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Romer Meets Kongsamut-Rebelo-Xie in a Nonbalanced Growth Model*

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Abstract

To comprehend the relationship scheme between human capital accumulation and structural change observed in developing and developed countries, the paper combines Romer (1990)'s endogenous technological change with Kongsamut, Rebelo and Xie (1997, 2001)'s structural change model and shows that human capital accumulation speeds up structural change in the early stage of structural change with relatively low human capital while slows down it in its later stage with relatively high human capital.

Keywords: Human Capital; Endogenous Growth; Structural Change.

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1 Introduction

A relationship scheme between human capital accumulation and the speed of structural change is observed in the real data. For developing countries, there are positive correlations between human capital accumulation and the speed of structural change; however, the corresponding correlations for developed countries are negative. Specifically, for each developing country in Table 1, along with the increase of per capita human capital, the time taken for ten (or so) percentage point reduction in the employment share of agriculture becomes less and less; on the contrary, for each developed country in Table 2, even though human capital increases, the time taken for two (or so) percentate point reduction in the agriculture employment share gets a little bit more. It may be important to find a formal model to explain the observed pattern of structural change.

Kongsamut, Rebelo and Xie (1997, 2001, henceforth KRX) develops a three-sector nonbalanced growth model to explain the Kuznets facts[1] and argues that the reason for structural change is the difference in the income elasticity of demand for the three final goods (i.e., agriculture, manufacturing and services). They do not discuss the speed of structural change and human capital plays no role in their model. Hence the original KRX model cannot explain the observed pattern about the speed of structural change. For the similar reasons, the supply-side literature to explain structural change, such as Baumol (1967), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Comin, Lashkari and Mestieri (2017), is not qualified for this job either. As is well known, Romer (1990) examines the growth effect of human capital in a neoclassical growth model by introducing a research and development sector and monopolistic competition. Therefore, combining the structural-change model developed by KRX with the endogenous growth model pioneered by Romer (1990) may have the potential to explain the observed pattern[2] of structural change.

By introducing Romer (1990)'s endogenous technological change into the multi-sector growth model pioneered by KRX, the paper shows that human capital affects the speed of structural change from two channels: the growth channel and other ones. The aggregate effects are examined to explain the observed pattern between human capital accumulation and the speed of structural change. If the stock of human capital is relatively low in the early stage of structural change, an increase in human capital will speed up structural change greatly. However, if the stock of human capital is relatively high in the later stage of structural change, then the structural-change effect of human capital may be negative. A numerical example is also given to develop more intuitions of the theoretical model.

[1]The Kuznets facts roughly refer to the massive reallocations of both labor relative weights in GDP from agriculture into manufacturing and services.

[2]In a closely related paper, Growiec, McAdam and Muck (2018) combines the Acemoglu (2003) two-sector model with the Romer (1990) endogenous technological change model to understand the empirical properties of US labor income share.

The rest of the paper is organized as follows. Section 2 presents our three-sector structural change model with endogenous technological changes. Section 3 examines the relationship between human capital accumulaiton and the dynamics of structural change on the generalized balanced growth path (GBGP). Section 4 concludes.

2 The Model

The production side of the economy consists of three sectors: a final-goods sector, an intermediate-goods sector, and a research sector. The final-goods sector is made up of three subsectors: agriculture, manufacturing and services. In each final-goods subsector, perfectly competitive firms produce a homogeneous final good using labor, human capital and all kinds of intermediate goods. Each subsector utilizes the constant return to scale Cobb-Douglas production function with different technological parameters and factor income shares, namely,

$$A_t = B_A \left(h_t^A H_{Yt}\right)^{\alpha} \left(N_t^A\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^A x_{jt}\right)^{1-\alpha-\beta} dj, \tag{1}$$

$$M_t + \dot{K}_t + \delta K_t = B_M \left(h_t^M H_{Yt}\right)^{\alpha} \left(N_t^M\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^M x_{jt}\right)^{1-\alpha-\beta} dj, \tag{2}$$

$$S_t = B_S \left(h_t^S H_{Yt}\right)^{\alpha} \left(N_t^S\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^S x_{jt}\right)^{1-\alpha-\beta} dj, \tag{3}$$

where $B_i > 0, i \in \{A, M, S\}$ are three technological parameters; α, β, $(1-\alpha-\beta) \in (0,1)$; H_{Yt} is the total amount of human capital used in the final-goods sector, h_t^i, $i \in \{A, M, S\}$ are the shares used in these three subsectors, $\sum_{i\in\{A,M,S\}} h_t^i = 1$; all of the labor force are employed in the final-goods sector and normalized to one, N_t^i, $i \in \{A, M, S\}$ stand for the shares/amounts employed in the three subsectors, $\sum_{i\in\{A,M,S\}} N_t^i = 1$; each subsector utilizes all kinds of intermediate-goods in its production, ϕ_{jt}^i stands for the demand share of the intermediate-good $j \in [0, \Upsilon_t]$ by subsector $i \in \{A, M, S\}$, $\sum_{i\in\{A,M,S\}} \phi_{jt}^i = 1$ for any $j \in [0, \Upsilon_t]$; the outputs of agriculture (A_t) and services (S_t) can be used for consumption, and the output of manufacturing can be consumed (M_t) or invested $\left(\dot{K}_t + \delta K_t\right)$, then equations (1), (2), and (3) are also market-clearing conditions for the three subsectors; the knowledge stock (Υ_t) of the economy is determined endogenously by the amount $(H_{\Upsilon t})$ of human capital utilized in the knowledge sector and the current knowledge stock (Υ_t); finally, the prices of the products of the three final-goods P_A, P_M, and P_S are positive.

The profit-maximization problem of each subsector i will be then discussed. By taking the price of its product P_{it}, the wage rates of the labor force and human capital w_{Lt}, w_{Ht}, and the prices of all intermediate-goods $\{p_{jt}\}$ as given, monopolist $i \in \{A, M, S\}$ solves the problem,

$$\max_{\left\{N_t^i, b_t^i, \phi_{jt}^i, j\in[0.\Upsilon_t]\right\}} P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj - w_{Lt} N_t^i - w_{Ht} h_t^i H_{Yt} - \int_{j=0}^{\Upsilon_t} p_{jt} \phi_{jt}^i x_{jt} dj.$$

The FOCs with respect to N_t^i, h_t^i, and ϕ_{jt}^i are as follows

$$P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \beta \left(N_t^i\right)^{\beta-1} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj = w_{Nt}, \tag{4}$$

$$\alpha P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha-1} \left(N_t^i\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj = w_{Ht}, \tag{5}$$

$$P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta} = p_{jt}. \tag{6}$$

The first two optimality conditions show that the marginal product values of labor force and human capital equal the wage rates of them in each subsector i. The third one displays that the equality between marginal revenue and marginal cost for any intermediate good j in each subsector i, which are also the (inverse) demand functions for any intermediate good j in each subsector i.

Knowledges are designs for the intermediate goods. Being created and granted a patent, a piece of knowledge can be used to produce a kind of intermediate good. The types of these intermediate goods are determined by the knowledge stock created by the research sector. Hence the amount of the types of intermediate-goods is essentially the knowledge stock Υ_t. Any kind of intermediate good is produced by a single monopolistic firm. The decision process of any monopolistic firm can be separated into two steps: first, he pays the price $P_{\Upsilon t}^j$ to buy the patent for producing intermediate good j in the competitive patents market, which is the sunk cost for the monopolistic firm. Since the patents market is competitive, the price of new design j is the present value of the profits flow $\{\pi_{j\tau}\}_{\tau=t}^{\infty}$ generated by monopolistic firm j, $P_{\Upsilon t}^j = \int_{\tau=t}^{\infty} \exp\left(-\int_{s=t}^{\tau} R_s ds\right) \pi_{j\tau} d\tau$. Second, in the monopoly pricing problem, monopolistic firm j rents capital (as variable costs) and produces intermediate goods j to satisfy the demand of the final-goods sector for its products. It is assumed that the unit cost for any intermediate good is the same $\eta\,(>0)$ units of capital. Then the profit-maximization problem of any monopolistic firm j can be summerized as:[3]

$$\pi_{jt} = \max_{p_{jt}, x_{jt}} p_{jt} x_{jt} - R_t \eta x_{jt}. \tag{7}$$

Optimization yields us the symmetric monopolistic pricing formula

$$p_{jt} = \frac{1}{1-\alpha-\beta} R_t \eta \equiv p_t. \tag{8}$$

Note that $R_t \eta$ is the marginal cost for producing additional unit of intermediate good, and $1/(1-\alpha-\beta)\,(>1)$ is the mark-up over marginal cost. In order to earn the monopoly profit, all monopolistic firms price their products over their marginal costs. Moreover, all monopolistic firms set the same monopoly price and hence earns the same monopoly profit.

[3] Note that the gross interest rate equals the net interest rate plus the depreciation rate, i.e., $R_t = r_t + \delta$. It is assumed that the monopoly firm bears the depreciation cost.

Since all production factors (capital, labor and all intermediate goods) of the final-goods sector are freely mobile, by utilizing the symmetric property of the profit-maximization problem of the intermediate-goods sector, we can derive the efficiency condition of production,

$$N_t^A = h_t^A = \phi_t^A, N_t^M = h_t^M = \phi_t^M, N_t^S = h_t^S = \phi_t^S, \tag{9}$$

which display that the optimal weights of labor, human capital and all intermediate goods employed in each subsector of the final-goods sector are equal. Furthermore, if we set $P_M = 1$ in equilibrium, then the (relative) prices of agriculture and services are constant, namely,

$$P_A = \frac{B_M}{B_A}, P_S = \frac{B_M}{B_S}. \tag{10}$$

The research sector uses human capital $H_{\Upsilon t}$ and the existing stock of knowledge Υ_t to produce new knowledge, with the following knowledge production function

$$\dot{\Upsilon}_t = \epsilon H_{\Upsilon t} \Upsilon_t, \tag{11}$$

where $\epsilon\,(> 0)$ is the productivity parameter. The knowledge production function shows that devoting more human capital to research leads to a higher production rate of new designs, and the larger the total stock of designs is, the higher the productivity of the researchers employed in the research sector will be. Due to its partially excludability and nonrivalry of consumption, the production of knowledge cannot be determined by the private maximizing behavior. The evolution of knowledge however follows the trajectory described by equation (11). In the paper, knowledge refers to designs for new intermediate goods. Then the accumulation of knowledge represents the increase of the types of intermediate goods.

Since human capital is freely mobile, no arbitrage requires it has the same rate of return among the research sector and three subsectors in the final-goods sector, namely,

$$P_{\Upsilon t} \epsilon \Upsilon_t = P_i B_i \alpha {H_{Yt}}^{\alpha-1} \Upsilon_t x_t^{1-\alpha-\beta}, i \in \{A, M, S\}. \tag{12}$$

The representative consumer makes consumption and asset accumulation decisions in order to maximize the discounted utility of consumption stream for three final goods, namely,

$$\max_{\{A_t, M_t, S_t, K_{t+1}\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w \right]^{1-\sigma} - 1}{1-\sigma} dt \right\}, \tag{13}$$

subject to the flow budget constraint (FBC):

$$\dot{K}_t = w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t, \tag{14}$$

where $\rho \in (0, 1)$ is the subjective time preference rate; $\sigma \in (0, +\infty)$ is the constant coefficient of relative risk aversion; $\overline{A}\,(> 0)$ is the level of subsistence consumption, $\overline{M}\,(> 0)$ and $\overline{S}\,(> 0)$

represent home production of manufacturing and services; $u, v, w \in (0,1)$ stand for the relative utility weights of consumption for agriculture, manufacturing and servies, satisfying $u+v+w=1$. Solving the utility-maximizing problem gives us the consumption Euler equation

$$\frac{\dot{M_t + \overline{M}}}{M_t + \overline{M}} \left(= \frac{\dot{A_t - \overline{A}}}{A_t - \overline{A}} = \frac{\dot{S_t + \overline{S}}}{S_t + \overline{S}} \right) = \frac{1}{\sigma} \left(R_t - \rho \right). \tag{15}$$

3 Human Capital and Structural Change on the GBGP

Since all intermediate-good firms are monopoly firms, the decentralized equilibrium of the multi-sector economy is a monopolistic competitive equilibrium, which is determined by (1)-(6), (8), (9), (11), (12), (14), (15) and the initial and transversality conditions. A generalized balanced growth path (GBGP) is defined as a trajectory along which the real interest rate is a constant, R^*. Imposing the knife-edge condition

$$\frac{\overline{A}}{B_A} = \frac{\overline{M}}{B_M} + \frac{\overline{S}}{B_S}, \tag{16}$$

we solve the stationary equilibrium as follows:

$$H_\Upsilon^* = \frac{\epsilon H - \rho \Lambda}{\epsilon \left(1 + \sigma \Lambda\right)}, \tag{17}$$

$$R^* = \frac{\epsilon H \sigma + \rho}{\sigma \Lambda + 1}, \tag{18}$$

$$g^* = \frac{\epsilon H - \Lambda \rho}{\sigma \Lambda + 1}, \tag{19}$$

where $\Lambda \equiv \alpha / \left(\alpha + \beta\right) \left(1 - \alpha - \beta\right)$. Equation (19) shows that the rate of economic growth depends on the total stock of human capital, time discount rate, and technological parameters of the research and final-goods sectors. The larger the total stock of human capital in the economy is, the more the human capital employed in the research sector becomes ($\partial H_\Upsilon^* / \partial H = 1 / \left(\sigma \Lambda + 1\right) > 0$), the faster knowledge accumulates ($\partial g_\Upsilon^* / \partial H = \epsilon / \left(\sigma \Lambda + 1\right) > 0$). Hence the rate of economic growth will be higher ($\partial g^* / \partial H = \epsilon / \left(\sigma \Lambda + 1\right) > 0$).

On the GBGP, using (1), (2), (3), (15), (19), and (16), we obtain the dynamic equations for the employment shares of the three final goods

$$\dot{N}_t^A = -g^* \frac{\overline{A}}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{A}}{B_A} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} < 0, \tag{20}$$

$$\dot{N}_t^M = g^* \frac{\overline{M}}{B_M H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{M}}{B_M} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0, \tag{21}$$

$$\dot{N}_t^S = g^* \frac{\overline{S}}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{S}}{B_S} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0. \tag{22}$$

Hence we have the following

Theorem 1 *A generalized balanced growth path (GBGP) with a constant equilibrium interest rate (18) and a constant equilibrium endogenous growth rate (19) exists whenever the knife-edge condition (16) holds. On the GBGP, as is implied by equations (20), (21), and (22), the employment and production shares decline in agriculture, rise in both manufacturing and services.*[4]

Theorem 1 shows that on the GBGP if technology changes and hence the economy grows endogenously, because the demand elasticity of income for agriculture is less than one and the demand elasticities for both manufacturing and services are larger than one, along with the growth of the economy, even though all the three final goods expand, the speeds of expansion for both manufacturing and services are larger than the one for agriculture, then the employment and production shares of agriculture decrease gradually, the ones for manufacturing and services increase correspondingly. Labor forces in the economy transfer from agriculture to both manufacturing and services, which displays that the industry structure upgrades gradually.

Corollary 1 *If* $H \to (\rho\Lambda/\epsilon)^{+}$, $\partial\left(-\dot{N}_t^A\right)/\partial H$ *and* $\partial\dot{N}_t^i/\partial H, i \in \{M, S\}$ *approach very large positive numbers for any finite* t*; if the parameter values satisfy* $H \in \left(\frac{\rho\Lambda}{\epsilon}, \frac{\rho(1+\Lambda)}{\epsilon(1-\sigma)}\right)$, $\sigma \in (0,1)$, *and* $t > t^c \equiv \frac{1+\sigma\Lambda}{\epsilon}\left[\frac{1-\alpha}{\alpha+\beta}\frac{1}{H+\rho/\epsilon\sigma} + \frac{\rho(1/\sigma+\Lambda)/\epsilon}{(H-\rho\Lambda/\epsilon)(H+\rho/\epsilon\sigma)}\right]$, *then* $\left(-\dot{N}_t^A\right)/\partial H < 0$, *and* $\partial\dot{N}_t^i/\partial H < 0, i \in \{M, S\}$.

From equations (20), (21), and (22), we can decompose the effects on structural changes of human capital into two different parts. The first part $(g^*/(\Upsilon_0 \exp(g^* t)))$ is through economic growth. Since the term $g^*/(\Upsilon_0 \exp(g^* t))$ is not a monotone function about g^*, even though the equilibrium growth rate g^* is increasing with respect to human capital H, i.e., $\partial g^*/\partial H > 0$, we are not sure of its effects on structural change. However, if there exist no innovations and growths, namely, $g^* = 0$, even though the demand elasticities among the three final goods are different, the industrial structure will not change, i.e., $\dot{N}_t^i = 0, i \in \{A, M, S\}$. Therefore, economic growth can be thought of as the fundamental driving force for structural change. The second part $\left(1/H_Y^{*\alpha} x^{*1-\alpha-\beta}\right)$ also displays ambiguous structural-change effect of human capital through other channels than growth. To examine the aggregate effects of human capital accumulation on the speed of structural change, we take the partial derivatives w.r.t H on both sides of (20), (21), and (22):

[4] We generalize the model to the case with different labor income shares and intermediate-good input shares among agrigulture, manufacturing and services. The derivations can be found in the online appendix or available upon request.

$$\frac{\partial\left(-\dot{N}_t^A\right)}{\partial H}=\underbrace{\left(-\dot{N}_t^A\right)}_{>0}\left[\frac{1}{H-(\rho\Lambda/\epsilon)}-\frac{\epsilon}{\sigma\Lambda+1}t+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right], \tag{23}$$

$$\frac{\partial\dot{N}_t^i}{\partial H}=\underbrace{\dot{N}_t^i}_{>0}\left[\frac{1}{H-(\rho\Lambda/\epsilon)}-\frac{\epsilon}{\sigma\Lambda+1}t+\frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right], i=M,S. \tag{24}$$

If the stock of human capital approaches $\rho\Lambda/\epsilon$ from the right, i.e., $H\to(\rho\Lambda/\epsilon)^+$, $\partial\left(-\dot{N}_t^A\right)/\partial H$ and $\partial\dot{N}_t^i/\partial H$ approach very large positive numbers for any finite t. That is to say, if the stock of human capital is relatively low, an increase of human capital will speed up structural changes greatly. This case may correspond to the observed pattern for developing countries in Table 1. For these countries with relatively low human capital, the structural-change effect of human capital accumulation is very large. On the other hand, we make the same assumptions of $H\in\left(\frac{\rho\Lambda}{\epsilon},\frac{\rho(1+\Lambda)}{\epsilon(1-\sigma)}\right)$ and $\sigma\in(0,1)$ as Romer (1990)[5] to guarantee both positive equilibrium growth rates and finite objectives. Then, if $t>t^c$, then both $\partial\left(-\dot{N}_t^A\right)/\partial H$ and $\partial\dot{N}_t^i/\partial H$ are negative, which shows that if the time t is so long that the process of structural change is almost completed and the stock of human capital is relatively high, the structural-change effect of human capital may be negative. For those developed countries with relatively high level of human capital, even though human capital still increases, there is very few space for structural change and the marginal contributions of human capital accumulation may be negative.

A numerical exercise makes the above results more clearly. Let $\overline{A}=1$, $A_0=2$, $B_A=4$, $\eta=1$, $\alpha=0.3$, $\beta=0.3$, $\sigma=0.8$, $\Upsilon_0=1$, $\overline{S}=0.5$, $S_0=0.5$, $B_S=2.5$. The stock of human capital in the economy is taken to be 0.2, 0.4, and 0.8, respectively. It is shown in Figures 1, 2 and 3 that (1) with higher level of human capital, the absolute values of the slopes for the employment (and production) shares of the three subsectors are larger in the first 20 periods, which displays that an increase of the stock of human capital speeds up structural change greatly; (2) on the contrary, in the last 20 periods, the absolute values of these slopes become smaller, which shows that the structural-change effect of human capital accumulation is negative.

Due to (21) and (22), we have the following

Corollary 2 *If the model parameters satisfy* $\frac{\overline{M}}{B_M}>\frac{\overline{S}}{B_S}$*, then the expansion of manufacturing is quicker than services; if the model parameters satisfy* $\frac{\overline{M}}{B_M}<\frac{\overline{S}}{B_S}$*, then the expansion of manufacturing is slower than services; If* $\frac{\overline{M}}{B_M}=\frac{\overline{S}}{B_S}$*, then the expansion speeds of both manufacturing and services are the same.*

[5] The two assumptions on H and σ are implicitly made in Romer (1990).

4 Conclusion

By introducing Romer (1990)'s endogenous technological change into the multi-sector growth model pioneered by KRX, the paper shows that human capital affects the speed of structural change from two channels: the growth channel and other ones. The aggregate effects are examined to explain the observed pattern in the data. If the stock of human capital is relatively low in the early stage of structural change, an increase in human capital accumulation will speed up structural change greatly. However, if the stock of human capital is relatively high in the later stage of structural change, then the structural-change effect of human capital may be negative. The combined model is helpful to comprehend the observed relationship scheme observed in developing and developed countries.

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5 Online Appendix (Not for Publication)

5.1 Derive the Production Efficiency Conditions

Substituting the demand functions for the intermediate good j into the objective function of the monopolistic problem yields us

$$\pi_{jt} = \max_{x_{jt}} P_i B_i \left(b_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta) \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta} x_{jt} - R_t \eta x_{jt}.$$

The necessary conditions w.r.t x_{jt} are

$$R_t \eta = P_i B_i \left(b_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} (1-\alpha-\beta)^2 \left(\phi_{jt}^i x_{jt}\right)^{-\alpha-\beta}, i \in \{A, M, S\}. \tag{25}$$

Combining the demand functions for intermediate goods and (25) yields us the monopoly pricing formula in the text. Due to (25), the demand functions for intermediate goods, and the monopoly price, we know that $\left\{\phi_{jt}^i x_{jt} : i \in \{A, M, S\}\right\}$ do not depend on j, that is, the optimal demand for each intermediate good is the same among the three subsectors. Since $\phi_{jt}^A x_{jt} + \phi_{jt}^M x_{jt} + \phi_{jt}^S x_{jt} = x_{jt}$ does not depend on j, i.e., $x_{jt} = x_t$, $\left\{\phi_{jt}^i : i \in \{A, M, S\}\right\}$ do not depend on j either. That is, these three subsectors in final-goods sector use the same share of each intermediate good, namely,

$$\phi_{jt}^A = \phi_t^A, \phi_{jt}^M = \phi_t^M, \phi_{jt}^S = \phi_t^S. \tag{26}$$

Furthermore, each monopoly firm earns the same monopoly profit, namely,

$$\pi_{jt} = (\alpha + \beta) p_t x_t = \pi_t, j \in [0, \Upsilon_t]. \tag{27}$$

Combining the two marginal productivity conditions for both labor and human capital leads to one efficiency condition in production:

$$\frac{h_t^A}{N_t^A} = \frac{h_t^M}{N_t^M} = \frac{h_t^S}{N_t^S} = 1, \tag{28}$$

which displays that at optimum each subsector in the final-goods sector utilizes the same weights for both labor and human capital. Combining the same two equations as above leads to another efficiency condition in production:

$$\frac{\phi_t^A}{N_t^A} = \frac{\phi_t^M}{N_t^M} = \frac{\phi_t^S}{N_t^S} = 1, \tag{29}$$

which shows that at optimum each subsector in the final-goods sector uses the same weights for both all intermediate goods and labor. Combining (28) and (29) yields the efficiency conditions for production

$$N_t^A = h_t^A = \phi_t^A, N_t^M = h_t^M = \phi_t^M, N_t^S = h_t^S = \phi_t^S. \tag{30}$$

5.2 Derive the Euler Equation and the MCE

The representative consumer makes consumption and asset accumulation decisions in order to maximize the discounted utility of consumption stream for three final goods, namely,

$$\max_{\{A_t, M_t, S_t, K_{t+1}\}} \left\{ \int_{t=0}^{\infty} e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{1-\sigma} - 1}{1-\sigma} dt \right\}, \tag{31}$$

subject to the flow budget constraint (FBC):

$$P_A A_t + P_M \left(M_t + \dot{K}_t\right) + P_S S_t = w_{Ht} H + w_{Nt} + R_t K_t, \tag{32}$$

and the initial asset stock K_0. Substituting $P_A = B_M/B_A, P_S = B_M/B_S$ and the definition $P_M = 1$ into (32) we obtain the FBC

$$\dot{K}_t = w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t. \tag{33}$$

Constructing the Hamiltonian

$$H(\cdot) \equiv e^{-\rho t} \frac{\left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{1-\sigma} - 1}{1-\sigma} + \lambda_t \left[w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t\right],$$

where λ_t is the present-value Hamilton multiplier. The first order necessary conditions are

$$e^{-\rho t} \left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{-\sigma} u \left(A_t - \overline{A}\right)^{u-1} \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w = \lambda_t \frac{B_M}{B_A}, \tag{34}$$

$$e^{-\rho t} \left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{-\sigma} \left(A_t - \overline{A}\right)^u v \left(M_t + \overline{M}\right)^{v-1} \left(S_t + \overline{S}\right)^w = \lambda_t, \tag{35}$$

$$e^{-\rho t} \left[\left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v \left(S_t + \overline{S}\right)^w\right]^{-\sigma} \left(A_t - \overline{A}\right)^u \left(M_t + \overline{M}\right)^v w \left(S_t + \overline{S}\right)^{w-1} = \lambda_t \frac{B_M}{B_S}, \tag{36}$$

$$\lambda_t R_t = -\dot{\lambda}_t, \tag{37}$$

$$\dot{K}_t = w_{Ht} H + w_{Nt} + R_t K_t - \frac{B_M}{B_A} A_t - M_t - \frac{B_M}{B_S} S_t, \tag{38}$$

together with the initial condition K_0 and the transversality condition $\lim_{t\to\infty} \lambda_t K_t = 0$.

From equations (34), (35), and (36), we know that

$$\frac{u}{v} \frac{M_t + \overline{M}}{A_t - \overline{A}} = \frac{B_M}{B_A}, \frac{w}{v} \frac{M_t + \overline{M}}{S_t + \overline{S}} = \frac{B_M}{B_S}. \tag{39}$$

Rearranging terms of equation (35), taking time derivative on both sides, and using (39) lead to

$$-\rho-\sigma\frac{\dot{\overline{M_t+\overline{M}}}}{M_t+\overline{M}}=\frac{\dot{\lambda}_t}{\lambda_t}. \tag{40}$$

Combining (37) and (40) gives us the Euler equation in the text

$$\frac{\dot{\overline{M_t+\overline{M}}}}{M_t+\overline{M}}\left(=\frac{\dot{\overline{A_t-\overline{A}}}}{A_t-\overline{A}}=\frac{\dot{\overline{S_t+\overline{S}}}}{S_t+\overline{S}}\right)=\frac{1}{\sigma}\left(R_t-\rho\right). \tag{41}$$

The monopolistic competitive equilibrium is described by the following

Theorem *A monopolistic competitive equibrium of the multi-sector economy is composed of equilibrium price sequences* $\left\{P_A, P_M, P_S, \left(p_{jt}, P^j_{\Upsilon t}\right)_{j\in[0,\Upsilon_t]}, w_{Ht}, w_{Lt}, R_t\right\}$ *and allocation sequences* $\left\{A_t, M_t, S_t, H_{Yt}, H_{\Upsilon t}, N^A_t, N^M_t, N^S_t, \phi^A_t, \phi^M_t, \phi^S_t, (x_{jt})_{j\in[0,\Upsilon_t]}\right\}$, *satisfying: (1) The representative consumer consumes and accumulates physical capital to maximize the objective function (31), subject to the FBC (32); (2) In the final-goods sector, given its production technology, each subsector chooses labor, human capital and all of the intermediate goods to maximize its profits; (3) Given the demand for its products, any intermediate-good monopoly firm* $j\in[0,\Upsilon_t]$ *chooses monopoly price* p_{jt} *to maximize its monopoly profit* π_{jt}*; (4) The research sector uses human capital* $H_{\Upsilon t}$ *and the existing knowledge stock* Υ_t *to develop new knowledge with the technology* $\dot{\Upsilon}_t=\epsilon H_{\Upsilon t}\Upsilon_t$*; (5) The markets for three final goods clear; (6) Labor market clears, i.e.,* $N^A_t+N^M_t+N^S_t=1$*; (7) The market for human captial clears, i.e.,* $\left(h^A_t+h^M_t+h^S_t\right)H_{Yt}+H_{\Upsilon t}=H_{Yt}+H_{\Upsilon t}=H$*; (8) Capital market clears, i.e.,* $K_t=\int_{j=0}^{\Upsilon_t}\eta x_{jt}dj$*; (9) Any patent market clears.*

5.3 Calculate the Growth Rate and Equilibrium Shares on the GBGP

Utilizing the demand functions for intermediate goods, the monopoly pricing formula, and the capital market clearing condition, we derive the intereat rate as

$$R_t=(1-\alpha-\beta)^2\,\eta^{\alpha+\beta-1}B_M {H_{Yt}}^{\alpha}\left(\frac{K_t}{T_t}\right)^{\alpha+\beta}. \tag{42}$$

Setting $R_t=R^*$ and using (41), we have

$$\frac{\dot{\overline{M_t+\overline{M}}}}{M_t+\overline{M}}=\frac{\dot{\overline{A_t-\overline{A}}}}{A_t-\overline{A}}=\frac{\dot{\overline{S_t+\overline{S}}}}{S_t+\overline{S}}=\frac{1}{\sigma}\left(R^*-\rho\right)\equiv g^*, \tag{43}$$

where $g^*\equiv\frac{1}{\sigma}\left(r^*-\rho\right)$ is defined as the growth rate of $A_t-\overline{A}$, $M_t+\overline{M}$, and $S_t+\overline{S}$ on GBGP. Since (K_t/T_t) is constant on GBGP, $K_t=\int_{j=0}^{\Upsilon_t}\eta x_{jt}dj$ and $x_{jt}=x_t$, we know that $x_t=K_t/\left(\eta\Upsilon_t\right)$ is constant, i.e., $x_t=x^*$. From the market-clearing condition of human capital and (42), we know that human capital employed both in the final-goods and research sectors are constant on GBGP, i.e., $H_{Yt} = H_Y^*$, $H_{\Upsilon t} = H_{\Upsilon}^*$. On the GBGP, we know that $P_{\Upsilon}^* = \pi^*/R^*$, and $P_{\Upsilon}^* = P_M B_M \left(\alpha/\epsilon\right) H_Y^{*\alpha-1} x^{*1-\alpha-\beta}$. Combining the two equations with the demand functions of intermediate goods, the monopoly pricing formula, and (27) yields

$$H_Y^* = \frac{\alpha R^*}{\epsilon \left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}, H_{\Upsilon}^* = H - \frac{\alpha R^*}{\epsilon \left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{44}$$

From the knowledge accumulation equation, we know that both knowledge stock and physical capital grow at the same rate, namely,

$$g_{\Upsilon}^* = g_K^* = g^* = \epsilon H - \frac{\alpha R^*}{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{45}$$

Since all endogenous variables grow at the same rate on GBGP, we have

$$\frac{1}{\sigma}\left(R^* - \rho\right) = \epsilon H - \frac{\alpha R^*}{\left(\alpha+\beta\right)\left(1-\alpha-\beta\right)}. \tag{46}$$

Solving (45) and (46) for R^* and g^* gives us

$$R^* = \frac{\epsilon H \sigma + \rho}{\sigma \Lambda + 1}, g^* = \frac{\epsilon H - \Lambda \rho}{\sigma \Lambda + 1}. \tag{47}$$

Combining the optimal monopoly price, (44), and (47), we obtain the equilibrium output of any intermediate good

$$x^* = \left[B_M \left(\frac{\Lambda}{\epsilon}\right)^{\alpha} \eta^{-1} \left(1-\alpha-\beta\right)^2 R^{*\alpha-1}\right]^{\frac{1}{\alpha+\beta}}. \tag{48}$$

Substituting (30) into the production functions the final-goods sector, we know that the employment shares of labor force in the three subsectors, on the GBGP, are

$$N_t^A = \frac{A_t}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t}, N_t^M = 1 - \left(\frac{A_t}{B_A} + \frac{S_t}{B_S}\right) \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t}, N_t^S = \frac{S_t}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t}. \tag{49}$$

Taking the time derivatives on both sides of these three equations leads to the change rates of the labor employment shares of agricuture, manufacturing and services

$$\dot{N_t^A} = -g^* \frac{\overline{A}}{B_A H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{A}}{B_A} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} < 0, \tag{50}$$

$$\dot{N_t^M} = g^* \frac{\overline{M}}{B_M H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{M}}{B_M} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0, \tag{51}$$

$$\dot{N_t^S} = g^* \frac{\overline{S}}{B_S H_Y^{*\alpha} x^{*1-\alpha-\beta} \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp\left(g^* t\right)} \frac{\overline{S}}{B_S} \frac{1}{H_Y^{*\alpha} x^{*1-\alpha-\beta}} > 0. \tag{52}$$

The production shares of the three subsector are defined as

$$\theta_i = \frac{P_i B_i \left(h_t^i H_{Yt}\right)^{\alpha} \left(N_t^i\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^i x_{jt}\right)^{1-\alpha-\beta} dj}{\sum_{k\in\{A,M,S\}} P_k B_k \left(h_t^k H_{Yt}\right)^{\alpha} \left(N_t^k\right)^{\beta} \int_{j=0}^{\Upsilon_t} \left(\phi_{jt}^k x_{jt}\right)^{1-\alpha-\beta} dj}, i \in \{A, M, S\}.$$

Substituting (30) and the relative prices into the above equations, we know that the labor employment shares equals the corresponding production shares in these three subsectors, namely,

$$N_t^i = \theta_i, i \in \{A, M, S\}.$$

Then we derive the comparative statics of an increase in human capital. Substituting (44), (47) and (48) into (50)-(52) leads to

$$\begin{aligned}
\dot{N}_t^A &= -c_{01} g^* \exp\left(-g^* t\right) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \\
\dot{N}_t^M &= c_{02} g^* \exp\left(-g^* t\right) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}}, \\
\dot{N}_t^M &= c_{03} g^* \exp\left(-g^* t\right) R^{*\frac{1-2\alpha-\beta}{\alpha+\beta}},
\end{aligned}$$

where

$$\begin{aligned}
c_{01} &= \frac{\overline{A}}{\Upsilon_0 B_A} \left(\Lambda/\epsilon\right)^{-\alpha} \left[B_M \left(\Lambda/\epsilon\right)^{\alpha} \eta^{-1} \left(1-\alpha-\beta\right)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))}, \\
c_{02} &= \frac{\overline{M}}{\Upsilon_0 B_M} \left(\Lambda/\epsilon\right)^{-\alpha} \left[B_M \left(\Lambda/\epsilon\right)^{\alpha} \eta^{-1} \left(1-\alpha-\beta\right)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))}, \\
c_{03} &= \frac{\overline{S}}{\Upsilon_0 B_S} \left(\Lambda/\epsilon\right)^{-\alpha} \left[B_M \left(\Lambda/\epsilon\right)^{\alpha} \eta^{-1} \left(1-\alpha-\beta\right)^2\right]^{((\alpha+\beta-1)/(\alpha+\beta))}.
\end{aligned}$$

Taking the partial derivatives on both sides of the above equations w.r.t H and arranging terms, we have

$$\frac{\partial\left(-\dot{N}_t^A\right)}{\partial H} = \left(-\dot{N}_t^A\right)\left[\frac{1}{H-(\rho\Lambda/\epsilon)} - \frac{\epsilon}{\sigma\Lambda+1}t + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right], \tag{53}$$

$$\frac{\partial \dot{N}_t^M}{\partial H} = \dot{N}_t^M \left[\frac{1}{H-(\rho\Lambda/\epsilon)} - \frac{\epsilon}{\sigma\Lambda+1}t + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right], \tag{54}$$

$$\frac{\partial \dot{N}_t^M}{\partial H} = \dot{N}_t^S \left[\frac{1}{H-(\rho\Lambda/\epsilon)} - \frac{\epsilon}{\sigma\Lambda+1}t + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right]. \tag{55}$$

If $H \to (\rho\Lambda/\epsilon)^+$, $\partial\left(-\dot{N}_t^A\right)/\partial H$ and $\partial \dot{N}_t^i/\partial H$ approach very large positive numbers. To avoid the infinity of the objective function on the GBGP, we impose the assumption

$$\rho - (1-\sigma) g^* = \rho - (1-\sigma)\frac{\epsilon H - \Lambda\rho}{\sigma\Lambda+1} > 0. \tag{56}$$

If $\sigma \in (1,+\infty)$, then it is satisfied by itself. If $\sigma \in (0,1)$, then it is equivalent to $H < \rho(1+\Lambda)/\epsilon(1-\sigma) \left(=\rho(\sigma\Lambda+1)/\epsilon(1-\sigma)+\rho\Lambda/\epsilon > \frac{\rho\Lambda}{\epsilon}\right)$. Then if $H \in (\rho\Lambda/\epsilon, \rho(1+\Lambda)/\epsilon(1-\sigma))$, then

$$\left[\frac{1}{H-(\rho\Lambda/\epsilon)} - \frac{\epsilon}{\sigma\Lambda+1}t + \frac{1-2\alpha-\beta}{\alpha+\beta}\frac{1}{H+(\rho/\epsilon\sigma)}\right] > 0 \tag{57}$$

is equivalent to

$$t > \frac{1+\sigma\Lambda}{\epsilon}\left[\frac{1-\alpha}{\alpha+\beta}\frac{1}{H+\rho/\epsilon\sigma} + \frac{\rho(1/\sigma+\Lambda)/\epsilon}{(H-\rho\Lambda/\epsilon)(H+\rho/\epsilon\sigma)}\right] \equiv t^c. \tag{58}$$

5.4 Derive the Generalized Model with Different Factor Income Shares

We generalize the benchmark model to include different factor income shares in the three subsectors (agriculture, manufacturing and services) of the final-goods sector. The production technology of the final-goods sector is changed as follows:

$$A_t = B_A\left(h_t^A H_{Yt}\right)^{\alpha_A}\left(N_t^A\right)^{\beta_A}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^A x_{jt}\right)^{1-\alpha_A-\beta_A}dj \equiv Y_{At}, \tag{59}$$

$$M_t + \dot{K}_t + \delta K_t = B_M\left(h_t^M H_{Yt}\right)^{\alpha_M}\left(N_t^M\right)^{\beta_M}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^M x_{jt}\right)^{1-\alpha_M-\beta_M}dj \equiv Y_{Mt}, \tag{60}$$

$$S_t = B_S\left(h_t^S H_{Yt}\right)^{\alpha_S}\left(N_t^S\right)^{\beta_S}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^S x_{jt}\right)^{1-\alpha_S-\beta_S}dj \equiv Y_{St}. \tag{61}$$

The profit-maximizing problem of each subsector i yields the following necessary conditions:

$$P_i B_i \alpha_i\left(h_t^i H_{Yt}\right)^{\alpha_i-1}\left(N_t^i\right)^{\beta_i}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^i x_{jt}\right)^{1-\alpha_i-\beta_i}dj = w_{Ht}, \tag{62}$$

$$P_i B_i\left(h_t^i H_{Yt}\right)^{\alpha_i}\beta_i\left(N_t^i\right)^{\beta_i-1}\int_{j=0}^{\Upsilon_t}\left(\phi_{jt}^i x_{jt}\right)^{1-\alpha_i-\beta_i}dj = w_{Nt}, \tag{63}$$

$$P_i B_i\left(h_t^i H_{Yt}\right)^{\alpha_i}\left(N_t^i\right)^{\beta i}(1-\alpha_i-\beta_i)\left(\phi_{jt}^i x_{jt}\right)^{-\alpha_i-\beta_i} = p_{jt}. \tag{64}$$

The monopoly pricing problem for each intermediate good j gives us the monopolistic pricing formula

$$p_{jt} = \frac{1}{1-\alpha_i-\beta_i}R_t\eta, i \in \{A, M, S\}, \tag{65}$$

which implies that the income shares of all intermediate goods employed in each subsector are the same, i.e.,

$$1-\alpha_i-\beta_i \equiv \gamma, \tag{66}$$

and all monopoly firms set the same price, i.e.,

$$p_{jt} = p_t. \tag{67}$$

Combining (64) and (67), we know that the optimal products of all intermediate goods are equal,

$$x_{jt} = x_t, \tag{68}$$

and each subsector i of the final-goods sector utilizes the same shares of all intermediate goods,

$$\phi_{jt}^i = \phi_t^i. \tag{69}$$

The research sector is the same as before. The consumer's utility maximization problem gives rise to the same Euler equation

$$\frac{\dot{\overline{M_t + \overline{M}}}}{M_t + \overline{M}} \left(= \frac{\dot{\overline{A_t - \overline{A}}}}{A_t - \overline{A}} = \frac{\dot{\overline{S_t + \overline{S}}}}{S_t + \overline{S}} \right) = \frac{1}{\sigma} \left(R_t - \rho \right). \tag{70}$$

Define

$$\frac{1}{q_t} \equiv \frac{\gamma}{\beta_A} \frac{N_t^i}{\phi_t^i}, \frac{1}{Q_t} \equiv \frac{\alpha_i}{\beta_i} \frac{N_t^i}{h_t^i}, i \in \{A, M, S\}. \tag{71}$$

Then we have that

$$\frac{1}{Q_t} = \sum_i \frac{\alpha_i}{\beta_i} N_t^i = \frac{\alpha_M}{\beta_M} - \left(\frac{\alpha_M}{\beta_M} - \frac{\alpha_A}{\beta_A} \right) N_t^A - \left(\frac{\alpha_M}{\beta_M} - \frac{\alpha_S}{\beta_S} \right) N_t^S, \tag{72}$$

$$\frac{1}{q_t} = \sum_i \frac{\gamma}{\beta_i} N_t^i = \frac{\gamma}{\beta_M} - \left(\frac{\gamma}{\beta_M} - \frac{\gamma}{\beta_A} \right) N_t^A - \left(\frac{\gamma}{\beta_M} - \frac{\gamma}{\beta_S} \right) N_t^S \tag{73}$$

To find the GBGP, by setting $R_t = R^*$ and utilizing (65)-(67) and (70), we know that $p^* = R^* \eta / \gamma$ and $\left(\dot{\overline{M_t + \overline{M}}} \right) / \left(M_t + \overline{M} \right) = \frac{1}{\sigma} \left(R^* - \rho \right) \equiv g^*$. Combining (64), (67)-(69), and (71) gives us

$$p^* = P_i B_i \gamma \left(\frac{\alpha_i}{\beta_i} Q_t H_{Yt} \right)^{\alpha_i} \left(\frac{\gamma}{\beta_i} q_t x_t \right)^{\gamma - 1}, i \in \{A, M, S\}. \tag{74}$$

Substituting (62), (63), and (71) into the flow budget constraint of the representative consumer, we obtain the following dynamic equation

$$\begin{aligned} \dot{K}_t = {} & B_M \alpha_M \left(\frac{\alpha_M}{\beta_M} Q_t H_{Yt} \right)^{\alpha_M - 1} \left(\frac{\gamma}{\beta_M} q_t x_t \right)^{\gamma} \Upsilon_t H + B_M \left(\frac{\alpha_M}{\beta_M} Q_t H_{Yt} \right)^{\alpha_M} \beta_M \left(\frac{\gamma}{\beta_M} q_t x_t \right)^{\gamma} \Upsilon_t \\ & + R_t K_t - P_A \left(A_t - \overline{A} \right) - \left(M_t + \overline{M} \right) - P_S \left(S_t + \overline{S} \right) + \left(-P_A \overline{A} + \overline{M} + P_S \overline{S} \right). \end{aligned}$$

We conjecture that if $P_{it} = P_i^*, i \in \{A, M, S\}$ are constant and the equality $-P_A^* \overline{A} + \overline{M} + P_S^* \overline{S} = 0$ holds on the GBGP, then each term on the both sides of the above equation expands at the same rate g^*. Due to (74), we have

$$0 = \alpha_i \left(\frac{\dot{Q}_t}{Q_t} + \frac{\dot{H}_{Yt}}{H_{Yt}} \right) + (\gamma - 1) \left(\frac{\dot{q}_t}{q_t} + \frac{\dot{x}_t}{x_t} \right), \tag{75}$$

$$Q_t H_{Yt} = c_{04} (q_t x_t)^{\frac{1-\gamma}{\alpha_i}}, \tag{76}$$

where

$$c_{04} = \left[\frac{p^*}{P_i B_i \gamma \left(\frac{\alpha_i}{\beta_i}\right)^{\alpha_i} \left(\frac{\gamma}{\beta_i}\right)^{\gamma-1}} \right]^{\frac{1}{\alpha_i}}.$$

Combining (75) and (76) yields us

$$\frac{\dot{Q}_t}{Q_t} + \frac{\dot{H}_{Yt}}{H_{Yt}} = \frac{\dot{q}_t}{q_t} + \frac{\dot{x}_t}{x_t} = 0, \tag{77}$$

which also displays that both $Q_t H_{Yt}$ and $q_t x_t$ are constant on the GBGP. Substituting (71) into (62) and then into no arbitrage condition $P_{\Upsilon t} \epsilon \Upsilon_t = w_{Ht}$ leads to

$$P_{\Upsilon t} = \frac{p^*}{\epsilon c_{04}} (x_t q_t)^{1-\frac{1-\gamma}{\alpha_M}},$$

which tells that $P_{\Upsilon t}$ is constant on the GBGP, i.e., $P_{\Upsilon t} = P^*_\Upsilon$. Then $\pi^* = R^* P^*_\Upsilon = (1-\gamma) p^* x^*$. Hence $x_t = x^*$ and $q_t = q^*$. Using (72) and (73), we know that

$$\frac{1}{Q_t} + \frac{1}{q_t} + 1 = \sum_i \frac{N^i_t}{\beta_i} = \frac{1}{\gamma}\frac{1}{q_t}, \tag{78}$$

which shows that $Q_t = Q^*$ and hence $H_{Yt} = H^*_Y$, $H_{\Upsilon t} = H - H^*_Y = H^*_\Upsilon$. By the knowledge accumulation equation $\dot{\Upsilon}_t = \epsilon H_{\Upsilon t} \Upsilon_t$, we know that $g^*_\Upsilon = \epsilon H^*_\Upsilon$ on the GBGP. Combining (62), (74), $R^* P^*_\Upsilon = (1-\gamma) p^* x^*$, and $P_{\Upsilon t} \epsilon \Upsilon_t = w_{Ht}$, we have that $H^*_Y = \frac{R^*}{\epsilon}\Omega^*$,where $\Omega^* \equiv q^* / (1-\gamma) Q^*$. Then $g^*_\Upsilon = \epsilon H^*_\Upsilon = \epsilon H - \Omega^* R^*$. Since $g^*_\Upsilon = \frac{1}{\sigma}(R^* - \rho) \equiv g^*$, we know that

$$R^* = \frac{\epsilon H \sigma + \rho}{1 + \sigma \Omega^*}, g^* = \frac{\epsilon H - \rho \Omega^*}{1 + \sigma \Omega^*}. \tag{79}$$

Then it is easy to derive the dynamic equations of the employment shares of agriculture, manufacturing and services on the GBGP, namely,

$$\dot{N}^A_t = -g^* \frac{\overline{A}}{B_A \left(\frac{\alpha_M}{\beta_M} Q^* H^*_Y\right)^{\alpha_M} \left(\frac{\gamma}{\beta_M} q^* x^*\right) \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{A}}{B_A} \frac{1}{\left(\frac{\alpha_A}{\beta_A} Q^* H^*_Y\right)^{\alpha_A} \left(\frac{\gamma}{\beta_A} q^* x^*\right)} < 0, \tag{80}$$

$$\dot{N}^M_t = g^* \frac{\overline{M}}{B_M \left(\frac{\alpha_M}{\beta_M} Q^* H^*_Y\right)^{\alpha_M} \left(\frac{\gamma}{\beta_M} q^* x^*\right) \Upsilon_t} = \frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{M}}{B_M} \frac{1}{\left(\frac{\alpha_M}{\beta_M} Q^* H^*_Y\right)^{\alpha_M} \left(\frac{\gamma}{\beta_M} q^* x^*\right)} > 0, \tag{81}$$

$$\dot{N}^S_t = g^* \frac{\overline{A}}{B_S \left(\frac{\alpha_S}{\beta_S} Q^* H^*_Y\right)^{\alpha_S} \left(\frac{\gamma}{\beta_S} q^* x^*\right) \Upsilon_t} = -\frac{g^*}{\Upsilon_0 \exp(g^* t)} \frac{\overline{A}}{B_S} \frac{1}{\left(\frac{\alpha_S}{\beta_S} Q^* H^*_Y\right)^{\alpha_S} \left(\frac{\gamma}{\beta_S} q^* x^*\right)} > 0. \tag{82}$$

Furthermore, the production shares of the three subsectors in the final-goods sector are not equal to their corresponding employment shares. Their production shares equal the same optimal input of each intermediate good in each subsector, namely,

$$\vartheta_i = \frac{\gamma q^*}{\beta_A} N_t^A = \phi_t^i. \tag{83}$$

If the factor income shares are the same in the three subsectors, i.e., $\alpha_A = \alpha_M = \alpha_S$ and $\beta_A = \beta_M = \beta_S$, then the above equilibrium results degenerate to the ones in the text.

To check our conjecture, we only need to solve the constant equilibrium prices $P_i^*, i \in \{A, M, S\}$ on the GBGP. Substituting (80), (81), and (82) into $\dot{N}_t^A + \dot{N}_t^M + \dot{N}_t^S = 0$ and using (63) lead to

$$-P_A^* \overline{A} \beta_A + \overline{M} \beta_M + P_S^* \overline{S} \beta_S = 0. \tag{84}$$

Now we solve the algebraic equations composed by $-P_A^* \overline{A} + \overline{M} + P_S^* \overline{S} = 0$ and (84) for P_A^* and P_S^*. Since $\beta_A \neq \beta_S$ (otherwise, the model degenerates to the original model with the same factor income shares), they can be solved as

$$P_A^* = \frac{\beta_M - \beta_S}{\beta_A - \beta_S} \frac{\overline{M}}{\overline{A}}, P_S^* = \frac{\beta_M - \beta_A}{\beta_A - \beta_S} \frac{\overline{M}}{\overline{S}}. \tag{85}$$

If the parameters satisfy $\beta_S < \beta_A < \beta_M$ or $\beta_M < \beta_A < \beta_S$, then they are positive.

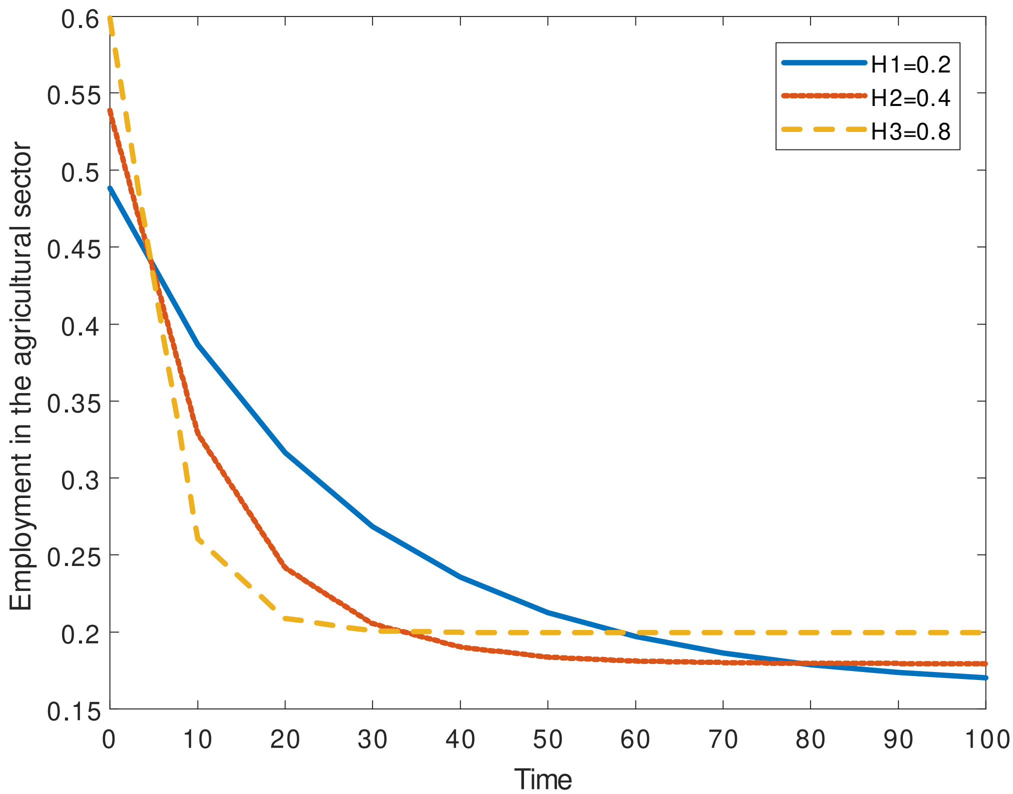

Figure 1: Human Capital and Employment in the Agricultural Sector

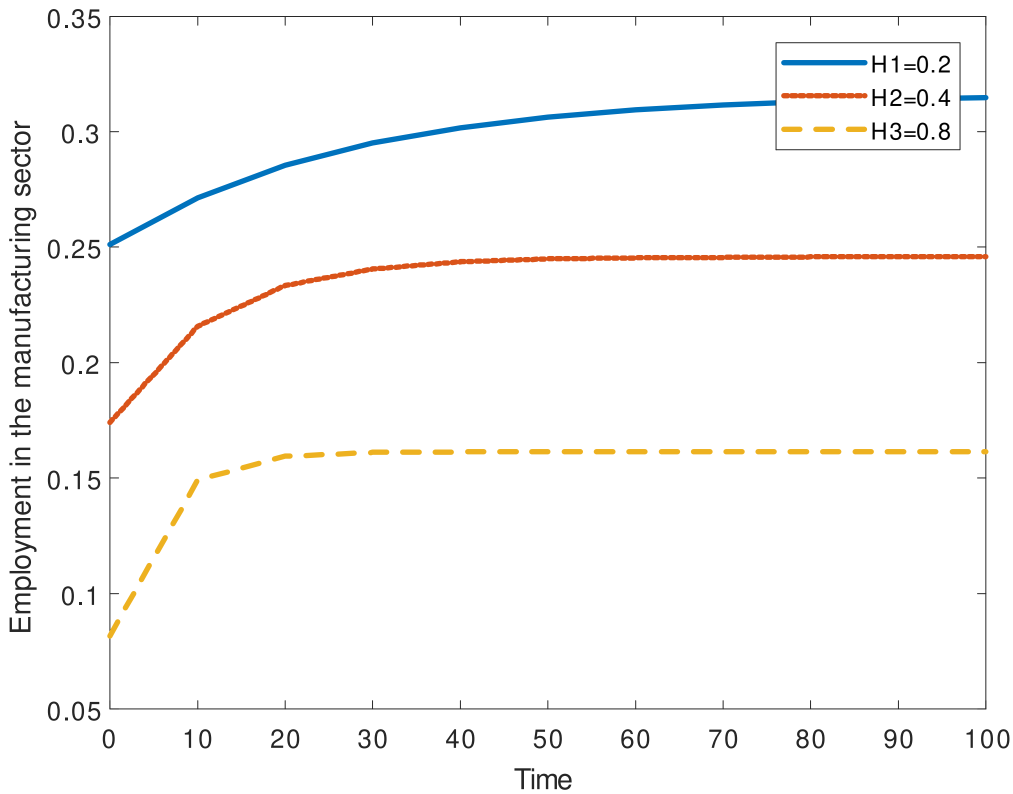

Figure 2: Human Capital and Employment in the Manufacturing Sector

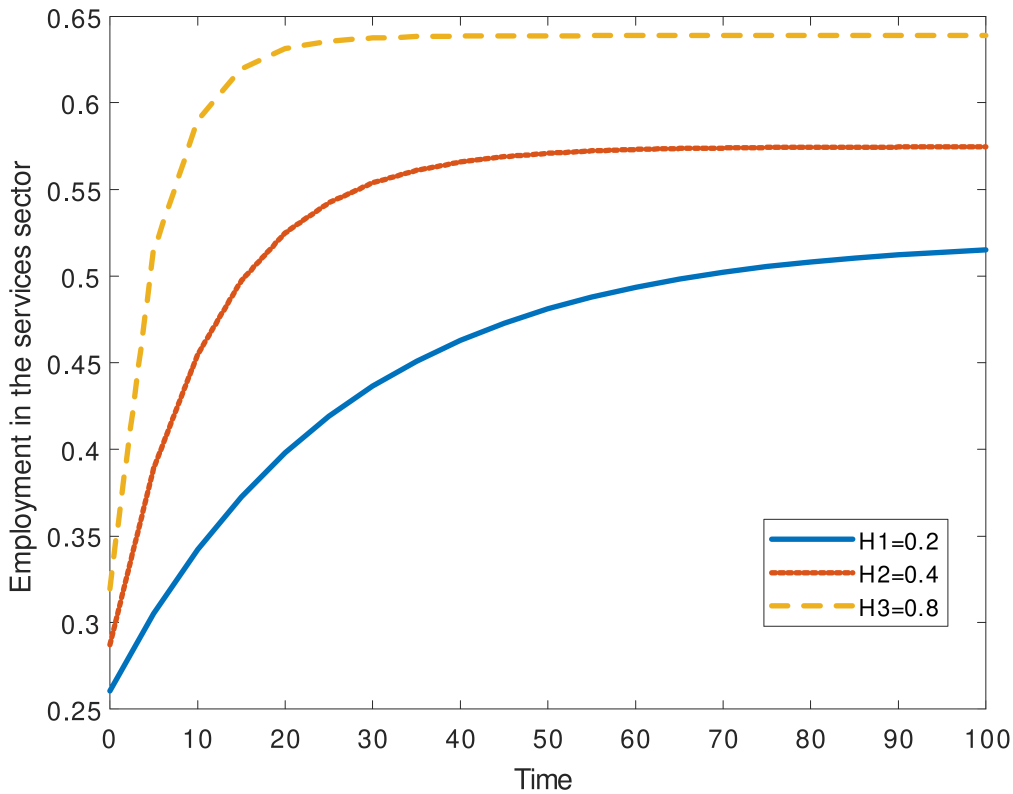

Figure 3: Human Capital and Employment in the Services Sector

Table 1

Declining Employment Shares and Average Years of Schooling for 5 Developing Countries.

Country	Declining Shares	Years	Years of Schooling
China	0.69-0.60	10	5.1
	0.59-0.50	11	6.7
	0.49-0.40	4	7.7
Thailand	0.79-0.72	14	4.2
	0.68-0.60	13	5.3
	0.60-0.50	6	6.5
India	0.72-0.70	24	3.1
	0.69-0.60	14	4.4
	0.59-0.54	10	5.7
Kenya	0.80-0.70	22	3.6
	0.69-0.61	4	6.1
	0.59-0.50	10	6.8
Indonesia	0.72-0.70	24	4.6
	0.69-0.60	14	6.5
	0.59-0.54	10	8.0

Source: Wittgenstein Centre for Demography and Global Human Capital (2015);
GGDC 10-Sector Database for sectoral employment data.

Table 2

Declining Employment Shares and Average Years of Schooling for 6 Developed Countries.

Country	Declining Shares	Years	Years of Schooling
Denmark	0.106-0.081	8	10.9
	0.079-0.060	9	11.4
	0.059-0.039	11	11.8
	0.038-0.027	13	12.2
France	0.138-0.115	3	7.2
	0.108-0.086	6	8.2
	0.083-0.061	8	9.0
	0.058-0.038	12	10.1
Italy	0.125-0.101	3	8.0
	0.100-0.080	4	8.6
	0.080-0.060	6	9.2
	0.057-0.037	15	10.2
Sweden	0.080-0.063	6	9.9
	0.063-0.042	12	10.8
	0.042-0.022	23	12.3
Japan	0.164-0.141	4	10.0
	0.139-0.118	4	10.6
	0.115-0.094	6	11.3
	0.090-0.071	6	12.0
	0.068-0.049	14	12.8
Korea	0.170-0.150	2	10.1
	0.140-0.117	2	10.9
	0.112-0.093	6	11.7
	0.089-0.069	7	12.5

Source: Wittgenstein Centre for Demography and Global Human Capital (2015);
GGDC 10-Sector Database for sectoral employment data.