Economic Transactions Govern Business Cycles

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Abstract

This paper presents the business cycle model based on treatment of economic agents as simple units of macroeconomics. Agents (banks, corporations, households, etc.) have numerous economic and financial variables like Assets, Credits, Debts, Consumption, etc. Agents perform economic and financial transactions with other agents. All agents are at risk but not for all agents risk assessments are performed now. Let's propose that risk assessment can be made for all agents and let's use agents risk ratings $\boldsymbol{x}$ as their coordinates. Agents coordinates for n risks define n-dimensional economic space. Economic and financial transactions between agents describe evolution of their economic and financial variables. Aggregations of economic or financial variables of agents in a unit volume at point $\boldsymbol{x}$ determine macro variables as functions of $\boldsymbol{x}$. Aggregations of transactions between agents in unit volumes at points $\boldsymbol{x}$ and $\boldsymbol{y}$ determine macro transactions as functions of $\boldsymbol{x}$ and $\boldsymbol{y}$. Macro transactions describe rate of change of macro variables at points $\boldsymbol{x}$ and $\boldsymbol{y}$. We derive economic equations that describe evolution of macro transactions. We show that business cycle fluctuations are consequence of these equations. We argue that business cycle fluctuations of particular macro variable can be treated as oscillations of "mean risk coordinates" of this economic variable. As example we study the business cycle determined by model interactions between transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ that provide Loans from Creditors at point $\boldsymbol{x}$ to Borrowers at point $\boldsymbol{y}$ and transactions $LR(t,\boldsymbol{x},\boldsymbol{y})$ that describe repayments from Borrowers at point $\boldsymbol{y}$ to Creditors at point $\boldsymbol{x}$. Starting with economic equations we derive the system of ordinary differential equations that describe the business cycle fluctuations of macro Credits $C(t)$ and macro Loan-Repayments $LR(t)$ of the entire economics.

Keywords: Business cycle; Economic Transactions; Risk Assessment; Economic Space
JEL: C02, C60, E32, F44, G00

1. Introduction.

"Serious efforts to explain business crises and depressions began amid the violent fluctuations in trade which followed the Napoleonic Wars" (Mitchell, 1927). Not much changed since Mitchell statement nearly a century ago. For decades description of business cycles remains essential macroeconomic problem: Tinbergen (1935), Schumpeter (1939), Smithies (1957), Morgenstern, (1959), Lucas (1980), Kydland and Prescott (1982), Plosser, (1989), Zarnowitz (1992), Lucas (1995), Diebold and Rudebusch, 1999; Rebelo (2005), Kiyotaki (2011), Schmitt-Grohé and Uribe (2012), Diebold and Yilmaz, 2013; Jorda, Schularick and Taylor (2016), Huggett (2017), Bordalo, Gennaioli and Shleifer (2017). "The incorporation of cyclical phenomena into the system of economic equilibrium with which they are in apparent contradiction remains the crucial problem of Trade Cycle Theory" (Hayek, 1933, quoted by Lucas, 1976). "Why aggregate variables undergo repeated fluctuations about trend, all of essentially the same character? Prior to Keynes' General Theory, the resolution of this question was regarded as one of the main outstanding challenges to economic research, and attempts to meet this challenge were called business cycle theory" (Lucas, 1995).

Risk assessment plays a special role for business cycle studies (Tallarini, 2000; Pesaran, Schuermann and Treutler, 2007; Mendoza and Yue, 2012; Diebold, 2012). Risk affect macroeconomic and finance development and stability (Huang, Zhou and Zhu, 2009; Nicolò and Lucchetta, 2011) and pricing models (Bollerslev and Zhang, 2003). Endogenous business cycle models within general equilibrium framework (Grandmont, 1985; Farmer and Woodford, 1997; Bilbiie, Ghironi and Melitz, 2012; Growiec, McAdam and Mućk, 2015; Engle, 2017) and relations between risk and business cycles counts hundreds of publications (Alvarez and Jermann, 1999; Tallarini, 2000; Pesaran, Schuermann and Treutler, 2007; Christiano, Motto and Rostagno, 2013). Actually current business cycle models follow general economic equilibrium framework (Lucas, 1975; Kydland and Prescott, 1982; 1991; Mullineux and Dickinson, 1992; Kiyotaki, 2011; Mendoza and Yue, 2012). "The real business cycle theory is a business cycle application of the Arrow-Debreu model, which is the standard general equilibrium theory of market economies." (Kiyotaki 2011).

However, complexity of economic processes and business cycles requires different approaches and approximations for their modeling. Any model only approximates real economic processes and it seems unbelievable that such complex phenomena as the business cycle can be described by single concept – general equilibrium (Arrow and Debreu, 1954;

Arrow, 1974; Kydland and Prescott, 1982; Lucas, 1995; Gintis, 2007; Ohanian, Prescott, Stokey, 2009; Starr 2011; Cardenete, Guerra, Sancho, 2012; Del Negro, et.al., 2013; Richter and Rubinstein, 2015). We propose that complexity and variability of business cycles requires description by approaches, which might be different from general equilibrium.

In this paper we present the business cycle model without using general equilibrium assumptions on state of markets, prices and etc. General Occam's razor principle (Baker, 2007) states: "Entities are not to be multiplied beyond necessity". In simple words: the less initial assumptions – the better. Instead of general equilibrium assumption we propose that econometrics can provide sufficient data to assess risks ratings for almost all agents of entire economics and estimate values of economic and financial transactions between agents. We do not specify particular risks under consideration and regard any economic or financial risks that impact economic processes. Economic and financial transactions between economic agents describe rate of change and evolution of agent's extensive variables like Credits and Debts, Assets and Investment and etc. We propose that all other economic variables should depend on such extensive variables and on economic transactions. Macroeconomic evolution and business cycle fluctuations are consequences of extremely complex economic processes. Any modeling and descriptions of macroeconomic evolution and business cycles require certain approximations and simplifications. In this paper we describe approximation of business cycle fluctuations that doesn't take into account influence of expectations and behavioral motivations (Simon, 1959; Grossman, 1980; Taylor, 1984; Dotsey and King, 1988; Jaimovich and Rebelo, 2007; Campbell, 2016; Thaler, 2016) on transactions and the business cycle. We shall describe impact of expectations on transactions and the business cycle fluctuations in forthcoming publications.

All extensive (additive) macro variables are composed by aggregation of corresponding extensive variables of agents. For example, macroeconomic Investment $I(t)$ equals sum of Investment (without doubling) of all agents. Credits $C(t)$ of entire economics equal sum of Credits provided by all agents. Actually, transactions between agents change their extensive variables. For example Credits transactions from agent A to agent B change total Credits provided by agent A and total Loans received by agent B. Description of transactions between agents allows model evolution of macro variables and, as we show below, can model the business cycle fluctuations.

Description of all transactions between economic agents is a very complex problem. To simplify it let's replace description of transactions between separate agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ by description of transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. To do that let's aggregate similar transactions between agents in a unit volume dV_x near risk point $\boldsymbol{x}$ and agents in a unit volume dV_y near risk point $\boldsymbol{y}$. Let assume that there are many agents in a unit volume dV_x and many agents in a unit volume dV_y . Let assume that scales of unit volumes dV_x and dV_y are small to compare with risk scales of entire economy. Risk scales of economy are defined by minimum or most secure risk grades and maximum or most risky grades of each particular risk. Such *roughening* of risk scales allows neglect granularity of separate agents and describe aggregate transactions between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ as certain economic "*transaction fluids*". Such simplification is *alike to* transition from kinetic description of multi particle system to hydrodynamic approximation. We develop the business cycle model that describe fluctuations of macro variables governed by macro transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. We model interactions between macro transactions by economic equations (see below (4.1-4.2) and (5.1.1-5.3)) and show that business cycle fluctuations are consequences of these equations.

As example of business cycle fluctuations we describe Credits provided by agents and Credits transactions between agents. Sum of Credits provided by all agents with risk coordinate $\boldsymbol{x}$ defines Credits $C(t,\boldsymbol{x})$ as function of t and $\boldsymbol{x}$. Aggregates of all transactions that describe Credit provided from Creditors at $\boldsymbol{x}$ to Borrowers at $\boldsymbol{y}$ define macro Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ as function of time and coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$. Evolution of Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ define evolution of Credits $C(t,\boldsymbol{x})$ as function of t and $\boldsymbol{x}$ and Loans $L(t,\boldsymbol{y})$ as function of t and $\boldsymbol{y}$. Total Credits $C(t)$ in economy equal sum of Credits of all agents in economy and that equals integral of Credits $C(t,\boldsymbol{x})$ by $d\boldsymbol{x}$ on economic space. Distribution of Credits $C(t,\boldsymbol{x})$ as function of $\boldsymbol{x}$ allows define mean Credits risk $\boldsymbol{X}_C(t)$ (3.7.3; 3.7.5) as mean risk $\boldsymbol{x}$ weighted by Credits $C(t,\boldsymbol{x})$ on economic space. Mean Credits risk $\boldsymbol{X}_C(t)$ can be treated *alike to* center of mass $\boldsymbol{X}_C(t)$ of a body with total Credit mass $C(t)$. Mean Credits risk $\boldsymbol{X}_C(t)$ is not a constant. $\boldsymbol{X}_C(t)$ changes due to variation of Creditors risks and changes of Credits provided by Creditors that are caused by economic and financial processes. Borders of economic domain (1) on economic space reduce motion of mean Credits risk $\boldsymbol{X}_C(t)$ and thus it should follow complex fluctuations on bounded economic domain (1). Fluctuations of mean Credits risk $\boldsymbol{X}_C(t)$ reflect business cycle processes and are accompanied by fluctuations of total Credits $C(t)$. As we show below, motion of mean Credits risk $\boldsymbol{X}_C(t)$ is governed by (see below (5.1.1-5.1.3; 5.2; 5.3)) complex evolution of Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$. Mean risk coordinates are different for different economic and financial variables and their mutual motions and interactions are very complex. Fluctuations of mean risk coordinates of different economic and financial variables reflect complex business cycle processes and accompanied by fluctuations of macro variables like Credits $C(t)$, Loans $L(t)$, Investment $I(t)$, and etc.

In Olkhov (2017d-e) we describe the business cycle model under the assumption that economic and financial transactions on economic space occur between agents with same risk coordinates only. Such approximation describes *local* transactions between agents on economic space. *Local* approximation allows simplify the problem and develop the business cycle model with *local* interactions between macro variables.

In real economics agents with risk rating $\boldsymbol{x}$ can conduct transactions – Credits, Investments and etc., to agents with any risk ratings $\boldsymbol{y}$. Transactions between agents with coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ display economic and financial "*action-at-a-distance*" between points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. That significantly complicates macroeconomic and the business cycle modeling. This paper describes "*action-at-a-distance*" transactions between agents with any risk coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$. We describe transactions by economic equations on economic space. Starting with these equations we derive a system of ordinary differential equations (ODE) that model the business cycle time fluctuations of macro variables.

The rest of the paper is organized as follows. In Section 2 we present model setup and give definitions of macro transactions (Olkhov 2017b; 2017c). In Section 3 we introduce a system of economic equations on macro-transactions and discuss their economic meaning (Olkhov 2017b; 2017c). In Section 4 we argue economic assumptions that allow describe business cycles aggregate fluctuations. As example of the business cycle we study a model interactions between macro Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ from Creditors at point $\boldsymbol{x}$ to Borrowers at point $\boldsymbol{y}$ and macro transactions $LR(t,\boldsymbol{x},\boldsymbol{y})$ of Repayments on Loans from Borrowers at point $\boldsymbol{y}$ to Creditors at point $\boldsymbol{x}$. We model these transactions by a system of economic equations and describe their evolution in a self-consistent manner. Starting with these equations we derive the system of ODE and derive simple solutions that describe the business cycle fluctuations around growth trend of Credits $C(t)$. Conclusions are in Section 5.

2. Model Setup

In this Section we explain meaning of economic space, macro variables as functions of coordinates $\boldsymbol{x}$ and introduce transactions between agents as functions of points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space (Olkhov, 2016a-b; 2017a-c). Let's regard any participants of economic and financial relations like banks, companies, households and etc., as economic agents. Agents have a lot of economic and financial variables like Assets, Credits, Consumption, Debts and Investment, Working Hours and Value-Added and etc. Aggregation of agents variables defines corresponding macroeconomic variables. For example aggregation of agents Investment defines macro Investment, aggregation of agents Consumption defines macro Consumption and etc. Thus description of agents variable allows model evolution and fluctuations of macro variables like Investment, Credits, GDP, Working Hours and etc., and describe different properties of Business Cycles. Economic and financial variables of agents are changed due to corresponding transactions between agents. For example Banks as Agents provide Credits to Borrowers and such transactions change amount of Credits provided by Banks and amount of Loans received by Borrowers. Hence description of transactions between agents provides opportunities to describe evolution of agents variables. Transactions between agents describe rate of change of corresponding variables and aggregations of transactions describe rate of change of macro variables. Thus modeling transactions helps for modeling business cycle fluctuations of macro variables. We propose describe evolution of transactions between agents as origin of business cycle fluctuations of macro variables.

It is obvious that any transactions between agents are determined by definite Expectations. Since Muth (1961) and Lucas (1972) importance and impact of Expectations on economic and financial evolution was studied in numerous papers and we refer (Kydland and Prescott, 1980; Brunnermeier and Parker, 2005; Greenwood and Shleifer, 2014; Manski, 2017) as only small part of these research. Macroeconomic evolution is very complex and any model of macroeconomics describes reality for certain approximation only. In current paper we simplify description of business cycles and neglect impact of Expectations on transactions between agents. Here we propose that transactions between agents depend on other transactions only. Such approximation allows describe transactions-driven business cycle model. We shall take into account impact of Expectations on business cycles and transactions between agents in forthcoming publications.

Let’s regard macroeconomics as ensemble of numerous economic agents. It is well known that all agents are under action of different risks. There are many economic and financial risks that impact agents economic evolution and their transactions. Impacts of different risks on agents economic evolution, agents risk management and risk assessment are studied in incredible amount of papers and we refer only few of them (Gupton, et al, 1997; Alvarez and Jermann, 1999; Diebold, 2012; Christiano et al, 2013; BIS, 2014; Skoglund and Chen, 2015; Engle, 2017). We don’t argue problems of risk assessment of agents in our paper but suggest that risk assessment methodologies can become a ground for macroeconomic modeling. Let’s outline that for decades international rating companies as Moody’s, Fitch, S&P (Metz and Cantor, 2007; Chane-Kon, et.al, 2010; Kraemer and Vazza, 2012) provide risk assessment and attribute risk ratings like *AAA, A, BB, C* and etc. to huge banks, international companies and so on. Up now rating agencies provide risk assessments for global banks and international corporations. Let's propose that it is possible to assess risk ratings for all agents of entire economics – as for global banks and corporations as for small companies and even households. That requires a lot of additional econometric and statistical data that are absent now. We hope that quality, accuracy and granularity of current U.S. National Income and Product Accounts system (Fox, et al., 2014) give us confidence that all econometric problems can be solved. Let's propose that our assumptions are fulfilled and it is possible evaluate risk assessments for all agents of entire economics. Risk ratings take values of risk grades and we propose regard these risk grades as points $x_1,...x_m$ of discrete space. Usage of risks ratings allows distribute economic agents over points $x_1,...x_m$ on discrete space. Let's call the space that map agents by their risk ratings $\boldsymbol{x}$ as economic space. Ratings of single risk distribute agents over points of one-dimensional discrete space. Assessments of two or three risks allow distribute agents on economic space with dimension two or three. It is obvious that number of risk grades, number of points *AAA, A, BB, C…* is determined by methodology of risk assessment. Let's assume that assessment methodology can be generalized to make risk grades continuous so, they fill certain interval *(0,X)* on space *R*. Let's take point *0* as most secure and point *X* as most risky grades. Value of most risky grade *X* always can be set as *X=1* but we use *X* notation for convenience. Let's assume that risk assessments of *n* risks define coordinates of agents on space R^n. Risk grades of *n* risks fill rectangle that define economic domain (1) on space R^n. For economics under action of *n* risks continuous risk ratings of economic agents fill economic domain

$$0 \leq x_i \ \leq X_i \,; i = 1, \dots n \tag{1}$$

on space R^n. As we mentioned above, risk grades X_i always can be set as $X_i=1$. Below we study economic and financial transactions and develop business cycle model for economics that is under the action of *n* risks on economic space R^n.

Transactions between agents change their extensive economic and financial variables. For example agent *A* can provide Credits to agent *B*. This transaction will change Credits provided by agent A and Loans received by agent *B*. Each transaction takes certain time *dt* and we consider transactions as rate or speed of change of corresponding variables. For example Credits transactions from agent *A* at moment *t* define rate of change of total Credits provided by agent *A* till moment *t* during time term *dt*. Let's call extensive economic or financial variables of two agents as *mutual* if output of one becomes an input of the other. For example, Credits as output of Creditors are *mutual* to Loans as input of Borrowers. Any exchange between agents by *mutual* variables is carried out by corresponding transactions. Any agent at point $\boldsymbol{x}$ may carry out transactions with agent at any point $\boldsymbol{y}$ on economic space. Different transactions define evolution of different couples of m*utual* variables. We regard agents as simple units of macroeconomics and propose treat agents *alike to* "economic particles" and economic or financial transactions between agents as interactions between "economic particles". For brevity let's further call economic agents as e-particles and economic space as e-space. Now let's present above considerations in a more formal manner.

2.1. Transactions between e-particles

As example let's treat Credits transactions *CL* that provide Loans from Creditors to Borrowers and follow Olkhov (2017b-c). Let's denote Credits transactions $cl_{1,2}(t,\boldsymbol{x},\boldsymbol{y})$ from e-particle *1* at point $\boldsymbol{x}$ to e-particle *2* at point $\boldsymbol{y}$. Credits transactions $cl_{1,2}(t,\boldsymbol{x},\boldsymbol{y})$ describe Credits provided by from e-particle *1* as Creditor at point $\boldsymbol{x}$ to Borrower at e-particle *2* as at point $\boldsymbol{y}$ during *dt*. Credits transactions $cl_{1,2}(t,\boldsymbol{x},\boldsymbol{y})$ describe issue of Credits and receiving of Loans. Let's call Credits and Loans as *mutual* variables. Let's state that all extensive economic or financial variables can be allocated as pairs of *mutual* variables or can be describes by *mutual* variables. Obviously, real economic processes are more complex and our assumption should be treated as *approximation*. For example, transactions may depend on *expectations* of agents and we shall study *expectations* problem in forthcoming paper. In this paper we develop approximation of economic processes and the business cycle model based on assumption that transactions describe dynamics of all extensive economic and financial variables of e-particles and hence determine evolution of all extensive macroeconomic and financial variables.

2.2 Macro transactions between points on economic space

Let's assume that transactions between e-particles at point $\boldsymbol{x}$ and e-particles at point $\boldsymbol{y}$ describe exchange of *mutual* variables Credits and Loans, Buy and Sell, and etc. Different transactions describe exchange by different *mutual* variables. For example *Buy-Sell (bs)* transactions with particular Commodities, Assets, Securities and etc. at time *t* describe exchange by amount *bs* of goods from e-particle *2* at point $\boldsymbol{y}$ to e-particle *1* at point $\boldsymbol{x}$ during time *dt*. Payment transaction for this particular amount *bs* of goods describe money transfer from e-particle *1* at point $\boldsymbol{x}$ as Buyer to e-particle *2* at point $\boldsymbol{y}$ as Seller. Description of transactions between separate e-particles is very complex problem. We propose that description of macroeconomic processes can be based on rougher model. To do that we suggest define economic and financial transactions between points of e-space. Main idea: let's replace precise description of transactions between separate e-particles by rougher description of transactions associated with points of e-space that don't distinguish separate e-particles. Such a roughening is already used in economics. For example aggregation of all Credits between agents of entire economics define macro Credit *C(t)* (see 3.6.2) provided in macroeconomics at moment *t* and equal macro Loans *L(t)* received in macroeconomics at moment *t*. Modeling transactions between all separate agents at points ***x*** and ***y*** on e-space establish too detailed picture. On the other hand description of variables like macro Credits *C(t)* as aggregates all transactions between all agents of entire economics gives too simplified economic model. We propose intermediate description of economy that aggregate transactions between agents that belong to domains near points ***x*** and ***y*** on risk e-space. Such approximation neglect granularity of separate e-particles but allows take into account distribution of transactions on e-space. Such approach is similar to transition from kinetic description of multi-particle system to hydrodynamic approximation in physics (Landau, Lifshitz, 1981; 1987; Resibois and De Leener, 1977). For example, let's define Credit transaction $CL(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ at point $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ as aggregate Credits from all e-particles at point ***x*** to all e-particles at point ***y***. As points ***x*** and ***y*** belong to *n*-dimensional e-space R^n then point $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ can be treated as a point of *2n*-dimensional e-space R^{2n}. Such roughening of transactions between e-particles describe transition from discreet description of transactions between separate e-particles to "*continuous media*" approximations of transactions between points ***x*** and ***y*** on e-space. Transactions as functions of $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ *2n*-dimensional e-space R^{2n} can be treated as "*transaction fluids*". For example Credits transactions between e-particles defines Credit "*transaction fluids*" *CL(t,z)*, Investment transactions define "*Investment fluid*" *I(t,z)*, Buy-Sell transactions with particular commodity, define "*Buy-Sell fluid*" *BS(t,z)* for particular commodity. Value of Credits *CL(t,z),* Investment *I(t,z),* Buy-Sell *BS(t,z)* transactions at point $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ play role *alike to* densities of *"transaction fluids*" similar to mass density of physical fluid (see 3.1; 3.4; 3.5). Velocity *(3.2-3.5.1)* of *"transaction fluid"* determine motion transactions carried by agents at points ***x*** and ***y***. For example, velocities of Credits transactions fluid $CL(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ are determined by velocities of Creditors along axes $\boldsymbol{x}=(x_1,..x_n)$ and by velocities of Borrowers along axes $\boldsymbol{y}=(y_1,..y_n)$. Evolution of such Credit *"transaction fluids*" can be described by economic equations (4.1-4.2) (Olkhov, 2016a-b; 2017a-d). Meaning of these equations is simple: economic equations (4.1) describe balance between left and right sides. Left side of equations (4.1) describes change of Credits density *CL(t,z)* in a unit volume on *2n*-dimensional e-space. Credits *CL(t,z)* in a unit volume can change due to its change in time as $\partial CL(t,z)/\partial t$ and due to flux $CL(t,z)\boldsymbol{v}(t,z)$ of Credits through surface of a unit volume. According to Divergence Theorem (Strauss 2008, p.179) surface integral for flux $CL(t,z)\boldsymbol{v}(t,z)$ equals volume integral for divergence of $CL(t,z)\boldsymbol{v}(t,z)$ and hence we obtain left side of equations (4.1). Here $\boldsymbol{v}(t,z)$ – velocity of Credits *"transaction fluids"* (3.1-3.5.1). Right side describes action of other transactions on evolution of Credits *"transaction fluids"* $CL(t,z)$. These equations reflect economic properties and relations between different transactions. Below we present above considerations in more formal way.

Let's assume that e-particles on e-space R^n at moment t have coordinates $\boldsymbol{x}=(x_1,\ldots x_n)$ and velocities $\boldsymbol{v}=(\upsilon_1,\ldots\upsilon_n)$. Velocities $\boldsymbol{v}=(\upsilon_1,\ldots\upsilon_n)$ describe change of e-particles risk coordinates during time dt. Let's assume that at moment t there are $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$ and $N(\boldsymbol{y})$ e-particles at point $\boldsymbol{y}$. Let's define that Credits transactions $cl_{i,j}(\boldsymbol{x},\boldsymbol{y})$ describe that e-particle i at point $\boldsymbol{x}$ provide Credit of amount $cl_{i,j}(\boldsymbol{x},\boldsymbol{y})$ and e-particle j at point $\boldsymbol{y}$ receive Loans of amount $cl_{i,j}(\boldsymbol{x},\boldsymbol{y})$ at moment t during time term dt. Let's take Credits transactions $cl(\boldsymbol{x},\boldsymbol{y})$ between points $\boldsymbol{x}$ and $\boldsymbol{y}$ as:

$$cl(t,\boldsymbol{x},\boldsymbol{y}) = \sum_{ij} cl_{ij}(t,\boldsymbol{x},\boldsymbol{y}); \quad i = 1,\ldots N(\boldsymbol{x});\ i = 1,\ldots N(\boldsymbol{y}) \tag{2.1}$$

$cl(t,\boldsymbol{x},\boldsymbol{y})$ equals growth of Credits provided by all e-particles at point $\boldsymbol{x}$ to all e-particles at point $\boldsymbol{y}$ at moment t and equals rise of Loans received by all e-particles at point $\boldsymbol{y}$ from all e-particles at point $\boldsymbol{x}$ at moment t during time dt. Transactions (2.1) between two points on e-space are random due to random number of e-particles at points $\boldsymbol{x}$ and $\boldsymbol{y}$ and random value of transactions between them. Evolution of Credit transaction $cl(t,\boldsymbol{x},\boldsymbol{y})$ depends on velocities $\boldsymbol{v}=(\boldsymbol{v}_x,\boldsymbol{v}_y)$ that describe change of risk ratings coordinates of e-particles involved in transactions at points $\boldsymbol{x}$ and $\boldsymbol{y}$. Such a treatment has *parallels* to definition of fluid velocity in hydrodynamics: motion of physical particles defines velocity of fluid (Landau and Lifshitz, 1981; Resibois and De Leener, 1977). Averaging procedure can be applied to additive variables only. Velocities of e-particles are not additive variables. To use averaging procedure let's introduce *additive* variables - transaction "impulses" $\boldsymbol{p} = (\boldsymbol{p}_x, \boldsymbol{p}_y)$ *alike to* impulses in physics (Olkhov, 2017b-c):

$$\boldsymbol{p}_{\boldsymbol{x}}(t,\boldsymbol{x},\boldsymbol{y}) = \sum_{i,j} cl_{ij}(t,\boldsymbol{x},\boldsymbol{y}) \cdot \boldsymbol{v}_{\boldsymbol{x}i}; \quad i = 1,\ldots N(\boldsymbol{x}); j = 1,\ldots N(\boldsymbol{y}) \tag{2.2}$$

$$\boldsymbol{p}_{\boldsymbol{y}}(t,\boldsymbol{x},\boldsymbol{y}) = \sum_{i,j} cl_{ij}(t,\boldsymbol{x},\boldsymbol{y}) \cdot \boldsymbol{v}_{\boldsymbol{y}j}; \quad i = 1,\ldots N(\boldsymbol{x}); j = 1,\ldots N(\boldsymbol{y}) \tag{2.3}$$

Here $\boldsymbol{v}_{xi}=(\upsilon_{1i},\ldots\upsilon_{ni})$ – velocities of e-particles at point $\boldsymbol{x}$ and $\boldsymbol{v}_{yj}=(\upsilon_{1j},\ldots\upsilon_{nj})$ – velocities of e-particles at point $\boldsymbol{y}$. Transactions *impulses* $\boldsymbol{p}_x$ and $\boldsymbol{p}_y$ are additive and admit averaging procedure by probability distribution. Transactions *impulses* p_{Xi} and p_{Yi}, $i=1,..n$ describe flow of Credits *"transaction fluid"* $cl(t,z=(\boldsymbol{x},\boldsymbol{y}))$ through unit surface in the direction of risks x_i for

Creditors and in the direction of y_i for Borrowers. Credits transactions $cl(t,\boldsymbol{x},\boldsymbol{y})$ (2.1) and transactions "impulses" $\boldsymbol{p_x}$ and $\boldsymbol{p}_y$ (2.2, 2.3) take random values due to random properties of transactions and motion of e-particles. To obtain regular mean impulses (Olkhov, 2017b, 2017c) let's average (2.1-2.3) by probability distribution function $f=f(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}); cl, \boldsymbol{p}=(\boldsymbol{p}_X,\boldsymbol{p}_Y);N(\boldsymbol{x}),N(\boldsymbol{y}))$ on $2n$-dimensional e-space R^{2n} that determine probability to observe Credits transactions with value cl at point $\boldsymbol{z}=(\boldsymbol{x}, \boldsymbol{y})$ between $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$ and $N(\boldsymbol{y})$ e-particles at point $\boldsymbol{y}$ with economic impulses $\boldsymbol{p} =(\boldsymbol{p}_x, \boldsymbol{p}_y)$ at time t. Averaging of Credits transactions and their transaction "impulses" by distribution function f determine mean "*transaction fluid" $CL(t,z)$* as functions of $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$. We do not argue here any properties of distribution function f. Mean macro Credits transactions $CL(\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ and "impulses" $\boldsymbol{P}=(\boldsymbol{P}_x,\boldsymbol{P}_y)$ take form:

$$CL(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})) = \sum_{N(x);N(y)} \int cl \;\; f\big(t,\boldsymbol{x},\boldsymbol{y};cl,\boldsymbol{p_x},\boldsymbol{p_y};N(\boldsymbol{x}),N(\boldsymbol{y})\big) dcl\, d\boldsymbol{p_x}\, d\boldsymbol{p_y} \quad (3.1)$$

$$\boldsymbol{P_x}(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})) = \sum_{N(x);N(y)} \int \boldsymbol{p_x} \;\; f\big(t,\boldsymbol{x},\boldsymbol{y};cl,\boldsymbol{p_x},\boldsymbol{p_y};N(\boldsymbol{x}),N(\boldsymbol{y})\big) dcl\, d\boldsymbol{p_x}\, d\boldsymbol{p_y} \quad (3.2)$$

$$\boldsymbol{P_y}(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})) = \sum_{N(x);N(y)} \int \boldsymbol{p_y} \;\; f\big(t,\boldsymbol{x},\boldsymbol{y};cl,\boldsymbol{p_x},\boldsymbol{p_y};N(\boldsymbol{x}),N(\boldsymbol{y})\big) dcl\, d\boldsymbol{p_x}\, d\boldsymbol{p_y} \quad (3.3)$$

Relations (3.1-3.3) define velocities $\boldsymbol{v}(t,z=(\boldsymbol{x},\boldsymbol{y}))=(\boldsymbol{v}_{\mathrm{x}}(t,z),\boldsymbol{v}_{\mathrm{y}}(t,z))$ of macro transactions as:

$$\boldsymbol{P_x}(t,\boldsymbol{z}) = CL(t,\boldsymbol{z})\boldsymbol{v_x}(t,\boldsymbol{z}) \quad (3.4)$$

$$\boldsymbol{P_y}(t,\boldsymbol{z}) = CL(t,\boldsymbol{z})\boldsymbol{v_y}(t,\boldsymbol{z}) \quad (3.5)$$

$$\boldsymbol{P}(t,\boldsymbol{z}) = \Big(\boldsymbol{P_x}(t,\boldsymbol{z});\boldsymbol{P_y}(t,\boldsymbol{z})\Big) \;\; ; \quad \boldsymbol{v}(t,\boldsymbol{z}) = \Big(\boldsymbol{v_x}(t,\boldsymbol{z});\boldsymbol{v_y}(t,\boldsymbol{z})\Big) \quad (3.5.1)$$

Let's repeat that macro transactions $CL(z=(\boldsymbol{x},\boldsymbol{y}))$ describe density of mean value of Credits transactions from all agents at point $\boldsymbol{x}$ to all agents at point $\boldsymbol{y}$. Impulses $\boldsymbol{P}=(\boldsymbol{P}_x,\boldsymbol{P}_y)$ describe flows of "*transaction fluids*" $CL(t,z=(\boldsymbol{x},\boldsymbol{y}))$ alike to flows of physical fluids with velocities $\boldsymbol{v}(t,z=(\boldsymbol{x},\boldsymbol{y}))=(\boldsymbol{v}_{\mathrm{x}}(t,z),\boldsymbol{v}_{\mathrm{y}}(t,z))$ on $2n$-dimensional e-space R^{2n}. Integral of Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ by variable $\boldsymbol{y}$ over e-space R^n defines rate of change all of Credits $C(t,\boldsymbol{x})$ from point $\boldsymbol{x}$ at moment t.

$$C(t,\boldsymbol{x}) = \int d\boldsymbol{y}\;\; CL(t,\boldsymbol{x},\boldsymbol{y}) \;\; ; \;\; L(t,\boldsymbol{y}) = \int d\boldsymbol{x}\;\; CL(t,\boldsymbol{x},\boldsymbol{y}) \quad (3.6.1)$$

Integral (3.6.1) also defines rate of change of all Loans $L(t,\boldsymbol{y})$ received at point $\boldsymbol{y}$. Integral of $CL(t,\boldsymbol{x},\boldsymbol{y})$ by variables $\boldsymbol{x}$ and $\boldsymbol{y}$ on e-space describes rate of change of total Credits $C(t)$ provided in economy and total Loans $L(t)$ received in economy at time t during time term dt:

$$C(t) = \int d\boldsymbol{x}\;\; C(t,\boldsymbol{x}) = \int d\boldsymbol{x}d\boldsymbol{y}\;\; CL(t,\boldsymbol{x},\boldsymbol{y}) = \int d\boldsymbol{y}\;\; L(t,\boldsymbol{y}) = L(t) \quad (3.6.2)$$

Relations (3.6.1; 3.6.2) show that Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ define evolution of Credits $C(t,\boldsymbol{x})$ provided from point $\boldsymbol{x}$ and total Credits $C(t)$ provided in economy at moment t and their *mutual* variables - Loans $L(t,\boldsymbol{y})$ received at point $\boldsymbol{y}$ and total Loans $L(t)$ received in macroeconomics at moment t.

Now let's introduce simple but important notion. As usual risk ratings are related with economic agents or particular Securities. Above we propose that it is possible estimate risk ratings of all agents of entire economics. For each macro variable let's define notion of mean risks. As example let us use macro Credits and Loans variables. Let's assume that e-particle 1 (Bank 1) with risk coordinate $\boldsymbol{x}$ at moment t issues Credits $C_1(t,\boldsymbol{x})$ and e-particle 2 (Bank 2) with risk coordinate $\boldsymbol{y}$ at moment t issues Credits $C_2(t,\boldsymbol{y})$. Coordinate $\boldsymbol{x}$ and $\boldsymbol{y}$ define risk ratings of Bank1 (e-particle1) and Bank 2 (e-particle 2). Let's state a question: What is risk rating – risk coordinate for group of both Banks? Two Banks issue Credits equal $C_1(t,\boldsymbol{x})+C_2(t,\boldsymbol{y})$. Let's define mean Credits risk coordinates $\boldsymbol{X}_{C1,2}(t)$ for two Banks as:

$$\boldsymbol{X}_{C1,2}(t) = \frac{\boldsymbol{x}C_1(t,\boldsymbol{x})+\boldsymbol{y}C_2(t,\boldsymbol{y})}{C_1(t,\boldsymbol{x})+C_2(t,\boldsymbol{y})} \quad or \quad \boldsymbol{X}_{C1,2}(t)\big(C_1(t,\boldsymbol{x}) + C_2(t,\boldsymbol{y})\big) = \boldsymbol{x}C_1(t,\boldsymbol{x}) + \boldsymbol{y}C_2(t,\boldsymbol{y}) \quad (3.7.1)$$

Above relations (3.7.1) define Credits mean risk coordinates as average of risk coordinates of agents weighted by value of Credits they issue at time t. Similar relations for Loans $L_1(t,\boldsymbol{x})$ and $L_2(t,\boldsymbol{y})$ received by e-particles 1 and 2 at points $\boldsymbol{x}$ and $\boldsymbol{y}$ define mean Loans risk $\boldsymbol{X}_{L1,2}(t)$ as:

$$\boldsymbol{X}_{L1,2}(t)\big(L_1(t,\boldsymbol{x}) + L_2(t,\boldsymbol{y})\big) = \boldsymbol{x}L_1(t,\boldsymbol{x}) + \boldsymbol{y}L_2(t,\boldsymbol{y}) \quad (3.7.2)$$

Thus different variables Credits $C(t,\boldsymbol{x})$ and Loans $L(t,\boldsymbol{x})$ determine different values of mean risk coordinates $\boldsymbol{X}_{C1,2}(t)$ and $\boldsymbol{X}_{L1,2}(t)$ respectively. Relations (3.7.1) are *alike to* center of Credits mass $\boldsymbol{X}_{C1,2}(t)$ of two physical particles with mass $C_1(t,\boldsymbol{x})$ at point $\boldsymbol{x}$ and mass $C_2(t,\boldsymbol{y})$ at point $\boldsymbol{y}$. For Credits $C(t,\boldsymbol{x})$ on e-space let's define Credits mean risk coordinates $\boldsymbol{X}_C(t)$ similar to relations (3.7.1) as integral over economic domain (1) taking into account total Credits $C(t)$ (3.6.2):

$$\boldsymbol{X}_C(t)C(t) = \int d\boldsymbol{x} \;\; \boldsymbol{x}\, C(t,\boldsymbol{x}) = \int d\boldsymbol{x}d\boldsymbol{y}\;\; \boldsymbol{x}\, CL(t,\boldsymbol{x},\boldsymbol{y}) \quad (3.7.3)$$

and mean Loan risk coordinates $\boldsymbol{X}_L(t)$ as

$$\boldsymbol{X}_L(t)L(t) = \int d\boldsymbol{y} \;\; \boldsymbol{y}\, L(t,\boldsymbol{y}) = \int d\boldsymbol{x}d\boldsymbol{y}\;\; \boldsymbol{y}\, CL(t,\boldsymbol{x},\boldsymbol{y}) \quad (3.7.4)$$

Mean Credits risk $\boldsymbol{X}_C(t)$ equals mean risk coordinates of total Credits $C(t)$ in economy. It is *alike to* center of mass coordinates $\boldsymbol{X}_C(t)$ of a body with total mass $C(t)$ and mass density $C(t,\boldsymbol{x})$. Mean risk $\boldsymbol{X}_L(t)$ defines mean Loans risk coordinates of total Loans $L(t)$ in economy. Let's repeat - mean Credit risk $\boldsymbol{X}_C(t)$ equals mean risk coordinates of e-particles averaged by Credits distribution $C(t,\boldsymbol{x})$. Mean Loans risk $\boldsymbol{X}_L(t)$ equals mean risk coordinates of e-particles averaged by Loans distribution $L(t,\boldsymbol{x})$. We underline that different economic variables - Investment $I(t,\boldsymbol{x})$, Assets $A(t,\boldsymbol{x})$ and etc. define different values of their mean risks. Let's remind that all variables are determined by corresponding economic transactions due to relations (3.6.1). Credits transactions mean risk of $CL(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ define mean risk of mutual variables for $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ as:

$$\{\boldsymbol{X}_C(t)C(t)\ ;\ L(t)\boldsymbol{X}_L(t)\} = \{\int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{x}\,CL(t,\boldsymbol{x},\boldsymbol{y})\ ;\ \int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{y}\,CL(t,\boldsymbol{x},\boldsymbol{y})\} \quad (3.7.5)$$

Relations (3.7.5) show that macro transactions like Credits transactions $CL(t,\boldsymbol{x},\boldsymbol{y})$ determine evolution of Credits mean risks $\boldsymbol{X}_C(t)$ and Loans mean risks $\boldsymbol{X}_L(t)$. The same statement is correct for mean risks determined by other macro transactions.

Why we attract attention to definition of mean risks? We propose that evolutions of mean risks for different macro variables describe ground for business cycle fluctuations of these variables. Let's take Credits $C(t,\boldsymbol{x})$ as example. Mean Credits risk $\boldsymbol{X}_C(t)$ is not a constant. It changes due to change of coordinates $\boldsymbol{x}$ and amount of Credits provided by e-particles (agents). Growth of risks of e-particles can increase and decline of risks can decrees mean Credits risk $\boldsymbol{X}_C(t)$. E-particles (economic agents) fill economic domain (1). Risk ratings of e-particles on economic domain (1) are bounded by minimum or most secure and maximum or most risky grades. Thus mean Credits risk $\boldsymbol{X}_C(t)$ as well as mean risks of any macro variable can't grow up or diminish steadily along each risk axes as their values are bounded on economic domain (1). Values of mean risks and value of Credits mean risk $\boldsymbol{X}_C(t)$ in particular along each risk axes should oscillate from certain minimum to maximum values and these fluctuations can be very complex.

We propose that business cycles correspond to fluctuations of mean risks of macro variables. Growth of mean Credits risk $\boldsymbol{X}_C(t)$ can correspond with growth of total Credits $C(t)$ provided in economy and decline of Credits mean risk can correspond with total Credits contraction. Cause, reason for mean risk change can be exogenous or endogenous. Risk change can be induced by technology shocks, political or regulatory decisions and etc. Reasons can be different but outcome should be the same – business cycles are governed by change of mean risks. Relations between mean Credits risk $\boldsymbol{X}_C(t)$ and value of total Credits $C(t)$ are much more complex but we repeat main statement: business cycles can be treated as fluctuations of mean risks for different macro variables.

As we show in (3.7.5) Credits macro transaction $CL(t,\boldsymbol{x},\boldsymbol{y})$ determine mean Credits $\boldsymbol{X}_C(t)$ and Loans $\boldsymbol{X}_L(t)$ risks. Below in Sec. 3, Sec.4 and in Appendix we describe model dynamics of Credits transaction $CL(t,\boldsymbol{x},\boldsymbol{y})$ on e-space by economic equations (5.1.1-5.1.3; 5.2; 5.3). Starting with these equations we derive the system of ODE (A.4; A.8.4-7; A.9.6-7) that describe business cycle fluctuations of macro Credits $C(t)$ provided in economy and total macro Loans $L(t)$ received in economy as consequences of fluctuations of mean Credits and Loans risks $\boldsymbol{X}_C(t)$ and $\boldsymbol{X}_L(t)$. Due to (3.6.1) total value of Credits $MC(t,\boldsymbol{x})$ provided from point $\boldsymbol{x}$ up to moment t equal:

$$\frac{\partial}{\partial t}MC(t,\boldsymbol{x}) = C(t,\boldsymbol{x}) \;\; ; \;\; MC(t,\boldsymbol{x}) = MC(0,\boldsymbol{x}) + \int_0^t d\tau \int d\boldsymbol{y} \;\, CL(\tau,\boldsymbol{x},\boldsymbol{y}) \tag{3.8}$$

Total value of Loans *ML(t,**y**)* received at point ***y*** up to moment *t*

$$\frac{\partial}{\partial t}ML(t,\boldsymbol{x}) = L(t,\boldsymbol{x}) \;\; ; \;\; ML(t,\boldsymbol{y}) = ML(0,\boldsymbol{y}) + \int_0^t d\tau \int d\boldsymbol{x} \;\, CL(\tau,\boldsymbol{x},\boldsymbol{y}) \tag{3.9}$$

Here *MC(0,**x**)* define initial values of Credits issued from point ***x*** on e-space. Relations that are similar to (3.6.1 - 3.9) define evolutions and fluctuations of all extensive economic and financial variables determined by macro transactions. Aggregate macro Credits *MC(t)* issued in entire economics equal (see 3.6.2; 3.8):

$$MC(t) = MC(0) + \int_0^t d\tau \int d\boldsymbol{x}d\boldsymbol{y} \;\, CL(\tau,\boldsymbol{x},\boldsymbol{y}) = \; MC(0) + \int_0^t d\tau \;\, C(\tau) \tag{3.10}$$

Thus to describe Business or Credit cycle fluctuations of *MC(t)* one should describe rate of change of total Credits *C(t)* and Credits transactions *CL(t,**x**,**y**)* (3.11)*:*

$$\frac{d}{dt}MC(t) = C(t) = \int d\boldsymbol{x}d\boldsymbol{y} \;\, CL(t,\boldsymbol{x},\boldsymbol{y}) \tag{3.11}$$

Oscillations of rate of change of Credits *C(t)* define business cycle fluctuations of macro Credits *MC(t).* Relations (3.1-3.11) establish basis for modeling business cycle fluctuations of economic and financial variables via description of macro transaction. Below we derive economic equations to describe evolution of Credit *"transaction fluid" CL(t,**x**,**y**).*

3. Economic equations on macro transactions

Macro transactions between points ***x*** and ***y*** on e-space determine evolution of macro variables (3.6.1 – 3.11). As example let's use Credits transactions to explain factors that cause change of macro "transaction fluids" (Olkhov, 2017b; 2017c). Value of Credits transactions *CL(t, **z**=(**x**,**y**))* (3.1) in a unit volume *dV* at point ***z**=(**x**,**y**)* can change due to two factors. First factor describes change of *CL(t,**z**)* in time as $\partial CL/\partial t$. Second factor describes change of *CL(t,**z**)* in a unit volume *dV* due to flux of transactions flow ***υ**CL* through surface of a unit volume. Divergence theorem (Strauss 2008, p.179) states that surface integral of flux ***υ**CL* through surface of a unit volume equals volume integral of divergence ***υ**CL*. Thus total change of transaction *CL(t,**z**)* in a unit volume *dV* equals

$$\frac{\partial CL}{\partial t} + \nabla \cdot (\boldsymbol{v}CL)$$

Here $\boldsymbol{\upsilon}=(\boldsymbol{\upsilon}_X,\boldsymbol{\upsilon}_Y)$ – velocity of transaction *CL(t,**z**=(**x**,**y**))* on *2n*-dimension e-space R^{2n} determined by (3.4-3.5), bold letters ***x, y,*** *z*, ***P, Q**₂* mean vectors, roman *t, CL* mean scalars and divergence equals:

$$\nabla \cdot (\boldsymbol{v}CL) = \sum\nolimits_{i=1,\dots n} \frac{\partial}{\partial x_i}\big(v_{xi}(t,\boldsymbol{x},\boldsymbol{y})CL(t,\boldsymbol{x},\boldsymbol{y})\big) + \sum\nolimits_{i=1,\dots n} \frac{\partial}{\partial y_i}\Big(v_{yi}(t,\boldsymbol{x},\boldsymbol{y})CL(t,\boldsymbol{x},\boldsymbol{y})\Big)$$

Change of transactions $CL(t,z)$ can be induced by action of different factors and we denote them as Q_1. Then equation on Credits transactions $CL(t,z=(\boldsymbol{x},\boldsymbol{y}))$ takes form:

$$\frac{\partial CL}{\partial t} + \nabla \cdot (\boldsymbol{v}CL) = Q_1 \qquad (4.1)$$

Equation (4.1) is a simple balance of factors that change $CL(t,z)$. Left side (4.1) describes how $CL(t,z)$ changes in a unit volume – due to change in time and due to flux through surface of a unit volume. Right side describes action of other factors. The same reasons define equation on transactions impulses $\boldsymbol{P}(t,z)=(\boldsymbol{P}_x(t,z)\ \boldsymbol{P}_y(t,z))$ determined by (3.2-3.3) as:

$$\frac{\partial \boldsymbol{P}}{\partial t} + \nabla \cdot (\boldsymbol{vP}) = \boldsymbol{Q}_2 \qquad (4.2)$$

Thus left side of (4.2) describes change of transaction impulses $\boldsymbol{P}(t,z)=(\boldsymbol{P}_x(t,z), \boldsymbol{P}_y(t,z))$ due to change in time $\partial \boldsymbol{P}/\partial t$ and due to flux $\boldsymbol{vP}$ through surface of unit volume that equal divergence $\nabla \cdot (\boldsymbol{vP})$. Right hand side $\boldsymbol{Q}_2$ describes action of other factors on evolution of transaction impulses $\boldsymbol{P}(t,z)$. Economic equations (4.1; 4.2) present a balance relations between changes of transactions $CL(t,z)$ and their impulses $\boldsymbol{P}(t,z)$ in the left side and action of other factors that can induce these changes in the right side.

To describe a particular economic model via equations (4.1; 4.2) let's determine direct form of right hand side Q_1 and $\boldsymbol{Q}_2$. Macro transactions $CL(t,z)$ and their impulses $\boldsymbol{P}(t,z)$ can depend on other transactions and on other economic factors like *expectations*, for example. In this paper we present the business cycle model in the approximation that takes into account interactions between different transactions only and neglects impact *expectations* and other economic factors. We shall study impact of *expectations* in forthcoming publications. Here we propose that all extensive macro variables are determined by macro transactions or depend on variables that are described by macro transactions.

Equations (4.1; 4.2) allow describe evolution of transactions under action of Q_1 and $\boldsymbol{Q}_2$ for two economic approximations. First economic approximation describes transactions and their mutual extensive variables under given exogenous impact determined by Q_1 and $\boldsymbol{Q}_2$. In other words one studies evolution of transactions under given action of exogenous factors Q_1 and $\boldsymbol{Q}_2$. The second approximation permits describe self-consistent evolution of transactions under their mutual interaction due to equations (4.1;4.2). Real economic and financial transactions depend on numerous factors and that makes description extremely complex. We propose to start with the simplest case that describes model mutual interactions between two transactions. For such a case evolution of transaction 1 is defined by left side of (4.1; 4.2) and is described by Q_1 and $\boldsymbol{Q}_2$ factors determined by transaction 2 and vice versa. Such approximation gives simples self-consistent model of mutual evolution of two interacting transactions and allows describe the business cycle model related to fluctuations of macro variables determined by these transactions. Below we study self-consistent model that describe mutual interaction between Credits *CL(t,z)* and Loan-Repayment *LR(t,z)* transactions. As consequences we describe the business cycle time fluctuations of macro Credits *C(t)* and macro Loans *L(t)*.

Let's study simplest case and assume that Credits transactions *CL(t,z)* in the left side of (4.1;4.2) depend on Q_1 and $\boldsymbol{Q}_2$ that determined by Loan-Repayment *LR(t,z)* transactions. Loan-Repayment *LR(t,z)* transactions describe payout on Credits by Borrowers from point $\boldsymbol{y}$ to Creditors at point $\boldsymbol{x}$. Let's describe evolution of Loan-Repayment *LR(t,z)* transactions by left side of equations similar to (4.1;4.2) with Q_1 and $\boldsymbol{Q}_2$ determined by Credits transactions *CL(t,z)*. We propose that Credits from point $\boldsymbol{x}$ to point $\boldsymbol{y}$ are provided at time *t* due to Loan-Repayments received at same time *t* and vice versa. Such assumptions simplify mutual dependence between Credits transactions *CL(t,z)* and Loan-Repayment *LR(t,z)* and allow describe the business cycle fluctuations of macro Credits *C(t)* issued at time *t*.

4 How macro transactions describe business cycles

In (Olkhov, 2017d-e) we proposed that agents perform only *local* economic or financial transactions with agents at same point $\boldsymbol{x}$. Such simplifications describe interactions between macro variables at point $\boldsymbol{x}$ by *local* operators. In this paper we model transactions that can occur between agents at arbitrary points $\boldsymbol{x}$ and $\boldsymbol{y}$. Such transactions describe *non-local* economic and financial "*action-at-a-distance*" between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ on e-space R^n. Below we describe the business cycle fluctuations determined by *non-local* Credit *CL(t,z)* and Loan-Repayment *LR(t,z)* transactions. Let's assume that *CL(t,z)* at point $z=(\boldsymbol{x},\boldsymbol{y})$ on e-space R^{2n} depend on Loan-Repayment *LR(t,z)* transactions and their impulses $\boldsymbol{L}(t,z)$ only and vice versa. Let's assume that Q_{11} for Continuity Equation (4.1) on macro transactions *CL(t,z)* at point *(t,z)* is proportional to scalar product of vector z and Loan-Repayment impulse $\boldsymbol{D}(t,z)$

$$Q_{11} = a\,\boldsymbol{z}\cdot\boldsymbol{D}(t,\boldsymbol{z}) = a(\,\boldsymbol{x}\cdot\boldsymbol{D}_{\boldsymbol{x}}(t,\boldsymbol{z}) + \boldsymbol{y}\cdot\boldsymbol{D}_{\boldsymbol{y}}(t,\boldsymbol{z}))$$

Loan-Repayment impulse $\boldsymbol{D}(t,z)$ and velocity $\boldsymbol{u}(t,z)$ are determined similar to (3.1-3.5.1). Let's assume that same relations define factor Q_{12} for Continuity Equation (4.1) on Loan-Repayment *LR(t,z)* macro transactions:

$$Q_{12} = b\,\boldsymbol{z}\cdot\boldsymbol{P}(t,\boldsymbol{z}) = b(\boldsymbol{x}\cdot\boldsymbol{P}_{\boldsymbol{x}}(t,\boldsymbol{z}) + \boldsymbol{y}\cdot\boldsymbol{P}_{\boldsymbol{y}}(t,\boldsymbol{z}))$$

Here *a* and *b* – const and Continuity Equations on transactions *CL(t,z)* and *LR(t,z)* take form:

$$\frac{\partial CL}{\partial t} + \nabla \cdot (\boldsymbol{v}CL) = Q_{11} = \ a\,\boldsymbol{z} \cdot \boldsymbol{D}(t,\boldsymbol{z}) = a\,(\boldsymbol{x} \cdot \boldsymbol{D}_x(t,\boldsymbol{z}) + \ \boldsymbol{y} \cdot \boldsymbol{D}_y(t,\boldsymbol{z})\,) \tag{5.1.1}$$

$$\frac{\partial LR}{\partial t} + \nabla \cdot (\boldsymbol{u}LR) = Q_{12} = \ b\,\boldsymbol{z} \cdot \boldsymbol{P}(t,\boldsymbol{z}) = \ b\,\left(\boldsymbol{x} \cdot \boldsymbol{P}_x(t,\boldsymbol{z}) + \ \boldsymbol{y} \cdot \boldsymbol{P}_y(t,\boldsymbol{z})\right) \tag{5.1.2}$$

$$\boldsymbol{P}(t,\boldsymbol{z}) = \ \boldsymbol{v}(t,\boldsymbol{z})CL(t,\boldsymbol{z}) \ \ ; \ \ \boldsymbol{D}(t,\boldsymbol{z}) = \boldsymbol{u}(t,\boldsymbol{z})LR(t,\boldsymbol{z}) \tag{5.1.3}$$

Economic meaning of (5.1.1-5.1.3) is as follows. *CL(t,z)* at point (*t,z*) grows up if Q_{11} is positive. A position vector *z* has origin at secure point *0* and points to risky point *z*. Hence for *a>0* positive value of $\boldsymbol{z} \cdot \boldsymbol{D}(t,\boldsymbol{x})$ models Loan-Repayment flow

$$\boldsymbol{D}(t,\boldsymbol{x}) = LR(t,\boldsymbol{x})\boldsymbol{u}(t,\boldsymbol{x})$$

in risky direction *z* and that can induce growth of Credits *CL(t,z)* to risky points. As well negative value of $\boldsymbol{z} \cdot \boldsymbol{D}(t,\boldsymbol{x})$ models Loan-Repayment flows from risky to secure domain and that can decrease Credits *CL(t,z)* as Creditors can prefer more secure Borrowers. This model simplifies Credit modeling as it neglect time gaps between providing Credits from $\boldsymbol{x}$ to $\boldsymbol{y}$ and Loan-Repayment received from Borrowers at $\boldsymbol{y}$ to Creditors at $\boldsymbol{x}$ and neglect other factors that can impact on providing Credits. To determine $\boldsymbol{Q_{21}}$ factor for (4.2) on Credit impulses ***P(t,z)*** let's assume that $\boldsymbol{Q_{21}}$ is a linear operator and in a matrix form takes form:

$$\boldsymbol{Q}_{21} = \ \widehat{\Omega}\boldsymbol{D}(t,\boldsymbol{z}) = \ \Omega_{ij}D_j(t,\boldsymbol{z})$$

Let's assume that $\boldsymbol{Q_{22}}$ factor that define Equations of Motion (4.2) on Loan-Repayment impulses ***L(t,z)*** is similar linear operator:

$$\boldsymbol{Q}_{22} = \ \widehat{\Phi}\boldsymbol{P}(t,\boldsymbol{z}) = \ \Phi_{ij}P_j(t,\boldsymbol{z})$$

and Equations of Motion for impulses ***P(t,z)*** and ***L(t,z)*** take form:

$$\frac{\partial \boldsymbol{P}}{\partial t} + \nabla \cdot (\boldsymbol{v}\,\boldsymbol{P}) = \boldsymbol{Q}_{21} = \Omega\boldsymbol{D}(t,\boldsymbol{z}) = \Omega_{ij}D_j(t,\boldsymbol{z}) = \ \Omega_{xij}D_{xj}(t,\boldsymbol{z}) + \Omega_{yij}D_{yj}(t,\boldsymbol{z}) \tag{5.2}$$

$$\frac{\partial \boldsymbol{D}}{\partial t} + \nabla \cdot (\boldsymbol{u}\,\boldsymbol{D}) = \boldsymbol{Q}_{22} = \Phi\boldsymbol{P}(t,\boldsymbol{z}) = \Phi_{ij}P_j(t,\boldsymbol{z}) = \Phi_{xij}P_{xj}(t,\boldsymbol{z}) + \Phi_{yij}P_{yj}(t,\boldsymbol{z}) \tag{5.3}$$

Equations (5.2-5.3) describe simple linear mutual dependence between transaction impulses ***P(t,z)*** and ***D(t,z)***. Economic meaning of equations (5.2; 5.3) can be explained as follows. Let's mention that integral of each component of impulses ***P(t,z)*** or its components $P_{xi}(t,z)$ and $P_{yi}(t,z)$ along axes x_i or y_i over *dz* define total macro impulses ***P(t)*** and its components $P_{xi}(t)$ or $P_{yi}(t)$ along risk axis x_i or y_i and due to (3.4; 3.5; A.6.3.1; A.6.3.2):

$$P_{xi}(t) = \int d\boldsymbol{z}\ P_{xi}(t,\boldsymbol{z} = (\boldsymbol{x},\boldsymbol{y})) = \int d\boldsymbol{x}d\boldsymbol{y}\ P_{xi}(t,\boldsymbol{x},\boldsymbol{y}) = C(t)v_{xi}(t) \tag{5.3.1}$$

$$\boldsymbol{P}(t) = \left(\boldsymbol{P}_C(t); \boldsymbol{P}_B(t)\right); \ \ \boldsymbol{P}_C(t) = \boldsymbol{P}_x(t)\,; \ \boldsymbol{P}_B(t) = \boldsymbol{P}_y(t) \tag{5.3.2}$$

Total impulses ***P(t)*** (5.3.2) have component of Creditors impulses $\boldsymbol{P}_C(t)=\boldsymbol{P}_x(t)$ along axes $\boldsymbol{x}$ and component $\boldsymbol{P}_B(t)=\boldsymbol{P}_y(t)$ of Borrowers impulses along axes ***y.*** Due to (A.4.2) total impulses (5.3.1) describe motion of macro Credits *C(t)* on e-space that describe change of mean Credits risk $X_{Ci}(t)$ (3.7.5) along each risk axes x_i. Motion of macro Credits *C(t)* on e-space is reduced by bounds of economic domain (1) along each risk axes. Thus motion of macro Credits C(t) in the risky direction should change with motion from risky to secure direction on economic domain (1). Thus total Credits impulses ***P(t)*** should fluctuate. Fluctuations of impulses ***P(t)*** describe motion of macro Credits *C(t)* from secure to risky domain and then from risky to secure. We regard business cycle fluctuations of macro variables as oscillations of their mean risks induced by corresponding fluctuations of their macro impulses. As we show below equations (5.2; 5.3) lead to equations (A.6.4-6.8) that describe fluctuations of total impulses ***P(t)*** and cause simple fluctuations of total Credits impulses ***P(t)***. For convenience we repeat definitions of macro Credits *C(t)*, Loan-Repayment *LR(t)* and impulses ***P(t)*** and ***D(t)***:

$$C(t) = \int dxdy\ CL(t,\boldsymbol{x},\boldsymbol{y}) \ \ ; \ \ LR(t) = \int dxdy\ LR(t,\boldsymbol{x},\boldsymbol{y}) \tag{5.4.1}$$

$$\boldsymbol{P}(t) = \int dxdy\ \boldsymbol{P}(t,\boldsymbol{x},\boldsymbol{y}) = \int dxdy\, CL(t,\boldsymbol{x},\boldsymbol{y})\, \boldsymbol{v}(t,\boldsymbol{x},\boldsymbol{y}) = C(t)\, \boldsymbol{v}(t) \tag{5.4.2}$$

$$\boldsymbol{D}(t) = \int dxdy\ \boldsymbol{D}(t,\boldsymbol{x},\boldsymbol{y}) = \int dxdy\, LR(t,\boldsymbol{x},\boldsymbol{y})\, \boldsymbol{u}(t,\boldsymbol{x},\boldsymbol{y}) = LR(t)\, \boldsymbol{u}(t) \tag{5.4.3}$$

To describe the business cycle fluctuations of macro Credits we start with system of equations (5.1.1-5.1.3) and equations (5.2; 5.3) on Credit *CL(t,z)* and Loan-Repayment *LR(t,z)* transactions and their impulses ***P(t,z)*** and ***D(t,z)***. From these equations we derive the system of ODE (Appendix: A.4; A.8.4-7; A.9.6-7) on aggregate variables *C(t), LR(t)* and present elementary solutions (A.10) for the business cycle fluctuations of macro variables under action of a single risk. The simplest case of business cycle fluctuations of total Credits *C(t)* under action of a single risk can be derived from (A.11) with *C(j)=const*, *j=0,1,2,3*:

$$C(t) = C(0) + a\,[C(1)\, sin\, \omega t + C(2)\, cos\, \nu t + C(3)\, exp\, \gamma t] \tag{6.1}$$

Due to (3.10; 6.1) macroeconomic Credits *MC(t)* provided in economy during time term [0,t]:

$$MC(t) = MC(0) + \left[C(0)t + a\frac{C(3)}{\gamma} exp\, \gamma t\right] + a\, [\frac{C(2)}{\nu} \sin \nu t - \frac{C(1)}{\omega} \cos \omega t\,] \tag{6.2}$$

Relations (6.1; 6.2) describe the business cycle fluctuations of total Credits *C(t).* Frequencies of business cycle fluctuations are determined by oscillations of Creditors impulses $P_x(t)$ with frequencies ω and oscillations of Borrowers impulses $P_y(t)$ with frequencies ν (Appendix, A.6.6-10; A.8.4-7; A.9.6-7). Business cycle fluctuations (6.1; 6.2) may happen about exponential growth trend *exp(γt)* (Appendix, A.9.5-7) and we take coefficient $\gamma = \max(\gamma_x, \gamma_y)$. Thus γ describes maximum growth trend induced by (A.8.6-7; A.9.1-2; A.10.1-2). Factors (A.8.6) are proportional to product of total Credits *C(t)* and square of transactions velocity $v^2(t)$ and we call them Credits "energy" because they *looks like* kinetic energy of a body with mass *C(t)* and square of velocity $v^2(t)$. However meaning of Credits "energy" have nothing common with meaning of energy in physics as no conservation laws are valid for this variable.

Macro Credits *MC(t)* during time term *[0,t]* are described by (6.2). If the initial value *C(0)* is not zero then macro Credits *MC(t)* has linear and exponential growth trend and oscillations with same frequencies ω and ν about these trends. Solutions (6.1) for Credits transactions *C(t)* and for Loan-Repayment transactions *LR(t)* present simplest form of Credit cycle fluctuations under the action of a single risk and simple interactions between two macro transactions (Appendix). Action of several risks can make Credit and Business cycle fluctuations more complex (A.11). If one neglect growth trend then business cycle fluctuations of Credits $C(t)$ under action of *n* risks can take form (A.11):

$$C(t) = C(0) + a\sum_{i=1}^{n}[\, C_{xi}(1)\sin\omega_i t + C_{xi}(2)\cos\omega_i t + \ C_{yi}(3)\sin\nu_i t + C_{yi}(4)\cos\nu_i t] \quad (6.3)$$

Relations (6.3) with frequencies ω_i reflect oscillations of Credit impulses ***P****(t)* along axes x_i, and frequencies ν_i along axes y_i, $i=1,..n$ on *2n* dimensional e-space $(\boldsymbol{x},\boldsymbol{y})$ (Appendix)

5. Conclusions

The business cycle fluctuations are extremely complex and their behavior is under permanent evolution due to development of entire economy. It is impossible establish single, precise, exact description of such alive phenomena and each model of business cycles should be based on definite assumptions and approximations. Occam's razor (Baker, 2007) principle states that the less initial assumptions are made by model - the better. We develop the business cycle model without assumptions of general equilibrium - no assumptions on state and evolution of markets, prices, etc. We describe macroeconomics and the business cycle on base of econometric observations and risk assessments. We propose that econometrics provide sufficient data for risk assessments of all agents of entire economics and use agent's ratings as their coordinates on economic space. Assessment of two or three risks defines agents risk coordinates on economic space with dimension 2 or 3. Risk coordinates distribute economic agents over points of economic space. All extensive economic or financial variables are defined as sum of corresponding variables of agents. Economic and financial transactions between agents are the only cause of evolution of agent's variables. Description of transactions between agents takes into account granularity of agents on economic space. To simplify economic model we move from description of transactions between agents to description of transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. That *looks like* transition from kinetics that takes into account granularity of physical particles to hydrodynamics that describes systems as continuous media or physical fluids and neglect granularity of physical particles (Landau, Lifshitz, 1981; 1987). We underline vital distinctions between economic and physical processes and remind that we use only *analogies* between economics and physics and don't apply direct physical results or equations to economic modeling. We aggregate transactions between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ and describe evolution of macro transactions by economic equations (4.1-4.2). Left side factors of (4.1-4.2) describe change of Credits transaction $CL(t,z)$ in time and due to flux through surface of a unit volume. Right side factors describe action of other factors on $CL(t,z)$. Motion of "*transaction fluids*" is determined by average collective velocity of agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively and variations of corresponding transactions between agents (3.2-3.5). Velocity of agents on economic space define change of risk ratings during time term dt. Reasons for risk change can be different. Risk change can be induced by endogenous or exogenous shocks, by technology or regulatory decisions or whatever. We don't discus here reasons for risk rating change. We describe consequences of risk coordinate evolution and show how they impact the business cycle. Agents of entire economics fill economic domain (1) on economic space that is bounded by most secure and most risky grades (1; A.1). Motion of "*transaction fluid*" causes change of corresponding mean risk $\boldsymbol{X}(t)$. For example motion of total Credits $C(t)$ is described by Credit impulse $\boldsymbol{P}_x(t)$ and causes motion of Credits mean risk $\boldsymbol{X}_C(t)$ (A.4.2). Motion of Credits mean risk $\boldsymbol{X}_C(t)$ can't go on steadily in one direction, as it will reach secure or risky boundaries of economic domain (1). Thus Credits mean risk $\boldsymbol{X}_C(t)$ should fluctuate and that should be accompanied by business cycle fluctuations of macro Credits $C(t)$. We propose that fluctuations of Credit mean risks $\boldsymbol{X}_C(t)$ reflect Credits cycle fluctuations.

To show benefits of our approach we present a simple model interactions between Credit $CL(t,z)$ and Loan-Repayment transactions $LR(t,z)$. We study model interactions between these transactions and derive system of economic equations (5.1.1-5.1.3; 5.2-5.3) in explicit and a self-consistent form. Starting with these economic equations we derive the system of ODE that describes business cycle time fluctuations (A.4; A.8.4-7; A.10.1-2). For simplest case of the business cycle fluctuations under action of single risk we present solutions for macro Credits $C(t)$ and $MC(t)$ (6.1; 6.2) We outline that system of ODE (A.4; A.8.4-7; A.10.1-2) contain equations for economic factors (A.8.6-8.7; A.9.1-9.2; A.10.1-10.2) that *looks like* kinetic energy. For example factors $ECx_i(t)$ and $ECy_i(t)$ (A.8.6) are proportional to product of total Credits $C(t)$ and square of velocity υ^2_{xi} and υ^2_{yi} along risk axes x_i or y_i and that is *looks like* kinetic energy of body with Credits mass $C(t)$ and square of velocity υ^2. Nevertheless these parallels have no further development it is very interesting that description of Credit cycle fluctuations requires equations (A.9.1-9.2) on factors (A.8.8-8.9) that are *alike to* Credits "*energy*".

Our approach has certain parallels to Leontief (1973) input-output analysis as he based his macroeconomic model on description of transactions between different industries. Meanwhile, breakdown of economics by Sectors and Industries does not define any metric space. Our model describe transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ of metric economic space. This "small" alterity permit define macro variables and macro transactions as functions of time and coordinates $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. It uncovers hidden complexity of macroeconomic processes and for sure requires usage of *likenesses and analogies* with mathematical physics methods and equations. Unfortunately, in our view, direct applications of mathematical physics methods have not enough economic meaning.

Comparison of our model with observed business cycles requires a lot of econometric data that could specify risk ratings of economic agents, their economic and financial variables, economic and financial transactions between agents. Lack of sufficient econometric data prevent comparisons of theoretical predictions of our business cycle model with econometric observations. Econometric assessment of our theory requires development of risk assessment methodology that allows estimate risk ratings for continuous risk grades. Usage of economic modeling on economic space requires methods that can estimate influence of particular risk on economic evolution and selection of n major risks that form representation of economic space. We propose that no principal obstacles can prevent development of econometrics in a way sufficient for modeling business cycles on economic space. We propose that our theory can help financial authorities and Central Banks, business communities and academic researchers to forecast and manage business cycles.

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Appendix

Economic Transactions and The Business Cycle Equations

Let's study transactions between agents on n-dimensional e-space R^n. We use standard notations: bold letters like ***P, v, x, y, z*** define vectors and roman *C, CL, X,…* - scalars. Vector $z=(\boldsymbol{x},\boldsymbol{y})$ is defined on $2n$-dimensional e-space R^{2n}. Scalar product:

$$\boldsymbol{z}\cdot\boldsymbol{P}=\boldsymbol{x}\cdot\boldsymbol{P_x}+\boldsymbol{y}\cdot\boldsymbol{P_y}=\sum\nolimits_{\boldsymbol{i=1,..n}} x_i\,P_{xi}+\sum\nolimits_{\boldsymbol{i=1,..n}} y_i\,P_{yi}$$

Divergence equals:

$$\nabla\cdot(\boldsymbol{v}f)=\nabla_x\cdot(\boldsymbol{v_x}f)+\nabla_y\cdot(\boldsymbol{v_y}f)=\sum\nolimits_{\boldsymbol{i=1,..n}}\frac{\partial}{\partial x_i}(v_{xi}f)+\sum\nolimits_{\boldsymbol{i=1,..n}}\frac{\partial}{\partial y_i}(v_{yi}f)$$

$$\nabla\cdot(\boldsymbol{vP})=\Big(\nabla\cdot(\boldsymbol{vP_x})\,;\nabla\cdot(\boldsymbol{v}P_y)\Big)=\Big(\nabla\cdot(\boldsymbol{v}P_{xj})\,;\nabla\cdot(\boldsymbol{v}P_{yj})\Big)\;;j=1,..n$$

Integral notations:

$$\int d\boldsymbol{z}=\int d\boldsymbol{x}\,d\boldsymbol{y}=\int dx_1\ldots dx_n dy_1\ldots dy_n$$

To derive a system of ODE on speed of total Credit *C(t)* and Loan-repayment *LR(t)* change let's start with equations (5.1.1*)*. Thus Credits transactions $CL(t,z=(\boldsymbol{x},\boldsymbol{y}))$ are determined on $2n$-dimensional e-space and economic domain (1) define $2n$-dimensional economic area $z=(\boldsymbol{x},\boldsymbol{y})$*:*

$$0\le x_i\;\le X_i\,;\;0\le y_i\;\le X_i\;i=1,\ldots n \qquad \text{(A.1)}$$

Let's remind that similar to (1) values of X_i can be set as $X_i=1$. To derive equations on *C(t)* (5.4.1) let's take integral by $dz=d\boldsymbol{x}d\boldsymbol{y}$ of equation (5.1.1):

$$\frac{d}{dt}C(t)=\frac{d}{dt}\int d\boldsymbol{z}\,CL(t,\boldsymbol{z})=-\int d\boldsymbol{z}\;\nabla\cdot\big(\boldsymbol{v}(t,\boldsymbol{z})CL(t,\boldsymbol{z})\big)+a\int d\boldsymbol{z}\;\boldsymbol{z}\cdot\boldsymbol{D}(t,\boldsymbol{z}) \qquad \text{(A.2.1)}$$

First integral in the right side (A.2.1) equals integral of divergence over $2n$ dimensional e-space and due to divergence theorem (Strauss 2008, p.179) equals integral of flux $\boldsymbol{v}CL$ through surface. Thus it equals zero as no economic or financial fluxes exist far from boundaries of economic domain (A.1).

$$\int d\boldsymbol{z}\;\nabla\cdot\big(\boldsymbol{v}(t,\boldsymbol{z})CL(t,\boldsymbol{z})\big)=0 \qquad \text{(A.2.2)}$$

Let's define *Pz(t)* and *Dz(t)* as*:*

$$Pz(t)=\int d\boldsymbol{z}\;\boldsymbol{P}(t,\boldsymbol{z})\cdot\boldsymbol{z}=\int d\boldsymbol{x}d\boldsymbol{y}\;\sum\nolimits_{i=1}^{n}x_iP_{xi}(t,\boldsymbol{x},\boldsymbol{y})+\int d\boldsymbol{x}d\boldsymbol{y}\sum\nolimits_{i=1}^{n}y_iP_{yi}(t,\boldsymbol{x},\boldsymbol{y}) \qquad \text{(A.3.1)}$$

$$Dz(t)=\int d\boldsymbol{z}\;\boldsymbol{D}(t,\boldsymbol{z})\cdot\boldsymbol{z}=\int d\boldsymbol{x}d\boldsymbol{y}\;\sum\nolimits_{i=1}^{n}x_iD_{xi}(t,\boldsymbol{x},\boldsymbol{y})+\int d\boldsymbol{x}d\boldsymbol{y}\sum\nolimits_{i=1}^{n}y_iD_{yi}(t,\boldsymbol{x},\boldsymbol{y}) \qquad \text{(A.3.2)}$$

Due to (5.1.1; 5.1.2; 5.4.1; A.2.1) equations on *C(t)* and *LR(t)* take form:

$$\frac{d}{dt}C(t)=a\,Dz(t)\quad;\quad\frac{d}{dt}LR(t)=b\,Pz(t) \qquad \text{(A.4)}$$

Equation (5.1.1) permits derive equation on Credits mean risk $\boldsymbol{X}_C(t)$ and Loans mean risk $\boldsymbol{X}_L(t)$ (3.7.3 - 3.7.5). Let's multiply (5.1.1) by z and take integral by $dz=d\boldsymbol{x}d\boldsymbol{y}$

$$\frac{d}{dt}\int d\boldsymbol{z}\, CL(t,\boldsymbol{z})\boldsymbol{z} = -\int d\boldsymbol{z}\;\; \boldsymbol{z}\,\nabla\cdot\big(\boldsymbol{v}(t,\boldsymbol{z})CL(t,\boldsymbol{z})\big) + a\int d\boldsymbol{z}\,\boldsymbol{z}\,(\boldsymbol{z}\cdot\boldsymbol{D}(t,\boldsymbol{z})) \qquad \text{(A.4.1)}$$

We refer (Olkhov, 2017d) for derivation of complete equations on mean risk. From (A.4.1) one can obtain:

$$\frac{d}{dt}C(t)\boldsymbol{X}_C(t) = \boldsymbol{P}_{\boldsymbol{x}}(t) + a\,(\boldsymbol{X}Dx(t) + \boldsymbol{X}Dy(t)) \qquad \text{(A.4.2)}$$

$$\frac{d}{dt}L(t)\boldsymbol{X}_L(t) = \boldsymbol{P}_{\boldsymbol{y}}(t) + a\,(\boldsymbol{Y}Dx(t) + \boldsymbol{Y}Dy(t)) \qquad \text{(A.4.3)}$$

$$\boldsymbol{X}Dx(t) = \int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{x}\,\big(\boldsymbol{x}\cdot\boldsymbol{D}_{\boldsymbol{x}}(t,\boldsymbol{z})\big) \;\;;\;\; \boldsymbol{X}Dy(t) = \int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{x}\,\big(\boldsymbol{y}\cdot\boldsymbol{D}_{\boldsymbol{y}}(t,\boldsymbol{z})\big)$$

$$\boldsymbol{Y}Dx(t) = \int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{y}\,\big(\boldsymbol{x}\cdot\boldsymbol{D}_{\boldsymbol{x}}(t,\boldsymbol{z})\big) \;\;;\;\; \boldsymbol{Y}Dy(t) = \int d\boldsymbol{x}d\boldsymbol{y}\,\boldsymbol{y}\,\big(\boldsymbol{y}\cdot\boldsymbol{D}_{\boldsymbol{y}}(t,\boldsymbol{z})\big)$$

Equations on factors *XDx(t), XDy(t), XDx(t), XDy(t)* can be derived similar to (Olkhov, 2017d) and for brevity we omit it here. In the absence of any interaction for $a=0$ equations (A.4.2; A.4.3) show that dynamics of $C(t)\boldsymbol{X}_C(t)$ and $L(t)\boldsymbol{X}_L(t)$ depends on $\boldsymbol{P}_x(t)$ and $\boldsymbol{P}_y(t)$

$$\frac{d}{dt}C(t)\boldsymbol{X}_C(t) = \boldsymbol{P}_{\boldsymbol{x}}(t) = C(t)\boldsymbol{v}_{\boldsymbol{x}}(t)\,;\; \frac{d}{dt}L(t)\boldsymbol{X}_L(t) = \boldsymbol{P}_{\boldsymbol{y}}(t) = L(t)\boldsymbol{v}_{\boldsymbol{y}}(t) \qquad \text{(A.4.4)}$$

Thus equations (A.6.6-6.8) that describe fluctuations of impulses $\boldsymbol{P}_x(t)$ and $\boldsymbol{P}_y(t)$ cause fluctuations of $C(t)\boldsymbol{X}_C(t)$ and $L(t)\boldsymbol{X}_L(t)$. Interactions between transactions (A.4.2; A.4.3) for $a\neq 0$ make these fluctuations much more complex. To avoid excess complexity here we don't derive complete system of ODE on $C(t)\boldsymbol{X}_C(t)$ and $L(t)\boldsymbol{X}_L(t)$.

To derive equations on *Pz(t)* and *Dz(t)* let's use equations on impulses $\boldsymbol{P}(t)$, $\boldsymbol{D}(t)$. Let's start with (5.3; 5.4). To simplify derivation of equations let's take matrix operators in equations (5.3; 5.4) in simplest diagonal form *(i=1,..n)*:

$$\Phi_{ij} = (\Phi_{xij};\Phi_{yij});\;\; \Phi_{ij}P_j = \big(\Phi_{xij}P_{xj};\Phi_{yij}P_{jy}\big) \qquad \text{(A.5.1)}$$

$$\Omega_{ij} = (\Omega_{xij};\,\Omega_{yij});\;\; \Omega_{ij}D_j = \big(\Omega_{xij}D_{xj};\Omega_{yij}D_{jy}\big) \qquad \text{(A.5.2)}$$

$$\Phi_{xij} = d_{xi}\delta_{ij} \;\;;\;\; \Phi_{yij} = d_{yi}\delta_{ij} \qquad \text{(A.5.3)}$$

$$\Omega_{xij} = c_{xi}\delta_{ij} \;\;;\;\; \Omega_{xij} = c_{xi}\delta_{ij} \qquad \text{(A.5.4)}$$

$$\Phi_{xij}P_{jx}(t,\boldsymbol{z}) = d_{xi}\delta_{ij}P_{xj}(t,\boldsymbol{z}) = d_{xi}P_{xi}(t,\boldsymbol{z}) \;;\; \Phi_{xij}P_{jx}(t,\boldsymbol{z}) = d_{yi}P_{yi}(t,\boldsymbol{z}) \qquad \text{(A.5.5)}$$

$$\Omega_{xij}D_{xj}(t,\boldsymbol{z}) = c_{xi}\delta_{ij}D_{xj}(t,\boldsymbol{z}) = c_{xi}D_{xi}(t,\boldsymbol{z}) \;;\; \Omega_{yij}D_{yj}(t,\boldsymbol{z}) = c_{yi}D_{yi}(t,\boldsymbol{z}) \qquad \text{(A.5.6)}$$

Thus equations (5.3; 5.4) take form *(i=1,..n):*

$$\frac{\partial P_{xi}}{\partial t} + \nabla\cdot(\boldsymbol{v}\,P_{xi}) = c_{xi}D_{xi}(t,\boldsymbol{z}) \;\;;\; \frac{\partial P_{yi}}{\partial t} + \nabla\cdot\big(\boldsymbol{v}\,P_{yi}\big) = c_{yi}D_{yi}(t,\boldsymbol{z}) \qquad \text{(A.6.1)}$$

$$\frac{\partial D_{xi}}{\partial t} + \nabla\cdot(\boldsymbol{u}\,D_{xi}) = d_{xi}P_{xi}(t,\boldsymbol{z}) \;\;; \frac{\partial D_{iy}}{\partial t} + \nabla\cdot\big(\boldsymbol{u}\,D_{yi}\big) = d_{yi}P_{yi}(t,\boldsymbol{z}) \qquad \text{(A.6.2)}$$

To derive equations on aggregate impulses $\boldsymbol{P}(t)$ and $\boldsymbol{D}(t)$ (5.4.2; 5.4.3) and their components P_{xi} , P_{yi} , D_{xi} , D_{yi} let's take integral by $d\boldsymbol{z}=d\boldsymbol{x}d\boldsymbol{y}$ of equation (A.5.3):

$$\frac{d}{dt}P_{xi}(t) = \frac{d}{dt}\int d\boldsymbol{z}\, P_{xi}(t,\boldsymbol{z}) = -\int d\boldsymbol{z}\, \nabla\cdot(\boldsymbol{v}\, P_{xi}) + c_{xi}\int d\boldsymbol{z}\, D_{xi}(t,\boldsymbol{z}) \qquad \text{(A.6.3)}$$

Due to relations (3.4;3.5) and similar relations concern impulses D_{xi} , D_{yi} obtain

$$P_{xi}(t) = \int d\boldsymbol{z}\, P_{xi}(t,\boldsymbol{z}) = C(t)v_{xi}(t);\ P_{yi}(t) = \int d\boldsymbol{z}\, P_{yi}(t,\boldsymbol{z}) = C(t)v_{yi}(t) \qquad \text{(A.6.3.1)}$$

$$D_{xi}(t) = \int d\boldsymbol{z}\, D_{xi}(t,\boldsymbol{z}) = LR(t)u_{xi}(t);\ D_{yi}(t) = \int d\boldsymbol{z}\, D_{yi}(t,\boldsymbol{z}) = LR(t)u_{yi}(t) \qquad \text{(A.6.3.2)}$$

Due to same reasons as (A.2.1) first integral in the right side (A.6.3) equals zero and equations (A.6.1; A.6.2) takes form ($i=1,..n$):

$$\frac{d}{dt}P_{xi}(t) = c_{xi}D_{xi}(t)\ \ ;\ \ \frac{d}{dt}D_{xi}(t) = d_{xi}P_{xi}(t) \qquad \text{(A.6.4)}$$

$$\frac{d}{dt}P_{yi}(t) = c_{yi}D_{yi}(t)\ \ ;\ \ \frac{d}{dt}D_{yi}(t) = d_{yi}P_{yi}(t) \qquad \text{(A.6.5)}$$

Due to (A.1) impulses $\boldsymbol{P}(t) = (P_{xi}(t),\ P_{yi}(t))$, $\boldsymbol{D}(t)=(D_{xi}(t),\ D_{yi}(t))$ (5.1.3) along each risk axes can't keep definite sign as in such a case they will reach max or min borders (A.1). Thus impulses along each axes must fluctuate and equations (A.6.4-6.5) and (A.6.6-6.8) describe simplest harmonique oscillations of impulses $\boldsymbol{P}(t)$ and $\boldsymbol{D}(t)$ (5.1.3) with frequencies ω_i, ν_i :

$$\omega_i^2 = -c_{xi}d_{xi} > 0\ ;\ \ \nu_i^2 = -c_{yi}d_{yi} > 0\ \ ;\ i = 1,..n \qquad \text{(A.6.6)}$$

$$\left[\frac{d^2}{dt^2} + \omega_i^2\right]P_{xi}(t) = 0\ ;\ \left[\frac{d^2}{dt^2} + \omega_i^2\right]D_{xi}(t) = 0 \qquad \text{(A.6.7)}$$

$$\left[\frac{d^2}{dt^2} + \nu_i^2\right]P_{yi}(t) = 0\ ;\ \left[\frac{d^2}{dt^2} + \nu_i^2\right]D_{yi}(t) = 0 \qquad \text{(A.6.8)}$$

Equations (A.6.6-A.6.8) describe simple harmonique oscillations of impulses $P_{xi}(t)$, $P_{yi}(t)$, $D_{xi}(t)$, $D_{yi}(t)$ along each risk axes with different frequencies ω_i, ν_i for $i=1,..n$. Frequencies ω_i, $i=1,..n$ describe possible oscillations related to fluctuations of transactions from Creditors along coordinates $\boldsymbol{x}=(x_1,..x_n)$. Frequencies ν_i, $i=1,..n$ describe oscillations due to Borrowers along coordinates $\boldsymbol{y}=(y_1,..y_n)$. Solutions of (A.6.7-8) have form:

$$P_{xi}(t) = P_{xi}(1)\sin\omega_i t + P_{xi}(2)\cos\omega_i t\, ; P_{yi}(t) = P_{yi}(1)\sin\nu_i t + P_{yi}(2)\cos\nu_i t \qquad \text{(A.6.9)}$$

$$D_{xi}(t) = D_{xi}(1)\sin\omega_i t + D_{xi}(2)\cos\omega_i t\, ; D_{yi}(t) = D_{yi}(1)\sin\nu_i t + D_{yi}(2)\cos\nu_i t \qquad \text{(A.6.10)}$$

Thus motions of Creditors and Borrowers on e-space induce oscillations (A.6.9-10) of macro transactions impulses with different frequencies ω_i and ν_i along risk axes x_i or y_i. To derive equations on $Pz(t)$ and $Dz(t)$ determined by (A.3.1;A.3.2) let's define their components $Pz_{xi}(t);Pz_{yi}(t);\ Dz_{xi}(t);Dz_{yi}(t)$ as:

$$Pz_{xi}(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, x_i P_{xi}(t,\boldsymbol{x},\boldsymbol{y})\ ;\ Pz_{yi}(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, y_i P_{yi}(t,\boldsymbol{x},\boldsymbol{y}) \qquad \text{(A.7.1)}$$

$$Dz_{xi}(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, x_i D_{xi}(t,\boldsymbol{x},\boldsymbol{y})\ ;\ Dz_{yi}(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, y_i D_{yi}(t,\boldsymbol{x},\boldsymbol{y}) \qquad \text{(A.7.2)}$$

Relations (A.3.1;A.3.2) can be presented as:

$$Pz(t) = \sum_{i=1}^{n} Pz_{xi}(t) + \sum_{i=1}^{n} Pz_{yi}(t) \tag{A.7.3}$$

$$Dz(t) = \sum_{i=1}^{n} Dz_{xi}(t) + \sum_{i=1}^{n} Dz_{yi}(t) \tag{A.7.4}$$

To define equations on $Pz_{xi}(t)$, $Pz_{yi}(t)$, $Dz_{xi}(t)$, $Dz_{yi}(t)$ use equations (A.6.1 ; A.6.2). Let's multiply equations (A.6.1) by x_i and take integral by $d\boldsymbol{x}d\boldsymbol{y}$

$$\frac{d}{dt}Pz_{xi}(t) = \frac{d}{dt}\int d\boldsymbol{x}d\boldsymbol{y}\, x_i P_{xi}(t,\boldsymbol{x},\boldsymbol{y}) = -\int d\boldsymbol{x}d\boldsymbol{y}\, x_i \nabla\cdot(\boldsymbol{v}\,P_{xi}) + c_{xi}\int d\boldsymbol{x}d\boldsymbol{y}\, x_i D_{xi}(t,\boldsymbol{z})$$

$$\int d\boldsymbol{x}d\boldsymbol{y}\, x_i \nabla\cdot(\boldsymbol{v}\,P_{xi})$$

$$= \int dx_{k\neq i} d\boldsymbol{y} \int dx_i\, x_i \frac{\partial}{\partial x_i}(v_{xi}P_{xi}) + \int dx_i\, x_i \int dx_{k\neq i} d\boldsymbol{y}\, \frac{\partial}{\partial x_{k\neq i}}(v_{xk\neq i}P_{xi})$$

Second integral equals zero due to same reasons as (A.2.1). Let's take first integral by parts:

$$\int dx_i\ x_i \frac{\partial}{\partial x_i}(v_{xi}P_{xi}) = \int dx_i \frac{\partial}{\partial x_i}(x_i v_{xi}P_{xi}) - \int dx_i\, v_{xi}P_{xi}$$

First integral in the right side equals zero and we obtain:

$$\int d\boldsymbol{x}d\boldsymbol{y}\, x_i \nabla\cdot(\boldsymbol{v}\,P_{xi}) = -\int d\boldsymbol{x}d\boldsymbol{y}\ v_i P_{xi} = -\int d\boldsymbol{x}d\boldsymbol{y}\ v_i^2(t,\boldsymbol{x},\boldsymbol{y})CL(t,\boldsymbol{x},\boldsymbol{y}) \tag{A8.1}$$

Let's denote as

$$ECx_i(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, v_{xi}^2(t,\boldsymbol{x},\boldsymbol{y})CL(t,\boldsymbol{x},\boldsymbol{y});\ ECy_i(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, v_{yi}^2(t,\boldsymbol{x},\boldsymbol{y})CL(t,\boldsymbol{x},\boldsymbol{y}) \tag{A.8.2}$$

$$ERx_i(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, u_{xi}^2(t,\boldsymbol{x},\boldsymbol{y})LR(t,\boldsymbol{x},\boldsymbol{y});\ ERy_i(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, u_{yi}^2(t,\boldsymbol{x},\boldsymbol{y})LR(t,\boldsymbol{x},\boldsymbol{y}) \tag{A.8.3}$$

Thus equations on $Pz_{xi}(t)$, $Pz_{yi}(t)$, $Dz_{xi}(t)$, $Dz_{yi}(t)$ take form:

$$\frac{d}{dt}Pz_{xi}(t) = ECx_i(t) + c_{xi}Dz_{xi}(t)\ ;\ \ \frac{d}{dt}Dz_{xi}(t) = ERx_i(t) + d_{xi}Pz_{xi}(t)$$

$$\frac{d}{dt}Pz_{yi}(t) = ECy_i(t) + c_{yi}Dz_{yi}(t)\ ;\ \ \frac{d}{dt}Dz_{yi}(t) = ERy_i(t) + d_{yi}Pz_{yi}(t)$$

Due to relations (A.6.6) above equations on $Pz_{xi}(t)$, $Pz_{yi}(t)$, $Dz_{xi}(t)$, $Dz_{yi}(t)$ can be presented as:

$$\left[\frac{d^2}{dt^2} + \omega_i^2\right]Pz_{xi}(t) = \frac{d}{dt}ECx_i(t) + c_{xi}ERx_i(t) \tag{A.8.4}$$

$$\left[\frac{d^2}{dt^2} + \omega_i^2\right]Dz_{xi}(t) = \frac{d}{dt}ERx_i(t) + d_{xi}ECx_i(t) \tag{A.8.5}$$

$$\left[\frac{d^2}{dt^2} + \nu_i^2\right]Pz_{yi}(t) = \frac{d}{dt}ECy_i(t) + c_{yi}ERy_i(t) \tag{A.8.6}$$

$$\left[\frac{d^2}{dt^2} + \nu_i^2\right]Dz_{yi}(t) = \frac{d}{dt}ERy_i(t) + d_{yi}ECy_i(t) \tag{A.8.7}$$

Equations (A.8.4-8.7) describe fluctuations of on $Pz_{xi}(t)$, $Pz_{yi}(t)$, $Dz_{xi}(t)$, $Dz_{yi}(t)$ with frequencies ω_i, ν_i under action of right-hand side factors ECx_i, ECy_i ERx_i, ERy_i (see below (A.9.1-10.2)). To close system of ODE (A.4; A.8.4-7) let's derive equations on $ECx_i(t)$, $ECy_i(t)$, $ERx_i(t)$, $ERy_i(t)$. Let's outline that relations (A.8.2-8.3; A.8.8-8.9) are proportional to product of Credits *C(t)* and squares of velocities $v^2(t)$ are thus are *alike to* of energy of Credits flow *C(t)* with velocity $\boldsymbol{v}$.

$$EC(t) = C(t)v^2(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, v^{\mathbf{2}}(t,\boldsymbol{x},\boldsymbol{y})CL(t,\boldsymbol{x},\boldsymbol{y}) = \sum_{i=1}^{n} ECx_i(t) + ECy_i(t) \quad \text{(A.8.8)}$$

$$ER(t) = LR(t)u^2(t) = \int d\boldsymbol{x}d\boldsymbol{y}\, u^{\mathbf{2}}(t,\boldsymbol{x},\boldsymbol{y})LR(t,\boldsymbol{x},\boldsymbol{y}) = \sum_{i=1}^{n} ERx_i(t) + ERy_i(t) \quad \text{(A.8.9)}$$

Let's regard *ECx$_i$(t)* and *ECy$_i$(t)* as components of *EC(t)* along each axes x_i and y_i. Let's repeat that relations $EC(t) = C(t)v^2(t)$ (A.8.8 – 8.9) are *alike to* kinetic energy of particle with mass *C(t)* and square velocity $v^2(t)$ and for convenience let's further call *EC(t)* and *ER(t)* as "energies" of corresponding flows. These similarities have no further analogies as no conservation laws on factors *EB(t)* and *ER(t)* exist. Equations on *ECx$_i$(t,z)* and *ECy$_i$(t,z)* take form similar to (4.1):

$$\frac{\partial}{\partial t}ECx_i(t,\boldsymbol{z}) + \nabla\cdot(\boldsymbol{v}\, ECx_i) = QECx_i \;\; ; \;\; \frac{\partial}{\partial t}ECy_i(t,\boldsymbol{z}) + \nabla\cdot(\boldsymbol{v}\, ECy_i) = QECy_i \quad \text{(A.9.1)}$$

$$\frac{\partial}{\partial t}ERx_i(t,\boldsymbol{z}) + \nabla\cdot(\boldsymbol{u}\, ERx_i) = QERx_i \;\; ; \;\; \frac{\partial}{\partial t}ERy_i(t,\boldsymbol{z}) + \nabla\cdot(\boldsymbol{u}\, ERy_i) = QERy_i \quad \text{(A.9.2)}$$

Let's propose that factors *QECx$_i$* take form of diagonal matrix as:

$$QECx_i = \mathrm{M}_{xij}ERx_j = \mu_{xi}\, ERx_i\, ; \;\; \mathrm{M}_{xij} = \mu_{xi}\delta_{ij} \quad \text{(A.9.3)}$$

$$QECy_i = \mathrm{M}_{yij}ERy_j = \mu_{yi}\, ERy_i\, ; \;\; \mathrm{M}_{yij} = \mu_{yi}\delta_{ij}\mu_{yi} \quad \text{(A.9.4)}$$

$$QERx_i = \mathrm{N}_{xij}ECx_j = \eta_{xi}ECx_i\, ; \; \mathrm{N}_{xij} = \eta_{xi}\delta_{ij} \quad \text{(F.9.5)}$$

$$QERy_i = \mathrm{N}_{yij}ECy_j = \eta_{yi}ECy_i\, ; \; \mathrm{N}_{yij} = \eta_{yi}\delta_{ij} \quad \text{(A.9.6)}$$

$$\gamma_{xi}^2 = \mu_{xi}\eta_{xi} > 0 \;\; ; \;\; \gamma_{yi}^2 = \mu_{yi}\eta_{yi} > 0 \quad \text{(A.9.7)}$$

Similar to derivation of equations on impulses *P$_{xi}$(t), P$_{yi}$(t), D$_{xi}$(t), D$_{yi}$(t)* (A.6.4-A.6.8) equations (A.9.1-7) give equations on *ECx$_i$(t), ECy$_i$(t), ERx$_i$(t), ERy$_i$(t)*:

$$\left[\frac{d^2}{dt^2} - \gamma_{xi}^2\right]EC_{xi}(t) = 0 \;\; ; \;\; \left[\frac{d^2}{dt^2} - \gamma_{xi}^2\right]ER_{xi}(t) = 0 \quad \text{(A.10.1)}$$

$$\left[\frac{d^2}{dt^2} - \gamma_{yi}^2\right]EC_{yi}(t) = 0 \;\; ; \;\; \left[\frac{d^2}{dt^2} - \gamma_{yi}^2\right]ER_{yi}(t) = 0 \quad \text{(A.10.2)}$$

Economic meaning of (A.9.1-A.9.7) is as follows: "energies" *ECx$_i$(t), ECy$_i$(t), ERx$_i$(t), ERy$_i$(t)* grow up or decay in time by exponent *exp(γ_{xi} t)* and *exp(γ_{yi} t)* that can be different for each risk axis *i=1,..n*. Here γ_{xi} define exponential growth or decay in time of *ECx$_i$(t)* induced by motion of Creditors along axes x_i and γ_{yi} and same time describe exponential growth or decrease in time of *ECy$_i$(t)* induced by motion of Borrowers along axes y_i. The same valid for *ERx$_i$(t), ERy$_i$(t)* respectively. Equations (A.4; A.8.4-7; A.10.1-2) describe a closed system of ODE that models time evolution of aggregate variables *C(t), LR(t), Pz$_{xi}$(t), Pz$_{yi}$(t), Dz$_{xi}$(t), Dz$_{yi}$(t), ECx$_i$(t), ECy$_i$(t), ERx$_i$(t), ERy$_i$(t)* and solutions (A.4; A.8.4-7; A.10.1-2) have form:

$$EC_{xi}(t) = EC_{xi}(1)\, exp\, \gamma_{xi}t + EC_{xi}(2)\, exp\, {-\gamma_{xi}t}$$

$$EC_{yi}(t) = EC_{yi}(1)\, exp\, \gamma_{yi}t + EC_{yi}(2)\, exp\, -\gamma_{yi}t$$

$$ER_{xi}(t) = ER_{xi}(1)\, exp\, \gamma_{xi}t + ER_{xi}(2)\, exp\, -\gamma_{xi}t$$

$$ER_{yi}(t) = ER_{yi}(1)\, exp\, \gamma_{yi}t + ER_{yi}(2)\, exp\, -\gamma_{yi}t$$

$$Pz_{xi}(t) = Pz_{xi}(1) \sin \omega_i t + Pz_{xi}(2) \cos \omega_i t + Pz_{xi}(3)\, exp\, \gamma_{xi}t + Pz_{xi}(4)\, exp\, -\gamma_{xi}t$$

$$Pz_{yi}(t) = Pz_{yi}(1) \sin \nu_i t + Pz_{yi}(2) \cos \nu_i t + Pz_{yi}(3)\, exp\, \gamma_{yi}t + Pz_{yi}(4)\, exp\, -\gamma_{yi}t$$

$$Dz_{xi}(t) = Dz_{xi}(1) \sin \omega_i t + Dz_{xi}(2) \cos \omega_i t + Dz_{xi}(3)\, exp\, \gamma_{xi}t + Dz_{xi}(4)\, exp\, -\gamma_{xi}t$$

$$Dz_{yi}(t) = Dz_{yi}(1) \sin \nu_i t + Dz_{yi}(2) \cos \nu_i t + Dz_{yi}(3)\, exp\, \gamma_{yi}t + Dz_{yi}(4)\, exp\, -\gamma_{yi}t$$

Total Credits *C(t)* as solution of (A.4; A.7.4) have form:

$$C(t) = C(0) \; + a \sum_{i=1}^{n} [\, C_{xi}(1) \sin \omega_i t + C_{xi}(2) \cos \omega_i t + C_{yi}(3) \sin \nu_i t + C_{yi}(4) \cos \nu_i t] +$$
$$a \sum_{i=1}^{n} [C_{xi}(5)\;\, exp\, \gamma_{xi}t + C_{xi}(6)\, exp\, -\gamma_{xi}t + C_{yi}(7)\, exp\, \gamma_{yi}t + C_{yi}(8)\, exp\, -\gamma_{yi}t] \qquad \text{(A.11)}$$

Simple but long relations define constants $C_{xi}(j)$, $C_{yi}(j)$, $j=0,..8$ that are determined by initial values and equations (A.4; A.8.4-7; A.10.1-2) and we omit them here. Similar relations are valid for total rate of Loan-Repayment *LR(t)* (5.4.1). Solutions (A.10) allow obtain simple relations on macro Credits *MC(t)* (3.10; 3.11).

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