cm} (5)

In the second period, firm D’s reaction function without inventory is defined by

$$R_D^2(s_D^2) = \arg \max_{(s_D^2 \geq 0)} \left[ \int_0^{s_D^2} P(x)dx - m_D s_D^2 - P(S^2)s_D^2 \right]$$  \hspace{1cm} (6)

and therefore its best response is shown by

$$\bar{R}_D^2(s_F^2) = \begin{cases} R_D^2(s_F^2) & \text{if } s_D^2 > I_D^1 \\ I_D^1 & \text{if } s_D^2 = I_D^1 \end{cases}$$  \hspace{1cm} (7)

Firm D maximizes economic welfare with respect to $s_D^i$, given $s_F^i$. The equilibrium solution needs to satisfy the following conditions: When inventory is zero, the first-order condition for firm D is

$$P - m_D - P^*s_F^i = 0$$  \hspace{1cm} (8)

and the second-order condition is

$$P' - P^*s_F^i < 0$$  \hspace{1cm} (9)

Therefore, we obtain

$$R_D^i(s_F^i) = \frac{P^*s_F^i}{P' - P^*s_F^i}$$  \hspace{1cm} (10)

In the first period, firm D’s reaction function is upward sloping. In the second period, firm
D’s best response also slopes upward for $s_D^2 > I_D^1$. This indicates that firm D treats $s_D^t$ as strategic complements.\(^1\)

Next, we derive firm F’s reaction functions from (4). In period one, since there is no inventory available, firm F’s reaction function is defined by

$$R_F^1(s_D^1) = \arg \max_{(s_f^1 > 0)} \left[ \frac{P(S^1)s_F^1 - m_Fs_F^1 - f_F^1}{k_F(s_F^1)} \right]$$

(11)

In period two, firm F’s reaction function without inventory is defined by

$$R_F^2(s_D^2) = \arg \max_{(s_f^2 > 0)} \left[ \frac{P(S^2)s_F^2 - m_Fs_F^2 - f_F^2}{k_F(s_F^2)} \right]$$

(12)

and therefore its best response is shown by

$$R_F^2(s_D^2) = \begin{cases} R_F^1(s_D^1) & \text{if } s_F^2 > I_F^1 \\ I_F^1 & \text{if } s_F^2 = I_F^1 \end{cases}$$

(13)

Firm F maximizes profit per capital with respect to $s_F^t$, given $s_D^t$. When inventory is zero, the first-order condition for firm F is

$$(P's_F^t + P - m_F)k_F - (Ps_F^t - m_Fs_F^t - f_F^t)k_F^t = 0$$

(14)

and the second-order condition is

$$(P's_F^t + 2P')k_F - (Ps_F^t - m_Fs_F^t - f_F^t)k_F^t < 0$$

(15)

In addition, we obtain

$$R_F'(s_D^t) = -\frac{P's_F^t k_F + P'(k_F - s_F^t k_F^t)}{(P's_F^t + 2P')k_F - (Ps_F^t - m_Fs_F^t - f_F^t)k_F^t}$$

(16)

\(^1\) The concept of strategic complements is due to Bulow, Geanakoplos, and Klemperer (1985).
Since \( k_F^* > 0 \), \( k_F - s_F^i k_F' < 0 \), so that \( P^* s_F^i k_F + P'(k_F - s_F^i k_F') \) is positive. This means that firm F treats \( s_F^i \) as strategic complements.

4. Results

In this section, we trace out the firms’ reaction curves in the model described in section 2. There is no inventory available in the first period, and further \( s_F^1 \) does not affect \( s_F^2 \) and \( s_D^2 \). Firm D’s and firm F’s reaction curves in the second period is decided by the level of \( I_F^1 \), and therefore we consider the second period.

We illustrate both firms’ reaction curves by using Figures 1-12. For explanations, the figures are drawn simply. We discuss the following four cases.

Case 1: Neither firm holds inventory.

Case 2: Only firm D holds inventory.

Case 3: Only firm F holds inventory.

Case 4: Each firm holds inventory.

We discuss these cases in orders.

Case 1
Both firms’ reaction curves are drawn in Figure 1, where $R_i^2$ denotes firm $i$’s second-period reaction curve with no inventory. Both firms’ reaction curves are upward sloping. The equilibrium is decided in Cournot fashion, i.e., the intersection of firm D’s and firm F’s second-period reaction curves gives us the equilibrium of the game. The reaction curves cross twice as depicted in Figure 1. Only point $N$ is a stable Cournot equilibrium, since in point $N'$ firm F’s second-period reaction curve crosses firm D’s from above. In the remainder of this paper, we delete the superscript 2 for brevity’s sake.

**Case 2**

This is the case in which only firm D holds inventory, and this case is illustrated in Figures 2-4. We consider the change of firm D’s best response curve, which is drawn in Figure 2. We suppose that firm D maintains the inventory level of $I_{D}^{1B}$ in period two. By holding inventory, firm D’s best response changes to (7). Firm D’s inventory holding creates a kink in its reaction curve at the level of $I_{D}^{1B}$. Therefore, firm D’s reaction curve becomes the kinked bold broken lines.

From Figure 2, we see that the inventory level of $I_{D}^{1B}$ changes the solution of the game. The intersection of the reaction curves is an equilibrium solution in the second period. That is, if firm D holds $I_{D}^{1B}$, then the solution occurs at $A$. We see that economic welfare is higher at $A$ than at $N'$, and $A$ is a stable solution.

We consider the situation drawn in Figure 3. If firm D maintains the inventory level of
in the second period, then its inventory holding creates a kink in its reaction curve at the level of $I_D^{IC}$. The intersection of the reaction curves is an equilibrium solution in period two. That is, if firm D holds $I_D^{IC}$, then the solution becomes $N'$. It is easy to see that $N'$ is an unstable equilibrium solution. If firm D maintains the inventory level of $I_D^{IC}$, then there is no stable solution.

We consider the situation drawn in Figure 4. If firm D holds $I_D^{IE}$ in period two, then its inventory holding creates a kink in its reaction curve at the level of $I_D^{IE}$. In this figure, the reaction curves cross at two points. Both $D$ and $N$ are stable solutions. We see that economic welfare is higher at $N$ than at $D$.

Case 3

This case is illustrated in Figures 5-7. Firstly, we consider the situation in Figure 5. We suppose that the firm F holds $I_F^{IG}$ in the second period. By holding inventory, firm F’s best response changes to (13). Firm F’s inventory holding creates a kink in its reaction curve at the level of $I_F^{IG}$. Therefore, firm F’s reaction curve becomes the kinked bold lines.

From Figure 5, we see that the inventory level of $I_F^{IG}$ changes the solution of the game. If firm F holds $I_F^{IG}$, then the solution is at $F$. However, we see that firm F’s profit per capacity is lower at $F$ than at $N'$.

Secondly, we consider the situation drawn in Figure 6. If firm F maintains the inventory level of $I_F^{IH}$ in period two, then its inventory holding creates a kink in its reaction curve at
the level of $I_{F}^{HL}$. The intersection of the reaction curves is an equilibrium solution in period two. That is, if firm F maintains the inventory level of $I_{F}^{HL}$, then the solution becomes $N'$. In this figure, there is no stable solution.

Thirdly, we consider the situation in Figure 7. If firm F holds $I_{F}^{LJ}$ in period two, then its inventory holding creates a kink in its reaction curve at the level of $I_{F}^{LJ}$. The best response curves cross at multiple points as in Figure 7.

**Case 4**

This case is illustrated in Figures 8-12. In this case, both firms use inventory holding as a strategic commitment device. Firstly, we consider the situation in Figure 8. Suppose that firm D maintains the inventory level of $I_{D}^{LB}$ in period two. By holding inventory, firm D’s best response changes to (7). Firm D’s inventory holding creates a kink in its reaction curve at the level of $I_{D}^{LB}$. Therefore, firm D’s reaction curve becomes the kinked bold broken lines. In addition, suppose that firm F holds $I_{F}^{LG}$ in period two. By holding inventory, firm F’s best response changes to (13). Firm F’s inventory holding creates a kink in its reaction curve at the level of $I_{F}^{LG}$. Therefore, firm F’s reaction curve becomes the kinked bold lines.

The solution is decided in Cournot fashion. From Figure 8, we see that inventory holding by each firm changes the solution of the game. Figure 8 indicates that there are three stable solutions.

Secondly, we consider the situation in Figure 9. If firm D maintains the inventory level of
$I^t_D$, then its best response curve becomes the bold broken lines, and if firm F holds $I^t_F$, then its best response curve becomes the bold lines. In Figure 9, both firms’ reaction curves do not cross each other. That is, the obvious outcome is that there is no stable solution.

Thirdly, we consider the situation in Figure 10. When firm D holds $I^{tE}_D$ in the second period, its quantity best response curve is kinked at the level of $I^{tE}_D$. In addition, inventory holding by firm F kinks its quantity best response curve. Therefore, firm D’s best response is depicted as the thick broken lines, while firm F’s best response curve is the thick lines. The firms’ best response curves cross twice as in Figure 10. It is obvious that both $N$ and $D$ are stable solutions.

Fourthly, we consider the situation drawn in Figure 11. Inventory holding by each firm kinks its quantity best response curve. If firms D and F hold $I^{tW}_D$ and $I^{tF}_F$ respectively, then firm D’s best response is depicted as the thick broken lines, and firm F’s best response curve is the thick lines. In this figure, the firms’ best response curves cross at multiple points. It is obvious that both $N$ and $M$ are stable solutions.

Fifthly, we consider the case drawn in Figure 12. We suppose that firms D and F hold $I^{tW}_D$ and $I^{tF}_F$, respectively. Therefore, firm D’s best response is depicted as the thick broken lines, while firm F’s best response curve is the thick lines. The firms’ best response curves cross four times as in Figure 12. We see that all these points are stable solutions.
5. Conclusion

We have considered a two-period mixed duopoly model in which there is a state-owned firm competing with a foreign joint-stock firm. Each firm is allowed to hold inventory as a strategic device. As a result, we have shown that there are multiple stable Cournot solutions in the international mixed duopoly model.

In the near future, we will extend our analysis by considering a mixed oligopoly model where a state-owned firm competes with both domestic and foreign joint-stock firms.

References


Figure 1: The reaction curves cross twice, but only point $N$ is a stable Cournot equilibrium.
Figure 2: Firm D’s best response is kinked at the level of $I_D^{1B}$. 
Figure 3: Point $N'$ is not a stable solution.
Figure 4: There are two stable solutions.
Figure 5: Firm F’s best response is kinked at the level of $I^G_F$. 
Figure 6: There is no stable solution.
Figure 7: There are two stable solutions.
Figure 8: There are three stable solutions.
Figure 9: There is no stable solution.
Figure 10: Both $N$ and $D$ are stable solutions.
Figure 11: Both $N$ and $M$ are stable solutions.
Figure 12: There are four stable solutions.