Cartel Stability under Quality Differentiation

Bos, Iwan and Marini, Marco A.

Maastricht University, University of Rome La Sapienza

31 July 2018

CARTEL STABILITY UNDER QUALITY DIFFERENTIATION

IWAN BOS AND MARCO A. MARINI

Abstract. This note considers cartel stability when the cartelized products are vertically differentiated. If market shares are maintained at pre-collusive levels, then the firm with the lowest competitive price-cost margin has the strongest incentive to deviate from the collusive agreement. The lowest-quality supplier has the tightest incentive constraint when the difference in unit production costs is sufficiently small.

Keywords: *Cartel Stability, Collusion, Vertical Differentiation.*

JEL Classification: D43, L13, L41.

1. Introduction

In this note, we examine cartel stability when the cartelized products are vertically differentiated. Goods or services are differentiated vertically when there is consensus among consumers about how to rank them quality-wise; comparing products A and B, all agree A to have a higher (perceived) value than B or *vice versa*. There might, however, still be a demand for lower-quality goods when buyers face budget constraints or differ in their willingness to pay for quality. This creates an incentive for suppliers to compete through offering different price-quality combinations.

One implication of this price-quality dispersion is that firms that consider colluding typically face heterogeneous incentive constraints. The fact that firms are induced to charge different prices, for example, affects both collusive and noncooperative profits. From a supply-side perspective, there commonly exists a positive relationship between the quality of a good and its production costs. This, too, impacts both sides of the constraint. It is therefore *a priori* unclear how quality differentiation impacts the sustainability of collusion.

The scarce literature on this topic provides mixed results and, moreover, does not consider the potential impact of cost heterogeneity.[1] Assuming identical costs, Häckner (1994) and Symeonidis (1999) both analyze an infinitely repeated vertically differentiated duopoly

Date: July 2018.

We appreciate the comments of Boris Ginzburg and participants at the 2018 Oligo Workshop in Athens. All opinions and errors are ours alone.

Iwan Bos, Department of Organization & Strategy, School of Business and Economics, Maastricht University. E-mail: i.bos@maastrichtuniversity.nl.

Marco A. Marini, Department of Social and Economic Sciences, University of Rome La Sapienza. E-mail: marini@dis.uniroma1.it.

[1] An extensive and detailed overview of this literature is provided by Marini (2018).

game.[2] Häckner (1994) finds that it is the high-quality supplier who has the strongest incentive to deviate, whereas Symeonidis (1999) establishes that the lowest-quality seller is the one most eager to leave the cartel.

In the following, we show that this cost assumption may not be innocuous. In the context of an n-firm infinitely repeated price-setting game, we allow production costs to increase with quality. Under the assumption that colluding firms maintain their pre-collusive market shares, we find that it is the competitive price-cost margin rather than the quality of the product that drives the incentive to deviate. Specifically, it is the supplier with the lowest noncooperative price-cost margin who has the strongest incentive to chisel on the cartel. Moreover, our analysis confirms the above-mentioned conclusion by Symeonidis (1999) when the difference in unit costs is sufficiently small.

The next section presents the model. Section 3 contains the main finding. Section 4 concludes.

2. Model

We consider an extended version of the classic vertical differentiation models of Mussa and Rosen (1978) and Gabszewicz and Thisse (1979). There is a given set of suppliers, denoted $N = \{1, \ldots, n\}$, who repeatedly interact over an infinite, discrete time horizon. In every period $t \in \mathbb{N}$, they simultaneously make price decisions with the aim to maximize the expected discounted sum of their profit stream. Firms face a common discount factor $\delta \in (0,1)$ and all prices set up until $t-1$ are assumed public knowledge.

Each firm $i \in N$ sells a single variant of the product with quality v_i. We assume $\infty > v_n > v_{n-1} > \ldots > v_1 > 0$ and refer to firm n as the *top firm*, firm 1 as the *bottom firm* and all others as *intermediate firms*. Unit production costs of firm $i \in N$ are given by the constant c_i and we suppose these costs to be positive and (weakly) increasing in quality, *i.e.*, $c_n \geq c_{n-1} \geq \ldots \geq c_1 > 0$.

Consumers have a valuation for the various product types of θ, which is uniformly distributed on $[\underline{\theta}, \overline{\theta}] \subset \mathcal{R}_{++}$. A higher θ corresponds to a higher gross utility when consuming variant v_i. Buyers purchase no more than one item so that someone 'located' at θ obtains the following utility

$$U(\theta) = \begin{cases} \theta v_i - p_i \text{ when buying from firm } i \\ 0 \text{ when not buying,} \end{cases} \tag{2.1}$$

where $p_i \in \left[0, \overline{\theta} v_n\right]$ is the price set by firm i.[3] Using (2.1), it can be easily verified that a consumer at $\theta_i \in [\underline{\theta}, \overline{\theta}]$ is indifferent between buying from, say, firm $i+1$ and firm i when

$$\theta_i(p_{i+1}, p_i) = \frac{p_{i+1} - p_i}{v_{i+1} - v_i}, \tag{2.2}$$

[2] Apart from being analytically convenient, the identical cost assumption can be defended on the grounds that the difference in quality may mainly come from upfront sunk investments in which case the impact on prices would be limited.

[3] Note that none of the buyers would buy at prices in excess of $\overline{\theta} v_n$.

for every $i = 1, 2, ..., n-1$. In the ensuing analysis, we further assume that the market is *covered* (*i.e.*, all consumers buy a product).[4]

Current profit of the bottom firm ($i = 1$) is therefore given by

$$\pi_1 (p_1, p_2) = (p_1 - c_1) \cdot (\theta_1 - \underline{\theta}) , \tag{2.3}$$

where $\theta_1 = \theta_1(p_2, p_1)$ is as specified by (2.2). For each intermediate firm ($i = 2, 3, ..., n-1$) profit is

$$\pi_i (p_{i-1}, p_i, p_{i+1}) = (p_i - c_i) \cdot (\theta_i - \theta_{i-1}) , \tag{2.4}$$

and for the top firm ($i = n$) it is

$$\pi_n (p_{n-1}, p_n) = (p_n - c_n) \cdot \left(\overline{\theta} - \theta_{n-1}\right) . \tag{2.5}$$

Before analyzing the infinitely repeated version of the above game, let us first consider the one-shot case in more detail. In this setting, each firm simultaneously picks a price to maximize its profit as specified in (2.3)-(2.5). Following the first-order conditions, this yields three types of best-response functions:

$$\widehat{p_1}(p_2) = \frac{1}{2} \left(p_2 + c_1 - \underline{\theta}(v_2 - v_1)\right) \tag{2.6}$$

for the bottom firm ($i = 1$). For each intermediate firm ($i = 2, 3, ..., n-1$), the best-reply is given by

$$\widehat{p_i}(p_{i-1}, p_{i+1}) = \frac{1}{2} \frac{p_{i-1}(v_{i+1} - v_i) + p_{i+1}(v_i - v_{i-1})}{(v_{i+1} - v_{i-1})} + \frac{1}{2} c_i. \tag{2.7}$$

The best-response function of the top firm ($i = n$) is

$$\widehat{p_n}(p_{n-1}) = \frac{1}{2} \left(p_{n-1} + c_n + \overline{\theta}(v_n - v_{n-1})\right) . \tag{2.8}$$

Since the action sets are compact and convex and the above best-reply functions are *contractions*, there exists a unique static Nash equilibrium price vector p^* for any finite number of firms.[5] Finally, we impose two more conditions to ensure that the equilibrium solution is interior (*i.e.*, all firms have a positive output at p^*) and that the market is indeed covered at the single-shot Nash equilibrium:

$$\overline{\theta} > \theta^*_{n-1} > \theta^*_{n-2} > \ldots > \theta^*_i > ... > \theta^*_1 > \underline{\theta} > \frac{p^*_1}{v_1} > 0, \tag{2.9}$$

where $\theta^*_i \equiv \theta_i \left(p^*_{i+1}, p^*_i\right)$ and $p^*_i \geq c_i$, for all $i \in N$.

[4] This is a common assumption in contributions that employ this type of spatial setting. See, for example, Tirole (1988, pp. 296-298) and Ecchia and Lambertini (1997).

[5] See, for instance, Friedman (1991, p.84). A sufficient condition for the contraction property to hold is (see, for example, Vives 2000, p.47):

$$\frac{\partial^2 \pi_i}{\partial \left(p_i\right)^2} + \sum_{j \neq i} \left| \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \right| < 0,$$

which, using (2.4) for all intermediate firms $i = 2, ..., n-1$, becomes

$$\frac{v_{i-1} - v_{i+1}}{\left(v_{i+1} - v_i\right)\left(v_i - v_{i-1}\right)} < 0,$$

which holds. The same applies for the top and the bottom firm.

3. Sustainability of Collusion

Within the above framework, we now study the sustainability of collusion assuming a standard grim-trigger punishment strategy. The incentive compatibility constraint (ICC) of a firm $i \in N$ is then given by:

$$\Omega_i \equiv \pi_i^c - (1-\delta) \cdot \pi_i^d - \delta \cdot \pi_i^* \geq 0, \tag{3.1}$$

where $\pi_i^c = \pi_i \left(p_{i-1}^c, p_i^c, p_{i+1}^c\right)$ is a firm's collusive payoff, $\pi_i^d = \pi_i \left(p_{i-1}^c, p_i^d, p_{i+1}^c\right)$, with $p_i^d = \widehat{p}_i(p_{i-1}^c, p_{i+1}^c)$, is deviation payoff and $\pi_i^* = \pi_i \left(p_{i-1}^*, p_i^*, p_{i+1}^*\right)$ is the Nash equilibrium payoff. Consequently, a collusive contract comprising the entire industry is sustainable only when $\Omega_i \geq 0$, for all $i \in N$.

In principle, this set-up allows for a plethora of sustainable collusive contracts. In the following, we limit ourselves to what is perhaps the simplest possible agreement. Specifically, we consider the maximization of total cartel profits without side payments under the assumption that firms maintain their market shares at pre-collusive levels. Such an agreement is appealing for several reasons. First, it seems a natural focal point in the issue of how to divide the market. Second, there have been quite a few cartels that employed such (or similar) market-sharing scheme.[6] Third, as will become clear in the following, it is arguably the most subtle arrangement in that firm behavior maintains a competitive appearance, thereby minimizing the possibility of cartel detection.

Let us first consider the case where none of the ICC's is binding (for which $\delta \to 1$ is sufficient). As an initial observation, notice that the fixed market share assumption implies that the price ranking should remain unaltered (*i.e.*, collusive prices are strictly increasing in quality). Next, note that the collusive price vector must contain a price for the lowest quality product that satisfies:

$$\underline{\theta} v_1 - p_1 = 0.$$

As prices are strategic complements, setting $p_1 < \underline{\theta} v_1$ would be suboptimal, whereas setting $p_1 > \underline{\theta} v_1$ is excluded by the covered market assumption.[7] Hence,

$$p_1^c = \underline{\theta} v_1.$$

The consumer who was indifferent between firm 1 and 2 absent collusion has now the following utility when buying from firm 1:

$$U(\theta_1^*) = \left(\frac{p_2^* - p_1^*}{v_2 - v_1}\right) v_1 - p_1^c.$$

Given fixed market shares, this then gives the highest possible collusive price for the product of firm 2.

$$U(\theta_1^*) = \left(\frac{p_2^* - p_1^*}{v_2 - v_1}\right) v_1 - p_1^c = \left(\frac{p_2^* - p_1^*}{v_2 - v_1}\right) v_2 - p_2^c.$$

Rearranging gives,

$$p_2^c = p_2^* + \left(p_1^c - p_1^*\right).$$

[6]See, for example, Harrington (2006).

[7]Albeit computationally more cumbersome, one in principle could relax the fixed market size assumption. This, however, would not alter the qualitative nature of our results.

A higher collusive price by firm 2 would mean that the customer on the boundary prefers firm 1, which contradicts market shares being fixed. Likewise, a lower price implies a decrease in sales for firm 1 and, therefore, cannot occur either. The unconstrained collusive prices for all other firms can be determined in a similar fashion. In general, the collusive price of firm $i \in N \backslash \{1\}$ is equal to the Nash price plus the price increase by the lowest-quality firm:

$$p_i^c = p_i^* + (p_1^c - p_1^*) \,. \tag{3.2}$$

Let us now consider the possibility that firms are less patient such that one or more ICC's may be binding. The next result shows that it is the supplier with the lowest price-cost margin absent collusion who has the tightest incentive constraint.

Proposition 1. *For any $i, j \in N$ and $j \neq i$, if $p_i^* - c_i > p_j^* - c_j$, then $\Omega_i > \Omega_j$.*

Proof. To begin, consider the ICC of an intermediate firm i:

$$\Omega_i \equiv \pi_i^c - (1-\delta) \cdot \pi_i^d - \delta \cdot \pi_i^* \geq 0,$$

which, for some collusive price p_i^c and fixing market shares at pre-collusive Nash levels, can be written as:

$$\Omega_i \equiv p_i^c - c_i - (1-\delta) \cdot \frac{\left(p_i^d - c_i\right) \cdot \left(\theta_i^d - \theta_{i-1}^d\right)}{\left(\theta_i^* - \theta_{i-1}^*\right)} - \delta \cdot (p_i^* - c_i) \geq 0,$$

where $\theta_i^d \equiv \theta_i\left(p_{i+1}^c, p_i^d\right)$ and $\theta_{i-1}^d \equiv \theta_i\left(p_i^d, p_{i-1}^c\right)$. Using $p_i^c = p_i^* + p_1^c - p_1^*$ and rearranging, this simplifies to:

$$\Omega_i \equiv p_1^c - p_1^* + (1-\delta) \cdot \left(p_i^* - c_i - \frac{\left(p_i^d - c_i\right) \cdot \left(\theta_i^d - \theta_{i-1}^d\right)}{\left(\theta_i^* - \theta_{i-1}^*\right)}\right) \geq 0.$$

Hence, as the first few terms are identical across firms, a difference is exclusively driven by the following part:

$$p_i^* - c_i - \frac{\left(p_i^d - c_i\right) \cdot \left(\theta_i^d - \theta_{i-1}^d\right)}{\left(\theta_i^* - \theta_{i-1}^*\right)}. \tag{3.3}$$

Notice that

$$\theta_i^* - \theta_{i-1}^* = \frac{(v_i - v_{i-1})\, p_{i+1}^* + (v_{i+1} - v_i)\, p_{i-1}^* - (v_{i+1} - v_{i-1})\, c_i}{2\,(v_{i+1} - v_i) \cdot (v_i - v_{i-1})},$$

and (using (3.2))

$$p_i^d = \frac{(v_i - v_{i-1}) \cdot \left(p_{i+1}^* + p_1^c - p_1^*\right) + (v_{i+1} - v_i) \cdot \left(p_{i-1}^* + p_1^c - p_1^*\right) + (v_{i+1} - v_{i-1}) \cdot c_i}{2\,(v_{i+1} - v_{i-1})}.$$

In turn, this gives

$$\theta_i^d - \theta_{i-1}^d = \frac{(v_i - v_{i-1}) \cdot \left(p_{i+1}^* + p_1^c - p_1^*\right) + (v_{i+1} - v_i) \cdot \left(p_{i-1}^* + p_1^c - p_1^*\right) - (v_{i+1} - v_{i-1}) \cdot c_i}{2\,(v_{i+1} - v_i) \cdot (v_i - v_{i-1})}.$$

Next, substituting in (3.3) yields:

$$p_i^* - c_i - \frac{\left((v_i - v_{i-1}) \cdot \left(p_1^c + p_{i+1}^* - p_1^*\right) + (v_{i+1} - v_i) \cdot \left(p_1^c + p_{i-1}^* - p_1^*\right) - (v_{i+1} - v_{i-1}) \cdot c_i\right)^2}{4\,(v_{i+1} - v_{i-1})^2 \cdot [p_i^* - c_i]},$$

which, after some manipulations, can be shown to equal

$$\frac{\left(p_i^* - c_i\right)^2 - \left(\frac{1}{2}\left(p_1^c - p_1^*\right) + p_i^* - c_i\right)^2}{p_i^* - c_i}.$$

Thus, at any given collusive price p_1^c, the only potential difference lies in the noncooperative price-cost margin: $p_i^* - c_i$. Let $x = p_i^* - c_i$ and $y = \frac{1}{2}\left(p_1^c - p_1^*\right)$ and evaluate:

$$\frac{x^2 - (y+x)^2}{x} = \frac{-y^2}{x} - 2y.$$

Taking the first derivative with respect to x gives

$$\left(\frac{y}{x}\right)^2 > 0.$$

We conclude that among intermediate firms the ICC is monotonic in the price-cost margin and that the firm with the lowest price-cost margin has the tightest ICC.

Next, consider firm n and firm $n-1$ for which the first part of the derivation is the same as before. It therefore suffices to evaluate the following inequality:

$$\frac{\left(p_n^* - c_n\right)^2 - \left(\frac{1}{2}\left(p_1^c - p_1^*\right) + p_n^* - c_n\right)^2}{p_n^* - c_n} > \frac{\left(p_{n-1}^* - c_{n-1}\right)^2 - \left(\frac{1}{2}\left(p_1^c - p_1^*\right) + p_{n-1}^* - c_{n-1}\right)^2}{p_{n-1}^* - c_{n-1}}.$$

Let $x = p_n^* - c_n$, $y = \frac{1}{2}\left(p_1^c - p_1^*\right)$ and $z = p_{n-1}^* - c_{n-1}$. Thus,

$$\frac{x^2 - (y+x)^2}{x} > \frac{z^2 - (y+z)^2}{z}.$$

Rearranging gives:

$$x > z \iff p_n^* - c_n > p_{n-1}^* - c_{n-1}.$$

Hence, it is the supplier with the smaller noncooperative price-cost margin (firm n or firm $n-1$) who has the tighter incentive constraint. In a similar fashion, it can be shown that $p_2^* - c_2 > p_1^* - c_1$ implies $\Omega_2 > \Omega_1$ and that $p_1^* - c_1 > p_2^* - c_2$ implies $\Omega_1 > \Omega_2$. We therefore conclude that if $p_i^* - c_i > p_j^* - c_j$, then $\Omega_i > \Omega_j$, for all $i, j \in N$ and $j \neq i$. □

A low competitive price-cost margin implies a relatively severe punishment in the event of a cartel break-down. This provides an incentive to abide by the agreement. What works against this force is that, with fixed market shares, these firms' collusive gains are also smaller. The previous proposition reveals that the latter effect strictly dominates the former. The next result follows immediately. In stating this result, let $\triangle c_{ij} = c_i - c_j$ for any firm $i, j \in N$ and $j \neq i$.

Corollary 1. *For any firm $i, j \in N$ and $j \neq i$, $\exists \mu \in \mathcal{R}_{++}$ such that if $\triangle c_{ij} < \mu$ and $v_i > v_j$, then $\Omega_i > \Omega_j$.*

Hence, if the difference in unit production costs is sufficiently small, then it is the lowest-quality supplier who has the tightest incentive constraint. Our analysis thus confirms the above-mentioned conclusion by Symeonidis (1999) in case quality heterogeneity is primarily driven by (sunk) fixed costs rather than variable costs.

4. Conclusion

Many markets are characterized by some degree of quality differentiation with corresponding firm heterogeneity in cost and demand. One implication of such differences is that colluding firms typically face non-identical incentive constraints. Existing literature on this topic focuses on demand differences, while ignoring the potential impact of cost heterogeneity. In this note, we considered how cartel stability is affected when unit costs are increasing in product quality. Under the assumption that colluding firms maintain their pre-collusive market shares, we found that the incentive to deviate from the collusive agreement is monotonic in the noncooperative price-cost margin. Specifically, the supplier with the lowest competitive mark-up is *ceteris paribus* most inclined to leave the cartel. Moreover, it is the lowest-quality seller who has the tightest incentive constraint when differences in unit costs are sufficiently small.

References

[1] Ecchia, Giulio and Luca Lambertini (1997), "Minimum Quality Standards and Collusion," *Journal of Industrial Economics*, 45(1), 101-113;

[2] Friedman, James W. (1991), *Game Theory with Applications to Economics*. Oxford University Press, Oxford;

[3] Gabszewicz, Jean J. and Jacques-François Thisse (1979), "Price Competition, Quality and Income Disparities," *Journal of Economic Theory*, 20(3), 340-359;

[4] Häckner, Jonas (1994), "Collusive pricing in markets for vertically differentiated products," *International Journal of Industrial Organization*, 12(2), 155-177;

[5] Harrington, Joseph E. Jr. (2006), "How do Cartels Operate?," *Foundations and Trends in Microeconomics*, 2(1), 1-105;

[6] Marini, Marco A. (2018), "Collusive Agreements in Vertically Differentiated Markets", in *Handbook of Game Theory and Industrial Organization*, *Volume 2: Applications*, L. C. Corchon and M. A. Marini (eds.), Edward Elgar, Chelthenam, UK, Northampton, MA, USA;

[7] Mussa, Michael and Sherwin Rosen (1978), "Monopoly and Product Quality," *Journal of Economic Theory*, 18(2), 301-317;

[8] Symeonidis, George (1999), "Cartel stability in advertising-intensive and R&D-intensive industries," *Economics Letters*, 62, 121-129;

[9] Tirole, Jean (1988), "*The Theory of Industrial Organization*," MIT Press, Cambridge, Massachusetts;

[10] Vives, X. (2000), *Oligopoly Pricing. Old Ideas and New Tools*. MIT Press, Cambridge, Massachusetts.