Excess burden of taxation and environmental policy mix with a consumer-friendly firm

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1. Introduction

During the last generation, market-based environmental regulations have been salient features of economic policies in polluting industries. The governments in developed and developing countries have continuously conducted indirect environmental policies such as emission taxes and tradable emission permits. Many economists have also shown that governments can promote social welfare by implementing market allocation of tradable emission permits or equivalently emission tax since it can minimize abatement costs when they differ between the regulated firms. For example, Borenstein (1988), Malueg (1990) and Sartzetakis (1997) showed that the tradable permits can increase social welfare in a sufficiently competitive production market with pollution when there are differences with respect to the abatement technologies among regulated firms. Due to its equivalence between tradable permits and emission taxes on the efficiency and welfare consequences, the widespread acceptance of tradable emission permits policy is getting increasing.[1]

On the other hand, climate change becomes a global challenge which requires not only setting the goals for the reduction of greenhouse gas emissions but also developing effective policy instruments to achieve them. During the global climate change negotiations, tradable emission permits have emerged as an essential policy tool. Nowadays, it becomes a successful international experiment for controlling a large amount of greenhouse gases in the world.[2]

However, the widespread acceptance of permits trading program generates an ongoing debate among economists on the efficiency of environmental and climate change policy. For example, Requate (1993), Sartzetakis (2004), Lee and Park (2005) and Garcia et al. (2018) demonstrated that if firms differ in both production and abatement technologies, the tradable permits cannot always assure efficiency. Hence, addressing the treatment of emission permits and offsets in both direct and indirect taxation is vital and practical. Failure to deal with potential tax obstacles could make the desired reductions in greenhouse gas emissions excessively costly and impede the global integration of carbon markets. Therefore, the appropriate choice of policy instruments will be the stringency of the effective policies and economic efficiency.

This study analyzes the policy interplay between the tradable emission permits and emission tax policies. Our approach links two existing research lines of related works. First, contrary to the behavioral assumption on the firms with homogeneous objectives of profit-maximization, we examine different objective functions between the firms and investigate how the heterogeneity of objectives affects the equivalence. In particular,

[1] Since the United States implemented experimental permits trading systems such as the sulfur dioxide (SO_2) trading program, in which allowances were freely allocated under the Clean Air Act, many countries including the European Union, Japan, Korea and China have introduced this program gradually. Stavins (1998), Kato (2006), Burtraw and McCormack (2017) introduced some useful real-world discussions on the tradable emission permits in the United Nations and the United States.

[2] The European Emission Trading System (EU ETS) established in 2005 already covers about half of total greenhouse gas emissions coming from Europe. With similar schemes under active consideration by a number of other countries, the share of total emissions from developed countries covered by "cap and trade" or other tradable permit regimes could triple in a few years. See OECD website (http://www.oecd.org/tax/tax-policy/).

we consider a production market which is characterized by the co-existence of for-profit firms and not-for-profit firms. We regard a not-for-profit firm as a consumer-friendly firm and define the objective of this firm as a combination of consumer surplus and its profits. This type of model formulation is one way of adopting corporate social responsibility (CSR) initiatives, in which it utilizes consumer surplus as a proxy of CSR concern.[3] Owing to the current expansion of CSR, the heterogeneity of objectives among the firms in this market configuration is as an essential part of our analysis.[4]

Second, contrary to the revenue-neutral property of grand-fathering allocation of permits, we take account the excess burden of taxation as a public finance, which can reduce or increase the welfare loss. On the one hand, we assume that the government utilizes a grandfathering approach for initial allocating of costless permits to the firms because it has by far been the dominant allocation approach in practice, both because it can offset the costs of emission reduction as well as for political reasons.[5] On the other hand, the emission tax revenue can provide double-dividend effect, which can be used for not only externality but for public finance to eliminate other distorting tax system such as income tax. However, the emission tax has also its own distorting effects on labor supply, which can have the excess burden as a tax on labor income.[6] Thus, the effect of the excess burden of taxation in environmental policies on the welfare consequences is an important part of our analysis.

In this study, we examine a Cournot duopoly market with a consumer-friendly firm in which both firms have the same abatement technologies and emit the same pollutants in the presence of excess burden of taxation. We then investigate the efficiency of policy mix between tradable permits and emission taxes. In particular, we analyze the interplay between the two policies and find the equivalent conditions for welfare

[3]Recently, driven by capital market and industry associations, CSR has become a global mainstream business strategy. For comprehensive discussions on recent CSR trends, see McWilliams and Siegel (2001), Benabou and Tirole (2010), Schreck (2011), Kitzmueller and Shimshack (2012) and Crifo and Forget (2015). Numerous theoretical studies that analyzed the CSR activities in different competition models include Goering (2012, 2014), Kopel and Brand (2012), Brand and Grothe (2013, 2015), Matsumura and Ogawa (2014), Kopel (2015), Lambertini and Tampieri (2015), Leal et al. (2018) and Garcia et al. (2018) among others.

[4]On the other hand, the analysis on mixed oligopolies where a welfare-maximizing public firm competes with profit-oriented private firms has also become richer and more diverse. For example, the effects of environmental policies under different market configurations are examined in Kato (2006), Ohori (2006), Wang and Wang (2009), Pal and Saha (2015), Xu et al. (2016a), Hsu et al. (2017), Xu and Lee (2018) and Lian et al. (2018). However, these analyses have focused on the various factors that affect the optimal privatization policies rather than the interplay between the environmental policies, which is main interest of our approach.

[5]Recently, it is proposed to use a consignment auction which combines aspects of free allocation and auctioning into one mechanism. See Stavins (1998), Fowlie (2010), Burtraw and McCormack (2017) and Khezr and MacKenzie (2018) for more discussions.

[6]Laffont and Tirole (1986) and Lin and Tan (1999) argued that the government's public policy might cause the welfare loss. For more discussion on the emission taxes, see Bovenberg and De Mooij (1994), Bovenberg and Goulder (1996) and Fullerton (1997). Regarding the excess burden of taxation in a mixed market where a public firm competes with private firms, see also Matsumura and Tomaru (2013, 2015) and Xu et al. (2016b).

consequences. We obtain two main results. First, for the parameters under which the firm does not sell all its emission quota, the government chooses the significant policy mix if the excess burden of taxation is large, whereas it chooses the single policy with permits otherwise. Second, for the parameters under which the firm sells all its emission quota, the government always chooses the significant policy mix. It shows that emission tax can be redundant and thus policy mix is degenerated when both excess burden of taxation and the degree of consumer-friendliness are low. It also shows that when the excess burden of taxation is significant, tradable permits policy with tax treatment is efficient to enhance welfare in the presence of a consumer-friendly firm. Finally, we show that when the degree of consumer-friendliness is sufficiently high in which a consumer friendly firm is strongly aggressive in production, environmental policy mix is efficient even in the tax revenue-neutral case where the excess burden of taxation does not matter.

The remainder of this paper is organized as follows. In section 2, we formulate a Cournot duopoly model with a consumer-friendly firm emitting same pollutants. We analyze the policy mix with tradable permits and tax in section 3. Section 4 concludes the paper.

2. Model

We consider a quantity-setting Cournot duopoly model. One of the firms is a consumer-friendly (CF) firm (hereafter referred to as firm 0) that cares for not only its profits but consumers surplus. The other is a for-profit (FP) firm (hereafter referred to as firm 1) that maximizes only its profits. Firms sell homogeneous output, $q_0 > 0$ and $q_1 > 0$, respectively, at the market clearing price $p(Q) = 1 - Q$ where $Q = q_0 + q_1$. We assume that both firms have identical technologies and the production cost function takes a quadratic form, $c(q_i) = q_i^2$, $i \in \{0, 1\}$.

Production leads to pollution, $e_i \geq 0$, but each firm can reduce pollution by undertaking abatement activities. We assume that each firm produces the same pollutants and has the same abatement technology. In particular, suppose that firm i chooses pollution abatement level $z_i > 0$. Then, the emission level can be reduced to $e_i = q_i - z_i$ by investing an amount of $\frac{z_i^2}{2}$ in abatement.

We consider a tradable permits regulation combined with a tax policy that the regulator uses to protect the environment. The government assigns an emission quota, $\xi_0 = \xi_1 = \xi > 0$, to each firm and allow it to trade emission permits at the market price. We assume that the emission market is perfectly competitive and thus the market price of permits, $\lambda > 0$, is determined by the market clearing price.[7] Thus, if we define the

[7]It implies that both firms do not have market powers in the competitive emission market and thus they behave as price takers. Note that both firms have the market power in the products market and thus they behave as price makers. On the other hand, we assume that an auctioneer in the competitive emission market can choose the emission market-clearing price which equates the total supply (which is given) with the demand of emissions. See Sartzetakis (2004), Lee and Park (2005) and Kato (2006). In Appendix F, we relax this assumption and consider a government-controlled competitive market, in which the emission market price is determined by the demand and total supply of quotas, which is controlled by the government. In fact,

net demand of firm i as $D_i = e_i - \xi$, total net demand of emission permits is zero at the market equilibrium $D_0 + D_1 = 0$. The government, as well, might impose a tax on the emission level, for which the tax rate is t. The resulting total tax revenue collected by the government is $T = t\sum_i e_i$. The profit of firm i is given by:

$$\pi_i = p \cdot q_i - q_i^2 - \frac{1}{2}z_i^2 - t \cdot e_i - \lambda \cdot (e_i - \xi), \quad i = 0, 1 \tag{1}$$

Note that the firm i buys emission permits if $e_i - \xi > 0$ while it sells if $e_i - \xi < 0$ in the market price of $\lambda > 0$.

We assume that the FP firm seeks only for profit maximization. However, the CF firm maximizes profits plus a fraction of consumer surplus, $CS = \frac{Q^2}{2}$. Thus, the payoff that CF firm maximizes is as follows[8]

$$V_0 = \pi_0 + \theta CS \tag{2}$$

The parameter $\theta \in (0, 1)$ measures the degree of concern on consumer surplus that the CF firm has, which is exogenously given.

The extent of environmental damage due to pollution by the industry is given by

$$ED = \frac{\left(\sum_{i=0}^{1} \xi_i\right)^2}{2} = 2\xi^2 \tag{3}$$

We assume that the government revenue from emission tax can be used to reduce tax rates in other markets, which results in the reduction of dead weight loss due to distortionary taxation. That is, we consider the excess burden of taxation, $\rho \cdot T$, where ρ represents the shadow cost of the government revenue, which can be used to reduce the excess burden of taxation as a public finance. Notice that ρ can be positive or negative, but we assume that $\rho > \rho_{so}(\theta) = \frac{-261+90\theta-11\theta^2}{540-198\theta+18\theta^2}$, which ensures the concavity of welfare function where $\rho_{so}(\theta) < 0$. It implies that the excess burden of taxation can cause either welfare-distorting effect or welfare-enhancing effect.

Then, the social welfare is the sum of consumer surplus (CS), the profits of both firms ($\pi_0 + \pi_1$) and the total taxes collected by the government (T) minus environmental damage (ED):

$$W = CS + \pi_0 + \pi_1 + (1 + \rho)T - ED \tag{4}$$

The game is played as follows: In the initial stage, the permit price, λ, is given *ex-ante* where net demand of emissions equals zero. Then, the government chooses emission quota ξ and tax rate t, expecting that λ is

as Burtraw and McCormack (2017) and Khezr and MacKenzie (2018) mentioned, the inefficiency of tradable emission permits may result from thin emission trading markets, weak price discovery, and regulatory or organizational complexities that hinder recognition of opportunity costs and innovation.

[8]If we regard a representative consumer as a green-consumer who takes care of environmental damage into her utility, as analyzed in Liu et al. (2015), we can define a net consumer surplus as $NCS = CS - ED$. Accordingly, we can also define the objective function of a consumer-friendly firm as $V_0 = \pi_0 + \theta NCS$, instead of $\pi_0 + \theta CS$, which might affect the analysis with a consumer-friendly firm in general. However, it is interesting to note that all the results in the analysis still hold because $ED = 2\xi^2$ in (3) is fixed in the 3rd and 2nd stages under the given quotas.

fixed, in the first stage. Having observed λ, ξ and t, both firms choose the abatement levels z_i in the second stage and outputs q_i in the last stage.[9]

3. Optimal policy mix

The last stage firms 0 and 1 choose their outputs to maximize (2) and (1), respectively. By solving these problems the equilibrium output as a function of the price of emission permits, λ, and tax rate, $t \geq 0$, is obtained:[10]

$$q_0 = \frac{(3+\theta)(1-t-\lambda)}{3(5-\theta)}, \quad q_1 = \frac{(3-\theta)(1-t-\lambda)}{3(5-\theta)}, \quad Q = \frac{2(1-t-\lambda)}{5-\theta} \tag{5}$$

In the second stage, firms choose abatement efforts to maximize their payoffs. Firm 0 chooses $z_0 \leq q_0$ (which implies $e_0 \geq 0$) that maximizes (2) while firm 1 chooses $z_1 \leq q_1$ (which implies $e_1 \geq 0$) that maximizes (1). Thus, the analysis should allow for corner solutions. Let

$$\Omega_0 = V_0 + \mu_0(q_0 - z_0) \quad \text{and} \quad \Omega_1 = \pi_1 + \mu_1(q_1 - z_1).$$

Then, the Kuhn-Tucker optimality conditions for firms $i = 0, 1$ are stated as

$$\frac{\partial \Omega_i}{\partial z_i} = 0, \quad \frac{\partial \Omega_i}{\partial \mu_i} = q_i - z_i \geq 0 \quad \text{and} \quad \mu_i(q_i - z_i) = 0 \quad \text{where} \quad \mu_i \geq 0 \quad \text{and} \quad z_i \geq 0. \tag{6}$$

These conditions are simplified as follows:

$$\mu_0 = \mu_1 = 0 \quad \text{for} \quad (\theta, \lambda, t) \in \mathcal{R}_1; z_0 = z_1 = t + \lambda \tag{7}$$

$$\mu_0 = 0, \mu_1 = -\frac{3-\theta-2(t+\lambda)(9-2\theta)}{3(5-\theta)} \quad \text{for} \quad (\theta, \lambda, t) \in \mathcal{R}_2; z_0 = t + \lambda, z_1 = q_1 \tag{8}$$

$$\mu_0 = -\frac{3+\theta-2(t+\lambda)(9-\theta)}{3(5-\theta)}, \mu_1 = -\frac{3-\theta-2(t+\lambda)(9-2\theta)}{3(5-\theta)} \quad \text{for} \quad (\theta, \lambda, t) \in \mathcal{R}_3; z_0 = q_0, z_1 = q_1 \tag{9}$$

where the regions $\mathcal{R}_i, i = 1, ..., 3$ are defined in Appendix A.[11]

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[9] In Appendix F, we consider a government-controlled competitive market where the market clearing permit price is determined after government chooses the emission quota and the tax rate, and then firms move later in which abatement choices are taken in the second stage and output choices in the last stage. This is similar to the case that transactions occur at the market clearing price in the financial markets where trade is centralized by a market operator. For more discussion on centralized competitive market, see Moreno and Ubeda (2006). In the appendix, we can show that main findings of our analysis still hold even under the given quotas determined by the government.

[10] It is interesting to note that the abatement choices in the second stage is not explicitly linked with the third stage equilibrium. It is so because the consumer-friendly firm does not care for its rival's profits and both firms use an end-of-pipe technology in their abatement activities, which is separable and additive to the production. We are thankful to an anonymous referee for pointing out this mechanism.

[11] In the analysis, we restrict our attention to the non-negative emission tax where $t \geq 0$ for practical implications. However, in Appendix G, we allow for emission subsidy where $t < 0$ and show that the policy mix is always assure the efficiency. This implies that two policies are likely to give rise to a higher welfare than one policy in general. We are thankful to an anonymous referee on this point.

Lemma 1. *Firm 0's emissions are always positive, i.e., $e_0 > 0$.*

Proof. $e_0 = 0$ only if $(\theta, \lambda, t) \in \mathcal{R}_3$. In this case, we also have $e_1 = 0$. Then, the net demand $\sum_{i=0}^{1} D_i = \sum_{i=0}^{1} (\xi - e_i) = 2\xi > 0$, which contradicts the assumption that the total net demand of emission permits is zero at the market equilibrium, which means that the case where $e_0 = 0$ never occurs. □

Lemma 1 allows us to focus in regions $\mathcal{R}_1$ and $\mathcal{R}_2$ that correspond to the cases where (i) none of the firms abates all contaminants, i.e., $z_0 = z_1 = t + \lambda$ and (ii) firm 1 abates all contaminants, i.e., $z_0 = t + \lambda$ and $z_1 = q_1$, respectively.

3.1. None of the firms abates all contaminant, i.e, $e_i > 0$, $i = 0, 1$ (Region $\mathcal{R}_1$)

In the first stage, the regulator choose simultaneously the emission quota $\xi > 0$ and the uniform tax $t \geq 0$ to maximize welfare given in (4).

$$Max \quad W = Q - \frac{Q^2}{2} - \sum_{i=0}^{1} q_i^2 - \frac{1}{2}\sum_{i=0}^{1} z_i^2 + \rho t \left(Q - \sum_{i=0}^{1} z_i\right) + \lambda\left(2\xi - \left(Q - \sum_{i=0}^{1} z_i\right)\right) - 2\xi^2$$
$$\text{s.t. } t \geq 0, \xi > 0 \tag{10}$$

The first order conditions are:

$$\frac{\partial W}{\partial \xi} = \lambda(2) - 4\xi = 0 \tag{11}$$

$$\frac{\partial W}{\partial t} = (1 - Q(t,\theta,\lambda))\frac{\partial Q}{\partial t} - 2\sum_{i=0}^{1} q_i(t,\theta,\lambda)\frac{\partial q_i}{\partial t} - \sum_{i=0}^{1} z_i(t,\theta,\lambda)\frac{\partial z_i}{\partial t} + (\rho t - \lambda)\left(\frac{\partial Q}{\partial t} - \sum_{i=0}^{1}\frac{\partial z_i}{\partial t}\right) \tag{12}$$
$$+ \rho\left(Q(t,\theta,\lambda) - \sum_{i=0}^{1} z_i(t,\theta,\lambda)\right) = 0$$

which yield

$$\xi = \frac{\lambda}{2} \quad \text{and} \quad t = max\{0, t_1^*\} \tag{13}$$

where $t_1^* = \frac{-\left(9-9\theta-2\theta^2\right)(1-\lambda)+\left(45-9\theta+\left(-270+99\theta-9\theta^2\right)\lambda\right)\rho}{261-90\theta+11\theta^2+(540-198\theta+18\theta^2)\rho}$. From (13), we have two cases: (i) significant policy mix, i.e, $t = t_1^* > 0$ and (ii) single policy with permits, i.e., $t = 0$.

3.1.1. Significant policy mix, i.e, $t_1^* > 0$

Making use of (5), (7) and (13) with $t = t_1^*$ we can find the permit price that clears the permits market $D_0 + D_1 = 0$ which yields:

$$\lambda^m = \frac{2\left(9(7 + 6\rho) - 2\theta^2 - 9\theta(2 + \rho)\right)}{H} \tag{14}$$

where $H = 9(101 + 132\rho) - 18\theta(17 + 23\rho) + \theta^2(29 + 36\rho) > 0$, From (14), the equilibrium tax, emission quota, output and abatement levels are obtained:

$$\begin{aligned} t_1^m &= \frac{3\left(-3(3-5\rho)+2\theta^2+3\theta(3-\rho)\right)}{H} \\ \xi^m &= \frac{9(7+6\rho)-2\theta^2-9\theta(2+\rho)}{H} \\ q_0^m &= \frac{3(3+\theta)(18+23\rho-\theta(3+4\rho))}{H} \\ q_1^m &= \frac{3(3-\theta)(18+23\rho-\theta(3+4\rho))}{H} \\ z_0^m &= z_1^m = \frac{9(11+17\rho)+2\theta^2-9\theta(1+3\rho)}{H} \end{aligned} \tag{15}$$

Finally, we have the resulting environmental damage and social welfare:

$$\begin{aligned} \pi_0^m &= \frac{\tau_0+\tau_1\rho+\tau_2\rho^2}{2H^2} \\ \pi_1^m &= \frac{\mu_0+\mu_1\rho+\mu_2\rho^2}{2H^2} \\ ED^m &= \frac{2\left(9(7+6\rho)-2\theta^2-9\theta(2+\rho)\right)}{H^2} \\ W^m &= \frac{3\left(\sigma_0+\sigma_1\rho+\sigma_2\rho^2+\sigma_3\rho^3\right)}{H^2} \end{aligned} \tag{16}$$

where τ_i, μ_i $(i = 0, 1, 2)$ and σ_j $(j = 0, 1, 2, 3)$ are as presented in Appendix B. Note that $\pi_i^m > 0$, for $i = 0, 1$. This implies that the policy mix might impose double financial burdens on the firms but its compliance cost might not be significant since they can earn non-negative profits under the environmental regulations.

3.1.2. Single policy with permits, i.e, $t = 0$

Making use of (5), (7) and (13) with $t = 0$ we can find the permit price that clears the permits market $D_0 + D_1 = 0$ which yields:

$$\lambda^s = \frac{2}{17-3\theta} \tag{17}$$

From (17), the equilibrium emission quota, output and abatement levels are obtained:

$$\begin{aligned} \xi^s &= \frac{1}{17-3\theta} \\ q_0^s &= \frac{3+\theta}{17-3\theta} \\ q_1^s &= \frac{3-\theta}{17-3\theta} \\ z_0^s &= z_1^s = \frac{2}{17-3\theta} \end{aligned} \tag{18}$$

Finally, we have the resulting environmental damage and social welfare:

$$\pi_0^s = \frac{2\left(11 - 3\theta - 2\theta^2\right)}{(17 - 3\theta)^2}$$
$$\pi_1^s = \frac{2\left(11 - 6\theta + \theta^2\right)}{(17 - 3\theta)^2}$$
$$ED^s = \frac{2}{(17 - 3\theta)^2}$$
$$W^s = \frac{2\left(30 - 9\theta - \theta^2\right)}{(17 - 3\theta)^2} \tag{19}$$

Note that single policy mix does not increase firm's compliance cost because it can earn non-negative profits under the tradable emission permits regulation.

Lemma 2. *In the case where* $e_1 > 0$, $t_1^m > 0$ *if* $\rho > \rho^m \equiv max\{\frac{9-9\theta-2\theta^2}{15-3\theta}, \frac{-\eta_4(\theta)+\sqrt{\eta_3(\theta)\eta_5(\theta)}}{36(6-\theta)(17-3\theta)^2}\}$ *where* $\eta_i (i = 3, 4, 5)$ *are as presented in Appendix C.*

Proof. From (15), we can show that $t_1^m > 0$ and $W^m > W^s$ if $\rho > \rho^m$; otherwise $t_1^m \leq 0$ or $W^s \geq W^m$. □

This lemma states that when both firms do not abate all emissions ($e_i > 0$), policy mix with emission tax treatment might be efficient under certain condition. It yields the following proposition:

Proposition 1. *When firm* 1 *doesn't sell all its emission quota under tradable emission permits policy, the government should choose policy mix with emission tax treatment only if the shadow cost of the excess burden of taxation is high. Otherwise, the government can choose single policy with tradable emission permits.*

This proposition implies that the excess burden of taxation has a significant role in determining the efficiency of policy mix in environmental regulation. In particular, when the heterogeneity between the firms is not significant and thus both firms actively compete in production and trade emissions, emission tax treatment is useful only when its double dividend effect is meaningful. It also implies that the emission tax should be always positive when the excess burden of taxation is significant irrespective of the degree of consumer-friendliness. Thus, the welfare effects of the excess burden of taxation interplay with the degree of consumer-friendliness. Note that even under the tax revenue-neutral case where $\rho = 0$, policy mix can be always optimal when $\theta \geq \frac{3}{4}\left(-3 + \sqrt{17}\right)$ so that firm 0 is more aggressive in production. (See regions in Appendix E.) It indicates that as the excess burden of taxation decreases, tax treatment with tradable emission permits policy is efficient when the heterogeneity between the firms is significant.

3.2. Firm 1 abates all contaminant $e_1 = 0$ *(Region* $\mathcal{R}_2$*)*

If $z_0 = t + \lambda$ and $z_1 = q_1$, at first stage the FOC (11) and (12) yield

$$\xi = \frac{\lambda}{2} \quad \text{and} \quad t = max\{0, t_2^*\} \tag{20}$$

where $t_2^* = \frac{-9+12\theta+5\theta^2-4\left(9-3\theta+2\theta^2\right)\lambda+3(5-\theta)(3+\theta-2(9-\theta)\lambda)\rho}{2(153-48\theta+7\theta^2+6(9-\theta)(5-\theta)\rho)}$. Then, from (20) we have the following lemma.

Lemma 3. *In the case where $e_1 = 0$, single policy with $t = 0$ is never optimal.*

Proof. Use $t = 0$, $z_0 = t + \lambda = \lambda$ and $z_1 = q_1 = \frac{(3-\theta)(1-\lambda)}{3(5-\theta)}$ to obtain the permits price value that solves $D_0 + D_1 = 0$: $\lambda = \lambda_0 = \frac{3-\theta}{33-5\theta}$. Notice that for any $0 < \theta < 1$, $(\theta, \lambda = \lambda_0, t = 0) \notin \mathcal{R}_2$, which means that there is no price that clears the permits market for these equilibrium outcomes. □

Thus, we have $t_2^* > 0$. Making use of (5), (8) and (20) we can find the permit price that clears the permits market $D_0 + D_1 = 0$ which yields:

$$\lambda^{m'} = \frac{(3+\theta)(3(4+3\rho) - \theta(4+\rho))}{\Delta} \tag{21}$$

where $\Delta = \theta^2(9 + 8\rho) - 12\theta(7 + 10\rho) + 9(35 + 48\rho) > 0$.

From (21), the equilibrium tax, emission quota, output and abatement levels are obtained:

$$\begin{aligned} t_2^{m'} &= \frac{(3+\theta)(\theta(11-3\rho) - 3(3-5\rho))}{2\Delta} \\ \xi^{m'} &= \frac{(3+\theta)(3(4+3\rho) - \theta(4+\rho))}{2\Delta} \\ q_0^{m'} &= \frac{(3+\theta)(3(13+17\rho) - \theta(5+7\rho))}{2\Delta} \\ q_1^{m'} = z_1^{m'} &= \frac{(3-\theta)(3(13+17\rho) - \theta(5+7\rho))}{2\Delta} \\ z_0^{m'} &= \frac{(3+\theta)(3(5+11\rho) + \theta(3-5\rho))}{2\Delta} \end{aligned} \tag{22}$$

Finally, we have the resulting environmental damage and social welfare:

$$\begin{aligned} \pi_0^{m'} &= \frac{(3+\theta)\left(\upsilon_0 + \upsilon_1\rho + \upsilon_2\rho^2\right)}{8\Delta^2} \\ \pi_1^{m'} &= \frac{\kappa_0 + \kappa_1\rho + \kappa_2\rho^2}{8\Delta^2} \\ ED^{m'} &= \frac{(3+\theta)^2(3(4+3\rho) - \theta(4+\rho))^2}{2\Delta^2} \\ W^{m'} &= \frac{3\left(\psi_0 + \psi_1\rho + \psi_2\rho^2 + \psi_3\rho^3\right)}{4\Delta^2} \end{aligned} \tag{23}$$

where υ_i, κ_i $(i = 0, 1, 2)$ and ψ_j $(j = 0, 1, 2, 3)$ are as presented in Appendix D. Note also that $\pi_i^{m'} > 0$, for $i = 0, 1$. This also implies that the compliance cost under the policy mix might not be significant.

Proposition 2. *When firm* 1 *sells all its emission quota under tradable emission permits policy, the government should choose policy mix with emission tax treatment irrespective of the excess burden of taxation.*

Proof. It follows from Lemma 3. Let $\rho^t = max\{\frac{-36+39\theta-\theta^2}{27-45+6\theta^2}, \rho^{w_0}, \rho^{w_1}\}$, where ρ^{w_0} is such that $W^{m'}(\rho^{w_0}, \theta) = W^s(\rho^{w_0}, \theta)$ and ρ^{w_1} is such that $t_2^{m'} > 0$ and $W^{m'}(\rho^{w_1}, \theta) = W^m(\rho^{w_1}, \theta)$. If we compare the welfare between (16) and (23), we can show that $W^{m'}$ is the highest if $\rho > \rho^t$ for any $\frac{3}{4}\left(5 - \sqrt{17}\right) < \theta < 1$. (See regions in Appendix E.) □

Note that firm 1 completely abates all contaminants, i.e., $z_1 = q_1$, only when θ is sufficiently high. When firm 0 is strongly aggressive in production, it will consume all the emission quotas in this industry. And firm 1 sells all its emission permits to firm 0 by abating all contaminants under tradable permits policy. In that case, emission permits market does not function efficiently because there is no room to reduce social cost by trading permits. Hence, it is natural to devise an appropriate tax treatment irrespective of the effect of excess burden of taxation. Note that the emission tax should be always positive when the degree of consumer-friendliness is significant. Therefore, the heterogeneity between the firms play a significant role in determining the optimal policy mix in environmental regulation.

4. Concluding remarks

This study considers an excess burden of taxation in a Cournot duopoly model with a consumer-friendly firm and examines environmental policy mix between tradable permits and emission taxes. We analyze the interplay between the two policies and find the equivalent conditions for welfare consequences. We show that emission tax can be redundant and thus policy mix is degenerated when both excess burden of taxation and the degree of consumer-friendliness are low. However, when the excess burden of taxation is significant, tradable permits policy with tax treatment is efficient to enhance welfare in the presence of a consumer-friendly firm. Finally, when the degree of consumer-friendliness is sufficiently high in which a consumer friendly firm is strongly aggressive in production, it consumes all emission permits and thus tradable permits policy with tax treatment is efficient even in the tax revenue-neutral case. Therefore, the mixture of the regulatory instruments matter for efficiency.

Our analysis shows that the CSR initiatives of the firms and the excess burden of taxation for the government can play significant roles in the design and implementation of environmental policy. However, it needs to be further examined in alternative settings under different market structures. This has to be left for future research.

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Appendix A. Regions

Define the following regions:

$$
\begin{aligned}
\mathcal{R}_1 =& \{(\theta,\lambda,t)\in\mathbb{R}^3 \mid 0<\theta<1, \quad 0<\lambda<\tfrac{3-\theta}{2(9-2\theta)}, \quad 0\le t<\tfrac{3-\theta-2\lambda(9-2\theta)}{2(9-2\theta)}\} \\
\mathcal{R}_2 =& \{(\theta,\lambda,t)\in\mathbb{R}^3 \mid 0<\theta<1, \quad 0<\lambda<\tfrac{3+\theta}{2(9-\theta)}, \quad \max\left\{\tfrac{3-\theta-2\lambda(9-2\theta)}{2(9-2\theta)},0\right\}\le t<\tfrac{3+\theta-2(9-\theta)\lambda}{2(9-\theta)}\} \\
\mathcal{R}_3 =& \{(\theta,\lambda,t)\in\mathbb{R}^3 \mid 0<\theta<1, \quad 0<\lambda\le 1, \quad \max\left\{\tfrac{3+\theta-2\lambda(9-\theta)}{2(9-\theta)},0\right\}\le t\le 1-\lambda\}
\end{aligned}
\tag{A.1}
$$

Appendix B. Values of $\tau_i(\theta)$, $\mu_i(\theta)$,$i = 0, 1, 2$ and $\sigma_j(\theta)$, $j = 0, 1, 2, 3$

$$
\begin{aligned}
\tau_0 &= 130653 - 808838\theta - 7983\theta^2 + 7056\theta^3 - 628\theta^4 \\
\tau_1 &= 325782 - 2012204\theta - 19854\theta^2 + 17748\theta^3 - 1728\theta^4 \\
\tau_2 &= 206469 - 1288898\theta - 11979\theta^2 + 11520\theta^3 - 1152\theta^4
\end{aligned} \tag{B.1}
$$

$$
\begin{aligned}
\mu_0 &= 130653 - 1158830\theta + 38673\theta^2 - 5580\theta^3 + 344\theta^4 \\
\mu_1 &= 325782 - 2906628\theta + 100026\theta^2 - 15300\theta^3 + 864\theta^4 \\
\mu_2 &= 206469 - 1860030\theta + 65025\theta^2 - 10080\theta^3 + 576\theta^4
\end{aligned} \tag{B.2}
$$

$$
\begin{aligned}
\sigma_0 &= 57267 - 356640\theta + 5517\theta^2 + 90\theta^3 - 58\theta^4 \\
\sigma_1 &= 148554 - 934744\theta + 14652\theta^2 + 252\theta^3 - 152\theta^4 \\
\sigma_2 &= 97983 - 621544\theta + 9807\theta^2 + 216\theta^3 - 96\theta^4 \\
\sigma_3 &= 1620 - 594\theta + 54\theta^2
\end{aligned} \tag{B.3}
$$

Appendix C. Values of $\eta_i(\theta)$

$$
\begin{aligned}
\eta_3 &= 1233 - 684\theta + 11\theta^2 + 12\theta^3, \\
\eta_4 &= 24741 - 112056\theta + 2083\theta^2 - 223\theta^3 + 12\theta^4, \\
\eta_5 &= 772569 - 5336646\theta + 1395508\theta^2 - 15958\theta^3 + 587\theta^4 + 12\theta^5
\end{aligned} \tag{C.1}
$$

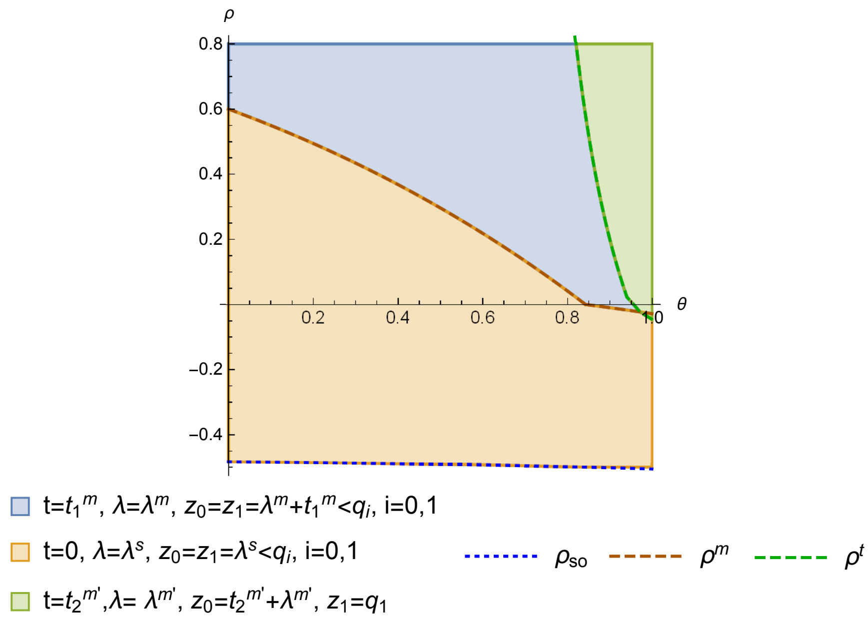
Appendix D. Values of $\upsilon_i(\theta)$, $\kappa_i(\theta)$,$i = 0, 1, 2$ and $\psi_j(\theta)$, $j = 0, 1, 2, 3$

$$
\begin{aligned}
\upsilon_0 &= 20655 - 169299\theta + 3345\theta^2 - 127\theta^3 \\
\upsilon_1 &= 53298 - 436650\theta + 8958\theta^2 - 558\theta^3 \\
\upsilon_2 &= 35451 - 291699\theta + 5985\theta^2 - 363\theta^3
\end{aligned} \tag{D.1}
$$

$$
\begin{aligned}
\kappa_0 &= 56781 - 371529\theta + 9666\theta^2 - 1704\theta^3 + 169\theta^4 \\
\kappa_1 &= 152118 - 1040049\theta + 27648\theta^2 - 3996\theta^3 + 234\theta^4 \\
\kappa_2 &= 103437 - 735489\theta + 19170\theta^2 - 2100\theta^3 + 81\theta^4
\end{aligned} \tag{D.2}
$$

$$\psi_0 = 27405 - 129778\theta + 11140\theta^2 + 146\theta^3 - 33\theta^4$$
$$\psi_1 = 74520 - 358556\theta + 28800\theta^2 + 528\theta^3 - 88\theta^4$$
$$\psi_2 = 51489 - 246778\theta + 16920\theta^2 + 470\theta^3 - 45\theta^4$$
$$\psi_3 = 810 + 288\theta - 60\theta^2 - 16\theta^3 + 2\theta^4 \tag{D.3}$$

Appendix E. Region space



Appendix F. The case when the government can control permits market price

In this Appendix, we assume that the emission market is a (government-controlled) centralized competitive market in which polluters trade the permits at the realization of market-clearing price. The government decides the total supply of permits and the tax rate in advance, by expecting the market price will be changed by its policy. Then, λ will be determined at the price where net demand is zero, given the fixed number of permits, determined by the government. After that, by observing λ, ξ and t, both polluters choose z_i in the second stage and q_i in the last stage. Then, the last and the second stages are the same as the original game, therefore, equations (5), (7)-(9) show the results of this stages. Lemma 1 also holds, then we only focus on regions $\mathcal{R}_1$ and $\mathcal{R}_2$.

Appendix F.1. None of the firms abates all contaminant, i.e, $e_i > 0$, $i = 0, 1$ (Region $\mathcal{R}_1$)

The government decides the total supply of permits and the tax rate in advance, by expecting the market price will be changed by its policy. This price would be determined where the net demand $D_0 + D_1 = 0$, then

$$\lambda_1 = \frac{1 - e(5-\theta)}{6-\theta} - t \tag{F.1}$$

Now the region $\mathcal{R}_1$ with $\lambda = \lambda_1$ could be rewritten as follows:

$$\mathcal{R}_1 = \left\{ 0 < \theta < 1, \frac{\theta}{2(9-2\theta)} < \xi < \frac{1}{5-\theta}, 0 \leq t < \frac{1-(5-\theta)\xi}{6-\theta} \right\}$$

.

In the first stage, the regulator choose simultaneously the emission quota $\xi > 0$ and the uniform tax $t \geq 0$ to maximize welfare given in (4).

$$\begin{aligned} Max \quad W &= Q - \frac{Q^2}{2} - \sum_{i=0}^{1} q_i^2 - \frac{1}{2}\sum_{i=0}^{1} z_i^2 + \rho t \left(Q - \sum_{i=0}^{1} z_i\right) + \lambda\left(2\xi - \left(Q - \sum_{i=0}^{1} z_i\right)\right) - 2\xi^2 \quad \text{(F.2)} \\ &= Q - \frac{Q^2}{2} - \sum_{i=0}^{1} q_i^2 - \frac{1}{2}\sum_{i=0}^{1} z_i^2 + \rho t(2\xi) - 2\xi^2 \\ &= \frac{63 - 18\theta - 2\theta^2}{9(6-\theta)^2} + \left(\frac{2(63-18\theta-2\theta^2)}{9(6-\theta)^2} + 2\rho t\right)\xi - \frac{(909 - 306\theta + 29\theta^2)}{9(6-\theta)^2}\xi^2 \end{aligned}$$

$s.t(\theta, \xi, t) \in \mathcal{R}_1$

Then we have the following results depending on the value of ρ and θ:

1. $\rho < 0$

   If $\rho < 0$, then W in (F.2) strictly decreases on t, then it must take the smallest possible value $t = 0$, this is a single policy with permits would be establish.

   (a) If $0 < \theta < \frac{3}{7}\left(12 - \sqrt{95}\right)$, then the maximum of W (with $t = 0$) is reached at $\xi = \frac{63-18\theta-2\theta^2}{909-306\theta+29\theta^2}$.

   (b) However, if $\frac{3}{7}\left(12 - \sqrt{95}\right) \leq \theta < 1$ then $W|_{t=0}$ decreases on ξ for any $\frac{\theta}{2(9-2\theta)} < \xi < \frac{1}{5-\theta}$, then $\xi \to \frac{\theta}{2(9-2\theta)}$

2. $\rho = 0$

   If $\rho = 0$, (F.2) does not depend on t, so it can take any value such that $0 \leq t < \frac{1-(5-\theta)\xi}{6-\theta}$.

   (a) If $0 < \theta < \frac{3}{7}\left(12 - \sqrt{95}\right)$, then the maximum is reached at $\xi = \frac{63-18\theta-2\theta^2}{909-306\theta+29\theta^2}$.

   (b) However, if $\frac{3}{7}\left(12 - \sqrt{95}\right) \leq \theta < 1$ then W decreases on ξ, then $\xi \to \frac{\theta}{2(9-2\theta)}$

3. $\rho > 0$

   If $\rho > 0$, then W strictly increases on t, then $t \to \frac{1-(5-\theta)\xi}{6-\theta}$ and
   $W \to W_{t_s} = \frac{63-18\theta-2\theta^2+\xi\left(126-36\theta-4\theta^2+(108-18\theta)\rho\right)-\xi^2\left(909-306\theta+29\theta^2+\left(540-198\theta+18\theta^2\right)\rho\right)}{9(6-\theta)^2}$

   (a) If $0 < \theta \leq \frac{3}{7}\left(12 - \sqrt{95}\right)$ or if $\frac{3}{7}\left(12 - \sqrt{95}\right) < \theta < 1$ and $\rho > \frac{-63+72\theta-7\theta^2}{54-42\theta+6\theta^2}$, W_{t_s} is maximized at $\xi = \frac{63-18\theta-2\theta^2+9(6-\theta)\rho}{909-306\theta+29\theta^2+(540-198\theta+18\theta^2)\rho}$.

(b) However, if $\frac{3}{7}\left(12-\sqrt{95}\right) < \theta < 1$ and $0 < \rho \leq \frac{-63+72\theta-7\theta^2}{54-42\theta+6\theta^2}$ then W_{t_s} decreases on ξ for any $\frac{\theta}{2(9-2\theta)} < \xi < \frac{1}{5-\theta}$, then $\xi \to \frac{\theta}{2(9-2\theta)}$

Therefore, Proposition 1 still holds: When firm 1 doesn't sell all its emission quota under tradable emission permits policy, the government chooses policy mix with emission tax treatment only if the shadow cost of the excess burden of taxation is high. Otherwise, the government chooses single policy with tradable emission permits.

Appendix F.2. Firm 1 abates all contaminant, i.e, $e_1 = 0$ (Region $\mathcal{R}_2$)

The government decides the total supply of permits and the tax rate in advance, by expecting the market price will be changed by its policy. This price would be determined where the net demand $D_0 + D_1 = 0$, then

$$\lambda_2 = \frac{3+\theta-6e(5-\theta)}{2(9-\theta)} - t \tag{F.3}$$

Now the region $\mathcal{R}_2$ with $\lambda = \lambda_2$ could be rewritten as follows:

$$\mathcal{R}_2 = \left\{0 < \theta < 1, 0 < e \leq \frac{\theta}{2(9-2\theta)}, 0 \leq t < \frac{3+\theta-(5-\theta)6e}{2(9-\theta)}\right\}$$

.

In the first stage, the regulator choose simultaneously the emission quota $\xi > 0$ and the uniform tax $t \geq 0$ to maximize welfare given in (4).

$$\begin{aligned} Max \quad W &= Q - \frac{Q^2}{2} - \sum_{i=0}^{1} q_i^2 - \frac{1}{2}\sum_{i=0}^{1} z_i^2 + \rho t\left(Q - \sum_{i=0}^{1} z_i\right) + \lambda\left(2\xi - \left(Q - \sum_{i=0}^{1} z_i\right)\right) - 2\xi^2 \quad \text{(F.4)} \\ &= Q - \frac{Q^2}{2} - \sum_{i=0}^{1} q_i^2 - \frac{1}{2}\sum_{i=0}^{1} z_i^2 + \rho t(2\xi) - 2\xi^2 \\ &= \frac{3\left(21-4\theta-\theta^2\right)}{4(9-\theta)^2} + 2\left(\frac{2\left(9-\theta^2\right)}{(9-\theta)^2} + t\rho\right)\xi - \frac{\left(3\left(105-28\theta+3\theta^2\right)\right)\xi^2}{(9-\theta)^2} \end{aligned}$$

s.t. $(\theta, \xi, t) \in \mathcal{R}_2$

Then we have the following results depending on the value of ρ and θ:

1. $\rho < 0$

   If $\rho < 0$, then W in (F.4) strictly decreases on t, then it must take the smallest possible value $t = 0$.

   (a) If $0 < \theta < \frac{3}{2}\left(13-3\sqrt{17}\right)$, then W (with $t = 0$) increases on ξ, then $\xi = \frac{\theta}{2(9-2\theta)}$.

   (b) However, if $\frac{3}{2}\left(13-3\sqrt{17}\right) < \theta < 1$ then the maximum of W (with $t = 0$) is reached at $\xi = \frac{2\left(9-\theta^2\right)}{3(105-28\theta+3\theta^2)}$.

2. $\rho = 0$

   If $\rho = 0$, (F.4) does not depend on t, so it can take any value such that $0 \leq t < \frac{3+\theta-(5-\theta)6e}{2(9-\theta)}$.

(a) If $0 < \theta < \frac{3}{2}\left(13 - 3\sqrt{17}\right)$, then W increases on ξ, then $\xi = \frac{\theta}{2(9-2\theta)}$.

(b) However, if $\frac{3}{2}\left(13 - 3\sqrt{17}\right) < \theta < 1$ then the maximum of W is reached at $\xi = \frac{2\left(9-\theta^2\right)}{3(105-28\theta+3\theta^2)}$.

3. $\rho > 0$

If $\rho > 0$, then W strictly increases on t, then $t \to \frac{3+\theta-(5-\theta)6\xi}{2(9-\theta)}$ and
$W \to W_{t_{s2}} = \frac{3\left(21-4\theta-\theta^2\right)}{4(9-\theta)^2} + (3+\theta)\left(\frac{4(3-\theta)}{(9-\theta)^2} + \frac{\rho}{9-\theta}\right)\xi - 3\left(\frac{105-28\theta+3\theta^2}{(9-\theta)^2} + \frac{2(5-\theta)\rho}{9-\theta}\right)\xi^2$

(a) If $0 < \theta \leq \frac{3}{2}\left(13 - 3\sqrt{17}\right)$ and $\rho > 0$ or if $\frac{3}{2}\left(13 - 3\sqrt{17}\right) < \theta < 1$ and $\rho > -\frac{36-39\theta+\theta^2}{27-27\theta+4\theta^2}$ then $W_{t_{s2}}$ increases on ξ for any $0 < \xi \leq \frac{\theta}{2(9-2\theta)}$, then $\xi = \frac{\theta}{2(9-2\theta)}$.

(b) However, if $\frac{3}{2}\left(13 - 3\sqrt{17}\right) < \theta < 1$ and $0 < \rho \leq -\frac{36-39\theta+\theta^2}{27-27\theta+4\theta^2}$ then $\xi = \frac{(3+\theta)(12-4\theta+(9-\theta)\rho)}{6(105-28\theta+3\theta^2+(90-28\theta+2\theta^2)\rho)}$.

Thus, Proposition 2 does not hold, but the optimal policy mix is still valid: When firm 1 sells all its emission quota under tradable emission permits policy, the government chooses policy mix with emission tax treatment only if the shadow cost of the excess burden of taxation is high. Otherwise, the government chooses single policy with tradable emission permits.

Appendix G. The case when the government can implement emission subsidy where $t < 0$

From Appendix A, we can define the following regions without any restriction on t:

$$\begin{aligned}
\mathcal{S}_1 &= \{(\theta,\lambda,t) \in \mathbb{R}^3 \mid 0 < \theta < 1, \quad \lambda > 0, \quad -\lambda \leq t < \tfrac{3-\theta-2\lambda(9-2\theta)}{2(9-2\theta)}\} \\
\mathcal{S}_2 &= \{(\theta,\lambda,t) \in \mathbb{R}^3 \mid 0 < \theta < 1, \quad \lambda > 0, \quad \tfrac{3-\theta-2\lambda(9-2\theta)}{2(9-2\theta)} \leq t < \tfrac{3+\theta-2\lambda(9-\theta)}{2(9-\theta)}\} \\
\mathcal{S}_3 &= \{(\theta,\lambda,t) \in \mathbb{R}^3 \mid 0 < \theta < 1, \quad \lambda > 0, \quad \tfrac{3+\theta-2\lambda(9-\theta)}{2(9-\theta)} \leq t \leq 1-\lambda\}
\end{aligned} \tag{G.1}$$

The government's problem is the same as in (4), except that there is no constraint on t, and W is concave if $\rho > \rho_{so}(\theta)$. When the constraint $\xi > 0$ must be satisfied, Lemma 1 still holds. Thus, if none of the firms abates all contaminant (Region $\mathcal{S}_1$), the emission quota and tax rate that satisfy $\frac{\partial W}{\partial \xi} = 0$ and $\frac{\partial W}{\partial t} = 0$ are: $\xi = \frac{\lambda}{2}$ and $t = t_1^*$. But, if firm 1 abates all contaminant (Region $\mathcal{S}_2$), the emission quota and tax rate that satisfy $\frac{\partial W}{\partial \xi} = 0$ and $\frac{\partial W}{\partial t} = 0$ are: $\xi = \frac{\lambda}{2}$ and $t = t_2^*$. Therefore, the policy mix with emission tax or subsidy always assures the efficiency.