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Abstract

This paper analyzes global dynamics in a macroeconomic model where both monetary and fiscal policies are nonlinear, consistent with empirical evidence. Nonlinear monetary policy, in which the nominal interest rate features an increasing marginal reaction to inflation, interacting with nonlinear fiscal policy, in which the primary budget surplus features an increasing marginal reaction to debt, gives rise to four steady-state equilibria. Each steady state exhibits in its neighborhood a pair of 'active'/'passive' monetary/fiscal policies *à la* Leeper-Woodford, and is typically investigated in isolation within linearized monetary models. We show that, when global nonlinear dynamics are taken into account, such steady states are endogenously connected. In particular, the global dynamics reveals the existence of infinite self-fulfilling paths that originate around the steady states locally displaying either monetary or fiscal 'dominance'—and thus locally delivering equilibrium determinacy—as well as around the unstable steady state with active monetary-fiscal policies, and that converge into an unintended high-debt/low-inflation (possibly deflation) attractor. Such global trajectories—bounded by two heteroclinic orbits connecting the three out-of-the-trap steady states—are, however, obscured if the four monetary-fiscal policy mixes are studied locally and disjointly.

JEL Classification : E52; E62; E63; D91; C61; C62.

Keywords : Nonlinear Monetary and Fiscal Policy Behavior; Evolutionary Macroeconomic Modelling; Multiple Equilibria; Global Nonlinear Dynamics; Debt-Deflation Traps.

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1 Introduction

The purpose of this paper is to analyze the interactions between monetary and fiscal policies from a nonlinear perspective. We hinge on fairly well-established empirical evidence showing the occurrence of nonlinear policy behavior in reaction to inflation and public debt. Central banks, on the one hand, tend to strengthen the adoption of corrective measures as inflation departs from the target through an *increasing* marginal response of nominal interest rates to upward pressures in inflation.[1] Reasons such as the zero lower bound problem on nominal interest rates (Benhabib, Schmitt-Grohé and Uribe, 2001), the loss in credibility as inflation rises (Neuenkirch and Tillmann, 2014), and the scope for asymmetric preferences (Cukierman and Muscatelli, 2008) are often advocated to explain the recourse to nonlinear 'Taylor rules'. Governments, on the other hand, tend to strengthen the adoption of corrective measures as fiscal imbalances deteriorate through an *increasing* marginal response of primary budget surpluses to the accumulation of debt.[2] Political-economy reasons such as political polarization, conflicting distributional objectives between different socioeconomic groups in relation to the burden of budgetary retrenchment, and political stalemate over the distribution of fiscal adjustments (Alesina and Drazen, 1991; Bertola and Drazen, 1993) are often advanced to explain the occurrence of postponed fiscal actions. The overall consequences for macroeconomic dynamics are widely unexplored. The present study is an effort to fill this gap.

We show that such nonlinearities in *both* monetary and fiscal policy actions enhance the multiplicity of steady-state equilibria and, as a consequence, fundamentally affect the implied dynamic interrelationships among macroeconomic policies. We demonstrate, specifically, that nonlinear monetary policy interacting with nonlinear fiscal policy necessarily gives rise to four steady-state equilibria.[3] Each steady state exhibits *in its neighborhood* a pair of 'active'/'passive' monetary/fiscal policies *à la* Leeper-Woodford (see, e.g., Leeper, 1991, Woodford, 2003, Canzoneri, Cumby, and Diba, 2011, and Leeper and Leith, 2016).[4] We prove that the steady states are endogenously connected from a global-dynamics perspective. In particular, there exists an infinite number of self-fulfilling paths originating around the steady states *locally* featured by either monetary or fiscal 'dominance' (Leeper and Leith, 2016)—and so locally featured by equilibrium determinacy associated to saddle-path stability—as well as around the unstable steady state with active monetary-fiscal policies, and *globally* spiraling into an unintended high-debt/low-inflation (possibly deflation) trap.

We find that the implied basin of attraction characterized by debt increases and disinflation turns to be bounded by two heteroclinic orbits connecting the three out-of-the-trap steady states. We demonstrate, in particular, that such heteroclinic orbits are generated by the *saddle manifolds* associated to the steady state with active monetary and passive fiscal policy, and to the steady state with passive monetary and active fiscal policy, respectively.

The foregoing global dynamic properties are obscured if one studies the overall four monetary-fiscal policy mixes locally and disjointly, as in the standard literature. Once nonlinearities compatible with empirical evidence are taken into account, on the other hand, it emerges that *neither* monetary dominance *nor* fiscal dominance can prevail as an equilibrium outcome. The existence of debt-disinflation attracting traps renders the equilibrium system globally indeterminate and the policy makers unable to 'pin down' the inflation rate.

In other words, the local determinacy results typically obtained under linear monetary and fiscal feedback rules disappear as soon as the two policy regimes turn to be globally affected by nonlinearities. From this perspective, the theoretical findings elucidated in this paper provide analytical foundations for the view that incorporating nonlinear policy behaviors into macroeconomic models might be essential for a general characterization of macroeconomics dynamics under monetary and fiscal state-contingent rules, reacting to the current fundamentals.

The paper is organized as follows. Section 2 lays out the paper's connections with the literature. Section 3 develops the dynamic model with nonlinear monetary and fiscal rules. Section 4 analyzes the steady states. Section 5 analyzes local and global dynamics, and establishes the main results. Section 6 discusses the implications of the main theoretical findings. Section 7 addresses the issue of results' robustness in the context of extended versions of our model. Section 8 summarizes the main conclusions and outlines possible directions for further theoretical work.

2 Related Literature and Model Choice

The present study is related to different strands of empirical and theoretical macroeconomic research. First, the paper is connected to the literature that documents the fact that central bank policy behavior can empirically be described by nonlinear feedback interest rate rules of Taylor's (1993) style. According to this hypothesis, monetary policy rules are characterized by an increasing marginal reaction of nominal short-term interest rates to upward deviations of inflation from the target. Dolado, Maria-Dolores and Ruge-Murcia (2004) find that the U.S. monetary policy regime after 1983 can be represented in terms of nonlinear Taylor rule due to the presence of asymmetric preferences, in the sense that, for a given magnitude, positive inflation deviations from the target are weighted more heavily than negative deviations. These findings are remarkably confirmed by the studies of Petersen (2007), Cukierman and Muscatelli (2008,) and Lee and Son (2013), in contrast with the linearity results originally due to Clarida, Galí and Gertler (2000). Analogous nonlinear monetary regimes are shown to have been robustly operative in the U.K. after the introduction of an inflation-targeting framework in 1992 (Martin and Milas, 2004; Taylor and Davradakis, 2006; Cukierman and Muscatelli, 2008; Castro, 2011), in several European countries—such as Germany, France and

Spain—before the introduction of the euro in 1999 (Dolado, Maria-Dolores and Naveira, 2000, 2005), and subsequently in the European Monetary Union (Dolado, Maria-Dolores and Naveira, 2005; Castro 2011; Kulikauskas, 2014). Monetary policy is further shown to have become systematically tighter as inflation rises in emerging Asian and Latin American countries (Miles and Schreyer, 2012, 2014; Ma, 2016; Shen, Lin and Guo, 2016). Beyond the occurrence of asymmetries in central banks preferences, further possible reasons advanced in the literature to justify the case for adopting a nonlinear monetary regime include the zero lower bound problem on nominal interest rates, emphasized in the seminal study of Benhabib, Schmitt-Grohé and Uribe (2001) about the plausibility of liquidity traps induced by Taylor-type rules, and the loss in central bank credibility as inflation increases, pointed out by Neuenkirch and Tillmann (2014).

Second, the paper is linked to the literature that finds that governments' policy behavior can be empirically characterized by nonlinear formulations of the primary-surplus feedback policy rules of the type originally proposed by Leeper (1991). The seminal work by Bohn (1998) shows that the U.S. historical primary budget surplus since 1916 can be described as a nonlinearly increasing function of debt. Additional evidence on nonlinear fiscal adjustments over the U.S. fiscal history is provided by Sarno (2001), Arestis, Cipollini and Fattouh (2004), and Cipollini, Fattouh and Mouratidis (2009). Such a type of 'delayed' budgetary policy measures when debt tends to accumulate overtime is also detected for the historical fiscal record in the U.K since 1919 (Considine and Gallagher, 2008; Arghyrou and Fan, 2013) and for European countries historically subject to fiscal imbalances—such as Italy, Spain, Portugal, Ireland and Greece (Bajo-Rubio, Diaz-Roldan and Esteve, 2004, 2006; Arghyrou and Luintel, 2007; Legrenzi and Milas, 2012a, 2012b, 2013; Piergallini and Postigliola, 2013). Postponed corrective actions in the conduct of fiscal policy are further shown to have occurred in Latin American and Caribbean countries (Chortareas, Kapetanios and Uctum, 2008). Remarkably, it is possible to identify several theoretical reasons for fiscal stabilization postponement. In particular, according to Alesina and Drazen (1991) and Bertola and Drazen (1993), the presence of political polarization, conflicting distributional objectives among different socioeconomic groups with respect to the burden of fiscal retrenchment, and political stalemate over how the burden of higher taxes or expenditure cuts should be allocated are likely to dampen the application of timely budgetary adjustments, up to a certain 'trigger point' at which a sufficiently pronounced consolidated fiscal action typically arises in order to rule out the widespread costs of a debt crisis.

Overall, our study is related to evolutionary macroeconomic modelling under at least three important dimensions emphasized by Nelson and Winter (1982), Radzicki and Sterman (1994), and Dosi and Nelson (1994), among others. First, our model examines the consequences of non-linear, non-optimal—in Ramsey's (1928) sense—*rule-guided* feedback behavior by policy makers, which is shown to cause *endogenous shifts* in both monetary and fiscal regimes. Such regimes are, on the other hand, typically studied locally and 'in isolation' by the standard literature (e.g., Leeper, 1991; Woodford, 2003). The approach developed in this paper is thus along the lines of the evolutionary macroeconomic model recently elaborated by Agliari, Naimzada and Pecora (2017), which is, however, entirely focused on the dynamic implications of nonlinear monetary policy rules. Second, the nonlinear rule-guided policy behavior gives rise to a nonlinear economic system that exhibits *multiple equilibria*. Remarkably, according to Dosi and Nelson (1994), 'behavior and achievement differ greatly across the possible equilibria'. Third, the global-dynamics analysis here performed shows the possible occurrence of off-target self-fulfilling patterns, leading to *unintended* outcomes, such as endogenous debt-deflation spirals. The presence of such unintended attractors *per se* prevents the economy from achieving globally welfare-maximizing equilibria. As a result, in view of the distinguishing features of the present setup—the occurrence of 'routines' (Nelson and Winter, 1982) in the monetary-fiscal policy setting, the emergence of multiple fixed points, and the existence of multiple off-target trajectories bringing about endogenous structural changes in macroeconomic policy regimes—our contribution attempts to provide an evolutionary interpretation of monetary-fiscal interactions.

3 The Model

We set forth a continuous-time macroeconomic environment *à la* Benhabib, Schmitt-Grohé and Uribe (2001) in order to show how a nonlinear monetary policy stance *combined* with a nonlinear fiscal policy stance can easily amplify the multiplicity of steady-state equilibria.

Consider an economy populated by a large number of identical infinitely lived households deriving utility from consumption and real money holdings. The lifetime utility function of the representative household is given by

$$\int_0^\infty e^{-rt} u(c(t), m(t)) dt, \tag{1}$$

where $r > 0$ indicates the rate of time preference, $c(t)$ consumption, and $m(t)$ real money balances at instant of time t. The utility function $u(\cdot, \cdot)$ is strictly increasing and strictly concave in both arguments. According to Reis (2007), consumption and real balances are Edgeworth complements, implying $u_{cm} > 0$. The representative household's instant budget constraint is given by

$$\dot{a}(t) = (R(t) - \pi(t)) a(t) - R(t) m(t) + y(t) - c(t) - \tau(t), \tag{2}$$

where $a(t)$ indicates real financial wealth, consisting of interest-bearing government bonds and money balances, $y(t)$ an endowment of perishable goods, $\tau(t)$ lump-sum taxes net of public transfers, $R(t)$ the nominal interest rate on bonds, and $\pi(t)$ the inflation rate. Households are subject to the borrowing limit condition precluding Ponzi's games, given by

$$\lim_{t\to\infty} e^{-\int_0^t [R(x)-\pi(x)]dx} a(t) \geq 0. \tag{3}$$

Thus, at optimum,

$$u_c(c(t), m(t)) = \lambda(t), \tag{4}$$

$$u_m(c(t), m(t)) = \lambda(t) R(t), \tag{5}$$

$$\dot{\lambda}(t) = \lambda(t)(r + \pi(t) - R(t)), \tag{6}$$

$$\lim_{t\to\infty} e^{-\int_0^t [R(x)-\pi(x)]dx} a(t) = 0, \tag{7}$$

where $\lambda(t)$ is the costate variable associated with the flow budget constraint.

Consider now the public sector. Consistently with Benhabib, Schmitt-Grohé and Uribe (2001) and the empirical literature discussed in the previous section, we assume that the monetary authority adopts an interest rate policy described by a nonlinear feedback rule of the form

$$R(t) = \Phi(\pi(t)), \tag{8}$$

where function $\Phi(\cdot)$ is continuous, strictly positive,[5] and obeys $\Phi'(\cdot), \Phi''(\cdot) > 0$.

The government's instant budget constraint is given by

$$\dot{a}(t) = (R(t) - \pi(t)) a(t) - s(t), \tag{9}$$

where $s(t) = \tau(t) + R(t) m(t)$ denotes the primary surplus inclusive of interest savings from the issuance of money.[6] Differently from the typical literature on monetary theory and policy, we consider the case in which not only the monetary policy stance, but also the fiscal policy stance followed by the government, displays nonlinearities. In particular, in order to capture the postponed fiscal adjustments detected empirically, the fiscal authority adjusts the primary surplus according to a non-linear feedback policy of the form

$$s(t) = \Omega(a(t)), \tag{10}$$

where function $\Omega(\cdot)$ is continuous, strictly positive,[7] and obeys $\Omega'(\cdot), \Omega''(\cdot) > 0$.

Equilibrium in the goods market requires $c(t) = y(t)$. Assume that the endowment is constant over time, that is, $y(t) = y$ for each $t \in [0, \infty)$, without loss of generality. Thus, from equations (4) and (5), it emerges that

$$\lambda(t) = L(R(t)), \tag{11}$$

with $L'(\cdot) = u_c / (u_{mm}/u_{cm} - u_m/u_c) < 0$. Combining (6), (8) and (11), equilibrium dynamics of inflation follow

$$\dot{\pi}(t) = -\frac{L(\Phi(\pi(t)))}{\Phi'(\pi(t)) L'(\Phi(\pi(t)))} (\Phi(\pi(t)) - \pi(t) - r). \tag{12}$$

Substituting (10) and (8) into (9), equilibrium dynamics of government liabilities follow

$$\dot{a}(t) = (\Phi(\pi(t)) - \pi(t)) a(t) - \Omega(a(t)). \tag{13}$$

4 Steady States and Active-Passive Monetary-Fiscal Policies

In this section, we develop the steady-state analysis and investigate the related properties in terms of monetary-fiscal policy regimes. In particular, the next proposition applies.

Proposition 1 *(Steady-State Analysis.) Suppose that both monetary and fiscal policies are nonlinear ($\Phi', \Phi'', \Omega', \Omega'' > 0$). Then, from the equilibrium system (12)-(13), there exist four steady states, $(\bar{a}, \bar{\pi})$, $(\hat{a}, \bar{\pi})$, $(\bar{a}, \hat{\pi})$, and $(\hat{a}, \hat{\pi})$, satisfying $\bar{a} < \hat{a}$ and $\bar{\pi} < \hat{\pi}$. Moreover, (a) at $(\bar{a}, \bar{\pi})$ monetary policy is passive and fiscal policy is active, (b) at $(\hat{a}, \bar{\pi})$*

monetary and fiscal policies are both passive, (c) at $(\bar{a}, \hat{\pi})$ monetary and fiscal policies are both active, and (d) at $(\hat{a}, \hat{\pi})$ monetary policy is active and fiscal policy is passive.

Proof. Setting $\dot{\pi}(t) = 0$ in (12) yields the Fisher equation, $\Phi(\pi) = r + \pi$. Because the monetary policy reaction function $\Phi(\cdot)$ is strictly positive and satisfies $\Phi'(\cdot), \Phi''(\cdot) > 0$, such a steady-state relation has two solutions, $\bar{\pi}$ and $\hat{\pi}$. Figure 1 shows the two steady-state equilibria for the inflation rate, which occur at the intersections of functions $\Phi(\pi)$ and $r + \pi$. Suppose that $\hat{\pi} > 0$ is the target inflation rate. Following Taylor (1993) and Benhabib, Schmitt-Grohé and Uribe (2001), assume also that, at the target inflation rate, monetary policy is 'active', that is, $\Phi'(\hat{\pi}) > 1$. According to this requirement, monetary authorities overreact to upward deviations of inflation from the target by increasing the nominal interest rate by more than one-for-one with respect to an increase in the inflation rate. Then, the alternative steady-state value $\bar{\pi} < \hat{\pi}$ must feature a relatively low—possibly negative—inflation rate. In addition, in the neighborhood of $\bar{\pi}$, monetary policy is necessarily 'passive', $\Phi'(\bar{\pi}) < 1$. Setting now $\dot{a}(t) = 0$ in (13) yields $\Omega(a) = ra$. Because the fiscal policy reaction function $\Omega(\cdot)$ is strictly positive and satisfies $\Omega'(\cdot), \Omega''(\cdot)$, this steady-state relation has two solutions, $\bar{a}, \hat{a} > 0$. Figure 2 shows the two steady-state equilibria for government liabilities, which occur at the intersections of functions $\Omega'(a)$ and ra. We have set $\bar{a} < \hat{a}$, which must imply $\Omega'(\bar{a}) < r$ and $\Omega'(\hat{a}) > r$. As a consequence, applying Leeper's (1991) terminology, fiscal policy is 'active' in the neighborhood of both $(\bar{a}, \bar{\pi})$ and $(\bar{a}, \hat{\pi})$, because, from (13), $\partial\dot{a}(t)/\partial a(t)|_{(\bar{a},\bar{\pi}),(\bar{a},\hat{\pi})} = r - \Omega'(\bar{a}) > 0$, *i.e.*, government liabilities *per se* tend to explode, and is 'passive' in the neighborhood of both $(\hat{a}, \bar{\pi})$ and $(\hat{a}, \hat{\pi})$, because, from (13), $\partial\dot{a}(t)/\partial a(t)|_{(\hat{a},\bar{\pi}),(\hat{a},\hat{\pi})} = r - \Omega'(\hat{a}) < 0$, *i.e.*, government liabilities *per se* tend to be stable. ■

Consequently, the foregoing steady-state analysis implies that the active monetary, passive fiscal policy regime, displaying 'monetary dominance' in Leeper and Leith's (2016) terminology, and the passive monetary, active fiscal policy regime, displaying 'fiscal dominance', cannot prevail globally. Beyond the well-known Leeper's dichotomy occurring at the steady states $(\bar{a}, \bar{\pi})$ and $(\hat{a}, \hat{\pi})$, there must exist—because of nonlinearities in *both* policies—other two steady states, $(\bar{a}, \hat{\pi})$ and $(\hat{a}, \bar{\pi})$, locally displaying an active monetary, active fiscal policy regime and a passive monetary, passive fiscal policy regime, respectively.

5 Local and Global Dynamics

The purpose of this section is to analyze the local and global equilibrium dynamics that emerge from our setup. The following propositions establish the main results.

Proposition 2 *(Local Analysis.) Suppose that both monetary and fiscal policies are nonlinear* $(\Phi', \Phi'', \Omega', \Omega'' > 0)$*. Then, from the equilibrium system (12)-(13), locally (a) the steady state* $(\bar{a}, \bar{\pi})$ *is a saddle point, (b) the steady state* $(\hat{a}, \bar{\pi})$ *is a sink, (c) the steady state* $(\bar{a}, \hat{\pi})$ *is a source, and (d) the steady state* $(\hat{a}, \hat{\pi})$ *is a saddle point.*

Proof. *(a)* Let $J_{(\bar{a},\bar{\pi})}$ be the Jacobian of (12)-(13) evaluated at $(\bar{a}, \bar{\pi})$. We have $\det J_{(\bar{a},\bar{\pi})} = \frac{-(r-\Omega'(\bar{a}))L(\Phi(\bar{\pi}))(\Phi'(\bar{\pi})-1)}{\Phi'(\bar{\pi})L'(\Phi(\bar{\pi}))} < 0$, since $\Omega'(\bar{a}) < r$, $L'(\Phi(\bar{\pi})) < 0$, and $\Phi'(\bar{\pi}) < 1$. Therefore, $(\bar{a}, \bar{\pi})$ is a saddle point, with the stable arm given by $\pi(t) = \bar{\pi} + \frac{-L(\Phi(\bar{\pi}))(\Phi'(\bar{\pi})-1)/(\Phi'(\bar{\pi})L'(\Phi(\bar{\pi})))-(r-\Omega'(\bar{a}))}{(\Phi'(\bar{\pi})-1)\bar{a}}(a(t) - \bar{a})$. *(b)* Let $J_{(\hat{a},\bar{\pi})}$ be the Jacobian evaluated at $(\hat{a}, \bar{\pi})$. We have $\mathrm{tr} J_{(\hat{a},\bar{\pi})} = (r - \Omega'(\hat{a})) - \frac{L(\Phi(\bar{\pi}))(\Phi'(\bar{\pi})-1)}{\Phi'(\bar{\pi})L'(\Phi(\bar{\pi}))} < 0$ and $\det J_{(\hat{a},\bar{\pi})} = \frac{-(r-\Omega'(\hat{a}))L(\Phi(\bar{\pi}))(\Phi'(\bar{\pi})-1)}{\Phi'(\bar{\pi})L'(\Phi(\bar{\pi}))} > 0$, for now $\Omega'(\hat{a}) > r$. This implies that $(\hat{a}, \bar{\pi})$ is a sink. *(c)* Let $J_{(\bar{a},\hat{\pi})}$ be the Jacobian evaluated at $(\bar{a}, \hat{\pi})$. We have $\mathrm{tr} J_{(\bar{a},\hat{\pi})} = (r - \Omega'(\bar{a})) - \frac{L(\Phi(\hat{\pi}))(\Phi'(\hat{\pi})-1)}{\Phi'(\hat{\pi})L'(\Phi(\hat{\pi}))} > 0$ and $\det J_{(\bar{a},\hat{\pi})} = \frac{-(r-\Omega'(\bar{a}))L(\Phi(\hat{\pi}))(\Phi'(\hat{\pi})-1)}{\Phi'(\hat{\pi})L'(\Phi(\hat{\pi}))} > 0$, since $\Omega'(\bar{a}) < r$, $L'(\Phi(\hat{\pi})) < 0$, and $\Phi'(\hat{\pi}) > 1$. Therefore, $(\bar{a}, \hat{\pi})$ is a source. *(d)* Let $J_{(\hat{a},\hat{\pi})}$ be the Jacobian evaluated at $(\hat{a}, \hat{\pi})$. We have $\det J_{(\hat{a},\hat{\pi})} = \frac{-(r-\Omega'(\hat{a}))L(\Phi(\hat{\pi}))(\Phi'(\hat{\pi})-1)}{\Phi'(\hat{\pi})L'(\Phi(\hat{\pi}))} < 0$, for now $\Omega'(\hat{a}) > r$. Thus, $(\hat{a}, \hat{\pi})$ is a saddle point, with the stable arm given by $\pi(t) = \hat{\pi}$. ■

Proposition 3 *(Global Analysis.) Suppose that both monetary and fiscal policies are nonlinear* $(\Phi', \Phi'', \Omega', \Omega'' > 0)$*. Then, from the equilibrium system (12)-(13), globally* *there exist infinite equilibrium paths originating in the neighborhood of the steady states* $(\bar{a}, \bar{\pi})$, $(\bar{a}, \hat{\pi})$ *and* $(\hat{a}, \hat{\pi})$, *and converging asymptotically to the steady state* $(\hat{a}, \bar{\pi})$; *the saddle manifolds associated with* $(\bar{a}, \bar{\pi})$ *and* $(\hat{a}, \hat{\pi})$ *give rise to three heteroclinic orbits connecting the four steady states; the two heteroclinic orbits associated to the stable saddle manifolds are the boundary of the basin of attraction of* $(\hat{a}, \bar{\pi})$.

Proof. Setting $\dot{\pi}(t) = 0$ in equation (12) yields two isoclines given by $\pi(t) = \bar{\pi}$ and $\pi(t) = \hat{\pi}$. In the phase plane $(a(t), \pi(t))$, they are horizontal, with $\hat{\pi} > \bar{\pi}$. Setting $\dot{a}(t) = 0$ yields $\Phi(\pi(t)) - \pi(t) = \frac{\Omega(a(t))}{a(t)}$. We have $\left.\frac{d\pi(t)}{da(t)}\right|_{\dot{a}(t)=0} = \frac{\Omega'(a(t)) - (\Omega(a(t))/a(t))}{(\Phi'(\pi(t))-1)a(t)}$, which is positive at $(\bar{a}, \bar{\pi})$ and $(\hat{a}, \hat{\pi})$, negative at $(\hat{a}, \bar{\pi})$ and $(\bar{a}, \hat{\pi})$, zero if $\Omega'(a(t)) = \Omega(a(t))/a(t)$ and tends to infinity as $\Phi'(\pi(t)) \to 1$. Let $\pi^* = \arg\min\limits_{\pi(t)} \{\Phi(\pi(t)) - \pi(t)\}$ and $a^* = \arg\min\limits_{a(t)} \left\{\frac{\Omega(a(t))}{a(t)}\right\}$. Therefore, in the phase plane $(a(t), \pi(t))$, we have: (Case I) if $\Phi(\pi^*) - \pi^* < \frac{\Omega(a^*)}{a^*}$, there are two isoclines $\dot{a}(t) = 0$, one U-shaped, connecting $(\bar{a}, \hat{\pi})$ and $(\hat{a}, \hat{\pi})$, the other inverted U-shaped, connecting $(\bar{a}, \bar{\pi})$ and $(\hat{a}, \bar{\pi})$; (Case II) if $\Phi(\pi^*) - \pi^* > \frac{\Omega(a^*)}{a^*}$, there are two isoclines $\dot{a}(t) = 0$, one U-shaped to the left, connecting $(\bar{a}, \bar{\pi})$ and $(\bar{a}, \hat{\pi})$, the other U-shaped to the right, connecting $(\hat{a}, \bar{\pi})$ and $(\hat{a}, \hat{\pi})$. From (12), $\dot{\pi}(t) > 0$ if either $\pi(t) < \bar{\pi}$ or $\pi(t) > \hat{\pi}$; $\dot{\pi}(t) < 0$ if $\bar{\pi} < \pi(t) < \hat{\pi}$. From (13), $\dot{a}(t) > (<) \, 0$ if $\Phi(\pi(t)) - \pi(t) > (<) \, \frac{\Omega(a(t))}{a(t)}$. Figure 3 shows the global dynamics for Case I. Figure 4 shows the global dynamics for Case II. In both cases, the stable arm of the saddle point passing through $(\bar{a}, \bar{\pi})$ has locally a positive slope, given by $\frac{-L(\Phi(\bar{\pi}))(\Phi'(\bar{\pi})-1)/\Phi'(\bar{\pi})L'(\Phi(\bar{\pi}))-(r-\Omega'(\bar{a}))}{(\Phi'(\bar{\pi})-1)\bar{a}}$, which is higher than the slope of the isocline $\dot{a}(t) = 0$ evaluated at $(\bar{a}, \bar{\pi})$, given by $\frac{\Omega'(\bar{a})-r}{(\Phi'(\bar{\pi})-1)\bar{a}}$. Because the steady state $(\bar{a}, \hat{\pi})$ is a source, there must exist one trajectory—the heteroclinic orbit Ψ_1—originating in the neighborhood of the steady state $(\bar{a}, \hat{\pi})$ and converging to the steady state $(\bar{a}, \bar{\pi})$. Such a saddle connection follows a non-monotonous path that must change direction when $\dot{a}(t) = 0$. For the same rationale, there further exists a second heteroclinic orbit, Ψ_2, originating in the neighborhood of the steady state $(\bar{a}, \hat{\pi})$ and converging to the steady state $(\hat{a}, \hat{\pi})$. The saddle connection Ψ_2 joining $(\bar{a}, \hat{\pi})$ and $(\hat{a}, \hat{\pi})$ is given by the isocline $\pi = \hat{\pi}$, which is also the stable arm of the saddle-path stable steady state $(\hat{a}, \hat{\pi})$. In addition, because the steady state $(\hat{a}, \bar{\pi})$ is a sink, there exists a third heteroclinic orbit, Ψ_3, originating in the neighborhood of the steady state $(\bar{a}, \bar{\pi})$ and converging to the steady state $(\hat{a}, \bar{\pi})$. In this case, the saddle connection Ψ_3 joining $(\bar{a}, \bar{\pi})$ and $(\hat{a}, \bar{\pi})$ is given by the isocline $\pi = \bar{\pi}$, which is also the unstable arm of the saddle-path exhibited by the steady state $(\bar{a}, \bar{\pi})$. Hence, in the neighborhood of the steady states $(\bar{a}, \bar{\pi})$, $(\bar{a}, \hat{\pi})$ and $(\hat{a}, \hat{\pi})$, for a given initial condition $a(0)$, there exists an infinite number of equilibrium initial values, for instance $\pi(0)_{11}$, $\pi(0)_{12}$, $\pi(0)_{21}$ and $\pi(0)_{22}$ in Figures 3 and 4, such that $(a(t), \pi(t))$ will converge asymptotically to the steady state $(\hat{a}, \bar{\pi})$. The heteroclinic orbits Ψ_1 and Ψ_2 are, as a result, the boundary of the basin of attraction of $(\hat{a}, \bar{\pi})$. ■

6 Discussion of the Results

The literature on monetary-fiscal policy interactions (e.g., Leeper, 1991, Woodford, 2003, Leeper and Leith, 2016) demonstrates that either an active monetary, passive fiscal regime or an active fiscal, passive monetary regime ensures equilibrium determinacy. In Figures 3 and 4, these cases of 'monetary dominance' or 'fiscal dominance', respectively, correspond to the existence of two saddle paths associated to the steady states $(\bar{a}, \bar{\pi})$ and $(\hat{a}, \hat{\pi})$.

If one focuses on local dynamics around either $(\bar{a}, \bar{\pi})$ or $(\hat{a}, \hat{\pi})$, for a given initial condition $a(0) \neq \bar{a}, \hat{a}$, there indeed exists a unique value of inflation—in the case of fiscal dominance given by $\pi(0)_F = \bar{\pi} + \frac{-L(\Phi(\bar{\pi}))(\Phi'(\bar{\pi})-1)/\Phi'(\bar{\pi})L'(\Phi(\bar{\pi}))-(r-\Omega'(\bar{a}))}{(\Phi'(\bar{\pi})-1)\bar{a}}(a(0) - \bar{a})$ and in the case of monetary dominance given by $\pi(0)_M = \hat{\pi}$—such that $(a(t), \pi(t))$ will converge to $(\bar{a}, \bar{\pi})$ or $(\hat{a}, \hat{\pi})$, respectively, as also shown in Figures 3 and 4.

However, $(\bar{a}, \bar{\pi})$ and $(\hat{a}, \hat{\pi})$ are not unique steady-state equilibria if the monetary and fiscal policy conduct is nonlinear. Proposition 2 shows that, when the monetary and the fiscal regimes are nonlinear, even if the steady state $(\bar{a}, \bar{\pi})$, exhibiting fiscal dominance, and the steady state $(\hat{a}, \hat{\pi})$, exhibiting monetary dominance, deliver locally a unique stable equilibrium, globally there exist infinite equilibrium paths originating in the neighborhood of $(\bar{a}, \bar{\pi})$, $(\bar{a}, \hat{\pi})$ and $(\hat{a}, \hat{\pi})$, and converging asymptotically to the high-debt/low-inflation steady state $(\hat{a}, \bar{\pi})$.

Inflation no longer needs to stay on a saddle path to guarantee global stability. As it emerges from Figures 3 and 4, all initial values $\pi(0)$ delimitated upwards by the saddle connections Ψ_1 and Ψ_2 do constitute equilibrium values that make $(a(t), \pi(t))$ converge to $(\hat{a}, \bar{\pi})$.

As a main consequence, the existence of the basin of attraction featured by debt increases and disinflation implies that the dynamic system is indeterminate even around the steady states usually displaying fiscal and monetary dominance. In other words, under nonlinear interest-rate and primary-surplus adjustments of the type empirically documented, *neither* monetary variables *nor* fiscal variables are viable to 'pin down' the inflation rate.

Given the results obtained, a relevant question that naturally arises at this point is the following. What can public authorities carry out in order to maintain the economy close to the desirable equilibria that ensure local stability and uniqueness? We shall show below that avoiding multiple debt-deflation spirals and, at the same time, guaranteeing global and local determinacy centered at a steady state that keeps the desirable features of the regime exhibiting monetary dominance—in particular, equilibrium uniqueness around the inflation target—would require a structural change in the fiscal policy behavior, along the lines suggested by Benhabib, Schmitt-Grohé and Uribe (2002).

Specifically, instead of displaying nonlinear adjustments of the primary budget surpluses to changes in debt according to rule (10), consider the case in which the fiscal authority also reacts to inflation according to

$$s(t) = \Psi(\pi(t))a(t), \tag{14}$$

where $\Psi'(\cdot) > 0$, $\Psi(\hat{\pi}) > 0$ and $\Psi(\bar{\pi}) < 0$. Such a fiscal policy rule prescribes the implementation of a fiscal stimulus should the economy embark on deflationary patterns. The law of motion of government liabilities thus becomes

$$\dot{a}(t) = (\Phi(\pi(t)) - \pi(t) - \Psi(\pi(t)))\, a(t), \tag{15}$$

whose solution is

$$a(t) = e^{\int_0^t [\Phi(\pi(x)) - \pi(x) - \Psi(\pi(x))]dx} a(0), \tag{16}$$

hence implying

$$\lim_{t\to\infty} e^{-\int_0^t [\Phi(\pi(x)) - \pi(x)]dx} a(t) = a(0) \lim_{t\to\infty} e^{-\int_0^t \Psi(\pi(x))dx}. \tag{17}$$

It follows that the tranversality condition is verified for a constant inflation path $\pi(t) = \hat{\pi}$, but is violated for an inflation path converging to $\bar{\pi}$, because $\Psi(\bar{\pi}) < 0$. In other words, in this case, off-target debt-deflation paths are ruled out as possible equilibrium outcomes by means of a 'Non-Ricardian' fiscal expansion as inflation starts to decrease.

7 Robustness of the Results

The setup so far analyzed conveys our lines of argument in a direct and transparent way. In this section, we shall incorporate two relevant extensions in order to address the issue of result robustness. Specifically, we move from an endowment to a production economy environment, first maintaining the assumption of flexible prices and second introducing sticky prices.

Let us initially extend the flexible-price economy to account for endogenous output. Production now requires labor, $h(t)$, via the technology $y(t) = f(h(t))$, where $f' > 0$, $f'' < 0$. The utility function of the representative household-firm unit is given by $\int_0^\infty e^{-rt} [u(c(t), m(t)) - v(h(t))] \, dt$, where $v', v'' > 0$. Optimality yields (4)-(7) jointly with

$$v'(h(t)) = \lambda(t) f'(h(t)). \tag{18}$$

In equilibrium, (4), (5) and (18) imply $y(t) = Y(R(t))$, with $Y' = u_{cm} f'/\Delta < 0$, and $\lambda(t) = L(R(t))$, with $L' = u_{cm}(v'' - \lambda f'')/f'\Delta < 0$, where $\Delta \equiv -f'\left(u_{cc}u_{mm} - u_{cm}^2\right) - [(v'' - \lambda f'')/f'] \times [u_{cm}(u_m/u_c) - u_{mm}] < 0$. Hence, one obtains the system (12)-(13) with analogous properties. The results obtained in Sections 3-5 are, therefore, qualitatively unchanged. The only difference is that, in this case, increases in inflation and thus in the nominal interest rate dampen the level of output via a 'real balance effect' *à la* Brock (1974).

Consider next the case of sticky prices. Each household-firm unit j now produces a differentiated good $y^j(t)$ via the production function $y^j(t) = f\left(h^j(t)\right)$, and faces a demand function of the form $y^d(t)\, d\left(P^j(t)/P(t)\right)$, where $y^d(t)$ indicates aggregate demand, $P^j(t)$ the product j's price, $P(t)$ the price level, and $d(\cdot)$ obeys $d' < 0$, $d(1) = 1$ and $d'(1) = -1$. Consistently with Rotemberg (1982), the lifetime utility is of the form

$$\int_0^\infty e^{-rt} \left[u(c^j(t), m^j(t)) - v\left(h^j(t)\right) - \frac{\rho}{2}\left(\frac{\dot{P}^j(t)}{P^j(t)} - \hat{\pi}\right)^2 \right] dt, \tag{19}$$

where ρ is a positive parameter. The budget constraint in real terms is now

$$\dot{a}^j(t) = (R(t) - \pi(t))\, a^j(t) - R(t)\, m^j(t) + \frac{P^j(t)}{P(t)} f\left(h^j(t)\right) - c^j(t) - \tau(t). \tag{20}$$

Let $\delta^j(t)$ be the multiplier associated with the constraint that output is demand-determined, $f\left(h^j(t)\right) = y^d(t)\, d\left(P^j(t)/P(t)\right)$, and set $\hat{\pi}^j(t) \equiv \dot{P}^j(t)/P^j(t)$. Optimality yields conditions (4)-(7) indexed by j and

$$v'\left(h^j(t)\right) = \left[\lambda^j(t)\frac{P^j(t)}{P(t)} - \delta^j(t)\right] f'\left(h^j(t)\right), \tag{21}$$

$$\dot{\pi}^j(t) = r\left(\pi^j(t) - \hat{\pi}\right) - \frac{1}{\rho}\left[\lambda^j(t)\frac{P^j(t)}{P(t)} f\left(h^j(t)\right) + \delta^j(t)\frac{P^j(t)}{P(t)} y^d(t)\, d'\left(\frac{P^j(t)}{P(t)}\right)\right]. \tag{22}$$

In the symmetric equilibrium, equations (4)-(5) imply $y(t) = Y(\lambda(t), R(t))$, with $Y_\lambda = -\left[u_{cm}(u_m/u_c) - u_{mm}\right]/\left(u_{cc}u_{mm} - u_{cm}^2\right) < 0$ and $Y_R = -\lambda/\left(u_{cc}u_{mm} - u_{cm}^2\right) < 0$. In this case, we thus obtain the system

$$\dot{\lambda}(t) = -\lambda(t)\left[\Phi(\pi(t)) - \pi(t) - r\right], \tag{23}$$

$$\begin{aligned} \dot{\pi}(t) &= r(\pi(t) - \hat{\pi}) - \frac{\lambda(t) Y(\lambda(t), \Phi(\pi(t)))}{\rho} \\ &\quad \times \left[1 + \varepsilon - \frac{\varepsilon v'\left(f^{-1}(Y(\lambda(t), \Phi(\pi(t))))\right)}{\lambda(t) f'\left(f^{-1}(Y(\lambda(t), \Phi(\pi(t))))\right)}\right], \end{aligned} \tag{24}$$

$$\dot{a}(t) = (\Phi(\pi(t)) - \pi(t))\, a(t) - \Omega(a(t)), \tag{25}$$

where $\varepsilon \equiv d'(1) < -1$. Let $\hat{\lambda}$ and $\bar{\lambda}$ be the steady-state levels of λ associated with $\hat{\pi}$ and $\bar{\pi}$, respectively, and, from (24), uniquely satisfying[8]

$$\frac{1+\varepsilon}{\varepsilon}\hat{\lambda} = \frac{v'\left(f^{-1}\left(Y\left(\hat{\lambda}, \Phi(\hat{\pi})\right)\right)\right)}{f'\left(f^{-1}\left(Y\left(\hat{\lambda}, \Phi(\hat{\pi})\right)\right)\right)}, \tag{26}$$

$$\frac{1+\varepsilon}{\varepsilon}\bar{\lambda} = \frac{v'\left(f^{-1}\left(Y\left(\bar{\lambda}, \Phi(\bar{\pi})\right)\right)\right)}{f'\left(f^{-1}\left(Y\left(\bar{\lambda}, \Phi(\bar{\pi})\right)\right)\right)} - \frac{\rho r(\hat{\pi} - \bar{\pi})}{\varepsilon Y\left(\bar{\lambda}, \Phi(\bar{\pi})\right)}. \tag{27}$$

Then, the next propositions apply.

Proposition 4 *(Local Analysis with Sticky Prices.) Suppose that both monetary and fiscal policies are nonlinear ($\Phi', \Phi'', \Omega', \Omega'' > 0$). Then, from the equilibrium system (23)-(25), locally (a) the steady state $\left(\bar{\lambda}, \bar{\pi}, \bar{a}\right)$ is a saddle point with a one-dimensional* *stable space, (b) the steady state* $\left(\bar{\lambda},\bar{\pi},\hat{a}\right)$ *is a saddle point with a two-dimensional stable space, (c) the steady state* $\left(\hat{\lambda},\hat{\pi},\bar{a}\right)$ *is a source, and (d) the steady state* $\left(\hat{\lambda},\hat{\pi},\hat{a}\right)$ *is a saddle point with a one-dimensional stable space.*

Proof. The Jacobian of (23)-(25) is block-diagonal. One eigenvalue is $r-\Omega'$ and the remaining two eigenvalues are obtained from the sub-matrix

$$K=\begin{bmatrix} 0 & -\lambda\left(\Phi'-1\right) \\ k_{21} & k_{22} \end{bmatrix}, \tag{28}$$

where $k_{21}=\frac{\varepsilon y}{\rho f'}\left[\left(v''-\frac{v'f''}{f'}\right)\frac{Y_\lambda}{f'}-\frac{v'}{\lambda}\right]>0$ and $k_{22}=r+\frac{\varepsilon y}{\rho(f')^2}\left(v''-\frac{v'f''}{f'}\right)Y_R\Phi'>0$. Since $\det K=k_{21}\lambda\left(\Phi'-1\right)\lesseqgtr 0$ if $\Phi'\lesseqgtr 1$, $\mathrm{tr}K=k_{22}>0$, the steady states $\left(\bar{\lambda},\bar{\pi},\bar{a}\right)$ and $\left(\bar{\lambda},\bar{\pi},\hat{a}\right)$ are saddle points with one- and two-dimensional stable spaces, respectively. Let μ_1 be the negative eigenvalue associated to (28). Then, the saddle-path solution around the $\left(\bar{\lambda},\bar{\pi},\bar{a}\right)$ yields $\lambda\left(t\right)=\bar{\lambda}-\frac{\bar{\lambda}\left(\Phi'\left(\bar{\pi}\right)-1\right)}{\mu_1}\left(\pi\left(t\right)-\bar{\pi}\right)$ and $\pi\left(t\right)=\bar{\pi}+\frac{\mu_1-\left[r-\Omega'\left(\bar{a}\right)\right]}{\left(\Phi'\left(\bar{\pi}\right)-1\right)a_1^*}\left(a\left(t\right)-\bar{a}\right)$, where $a\left(t\right)=\bar{a}+\left(a\left(0\right)-\bar{a}\right)e^{\mu_1 t}$. The steady states $\left(\hat{\lambda},\hat{\pi},\bar{a}\right)$ and $\left(\hat{\lambda},\hat{\pi},\hat{a}\right)$ are a source and a saddle point with a one-dimensional stable space, respectively. The saddle-path solution around $\left(\hat{\lambda},\hat{\pi},\hat{a}\right)$ is given by $\lambda\left(t\right)=\hat{\lambda}$, $\pi\left(t\right)=\hat{\pi}$, and $a\left(t\right)=\hat{a}+\left(a\left(0\right)-\hat{a}\right)e^{\left[r-\Omega'\left(\hat{a}\right)\right]t}$. ■

Proposition 5 *(Global Analysis with Sticky Prices.) Suppose that both monetary and fiscal policies are nonlinear* ($\Phi',\Phi'',\Omega',\Omega''>0$)*. Then, from the equilibrium system (23)-(25), globally there exist infinite equilibrium paths originating in the neighborhood of the steady states* $\left(\bar{\lambda},\bar{\pi},\bar{a}\right)$, $\left(\hat{\lambda},\hat{\pi},\bar{a}\right)$ *and* $\left(\hat{\lambda},\hat{\pi},\hat{a}\right)$*, and converging asymptotically to the steady state* $\left(\bar{\lambda},\bar{\pi},\hat{a}\right)$*; the saddle manifolds associated to* $\left(\bar{\lambda},\bar{\pi},\bar{a}\right)$ *and* $\left(\hat{\lambda},\hat{\pi},\hat{a}\right)$ *give rise to three types of heteroclinic orbits connecting the four steady states; the two heteroclinic orbits associated to the stable saddle manifolds are the boundary of the basin of attraction of* $\left(\bar{\lambda},\bar{\pi},\hat{a}\right)$.

Proof. From (25), $\frac{d\dot{a}(t)}{d\pi(t)}=\left(\Phi'-1\right)a\left(t\right)\lesseqgtr 0$ if $\Phi'\lesseqgtr 1$. This implies that, in the neighborhood of $\left(\bar{\lambda}, \bar{\pi}, \bar{a}\right)$, for a given initial condition $a(0)$, there exists an infinite number of equilibrium initial values $\pi(0) < \pi(0)_F = \bar{\pi} + \frac{\mu_1 - [r - \Omega'(\bar{a})]}{(\Phi'(\bar{\pi})-1)\bar{a}} (a(0) - \bar{a})$, such that $(\lambda(t), \pi(t), a(t))$ will converge asymptotically to $\left(\bar{\lambda}, \bar{\pi}, \hat{a}\right)$ along the saddle path associated with the submatrix (28), given by $\lambda(t) = \bar{\lambda} - \frac{\bar{\lambda}(\Phi'(\bar{\pi})-1)}{\mu_1} (\pi(t) - \bar{\pi})$ around both $\left(\bar{\lambda}, \bar{\pi}, \bar{a}\right)$ and $\left(\bar{\lambda}, \bar{\pi}, \hat{a}\right)$. The saddle manifold associated with $\left(\bar{\lambda}, \bar{\pi}, \bar{a}\right)$ is thus the boundary of the basin of attraction of $\left(\bar{\lambda}, \bar{\pi}, \hat{a}\right)$. Since $\left.\frac{d\dot{a}(t)}{da(t)}\right|_{\left(\bar{\lambda}, \bar{\pi}\right)} = r - \Omega'(a(t))$ is positive at $\bar{a}$ and negative at $\hat{a}$, there exists a second type of heteroclinic orbit joining the two steady states $\left(\bar{\lambda}, \bar{\pi}, \bar{a}\right)$ and $\left(\bar{\lambda}, \bar{\pi}, \hat{a}\right)$, along $\lambda(t) = \bar{\lambda}$ and $\pi(t) = \bar{\pi}$, which are also the stable arms of the saddle point $\left(\bar{\lambda}, \bar{\pi}, \hat{a}\right)$. Since, finally, $\left(\hat{\lambda}, \hat{\pi}, \bar{a}\right)$ is a source and $\left(\hat{\lambda}, \hat{\pi}, \hat{a}\right)$ is a saddle point with a one-dimensional stable space, there exists a third type of heteroclinic orbit joining the two steady states $\left(\hat{\lambda}, \hat{\pi}, \bar{a}\right)$ and $\left(\hat{\lambda}, \hat{\pi}, \hat{a}\right)$, along $\lambda(t) = \hat{\lambda}$ and $\pi(t) = \hat{\pi}$, which are also the stable arms of the saddle point $\left(\hat{\lambda}, \hat{\pi}, \hat{a}\right)$. As a result, there exist infinite equilibrium paths originating in the neighborhood of both $\left(\hat{\lambda}, \hat{\pi}, \bar{a}\right)$ and $\left(\hat{\lambda}, \hat{\pi}, \hat{a}\right)$, and converging asymptotically to the steady state $\left(\bar{\lambda}, \bar{\pi}, \hat{a}\right)$. ■

Hence, from Propositions 4-5, the presence of price stickiness in the framework and the implied sluggish adjustments associated with the presence of a Phillips curve do not alter in any essential dimension the root of the main results obtained in the baseline model with flexible prices.

8 Conclusions

The issue of interaction between monetary and fiscal policies is a central topic in macroeconomic theory, but is largely uninvestigated when both policies are nonlinear, as supported by much empirical evidence.

The present paper has three main conclusions. First, nonlinear monetary policy, in which the nominal interest rate displays an increasing marginal response to inflation, interacting with nonlinear fiscal policy, in which the primary surplus displays an increasing marginal response to debt, generates four steady-state equilibria. In particular, each steady state features in its neighborhood a pair of 'active'/'passive' monetary/fiscal policies *à la* Leeper-Woodford.

Second, the steady states are endogenously connected. In particular, the dynamic analysis shows the existence of infinite self-fulfilling paths that originate around the steady states locally displaying either monetary or fiscal dominance, hence locally sustaining equilibrium determinacy, but that globally converge into an unintended high-debt/low-inflation (possibly deflation) trap.

Third, and relatedly, high debt-deflation traps *à la* Fisher can naturally occur because of the interaction of nonlinear monetary-fiscal policy behavior, without recourse to more complicated derivations. Such global trajectories are, nevertheless, overlooked if the four monetary-fiscal policy mixes are investigated locally and separately. This paper shows that globally there exist heteroclinic orbits, *i.e.*, saddle connections among steady states—arising from the saddle paths in which inflation is typically pinned down by monetary or fiscal variables within standard *linear* formulations of monetary-fiscal interrelationships—which, in the present context, constitute the boundary of the basin of attraction.

To keep the theoretical investigation compact and convey our line of argument in a transparent way—directly comparable with Leeper's seminal work—we have first abstracted from the presence of price stickiness. Under sticky prices, however, we have shown that the implied sluggish adjustments associated with the presence of a Phillips curve do not affect the essence of the analysis in any fundamental dimension.

The analytical results presented in this paper, in conclusion, imply that accounting for the observed occurrence for nonlinearities in both central bank and government actions is essential for a comprehensive characterization of the issue of equilibrium dynamics under monetary and fiscal feedback policy rules. Possible extensions of the present setup aimed to internalize, for example, distortionary taxation, the maturity structure of government debt, sovereign risk, and/or agents' learning, may be the focus of further research. The simplified framework we have presented could then be employed as an useful benchmark for more complex analysis along these lines.

Notes

[1]As the discussion of the related literature in the following section shall point out, the body of studies empirically supporting the scope for nonlinear feedback interest rate policy rules is large. It includes, for instance, Dolado, Maria-Dolores and Naveira (2000, 2005), Dolado, Maria-Dolores and Ruge-Murcia (2004), Martin and Milas (2004), Taylor and Davradakis (2006), Petersen (2007), Cukierman and Muscatelli (2008), Hayat and Mishra (2010), Castro (2011), Klose (2011), Miles and Schreyer (2012, 2014), Lee and Son (2013), Kulikauskas (2014), Naraidoo and Paya (2014), Neuenkirch and Tillmann (2014), Sznajderska (2014), Ma (2016), and Shen, Lin and Guo (2016).

[2]The empirical research detecting nonlinear fiscal adjustments is also extensive. It includes, for example, Bohn (1998), Sarno (2001), Arestis, Cipollini and Fattouh (2004), Bajo-Rubio, Diaz-Roldan and Esteve (2004, 2006), Arghyrou and Luintel (2007), Chortareas, Kapetanios and Uctum (2008), Considine and Gallagher (2008), Cipollini, Fattouh and Mouratidis (2009), Legrenzi and Milas (2012a, 2012b, 2013), Arghyrou and Fan (2013), and Piergallini and Postigliola (2013).

[3]Piergallini (2016) shows that a fiscal policy displaying convex nonlinearity in the surplus-debt relationship is an *independent* source of multiplicity of steady-state equilibria. To establish this result, he assumes a conventional linear Taylor rule. By contrast, in this paper we attempt to analyze the dynamic effects of a nonlinear behavior in fiscal policy conduct *interacting* with a nonlinear behavior in monetary policy conduct.

[4]Fiscal policy is 'passive' ('active') in Leeper's (1991) sense when the primary budget surplus set by the government brings about local stability (instability) of government liabilities for all stable paths of the other endogenous variables—such as inflation and output—in the neighborhood of a steady state. Monetary policy is 'active' ('passive') in Leeper's (1991) sense when the nominal interest rate set by the central bank increases by more (less) than one-for-one with re-

spect to an increase in the inflation rate, thereby verifying (violating) the Taylor (1993) principle (Woodford, 2003). See Canzoneri, Cumby, and Diba (2011), and Leeper and Leith (2016) for comprehensive analyses and literature reviews on the interactions between monetary and fiscal policies.

[5]This assumption is meant to guarantee that, at the steady states, real money demand is bounded, as in Benhabib, Schmitt-Grohé and Uribe (2001).

[6]For simplicity and without loss of generality, we set public consumption equal to zero.

[7]This assumption is meant to ensure that, at the steady states, government liabilities are strictly positive.

[8]From (27), uniqueness of $\bar{\lambda}$ holds under standard functional forms for preferences and technology. See Benhabib, Schmitt-Grohé and Uribe (2001).

Compliance with Ethical Standards

The author declares that he has no conflict of interest.

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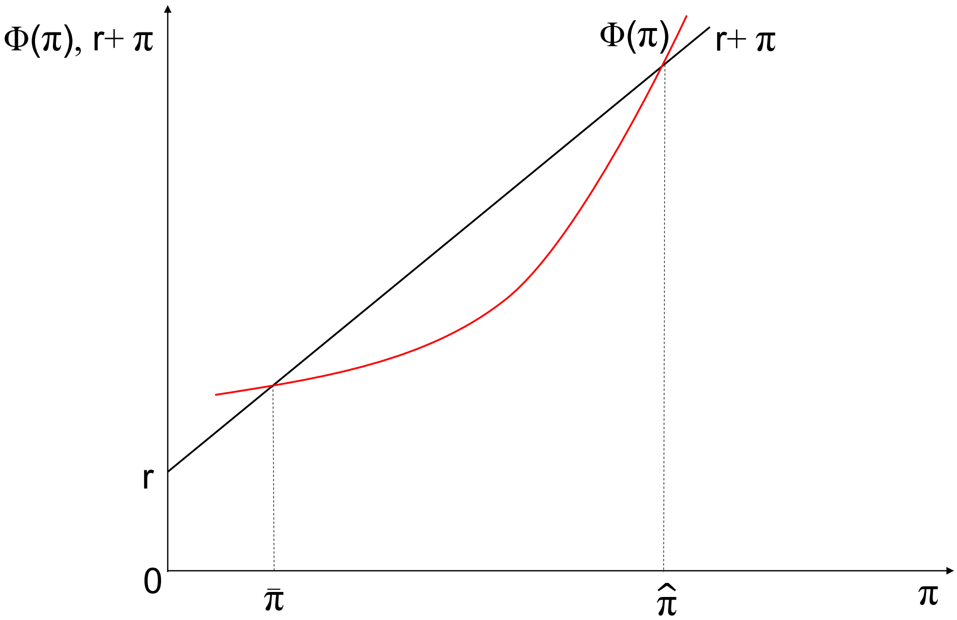

Figure 1: Multiple steady states under a nonlinear monetary regime

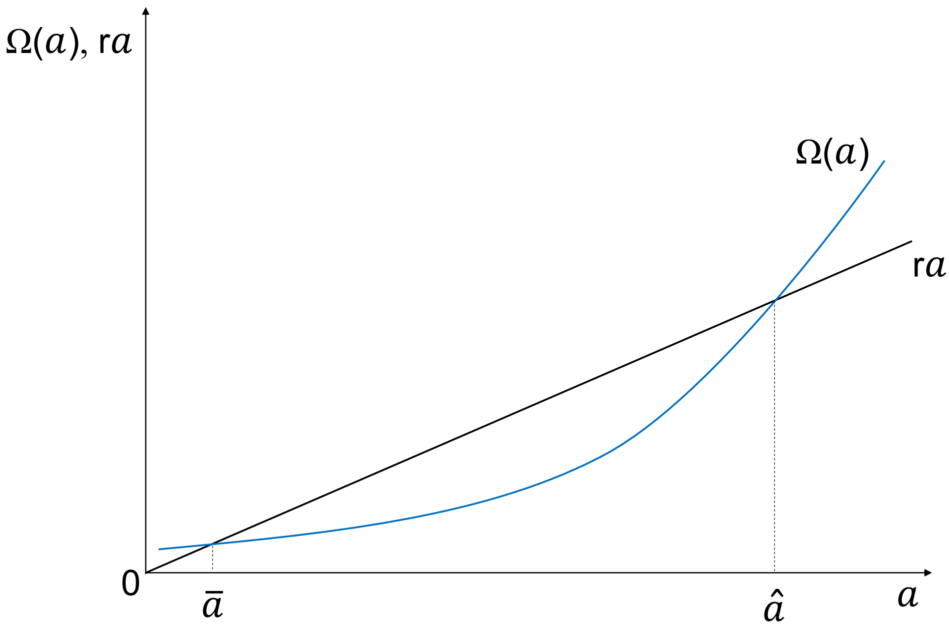

Figure 2: Multiple steady states under a nonlinear fiscal regime

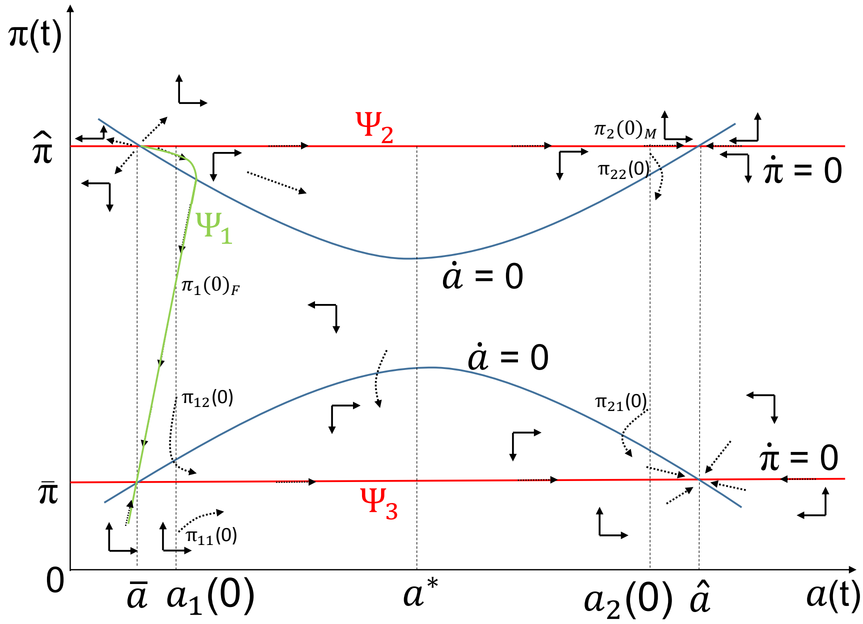

Figure 3: Dynamic behavior of $(a\,(t)\,,\pi\,(t))$ with nonlinear monetary and fiscal regimes for Case I: $\Phi(\pi^{*})-\pi^{*}<\frac{\Omega(a^{*})}{a^{*}}$, where $\pi^{*}=\underset{\pi(t)}{\arg\min}\left\{\Phi(\pi\,(t))-\pi\,(t)\right\}$ and $a^{*}=\underset{a(t)}{\arg\min}\left\{\frac{\Omega(a(t))}{a(t)}\right\}$

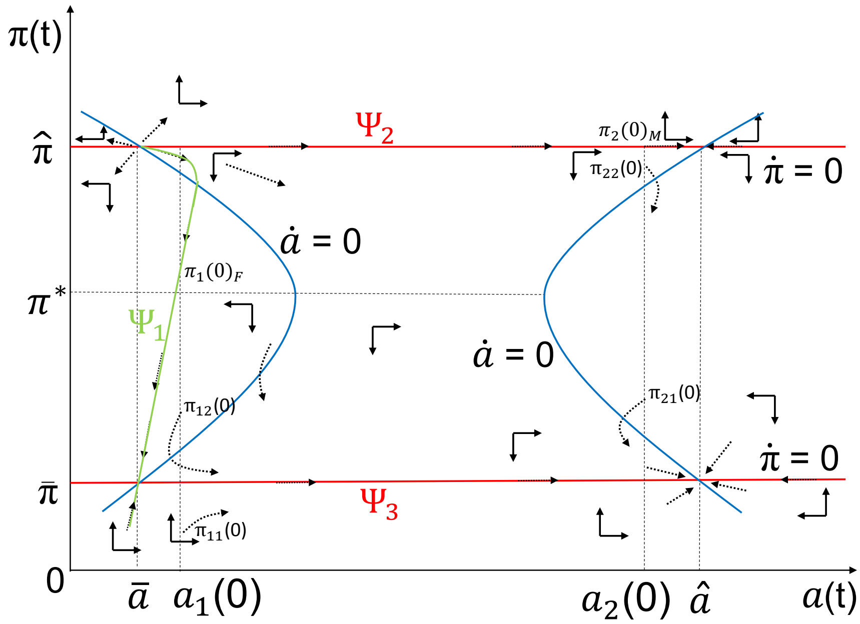

Figure 4: Dynamic behavior of $(a(t), \pi(t))$ with nonlinear monetary and fiscal regimes for Case II: $\Phi(\pi^*) - \pi^* > \frac{\Omega(a^*)}{a^*}$, where $\pi^* = \underset{\pi(t)}{\arg\min}\{\Phi(\pi(t)) - \pi(t)\}$ and $a^* = \underset{a(t)}{\arg\min}\left\{\frac{\Omega(a(t))}{a(t)}\right\}$