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Abstract

Sparked by Baumol's revenue- versus profit-maximizing models of the firm, this paper using a growth model shows that if a nation seeks GDP-maximizing growth with capital expansion as driving force, the model could work only under the assumption that the consumers' aversion to under-consumption, an unavoidable consequence of over-investment, remains constant, and the effects of under-consumption are not accumulative. Otherwise, it has to decelerate growth and ultimately converges to the neoclassical growth model with consumption optimality. The empirical evidence on growth models of ex-Soviet Union, China and Eastern Asia are examined to explore the extent to which the model captures the real world.

JEL codes: E20, O40, O50.

Key words: GDP-maximizing growth; under-consumption effects; transition between steady states; sustainability; Chinese model.

1. Introduction

Baumol (1959) has put forward the original idea of a firm that maximizes revenue instead of profit. While profit maximization is the goal of the owners, the separation between ownership and control within the modern firm gives managers a large amount of discretional power to deviate from it, because the sale is the prevailing indicator of the competitive position within the industry, and the increase in revenue is a sign of managerial success and most managers' remunerations depend on sales.

Logically this idea can be extended to think about a nation's economic growth. In the neoclassical tradition, economic growth is analyzed in terms of consumption optimality.

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Consumption, via a utility function, is the sole objective to maximize, while capital is just treated as a means to generate consumption goods. One of the major neoclassical models, the Ramsey (1928) model, provides for an endogenous determination of the savings rate. It was further developed by Cass (1965) and Koopmans (1965), and became known as the Ramsey-Cass-Koopmans (RCK) model.

The government of a nation, or, in a conventional appellation, the social planner can maximize consumption just like a firm maximizing profit. It may also maximize GDP just as a firm maximizing revenue. In the absence of foreign exchange, GDP consists of capital and consumption. GDP maximization seemingly attributes equal importance to capital and consumption. But since capital is the sole generator of GDP and consumption is, from the point of view of forming capital, a cost to minimize, capital formation through investment takes a leading place. Hence, GDP maximization is in essence capital maximization constrained by a certain level of consumption.

But what could motivate the social planer to maximize GDP instead of consumption? GDP growth is a generally recognized indicator of governance performance. This can reinforce the legitimacy of an autocratic government. Even a democratic government can search for the goal of GDP maximization since its election depends on employment, which is linked with GDP growth. If following the Public choice view, the representative government tends to become a revenue-maximizing Leviathan (Brennan and Buchanan 1980), and the senior bureaucrats seek to maximize the budget and the output of the bureau (Niskanen 1971). GDP maximization is a deviation from the consumers' goal of consumption optimality.

Dealing with this issue is not just an intellectual exercise, but has meaningful empirical implication. This will be shown after the theoretical analysis.

The objective of this study is to build a GDP-maximizing model (GDP max model) to answer two questions: 1) if the social planer maximizes GDP, what would be the steady-state GDP, consumption, and capital formation relative to a neoclassical RCK model? 2) under what condition is the model sustainable? For the comparative sake, the GDP max model is made tractable and as close as possible in structure to a RCK model. Except those qualifying the specificities in the GDP max model, for both models the variables are the same, the basic structures of the equations are comparable and all parameters are set to the same values. In this way, all results can be derived in terms of a comparison with the latter, just like the need in the profit-maximizing model, as a benchmark for a firm's revenue-maximizing model.

The remainder of this study is organized as follows. Section 2 builds the model. Section 3 analyzes the model sustainability, adjustments and resulting adjusted transition trajectory. Section 4 investigates its applications. Section 5 concludes.

2. The Model

2.1.Outline of the RCK Model

In what follows, the equations noted as A (e.g., 1A) are from the RCK model; those noted as B (e.g., 1B) are their equivalents in the GDP max model. To distinguish the variables in the two models, capital and investment are labeled as k and $\dot{k}$ in the GDP max model; their equivalents in the RCK model are labeled as $\bar{k}$ and $\dot{\bar{k}}$. Finally, all steady-state values are labeled * (e.g., k^* and $\bar{k}^*$).

Assume an economy with population = L and without population growth. In the simplest RCK model, the social planner maximizes a time-discounted U($\bar{c}$) =log($\bar{c}$), the utility function of a representative family over a period T, subject to:

$$\dot{\bar{k}} = \bar{k}^{\alpha} - \bar{c} - \delta\bar{k} \tag{1A}$$

where $\bar{c}$ is consumption, $\bar{k}$ is capital and $\bar{q} = \bar{k}^{\alpha}$ is production with a Cobb-Douglas production function. All of these variables are in terms of per capita. δ is the capital discount rate. With the utility function (1A) and a current-value Hamiltonian, another key equation of motion is derived:

$$\dot{\bar{c}} = [\alpha\bar{k}^{\alpha-1} - (\delta + \rho)]\bar{c} \tag{2A}$$

where ρ is the time preference rate. The procedure to derive equation (2A) (Barro and Sala-i-Martin 2003, A.3.8) is omitted for briefness.

The steady-state values of $\bar{c}$ and $\bar{k}$ are found by setting equations (1A) and (2A) = 0. Equation (2A) ensures that, at the steady state, the marginal product of $\bar{k}^*$ amounts to $\delta + \rho$. It follows that:

$$\bar{k}^* = (\frac{\alpha}{\delta+\rho})^{\frac{1}{1-\alpha}} \tag{3A}$$

$$\bar{c}^* = (\frac{\alpha}{\delta+\rho})^{\frac{\alpha}{1-\alpha}} - \delta(\frac{\alpha}{\delta+\rho})^{\frac{1}{1-\alpha}} \tag{4A}$$

2.2.Structure of the Equations

Now the broad structure of the GDP max model is spelled out. The social planner maximizes a time discounted utility function U(q) over a period T:

$$q = k^{\alpha} - (1 - \frac{c}{\bar{c}})^{\frac{1}{\sigma}}\bar{c} \qquad (0 < \sigma \leq 1; (1 - \frac{c}{\bar{c}})^{\frac{1}{\sigma}} = 0 \; if \; c > \bar{c}) \qquad (5)$$

where q , the production per capita, is, like in the RCK model, a simplified version of GDP per capita. In what follows, the notation gdp in the model means GDP per capita, i.e., q≡gdp.

In this production function, in addition to k^{α}, a constraint, which can be called under-consumption constraint, is put. Some justification is needed. If the social planner is able to deviate from consumption optimality, the economy is no longer entirely governed by the market, because if applying the marginal pricing principle, the social planner has not any possibility to maximize GDP. While in an RCK model, the choice between maximizing current consumption and sacrificing it for future consumption is a natural trade-off, in a GDP max model, the social planner, with this discretional power, has unlimited desire to expand capital and minimize consumption since the unique driving force acting on GDP growth is investment. Some trade-off must be made. On the base of the real-world observations, our approach brings into the picture the negative under-consumption effects in terms of losses in GDP, to influence the social planner's calculations.

The term $(1 - {}^{c}/_{\bar{c}})^{\frac{1}{\sigma}}\bar{c}$ is set to measure the loss in GDP due to the under-consumption effects. c and $\bar{c}$ are consumptions in the GDP max and RCK models, respectively. Under-consumption is defined by $c/\bar{c}$. The higher the $c/\bar{c}$, the lower is the under-consumption level. The total effect is scaled by $\bar{c}$. σ is a parameter that scales up the loss in GDP attributable to the under-consumption effects. It can be interpreted as the degree of aversion of under-consumption. With the same level of under-consumption, a larger σ leads to a higher loss in GDP. Concretely, the social planner uses both models to obtain the steady-state under-consumption level and other key steady-state values for comparison.

In our sense, as GDP maximization implies an over-expansion of capital (from the criterion of consumption optimality) at the cost of consumption, the inescapable consequence is under-consumption.[2] Under-consumption mainly has two categories of effects. The first is the disincentive effect. The aversion to under-consumption results in a direct production loss.[3] A second category of the effects of under-consumption is the physiological effects due to under-consumption. The deficiencies in adequate health care reduce the effective labor force for production. The costs due to health and environmental degradation are equivalent to the loss of GDP.[4]

The values of $c/\bar{c}$ are needed to be specified into two categories: $c/\bar{c} < 1$ and $c/\bar{c} \geq 1$. $c/\bar{c} < 1$ is the normal case in which the trade-off is effective for the social planner to choose an optimal value of c. $c/\bar{c} \geq 1$ is an extreme case in which under-consumption and the tradeoff disappear. It can be argued that, to the extent that the marginal loss in GDP of one unit of forgone c, due to under-consumption, is lower than that of the marginal contribution to GDP via increasing k by this unit of forgone c (as will be shown, this is the condition for the GDP max model to maintain), the social planner always has an interest to cut c to be less than $\bar{c}$. Whenever the social planner maximizes GDP and has a choice in c, c could not be higher than $\bar{c}$, because that would be a contradiction with the logic of GDP maximization. For this reason, we must specify equation (5) as:

$$q = k^{\alpha} - (1 - \tfrac{c}{\bar{c}})^{\frac{1}{\sigma}}\bar{c} \qquad (0 < \sigma \leq 1;\ c < \bar{c}) \tag{5'}$$

With the production function defined in equation (5'), the equation of motion for investment is:

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[2] Besides the general under-consumption as a natural consequence of capital expansion, some consumption in the form of basic public goods, especially those for health care, environment improvement and poverty reduction, of which the importance has been increasing in a modern economy, are under-provided. More illustrations are provided in the section on empirical evidence.

[3] In this respect, it will be enough to recall the role played by loafing on the job and absenteeism in the breakdown of the economy of the ex-Soviet Union, in which the shortage of consumption goods was phenomenal (Filtzer 1996).

[4] The World Bank (2013, 249) estimated that in 2008, China's environmental degradation and resource depletion were valued at approximately 9% of GDP, over ten times higher than the corresponding levels in South Korea and Japan. In this sense, the real GDP of China could be estimated as 9% lower if assuming that these damages must be repaired.

$$\dot{k} = k^{\alpha} - (1 - \frac{c}{\bar{c}})^{\frac{1}{\sigma}}\bar{c} - c - \delta k \tag{1B}$$

As such, the social planner is facing a trade-off between under-consumption and the pro-investment effects of the control variable c.

For the sake of tractability, the objective function is set in the simplest form: U(gdp)=gdp. The current-value Hamiltonian for GDP maximization is:

$$H = k^{\alpha} - \left(1 - \frac{c}{\bar{c}}\right)^{\frac{1}{\sigma}} \bar{c} + m[k^{\alpha} - \left(1 - \frac{c}{\bar{c}}\right)^{\frac{1}{\sigma}} \bar{c} - c - \delta k] \tag{6}$$

where $m = \lambda e^{\rho t}$ is the shadow price λ in the current value.

The first-order condition with respect to c is:

$$\frac{\partial H}{\partial c} = \frac{1}{\sigma}(1 - \frac{c}{\bar{c}})^{\frac{1-\sigma}{\sigma}} + m\left[\frac{1}{\sigma}\left(1 - \frac{c}{\bar{c}}\right)^{\frac{1-\sigma}{\sigma}} - 1\right] = 0 \tag{7}$$

hence:

$$m = B/(1 - B) \tag{8}$$

where:

$$B = \frac{1}{\sigma}(1 - \frac{c}{\bar{c}})^{\frac{1-\sigma}{\sigma}} > 0 \text{ and } \neq 1 \tag{9}$$

$B = \frac{\partial q}{\partial c}$ is the induced marginal loss of GDP by forgoing one unit of c, due to under-consumption. Both $B \neq 0$ and $m \neq 0$ come from equation (5’), in which $c < \bar{c}$. B=1 is assumed to be not permissible; otherwise equation (8) will be undefined. B has central importance in our analysis and will be repetitively resorted to.

Using equation (7) and differentiating m with respect to t:

$$\dot{m} = -\frac{(\frac{1-\sigma}{\sigma})B\dot{c}}{(1-B)^2(\bar{c}-c)} \tag{10}$$

Another first-order condition with respect to k is:

$$\frac{\partial H}{\partial k} = \alpha k^{\alpha-1} - m(\delta + \rho) = -\dot{m} \tag{11}$$

Putting equations (8) and (10) into (11) to replace m and $\dot{m}$, and rearranging, an equation of motion for c comparable to that in the RCK model (2A) is obtained as:

$$\dot{c}[\frac{\left(\frac{1-\sigma}{\sigma}\right)B}{(1-B)^2(\bar{c}-c)}] = [\alpha k^{\alpha-1} - \frac{B}{1-B}(\delta + \rho)] \tag{2B}$$

2.3.The Steady State

Setting $\dot{c}$ and $\dot{k}$=0, the steady-state values of k^* and c^* are, respectively:

$$k^* = (\frac{1-B^*}{B^*})^{\frac{1}{1-\alpha}}\bar{k}^* \tag{3B}$$

$$c^* = (\frac{1-B^*}{B^*})^{\frac{\alpha}{1-\alpha}}\bar{k}^{*\alpha} - \left(\frac{1-B^*}{B^*}\right)^{\frac{1}{1-\alpha}}\delta\bar{k}^* - (1-\frac{c^*}{\bar{c}^*})^{\frac{1}{\sigma}}\bar{c}^* \tag{4B}$$

where:

$$B^* = \frac{1}{\sigma}(1-\frac{c^*}{\bar{c}^*})^{\frac{1-\sigma}{\sigma}} > 0 \; and \neq 1 \tag{9'}$$

Equations (3B) and (4B) are not in a reduced form. However, the comparisons between the steady-state values of c and k in the two models becomes plausible, thanks to the similarity in the structure of the equations determining their steady-state values and their identical parameters α, δ and ρ. These comparisons and the comparison of the steady-state GDP between the two models are presented in the subsequent section.

3. Analysis

3.1. Sustainability

A least controversial concept of sustainability is adopted: the GDP max model is unsustainable unless its GDP is higher than that in the RCK model. In other words, unsustainability means that the GDP max model must convert to consumption max model since the latter brings at least the same level of GDP. This definition has two justifications: 1) it is inductively appealing as a GDP max model loses its sense of existence once it no longer has an advance in GDP; and 2) it is compatible with such popular criteria as environmental considerations, as the latter is covered by the under-consumption effects that are dealt with in the model.

Checking equations (3A), (3B), (4A) and (4B), the differences in the steady-states values of consumption and capital between the two models are mainly determined by $(1-B^{*)}/B^*$. Hence, the key factor for the comparison is B^*, defined by equation (9'). The steady-state GDP values are:

$$\overline{gdp}^* = \bar{k}^{*\alpha} = (\frac{\alpha}{\delta+\rho})^{\frac{\alpha}{1-\alpha}} \tag{12A}$$

$$gdp^* = k^{*\alpha} - \left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}}\bar{c}^* = (\frac{1-B^*}{B^*})^{\frac{\alpha}{1-\alpha}}\overline{gdp}^* - \left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}}\bar{c}^* \tag{12B}$$

Equations (12A) and (12B) give rise to a simpler expression of sustainability. Assuming, in the case of consumption optimization, a share θ of $\overline{gdp}^*$ is used for $\bar{c}^*$:

$$gdp^* = [(\frac{1-B^*}{B^*})^{\frac{\alpha}{1-\alpha}} - \theta\left(1 - \frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}}]\ \overline{gdp}^* \tag{13}$$

Without taking into account the second terms in the bracket on the right hand of equation (13), B^* must be, at least, <0.5 for the GDP max model being sustainable. This implies, by virtue of equation (8), that at the steady state, $m^* < 1$. Thus, the following proposition is put forward:

Proposition 1: a necessary condition for the GDP max model being sustainable is that, the steady-state current-value shadow price, $m^* = \frac{B^*}{1-B^*} < 1$.

Why must B^* be <0.5 or $m^* < 1$? Given that $B = \partial gdp/\partial c$ is the induced marginal loss in GDP by one unit of foregone consumption due to under-consumption, from the first-order condition equation (7), if m<1, then (1-B)>B. This means that the marginal contribution to GDP via increasing k by one unit of forgone consumption is larger than B.

B^* <0.5 is just the necessary condition, or the minimum requirement. This necessary condition is important because as shown, it results in a meaningful theoretical interpretation. From equation (13), the sufficient condition for the model being sustainable is: $(\frac{1-B^*}{B^*})^{\frac{\alpha}{1-\alpha}} - \theta\left(1 - \frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}}$>1. Under this condition, the boundary value of B^* for the sustainability is still lower than just <0.5. But to simplify the presentation in the following theoretical analysis, the boundary value of B^* for the sustainability is referred as being 0.5.

Using equations (3A), (3B), (4A) and (4B), the GDP max model achieves the unique steady-state values of c^* and k^*. We are able to compare the steady-state values of k, c and GDP between the two models. Comparing equations (3A) and (3B), as B^*<0.5, then $k^* > \bar{k}^*$ follows. Comparing equations (4A) and (4B), $c^* < \bar{c}^*$ because B^* <0.5 and $\left(\frac{1-B^*}{B^*}\right)^{\frac{\alpha}{1-\alpha}} < \left(\frac{1-B^*}{B^*}\right)^{\frac{1}{1-\alpha}}$. Therefore, in their steady states, capital and GDP are higher, although consumption is lower in the GDP max model than in the RCK model.

With equation (13), the sustainability of the GDP max model is governed by B^*. The next question becomes: "What determines $B^* < 0.5$?"

Checking equation (9'), on the one hand, logically B^* must be assumed to be a positive function of σ. When the value of the parameter scaling up the under-consumption effects is larger, the larger is the marginal production loss due to under-consumption.

$$\frac{\partial B^*}{\partial \sigma} > 0 \tag{14}$$

On the other hand, B^* is a negative function of $c^*/\bar{c}^*$, because:

$$\frac{\partial B^*}{\partial \frac{c^*}{\bar{c}^*}} = -\frac{(1-\sigma)/_{\sigma}}{1-\frac{c^*}{\bar{c}^*}} B^* < 0 \tag{15}$$

Since $\frac{\partial B^*}{\partial \sigma} > 0$, it follows that, inasmuch as $\sigma < 1$ and is constant, to the extent that the negative effect on B^* of the increasing $c^*/\bar{c}^*$ sufficiently offsets the positive effect of σ so that $gdp^*/\overline{gdp}^* > 1$, the GDP max model could always remain sustainable. With a low σ, its corresponding $c^*/\bar{c}^*$ can be fixed at a low level; thus B^* stays at a value that is much lower than 0.5. With a higher σ, its corresponding $c^*/\bar{c}^*$ must be fixed at a higher level; thus, B^* approaches 0.5, but the model always maintains sustainability. Needless to say that once σ is so high that it is close to 1, $c^*/\bar{c}^*$ has to be set close to 1. In this case, the GDP max model becomes very close to the RCK model, even if in theory it is still sustainable.

3.2. Transition between Steady States with Increasing $\boldsymbol{\sigma}$

Our discussion on the relationship between σ and $c^*/\bar{c}^*$ has so far been based on a constant σ. However, this can be shown to be an unrealistic postulation for two reasons.

The first reason is a rising σ reflects the increase of the aversion to under-consumption over time. Two arguments support the proposal that this aversion increases over time.

The first argument is that, along with income growth, the propensity for consumption rises because: 1) as the ultimate aim of production is consumption, the desire to consume increases; and 2) the capability to afford a higher consumption level rises; and 3) the demand for necessary goods decreases, whereas the demand for high-quality, comfort-linked luxury and leisure goods increases. The latter being more expensive, the share of consumption in income must increase. Along with higher income, people are also more sensible to the "public bad" that affects their health and living environment; this is another category of consumption that becomes increasingly expensive. Corresponding to the increasing propensity for consumption, the share of consumption in income rises with income growth; this statement is supported by empirical tests.[5] Therefore, with increasing propensity for consumption, the aversion to the same degree of under-consumption increases over time.

The second argument is that the "international demonstration effect" is intensified along with income growth. This influential concept was formulated by Nurkse (1953). He believed that in developing nations, people "come into contact with superior goods or superior patterns of consumption, with new articles or new ways of meeting old wants." As a result, these people are "apt to feel after a while a certain restlessness and dissatisfaction. Their knowledge is extended, their imagination stimulated; new desires are aroused" (Nurkse 1953, quoted in Kattel, Kregel, and Reinert 2009, 141). This international demonstration effect is tendentiously intensified with an increase in internationalization that is correlated with the development level of a country.

Second reason on why σ increasing over time captures the real world is a rising σ reflects the accumulative physiological effects of under-consumption. In this respect, some intuitively appealing interpretations are available. The costs of health care are lower if it takes place earlier. At time t, to prevent any physiological harm with the severity s_t, either due to the shortage of basic consuming goods, the deficiency in health care or environmental degradation, the cost is assumed to be c_t. If the treatment is postponed to t+1, the severity becomes s_{t+1} with $s_{t+1}>s_t$, and it follows that $c_{t+1}>c_t$. Therefore, the same level of under-consumption has, over time, a stronger physiological effect.

With an evolving parameter σ, a long-term comparative dimension is introduced into the dynamic analysis. This junction is inductively appealing since a dynamic analysis involves long-term evolution, and the parameters that specify the model are more likely to evolve. The method used subsequently is called as the transition between steady states after a change in a structural parameter (Novales, Fernandez, and Ruiz 2009).

[5] Using the world development indicators (World Bank 2014) for 198 countries from 1960 to 2012, it was found that ruling out those with a GDP per capita (at a constant price) that is lower than 1,000 USD (that inductively must invariably spend an unusually high share of income on necessary goods), the consumption to GDP ratio rises along with the GDP per capita. The regression results are available upon request.

Having shown the realism of the premise that σ rises over time, observing equation (13), and given $B^* = \frac{1}{\sigma}(1 - \frac{c^*}{\bar{c}^*})^{\frac{1-\sigma}{\sigma}}$, the key process is with rising σ, to keep maximizing gdp^*, how the under-consumption rate $c^*/\bar{c}^*$ is moved.

As shown in Appendix A.1., totally differentiating gdp^* with respect to σ and $c^*/\bar{c}^*$ and setting $dgdp^* = 0$ yields $d\frac{c^*}{\bar{c}^*}/d\sigma > 0$. Whenever σ rises, $c^*/\bar{c}^*$ always must rise. With rising σ, as the renewed steady-state values of c^* are successively higher, leading to higher B^*, consequently, their k^* (via equation (3B)) and gdp^* (via equation (13)) are successively lower. Their sustainability and relative growth rates during the transition are determined by their gdp^*, relating to $\overline{gdp}^*$. All of these define the shifting steady states and can be expressed by a new proposition.

Proposition 2: In the presence of an increasing σ, to maximize GDP, the shifting steady states of the model determine the growth rates to decelerate. Ultimately, when σ rises to a certain level, the GDP max model converges to the steady state of the RCK model.

The sustainability requires B^* to be sufficiently low so that (1-B^*)>B^*. Whenever σ rises, if $c^*/\bar{c}^*$ remains unchanged, B^*will increase towards the boundary value fixed by Proposition 1 and the sustainability of the model is at stake. Consequently, to keep maximizing GDP, consumption must be raised.

When σ rises from a low base, as under-consumption leads to low GDP loss, $c^*/\bar{c}^*$ is only required to slightly increase. Accordingly, $B^*$is kept much lower than 0.5. Thus, through a dynamic force that results in a fairly modest decrease of k^*, the model maintains a high growth at the transition stage and the sustainability of the model is not a concern. This makes very intuitive sense in our model. At the early development stages, peoples' aversion to under-consumption is low and under-consumption manifests a weak accumulative effect. Therefore, consumption growth can be fairly low.

When σ rises from a medium base, since the ratio $c^*/\bar{c}^*$ is now required to attain a level higher relative to the case of a low σ, by inducing a larger decrease in the steady-state capital, the model loses much of its force, and a slowdown is expected. The concern is to stimulate the economy and not slowing down too much. But B^* is still lower than 0.5 and the sustainability is not a concern.

When σ rises from a high base, $c^*/\bar{c}^*$ is needed to come up to one. Thus, relating to the RCK model, there is no longer sufficient capital formation derived from under-consumption. Not only does relative growth fall at a minimal level, but as $B^* \to 0.5$, from equations 3(A) and 3(B), $k^* \to \bar{k}^*$, and from equation (13), $gdp^* \to \overline{gdp}^*$. In other words, the model is not sustainable and converges to the RCK model.

3.3. A Numerical Simulation

This simulation of the transition between steady states, in the presence of an increasing σ, is illustrated in Table 1. First, using equations (3A), (3B), (4A), (4B) and (9'), the optimal ratios $\mathrm{c}^*/\bar{\mathrm{c}}^*$ and $\mathrm{k}^*/\bar{\mathrm{k}}^*$ as well as B^* are simulated. Afterwards, using equation (13), the corresponding ratios $\mathrm{gdp}^*/\overline{\mathrm{gdp}}^*$ are obtained.

Table 1. Simulation results

σ	$c^*/\bar{c}^*$	$k^*/\bar{k}^*$	$gdp^*/\overline{gdp}^*$	B^*
0.1	0.333	5.649	1.987	0.261
0.2	0.514	4.844	1.861	0.280
0.3	0.645	4.217	1.757	0.297
0.4	0.749	3.689	1.664	0.314
0.5	0.834	3.228	1.579	0.331
0.6	0.904	2.819	1.500	0.349
0.7	0.958	2.470	1.428	0.368
0.8	0.991	2.226	1.375	0.382
0.9	0.99993	2.15838	1.36033	0.38660
0.93	0.99999	2.15778	1.36020	0.38664
0.94	X	X	X	X

Notes: These results are calculated on the basis of equations (3A), (3B), (4A), (4B), (9') and (13) by using the representative values of $\delta = 0.07$, $\rho = 0.02$ and $\alpha = 0.4$. B^* is the steady-state marginal loss of GDP due to under-consumption. $c^*/\bar{c}^*$ is the ratio of steady-state consumption per capita derived from the GDP max model to that from the RCK model. $k^*/\bar{k}^*$ is the ratio of steady-state capital per capita derived from the GDP max model to that from the RCK model. $gdp^*/\overline{gdp}^*$ is the ratio of steady-state GDP per capita derived from the GDP max model to that from the RCK model. Both the computations of $\mathrm{gdp}^*/\overline{\mathrm{gdp}}^*$ from (13) and $\mathrm{c}^*/\overline{\mathrm{c}}^*$ from (4B) use $\theta =$ 0.69, computed with $\theta = (\overline{\mathrm{gdp}}^* - \delta\overline{\mathrm{k}}^*)/\overline{\mathrm{gdp}}^*$. To compute $\mathrm{c}^*/\overline{\mathrm{c}}^*$, (4B) is reformulated in:

$$\mathrm{Min}\ \left(\frac{1-\mathrm{B}^*}{\mathrm{B}^*}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{1-\mathrm{B}^*}{\mathrm{B}^*}\right)^{\frac{1}{1-\alpha}}\delta\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}} - \left(1-\frac{\mathrm{c}^*}{\overline{\mathrm{c}}^*}\right)^{\frac{1}{\sigma}}\overline{\mathrm{c}}^* - \mathrm{c}^*,$$

and the relationship $\mathrm{c}^* = \frac{\mathrm{c}^*}{\overline{\mathrm{c}}^*}\theta\overline{\mathrm{gdp}}^*$ is employed to replace c^*. The X in the last line indicates that no optimal solution is yielded.

From Table 1, in line with Proposition 2, along with the rising σ, the ratio $\mathrm{c}^*/\overline{\mathrm{c}}^*$ as well as the B^* progressively rise; the ratio $\mathrm{k}^*/\overline{\mathrm{k}}^*$ falls. In the beginning, $\mathrm{gdp}^*/\overline{\mathrm{gdp}}^*$ is high but it progressively decreases. Finally, when σ reaches a certain high level (to 0.94 in this case), the simulation is no longer able to produce an optimal solution. This means that, at some point between $0.93 < \sigma < 0.94$, B^* has reached the boundary value for sustainability and the model is no longer sustainable.

3.4.Adjustments during Transition

So far, the steady-state analysis is supported by mathematical solutions. As solving for the values of c, k and q during the transition is not viable, then drawing the phase diagrams remains the sole solution. The analysis at the transition stage is crucial as the values of the variables during the transition exhibit dynamic behavior. And more importantly, the adjustments occur during the transition, not at the steady state. The social planner, in the function of rising σ, will spontaneously switch to the new transition paths that point to the corresponding new steady states. Therefore, the adjusting process must be illustrated during the transition. It consists of a phase diagram presentation of the comparative dynamics.

As our task is to undertake a two-model comparison and comparative dynamic analysis with phase diagrams, an appealing method is to put two or more phase diagrams comparable in value and time into the same figure. The difficulty comes from the expositions in a timely comparable dimension. Given that under-consumption is measured in relation to consumption in the RCK model and $\bar{c}$ must go into the GDP max model, the determination of the under-consumption ratio, $c/\bar{c}$, requires that c and $\bar{c}$ are matched at the same time point. We can show that in the cases where the two models have the same or different transition speeds for getting to their steady states, the time matching problem during the transition can be geometrically solved. But to save space, these demonstrations are omitted and are available upon request. To be able to fulfill the objective stated for this study, in this and subsequent sections, it is assumed that the two models have no time lag and both get to their steady states at the same time.

Figure 1 depicts the situation in which σ is constant, hence the GDP max model works without adjustments; in Figure 2, σ is increasing and the GDP max model is with the adjustments.

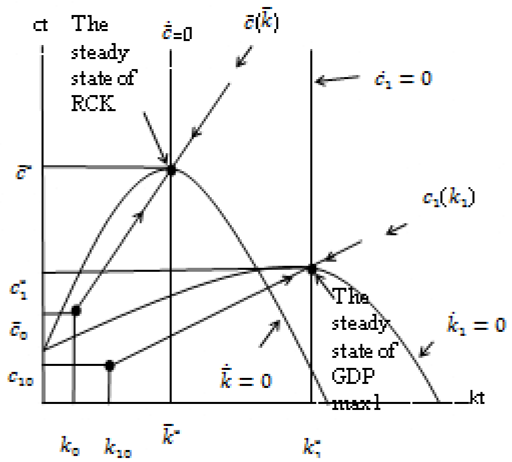

Figure 1. The phase diagrams of the RCK and GDP max models without adjustments.

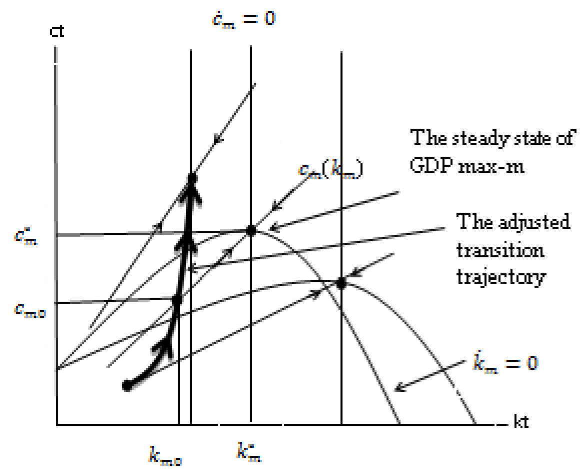

Figure 2. The phase diagrams of the RCK and GDP max models with adjustments.

In Figure 1, two phase diagrams are separately drawn for the RCK and GDP max models. With the two phase diagrams in the same figure, the arrows indicating the directions of motion in each region and the unstable arms must be omitted for the sake of avoiding confusion. Thus, the saddle-path stability exhibited by the RCK model, as well as by the GDP max model, is not indicated by the arrows. [6]

In the RCK model, $\dot{\bar{c}} = 0$ and $\dot{\bar{k}} = 0$ loci divide the space into four regions. The intersection between the line $\dot{\bar{c}} = 0$ and the curve $\dot{\bar{k}} = 0$ determines the steady state. The stable arm labeled $\bar{c}(\bar{k})$ is upward-sloping and goes through the origin and the steady state. Along the transition, $\bar{c}$ and $\bar{k}$ are increasing.

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[6] From equations (1A) and (2A), both $\partial\dot{\bar{k}}/\partial\bar{c} = -1 < 0$ and $\partial\dot{\bar{c}}/\partial\bar{k} = -\alpha(1-\alpha)\bar{k}^{\alpha-2} < 0$ determine the saddle-path stability of the RCK model. The saddle-path stability of the GDP max model is determined in the same way. From equations (1B) and (2B), $\partial\dot{k}/\partial c = -1 + B < 0$ (given the fact that the sustainability constraint requires B<0.5), and $\partial\dot{c}/\partial k = -\alpha(1-\alpha)k^{\alpha-2}(1-B)^2(\bar{c}-c)/[\left(\frac{1-\sigma}{\sigma}\right)B] < 0$.

The GDP max model, before the adjustments, is labeled GDP max1. Its intersection between the line $\dot{c_1} = 0$ and the curve $\dot{k_1} = 0$ determines the steady state. The stable arm labeled $c_1(k_1)$ points towards the steady state. Along the transition, c and k are increasing.

To trace the initial point of the GDP max1 (c_{10}, k_{10}) relating to initial point of the RCK model ($\bar{c}_0, \bar{k}_0$), making use of the fact that the initial endowment (W_0) is given and $W_0 = \bar{c}_0 + \bar{k}_0$=$c_{10}$+$k_{10}$, then if $\bar{c}_0$ is much higher than c_{10}, k_{10} must be proportionally higher than $\bar{k}_0$.

It was previously shown that $k^* > \bar{k}^*$ and $c^* < \bar{c}^*$. As a result, line $\dot{c_1} = 0$ is on the right of line $\dot{\bar{c}}$=0 and curve $\dot{k_1} = 0$ is below curve $\dot{\bar{k}}$=0.

In Figure 2, together with the two phase diagrams already introduced in Figure 1, a third phase diagram is added: a representative adjustment of the GDP max model. Implied by Proposition 2, in the presence of a regularly increasing σ, to maximize GDP, the social planner has to adjust periodically. If the planner adjusts m times, there will be m in-between steady states between the GDP max1 and the RCK models. Here, they are represented by just one case labeled m.

Considering that, after the adjustment, $k_m^* < k_1^*$ and $c_m^* > c_1^*$,[7] then, line $\dot{c}_m = 0$ is on the left of $\dot{c}_1 = 0$ and on the right of $\dot{\bar{c}}$=0, and curve $\dot{k}_m = 0$ is above $\dot{k}_1 = 0$ and below $\dot{\bar{k}} = 0$. The stable arm $c_m(k_m)$ is at the intermediate level between $\bar{c}(\bar{k})$ and $c_1(k_1)$.

With m adjustments and connecting all the successive starting points of m+2 stable arms, a convex and upward-sloped curve labeled the adjusted transition trajectory is formed. This construction is a phase diagram application of the comparative dynamic analysis. To save space, we omit this more rigorous presentation because the convex and upward-sloped form of this trajectory is already observed in Figure 2. Consequently, a new proposition is offered.

Proposition 3: In the presence of a regularly increasing σ, to maximize GDP, the social planner has to adjust its transition trajectory, and accordingly, is constituted an adjusted transition trajectory that is a convex upward sloping curve pointing to the steady state of the RCK model.

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[7] $k_m^* < k_1^*$, because as $\partial B/\partial\sigma > 0$, from (3B), this result is obtained. $c_m^* > c_1^*$ is by virtue of equation (16) in Appendix A, ensuring $d\frac{c^*}{\bar{c}^*}/d\sigma$ >0.

The new insight into the GDP max model provided by this proposition is that, while in Proposition 2, the deceleration of the GDP growth at the transition stage is only indirectly inferred from the steady-state results, in Proposition 3, this deceleration is directly implied by the shape of the adjusted transition trajectory. Since the slope of the trajectory reflects the marginal relationship between consumption and capital, the convex upward shape means that consumption rises with an increasing speed while capital increases in a decreasing cadence. Correspondingly, the GDP growth decelerates over time.

Why does the adjusted transition trajectory take on a convex upward shape and lead the GDP growth to decelerate? The very reason resides in the relationship between σ and under-consumption within B. In Appendix B, through totally differentiating B, the first derivative is $\frac{\partial(c/\bar{c})}{\partial\sigma} > 0$. When σ is low, the second derivative $\frac{\partial^2(c/\bar{c})}{\partial\sigma^2}$ <0, while $\frac{\partial^2(c/\bar{c})}{\partial\sigma^2} > 0$ when σ increases and reaches a certain level, implying that the curve $c/\bar{c}$ over σ is firstly concave and next convex.

This makes very intuitive sense in our model. At the early development stages, peoples' aversion to under-consumption is low and under-consumption manifests a weak accumulative effect. Therefore, consumption growth can be fairly low. As the aversion and accumulation go with development, consumption growth is required to accelerate.

4. Empirical evidence: slowdown or breakdown?

It has been shown that constrained by under-consumption, the GDP max model could be high in growth rate for some period, but at last unsustainable. To what extent does the GDP max model capture the real world? For answering this question, two criterions are applied to identify the countries having visible GDP-maximizing inclination: 1) they meet weak or are free of democratic control; 2) they have an unusual GDP growth period with high capital formation share in GDP.

With these criterions, ex-Soviet Union was an extreme case of such model. It essentially consisted of minimizing population's consumption and of mobilizing as more as possible economic resources for industrialization, especially for the heavy industries, in order to win international competition, above all military competition with the USA. While the ex-Soviet Union became one of the leading industrial nations of the world, it suffered severe shortage of consumer goods before broken down in the end 1980s. If the economy is unable to realize the adjustment, as all $c/\bar{c}$ ratios, other than that along the steady arm will lead to unstable states, the breakdown will occur. This may be one of the explanations of the collapse of the ex-Soviet economy. The most important feature of this economy is the predominant role of its military industry. Gaidar (2006) explained the end of Soviet empire by the distorted nature of the Soviet economy, and in particular an extremely high proportion of resources allocated to serve the needs of the military-industrial complex. In the late 1980s, the Soviet Union devoted a quarter of its gross economic output to the defense sector. The military-industrial complex employed at least one of every five adults in the Soviet Union. In 1989, one-fourth of the entire Soviet population was engaged in military activities.[8] If the heavy industries were in civil usage, there was a possibility to convert to consumption production. Military industries, however, were often "locked in" specific usage, and at the moment where the products lost the demand, the productive system was so specific that its value went to zero. The failure of adjustments provoked a sudden and dramatic drop of its GDP.

The Eastern Asian model covering around ten countries in the region is close to the GDP max model. The two most eminent representatives are Japan and China. While Japanese firms have been qualified as revenue-maximizing (Uekusa and Caves, 1976, and Komiya, 1992), following Morishima (2000), the Japanese government was strong because the opposition was weak. Democracy in the true sense was thus not developed. The state influenced the economy through its privileged links with the dominant banking and industrial conglomerates (the zaibatsu**),** supported economic nationalism and helped Japanese firms with exportation and taking market shares in international competition. Therefore, the state had a pronounced pro-investment and GDP growth tendency.

In current times, China is the most representative rising star in terms of maximizing GDP growth. With its double heritage from the central-planning system and Eastern Asian culture, and in the absence of democratic control, the state remains the major player in the economy. Chinese government admits officially the existence of the "GDP worship" in China in the past.

Table 2 depicts that both Japan and South Korea, in spite of their GDP growth oriented inclination during a certain period, had their capital to GDP ratios fall and approach the world mean level, implying the unsustainability and a stepping back to consumption optimization (in South Korea, this ratio dropped to 27.55% in 2012).

[8] http://en.wikipedia.org/wiki/Military_history_of_the_Soviet_Union.

Table 2. Shares of gross capital formation in GDP (%)

Period	71-75	76-80	81-85	86-90	91-95	96-00	01-05	06-10
China (Consumption/GDP)	29.10 (70.95)	32.66 (67.50)	34.91 (65.16)	37.50 (63.53)	39.72 (59.03)	38.21 (58.87)	39.30 (57.80)	44.55 (49.13)
Japan	36.25	31.71	29.40	29.91	30.10	26.79	23.21	21.75
South Korea	26.47	30.84	29.88	32.03	37.48	32.87	29.74	29.29
USA	21.98	23.14	23.51	23.01	20.72	22.48	22.43	20.93
World average level	22.40	24.29	24.16	21.92	22.56	22.52	21.77	23.93

Source: Calculated on the basis of the world development indicators (World Bank, 2014).

In most periods, not only were China's shares higher than the world mean level and that of the USA, but its most recent level was higher than that of Japan and South Korea in their growth peaks. This growing trend that began in the 1970s continues to maintain itself. As shown in Table 2, inversely correlated with the capital to GDP ratio, China's share of consumption in GDP has been in continuous decline and has reached below the 50% point in 2006-2010, significantly lower than the world mean level (over 75%).

Could the model be valid to forecast what will happen for China? Several factors could drive China to decelerate its capital expansion and, hence, GDP growth. First, as IMF (2012) estimated, its rate of utilization of production capacity has been as low as 60%. Second, it has accumulated an enormous burden of health care deficiency and of environment degradation. Since long, the spending on health infrastructure and insurance has been excessively low. According to World Health Organization data, in 2012, China's total health expenditures in GDP were 5.4%, while the mean value in the world was 10.1%. The government expenditures spent in the economies of energy and the environment in terms of GDP percentage were only 1.5% in 2012. This value is lower than that of most countries (2-3%). Environmental deterioration has reached the dangerous level (cf. World Bank 2013, chapter 3).

Whereas in a lot of countries in which some public expenditures on consumption, especially those for health care, environment improvement and poverty reduction, consist of the main items of their budgets and the cause of their huge budget deficits, China has "economized" them and put them all in financing their capital expansion. After more than 30 years of extensive growth through constraining people's consumption, it is now time for China to deal with the consequences that have been accumulated, or expressed in an alternative manner, to return to joining the "normal" consumption maximizing world.

5. Conclusion

The GDP-maximizing model, constrained by consumers' rising aversion to under-consumption and the accumulative under-consumption effects, was shown to go along a convex upward adjusted transition trajectory from high growth to slowing down, and finally converges to the RCK model. To explain simply, the maximizing GDP growth worked well during the first period in which the aversion to under-consumption was low and under-consumption had not yet manifested significant accumulative effects. But this aversion and the accumulation go with economic growth, and inevitably lead the government to reduce under-consumption in order to maintain GDP growth. Consequently, the capital expansion must decelerate and GDP growth progressively converges to the level of a maximizing consumption model.

The study provided one explanation for the breakdown of the ex-Soviet economy: the incapability to convert the heavy and military industrial sectors into the production of consumption goods. It also challenges the view that through long-lasting high GDP growth driven by the expansion of capital, China will become the world economic leader.

Appendixes

A.1. Deriving the Relationship between σ and $\mathbf{c}^*/\bar{\mathbf{c}}^*$ at the Steady State

From equation (13), totally differentiating gdp^* with respect to σ and $\frac{c^*}{\bar{c}^*}$, setting $gdp^* = 0$, and rearranging, the following equation is obtained:

$$\frac{d\frac{c^*}{\bar{c}^*}}{d\sigma} = \frac{-(\frac{\alpha}{1-\alpha})(\frac{1-B^*}{B^*})^{\frac{2\alpha-1}{1-\alpha}}\left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{\sigma-1}{\sigma}}\left[1+\frac{1}{\sigma}\ln\left(1-\frac{c^*}{\bar{c}^*}\right)\right]-\theta\frac{1}{\sigma^2}\left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}}\ln\left(1-\frac{c^*}{\bar{c}^*}\right)}{\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-B^*}{B^*}\right)^{\frac{2\alpha-1}{1-\alpha}}(1-\sigma)\left(1-\frac{c^*}{\bar{c}^*}\right)^{-\frac{1}{\sigma}}+\theta\frac{1}{\sigma}\left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1-\sigma}{\sigma}}}>0 \quad \text{(A1)}$$

To prove this positivity, as by equation (14), $\partial B^*/\partial\sigma > 0$, and:

$$\frac{\partial B^*}{\partial \sigma} = -\frac{1}{\sigma}B^*[1+\frac{1}{\sigma}\ln\left(1-\frac{c^*}{\bar{c}^*}\right)] \quad \text{(A2)}$$

Given $\ln\left(1-{}^{c}/_{\bar{c}}\right) < 0$, it must be that:

$$\frac{1}{\sigma}\ln\left(1-\frac{c}{\bar{c}}\right) < -1 \tag{A3}$$

By equation (A3), the numerator in equation (A1) is positive. It follows that, $d\frac{c^*}{\bar{c}^*}/d\sigma > 0$. Concretely, equation (A3) fixes some reasonable constraints and requires that, e.g., when σ =0.4, $c/\bar{c}$ must be higher than 1/3, and when σ =0.7, $c/\bar{c}$ must be higher than ½. It allows for avoiding some unreasonable extreme cases in which the ratio $c/\bar{c}$ is too low.

A.2. Deriving the Relationship between σ and $c/\bar{c}$ during Transition

The first derivative is obtained by totally differentiating B in equation (9), with respect to σ and $c/\bar{c}$. Putting dB=0 yields:

$$\frac{d\frac{c}{\bar{c}}}{d\sigma} = -\frac{\left(1-\frac{c}{\bar{c}}\right)}{1-\sigma}\left[1+\frac{1}{\sigma}\ln\left(1-\frac{c}{\bar{c}}\right)\right] > 0 \tag{19}$$

in which, by virtue of equation (18), $\frac{1}{\sigma}\ln\left(1-\frac{c}{\bar{c}}\right) < -1$.

To find the second derivative, deriving equation (19) with respect to σ results in:

$$\frac{\partial^2(c/\bar{c})}{\partial\sigma^2} = \frac{\left(1-\frac{c}{\bar{c}}\right)}{1-\sigma}\left[-\frac{1+\frac{1}{\sigma}\ln\left(1-\frac{c}{\bar{c}}\right)}{1-\sigma} + \frac{\frac{1}{\sigma}\ln\left(1-\frac{c}{\bar{c}}\right)}{\sigma}\right] \tag{20}$$

Always by $\frac{1}{\sigma}\ln(1-c/\bar{c}) < -1$, the sign of this second derivative hinges on which of the two terms in the bracket is larger. When σ is low, the denominator of the first term is larger than that of the second term, so that $\frac{\partial^2(c/\bar{c})}{\partial\sigma^2} < 0$, and the part of the adjusted transition trajectory associated with a lower σ is concave upward. When σ rises and reaches a certain level, the denominator of the first term is smaller than that of the second term, so that $\frac{\partial^2(c/\bar{c})}{\partial\sigma^2} > 0$, and the part of the adjusted transition trajectory associated with a higher σ is convex upward-shaped.

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