Welfare economic foundation of hoarding loss by money circulation optimization

Miura, Shinji

Independent

13 August 2018

Welfare economic foundation of hoarding loss by money circulation optimization

Shinji Miura (Independent. Gifu, Japan)

Abstract

Saving brings an economic loss. This is one of the basic propositions of the under-consumption theory. This paper aims to give a welfare economic foundation of this proposition through an optimization method considering money circulation in the case where a type of saving is limited to hoarding. If price is fixed, a non-hoarding state is a necessary condition for Pareto efficiency. However, individual agents who prefer future expenditure hoard money, thus individual rational behavior brings about a Pareto inefficient state. This irrationality of rationality occurs because of a qualitative difference of the budget constraint between the whole society and an individual agent. The former's constraint incorporates a truth that hoarding decreases other's revenue, whereas the latter's does not. Selfish individual agents make a decision with an ignorance of this relational truth because their interest is limited to their private range. As a result, agents fall into an irrational situation despite their rational judgment.

Keywords: Money Circulation, Welfare Economics, Under-Consumption, Paradox of Thrift, Intertemporal Choice.

1. Introduction

Saving brings an economic loss even though it is often regarded as a virtue. This proposition, known as the paradox of thrift, is one of main elements of the under-consumption theory. This theory has been known since the early nineteenth century, and became popular especially in the interwar period (Haberler, 1964, pp.118-141; Klein, 1966, pp.124-152; Nash and Gramm, 1969; Bleaney, 1976; Dimand, 1991; Allgoewer, 2002; Clark, 2008; Schneider, 2008).[1]

The under-consumption theory based on recognition of money circulation was already attempted by some pioneers including Nicholas August Ludwig Jacob Johannsen (Hegeland, 1954, pp.5-14; Schneider, 1962, pp.131-134; Marget, 1964, pp.329-336; Allsbrock, 1986; Hagemann and Rühl, 1990; Rühl, 2000), the pair of William Trufant Foster and Waddill Catchings (Gleason, 1959; Carlson, 1962; Tavlas, 1976; Dimand, 2008a, 2008b),[2] and two German economists, Ferdinand Grünig and Carl Föhl (Pedersen, 1954, 1957; Schneider, 1962, pp.156-159; Rothschild, 1964, pp.8-11; Ambrosi, 1996).[3]

The course of these pioneers is basically correct, but its precise foundation

[1] The under-consumption theory was also spread in Japan of the day. In July 1932, when the Great Depression was attacking Japan, Takao Izeki wrote a little overdrawn comment as follows. "There is no person denying that a main cause of this current worldwide depression is a lack of purchasing power in a general consumption class." (Izeki, 1932, p.210) Further, in May 1934, Yasuma Takata discussed the paradox of thrift while examining a denial of the paradox by Friedrich August von Hayek (Hayek, 1931; Takata, 1934).
[2] The author owes a knowledge regarding Foster and Catchings largely to Sasahara (1981) written in Japanese.
[3] The author owes a knowledge regarding Föhl largely to Ito (1952) and Nagasawa (1968) written in Japanese.

still seems to be insufficient. The research of money circulation structure has been continued sometimes independently of the under-consumption theory. Above all, the money circulation equation by Mária Augustinovics in Hungary, which was invented to support a planned economy being affected by the economic input-output theory, is a valuable research work (Augustinovics, 1965). The author developed the equation in the preceding papers (Miura, 2014, 2015a).

However, the money circulation equation is a pure description of the movement of money without pursuing the causes of the movement. Under-consumption is an ethical problem of economic society, but mere description of the movement of money cannot give any evaluation criterion. For the foundation of under-consumption, we need another method.

We think that a social evaluation criterion should be basically based on individual utility. The new welfare economics after Vilfredo Pareto provides a clear method to analyze this idea with using the concept of Pareto efficiency. For the sake of a normative analysis of a monetary economy, we need a method to connect a money circulation analysis with the welfare economics. This method is a money circulation optimization theory. We aim to construct this theory and authenticate the paradox of thrift.

Our attempt can be regarded as a micro-foundation of the paradox of thrift. Recently, there exist some such research efforts including Christiano et al. (2011), Eggertsson (2011), Chamley (2011), Huo & Ríos-Rull (2013). However, they do not prove that saving brings a Pareto inefficient state.

The new welfare economics has usually been connected with the general equilibrium theory so far. This connection derives the first fundamental theorem of welfare economics, which insists that a market economy is realized as a Pareto efficient state (Arrow, 1951; Debreu, 1959, pp.90-97; Stiglitz, 1991; Blaug, 2007; Feldman, 2008). Against this theorem, an authentic under-consumption theory should clarify that saving causes a Pareto inefficient state.

The following description by Paul Anthony Samuelson, who spread the term of the paradox of thrift on a contemporary economics, is notable.

"The individual who saves cuts down on his consumption. He passes on less purchasing power than before. Therefore, someone else's income is reduced. For one man's outgo is another man's income." (Samuelson, 1948, p.271)

Causality between saving and expenditure and revenue shown in this quotation is too important for understanding the paradox of thrift. We succeed to this recognition,[4] but Samuelson's limitation was that he could not found the paradox by welfare economics.

For a foundation of under-consumption by expenditure optimization, the

---

[4] However, we cannot approve the next sentences which follow the quotation in the text. "If one individual succeeds in saving more, it is because someone else is forced to dissave. If one individual succeeds in hoarding more money, someone else must do without." (Samuelson, 1948, p.271) This seems to be affected by the following statement by John Maynard Keynes. "although the amount of his own saving is unlikely to have any significant influence on his own income, the reactions of the amount of his consumption on the incomes of others makes it impossible for all individuals simultaneously to save any given sums." (Keynes, 1936, p.84) This idea is grounded on the identity between saving and investment, which Keynesians espouse. Yet, this proposition is incorrect. The existence of investment is not necessary for that of saving. In the following part of this paper, we will show it by proving that saving can exist without investment.

dual decision hypothesis by Robert Wayne Clower is also a remarkable contribution (Clower, 1965). It implies that realized revenue used in an expenditure optimization should not be regarded as being decided by a level of supply resources of commodities. Although this is a superior critique against the general equilibrium theory, a weak point of the hypothesis is that it does not clarify a decision principle of the realized revenue in lieu of supply resources.

Robert Barro and Herschel Grossman developed the dual decision hypothesis through combining it with Don Patinkin's analysis about labor markets and constructed the general disequilibrium model (Barro and Grossman, 1971). But their model only regards Clower's realized revenue as labor supply and Patinkin's revenue as labor demand. It does not clarify a cause of the realized revenue.

Clower's realized revenue ought to be connected with Samuelson's preceding causality. In a monetary economy, the realized revenue is decided by a money flow from expenditure, whereas expenditure is affected by a money flow from revenue through a decision-making process under the budget constraint. A money circulation structure is composed of these bidirectional flows, and these flows bring bidirectional causalities between expenditure and revenue. Our money circulation optimization is a method to optimize expenditure while considering the bidirectional causalities.[5]

[5] In order to understand the content of this paper easily, the author recommends that you preliminarily read Miura (2016), which he wrote as an introduction of this paper.

2. Basic Assumptions

In this paper, we will discuss a primitive monetary economy. This primitive economy is not a faithful description of our monetary economy. However, for the purpose of clearly understanding the essence of the under-consumption problem, we believe that it is appropriate to analyze a cause of under-consumption in a primitive economy as a primary approach.

We will clarify a basic assumption of this paper.

We decide that a target group for description is called the relevant society. The relevant society consists of finite number of agents. We give natural numbers to each agent of the relevant society. The relevant society is denoted by a set of agents as $N=\{1,\ldots,n\}$.

The target term for description is called the relevant term. We assume that the relevant term is of finite length, thus the relevant term always has its term beginning and term end. The sphere which includes the relevant society and the relevant term is called the relevant space-time.

We assume that money is always used in all exchanges in the relevant space-time. In other words, the world we discuss is a monetary economy. Barter trading does not exist in our assumed economy.

We define expenditure as transferring money to the relevant space-time, and revenue as money being transferred from the relevant space-time. Further, we put a supposition that production and disappearance of money do not occur in the relevant space-time. We also suppose that money transfer between the relevant society and its outside does not exist in the relevant term. By these suppositions, the money stock in the relevant space-time is constant.[6]

Note that there is an economic theory called the monetary circuit theory, which has been mainly developed by French and Italian economists. It shares our interest in respect of emphasizing circulation in order to understand a monetary economy. Another feature of the circuit theory is to emphasize credit creation and destruction of money by banks (Graziani, 1990, pp.12-14; Deleplace and Nell, 1996, p.11; Rochon and Rossi, 2003, p.xxxiii). From this perspective, our supposition that money is constant may be hard to be justified.

We do not deny that credit money is an important issue of a modern economy, but we believe that our supposition is justified as a primary approach for money circulation research.

Perishable goods disappear as soon as they are used, thus production of the goods is worth consideration. Durable goods do not disappear for a long time, but often show deterioration with their use, thus production of the goods also needs to be considered.

On the other hand, if money is used, it is only transferred to another agent and does not disappear. It does not show deterioration through the transfer, either. Hence, money can circulate for a long time. Circuit theorists recognize money circulation as a complete process from creation to destruction, but we do not know clearly when the circulation begins and

---

[6] Strictly speaking, a further assumption is necessary for constancy of the money stock. See p.16 of this paper.

ends. Money circulation is a semi-permanent process, thus creation and destruction are not primary issues of the money circulation. Therefore, a primary research to inquire into the circulation should regard money as an eternal existence.

Further, there is only one kind of consumption goods in our primitive economy. We assume that money transfer is limited to the transfer through the trading of consumption goods. Although the existence of production factors including labor is permitted, we suppose they are not traded. Our assumption can also be expressed that all agents in the relevant space-time are individual proprietors who sell only consumption goods. All other money transfers including tax payment and donation are assumed not to exist.

Note that a type of saving we can treat is limited by this assumption. Phenomena which we usually call saving are commonly disposal methods of money which enable the future expenditure of a saver.[7] This view of saving is based on an individual usage of an economic agent. However, if we pay attention to a social movement of money, we can classify savings into two different types. One type of saving is hoarding, which is not transferred while saving is executed. However, the other saving types including deposit, loan and equity investment are transferred from their owner to another agent while saving is executed. For this reason, we call the latter savings collectively transfer saving. People who are interested in movement of

[7] This definition can be applied only for a saving of money. We can define saving of usual commodity similarly as a disposal method of commodity which enables the future use of a saver. However, this paper discusses only a saving of money.

money must notice this difference between the two savings.[8]

Our assumption that money transfer is limited to the transfer for consumption goods eliminates the existence of transfer saving. A saving method in our primitive economy is limited to hoarding, which is the simplest method of saving. Even though transfer saving is an important element of a modern economy, our assumption that saving methods are limited to hoarding will enable us to understand a welfare economic essence of under-consumption phenomenon simply and clearly.

Next, we introduce an idea of the disposal irreversibility principle. This represents that revenue can be expended neither before nor exactly at the same time when revenue is received. This principle is substantially the same as to reject a possibility of time travel into the past or exactly simultaneous teleportation. Since these possibilities are not approved by the present state of scientific knowledge (Gott, 2002; Davies, 2002; Nemiroff and Wilson, 2014), this is a scientifically justifiable principle.

If we do not consider the disposal irreversibility principle, aporias which we named two missing problems are caused in a monetary budget constraint. We have to incorporate this principle into the budget constraint to avoid the aporias, as elaborately discussed in the author's preceding paper (Miura, 2015b). We will clarify this irreversibility budget constraint concretely based

[8] Henry Abbati, who was an under-consumptionist acting in the interwar period, emphasized to distinguish between hoarding and transfer saving from an early day, though he was captured by a traditional prejudice which tends to identify transfer saving with investment (Abbati, 1924, p.101, n.2). The author owes a knowledge regarding Abbati's this recognition to Kojima (1995, pp.122-123) written in Japanese. Also note that Dennis Holme Robertson comments on Abbati's under-consumption theory in Robertson (1932).

on our supposition of the primitive economy.[9]

Based on the disposal irreversibility principle, if the relevant term is divided into very short terms, revenue in the term cannot be expended in the same term. We call such a short term a basic-term. We give natural numbers to each basic-term. We set the numbers of basic-terms such that smaller numbers correspond to the earlier basic-terms and larger numbers correspond to later basic-terms. Note that the relevant term can be divided into finite basic-terms by the principle (Miura, 2014, pp.194-195; Miura, 2015b, p.93, n.19). Based on this, the relevant term is denoted by a set of basic-terms as $T=\{1,\dots,t\}$.

The budget constraint of each agent is defined for every basic-term.

Since we assume that money is not produced nor transferred from the outside, the budget of the first basic-term is limited to money which an agent possesses at the beginning of the relevant term. A part of the budget is expended. Considering our assumption that money does not disappear or is not transferred to the outside, we define the non-expended part of the budget as a hoarding (Miura, 2015b, p.94).[10] Then, the budget constraint of Agent *i* in the first basic-term is denoted

$$X_{i(1)}+H_{i(1)}=\Psi_i,$$

where Ψ_i is a quantity of money which Agent *i* possesses at the beginning of

[9] An idea of the irreversibility budget constraint was primitively suggested by Dennis Holme Robertson (Robertson, 1933; Keynes et al., 1933; Metzler, 1948; Van Eeghen, 2014), and was developed by Sho Chieh Tsiang and others (Tsiang, 1966; Kohn, 1981; Kohn, 1988; Laidler, 1989).

[10] Based on the disposal irreversibility principle, non-expendable money in a basic-term cannot be included in the budget of the term even if it exists in the term. Accordingly, revenue in a basic-term cannot become hoarding in the same term.

the relevant term; $X_{i(v)}$ is the expenditure quantity of Agent *i* in Basic-term *v*; and $H_{i(v)}$ is the hoarding quantity of Agent *i* in Basic-term *v*.

The hoarding in a basic-term becomes a part of the budget in the following basic-term. The budget also includes revenue received in the preceding basic-term. Components of the budget are limited to these two because we suppose that money is not produced nor transferred from the outside. Then, a part of the budget is expended, and a remaining part is hoarded. Usages of the budget are limited to these two because we suppose that money does not disappear nor transfer to the outside. Then, the budget constraint in Basic-term *v* which is after the first basic-term is denoted

$$X_{i(v)}+H_{i(v)}=H_{i(v-1)}+Y_{i(v-1)},$$

where $Y_{i(v)}$ is the revenue quantity of Agent *i* in Basic-term *v*. Note that the time of revenue and expenditure differs in this constraint. This is the essence of time irreversible disposal.

Hereby, the budget constraints of each basic-term have been determined. We will also clarify how money possessed at the end of the relevant term is decided. Note that hoarding and revenue in the last basic-term cannot be expended in the relevant term. They are forced to be possessed at the term end. Since the other disposable quantities of money have been already included in each budget constraint, they are all of elements of the term end possession. Hence, if we let Ω_i be a quantity of money which Agent *i* possesses at the end of the relevant term,

$$\Omega_i=H_{i(t)}+Y_{i(t)},$$

is satisfied. We call this the term end settlement formula.

A set of the budget constraints of all basic-terms and the settlement formula is the irreversibility budget constraint.

Next, we clarify an assumption regarding individual utility. We assume that all individual agents included in the relevant society shared a finite planning period. Then, we regard the shared planning period as the relevant term. Each individual agent obtains utility from consumption goods exchanged in each basic-term included in the relevant term. Therefore, if we let $C_{i(v)}$ be the quantity of consumption goods obtained by Agent *i* in Basic-term *v*, a utility function of Agent *i* is denoted by $U_i[C_{i(1)},\ldots,C_{i(t)}]$.

Further, we assume that the utility increases if the quantity of consumption goods in each basic-term increases, as far as it does not change the quantity of consumption goods in another basic-term. That is, we put the assumption of non-satiation of utility regarding consumption. Mathematically, the non-satiation of consumption is expressed as $\partial U_i/\partial C_{i(v)}>0$ for any Agent *i* and any Basic-term *v*.

Next, we introduce the concept of price. The price of a basic-term is defined as the expenditure of the basic-term per unit of consumption goods exchanged with the expenditure. Based on this definition, price is always positive value as far as exchange is executed. Then, exchange in a basic-term is assumed to be executed under the common price. Let $P_{(v)}$ be the common price in Basic-term *v*. The formula $X_{i(v)}=P_{(v)}C_{i(v)}$ holds by this assumption, and $C_{i(v)}=X_{i(v)}/P_{(v)}$ also holds.

In this paper, we suppose that price of a basic-term is never affected even if expenditure of any basic-term changes.[11] Note that this supposition of price fixedness does not eliminate a possibility of temporal variation of price. Then, we suppose that an effect on real consumption by nominal expenditure occurs only through price level. Based on this supposition, we derive a relationship between consumption and expenditure.

Theorem 2.1. $\partial C_{i(v)}/\partial X_{i(v)}>0$ holds for any Agent i and any Basic-term v.

Proof. Due to the formula $C_{i(v)}=X_{i(v)}/P_{(v)}$ and the supposition of price fixedness, we obtain $\partial C_{i(v)}/\partial X_{i(v)}=1/P_{(v)}$. Since $P_{(v)}>0$ holds, $\partial C_{i(v)}/\partial X_{i(v)}>0$ is satisfied. [Q.E.D.]

Moreover, we put a supposition that utility obtained from nominal expenditure is determined only by utility obtained from real consumption which is directly exchanged with the expenditure. Based on this supposition, we justify that $\partial U_i/\partial X_{i(v)}=(\partial U_i/\partial C_{i(v)})(\partial C_{i(v)}/\partial X_{i(v)})$ holds between the utility function and nominal expenditure.

Utility regarded as a function of nominal expenditure is called the nominal utility function in this paper. We can derive the following theorem regarding the nominal utility function.

Theorem 2.2. $\partial U_i/\partial X_{i(v)}>0$ holds for any Agent i and any Basic-term v.

Proof. $\partial U_i/\partial C_{i(v)}>0$ holds by the non-satiation of consumption, and $\partial C_{i(v)}/\partial X_{i(v)}>0$ holds by Theorem 2.1. Hence, $(\partial U_i/\partial C_{i(v)})(\partial C_{i(v)}/\partial X_{i(v)})>0$ is

[11] The author previously planned to release a paper which discusses a case where expenditure affects price (See Miura, 2016, p.287), but he does not intend to accomplish it now.

derived. Since we suppose that $\partial U_i/\partial X_{i(v)}=(\partial U_i/\partial C_{i(v)})(\partial C_{i(v)}/\partial X_{i(v)})$ holds, we obtain $\partial U_i/\partial X_{i(v)}>0$. [Q.E.D.]

Theorem 2.2 represents that the nominal utility function of an individual agent increases monotonically. We call this the monotonicity of the nominal utility function.

3. Necessary Condition for Pareto Efficiency

Although we have discussed the budget constraint of an individual agent in the preceding section, the constraint for expenditure can be defined not only per individual unit but also in the whole society. We derive the irreversibility budget constraint of the whole society by aggregating those of all individual agents. However, we decide that the quantities of the whole society are denoted by removing subscripts. The constraint is the set of the following formulas.

$$X_{(1)}+H_{(1)}=\Psi.$$

$$X_{(v)}+H_{(v)}=H_{(v-1)}+Y_{(v-1)}.$$

$$\Omega=H_{(t)}+Y_{(t)}.$$

Here, we pay attention to the relationship between expenditure and revenue. The irreversibility constraint only expresses a money flow from revenue to expenditure but does not express a money flow from expenditure to revenue. We call the latter flow the expenditure reflux phenomenon. Money can circulate because the both flows exist. Therefore, we have to consider the both flows to represent money circulation completely. Such a consideration is indeed the essence of our money circulation optimization.

Note that expenditure is defined as transferring money to the relevant space-time, and revenue is defined as money being transferred from the relevant space-time. Hence, the two concepts describe the same event namely money transfer grasped from different perspectives. Money expended by someone must be received by another. As a result, the sum of expenditure and that of revenue are naturally equal. We call this the law of transfer equality. Using symbols, this law is denoted as $X_{(v)}=Y_{(v)}$ for any Basic-term v.

This law can also be interpreted as a quantitative expression of the expenditure reflux of the whole society (Miura, 2015b, p.98). It represents that variation of expenditure causes the same amount of variation of revenue in the whole society. The budget constraint manages to represent money circulation completely by incorporating the law. Then, we substitute the law of transfer equality for the irreversibility budget constraint. It is the irreversibility reflux budget constraint of the whole society. Since this name is too long, we call it merely the whole budget constraint hereafter. This is an authentic monetary budget constraint of the whole society.

Theorem 3.1. $X_{(v)}+H_{(v)}=\Psi$, $H_{(v)}+Y_{(v)}=\Psi$ and $\Omega=\Psi$ for any Basic-term v.

Proof. We prove this by mathematical induction.

In the case v is the first basic-term, $X_{(1)}+H_{(1)}=\Psi$ is immediately derived from the budget constraint in the first basic-term. In the case v is a following basic-term, we suppose that $X_{(v-1)}+H_{(v-1)}=\Psi$ holds. By this supposition and the law of transfer equality in Basic-term *v−1*, $H_{(v-1)}+Y_{(v-1)}=\Psi$ is satisfied. Substituting this equation for the budget constraint in the following basic-term, we can derive $X_{(v)}+H_{(v)}=\Psi$. Moreover, from this formula and the law of transfer equality, we can derive $H_{(v)}+Y_{(v)}=\Psi$. This conclusion and the term end settlement formula derive $\Omega=\Psi$. [Q.E.D.]

Due to Theorem 3.1, the whole budget constraint can be denoted as $X_{(v)}+H_{(v)}=\Psi$ for any Basic-term *v*. Further, expenditure and hoarding are non-negative by their economic meaning, thus $X_{(v)}\geq 0$ and $H_{(v)}\geq 0$ holds. Therefore, we can derive $0\leq X_{(v)}\leq\Psi$ for any Basic-term *v*. Hereafter, we sometimes indicate this constraint as the whole budget constraint.

Since we assume that money is not produced nor transferred to the outside, Ψ also represents the money stock in the relevant space-time. Accordingly, the whole budget constraint is nothing but a constraint by money stock of the whole society.

This constraint is substantially the same as the cash-in-advance constraint applied to the whole society.[12] However, it should be noted that it is not an axiom, but a theorem derived from the irreversibility budget constraint and the law of transfer equality.

If we substituted the law of transfer equality for the budget constraint without considering the disposal irreversibility principle, expenditure would

[12] The cash-in-advance constraint was primitively suggested by Karl Brunner, and was developed by Mario Henrique Simonsen, Robert Wayne Clower and others. (Brunner, 1951, pp.167-171; Clower, 1967; Boianovsky, 2002).

disappear from the constraint. This is a reflection that, if money can be disposed time reversibly, agents can continue to expend unlimitedly and money can circulate infinitely in a temporally closed place. In this case, money stock does not become a constraint for expenditure, and there exists a possibility that expenditure is not derived as a finite value. However, the irreversibility budget constraint enforces that agents cannot expend repeatedly in one basic-term, thus finiteness of expenditure is guaranteed.[13] A constraint for the whole expenditure by money stock is a product of the time irreversible disposal in a money circulation structure.

Careful readers will notice that it also implies the constancy of money stock is a product of time irreversibility. We explain this strictly. $H_{(v)}+Y_{(v)}$ represents the whole quantity of money which is possessed at the end of Basic-term *v* and at the beginning of Basic-term *v+1* (Miura, 2015b, p.94). Furthermore, Ω represents money stock at term end. Accordingly, Theorem 3.1 expresses that the amount of money in the whole society is constant in the term beginning quantity Ψ. If our world were time reversible, the same money could exist simultaneously, thus we cannot derive the theorem and the constancy of money stock cannot be proved.

You should recognize a significance of the disposal irreversibility principle for the money circulation theory.

Note that the whole budget constraint expresses an expendable range of

[13] This is an example of the second missing problem and its solution discussed in Miura (2015b). The discussion regarding the jinn particle in a physics world may promote an understanding of this issue (Lossev and Novikov, 1992; Gott, 2002, pp.20-24).

the whole society constrained by money stock, but this is not a unique constraint for the whole expenditure. We assume that money is expended only for exchange with consumption goods, thus feasible expenditure is constrained by stocks of the exchanged goods. In order to express this constraint, we define a real commodity supply set as a combination of commodities in each basic-term which can be supplied.

Moreover, in order to enable us to compare it with nominal expenditure, we define a nominal commodity supply set as follows. Let $C_{(v)}$ be the quantity of consumption goods aggregated among the whole society in Basic-term *v*. Note that $C_{(v)}=X_{(v)}/P_{(v)}$ holds. Based on this, a combination of expenditure $(X_{(1)},\ldots,X_{(t)})$ belongs to the nominal commodity supply set if and only if a combination of consumption goods $(X_{(1)}/P_{(1)},\ldots, X_{(t)}/P_{(t)})$ belongs to the real commodity supply set.

The nominal commodity supply set represents expenditure quantity which is needed to exchange supplied commodity. The feasible expenditure of the whole society must belong to both the whole budget constraint and the nominal supply set. Therefore, we define an intersection of these as a feasible expenditure set.

In the case that $X_{(v)}=\Psi$ holds for any Basic-term *v*, $H_{(v)}=0$ also holds for any Basic-term *v* by the whole budget constraint. We call this the non-hoarding state. If the non-hoarding state is not included into the nominal supply set, it does not belong to the feasible expenditure set. In this case, money must be hoarded because commodities are sold out even by given low expenditure. We call this undersupplied hoarding.

Considering $X_{(v)}=P_{(v)}C_{(v)}$ holds, the nominal supply set becomes small if price becomes low. On the other hand, the whole budget constraint does not vary by a price change. Therefore, commodities are easily sold out if price is too low. As a result, undersupplied hoarding is easy to occur.

Even if undersupplied hoarding occurs, all supplied commodities can be consumed. Therefore, this is not a hoarding which brings an economic loss. We should pay attention that all types of hoarding do not cause a loss.

From here, we suppose that the whole budget constraint is always included into the nominal commodity supply set. By this supposition, the whole budget constraint accords with the feasible expenditure set. This supposition concludes that the undersupplied hoarding does not occur.

Then, we prove the following theorem regarding Pareto efficiency defined in the feasible expenditure set.

Theorem 3.2. We assume that the feasible expenditure set accords with the whole budget constraint. Further, price is supposed to be fixed in the range of the whole constraint. The non-hoarding state is a necessary condition for Pareto efficiency in the feasible expenditure set.

Proof. Let $(X_{(1)}^*,\ldots,X_{(t)}^*)$ be a state which satisfies the whole budget constraint and in which money is hoarded. In this state, there exists an agent who hoards money in a basic-term. Let *j* be an agent who hoards money and *w* be a basic-term in which Agent *j* hoards money. $H_{j(w)}^*>0$ is satisfied by this definition. Due to the whole budget constraint and the non-negativity of expenditure and hoarding, $0 \leq (X_{j(w)}{}^* + H_{j(w)}{}^*) + \sum_{i \neq j} X_{i(w)}{}^* \leq \Psi$ holds for Basic-term *w* and $0 \leq X_{(v)}{}^* \leq \Psi$ holds for any Basic-term *v*.

Then, we define a state $(X_{(1)}{}^{**}, \ldots, X_{(t)}{}^{**})$ such that $X_{i(v)}{}^{**} = X_{i(v)}{}^*$ if $v \neq w$ or $i \neq j$ and $X_{j(w)}{}^{**} = X_{j(w)}{}^* + H_{j(w)}{}^*$. Note that $0 \leq X_{(v)}{}^{**} = X_{(v)}{}^* \leq \Psi$ is satisfied if $v \neq w$ and $0 \leq (X_{j(w)}{}^* + H_{j(w)}{}^*) + \sum_{i \neq j} X_{i(w)}{}^* = X_{j(w)}{}^{**} + \sum_{i \neq j} X_{i(w)}{}^{**} \leq \Psi$ is also satisfied. Therefore, we can confirm that $(X_{(1)}{}^{**}, \ldots, X_{(t)}{}^{**})$ satisfies the whole budget constraint, which accords with the feasible expenditure set.

Considering $H_{j(w)}{}^* > 0$ holds, we can derive $X_{j(w)}{}^{**} = X_{j(w)}{}^* + H_{j(w)}{}^* > X_{j(w)}{}^*$. Further, $X_{j(v)}{}^{**} = X_{j(v)}{}^*$ is satisfied if $v \neq w$. By Theorem 2.2, we can see that $U_j[X_{j(1)}{}^{**}, \ldots, X_{j(t)}{}^{**}] > U_j[X_{j(1)}{}^*, \ldots, X_{j(t)}{}^*]$ holds. On the other hand, $X_{i(v)}{}^{**} = X_{i(v)}{}^*$ holds for any Basic-term *v* and any Agent *i* excluding Agent *j*. Theorem 2.2 implies that utility of an agent does not vary if expenditure of the agent does not vary in any basic-term. Hence, $U_i[X_{i(1)}{}^{**}, \ldots, X_{i(t)}{}^{**}] = U_i[X_{i(1)}{}^*, \ldots, X_{i(t)}{}^*]$ holds if $i \neq j$. The utility of Agent *j* in $(X_{(1)}{}^{**}, \ldots, X_{(t)}{}^{**})$ increases than that in $(X_{(1)}{}^*, \ldots, X_{(t)}{}^*)$, which represents an arbitrary hoarding state, without changing the utilities of the other agents. Therefore, any hoarding state is Pareto inefficient in the feasible expenditure set. [Q.E.D.]

We interpret Theorem 3.2 qualitatively while considering consistency of money stock. For example, foods disappear by its use, thus it is beneficial that foods are hoarded for the future use. However, money does not disappear and it is not degraded by its use in the whole society. Money which is once used is transferred to the other and can be used again. Hence, hoarding of money is merely a loss of opportunity for use, thus it is an ineffective behavior.

Note that, if the nominal commodity supply set is included in the whole budget constraint, the supply set accords with the feasible expenditure set. In this case, a Pareto efficient state in the former set accords with the state in the latter set.

However, we assume that an inclusion relation between the supply set and the whole budget constraint is opposite. The two efficient states can accord even if our assumed inclusion relation holds, but it is not always valid. There is a possibility that the two efficient states are different. The Pareto efficiency in the supply set is a pure efficiency of real consumption, but feasibility of this efficiency can be blocked because the money stock in the whole society is too short to execute expenditure needed to purchase all supplied commodities. We call this economic loss a money shortage loss.

Considering $X_{(v)}=P_{(v)}C_{(v)}$ holds, the nominal supply set becomes large if price becomes high level. On the other hand, the whole budget constraint does not vary by a price change. Therefore, the supply set is hard to be included in the whole budget constraint if price is too high. As a result, the money shortage loss is easy to occur.

The money shortage loss is a cause of under-consumption, but it is not its unique cause. Even if the money shortage loss does not exist, another economic loss can occur because of hoarding. Hereafter, we examine this possibility.

4. Various Causes of Hoarding

The non-hoarding state is a necessary condition for Pareto efficiency in our primitive economy. Then, is there a possibility that agents hoard money and an economic loss is brought? In order to pursue a possibility of this hoarding loss, we aim to examine causes of hoarding.

As mentioned above, we eliminate a possibility of the undersupplied hoarding. If we accept this possibility, we can explain the existence of hoarding. However, all supplied commodities can be consumed even if this hoarding occurs, thus it is not a hoarding which brings an economic loss. In order to found the hoarding loss, we should pursue another cause of hoarding.

We can doubt the assumption of the non-satiation of consumption. A satiation of consumption can be realistic, and there may be an agent whose utility is satiated in a medium of the budget constraint. In this case, they hoard money because their utility is not increased through more expenditure. However, since this satiated hoarding does not bring a decrease of utility, it is not also an economic loss.

Note that realization of exchange is not guaranteed only by the existences of potential buyer and seller. Feasible expenditure is constrained by many environmental conditions such as a bodily condition, a geographical condition, a communication condition, a linguistic condition and a political condition (Miura, 2015b, pp.89-90). Nevertheless, these environmental constraints have never been considered yet excluding the disposal irreversibility principle. Some of these may make agents be impossible to exchange money. In this case, agents are forced to hoard money.

Since many environment constraints do not seem to affect utility, they may be able to found the hoarding loss. Nevertheless, there seems to be some environment constraints which affect a utility. For example, human beings need not any food if they can live without eating, and they will not desire any car if they need not move anywhere. If these living conditions of human beings change, they would tend to hoard easily. Such an environmental hoarding is a kind of the satiated hoardings, thus it cannot found the hoarding loss.

Even if environment constraints do not affect utility and an environmental hoarding brings an economic loss, the hoarding is hard to be called a form of hoarding based on a positive intention. Not only the environmental hoarding but also the undersupplied hoarding and the satiated hoarding are not caused by abstinent intention. The paradox of thrift is called a paradox because saving is regarded as a virtue based on an abstinent intention. Hence, for an explanation of the paradox of thrift, we should explain the existence of hoarding by an intentional abstinent.

Until now, we have assumed that hoarding does not bring utility. If we change this assumption and incorporate hoarding in the utility function similar to the idea of the money-in-the-utility function (Feenstra, 1992; Walsh, 2003, pp.43-59), we will be able to explain the existence of the intentional hoarding easily.

However, in the case that hoarding is included in the individual utility, it seems to be difficult to regard the hoarding as an economic loss. But we should not lightly judge so because situations which environ an individual and a society can be different. We ought to examine relationships between individual utility and social welfare considering this difference of both situations. In order to judge this issue, we need to think about a ground of justifiability that hoarding brings utility.

What is a feature of that hoarding gives utility? It is not a utility obtained from a future consumption because it is already included in the utility function. There exists a fact that some people collect rare coins and bills as a hobby. In this case, it is not doubtful that they obtain utility from hoarding. Some people may hoard money based on asceticism morality. This can also be interpreted as one of the cases that hoarding gives utility. If hoarding occurs by these reasons, it ought to be considered in social welfare, thus it is inappropriate to found the hoarding loss. For the purpose, we must search for another reason.

Paul Davidson justified the utility of money by an uncertainty (Davidson, 2012, p.354). It can be one of main reasons of hoarding, and there exist research works of the precautionary saving, which is a saving based on uncertainty (Kimball, 1990; Carroll and Kimball, 2008). If uncertainty is caused by relationships between others, it is inherent to individual agents. Accordingly, there may be no problem even if hoarding is not considered in social welfare.

Uncertainty seems to be a kind of individual irrationality. We do not deny that the individual irrationality including uncertainty is an important issue for understanding a realistic economy. It is worthy of note that Bruce Greenwald and Joseph Stiglitz showed that incomplete information or incomplete markets are not Pareto efficient (Greenwald and Stiglitz, 1986). However, if the hoarding loss is caused only by individual irrationalities, the loss would disappear when individual intelligence improves to the maximum.

But in fact, a society can be irrational even if individual agents are entirely rational. The improvement of individual intelligence will not perfectly solve an economic loss. There exists a limitation to a solution of social irrationality by individual rationality. In order to show it, we strategically adopt an assumption of rational agents with perfect information. The same as before, we assume that hoarding is not included in the utility function for a definite foundation of the hoarding loss. Then, we will search for occurrence conditions of abstinent hoarding by an individual exercising rational judgment.

5. Foundation of Hoarding Loss

Our money circulation optimization is an optimal method of expenditure which reflects two money flows between expenditure and revenue. The flow from revenue to expenditure is expressed as an expenditure optimization under the budget constraint, which includes revenue. The flow from expenditure to revenue is expressed by the law of transfer equality, which is a quantitative expression of the whole expenditure reflux. Based on this idea, an individual optimal solution of the money circulation optimization is defined as an expenditure optimal solution by an individual decision-making which is consistent with the law of transfer equality.

First, we examine an individual decision-making. We already confirmed the monotonicity of the nominal utility function in Theorem 2.2. That is, $\partial U_i/\partial X_{i(v)}>0$ holds for any Agent *i* and any Basic-term *v*. Moreover, we add two new suppositions to the nominal marginal utility.

First, we suppose that the nominal marginal utility is diminishing. Mathematically, this supposition is denoted by $\partial^2 U_i/\partial X_{i(v)}{}^2<0$ for any Agent *i* and any Basic-term *v*.

Second, we suppose that the nominal marginal utility of a basic-term is independent of expenditure of another basic-term. Mathematically, this supposition is denoted by $\partial^2 U_i/\partial X_{i(v)}X_{i(w)}=0$ between different Basic-terms *v* and *w* for any Agent *i*. Due to this supposition, nominal marginal utility of a basic-term is only a function of expenditure of the same basic-term. Based on this, we can denote the nominal marginal utility as $\partial U_i/\partial X_{i(v)}=U_{i(v)}[X_{i(v)}]$.

Next, we consider the budget constraint which each individual agent faces. For a simple analysis, we ignore environmental constraints excluding the disposal irreversibility principle. Further, we have to consider causality between expenditure and revenue for an individual agent.

If an individual agent expends, their revenue is not guaranteed to increase. Inversely, even if the individual agent receives money, they do not need to expend it. Therefore, expenditure and revenue are two different events for an individual agent. Based on this, we assume that the individual agent maximizes their utility under the irreversibility budget constraint with recognition that expenditure does not affect revenue.[14]

Eventually, Agent *i* maximizes $U_i[X_{i(1)},\dots,X_{i(t)}]$ subject to $X_{i(1)}+H_{i(1)}=\Psi_i$ and $X_{i(v)}+H_{i(v)}=H_{i(v-1)}+Y_{i(v-1)}$ such that $v\geq 2$. In addition, the non-negativity constraints, $X_{i(v)}\geq 0$ and $H_{i(v)}\geq 0$, must hold for any Basic-term *v*.

Expenditure and hoarding are both variables of this optimization. We aim to delete the latter variable. Adding up constraints from the first basic-term to Basic-term *v*, we obtain the following formula.

$$X_{i(1)}+\dots+X_{i(v-1)}+X_{i(v)}+H_{i(v)}=\Psi_i+Y_{i(1)}+\dots+Y_{i(v-1)}.$$ [15]

We replace this with the constraint in Basic-term *v*. Applying the non-negativity constraint of $H_{i(v)}$ to this constraint, we can derive

$$X_{i(1)}+\dots+X_{i(v-1)}+X_{i(v)}\leq\Psi_i+Y_{i(1)}+\dots+Y_{i(v-1)}.$$

Agent *i* maximizes $U_i[X_{i(1)},\dots,X_{i(t)}]$ under this constraint and the non-negativity of expenditure applied to all basic-terms. A solution of this maximum problem is necessary to satisfy the Karush-Kuhn-Tucker (KKT)

[14] In reality, the expenditure reflux occurs also in an individual unit, thus expenditure of an individual agent affects revenue of the same agent. Based on this recognition, the author suggested the irreversibility reflux budget constraint of an individual agent (Miura, 2015b, p.98). However, the constraint was schemed as a normative constraint not as a realistic constraint. Since an empirical recognition of the individual reflux is too difficult, any real agent does not seem to make their decision considering the reflux. Therefore, our supposition that the individual budget constraint does not incorporate the expenditure reflux seems to be appropriate for a realistic decision-making.

[15] This is an original formula which is used to derive the irreversibility budget constraint (Miura, 2015b, pp.94-95).

conditions.[16] We aim to clarify these conditions concretely.

We define a Lagrangian function of Agent *i* as follows.

$$L_i=U_i[X_{i(1)},\ldots,X_{i(t)}]$$
$$+\sum_{v\in T}\lambda_{i(v)}(\Psi_i+Y_{i(1)}+\ldots+Y_{i(v-1)}-X_{i(1)}-\ldots-X_{i(v-1)}-X_{i(v)})+\sum_{v\in T}\theta_{i(v)}X_{i(v)}$$

Note that $\lambda_{i(v)}$ and $\theta_{i(v)}$ refer to the Lagrange multipliers. Let $X_{i(v)}{}^*$ be an optimal solution of $X_{i(v)}$. By the KKT conditions, we obtain

$$U_{i(v)}[X_{i(v)}{}^*]=\lambda_{i(v)}+\ldots+\lambda_{i(t)}-\theta_{i(v)},$$
$$X_{i(1)}{}^*+\ldots+X_{i(v-1)}{}^*+X_{i(v)}{}^*\leq\Psi_i+Y_{i(1)}+\ldots+Y_{i(v-1)},\ X_{i(v)}{}^*\geq 0$$
$$\lambda_{i(v)}(\Psi_i+Y_{i(1)}+\ldots+Y_{i(v-1)}-X_{i(1)}{}^*-\ldots-X_{i(v-1)}{}^*-X_{i(v)}{}^*)=0,$$
$$\theta_{i(v)}X_{i(v)}{}^*=0,\ \lambda_{i(v)}\geq 0,\ \theta_{i(v)}\geq 0.$$

Theorem 5.1. $\lambda_{i(t)}>0$ for any Agent *i*.

Proof. $U_{i(t)}[X_{i(t)}{}^*]=\lambda_{i(t)}-\theta_{i(t)}$ holds by the KKT condition, and $U_{i(t)}[X_{i(t)}{}^*]>0$ is satisfied by the monotonicity of utility in Theorem 2.2. Therefore, we obtain $\lambda_{i(t)}>\theta_{i(t)}$. Since $\theta_{i(t)}\geq 0$ holds by the KKT condition, $\lambda_{i(t)}>0$ is derived. [Q.E.D.]

In order to simplify an analysis, we hereafter assume that $\theta_{i(v)}=0$ holds generally. This assumption requests that the non-negativity of expenditure does not become an active constraint. If it becomes an active constraint, $X_{i(v)}{}^*=0$ must hold. We require that expenditure of all agents in any basic-term ought to be derived as a positive value. This does not seem so unnatural assumption because any agents need to expend money a little as far as they continue to live in relevant space-time.[17]

---

[16] Regarding the general KKT conditions, refer to texts of mathematical optimization theory including Bertsekas (1999, p.316).

[17] Someone may object like this. Human beings have to live with doing other than trading. They also have to sleep. Hence, any human beings have a time not to expend money. This objection is realistically appropriate. Yet, we declare that we ignore

Based on this, we can prove the following theorem.

Theorem 5.2. Let $v<w$. $U_{i(v)}[X_{i(v)}^*] \geq U_{i(w)}[X_{i(w)}^*]$ holds generally. Moreover, $U_{i(v)}[X_{i(v)}^*]=U_{i(w)}[X_{i(w)}^*]$ holds if and only if $\lambda_{i(r)}=0$ for $v \leq r \leq w-1$.

Proof. From the KKT conditions considering $\theta_{i(v)}=\theta_{i(w)}=0$, we can derive

$$U_{i(v)}[X_{i(v)}^*]-U_{i(w)}[X_{i(w)}^*]=\lambda_{i(v)}+\ldots+\lambda_{i(w-1)}.$$

Since all Lagrange multipliers are always non-negative by the KKT conditions, $U_{i(v)}[X_{i(v)}^*] \geq U_{i(w)}[X_{i(w)}^*]$ holds generally. Further, if $\lambda_{i(r)}=0$ holds for $v \leq r \leq w-1$ generally, it is obvious that $U_{i(v)}[X_{i(v)}^*]=U_{i(w)}[X_{i(w)}^*]$ holds. If there exists r such that $\lambda_{i(r)}>0$, considering with the non-negativity of all Lagrange multipliers, $U_{i(v)}[X_{i(v)}^*]>U_{i(w)}[X_{i(w)}^*]$ is satisfied. [Q.E.D.]

As mentioned above, the optimal solution of the money circulation optimization must be consistent with the law of transfer equality. In order to secure the consistency, we aggregate the individual optimal solutions and apply the law of transfer equality to the aggregated solution.

Note that the whole budget constraint is derived from the aggregated irreversibility budget constraint and the law of transfer equality. Accordingly, the aggregated solution must also satisfy the whole budget constraint for the optimal solution is consistent with the law of transfer equality.

Let $X_{(v)}^*$ and $H_{(v)}^*$ be aggregated optimal solutions of expenditure and hoarding. We can prove the following theorems regarding the aggregated solutions.

---

environmental constraints excluding time irreversible disposal in p.25 of this paper, thus we believe that we can evade such an objection.

Theorem 5.3. In any Basic-term v, if $\lambda_{i(v)}>0$ holds for any Agent i, $X_{(v)}^*=\Psi$ and $H_{(v)}^*=0$ are satisfied.

Proof. By the KKT conditions, the assumption derives that

$$\Psi_i+Y_{i(1)}+\ldots+Y_{i(v-1)}-X_{i(1)}^*-\ldots-X_{i(v-1)}^*-X_{i(v)}^*=0,$$

holds for any Agent i. Aggregating this equation of all agents, we obtain

$$\Psi+Y_{(1)}+\ldots+Y_{(v-1)}-X_{(1)}^*-\ldots-X_{(v-1)}^*-X_{(v)}^*=0.$$

Further, $Y_{(w)}=X_{(w)}^*$ for $1\leq w\leq v-1$ holds by the law of transfer equality. As a result, $X_{(v)}^*=\Psi$ is derived. Moreover, since $H_{(v)}^*=\Psi-X_{(v)}^*$ holds because of the whole budget constraint, we can also derive $H_{(v)}^*=0$. [Q.E.D.]

Theorem 5.4. $X_{(t)}^*=\Psi$ and $H_{(t)}^*=0$.

Proof. This is derived from Theorems 5.1 and 5.3. [Q.E.D.]

Theorem 5.4 shows that anyone does not hoard money at all in an optimal state of the last basic-term. This is intuitively because hoarding of the last basic-term cannot be expended in the planning period any more thus it is judged as ineffective. If agents set one basic-term for the planning period, the relevant term consists of only the last basic-term, so Theorem 5.4 represents that anyone does not hoard money in this case. We can see that the hoarding loss does not occur if all expenditure plans are executed momentarily. Yet, humans are beings living in a long time, thus momentary expenditure planning is worthless to consider. Hereafter, we examine a case where the planning period consists of plural basic-terms.

In order to simplify an analysis, we introduce a new supposition that the Lagrange multipliers of the same basic-term share a sign among all agents.

That is, $\lambda_{i(v)}=0$ holds for any Agent *i* if there exists Agent *j* such that $\lambda_{j(v)}=0$, and $\lambda_{i(v)}>0$ holds for any Agent *i* if there exists Agent *j* such that $\lambda_{j(v)}>0$.

Then, we define some types of time preference regarding expenditure by comparing nominal marginal utilities of different basic-terms obtained from the same amount of expenditure. We will classify agents into three time preference types.

Let $v<w$ hold.

If $U_{i(v)}[X]>U_{i(w)}[X]$ is satisfied, past expenditure gives larger marginal utility than future expenditure at an expenditure level X. In this case, it is defined as Agent *i* preferring past expenditure between Basic-terms *v* and *w* at an expenditure level X.

If $U_{i(v)}[X]<U_{i(w)}[X]$ is satisfied, future expenditure gives larger marginal utility than past expenditure at an expenditure level X. In this case, it is defined as Agent *i* preferring future expenditure between Basic-terms *v* and *w* at an expenditure level X.

If $U_{i(v)}[X]=U_{i(w)}[X]$ holds, past expenditure and future expenditure gives the same marginal utility at an expenditure level X. In this case, it is defined as Agent *i* preferring expenditure time neutrally between Basic-terms *v* and *w* at an expenditure level X.

Then, we suppose a uniformity of time preference regarding expenditure level. This uniformity means that time preference does not depend on the expenditure level. If an agent in a basic-term prefers the past expenditure at an expenditure level, the agent also prefers the past expenditure in any another expenditure level. In the case that the agent prefers the future or time neutrality, the same relationship is supposed. Uniformity may not be appropriate in reality, but this paper supposes to simplify analysis.

Based on the supposition of the uniformity, the individual agent is called a past preference type if the agent prefers past expenditure. Moreover, the agent is called a future preference type if the agent prefers future expenditure, and the agent is called a time neutral preference type if the agent prefers expenditure time neutrally.

This paper analyzes a case where all agents have similar preference. In this case, we can derive the following theorems.

Theorem 5.5. Assume that there exist Basic-terms v and w such that $v<w$, $X_{(w)}*=\Psi$ holds, and time preference type of all agents between v and w is the time neutral preference type. Moreover, if there exists Basic-term r such that $v+1\leq r\leq w-1$, $\lambda_{i(r)}=0$ is assumed to be all satisfied. In this case, $\lambda_{i(v)}=0$, $X_{(v)}*=\Psi$ and $H_{(v)}*=0$ hold.

Proof. We first prove that there exists an agent such that $\lambda_{i(v)}=0$ by reduction ad absurdum. Suppose that $\lambda_{i(v)}>0$ holds for all agents. From Theorem 5.2, $U_{i(v)}[X_{i(v)}*]>U_{i(w)}[X_{i(w)}*]$ holds for any Agent i. Since all agents are assumed to be the neutral preference type between Basic-term v and w,

$$U_{i(w)}[X_{i(v)}*]=U_{i(v)}[X_{i(v)}*]>U_{i(w)}[X_{i(w)}*]=U_{i(v)}[X_{i(w)}*]$$

holds generally. Due to the diminishment of marginal utility, the conditions $U_{i(w)}[X_{i(v)}*]>U_{i(w)}[X_{i(w)}*]$ and $U_{i(v)}[X_{i(v)}*]>U_{i(v)}[X_{i(w)}*]$ derive $X_{i(v)}*<X_{i(w)}*$. This inequality is satisfied in any agents. Aggregating these inequalities of all

agents, we obtain $X_{(v)}^* < X_{(w)}^*$. Since $X_{(w)}^* \leq \Psi$ holds generally by the whole budget constraint, $X_{(v)}^* < \Psi$ is derived. However, the supposition $\lambda_{i(v)} > 0$ for all agents derives $X_{(v)}^* = \Psi$ by Theorem 5.3. This is a contradiction. The supposition is denied by reduction ad absurdum.

Based on the sharing the sign of Lagrange multiplier, a remained possibility is $\lambda_{i(v)} = 0$ for all agents. Considering that $\lambda_{i(r)} = 0$ is assumed to be all satisfied for $v+1 \leq r \leq w-1$, $\lambda_{i(r)} = 0$ holds for any $v \leq r \leq w-1$. From Theorem 5.2, $U_{i(v)}[X_{i(v)}^*] = U_{i(w)}[X_{i(w)}^*]$ holds for any Agent *i*. All agents are assumed to be the neutral preference type between Basic-term *v* and *w*, thus

$$U_{i(w)}[X_{i(v)}^*] = U_{i(v)}[X_{i(v)}^*] = U_{i(w)}[X_{i(w)}^*] = U_{i(v)}[X_{i(w)}^*]$$

holds generally. Due to the diminishment of marginal utility, the conditions $U_{i(w)}[X_{i(v)}^*] = U_{i(w)}[X_{i(w)}^*]$ and $U_{i(v)}[X_{i(v)}^*] = U_{i(v)}[X_{i(w)}^*]$ derive $X_{i(v)}^* = X_{i(w)}^*$. This equation is satisfied in any agents. Aggregating these equations of all agents, we obtain $X_{(v)}^* = X_{(w)}^*$. Since $X_{(w)}^* = \Psi$ is assumed, we obtain $X_{(v)}^* = \Psi$. Moreover, $H_{(v)}^* = 0$ is derived by the whole budget constraint. [Q.E.D.]

Theorem 5.6. Assume that there exist Basic-terms *v* and *w* such that $v < w$, and $X_{(w)}^* = \Psi$ holds. Further, all agents are the time neutral or past preference type between *v* and *w*, and at least one agent is the latter type. Moreover, if there exists Basic-term *r* such that $v+1 \leq r \leq w-1$, $\lambda_{i(r)} = 0$ is assumed to be all satisfied. In this case, $\lambda_{i(v)} > 0$, $X_{(v)}^* = \Psi$ and $H_{(v)}^* = 0$ hold.

Proof. We prove $\lambda_{i(v)} > 0$ for any Agent *i* by reduction ad absurdum. According to the sharing the sign of Lagrange multiplier, we suppose $\lambda_{i(v)} = 0$ holds for any agent. Considering that $\lambda_{i(r)} = 0$ is assumed to be all satisfied for

$v+1 \leq r \leq w-1$, $\lambda_{i(r)}=0$ holds for any $v \leq r \leq w-1$. Due to Theorem 5.2, $U_{i(v)}[X_{i(v)}{}^*]=U_{i(w)}[X_{i(w)}{}^*]$ holds for any Agent i. All agents are assumed to be the neutral or past preference type between Basic-term v and w, thus

$$U_{i(w)}[X_{i(v)}{}^*] \leq U_{i(v)}[X_{i(v)}{}^*]=U_{i(w)}[X_{i(w)}{}^*] \leq U_{i(v)}[X_{i(w)}{}^*]$$

holds for any Agent i, and there exists Agent j such that

$$U_{j(w)}[X_{j(v)}{}^*] < U_{j(v)}[X_{j(v)}{}^*]=U_{j(w)}[X_{j(w)}{}^*] < U_{j(v)}[X_{j(w)}{}^*]$$

Due to these and the diminishment of marginal utility, we can derive $X_{i(v)}{}^* \geq X_{i(w)}{}^*$ for any Agent i and $X_{j(v)}{}^* > X_{j(w)}{}^*$ for one Agent j. Aggregating these inequalities of all agents, we obtain $X_{(v)}{}^* > X_{(w)}{}^*$. Since $X_{(w)}{}^*=\Psi$ is assumed, $X_{(v)}{}^* > \Psi$ holds, but this contradicts the whole budget constraint. $\lambda_{i(v)}>0$ for any agents has been proved by reduction ad absurdum and the sharing of the sign of Lagrange multiplier. $X_{(v)}{}^*=\Psi$ and $H_{(v)}{}^*=0$ is derived from this proved proposition and Theorem 5.3. [Q.E.D.]

Note that $X_{(t)}{}^*=\Psi$ is satisfied by Theorem 5.4. Hence, Theorems 5.5 and 5.6 seem to clarify that hoarding never occurs if agents are the time neutral or past preference type. However, should we accept this conclusion simply?

Suppose that all assumptions of Theorem 5.5 hold. As shown in the proof of the theorem, $X_{i(v)}{}^*=X_{i(w)}{}^*$ holds for all agents. Also by the theorem, $H_{i(v)}{}^*=H_{i(w)}{}^*=0$ holds generally. Hence, $X_{i(v)}{}^*+H_{i(v)}{}^*=X_{i(w)}{}^*+H_{i(w)}{}^*$ is satisfied for any agents. Since the sum of expenditure and hoarding in the same basic-term accords with the budget of the term,[18] this conclusion means that budgets of the two basic-terms must be equal in all agents.

[18] See pp.9-10 of this paper.

Yet, this seems to be an extremely rare case. The budget in each basic-term ought to be decided depending on an initial state and distribution.[19]

Why have we derived such a rare case? It seems to be because of our supposition that all agents share a sign of Lagrange multipliers. We rethink this supposition. Due to the KKT conditions, a value of Lagrange multiplier connects with that of marginal utility of optimal state. Hence, the supposition seems to request a kind of homogeneity of situation among all agents. As a result, the supposition may make it impossible to analyze an effect of distribution.

This supposition of homogeneity seems to be similar to the so-called assumption of the representative agent. Although it is widely used in contemporary macroeconomics, there exists critique against it (Kirman, 1992; Carroll, 2000; An et al., 2009). It has an advantage of simple analysis, thus we also adopted it for an introductory explanation for our money circulation optimization (Miura, 2016). However, we should also recognize that the representative agent model makes it impossible to analyze an effect of distribution. In addition, it tends to blind us to difference in situation between an individual and society. This difference is indeed the essence of hoarding loss we intend to prove.[20] For more advanced research of economic theory, we should abandon the representative agent model.

We will get back on track. If we accept that Lagrange multipliers can be

---

[19] This is implied by the solution of the money circulation equation regarding the end money (Miura, 2014, p.192).

[20] When we used the representative agent as an introduction of hoarding loss, we warned you not to lose sight of this difference (Miura, 2016, p.274).

different signs, there still exists a possibility that some agents hoard money. We should not judge that hoarding never occurs if agents prefer time neutrality by the preceding theorems. This is also the same in the case that they prefer the past expenditure. Nevertheless, it is not easy to prove the existence of hoarding affected by a distribution in the case of the neutral and past preference type.[21]

However, if agents prefer the future expenditure, we can prove the existence of hoarding without considering a distribution as shown in the following theorem. Note that this theorem does not depend on the supposition of sharing a sign of Lagrange multipliers.

Theorem 5.7. Assume that there exist Basic-terms v and w such that $v \prec w$ which satisfy conditions that all agents are the time neutral or future preference type between v and w, and at least one agent is the latter type. In this case, $X_{(v)}^* < \Psi$ and $H_{(v)}^* > 0$ hold.

Proof. Since $v \prec w$ and Theorem 5.2 hold, $U_{i(v)}[X_{i(v)}^*] \geq U_{i(w)}[X_{i(w)}^*]$ is satisfied generally. Considering this and the supposed conditions,

$$U_{i(w)}[X_{i(v)}^*] \geq U_{i(v)}[X_{i(v)}^*] \geq U_{i(w)}[X_{i(w)}^*] \geq U_{i(v)}[X_{i(w)}^*]$$

holds for any Agent i, and there exists Agent j such that

$$U_{j(w)}[X_{j(v)}^*] > U_{j(v)}[X_{j(v)}^*] \geq U_{j(w)}[X_{j(w)}^*] > U_{j(v)}[X_{j(w)}^*]$$

Due to these and the diminishment of marginal utility, we can derive $X_{i(v)}^* \leq X_{i(w)}^*$ for any Agent i and $X_{j(v)}^* < X_{j(w)}^*$ for one Agent j. If we aggregate

[21] The author already did elementary research into this issue, and he previously planned to release a paper about it (See Miura, 2016, p.287). Yet, he does not intend to accomplish it now.

these inequalities of all agents, we obtain $X_{(v)}^* < X_{(w)}^*$. $X_{(w)}^* \leq \Psi$ holds generally by the whole budget constraint, thus $X_{(v)}^* < \Psi$ is derived. Also by the whole constraint, we obtain $H_{(v)}^* > 0$. [Q.E.D.]

Theorem 5.7 represents that hoarding occurs without any supposition if all agents similarly prefer the future expenditure. On the other hand, Theorem 3.2 shows that the non-hoarding state is a necessary condition for Pareto efficiency if price is fixed. We can conclude that the Pareto inefficient hoarding loss is realized by rational abstinence of individual agents.

This hoarding loss is directly connected with a decrease of nominal expenditure. Nevertheless, the decrease of nominal expenditure corresponds to that of real consumption under the fixed price as shown in Theorem 2.1. Hence, this is also a realization of under-consumption.

6. Concluding Comments

We have clarified that hoarding is judged beneficial by individual agents but it brings an economic loss to the whole society. This paradoxical phenomenon is caused by a qualitative difference of the budget constraint between the whole society and an individual agent. This difference depends on whether the law of transfer equality is incorporated in the constraint beforehand or not. Although the law is considered in the individual optimal solution, we must pay attention that it is considered after the optimization, not before. This implies that individual agents make a decision with ignoring the law subjectively but it still has an effect objectively.

Hoarding directly has an effect of increasing the future budget. Hoarding occurs together with a decrease of expenditure, but the decrease has no effect on the future budget of an individual agent who hoards money. As a whole, hoarding has an effect of increasing the future budget. This increase of the future budget enables an increase of the future expenditure. Since abstinent agents expect such causality, they hoard money. The expectation of this causality seems a main reason why saving is regarded as a virtue.

But as a fact of the whole society, this causality does not exist. Even in the whole society, hoarding has a direct effect to increase the future budget. Nevertheless, money which someone expends must be received by someone. This is a content of the whole expenditure reflux. Due to the reflux, a decrease of the whole expenditure, which occurs together with hoarding, brings the same decrease of the whole revenue. Since the whole revenue is included in the whole future budget, the decrease of revenue has an effect to decrease the budget. This effect neutralizes the direct effect of hoarding which increases the budget.

The law of transfer equality is a quantitative expression of the whole expenditure reflux. Since the whole budget constraint incorporates the law, it considers this neutralization. As the constraint shows, the whole budget is always decided only by money stock, and hoarding has no effect to vary the whole future budget. Therefore, judged from view of the whole society, it is an ineffective behavior which merely misses an opportunity of the past expenditure.

If individual agents made a decision considering the law of transfer equality beforehand, this ineffectiveness would not occur. But as a matter of fact, they make a decision ignoring the law despite its objective truthfulness. Why do agents ignore the truth?

For an individual agent, expenditure is a transfer from them to others, and revenue is a transfer from others to them. They are separate events. Hence, the separateness between expenditure and revenue is certainly a truth provided that judged in a private range. Individual agents do not make a decision based on an erroneous factual judgment.

However, this separateness is only an individual truth but not a social truth. Grasped from the view of the whole society, expenditure and revenue are the same events namely money transfer. A quantitative expression of this sameness is nothing but the law of transfer equality.

The individual separateness and the social sameness between expenditure and revenue seem to be a contradiction, but it is a superficial view. The reason why they do not contradict is in the existence of others. Compatibleness between the individual separateness and the social sameness can be explained by a fact that expenditure of an agent is certainly separate from their own revenue but is the same as their others' revenue.

Nevertheless, the interest of selfish individual agents is limited to their private range. Accordingly, they forget about a fact that their expenditure affects their others' revenue. Although they consider that their hoarding certainly increases their own utility, they do not consider that the hoarding lowers their others' revenue and decreases the others' utility. If they are pleased that their expenditure raises their others' revenue and then increases the others' utility just like, increasing in their own utility, the hoarding loss would not occur. Yet, selfish agents do not regard their others' utility as their own utility. They take no notice even if their decision damages their others. It is the essence of selfish decision-making.

Thinking from another point of view, a relational truth among agents never reflects in the individual decision-making processes. Individuals are rational in the sense that they make a decision while considering a correct truth. However, they are irrational in the sense that they do not make a decision while considering all truth owing to ignorance of the relational truth. As a result, they fall into an irrational situation despite their rational judgment.

Note that we never intend to blame hoarding as a foolish behavior. We ask the readers to pay attention to a possibility that, if an individual hoards money, their utility can increase though their others' utility decreases. The author himself also hoards money even though he knows the hoarding is a cause of social inefficiency. Even if he quits hoarding and expends all of his money for efficiency of society, his others do not guarantee his future budget which he can gain in the case he hoards money. Expending money indiscriminately depletes his future budget and makes him impossible to live in the future. As long as a monetary economy is grounded on the principle of personal responsibility, someone's expenditure behavior for social efficiency has a possibility to destroy their life. We never require a sacrifice of an individual life for social efficiency.

A message of our research is that we should grasp a structural problem in a monetary economy grounded on personal responsibility. Money is a product of the relation namely exchange. Money cannot exist if people live in isolation. Then, a fact that contemporary people cannot live without money is evidence that they cannot live without others. Nevertheless, their decision-makings in a monetary economy are isolated from each other. Accordingly, people with personal responsibility are forced to live depending on others beyond their control, then coexistence with distrust and dependence covers economy and causes social inefficiency.

This discrepant structure of isolation in relation is not only an essence of our monetary economy but also that of our life. An individual human being cannot live without others physically, but the others we have to live with are a being separated from self psychologically.

Others are existences who give me fears. However, even if I intend to escape from others, they can chase anywhere I can go. Others are also existences who give me a favor such as food, cloth, shelter, amusements, and love. Nevertheless, they do not always accept my earnest request.

I must live with others, but the fateful others are beyond my control. My fate is forced to be affected by their behavior which I cannot decide. Others' freedom is my unfreedom. I have to live with feeling my powerlessness that

I am forced to entrust my life to others. On the other hand, it is your fate that you have to accept my choice forced in such an environment even if it damages you.

An invisible wall rises without limit between me and you, who are always with me. This structure of isolation in relation is the cause of suffering of our life, but it indeed reflects that the human being is not the whole but a part of the world. Suffered by its other parts, a partial existence becomes conscious of its essence that it is a partial existence, namely it is not God. This consciousness may be a fate which God imposes on me.

I must live this fate as a non-divine being.

[References]

Abbati, H., 1924. The Unclaimed Wealth: How Money Stops Production. George Allen & Unwin, London.

Allgoewer, E., 2002. Underconsumption theories and Keynesian economics: Interpretations of the Great Depression. Discussion paper no. 2002-14 of University of St. Gallen. http://www1.vwa.unisg.ch/repec/usg/dp2002/dp0214allgoewer_ganz.pdf

Allsbrook, O.O., 1986. N.A.L.J. Johannsen: An early monetarist. Journal of Institutional and Theoretical Economics (JITE) / Zeitschrift für die gesamte Staatswissenschaft. 142, 431-437. http://jstor.org/stable/40750877

Ambrosi, G.M., 1996. Carl Föhl on economic activity and money.

An, S., Chang, Y., Kim, S.B., 2009. Can a representative-agent model represent a heterogeneous-agent economy. American Economic Journal: Macroeconomics. 1-2, 29-54. http://www.jstor.org/stable/25760273

Arrow, K.J., 1951. An extension of the basic theorems of classical welfare economics, in: Neyman, J. (Ed.), Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. University of California Press, Berkeley, pp. 507-522.

Augustinovics, M., 1965. A model of money-circulation. Economics of Planning. 5, 44-57. http://dx.doi.org/10.1007/BF02424905

Barro, R.J., Grossman, H.I., 1971. A general disequilibrium model of income and employment. The American Economic Review. 61-1, 82-93. http://www.jstor.org/stable/1910543

Bertsekas, D.P., 1999, Nonlinear Programming, second ed. Athena Scientific, Belmont, Massachusetts.

Blaug, M., 2007. The fundamental theorems of modern welfare economics, historically contemplated. History of Political Economy. 39, 185-207. http://dx.doi.org/10.1215/00182702-2007-001

Bleaney, M.F., 1976. Underconsumption Theories: A History of Critical Analysis. Lawrence and Wishart, London.

Boianovsky, M., 2002. Simonsen and the early history of the cash-in-advance approach. The European Journal of the History of Economic Thought. 9, 57-71. http://dx.doi.org/10.1080/09672560110103388

Brunner, K., 1951. Inconsistency and indeterminacy in classical economics. Econometrica. 19, 152-173. http://dx.doi.org/10.2307/1905731

Carlson, J.A., 1962. Foster and Catchings: A mathematical reappraisal. Journal of Political Economy. 70, 400-402. http://www.jstor.org/stable/1861737

Carroll, C.D., 2000. Requiem for the representative consumer? Aggregate implications of microeconomic consumption behavior. The American Economic Review. 90-2, 110-115. http://www.jstor.org/stable/117202

Carroll, C.D. and Kimball, M.S., 2008. Precautionary saving and precautionary wealth, in: Durlauf, S.N. & Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, second ed. Palgrave Macmillan, Basingstoke, volume 6, pp. 579-585. http://dx.doi.org/10.1057/9780230226203.1325

Chamley, C., 2011. A paradox of thrift in general equilibrium without forward market. Journal of the European Economic Association. 10, 1215-1235. http://dx.doi.org/10.1111/j.1542-4774.2012.01097.x

Christiano, L., Eichenbaum, M., Rebelo, S., 2011. When is the government spending multiplier large? Journal of Political Economy. 119, 78-121. http://dx.doi.org/10.1086/659312

Clark, D., 2008. Monetary cranks, in: Durlauf, S.N. & Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, second ed. Palgrave Macmillan, Basingstoke, volume 5, pp. 697-699. http://dx.doi.org/10.1057/9780230226203.1120

Clower, R., 1965. The Keynesian counterrevolution: A theoretical appraisal, in: Hahn F.H., Brechling F.P.R. (Eds.), The Theory of Interest Rates: Proceedings of a Conference Held by the International Economic Association. Macmillan, London, pp. 103-125.

Clower, R.W., 1967. A reconsideration of the microfoundations of monetary theory. Economic Inquiry. 6, 1-8.
http://dx.doi.org/10.1111/j.1465-7295.1967.tb01171.x

Davidson, P., 2012. Keynes' General Theory, in: King, J.E. (Ed.), The Elgar Companion to Post Keynesian Economics, second ed. Edward Elgar, Cheltenham, pp. 347-355.

Davies, P., 2002. How to Build a Time Machine. Penguin, London.

Debreu, G., 1959. Theory of Value: An Axiomatic Analysis of Economic Equilibrium. John Wiley & Sons, Inc., New York.

Deleplace, G., Nell, E.J. (Eds.), 1996. Motion in Money: The Post Keynesian and Circulation Approaches. Macmillan, Basingstoke.

Dimand, R.W., 1991. Cranks, heretics and macroeconomics in the 1930s. History of Economics Review. 16, 11-30.
http://www.hetsa.org.au/pdf-back/16-A-2.pdf

Dimand, R.W., 2008a. Catchings, Wadill (1879-1967), in: Durlauf, S.N. & Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, second ed. Palgrave Macmillan, Basingstoke, volume 1, pp. 711-712.
http://dx.doi.org/10.1057/9780230226203.0206

Dimand, R.W., 2008b. Foster, William Trufant (1879-1950), in: Durlauf, S.N. & Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, second ed. Palgrave Macmillan, Basingstoke, volume 3, pp. 471-472. http://dx.doi.org/10.1057/9780230226203.0597

Eggertsson, G.B., 2011. What fiscal policy is effective at zero interest rates? NBER Macroeconomics Annual 2010. 25, 59-112. http://www.nber.org/chapters/c12027

Feenstra, R.C., 1992. Money in the utility function, in: Neuman P., Milgate M., Eatwell. J., (Eds.), The New Palgrave Dictionary of Money and Finance. Macmillan, London, volume 2, pp.790-791.

Feldman, A.M., 2008, Welfare economics, in: Durlauf, S.N. & Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, second ed. Palgrave Macmillan, Basingstoke, volume 8, pp. 721-730. http://dx.doi.org/10.1057/9780230226203.1826

Gleason, A.H., 1959. Foster and Catchings: A reappraisal. Journal of Political Economy. 67, 156-172. http://www.jstor.org/stable/1825391

Gott, R., 2002. Time Travel in Einstein's Universe: The Physical Possibilities of Travel through Time. Phoenix, London.

Graziani, A., 1990. The theory of monetary circuit. Économie et Sociétés. 24, 7-36.

Greenwald, B.C., Stiglitz, J.E., 1986, Externalities in Economies with Imperfect Information and Incomplete Markets. *The Quarterly Journal of Economics*, 101(2), 229-264. http://dx.doi.org/10.2307/1891114

Haberler, G., 1964. Prosperity and Depression: A Theoretical Analysis of Cyclical Movements, fifth ed. George Allen & Unwin, London.

Hagemann, H., Rühl, C., 1990. Nicholas Johannsen and Keynes's "finance motive". Journal of Institutional and Theoretical Economics (JITE) / Zeitschrift für die gesamte Staatswissenschaft. 146, 445-469. http://jstor.org/stable/40751327

Hayek, F.A., 1931. The "paradox" of saving. Economica. 11, 125-169. http://dx.doi.org/10.2307/2547921

Hegeland, H., 1954. The Multiplier Theory. C.W.K. Gleerup, Lund.

Huo, Z., Ríos-Rull, J.V., 2013. Paradox of thrift recession. NBER Working Paper, No.19443. http://www.nber.org/papers/w19443

Ito, Z., 1952. Mr. Föhl's theory of public investment. Bulletin of Yamagata University (Humanities). 2, 201-224. (In Japanese)

Izeki, T., 1932. Theory of Japanese Controlled Economy. Kokusai Keizai Shinposha and Takase Shobo. (In Japanese)

Keynes, J.M., Hawtrey, R.G., Robertson D.H. 1933. Mr. Robertson on "saving and hoarding". The Economic Journal. 43, 699-712. http://dx.doi.org/10.2307/2224530

Keynes, J.M., 1936. The General Theory of Employment Interest and Money. Macmillan, London.

Kimball, M.S., 1990. Precautionary saving in the small and in the large. Econometrica. 58, 53-73. http://dx.doi.org/10.2307/2938334

Kirman, A.P., 1992. Whom or what does the representative individual represent? The Journal of Economic Perspectives. 6-2, 117-136. http://www.jstor.org/stable/2138411

Klein, L.R., 1966. The Keynesian Revolution, second ed. Macmillan, New York.

Kojima, H., 1995. Abbati's theory of effective demand (2): Johannsen, Hobson, and Abbati on saving. Keizai Ronso. 156-2/3, 27-65. (In Japanese) http://hdl.handle.net/2433/45001

Kohn, M., 1981. In defense of the finance constraint. Economic Inquiry. 19, 175-195. http://dx.doi.org/10.1111/j.1465-7295.1981.tb01129.x

Kohn, M., 1988. The finance constraint theory of money: A progress report. The Jerome Levy Economics Institute Working Paper No.5. http://dx.doi.org/10.2139/ssrn.191371

Laidler, D., 1989. The context of S.C. Tsiang's monetary economics, in: Kohn, M. (Ed.), Finance Constraints and the Theory of Money: Selected Papers. Academic Press, San Diego, pp. 359-375.

Lossev, A., Novikov, I.D., 1992. The jinn of the time machine: Non-trivial self-consistent solutions. Classical and Quantum Gravity. 9, 2309-2321. http://dx.doi.org/10.1088/0264-9381/9/10/014

Marget, A.W., 1966. The Theory of Prices: A Re-Examination of the Central Problem of Monetary Theory. Augustus M. Kelley Publishers, New York, volume 1.

Metzler, L.A., 1948. Three lags in the circular flow of income, in: Income, Employment, and Public Policy: Essays in Honor of Alvin H. Hansen. W.W. Norton Company Inc., New York, pp. 11-32.

Miura, S., 2014. Money circulation equation considering time irreversibility. Advances in Linear Algebra & Matrix Theory. 4, 187-200. http://dx.doi.org/10.4236/alamt.2014.44016

Miura, S., 2015a. Quantification of revenue induction and expenditure reflux in a monetary economy. Advances in Linear Algebra & Matrix Theory. 5, 25-35. http://dx.doi.org/10.4236/alamt.2015.51003

Miura, S., 2015b. Monetary budget constraint and its missing problems. Open Journal of Optimization. 4, 87-99. http://dx.doi.org/10.4236/ojop.2015.43011

Miura, S., 2016. Graphic explanation for welfare economic foundation of hoarding loss. Journal of Economics Bibliography. 3, 265-289. http://www.kspjournals.org/index.php/JEB/article/view/763

Nagasawa, K., 1968. Carl Föhl. The Hitotsubashi Review. 59, 507-527. (In Japanese) http://hdl.handle.net/10086/2640

Nash, R.T., Gramm, W.P., 1969. A neglected early statement of the paradox of thrift. History of Political Economy. 1, 395-400. http://dx.doi.org/10.1215/00182702-1-2-395

Nemiroff, R., Wilson, T., 2014. Searching the internet for evidence of time travelers. The Winnower 3:e140984.46096. http://www.dx.doi.org/10.15200/winn.140984.46096

Pedersen, J., 1954. The creation of money income. Weltwirtschaftliches Archiv. 72, 155-178. http://www.jstor.org/stable/40433458

Pedersen, J., 1957. Remarks to Carl Föhl's "Geldschöpfung und Wirtschaftskreislauf". Weltwirtschaftliches Archiv. 78, 1-10.

http://www.jstor.org/stable/40433458

Robertson, D.H., 1932. Reviewed work: "The Economic Lessons of 1929-31: The First Eleven Bulletins of the Unclaimed Wealth Utilisation Committee of Geneva. by A.H. Abbati, T.E. Gregory." The Economic Journal. 42, 612-614. http://dx.doi.org/10.2307/2223788

Robertson, D.H., 1933. Saving and hoarding. The Economic Journal. 43, 399-413. http://dx.doi.org/10.2307/2224283

Rochon, L.P., Rossi, S. (Eds.), 2003. Modern Theories of Money: The Nature and Role of Money in Capitalist Economies. Edward Elgar Publishing, Cheltenham.

Rothschild, K.W., 1964. The old and the new: Some recent trends in the literature of German economics. The American Economic Review. 54-2, 1-33. http://www.jstor.org/stable/1809373

Rühl, C., 2000. Nicholas August Ludwig Jacob Johannsen (1844-1928), in: Arestis, P., Sawyer, M. (Eds.), A Biographical Dictionary of Dissenting Economists, second ed. Edward Elgar, Cheltenham, pp. 327-335.

Samuelson, P.A., 1948. Economics: An Introductory Analysis, first ed. McGrow Hill, New York.

Sasahara, S., 1981. The under-consumption theory of Foster and Catchings: Its feature and political implication. Keizaigaku Ronso. 22, 1-40. (In Japanese)

Schneider, E., 1962. Money, Income and Employment. Translated by Kurt Klappholz, George Allen & Unwin, London.

Schneider, M., 2008. Underconsumptionism, in: Durlauf, S.N. & Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, second ed. Palgrave Macmillan, Basingstoke, volume 8, pp. 456-459. http://dx.doi.org/10.1057/9780230226203.1756

Stiglitz, J.E., 1991. The invisible hand and modern welfare economics. NBER Working Paper, No.3641. http://dx.doi.org/10.3386/w3641

Takata, Y., 1934. Regarding the contradiction of thrift: Doubt against the idea of thrift praise by Hayek. Keizai Ronso. 38, 945-963. (In Japanese) http://hdl.handle.net/2433/130449

Tavlas, G.S., 1976. Some further observations on the monetary economics of Chicagoans and Non-Chicagoans. Southern Economic Journal. 42, 685-692. http://dx.doi.org/10.2307/1056261

Tsiang, S.C., 1966. Warlas' law, Say's law and liquidity preference in general equilibrium analysis. International Economic Review. 7, 329-345. http://www.jstor.org/stable/1814282

Van Eeghen, P.H., 2014. Why DSGE analysis cannot accurately model financial-real sector interaction. Real-World Economics Review. 17, 17-40. http://www.paecon.net/PAEReview/issue67/Eeghen67.pdf

Walsh, C.E., 2003. Monetary Theory and Policy, second ed. The MIT Press, Cambridge, Massachusetts.