Financial Deepening in a Two-Sector Endogenous Growth Model with Productivity Heterogeneity

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Abstract

We develop a tractable two-sector endogenous growth model in which heterogeneous entrepreneurs face borrowing constraints and the government collects tax to fund public eduction. This model is isomorphic to a Uzawa-Lucas model and there exists a balanced-growth path equilibrium in which the growth rate depends on the financial deepening level. We show that the policy tax rate exerts inverted U-shaped effects on the growth rate. Additionally, at the optimal policy tax rates the model's predictions are consistent with correlational regularities documented from 35 OECD countries with regards to financial deepening, factor accumulation and working hours.

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1 Introduction

This paper explores the effects of financial deepening on factor accumulation and growth dynamics in an economy with firm heterogeneity and financial imperfection. It is motivated by regularities documented from all 35 OECD countries using macroeconomic data from the Penn World Table 9.0 (PWT 9.0), updated Financial Development and Structure Data-set, and Barro-Lee Education Attainment Data-set. [1] Particularly, there are significant differences in the level of financial deepening among 35 OECD countries; the Private Credit indicator that reflects financial deepening,[2] ranges from 18% in Mexico to 187% in the USA. Figure (1) shows that among OECD countries, those economies with deeper financial markets tend to:

(i) Have higher levels of physical capital stock per employed person and higher average level of human capital.

(ii) Have less average annual hour worked and spend more time on education attainment activities. [3]

(iii) Have higher levels of Total Factor Productivity (TFP).

(iv) Have higher relative physical capital intensity, namely higher ratio of physical capital stock per employed person over the corresponding human capital level.

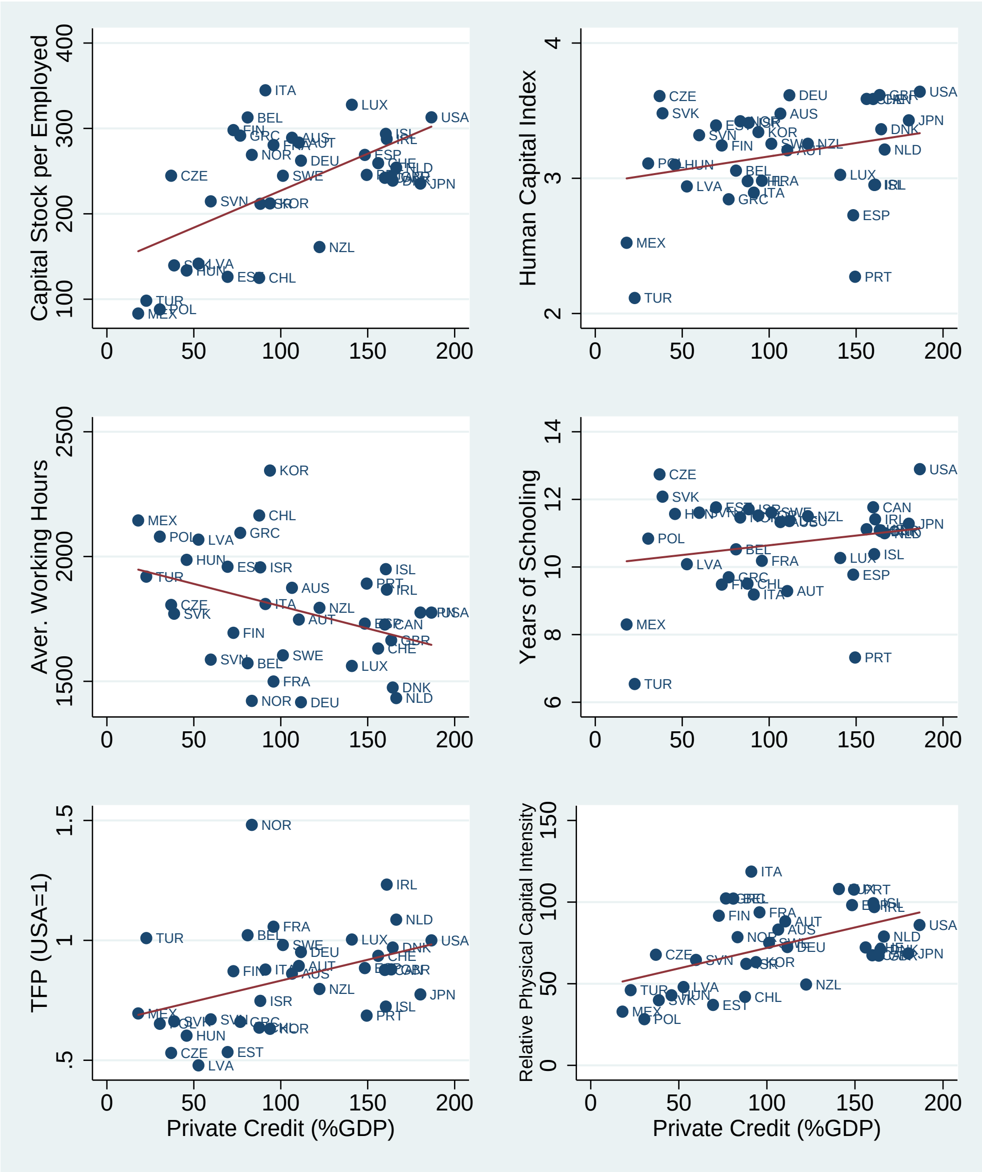
To explain these aforementioned regularities, this paper develops a tractable two-sector endogenous growth model in which heterogeneous entrepreneurs face borrowing constraints and the government collects tax to fund public eduction. It particularly focuses on the interactions between firm heterogeneity and the efficiency of the financial markets in allocating capital. To be more specific, because of financial imperfections all entrepreneurs including those with the highest productivity face

[1] See the Appendix A for the description of these Data-sets.

[2] Private Credit is the ratio of total credit issued by depository and other financial institutions to the private sector over GDP. Similar results using another financial deepening indicator, Liquid Liabilities (M3) over GDP, are available upon request.

[3] The average years of total schooling for the age group beyond 15 can be considered as a proxy for both the level of human capital and the average time engaging in education.

Figure 1: Correlational Regularities in OECD Countries



Notes: The horizontal axis in all sub-panels denotes Private Credit Indicator and its correlation coefficients (p-values) with other variables in the subpanels (across first) are 0.59 (0.00), 0.26 (0.12), -0.38 (0,02), 0.20 (0.24), 0.41 (0.02), 0.51 (0.00), respectively. Relative physical capital intensity is defined as ratio of physical capital stock per employed person over the corresponding human capital level See the Appendix A for data source and description.

borrowing constraints. Hence, highly productive entrepreneurs are unable to borrow capital to a desired level to extend production while less productive ones participate in producing goods. Consequently, capital is not efficiently allocated and the aggregated productivity is not optimal. [4] As the financial markets deepen, the borrowing constraint is gradually relaxed and more capital is allocated to more productive producers. As a result, the threshold of entrepreneurs who are active in producing goods, and aggregate productivity of the economy increase along with financial depth.

Productivity improvement in turn influences the aggregate dynamics of the economy. An increase in the aggregate productivity increases the capital returns, hence facilitating the accumulation of physical capital and raising the government's tax revenues. Productivity also increases the wage rates, so encourages workers to spend more time on the education activities, leading to faster human capital accumulation. As a result, a higher level of financial deepening leads to a higher aggregate productivity, more time spent on education, and a faster growth rate. In addition, it is shown that if the government's objective is to maximize the growth rates, the policy tax rates should be an increasing function of the financial deepening level. In general, a higher policy rate rate dampens the accumulation of physical capital but leads to higher government expenditure on education, hence faster accumulation of human capital. However, it turns out that the growth effects of financial deepening on physical capital accumulation, the effects that arises from aggregate productivity improvements, still dominates the taxation effects. Therefore, the relative physical capital intensity is higher in economies with deeper financial markets.

Related Literature: This paper belongs to the large literature that studies the relationships between financial market frictions and economic development (See, e.g, Levine 2005 and Banerjee and Duflo, 2005 for a detailed survey). It is closely related to the branch of literature studying the interactions between entrepreneurship, financial frictions, and productivity. [5] It contributes to this branch of literature by

[4] When there are no borrowing constraint, only entrepreneurs with the highest productivity produce goods, while all others just save and lend capital. Hence, capital is efficiently allocated to the most productive entrepreneurs and aggregate measured productivity is optimal.

[5] See Buera et al. (2015) for a thorough review of this literature.

examining the impacts of financial deepening in a tractable two-sector endogenous growth model with human capital accumulation.

This paper is closely related to and develops from the theoretical framework of Moll (2014), who explores the role of self-financing in a general equilibrium environment with heterogeneous productivity and financial imperfections. Specifically, Moll (2014) introduces productivity persistence into the previous work of Angeletos (2007) and Kiyotaki and Moore (2012), and shows that entrepreneurs can possibly accumulate enough internal funds to overcome borrowing constraints. As a result, self-financing can undo capital mis-allocation and reduce the long-run steady state TFP losses when the shocks are sufficiently persistent.

This paper extends the theoretical model of Moll (2014) by incorporating human capital and address the effects of financial deepening in a two-sector endogenous model. Unlike the model in Moll (2014), which is a one-sector model and is isomorphic to the Solow model, this model is isomorphic to a Uzawa-Lucas model. Specifically, we assume that human capital is accumulated by the worker's efforts, the existing level of human capital stock and also public eduction provided by the government who collects taxes. By incorporating human capital and taxation, this paper can address the effects of financial deepening on the accumulation of both physical and human capital and also working hours. It can also analyze the effects of taxation on the aggregate dynamics. This paper contributes to the two-sector endogenous growth literature by showing that in the presence of financial imperfections and firm heterogeneity, capital taxation exerts U-shaped effects on the balanced growth path rate and that optimal policy rates and balanced-growth path rates are increasing functions of the level of financial deepening. [6]

This paper is organized as follows. Section 2 sets up the model economy. Section 3 defines and analyzes the aggregate equilibrium and the balanced-growth path equilibrium; it then discusses the effects of government's policy tax rate and shows that the model's predictions are consistent with OECD data. Section 4 concludes.

[6] See for example Mino (2015) for the discussion of AK endogenous growth model with productivity heterogeneity.

2 The Model

This is a two-sector growth model. In particular, there is a representative worker and a unit measure of entrepreneurs. Each entrepreneur owns a private firm and are indexed by their productivity, z, and their *taxable* personal wealth, a. Productivity heterogeneous firms combines *effective labor* and physical capital to produce homogenous final goods used for both consumption and physical capital accumulation. Human capital is accumulated by the representative worker's efforts, the existing level of human capital stock and also public eduction provided by the government. The final goods sector is developed from Moll (2014) with efficiency labor.

2.1 The Representative Worker

The representative workers has the following preferences,

$$\int_0^\infty e^{-\rho t} \log c_w(t) dt \tag{2.1}$$

where ρ is the discount rate and c_w is the worker's consumption.

At time t, this worker is endowed with one unit of non-leisure time and has accumulated the level $h(t)$ of human capital. He then chooses a fraction $u(t)$ of his time to supply $u(t)h(t)$ efficiency units of labor at a competitive labor market at a wage $w(t)$. The worker uses the left fraction of his time $1 - u(t)$ to accumulate his level of human capital via as the following function:

$$\dot{h} = b(1-u)h^\phi g_e^{1-\phi} \tag{2.2}$$

where $\phi \in (0, 1)$, b is a parameter that denotes the efficiency of accumulating human capital and g_e is the level of government expenditures on public education that the worker takes as given.

There are two main features that distinguishes human capital accumulation in (2.2) from the existing literature. One is ϕ is less than one, which implies that the production of human capital exhibits diminishing returns to existing level of human capital stock, $h(t)$. This differs from the traditional Uzawa-Lucas model setting with $\phi = 1$. The other one is that the accumulation of human capital depends not only on private investment input, $1 - u(t)$ and $h(t)$ but also on the level of public investment/expenditure on education, $g_e(t)$.

As in Moll (2014), we assume that the worker does not have access to financial markets and consume all their labor incomes. Hence, his budget constraint is as follows:[7]

$$c_w(t) = u(t)h(t)w(t) \tag{2.3}$$

The worker optimizes the sum of discounted utilities from consumption (2.1) subject to the budget constraint (2.3) and his human capital accumulation (2.2) while taking the wage rate $w(t)$ and the amount of public spending on education $g_e(t)$ as given. The optimization conditions imply that optimal allocating time to work, $u(t)$, will obey the following differential equations:

$$\frac{\dot{u}}{u} - buh^{\phi-1}g_e^{1-\phi} + \rho = 0 \tag{2.4}$$

2.2 Entrepreneurs

All entrepreneurs have the same preferences,

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c_e(t) dt \tag{2.5}$$

where c_e expresses entrepreneur's consumption.

Each entrepreneur owns a firm that produces homogenous final goods. At each time, t, this firm employs n^d efficiency units of labor from workers at the wage rate, $w(t)$ and rents k^d units of physical capital from a competitive capital market at the rental rate, $r(t)$, to produce final goods with the following production technology,

$$y = f(z, k, n) = (zk)^\alpha n^{1-\alpha} \tag{2.6}$$

where $\alpha \in (0, 1)$ and z denotes idiosyncratic productivity and can be interpreted as individual entrepreneurial ability or efficiency level of capital. We assume that

[7] An important implication of this assumption is that since workers do not participate in the capital markets, their behavior is not directly affected by the the process of capital accumulation.

idiosyncratic productivity z is *i.i.d* over time as well as across entrepreneurs with the distribution function $G(z)$ and the corresponding density distribution $g(z)$. [8]

Each time, an entrepreneur receives profits from his private firm and capital returns from his personal assets but need to pay wealth tax to the government. [9] Hence, an entrepreneur's wealth $a(t)$ evolves as follows: [10]

$$\dot{a} = \pi(z, k, n) + ra - \tau a - c_e \tag{2.7}$$

where $\pi(z, k, n) \equiv f(z, k, n) - wn - rk$, denotes his firm profits and τ is the policy tax rate imposed by the government.

The entrepreneur at the same time faces the following borrowing constraint:

$$\frac{k}{a} \leq \lambda \tag{2.8}$$

where $\lambda \geq 1$ denotes the maximum borrowing leverage ratio that reflects the degree of financial deepening.

The borrowing constraint, (2.8), equivalently to $k \leq \lambda a$, states that the maximum amount of capital an individual entrepreneur can borrow is limited by the amount of his personal assets, a, and the efficiency of the financial markets reflected by the maximum borrowing leverage ratio λ. In particular, $\lambda = 1$ expresses financial autarky where entrepreneurs are completely capital self-financed whereas $\lambda = \infty$ denotes perfect financial markets where entrepreneurs can borrow freely.[11]

Each entrepreneur maximizes his private firm's profits subject to the technology (2.6) and the borrowing constraint (2.8) as follows,

$$\Pi(a, z) = \max_{k,n} \{f(z, k, n) - wn - rk\}$$
$$s.t. \quad k \leq \lambda a$$

[8] The law of large numbers then implies that the population share of type z entrepreneurs is stationary and deterministic.

[9] As it will be shown later, entrepreneur consumption is also linear in his wealth so the imposition of consumption tax on on entrepreneurs will lead to similar quantitative results.

[10] It is straightforward to show that rental capital market setting is equivalent to a setup where entrepreneurs own and accumulate capital and are allowed to trade a risk-free bond.

[11] Buera and Shin (2013) and Moll (2014) for further discussions of this borrowing constraint.

The optimization conditions of this problem imply that individual entrepreneurs' capital and effective labor demands and profits are linear in personal wealth, and there is a productivity cutoff for active entrepreneurs $\underline{z}$ as follows:

$$k^d(a,z) = \begin{cases} \lambda a & \text{for} \quad z \geq \underline{z} \\ 0 & \text{for} \quad z < \underline{z} \end{cases} \tag{2.9}$$

$$n^d(a,z) = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} z\lambda a \tag{2.10}$$

$$\Pi(a,z) = \max\{z\pi - r, 0\}\,\lambda a \tag{2.11}$$

$$\underline{z}\pi = r \tag{2.12}$$

$$\pi \equiv \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \tag{2.13}$$

Entrepreneurs also maximize the expected sum of discounted utilities from consumption (2.5) subject to the budget constraint (2.7), which can now be rewritten as:

$$\dot{a} = [\lambda \max\{z\pi - r, 0\} + r - \tau]\, a - c_e \tag{2.14}$$

Let $V(a,z,t)$ be the value function of this optimality problem, then the Hamilton-Jacobi-Bellman equation is set as follows,

$$\rho V(a,z,t) = \max_{c_e(t)} \left\{ log(c_e(t)) + \frac{1}{dt}\mathbb{E}_{a,z}\left[dV(a,z,t)\right] \quad \text{s.t.} \quad (2.14) \right\}$$

This in turn implies the optimal consumption rule, $c(t) = \rho a(t)$.[12] Consequently, we obtain the following optimal saving policy function, which is linear in wealth.

$$\dot{a} = s(z)a, \quad where: \tag{2.15}$$
$$s(z) = \lambda \max\{z\pi - r, 0\} + r - \tau - \rho_e$$

[12]This optimal rule can be confirmed as follows. First guess that the value function takes the form $V(a,z,t) = B\left[v(z,t) + loga\right]$, where B is an undermined constant. Substitute this form back to the Hamilton-Jacobi-Bellman equation (2.15) and take first order condition to obtain $c = a/B$. Substitute back in and then apply the envelop theorem to obtain $B = 1/\rho$.

2.3 The Government

For simplicity, we assume that each time the government collects tax from the entrepreneurs at a policy tax rate τ and spends on public education for the accumulation of human capital. The budget constraint of the government then states:

$$\int \tau a d\Phi_t(a, z) = g_e \tag{2.16}$$

where $\Phi_t(a, z)$ is the joint distribution of productivity and wealth at time t.

3 The Aggregate Equilibrium Dynamics

An equilibrium in this economy is sequences of factor prices and corresponding quantities such that (1) each worker and entrepreneur maximize their expected sum of discounted utilities subject to their corresponding budget constraint taking as given equilibrium prices and a tax policy, (2) the government budget constraint (2.16) balances and (3) the factor markets clear at each point in time as follows,

$$\int k_t^d(a, z, Y) d\Phi_t(a, z) = \int a d\Phi_t(a, z) \equiv k(t) \tag{3.17}$$

$$\int n_t^d(a, z, Y) d\Phi_t(a, z) = uh \tag{3.18}$$

3.1 The Aggregate Equilibrium

The aggregate equilibrium dynamics of the model economy can be summarized up in the following Proposition. [13]

Proposition 1. *For a given policy tax rate τ, the dynamics of the aggregates and*

[13] See the Appendix B for the detailed derivation.

the fraction of non-leisure time assigned to goods production can be expressed as:

$$y = Ak^{\alpha}(uh)^{1-\alpha} \tag{3.19}$$

$$\dot{k} = \alpha Ak^{\alpha}(uh)^{1-\alpha} - (\rho + \tau)k \tag{3.20}$$

$$\dot{h} = b(1-u)\tau^{1-\phi}h^{\phi}k^{1-\phi} \tag{3.21}$$

$$\frac{\dot{u}}{u} - bu\tau^{1-\phi}h^{\phi-1}k^{1-\phi} + \rho = 0 \tag{3.22}$$

where the measured aggregate productivity level, the productivity cutoff, and the effective tax rate are determined by

$$A(t) \equiv \mathbb{E}_{\omega,t}\left[z|z \geq \underline{z}\right]^{\alpha} = \left(\lambda \int_{\underline{z}}^{\infty} zg(z)dz\right)^{\alpha} \tag{3.23}$$

$$1 = \lambda\left(1 - G(\underline{z})\right) \tag{3.24}$$

Finally, the wage rates and the capital return are given by,

$$w = (1-\alpha)Ak^{\alpha}(uh)^{-\alpha} \tag{3.25}$$

$$r = \alpha \frac{\underline{z}}{\mathbb{E}_{\omega,t}\left[z|z \geq \underline{z}\right]} Ak^{\alpha-1}(uh)^{1-\alpha} \tag{3.26}$$

The relationship between financial deepening and the aggregate measure productivity can be explained by using (3.23). Suppose that there is no borrowing constraint. Then, because a firm's profits are linear in capital as in (2.11), only entrepreneurs with the highest productivity, z_{max}, produce. Therefore, it is straightforward from (2.6) that the aggregate production function becomes,

$$Y = \bar{A}k^{\alpha}(uh)^{1-\alpha} \tag{3.27}$$

where $\bar{A} = (z_{max})^{\alpha}$ is measured productivity in the absence of borrowing constraint.

As $\mathbb{E}_{\omega,t}\left[z|z \geq \underline{z}\right] \leq z_{max}$, $A(t) \leq \bar{A}$. Hence, there are TFP losses in the presence of financial frictions and the magnitude of the losses is smaller when the productivity cutoff, $\underline{z}$, is higher. The equation (3.24) implies that as the financial markets deepen; namely λ increases, only producers with relatively high productivity remain active in producing goods and the aggregate measured productivity improves.

By contrast, when the financial markets are completely shutdown and all entrepreneurs are self-financed by their personal wealth, i.e., $\lambda = 1$, the measured productivity is at its lower bound, $\underline{A}$. To see this, first substitute $\lambda = 1$ into the equation (3.24) we obtain that the productivity cutoff $\underline{z}$ is at its lower bound, 0. This implies that all entrepreneurs including those are least productive participate in productions by using their own personal funds. Hence, the lower bound for the measured productivity is equal to the average productivity of all entrepreneurs is,

$$\underline{A} = \left(\int_0^\infty z g(z) dz \right)^\alpha \tag{3.28}$$

For further analytical exploration, we make another assumption about the idiosyncratic productivity z. Specifically, each moment entrepreneurs draw z from a Pareto distribution whose distribution function is given by,

$$G(z) = \begin{cases} 1 - \left(\frac{1}{z}\right)^\varphi & z \geq 1 \\ 0 & z < 1 \end{cases} \tag{3.29}$$

where $\varphi > 1$ is the shape parameter. Smaller φ corresponds to a heavier tail of the productivity distribution, i.e., a higher fraction of very productive entrepreneurs, therefore, implying a higher degree of productivity heterogeneity among entrepreneurs.

The productivity density function $g(z)$ for $z \geq 1$ is hence $\varphi z^{-\varphi-1}$. Consequently, weighted average of productivity of active entrepreneurs $\mathbb{E}_{\omega,t}\left[z | z \geq \underline{z}\right] = \frac{\varphi}{\varphi-1}\underline{z}$ and the measured productivity can be expressed as follows:

$$A(t) \equiv \mathbb{E}_{\omega,t}\left[z | z \geq \underline{z}\right]^\alpha = \left(\frac{\varphi}{\varphi - 1} \underline{z} \right)^\alpha \tag{3.30}$$

The capital market clearing equation (3.24) then implies the productivity cutoff $\underline{z}$ is $\lambda^{\frac{1}{\varphi}}$ and the measured TFP becomes,

$$A = \left(\frac{\varphi}{\varphi - 1} \right)^\alpha \lambda^{\frac{\alpha}{\varphi}} \tag{3.31}$$

It is straightforward that a higher λ that reflects deeper financial markets implies higher measured productivity A.

3.2 The Balanced-Growth Path Equilibrium

Define the relative physical capital intensity $\kappa = \frac{k}{h}$ as the ratio of physical capital stock (per worker) over the average level of human capital and a *balanced-growth path equilibrium* of this economy is established when,

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \gamma, \quad \text{for all t} \tag{3.32}$$

where γ denotes the balanced-growth rate.

Substituting κ and γ into (3.20), (3.21), (3.22) and note that under a balanced-growth path equilibrium, u and κ stay constant over time, we obtain the following 3 equations for 3 variables, κ, γ and u as,

$$\begin{aligned}
&\alpha \left(\frac{\varphi}{\varphi - 1}\right)^{\alpha} \lambda^{\frac{\alpha}{\varphi}} \left(\frac{\kappa}{u}\right)^{\alpha-1} - (\rho + \tau) = \gamma \\
&b(1-u)\tau^{1-\phi}\kappa^{1-\phi} = \gamma \\
&-bu\tau^{1-\phi}\kappa^{1-\phi} + \rho = 0
\end{aligned}$$

Consequently, the balanced-growth equilibrium fraction of working hour, u and physical-human capital ratio κ are given by:

$$u = \frac{\rho}{\gamma + \rho} \tag{3.33}$$

$$\kappa = \frac{1}{\tau}\left(\frac{\gamma + \rho}{b}\right)^{\frac{1}{1-\phi}} \tag{3.34}$$

and the balanced-growth rate γ are determined by the following equation,

$$(\gamma + \rho + \tau)(\gamma + \rho)^{\chi(2-\phi)} = \alpha\rho^{1-\alpha} b^{\chi} \tau^{1-\alpha} \left(\frac{\varphi}{\varphi - 1}\right)^{\alpha} \lambda^{\frac{\alpha}{\varphi}} \tag{3.35}$$

where $\chi \equiv \frac{1-\alpha}{1-\phi}$

Proposition 2. *When the following condition*

$$\frac{\rho^{\alpha+\chi}}{\alpha^{\alpha+1}(1-\alpha)^{1-\alpha}} \leq \left(\frac{\varphi}{\varphi - 1}\right)^{\alpha} b^{\chi} \lambda^{\frac{\alpha}{\varphi}} \tag{3.36}$$

is satisfied then there exists a policy tax rate such that the economy converges to a positive balanced-growth path rate equilibrium. Under this equilibrium, economies with deeper financial markets grow with higher growth rates.

Intuitively, this Proposition states that when financial markets are sufficiently deep, high λ, and/or the human capital accumulation is sufficiently efficient, high b, so that the condition (3.36) is satisfied then there exists a balanced-growth path rate equilibrium.

Proof. Step 1: We first show that if (3.36) is satisfied then there exists τ so that the following condition is satisfied.

$$\frac{\rho+\tau}{\tau^{1-\alpha}} < \alpha\rho^{-\chi}\left(\frac{\varphi}{\varphi-1}\right)^{\alpha} b^{\chi}\lambda^{\frac{\alpha}{\varphi}} \tag{3.37}$$

Denote the LHS of (3.37) as $m(\tau)$, it is straightforward that for $\tau > 0$, $m(\tau)$ is an U-shaped function of τ and is minimized at $\tilde{\tau} = \frac{1-\alpha}{\alpha}\rho$ with $m(\tilde{\tau}) = \frac{\rho^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$. Rewrite the condition (3.36) as,

$$\frac{\rho^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \leq \alpha\rho^{-\chi}\left(\frac{\varphi}{\varphi-1}\right)^{\alpha} b^{\chi}\lambda^{\frac{\alpha}{\varphi}}$$

which implies that the horizontal line representing RHS of (3.37) interacts the vertical axis at a point higher than $m(\tilde{\tau})$. Therefore, if τ belongs to $(\underline{\tau}, \bar{\tau})$, where $\underline{\tau}, \bar{\tau}$ are two solutions of the following equation

$$\frac{\rho+\tau}{\tau^{1-\alpha}} = \alpha\rho^{-\chi}\left(\frac{\varphi}{\varphi-1}\right)^{\alpha} b^{\chi}\lambda^{\frac{\alpha}{\varphi}} \tag{3.38}$$

then the condition (3.37) is satisfied.

Step 2: If $\tau \in (\underline{\tau}, \bar{\tau})$ so that (3.37) is satisfied then there exists a positive solution for (3.35). Denote the LHS of (3.35) as $\Psi(\gamma)$, then

$$\Psi(\gamma) \geq 0, \frac{d\Psi(\gamma)}{d\gamma} > 0 \quad \text{for all} \quad \gamma \geq -\rho$$

namely, the curve that represents the LHS of (3.35) is non-negative and strictly increasing for all $\gamma \geq -\rho$. Because the RHS of (3.35) is also a non-negative number, (3.35) always has at least one solution. Rewrite (3.37) as,

$$(\rho+\tau)\,\rho^{\chi(2-\phi)} < \alpha\rho^{1-\alpha}\tau^{1-\alpha}\left(\frac{\varphi}{\varphi-1}\right)^{\alpha} b^{\chi}\lambda^{\frac{\alpha}{\varphi}}$$

which equivalently states that the horizontal line representing the RHS of (3.35) interacts the vertical axis at a point above $\Psi(0)$. Consequently, there exists a unique non-negative solution $0 \leq \gamma$ for the equation (3.35).

The proof that under the balanced-growth path equilibrium, a higher λ leads to a higher growth rate and a lower fraction of working hours is straightforward from the two equations (3.33) and (3.35) that determine γ and u.

□

3.3 Taxation

Proposition 2 also states that the policy tax rate τ should not be too low, namely not lower than $\underline{\tau}$, and also not be too high, namely not higher than $\bar{\tau}$, so that the economy can converge to a balanced-growth path equilibrium. Intuitively, a very high policy rate rate (on entrepreneurs) will dampen the accumulation of physical capital whereas a very low tax rate will lead to low government expenditure on education, hence slow accumulation of human capital.

In addition, Equation (3.34) implies that the relative physical capital intensity depends not only on the growth rate γ but also on the policy tax rate τ. A higher level of financial deepening increases the growth rate, hence raising the relative physical capital intensity while a higher physical capital tax rate lowers the intensity. Therefore, we need to investigate the effects of policy tax rate on the aggregate dynamics and also specify how the government chooses the policy tax rate for further analysis.

Proposition 3. *When the economy is on its balanced-growth path equilibrium, the policy tax rate exerts inverted U-shaped effects on the growth rate.*

Proof. Denote $\gamma(\tau)$ as the solution of equation (3.35). Substituting $\gamma(\tau)$ back and then differentiating the equation with respect to τ we obtain,

$$\frac{d\gamma(\tau)}{d\tau} = \mu(\tau)\left[\gamma(\tau) - \nu(\tau)\right] \tag{3.39}$$

where

$$\mu(\tau) \equiv \frac{1-\alpha}{\tau}\left[1+\chi(2-\phi)\left(\frac{\gamma(\tau)+\rho+\tau}{\gamma(\tau)+\rho}\right)\right]^{-1} \tag{3.40}$$

$$\nu(\tau) \equiv \frac{\alpha}{1-\alpha}\tau - \rho \tag{3.41}$$

Note that because $\gamma(\tau), \mu(\tau)$, and $\nu(\tau)$ are continuous functions of τ for all $\tau \in (\underline{\tau}, \bar{\tau})$, $\frac{d\gamma(\tau)}{d\tau}$ is a continuous function of τ. Additionally, since $\mu(\tau) > 0$ for all $\tau \in (\underline{\tau}, \bar{\tau})$, the function's sign of the derivative of $\gamma(\tau)$, namely $\frac{d\gamma(\tau)}{d\tau}$, depends only on the magnitude relation between $\gamma(\tau)$ and $\nu(\tau)$. Particularly, $\gamma(\tau)$ is an increasing function of τ when the curve representing $\gamma(\tau)$ is above the positive sloping line representing $\nu(\tau)$. When the curve representing $\gamma(\tau)$ is below that of $\nu(\tau)$, $\gamma(\tau)$ is a decreasing function of τ.

Also notice that $\frac{d\nu(\tau)}{d\tau} = \frac{\alpha}{1-\alpha} > 0$ so $\nu(\tau)$ is a strictly increasing function of τ. Therefore, the two curves can not intersect tangentially, namely if the two curves intersect, they must pass through each other. The reason is that at any possible intersection point the slope of $\gamma(\tau)$, by definition, must be equal to zero while the slope of the line representing $\nu(\tau)$ is always positive. Moreover, if there exists an intersection point then on the left (right) of this point, the curve representing $\gamma(\tau)$ must lie above (below) that of $\nu(\tau)$ hence $\gamma(\tau)$ must be increasing (decreasing) with τ. [14] As a result, the two curves can possibly intersect at most one possible point.

Recall from the definition of $\tilde{\tau}$, $\underline{\tau}$, and $\bar{\tau}$ in (3.38) that,

$$\begin{aligned}\gamma(\underline{\tau}) =& \gamma(\bar{\tau}) = 0 \\ \underline{\tau} < \tilde{\tau} \equiv& \frac{1-\alpha}{\alpha}\rho < \bar{\tau}\end{aligned} \tag{3.42}$$

[14] Suppose the curve representing $\gamma(\tau)$ lies below that of $\nu(\tau)$ on the left of an interaction point then by definition $\gamma(\tau)$ must be decreasing on the left of an interaction point. However, $\nu(\tau)$ is always strictly increasing with all τ and if the curve representing $\gamma(\tau)$ lies below that of $\nu(\tau)$, $\gamma(\tau)$ must be increasing so that it can interact with the curve representing $\nu(\tau)$, which is contradictory.

Therefore,

$$
\begin{aligned}
\left.\frac{d\gamma(\tau)}{d\tau}\right|_{\tau=\underline{\tau}} &= \mu(\underline{\tau})\left[0-\nu(\underline{\tau})\right] = \mu(\underline{\tau})\left[\rho - \frac{\alpha}{1-\alpha}\underline{\tau}\right] > 0 \\
\left.\frac{d\gamma(\tau)}{d\tau}\right|_{\tau=\bar{\tau}} &= \mu(\bar{\tau})\left[0-\nu(\bar{\tau})\right] = \mu(\bar{\tau})\left[\rho - \frac{\alpha}{1-\alpha}\bar{\tau}\right] < 0
\end{aligned} \tag{3.43}
$$

which states that at $\tau = \underline{\tau}(\bar{\tau})$, the curve representing $\gamma(\tau)$ positions above (below) the positive sloped line representing $\nu(\tau)$. Additionally, there exists $\tau^* \in (\underline{\tau}, \bar{\tau})$ such that

$$
\left.\frac{d\gamma(\tau)}{d\tau}\right|_{\tau=\tau^*} = 0 \quad \text{or} \quad \gamma(\tau^*) = \nu(\tau^*) = \frac{\alpha}{1-\alpha}\tau^* - \rho \tag{3.44}
$$

or equivalently the curve representing $\gamma(\tau)$ intersects the positive sloped line representing $\nu(\tau)$ at τ^*. As argued above, the two curves can only intersect at most one possible point, therefore,

$$
\frac{d\gamma(\tau)}{d\tau} > 0 \quad \text{for} \quad \tau \in (\underline{\tau}, \tau^*), \qquad \frac{d\gamma(\tau)}{d\tau} < 0 \quad \text{for} \quad \tau \in (\tau^*, \bar{\tau}) \tag{3.45}
$$

□

Corollary 1. *At the optimal policy tax rate that maximizes the balanced-growth path rate, the relative physical capital intensity increases with financial deepening level.*

Proof. Substituting equation (3.44) back into (3.35) to obtain the following equation that determines the optimal policy tax rate, τ^*.

$$
\tau^\alpha \left(\frac{\alpha}{1-\alpha}\tau\right)^{\chi(2-\phi)} = \alpha(1-\alpha)\rho^{1-\alpha}\left(\frac{\varphi}{\varphi-1}\right)^\alpha b^\chi \lambda^{\frac{\alpha}{\varphi}} \tag{3.46}
$$

$$
\tau^{1+\chi} = \alpha^{\alpha-\chi}(1-\alpha)^{2-\alpha+\chi}\rho^{1-\alpha}\left(\frac{\varphi}{\varphi-1}\right)^\alpha b^\chi \lambda^{\frac{\alpha}{\varphi}} \tag{3.47}
$$

$$
\tau^* = \theta\lambda^{\frac{\alpha}{\varphi(1+\chi)}} \tag{3.48}
$$

where $\theta \equiv \left[\alpha^{\alpha-\chi}(1-\alpha)^{2-\alpha+\chi}\rho^{1-\alpha}b^\chi\left(\frac{\varphi}{\varphi-1}\right)^\alpha\right]^{\frac{1}{1+\chi}}$

The optimal balanced growth path rate and the corresponding relative physical capital intensity are then obtained as follows,

$$\gamma^* = \frac{\alpha}{1-\alpha}\theta\lambda^{\frac{\alpha}{\varphi(1+\chi)}} - \rho \tag{3.49}$$

$$\kappa^* = \frac{1}{\tau}\left(\frac{\gamma+\rho}{b}\right)^{\frac{1}{1-\phi}} = \left[\frac{\alpha}{b(1-\alpha)}\theta^{\phi}\right]^{\frac{1}{1-\phi}} \lambda^{\frac{\alpha\phi}{\varphi(2-\alpha-\phi)}} \tag{3.50}$$

It is straightforward that τ^*, γ^*, and κ^* are all increasing functions of parameter λ that reflects financial deepening level. $\square$

The results of this corollary imply that when the government chooses the policy tax rate to maximize the growth rate then economies with deeper financial markets will use physical capital more intensively and work less. These predictions are consistent with regularities documented from 35 OECD countries.

4 Concluding remarks

This paper starts by documenting selected correlational regularities from 35 OECD countries, especially the fact that economies with deeper financial markets tend to have higher physical-human capital ratios and less average working hours. To understand these regularities, the paper develops a tractable two-sector endogenous growth model in which heterogeneous entrepreneurs face borrowing constraints and the government collects tax to fund public eduction. It then shows that this model is isomorphic to a Uzawa-Lucas model and discusses the conditions under which there exists a balanced-growth path equilibrium. Finally, the paper demonstrates that when the government chooses the policy tax rates to maximize growth rates, this model's predictions are consistent with the regularities documented from OECD countries.

A Appendix A: Data-set Description

1. Data for years of total schooling the age group beyond 15 is taken from Barro-Lee Education Attainment Data-set: Barro, Robert and Jong-Wha Lee, 2013, “A New Data Set of Educational Attainment in the World, 1950-2010.” Journal of Development Economics, vol 104, pp.184-198, available for download at `http://www.barrolee.com/`. The Years of Schooling variable is obtained by taking the average of years of total schooling from 2001-10.

2. Financial data is taken from Financial Development and Structure Data-set (Updated Nov. 2013): Thorsten Beck , Asli Demirguc-Kunt , Ross Eric Levine , Martin Cihak and Erik H.B. Feyen, available for download at `http://siteresources.worldbank.org/INTRES/Resources/469232-1107449512766/FinStructure_November_2013.xlsx`. The Private Credit Indicator that reflects financial deepening level is then computed by taking the average of the ratio of total credit issued by depository and other financial institutions to the private sector over GDP from 2001-10.

3. Data for all other variables are obtained from The Penn World Table 9.0 (PWT 9.0): Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), “The Next Generation of the Penn World Table” American Economic Review, 105(10), 3150-3182, available for download at `www.ggdc.net/pwt`. In particular, capital stock, ck, is at current PPPs in bil. 2011US$; TFP level, ctfp, is at current PPPs where USA is equal to one; human capital index, hc, is computed based on years of schooling and returns to education; employment, emp, is the number of persons engaged in millions; average working hours, avh, is the average annual hours worked by persons engaged. All indicators in this paper are then computed by taking the average of the corresponding variables from 2001-10.

B Appendix B: Workers and Firms' Optimization

The Representative Worker: The representative worker optimizes the following sum of discounted utilities from consumption

$$\int_0^\infty e^{-\rho t} \log c_w(t) dt$$

subject to

$$\dot{h} = b(1-u)h^\phi g_e^{1-\phi}$$
$$c_w(t) = u(t)h(t)w(t)$$

After substituting the budget constraint into the instantaneous utility function and denoting the current-value costate variable by $\mu(t)$, the current-value Hamiltonian for the representative worker' optimization problem can be expressed as:

$$H(u,h,\lambda) = \log[wuh] + \mu\left[b(1-u)h^\phi g_e^{1-\phi}\right]$$

The F.O.Cs of this optimization state:

$$\frac{1}{u} = \mu b h^\phi g_e^{1-\phi}$$
$$\frac{1}{h} + \mu\left(\phi b(1-u)h^{\phi-1} g_e^{1-\phi}\right) = \rho\mu - \dot{\mu}$$
$$\lim_{t\to\infty} e^{-\rho t}\mu(t)h(t) = 0$$

These F.O.Cs together with the evolution equation for h imply that optimal allocating time to work, $u(t)$, will obey the following differential equations:

$$\frac{\dot{u}}{u} - buh^{\phi-1} g_e^{1-\phi} + \rho = 0$$

Firms' Problem: Each entrepreneur maximizes the profit of his private firm subject to the technology (2.6) and his borrowing constraint (2.8), hence the profit of an entrepreneur can be expressed as follows:

$$\Pi(a,z) = \max_{k,n}\{f(z,k,n) - wn - rk\}$$
$$s.t. \quad k \le \lambda a$$

The first order condition of this problem with respect to labor states:

$$(1-\alpha)(zk)^{\alpha}n^{-\alpha} = w$$

The implied labor demand then is, $n^d = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}(zk)$. After substituting the optimal labor demand into the technology equation (2.6) we then obtain the production function that is linear in individual entrepreneurs capital input as follows:

$$F(z,k) = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}(zk)$$

Consequently, an entrepreneur's profits can be expressed as:

$$\Pi(a,z) = \max_{k}\left\{\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} zk - rk\right\}$$
$$s.t. \quad k \leq \lambda a$$

This entrepreneur's profit is linear in capital input, hence implying capital demand, labor demand and the productivity cutoff as in (2.9).

C Appendix C: Proof of the Proposition 1

In this economy, the aggregate output denoted by y can be obtained by summing the amounts of homogenous final goods produced by all active entrepreneurs, i.e., entrepreneurs with idiosyncratic productivity higher than the cut-off $\underline{z}$:

$$\begin{aligned}
y = &\int f(z,k,l)d\Phi_t(a,z) = \iint \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z\lambda a g(z)\psi(a)dadz \\
=&\frac{\pi}{\alpha}\lambda\int a\psi(a)da\int zg(z)dz = \frac{\pi}{\alpha}\lambda k\int_{\underline{z}}^{\infty} zg(z)dz \\
=&\frac{\pi}{\alpha}\lambda X k, \quad \text{where} \quad X \equiv \int_{\underline{z}}^{\infty} zg(z)dz
\end{aligned} \tag{C.51}$$

Substituting the optimal labor demand (2.10) into the labor market clearing condition (3.18), we obtain

$$uh = \int n^d(a,z)d\Phi(a,z) = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}}\lambda X k$$

which then implies that:

$$\pi = \alpha \left(\lambda X\right)^{\alpha-1} k^{\alpha-1}(uh)^{1-\alpha} \tag{C.52}$$

Plugging this equation back to $(C.51)$ we obtain the aggregate output as follows:

$$y = \left(\lambda X\right)^{\alpha} u^{1-\alpha} k^{[1+\alpha-1]}(h)^{1-\alpha} = Ak^{\alpha}(uh)^{1-\alpha}$$

where A is the endogenous measured TFP

$$A(t) \equiv \left(\lambda X\right)^{\alpha} = \left(\frac{\int_{\underline{z}}^{\infty} zg(z)dz}{(1-G(\underline{z}))}\right)^{\alpha} = \mathbb{E}_{\omega}\left[z|z \geq \underline{z}\right]^{\alpha}$$

The wage rate, w, can be obtained by substituting $(C.52)$ into the definition of π (2.13):

$$w = (1-\alpha)Ak^{\alpha}(uh)^{-\alpha}$$

Similarly, the capital return, r, can be expressed as:

$$r = \alpha \frac{\underline{z}}{\mathbb{E}_{\omega,t}\left[z|z \geq \underline{z}\right]} Ak^{\alpha-1}(uh)^{1-\alpha}$$

The budget constraint of the government becomes

$$g_e = \int \tau a d\Phi_t(a,z) = \iint \tau a g(z)\psi(a) da dz = k\tau$$

We then derive the dynamic equation of the aggregate capital stock, k, by first aggregating wealth of all entrepreneurs

$$\begin{aligned}\frac{\dot{k}}{k} &= \frac{1}{k}\int \dot{a} d\Phi(a,z) = \frac{1}{k}\iint \dot{a} g(z)\psi(a) da dz \\ &= \frac{1}{k}\iint \left(\lambda \max\left\{z\pi - r, 0\right\} + r - \tau - \rho\right) a g(z)\psi(a) da dz \\ &= \int_0^{\infty} \left(\lambda \max\left\{z\pi - r, 0\right\} + r - \tau - \rho\right) g(z) dz\end{aligned}$$

and then dividing entrepreneurs into the inactive group ($z < \underline{z}$) and the active group ($z \geq \underline{z}$), therefore

$$\begin{aligned}\frac{\dot{k}}{k} &= r - \rho - \tau + \int_{\underline{z}}^{\infty} \lambda\left\{z\pi - r\right\} g(z)dz = r - \rho - \tau + \pi\lambda \int_{\underline{z}}^{\infty} zg(z)dz - r\lambda \int_{\underline{z}}^{\infty} g(z)dz \\ &= \pi\lambda X + r - \rho - \tau - r = \alpha\left(\lambda X\right)^{\alpha-1} k^{\alpha-1}\lambda X - \rho = \alpha A k^{\alpha-1}(uh)^{1-\alpha} - \rho - \tau\end{aligned}$$

where the third and forth equal signs are implied by the definition of X in (3.19), $\underline{z}$ in (2.12), and π in $(C.52)$.

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