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# **Wage-rise contract and labour-managed Cournot oligopoly with complementary goods**

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## **Abstract**

This paper considers a quantity-setting oligopoly model with complementary goods where labour-managed firms are allowed to offer wage-rise contracts as a strategic commitment. The following two stages are considered. In the first stage, each firm independently decides whether or not to adopt a wage-rise contract as a strategic commitment device. In the second stage, each firm independently chooses and sells its actual output. The paper analyses the equilibrium of the labour-managed oligopoly model.

Keywords: Cournot competition; Labour-managed oligopoly; Wage-rise contract; Complementary goods

JEL classification: C72; D21; L13

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# 1. Introduction

After the Second World War, the right to manage the firm in the former Yugoslavia was, within the limits determined by law, in the hands of its employees (Furubotn and Pejovich, 1970). The first theoretical analysis of firm behaviour in the former Yugoslavia was done by Ward (1958). Since the seminal work by Ward (1958), a great many researchers have analysed the behaviour of labour-managed firms. For instance, Laffont and Moreaux (1985) study the welfare properties of free-entry Cournot equilibria in labour-managed market economies and demonstrate that Cournot equilibria are efficient provided that the market is sufficiently large. Okuguchi (1986) compares the Cournot and Bertrand equilibrium prices for the labour-managed oligopoly under product differentiation and demonstrates that the Cournot equilibrium prices are not lower than the Bertrand ones. Zhang (1993) uses the framework of Dixit (1980) and Bulow, Geanakoplos and Klemperer (1985a) and studies whether labour-managed firms can hold excess capacity to deter entry and whether they are more or less likely to do so than profit-maximizing firms under identical conditions. He demonstrates that it is more likely for labour-managed firms than for profit-maximizing firms to keep excess capacity to deter entry. Okuguchi (1993) examines two models of duopoly with product differentiation and with only labour-managed firms, in one of which two firms' strategies are outputs (labour-managed Cournot duopoly) and prices become strategic variables in the other (labour-managed Bertrand duopoly). He demonstrates that if two firms are symmetric, leadership is not as advantageous as followership in both labour-managed Cournot-Stackelberg

and Bertrand-Stackelberg duopolies with product differentiation. Lambertini and Rossini (1998) analyse the behaviour of labour-managed firms in a two-stage Cournot duopoly model with capital strategic interaction and demonstrate that labour-managed firms choose their capital commitments according to the level of interest rate, unlike what usually happens when only profit-maximizing firms operate in the market. Lambertini (2001) considers a spatial differentiation duopoly model and demonstrates that if both firms are labour-managed, there exists a (symmetric) subgame perfect equilibrium in pure strategies with firms located at the first and third quartiles, if and only if the setup cost is low enough. Ireland (2003) compares the behaviour of profit-maximizing firms with that of labour-managed firms in Bertrand oligopoly competition and shows that labour-managed firms price lower than profit-maximizing firms. In addition, Ohnishi (2007) uses a quantity-setting model with substitute goods where two labour-managed firms are allowed to offer wage-rise contracts as a strategic commitment and shows that there is an equilibrium in which at least one labour-managed firm adopts the contract. There are also many other related excellent research works

We examine a Cournot oligopoly model in which labour-managed firms produce complementary goods and can offer wage-rise contracts as a strategic commitment. To the best of the author's knowledge, there is no previous work dealing with such economic situation.

The purpose of this study is to present the equilibrium of a Cournot oligopoly model with complementary goods where labour-managed firms are allowed to offer wage-rise contracts as a strategic device. We find that our results are quite different from the results obtained from the

labour-managed Cournot oligopoly model with substitute goods.

The rest of this paper is organized as follows. In Section 2, we formulate the model. Section 3 presents the equilibrium of the model. Finally, Section 4 concludes the paper.

## 2. The model

Let us consider a market in which there are  $n$  labour-managed firms producing complementary goods. There is no possibility of entry or exit.

The market is modelled by means of the following two-stage game. In the first stage, each labour-managed firm independently decides whether or not to adopt a wage-rise contract as a strategic commitment device. If labour-managed firm  $i$  ( $i = 1, 2, \dots, n$ ) does so, then it chooses an output level  $x_i^* \geq 0$  and a wage premium rate  $t_i > 0$ . In addition, labour-managed firm  $i$  agrees to pay each employee a wage premium uniformly if it actually produces more than  $x_i^*$ . In the second stage, each labour-managed firm independently chooses and sells its actual output  $x_i > 0$ .

Therefore, firm  $i$ 's income per worker is given by

$$\psi_i = \begin{cases} \frac{p_i(x_1, x_2, \dots, x_n)x_i - r_i k_i(x_i) - f_i}{l_i(x_i)} & \text{if } x_i \leq x_i^*, \\ \frac{p_i(x_1, x_2, \dots, x_n)x_i - r_i k_i(x_i) - (x_i - x_i^*)t_i - f_i}{l^i(x^i)} & \text{if } x_i \geq x_i^*, \end{cases} \quad (1)$$

where  $p_i : \mathbb{R}_{++}^n \rightarrow \mathbb{R}_{++}$  denotes firm  $i$ 's inverse demand function,  $x_i \in (0, \infty)$  is firm  $i$ 's

quantity,  $r_i \in (0, \infty)$  is firm  $i$ 's unit cost of capital for,  $k_i : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$  is firm  $i$ 's capital input function,  $f_i \in (0, \infty)$  is firm  $i$ 's fixed cost, and  $l_i : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$  is firm  $i$ 's labour input function.

We assume that there is a unique Cournot equilibrium and each firm's price, output and income per worker are positive in the equilibrium. In addition, the following assumptions are made.

Assumption 1:  $p_i$  is twice continuously differentiable with bounded derivatives,  $\partial p_i / \partial x_i < 0$  (downward-sloping demand), and  $\partial p_i / \partial x_j > 0$  (complementary goods) ( $i, j = 1, 2, \dots, n; i \neq j$ ).

Assumption 2:  $\left| \frac{\partial p_i}{\partial x_i} \right| > \sum_{j \neq i} \left| \frac{\partial p_i}{\partial x_j} \right|$ .

Assumption 3:  $\frac{dk_i}{dx_i} > 0$  and  $\frac{d^2 k_i}{dx_i^2} > 0$ .

Assumption 4:  $\frac{dl_i}{dx_i} > 0$  and  $\frac{d^2 l_i}{dx_i^2} > 0$ .

These are fairly standard assumptions in Cournot oligopoly games except complementary goods. Assumption 1 states that demand is downward-sloping. Assumption 2 means that firm  $i$ 's own effects of quantity on demand exceed firm  $j$ 's cross effects. Throughout this paper, we use subgame perfection as an equilibrium concept.

### 3. Results

We begin by giving supplementary explanations of the model introduced in the previous section. We derive labour-managed firm  $i$ 's reaction functions in quantities from (1). If firm  $i$  does not offer a wage-rise contract, its reaction function is defined by

$$R_i(x_{-i}) = \arg \max_{x_i > 0} \left[ \frac{p_i(x_1, x_2, \dots, x_n)x_i - r_i k_i(x_i) - f_i}{l_i(x_i)} \right], \quad (2)$$

where  $x_{-i} = (x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$ . On the other hand, if firm  $i$  offers a wage-rise contract and produces  $x_i \geq x_i^*$ , its reaction function is defined by

$$R_i^*(x_{-i}) = \arg \max_{x_i > 0} \left[ \frac{p_i(x_1, x_2, \dots, x_n)x_i - r_i k_i(x_i) - (x_i - x_i^*)t_i - f_i}{l_i(x_i)} \right]. \quad (3)$$

Therefore, if firm  $i$  sets  $x_i^*$  and offers a wage-rise contract, its best response is as follows:

$$\hat{R}_i(x_{-i}) = \begin{cases} R_i(x_{-i}) & \text{if } x_i < x_i^*, \\ x_i^* & \text{if } x_i = x_i^*, \\ R_i^*(x_{-i}) & \text{if } x_i > x_i^*. \end{cases} \quad (4)$$

We now state the following lemma.

**Lemma 1:** Both  $R_i(x_{-i})$  and  $R_i^*(x_{-i})$  are downward sloping.

Proof: Firm  $i$ 's aim is to maximize income per worker with respect to its own output, given the other firms' output. The Cournot solution needs to satisfy the following conditions: If firm  $i$  does

not offer a wage-rise contract as a strategic device, the first-order condition for income per worker maximization is

$$\left( \frac{\partial p_i}{\partial x_i} x_i + p_i - r_i \frac{dk_i}{dx_i} \right) l_i - (p_i x_i - r_i k_i - f_i) \frac{dl_i}{dx_i} = 0, \quad (5)$$

and the second-order condition is

$$\left( \frac{\partial^2 p_i}{\partial x_i \partial x_i} x_i + 2 \frac{\partial p_i}{\partial x_i} - r_i \frac{d^2 k_i}{dx_i^2} \right) l_i - (p_i x_i - r_i k_i - f_i) \frac{d^2 l_i}{dx_i^2} < 0. \quad (6)$$

On the other hand, if firm  $i$  adopts a wage-rise contract as a strategic device and produces  $x_i \geq x_i^*$ , the first-order condition for income per worker maximization is

$$\left( \frac{\partial p_i}{\partial x_i} x_i + p_i - r_i \frac{dk_i}{dx_i} - t_i \right) l_i - (p_i x_i - r_i k_i - t_i x_i + t_i x_i^* - f_i) \frac{dl_i}{dx_i} = 0, \quad (7)$$

and the second-order condition is

$$\left( \frac{\partial^2 p_i}{\partial x_i \partial x_i} x_i + 2 \frac{\partial p_i}{\partial x_i} - r_i \frac{d^2 k_i}{dx_i^2} \right) l_i - (p_i x_i - r_i k_i - t_i x_i + t_i x_i^* - f_i) \frac{d^2 l_i}{dx_i^2} < 0. \quad (8)$$

Moreover, we obtain

$$R'_i(x_{-i}) = - \frac{\frac{\partial^2 p_i}{\partial x_i \partial x_j} x_i l_i + \frac{\partial p_i}{\partial x_j} \left( l_i - x_i \frac{dl_i}{dx_i} \right)}{\left( \frac{\partial^2 p_i}{\partial x_i \partial x_i} x_i + 2 \frac{\partial p_i}{\partial x_i} - r_i \frac{d^2 k_i}{dx_i^2} \right) l_i - (p_i x_i - r_i k_i - f_i) \frac{d^2 l_i}{dx_i^2}} \quad (9)$$

and

$$R_i^{*'}(x_{-i}) = - \frac{\frac{\partial^2 p_i}{\partial x_i \partial x_j} x_i l_i + \frac{\partial p_i}{\partial x_j} \left( l_i - x_i \frac{dl_i}{dx_i} \right)}{\left( \frac{\partial^2 p_i}{\partial x_i \partial x_i} x_i + 2 \frac{\partial p_i}{\partial x_i} - r_i \frac{d^2 k_i}{dx_i^2} \right) l_i - (p_i x_i - r_i k_i - t_i x_i + t_i x_i^* - f_i) \frac{d^2 l_i}{dx_i^2}}. \quad (10)$$



Since  $\frac{d^2 l_i}{dx_i^2} > 0$ ,  $l_i - x_i \frac{dl_i}{dx_i} < 0$ , and hence  $\frac{\partial^2 p_i}{\partial x_i \partial x_j} x_i l_i + \frac{\partial p_i}{\partial x_j} \left( l_i - x_i \frac{dl_i}{dx_i} \right)$  is positive. Q.E.D.

Secondly, we present the following lemmas, which provide characterizations of wage-rise contracts as a strategic commitment.

**Lemma 2:** If labour-managed firm  $i$  adopts a wage-rise contract and an equilibrium is achieved, then at equilibrium  $x_i = x_i^*$ .

Proof: First, consider the possibility that  $x_i > x_i^*$  in equilibrium. From (1), firm  $i$ 's income-per-worker is

$$\psi_i = \frac{p_i(x_1, x_2, \dots, x_n)x_i - r_i k_i(x_i) - (x_i - x_i^*)t_i - f_i}{l^i(x^i)}.$$

Here, firm  $i$  can increase income per worker by increasing  $x_i^*$ , and the equilibrium solution does not change in  $x_i \geq x_i^*$ . Hence,  $x_i > x_i^*$  does not result in an equilibrium.

Next, consider the possibility that  $x_i < x_i^*$  in equilibrium. From (1), we see that it is impossible for firm  $i$  to change its output in equilibrium because such a strategy is not credible. That is, wage-rise contracts do not function as a strategic commitment device. Q.E.D.

**Lemma 3:** Labour-managed firm  $i$ 's optimal output is smaller when it adopts a wage-rise contract as a strategic commitment than when it does not.

Proof: From (1), we see that the contract will never decrease the marginal cost of firm  $i$ . The first-order condition for firm  $i$  when its marginal cost is  $r_i \partial k_i / \partial x_i$  is (5), and the first-order condition for firm  $i$  when its marginal cost is  $r_i \partial k_i / \partial x_i + t_i$  is (7). Here,  $t_i$  is positive. Lemma 2 shows that firm  $i$ 's optimal output when it adopts the contract coincides with  $x_i^*$ . To satisfy (7),  $\left( \frac{\partial^2 p_i}{\partial x_i \partial x_i} x_i + 2 \frac{\partial p_i}{\partial x_i} - r_i \frac{d^2 k_i}{dx_i^2} \right) l_i - (p_i x_i - r_i k_i - f_i) \frac{d^2 l_i}{dx_i^2}$  needs to be positive. Thus, firm  $i$ 's income-per-worker-maximizing output is smaller when its marginal cost is  $r_i \partial k_i / \partial x_i + t_i$  than when its marginal cost is  $r_i \partial k_i / \partial x_i$ . Q.E.D.

We now present the equilibrium of the model introduced in the previous section. Both  $R_i(x_{-i})$  and  $R_i^*(x_{-i})$  slope downwards. If none of the firms provides a wage-rise contract to its workers, then the unique equilibrium occurs at the intersection of  $R_1(x_{-1}), R_2(x_{-2}), \dots, R_n(x_{-n})$ . Therefore, if firm  $i$  chooses  $x_i^*$  and offers a wage-rise contract, then its marginal cost of output has a discontinuity at  $x_i = x_i^*$ . Lemma 3 states that the provision of a wage-rise contract by firm  $i$  decreases its optimal quantity. The labour-managed firms choose quantities in a Cournot fashion. We now characterize the equilibrium of the model in the following proposition.

**Proposition 1:** In the equilibrium of the labour-managed oligopoly model with complementary goods, none of the labour-managed firms offers a wage-rise contract as a strategic commitment.

Proof: We consider firm  $i$ 's Stackelberg leader output. If firm  $i$  is the Stackelberg leader, then it selects  $x_i$  and the other firms select their outputs after observing  $x_i$ . When firm  $i$  is the Stackelberg leader, it maximizes  $\psi(x_i, R_{-i}(x_i))$  with respect to  $x_i$ . Therefore, firm  $i$ 's Stackelberg leader output satisfies the following first-order condition:

$$\left( \frac{\partial p_i}{\partial x_i} x_i + p_i - r_i \frac{dk_i}{dx_i} \right) l_i - (p_i x_i - r_i k_i - f_i) \frac{dl_i}{dx_i} + \frac{\partial p_i}{\partial x_i} x_i \frac{dR_{-i}}{dx_i} l_i = 0. \quad (11)$$

Here  $\frac{\partial p_i}{\partial x_i} < 0$  (Assumption 1) and  $\frac{dR_{-i}}{dx_i} < 0$  (Lemma 2). To satisfy (11),

$\left( \frac{\partial p_i}{\partial x_i} x_i + p_i - r_i \frac{dk_i}{dx_i} \right) l_i - (p_i x_i - r_i k_i - f_i) \frac{dl_i}{dx_i}$  needs to be negative. Hence, each firm's

Stackelberg leader output is higher than its Cournot output.

On the other hand, Lemma 3 states that firm  $i$ 's optimal output is smaller when it adopts a wage-rise contract as a strategic commitment than when it does not. In addition, Lemma 1 means that  $R_i(x_{-i})$  is always downward sloping. Thus, the proposition is proved. Q.E.D.

Proposition 1 means that the equilibrium coincides with the Cournot solution with no wage-rise contracts.

## 4. Conclusion

We have shown the equilibrium of quantity-setting oligopoly competition in which labour-managed firms can offer wage-rise contracts as a strategic commitment device. Ohnishi (2007) examines a labour-managed Cournot duopoly model with substitute goods shows that there is an equilibrium in which at least one labour-managed firm offers a wage-rise contract as a strategic commitment. As a result, we find that our results are quite different from the results obtained from the labour-managed Cournot oligopoly model with substitute goods.

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