Non-Altruistic Equilibria

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Abstract

Which choice will a player make if he can make one of two choices in which his own payoffs are equal, but his rival's payoffs are not equal, *i.e.* one with a large payoff for his rival and the other with a small payoff for his rival? This paper introduces non-altruistic equilibria for normal form games and extensive form non-altruistic equilibria for extensive form games as equilibrium concepts of noncooperative games by discussing such a problem and examines the connections between their equilibrium concepts and Nash and subgame perfect equilibria that are important and frequently encountered equilibrium concepts.

Keywords: Normal form game; extensive form game; non-altruistic equilibrium.

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1. Introduction

In the early 1950s, Nash (1950, 1951) defined noncooperative games and proposed an equilibrium concept, which is still the most prominent and most frequently encountered equilibrium concept today. In the middle of the 1960s, Selten (1965) pointed out the problem of Nash equilibria in extensive form games and proposed subgame perfect equilibria as a valid equilibrium concept for extensive form games. Afterwards, a great many equilibrium concepts for noncooperative games have also been proposed as refinements of Nash's equilibrium concept. Of these, Aumann's (1974, 1987) correlate equilibrium, Selten's (1975) trembling-hand perfect equilibrium, Myerson's (1978) proper equilibrium, Kreps and Wilson's (1982) sequential equilibrium, van Damme's (1984) quasi-perfect equilibrium, Kohlberg and Mertens's (1986) Mertens-stable equilibrium, Bernheim, Peleg, and Whinston's (1987) coalition-proof Nash equilibrium, Cho and Kreps's (1987) perfect Bayesian equilibrium, Maskin and Tirole's (1988a, 1988b) Markov perfect equilibrium, Fudenberg and Levine's (1993) self-confirming equilibrium, and Mckelvey and Palfrey's (1995, 1998) quantal response equilibrium are extremely well known.

Moreover, De Marco and Morgan (2008) introduce slightly altruistic equilibrium as a refinement concept for Nash equilibria. Rusinowska (2002) assumes that players can be either jealous or friendly towards their opponents' payments and considers a bargain as a non-cooperative game.

We now consider the following simple example in Figure 1. This example is a simple two-by-two game where player i $(i=1,2)$ has two possible actions in his action sets (A_i and B_i). The payoffs to the two players when a particular pair of actions is picked are given in the appropriate cell of the matrix. The first entry in each cell is player 1's payoff for the corresponding strategy profile; the second is player 2's. This example has two Nash equilibria: (A_1, A_2) and (A_1, B_2). The equilibrium payoffs for (A_1, A_2) are (10, 1). At this time, player 1's payoff is 10, while player 2's payoff is only 1. It can be said that this result is not good for player 2 if both players are in a rival relationship. Therefore, since player 2's own payoffs are equal even if player 2 picks either of choices A or B, player 2 will prefer to pick B, in which player 1's payoff is lower. At this time, the equilibrium is only (A_1, B_2).

We propose non-altruistic equilibria for normal form games, and extensive form non-altruistic equilibria for extensive form games as criteria for equilibria of noncooperative games. The basic idea behind their equilibrium concepts runs as follows. Consider a situation that involves two players, denoted 1 and 2. Player 2 can make one of two choices in which his own payoffs are equal, but player 1's payoffs are not equal, *i.e.* one with a large payoff for player 1 and the other with a small payoff for player 1. Which choice will player 2 make? If player 2 does not collude with player 1, then player 2 may make the choice that makes player 1's payoff the lowest. Furthermore, in player 1's decision before player 2 faces the choices, it can be said that the result is clear if player 1 makes the choice that makes player 2's payoff the lowest. We will obtain new equilibria by discussing such a problem.

The purpose of this paper is to propose non-altruistic equilibria for normal form games and extensive form non-altruistic equilibria for extensive form games as equilibrium concepts of noncooperative games, and to show the connections between their equilibrium concepts and Nash and subgame perfect equilibria that are important and frequently encountered equilibrium concepts.

The remainder of this paper is structured as follows. In Section 2, we introduce and examine non-altruistic equilibria as a criterion for equilibria. Section 3 illustrates non-altruistic equilibria using some examples. Section 4 describes an extensive form game, and introduces extensive form non-altruistic equilibria as a criterion for equilibria. Section 5 illustrates extensive form non-altruistic equilibria using an example. Section 6 concludes the paper. Appendix provides a different definition of non-altruistic equilibrium and compares this definition with that given in Section 2.

2. Non-altruistic equilibria

Let us consider a game $G = (N, (S_i)_{i\in N}, (h_i)_{i\in N})$ with a finite player set $N = \{1, 2, ..., n\}$. S_i is a finite pure strategy set for each player $i \in N$, and S is used to denote $S_1 \times ... \times S_n$. Each player i has a payoff function $h_i(s_i, s_{-i})$ depending on the strategy combination (s_i, s_{-i}) played. The notation s_{-i} indicates a strategy combination $(s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$ for all players except player i.

We present the following definition as a criterion for equilibria in normal form games.

Definition 1: In G, a strategy combination $s^* = (s_1^*, ..., s_n^*)$ constitutes a non-altruistic equilibrium if and only if, every $i, j \in N; i \neq j$,

$h_i(s_i^*, s_{-i}^*) > h_i(s_i, s_{-i}^*)$, or

for $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$, $\sum_{j\in N; j\neq i} h_j(s_i^*, s_{-i}^*) \leq \sum_{j\in N; j\neq i} h_j(s_i, s_{-i}^*) \qquad \forall s_i \in S_i$.

In non-altruistic equilibria, each player gives high priority to his own payoff maximization and also conducts the other players' payoff minimizing behavior. That is, if each player faces choices that his own payoffs are equal under his own payoff maximizing behavior, then he will make the choice that gives the lowest payoff for the other players.

Consider now the following proposition.

Proposition 1: In G, every non-altruistic equilibrium is also a Nash equilibrium, but not vice versa.

Proof: It is clear that a non-altruistic equilibrium that satisfies $h_i(s_i^*, s_{-i}^*) > h_i(s_i, s_{-i}^*)$ also a Nash equilibrium and vice versa.

Therefore, we prove the case of $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$. Every strategy profile that satisfies $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$ is a Nash equilibrium. However, a non-altruistic equilibrium has to satisfy both $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$ and $\sum_{j \in N; j \neq i} h_j(s_i^*, s_{-i}^*) \leq \sum_{j \in N; j \neq i} h_j(s_i, s_{-i}^*)$. In other words, a strategy profile that satisfies $\sum_{j \in N; j \neq i} h_j(s_i^*, s_{-i}^*) > \sum_{j \in N; j \neq i} h_j(s_i, s_{-i}^*)$ for $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$ is a Nash equilibrium, and not a non-altruistic equilibrium (see Figure 1). This completes the proof of Proposition 1. *Q.E.D.*

3. Examples of the normal form representation

We use three examples in order to illustrate non-altruistic equilibria.

3.1. Example 1

Consider the example depicted in Figure 2. This example is a three-player game, where player $i (i = 1, 2, 3)$ has two possible actions in his action sets (A_i and B_i). The first entry in each cell is player 1's payoff for the corresponding strategy profile; the second is player 2's; the third is player 3's. In this example, it is clear that the non-altruistic equilibrium is (A_1, A_2, A_3), and the non-altruistic equilibrium payoffs are $(4, 4, 4)$. Furthermore, the Nash equilibrium is (A_1, A_2, A_3) as well. In the example, the non-altruistic equilibrium coincides with the Nash equilibrium.

3.2. Example 2

Consider the following example in Figure 3. The second example is a three-player game, where player $i(i=1,2,3)$ has three possible actions in his action sets (A_i, B_i, and C_i). In this example, the Nash equilibrium is both (A_1, A_2, A_3) and (C_1, C_2, C_3), whereas the non-altruistic equilibrium is only (C_1, C_2, C_3). Although (A_1, A_2, A_3) is a Nash equilibrium, it is not a non-altruistic equilibrium because player 1 will deviate to B. That is, player 1 will choose B because the sum of players 2 and 3's payoffs from player 1's choice of A is smaller than that of players 2 and 3's payoffs from player 1's choice of B. The strategy profile (B_1, A_2, A_3) is no non-altruistic equilibrium because player 3 will deviate to B. In each of twenty-seven cells, when a similar discussion is repeated, we know that (C_1, C_2, C_3) is a non-altruistic equilibrium because each player will not deviate, and the non-altruistic equilibrium payoffs are (5, 5, 5).

3.3. Example 3

Consider the game in Figure 4. The third example is a three-player game, where player $i(i=1,2,3)$ has two possible actions in his action sets (A_i and B_i). In this example, the Nash equilibrium is (B_1, B_2, B_3). However, player 3's payoffs from (B_1, B_2, A_3) and (B_1, B_2, B_3) are equal, and the sum of players 1 and 2's payoffs from (B_1, B_2, B_3) is lower than that of players 1 and 2's payoffs from (B_1, B_2, A_3). Therefore, in non-altruistic strategies, player 3 will prefer (B_1, B_2, A_3) to (B_1, B_2, B_3), and the strategic profile (B_1, B_2, B_3) is no non-altruistic equilibrium. Furthermore, player 2's payoff from (B_1, B_2, A_3) is lower than player 2's payoff from (B_1, A_2, A_3). Hence, we can see that this game has no non-altruistic equilibrium by confirming the same as the discussion above.

4. Extensive form non-altruistic equilibria

A finite *n*-person extensive form game is described as a four-tuple $\Gamma = (T, P, U, h)$, where the constituents are as follows. The game tree T is a finite tree consisting of nodes and branches. The player partition $P = (P_0, P_1 ..., P_n)$ is a partition of decision points into player sets, where $P_i (i = 1, 2, ..., n)$ is a player in the game, and player 0 is the chance player responsible for the random moves in the game. The information partition $U = (U_0, U_1, ..., U_n)$ is a partition of $P = (P_0, P_1, ..., P_n)$, and U_0 and $U_i (i = 1, 2, ..., n)$ are called the information sets of player 0 and player i, respectively. The payoff function $h(z) = (h_1(z), ..., h_n(z))$ is real numbers corresponding to every end point z. When a play reaches z, each player $i (i = 1, 2, ..., n)$ gets the payoff $h_i(z)$.

We introduce the following terminology in order to examine a finite extensive form game Γ. A pure strategy s_i of player i is a function that assigns a choice at u to every $u \in U_i$, and the set of all pure strategies of player i is denoted by S_i. A strategy combination $s = (s_1, ..., s_n)$ is a set of *n*-tuple pure strategies. Hence, a normal form game $G(\Gamma) = (s_1, ..., s_n; h_1(s), ..., h_n(s))$ of Γ is denoted by the expected payoffs of a strategy combination. The non-altruistic equilibrium of Γ can be defined in much the same way as the non-altruistic equilibrium of G.

It is well known that subgame perfect equilibria are a refinement of Nash equilibria, *i.e.* subgame perfect equilibria are proposed to compensate for the problem of Nash equilibria in extensive form games.

Now, we define non-altruistic equilibria in extensive form.

Definition 2: In Γ, a strategy combination $s^* = (s_1^*, \ldots, s_n^*)$ constitutes an extensive form non-altruistic equilibrium if and only if it induces a non-altruistic equilibrium in every subgame of Γ.

This definition means that an extensive form non-altruistic equilibrium occurs when every player plays a non-altruistic equilibrium in every subgame of Γ.

We present the following proposition.

Proposition 2: Every Γ has at least one extensive form non-altruistic equilibrium.

Proof: We will prove this positive by induction on the length of Γ, *i.e.* on the maximum number of moves in a play of Γ. Consider a game Γ with m nodes, and u^0 is the starting node of the game tree T. Let u^1, u^2, u^3, …, u^k denote the nodes that are connected to u^0 by a branch, and let T^1, T^2, T^3, …, T^k be the disjoint subtrees of T starting at these nodes. Each T^k corresponds to Γ^k. Therefore, Γ^k possesses, by the induction hypothesis, an equilibrium solution $s^k = (s_i^k)_{i \in N}$ in pure strategies. First, consider the case of only the starting node and end points. If u^0 is a chance node, then a branch is decided, and this leads to $s^k = (s_i^k)_{i \in N}$ for player i. If u^0 is a node of player *i*, then player *i* chooses a branch that satisfies:

$h_i(s_i^*, s_{-i}^*) > h_i(s_i, s_{-i}^*)$, or

$$\text{for } h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*),\ \sum_{j\in N; j\neq i} h_j(s_i^*, s_{-i}^*) \leq \sum_{j\in N; j\neq i} h_j(s_i, s_{-i}^*) \qquad \forall s_i \in S_i \tag{1}$$

Therefore, Proposition 2 follows in this case.

Second, consider games that have nodes besides the starting node. Assume that the result is true for any game with less than m nodes. Consider a game Γ with m nodes. If a node is a chance node, then a branch is decided, and this leads to $s^k = (s_i^k)_{i\in N}$ for player i. On the other hand, if a node is a node of player i, then player i chooses a branch k that leads to Γ^k where his strategy satisfies (1). Thus, it is evident that s is a pure equilibrium solution of Γ. *Q.E.D.*

Next, we discuss the connections between extensive form non-altruistic and subgame perfect equilibria.

Proposition 3: In Γ, an extensive form non-altruistic equilibrium that satisfies $h_i(s^*) > h_i(s_i, s_{-i}^*)$ is also a subgame perfect equilibrium, and vice versa.

Proof: It is clear that a non-altruistic equilibrium that satisfies $h_i(s^*) > h_i(s_i, s_{-i}^*)$ is also a Nash equilibrium and vice versa. A strategy combination is a Nash equilibrium (a non-altruistic equilibrium) if the players' strategies constitute a Nash equilibrium (a non-altruistic equilibrium) in the entire game, and a strategy combination is a subgame perfect equilibrium (an extensive form non-altruistic equilibrium) if the players' strategies constitute a Nash equilibrium (a non-altruistic equilibrium) in every subgame including the entire game. Thus, the set of extensive form non-altruistic equilibria coincides with the set of subgame perfect equilibria. Thus, Proposition 3 is true. *Q.E.D.*

Proposition 4: In Γ, every extensive form non-altruistic equilibrium is also a subgame perfect equilibrium, but not vice versa.

Proof: Proposition 3 shows that in the case of $h_i(s^*) > h_i(s_i, s_{-i}^*)$ the set of extensive form non-altruistic equilibria coincides with the set of subgame perfect equilibria.

Therefore, we prove the case of $h_i(s^*) = h_i(s_i, s_{-i}^*)$. A strategy profile that satisfies $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$ is a Nash equilibrium. However, a non-altruistic equilibrium need to satisfy both $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$ and $\sum_{j \in N; j \neq i} h_j(s_i^*, s_{-i}^*) \leq \sum_{j \in N; j \neq i} h_j(s_i, s_{-i}^*)$. In other words, a strategy profile that satisfies $\sum_{j \in N; j \neq i} h_j(s_i^*, s_{-i}^*) > \sum_{j \in N; j \neq i} h_j(s_i, s_{-i}^*)$ for $h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*)$ is a Nash equilibrium, but not a non-altruistic equilibrium. A strategy combination is a subgame perfect equilibrium (an extensive form non-altruistic equilibrium) if the players' strategies constitute a Nash equilibrium (a non-altruistic equilibrium) in every subgame including the entire game, and the proposition follows. *Q.E.D.*

5. An example of the extensive form representation

Let us consider the example depicted in Figure 5. In this game, player 1 moves at node u^0 and player 2 moves at node u^1 or node u^2. In this figure, δ is a real number larger than 0 and smaller than 10. The normal form $G(\Gamma)$ of the extensive form game runs as Figure 6. Here, player 1 picks rows and player 2 picks columns. This game has four Nash equilibria: (A, AA), (A, AB), (B, BA)

and (B, BB), and two subgame perfect equilibria: (A, AB) and (B, BB).

What happens to the equilibrium in this game if there is no agreement on player 2's choice with the highest payoff of player 1 among choices with the same payoff for player 2? In the strategy profile (A, AB), player 1's payoff is 10, while player 2's payoff is zero. Here, if player 2 chooses B, then the equilibrium payoffs are $(0, 0)$ and player 1's payoff decreases greatly. If player 1 chooses B and player 2 also chooses B, then player 2's payoff is 10. On the other hand, if player 1 chooses A, then even if player 2 chooses either A or B, player 2's payoff is zero. If player 1 chooses A, player 2 will not receive any payoff more than zero. That is, it can be said that the action by which player 1 chooses A is a hostile one against player 2. Hence, if player 1 chooses A, player 2 will choose B. At this time, it can be said that player 1's payoff when he chooses A is zero.

In addition, if player 2 threatens player 1 that if player 1 chooses A, player 2 will choose B, then this threat will be credible because of $10 > 0$, and therefore player 1 should not choose A. That is, it can be thought that this result is an equilibrium, and the equilibrium payoffs are $(10 - \delta, 10)$.

In this game, the extensive form non-altruistic equilibrium is only (B, BB). This extensive form game has three subgames: (i) the entire game starting at node u^0, (ii) the subgame starting at node u^1 and (iii) the subgame starting at node u^2. From Definition 2, we can see that (A, AB) is no extensive form non-altruistic equilibrium because it is only non-altruistic equilibria in subgames (i) and (iii). On the other hand, (B, BB) is an extensive form non-altruistic equilibrium because it is non-altruistic equilibria in all three subgames.

6. Conclusion

We have proposed non-altruistic equilibria as a criterion for equilibria of normal form games and have shown that the following cases exist: (i) every non-altruistic equilibrium is also a Nash equilibrium but vice versa, and (iii) not every normal form game has a non-altruistic equilibrium. In addition, we have proposed extensive form non-altruistic equilibria as a criterion for equilibria of finite extensive form games and have shown that (i) every finite extensive form game has at least one extensive form non-altruistic equilibrium, and (ii) every extensive form non-altruistic equilibria is a subgame perfect equilibria but not vice versa. It is impossible to find mixed-strategy non-altruistic equilibria in normal form games. However, we will study non-altruistic equilibria using methods instead of mixed strategies in the future.

Appendix

In section 2, we defined a non-altruistic equilibrium. However, we can also define it as follows:

Definition 2': In G, a strategy combination $s^* = (s_1^*, \ldots, s_n^*)$ constitutes a non-altruistic equilibrium if and only if, every $i, j \in N; i \neq j$,

$$h_i(s_i^*, s_{-i}^*) > h_i(s_i, s_{-i}^*)\text{, or}$$

$$\text{for } h_i(s_i^*, s_{-i}^*) = h_i(s_i, s_{-i}^*),\ h_j(s_i^*, s_{-i}^*) \le h_j(s_i, s_{-i}^*) \qquad \forall s_i \in S_i .$$

In this definition, each player gives higher priority to conduct the other players' payoff minimizing behavior. We illustrate the difference between Definition 1 and Definition 1' using the following payoff matrices.

First, consider Figure 7. This example is a three-player game where each player i $(i = 1, 2, 3)$ has two possible actions in his action sets (A_i and B_i). This game has two Nash equilibria: (B_1, B_2, A_3) and (B_1, B_2, B_3). However, in (B_1, B_2, A_3), player 1's payoff is 1 and player 2's payoff is 4. In (B_1, B_2, B_3), player 1's and player 2's payoffs both are 5. By Definition 1 (Definition 1'), player 3 will pick A, not B. Therefore, the non-altruistic equilibrium is (B_1, B_2, A_3), and the equilibrium payoffs are (1, 4, 5), which is worse for player 1 and player 2 than (5, 5, 5).

Next, consider Figure 8. This example also has two Nash equilibria: (B_1, B_2, A_3) and (B_1, B_2, B_3). In (B_1, B_2, A_3), player 1's payoff is $1\ (< 5)$, and player 2's payoff is $6\ (> 5)$. The sum of player 1's and player 2's payoffs is 7. On the other hand, in (B_1, B_2, B_3), the sum of player 1's and player 2's payoffs is 10.

If the game is played according to Definition 1, player 3 picks A, not B, and the equilibrium payoffs are (1, 6, 5), which is worse only for player 1 than (5, 5, 5). However, if the game is played according to Definition 1', then player 3 picks B, not A, and the non-altruistic equilibrium is (B_1, B_2, B_3).

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	A_2	B_2
A_1	10, 1	1, 1
B_1	5, 0	0, 0

Figure 1: A two-player game with two action sets (*A* and *B*).

A_3

	A_2	B_2
A_1	4, 4, 4	3, 3, 3
B_1	3, 3, 3	2, 2, 2

B_3

	A_2	B_2
A_1	2, 2, 2	1, 1, 1
B_1	1, 1, 1	0, 0, 0

Figure 2: A three-player game with two action sets (*A* and *B*).

A_3

	A_2	B_2	C_2
A_1	8, 8, 8	7, 7, 7	6, 6, 6
B_1	8, 7, 0	6, 6, 6	5, 5, 5
C_1	6, 6, 6	5, 5, 5	4, 4, 4

B_3

	A_2	B_2	C_2
A_1	5, 5, 5	4, 4, 4	3, 3, 3
B_1	4, 4, 4	3, 3, 3	2, 2, 2
C_1	3, 3, 3	2, 2, 2	1, 1, 1

C_3

	A_2	B_2	C_2
A_1	2, 2, 2	1, 1, 1	0, 0, 0
B_1	1, 1, 1	0, 0, 0	3, 3, 3
C_1	0, 0, 0	3, 3, 3	5, 5, 5

Figure 3: A three-player game with three action sets (*A*, *B*, and *C*).

A_3

	A_2	B_2
A_1	0, 0, 0	1, 1, 1
B_1	1, 1, 1	5, 0, 3

B_3

	A_2	B_2
A_1	1, 1, 1	2, 2, 2
B_1	2, 2, 2	3, 3, 3

Figure 4: A three-player game with two action sets (*A* and *B*).

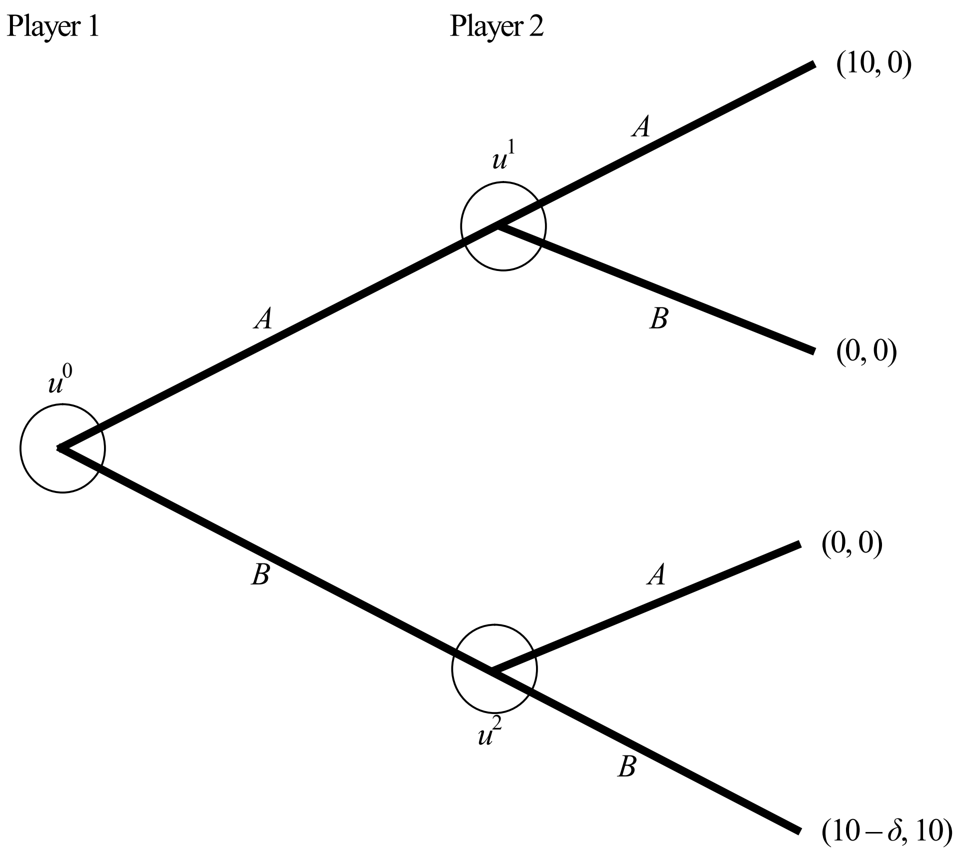

Figure 5: An extensive form game.

	AA	*AB*	*BA*	*BB*
A	10, 0	10, 0	0, 0	0, 0
B	0, 0	0, 0	$10-\delta$, 10	$10-\delta$, 10

Figure 6: A normal form game.

		A_2	B_2
A_3	A_1	0, 0, 0	1, 1, 1
	B_1	1, 1, 1	1, 4, 5

		A_2	B_2
B_3	A_1	0, 0, 0	1, 1, 1
	B_1	1, 1, 1	5, 5, 5

Figure 7: Player 2's payoff of 4 in (B_1, B_2, A_3).

		A_2	B_2
A_3	A_1	0, 0, 0	1, 1, 1
	B_1	1, 1, 1	1, 6, 5

		A_2	B_2
B_3	A_1	0, 0, 0	1, 1, 1
	B_1	1, 1, 1	5, 5, 5

Figure 8: Player 2's payoff of 6 in (B_1, B_2, A_3).