Trade Liberalization, Technology Diffusion, and Productivity

Kishi, Keiichi and Okada, Keisuke

Kansai University

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Keiichi Kishi[†] Keisuke Okada[‡]

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Abstract

This study develops the international trade theory of technology diffusion with heterogeneous firms. Each new entrant randomly searches for and meets incumbents and then adopts their existing technology. As in previous international trade models based on firm heterogeneity, trade liberalization induces the least productive firms to exit, and then the resources can be reallocated toward more productive firms. However, we show that this resource reallocation effect is mitigated by the entry of low-productive firms. Trade liberalization facilitates the diffusion of existing low-productive technologies to new entrants, which shifts the weight in the productivity distribution from the upper tail area to the area around the least productivity. Thus, some resources can be reallocated toward low-productive firms. In addition, trade liberalization reduces domestically produced varieties. Consequently, we show the non-monotonic relationship between trade liberalization and aggregate productivity.

JEL classification: F12, L11, O33

Keywords: International trade, Innovation, Productivity distribution

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[†]Faculty of Economics, Kansai University, 3-3-35 Yamate-cho, Suita-shi, Osaka 564-8680, Japan, E-mail address: kishi@kansai-u.ac.jp

[‡]Faculty of Economics, Kansai University, 3-3-35 Yamate-cho, Suita-shi, Osaka 564-8680, Japan, E-mail address: k_okada@kansai-u.ac.jp

1 Introduction

Trade liberalization has been a central issue in international economics and the costs and benefits of globalization have been an interesting subject for economists and policymakers. International trade models with heterogeneous firms such as those proposed by Melitz (2003) and Melitz and Ottaviano (2008) show that trade liberalization causes low-productive firms to exit because of intense market selection. As a consequence, resources reallocate to high-productive firms under the so-called "resource reallocation effect," which contributes to increasing aggregate productivity at the industry level. Many empirical studies confirm the resource reallocation effect (or market selection effect) caused by trade liberalization.

Based on the foregoing, this study investigates the effects of trade liberalization on productivity, considering the entry and exit of firms. Because the entry and exit of firms occur constantly, it is important to consider what kinds of firms contribute to the productivity gain. Aggregate productivity is affected not only by the market shares of surviving firms but also by the entry and exit of firms. By proposing a new method of productivity decomposition based on Olley and Pakes (1996), Melitz and Polanec (2015) show that the contribution of entering firms in Slovenian manufacturing sectors after economic reforms (e.g., the liberalization of prices and wages, deregulation of firm entry, and privatization of state-owned firms) is overvalued compared with that in previous studies. Specifically, among surviving, entering, and exiting firms, the contribution of entering firms to labor productivity is negative and the contribution to total factor productivity is nil. This result indicates that new entrant firms, which tend to be small and therefore most likely to leave the market (World Trade Organization, 2016), do not necessarily have high productivity after economic reforms. In this study, we refer to this as the "low-productive entrant effect."

By considering this low-productive entrant effect explicitly, we examine how trade liberalization affects productivity. In the international trade model presented herein, constructed based on the model of Melitz (2003) with technology diffusion á la Aghion and Howitt (1998, Ch. 3), Lucas and Moll (2014), and Perla and Tonetti (2014), trade liberalization causes both the resource reallocation effect (owing to the market selection effect) and the low-productive entrant effect. Each new entrant randomly searches for and meets an incumbent and then learns its existing technology. Some new entrants succeed in adopting the frontier (the most productive) technology, as in Aghion and Howitt (1998, Ch. 3). The other new entrants adopt existing non-frontier technologies, as in Lucas and Moll (2014) and Perla and Tonetti (2014). Then, the model generates the *endogenous* stationary Pareto productivity distribution.[1] Therefore, trade liberalization changes the shape (Pareto exponent) of the distribution. Specifically, we show that trade liberalization increases the Pareto exponent because of the low-productive entrant effect. That is, trade liberalization induces the entry of low-productive firms, which shifts the weight in the productivity distribution from the upper tail area to the area around the least productivity (minimum support of the distribution). Therefore, the endogenous response of the distribution has a negative effect on aggregate productivity. On the contrary, as in the Melitz (2003) model, trade liberalization induces the least productive firms to exit the market, which causes the reallocation of resources to high-productive

[1] Nigai (2017) empirically shows that the log-normal distribution fits the data on the French firms' productivity measure for the majority of the support, while the Pareto distribution provides a better fit for the upper-right tail of the productivity distribution. Many related theoretical studies examine the Pareto distribution in the closed economy, such as Luttmer (2007, 2012), Staley (2011), Acemoglu and Cao (2015), Kishi (2016), König et al. (2016), and Benhabib et al. (2017).

firms, thereby affecting aggregate productivity positively. In summary, the resource reallocation effect is mitigated by the low-productive entrant effect. In other words, some resources are reallocated toward low-productive new entrants. Further, as shown in the Melitz (2003) model, trade liberalization reduces the number of domestic varieties, which also has a negative effect on aggregate productivity. Then, the net effect of trade liberalization on aggregate productivity depends on whether the resource reallocation effect dominates the low-productive entrant effect and the reduction in domestic varieties. Indeed, we numerically demonstrate the non-monotonic relationship between trade liberalization and aggregate productivity.

The low-productive entrant effect also worsens welfare because of the negative effect on aggregate productivity. However, we numerically show that trade liberalization increases welfare in the steady-state level. In the model, real income is the welfare measure in the steady state. Then, two competing effects on real income exist. First, trade liberalization reduces the ideal price index because of the dominant resource reallocation effect and the increase in the share of imported varieties, which has a positive effect on real income. Second, trade liberalization may reduce average assets per capita because of the low-productive entrant effect and reduction in domestic varieties. Then, trade liberalization reduces asset income, which has a negative effect on real income. Consequently, we numerically show that the effect of the reduction in the price index on real income dominates the reduction in asset income and thus trade liberalization increases real income and welfare in the steady-state level.

The theoretical literature is silent on the impact of trade liberalization on the shape (Pareto exponent) of the productivity distribution because of the assumption of an *exogenous* productivity distribution. In the Melitz (2003) model, new entrants draw productivity from the exogenous distribution. Then, the resource reallocation effect due to trade liberalization only increases the minimum support on the productivity distribution because of the exit of the least productive firms. Therefore, Melitz (2003) emphasizes the contribution of exiting firms to aggregate productivity. In Lucas and Moll (2014) and Perla and Tonetti (2014), the shape (Pareto exponent) of the stationary Pareto productivity distribution is exogenously determined. Therefore, the model of Perla et al. (2015), which is a heterogeneous firms' trade model with technology diffusion à la Lucas and Moll (2014) and Perla and Tonetti (2014), derives the exogenous Pareto productivity distribution. Similarly, by combining the reduced-form fashion of the technology diffusion of Lucas and Moll (2014) and Perla and Tonetti (2014) with the Melitz (2003) model, Sampson (2016) also derives the exogenous Pareto productivity distribution. Impullitti et al. (2013), who combine the Luttmer (2007, 2012) model with the Melitz (2003) model, also derive the Pareto distribution, where trade liberalization does not affect the Pareto exponent. Further, trade models with heterogeneous firms usually suppose that the entrant draws productivity from an exogenous Pareto distribution (see the examples by Helpman et al. (2004), Melitz and Ottaviano (2008), Demidova and Rodríguez-Clare (2009, 2013), Bustos (2011), Rubini (2014), Melitz and Redding (2015), Ourens (2016), and Sampson (2016)). Then, in the literature, trade liberalization does not affect the Pareto exponent. On the contrary, by applying the method developed by Kishi (2016) and Benhabib et al. (2017), this study constructs the Melitz (2003) model with an endogenous Pareto distribution. The method described in Kishi (2016) and Benhabib et al. (2017) aims to add the growth and adoption of frontier technology into the technology diffusion model of Lucas and Moll (2014) and Perla and Tonetti (2014). This is a tractable way in which to generate an endogenous Pareto exponent. Then, we can analytically clarify the impact of trade liberalization on the distribution through the entry of low-productive firms. By virtue of the endogenous distribution, this study thus incorporates not only the effect of exiting firms on productivity but also the effect of entering firms on productivity.

An extensive body of empirical research has examined the relationship between trade liberalization and productivity both in developed and in developing countries. Studies of developed countries include Bernard et al. (2003) and Bernard et al. (2006) for the United States and Trefler (2004), Lileeva (2008), and Lileeva and Trefler (2010) for Canada. Other studies targeting developing countries include Pavcnik (2002) for Chile, Amiti and Konings (2007) for Indonesia, Fernandes (2007) for Colombia, Goldberg et al. (2010), Nataraj (2011), and Topalova and Khandelwal (2011) for India, Bustos (2011) for Argentina, and Yu (2015) for China.

Most of these studies confirm that trade liberalization increases productivity monotonically at the industry and firm (or plant) levels. There are likely to be two channels through which trade liberalization increases productivity. One is the reallocation of resources from low-productive sectors to high-productive ones as the Melitz (2003) model predicts. Empirical studies such as Pavcnik (2002) and Bernard et al. (2006) support this view. Bernard et al. (2006), for example, use data on U.S. manufacturing industries and plants from 1977 to 2001 to examine the effects of industry-level tariffs and transportation costs. They find that a reduction in trade costs increases industry productivity and that these gains are attributed to the reallocation to high-productive plants within industries. The other is the adoption of more advanced technologies. Bustos (2011) uses data on Argentinean firms and finds that a regional free trade agreement (FTA), MERCOSUR, induces firms to increase investment in technology.[2][3]

Although, to the best of our knowledge, no studies explicitly examine the non-monotonic relationship between trade liberalization and productivity, some studies imply such a relationship. Trefler (2004) examines the effects of the Canada-U.S. FTA on the labor productivity of Canadian manufacturing at the industry and plant levels. He shows that U.S. tariff concessions do not have a significant impact on labor productivity at the industry level, although they significantly increase labor productivity at the plant level.[4] As a reason for this result, he points out that U.S. tariff concessions may promote the entry of less productive plants. In this study, we use Trefler's (2004) dataset and confirm the negative impact of trade liberalization on labor productivity in certain circumstances, as our model predicts.

The contributions of this study to existing research are summarized as follows. First, by incorporating the process of technology diffusion, we derive the endogenous stationary Pareto productivity distribution. Second, we show that trade liberalization changes the shape (Pareto exponent) of the distribution, which is consistent with the low-productive entrant effect through trade liberalization. Finally, we quantitatively

[2]Yeaple (2005), Ederington and McCalman (2008), Lileeva and Trefler (2010), and Bustos (2011) theoretically show that trade liberalization induces some firms to invest in new technologies, which increases plant-level productivity.

[3]Recently, empirical studies have paid attention to tariff reductions in final goods and imported intermediate goods as measures of trade liberalization. On the one hand, lowering output tariffs leads to competition, which raises firms' productivity. On the other hand, reductions in input tariffs cause firms to increase efficiency because they can obtain cheaper imported inputs. Most studies show that both input and output tariff reductions have a positive impact on productivity at the firm level (e.g., Amiti and Konings, 2007; Nataraj, 2011; Topalova and Khandelwal, 2011; Yu, 2015).

[4]Trefler (2004) shows that Canadian tariff concessions significantly increase labor productivity at the industry level, which is viewed as empirical evidence of the result predicted by Melitz (2003). However, Segerstrom and Sugita (2015) show that the opposite relationship holds under a multi-industry version of the Melitz (2003) model.

assess the net effect of trade liberalization on aggregate productivity that consists of the resource reallocation effect, low-productive entrant effect, and reduction in domestic varieties. Then, we numerically demonstrate the non-monotonic relationship between trade liberalization and aggregate productivity.

The remainder of the paper is organized as follows. In Section 2, we outline the trade model of Melitz (2003) with technology diffusion à la Aghion and Howitt (1998, Ch. 3), Lucas and Moll (2014), and Perla and Tonetti (2014). In Section 3, we derive the productivity distribution across active firms to yield average productivity. In Section 4, we analytically show the impact of trade liberalization on average productivity. In Section 5, we numerically investigate the impact of trade liberalization on average and aggregate productivities as well as welfare. Section 6 revisits Trefler's (2004) study and confirms the negative effect of trade liberalization in certain circumstances. Section 7 concludes.

2 Theoretical framework

We develop an international trade model based on technology diffusion. There are $1 + N$ symmetric countries. When $N = 0$, the model describes an autarkic economy, whereas when $N > 0$, the model describes an open economy. Time is continuous. We focus on the balanced-growth equilibrium in which all endogenous variables grow at constant rates. We omit the country index when representing the country's variables since we focus on symmetric countries.

2.1 Household

There is a representative household with the following utility:

$$U = \int_0^\infty e^{-\rho t} L \frac{c(t)^{1-\theta} - 1}{1 - \theta} dt, \tag{1}$$

where $\rho > 0$ is the subjective discount rate, $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, $c(t)$ denotes consumption per capita at time t, and L is the population size, which is constant over time. Hereafter, we omit time t whenever no ambiguity results.

The household's budget constraint expressed in per capita terms is

$$\dot{a} = 1 + ra - Pc, \tag{2}$$

where a is the value of assets per capita, 1 represents the wage rate, which is normalized to unity (i.e., we take labor as the numéraire), r is the interest rate, and P is the price of the consumption good.

The representative household's optimization problem implies the well-known Euler equation for consumption:

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left(r - \rho - \frac{\dot{P}}{P} \right) \tag{3}$$

and the transversality condition:

$$\lim_{t \to \infty} \frac{e^{-\rho t} c^{-\theta} a}{P} = 0. \tag{4}$$

According to Eq. (2), the growth rates of both a and Pc must be zero in the balanced-growth equilibrium:

$$0 = \frac{\dot{a}}{a} = \frac{\dot{P}}{P} + \frac{\dot{c}}{c}. \tag{5}$$

Then, the budget constraint (2) can be rewritten as

$$Pc = 1 + ra. \tag{6}$$

That is, expenditure Pc for consumption equals the sum of wage income 1 and asset income ra in the balanced-growth equilibrium.

2.2 Final good

The final good is produced by using the continuum of intermediate goods under perfect competition, according to the following production function:

$$Q = \left(\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

where Q is the output of the final good, $q(\omega)$ is the intermediate input of variety $\omega \in \Omega$ used in the production of the final good, Ω is the set of the varieties of the intermediate good available for the production of the final good in a typical country, and $\sigma > 1$ is the elasticity of substitution between any two intermediate goods. The conditional factor demand function for $q(\omega)$ derived from Eq. (7) is

$$q(\omega) = \left(\frac{P}{p(\omega)} \right)^{\sigma} Q, \tag{8}$$

where $p(\omega)$ is the price of the intermediate good ω and the price of the final good is

$$P = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}. \tag{9}$$

The final good can be used only for consumption, and thus its market-clearing condition is

$$Q = cL. \tag{10}$$

2.3 Intermediate good

The intermediate good is produced by monopolistically competitive firms using the labor force. Each intermediate firm can sell the product in both domestic and foreign markets. The production structure for each good is equivalent to that in Melitz (2003). The production of the intermediate good involves both fixed and variable costs: to produce q units of output for the domestic market, $f + q/\varphi$ units of labor are required, where $f > 0$ is the fixed cost measured in units of labor and φ is productivity. By contrast, to export q units of output into a foreign market, $f_x + \tau q/\varphi$ units of labor are required, where $f_x > 0$ is the fixed cost measured in units of labor and $\tau > 1$ represents the standard iceberg cost; in other words,

τ units of a good must be shipped for 1 unit to arrive in a foreign country. We omit the notation ω, as it is sufficient to identify each firm by its productivity φ.

Under Eq. (8), firms whose productivity is φ choose the domestic price to maximize the domestic profit. Then, the domestic profit for a firm whose productivity is φ is given by

$$\Pi_d(\varphi) = \max\left\{0, \left(\frac{1}{\sigma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \varphi^{\sigma-1}P^{\sigma}Q - f\right\}. \tag{11}$$

From Eq. (11), the exit cutoff productivity φ^*, where firms exit if and only if $\varphi < \varphi^*$, is given by

$$\Pi_d(\varphi^*) = 0 \iff (\varphi^*)^{\sigma-1} = (\sigma-1)\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{f}{P^{\sigma}Q}\right). \tag{12}$$

Similarly, firms whose productivity is φ choose the export price to maximize the export profit for a typical country. Then, the export profit for a firm whose productivity is φ is given by

$$\Pi_x(\varphi) = \max\left\{0, \left(\frac{1}{\sigma-1}\right)\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{1}{\tau}\right)^{\sigma-1} \varphi^{\sigma-1}P^{\sigma}Q - f_x\right\}. \tag{13}$$

From Eq. (13), threshold productivity φ_x^*, where firms choose to export if and only if $\varphi \geq \varphi_x^*$, is given by

$$\Pi_x(\varphi_x^*) = 0 \iff (\varphi_x^*)^{\sigma-1} = (\sigma-1)\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\tau^{\sigma-1}\left(\frac{f_x}{P^{\sigma}Q}\right). \tag{14}$$

To simplify the analysis, define the log of relative productivity $\phi \equiv \ln(\varphi/\varphi^*)$. Below, we simply refer to ϕ as either relative productivity or productivity whenever no ambiguity results. An active firm's productivity φ is higher than exit cutoff φ^*; therefore, $\phi \geq 0$ holds for all active firms. Now, we can rewrite the domestic profit (11) and export profit (13) by using ϕ as follows:

$$\pi_d(\phi) = \begin{cases} f[e^{(\sigma-1)\phi} - 1] & \text{for } \phi \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$$\pi_x(\phi) = \begin{cases} \left(\frac{1}{\tau}\right)^{\sigma-1} f[e^{(\sigma-1)\phi} - e^{(\sigma-1)\phi_x^*}] & \text{for } \phi \geq \phi_x^* \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $\pi_d(\phi) \equiv \Pi_d(\varphi^* e^{\phi})$, $\pi_x(\phi) \equiv \Pi_x(\varphi^* e^{\phi})$, and

$$\phi_x^* \equiv \ln\left(\frac{\varphi_x^*}{\varphi^*}\right) = \left(\frac{1}{\sigma-1}\right)\ln\left(\frac{\tau^{\sigma-1}f_x}{f}\right) \geq 0. \tag{17}$$

Now, we suppose that $\phi_x^* \geq 0 \iff \tau^{\sigma-1}f_x \geq f$ holds, which ensures that the absolute export cutoff φ_x^* is larger than absolute exit cutoff φ^* in the equilibrium. For convenience, we restate the following assumption.

Assumption 1 *The iceberg cost and fixed cost of exporting a variety is sufficiently large:*

$$\tau^{\sigma-1} f_x \geq f. \tag{18}$$

Total profits for the firm whose productivity is ϕ equal the sum of the domestic profit (15) and export profit (16) across all foreign markets:

$$\pi(\phi) \equiv \pi_d(\phi) + N\pi_x(\phi). \tag{19}$$

By using the definition of ϕ, we can derive the domestic price $p_d(\phi)$ and the export price $p_x(\phi)$ as follows:

$$p_d(\phi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{e^{\phi}\varphi^*} \text{ for } \phi \geq 0 \tag{20}$$

$$p_x(\phi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau}{e^{\phi}\varphi^*} \text{ for } \phi \geq \phi_x^*. \tag{21}$$

Next, we derive the amount $l_d(\phi)$ of labor required for production for the domestic market and the amount $l_x(\phi)$ of labor required for production for a foreign market as follows:

$$l_d(\phi) = \begin{cases} f[(\sigma-1)e^{(\sigma-1)\phi}+1] & \text{for } \phi \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

$$l_x(\phi) = \begin{cases} \left(\frac{1}{\tau}\right)^{\sigma-1} f[(\sigma-1)e^{(\sigma-1)\phi}+e^{(\sigma-1)\phi_x^*}] & \text{for } \phi \geq \phi_x^* \\ 0 & \text{otherwise.} \end{cases} \tag{23}$$

Then, total employment used for production by a firm with productivity ϕ is as follows:

$$l(\phi) \equiv l_d(\phi) + Nl_x(\phi). \tag{24}$$

2.4 Frontier technology

The formation of the frontier technology follows Aghion and Howitt (1998, Ch. 3), Acemoglu et al. (2006), and Kishi (2016). Assume that a certain technology $\bar{\varphi}(t)$ exists, whose initial value corresponds to the world frontier technology in the initial period, that is, $\bar{\varphi}(0) = \max\{\varphi(\omega)|\omega \in \Omega_W(0)\}$, where $\Omega_W(0)$ represents the set of all the varieties of goods in the world (all countries) at time $t = 0$. Define g as the growth rate of the technology, that is, $g \equiv \dot{\bar{\varphi}}(t)/\bar{\varphi}(t)$, which is an exogenous variable. We can show that the technology coincides with the world frontier technology in all time periods, that is, $\bar{\varphi}(t) = \max\{\varphi(\omega)|\omega \in \Omega_W(t)\}$ for all $t \geq 0$, because later we show that some varieties ω always exist whose technological level is $\bar{\varphi}(t)$, and productivity $\varphi(\omega)$ cannot exceed $\bar{\varphi}(t)$ for all t. It does not matter whether we define $\bar{\varphi}$ as the domestic frontier technology or world frontier technology since we consider symmetric countries.

Define $\mu(\phi)$ as the stationary probability density function of an active firm's log of relative productivity ϕ. The support of the distribution is $\phi \in [0, \bar{\phi}]$, because the minimum support is $\ln[\varphi^*(t)/\varphi^*(t)] = 0$ and the maximum support is $\bar{\phi} \equiv \ln[\bar{\varphi}(t)/\varphi^*(t)]$. If the stationary distribution $\mu(\phi)$ exists, the support of the distribution must be constant over time. Therefore, the growth rate of $\varphi^*(t)$ must be equal to that of $\bar{\varphi}(t)$ in the balanced-growth equilibrium, that is, $g = \dot{\varphi}^*(t)/\varphi^*(t)$.

2.5 Technology diffusion and innovation

Innovations result from research and development (R&D) activity. By employing the fixed amount $f_e > 0$ of labor, a new entrant (R&D firm) produces one unit of a new intermediate product. The productivity of the new product is a random variable that takes either the frontier technology or the non-frontier technologies owned by existing firms. More precisely, as in Kishi (2016) and Benhabib et al. (2017), we consider the following R&D activities, which combine the elements of Aghion and Howitt (1998, Ch. 3), Lucas and Moll (2014), and Perla and Tonetti (2014). New entrants can attain frontier technology $\bar{\varphi}$ with exogenous probability $p \in (0, 1)$. This setup follows Aghion and Howitt (1998, Ch. 3). However, with probability $1 - p$, new entrants draw their productivity from the endogenous domestic productivity distribution $\mu(\phi)$. This setup follows Lucas and Moll (2014) and Perla and Tonetti (2014). It does not matter whether new entrants draw their productivity from either the domestic or the global productivity distribution because we focus on symmetric countries.

In summary, each new entrant tries to improve its new product productivities (e.g., technological level, product management, product quality) by adopting the knowledge of existing firms. These knowledge adoptions (diffusions) create new products equipped with frontier knowledge with probability p. However, the knowledge adoptions are incomplete with probability $1-p$ in the sense that they create new products whose individual productivities are below the frontier.

2.6 Value of the firm and entry

Define the value of the firm with relative productivity ϕ as $v(\phi)$, which is the sum of discounted total profits (19). Given that firm exits form the market when productivity ϕ reaches the exit cutoff $\phi = 0$, the value of the firm must follow the following Hamilton-Jacobi-Bellman (HJB) equation:[5]

$$rv(\phi) = \pi(\phi) - gv'(\phi) \text{ for all } \phi > 0, \tag{25}$$

the value-matching condition

$$v(0) = 0, \tag{26}$$

and the smooth-pasting condition

$$v'(0) = 0. \tag{27}$$

The value-matching condition (26) ensures that the firm at the exit cutoff $\phi = 0$ is indifferent between continuation, whose value is $v(0)$, and exit, whose value is zero. According to Eqs. (15), (16), and (19), Eq. (25) at exit cutoff $\phi = 0$ becomes $rv(0) = -gv'(0)$. Then, to satisfy the value-matching condition (26), we require the additional condition $v'(0) = 0$, which is the smooth-pasting condition (27).[6]

[5] The second term on the right-hand side of Eq. (25) represents the capital gain $dv(\phi)/dt = -gv'(\phi)$.

[6] See, for example, Dixit and Pindyck (1994) and Stokey (2009) for the smooth-pasting condition.

Since total profits (19) are the sum of domestic profit $\pi_d(\phi)$ and export profit $N\pi_x(\phi)$ globally, the value function $v(\phi)$ can be written as

$$v(\phi) = v_d(\phi) + Nv_x(\phi), \tag{28}$$

where $v_d(\phi)$ is the sum of discounted domestic profits $\pi_d(\phi)$ and $v_x(\phi)$ is the sum of discounted export profits $\pi_x(\phi)$. Then, $v_d(\phi)$ must follow the following HJB equation:

$$rv_d(\phi) = \pi_d(\phi) - gv_d'(\phi) \text{ for all } \phi > 0, \tag{29}$$

the value-matching condition

$$v_d(0) = 0, \tag{30}$$

and the smooth-pasting condition

$$v_d'(0) = 0. \tag{31}$$

Given that each firm exports the product if and only if $\phi \geq \phi_x^*$, $v_x(\phi)$ must follow the following HJB equation:

$$rv_x(\phi) = \pi_x(\phi) - gv_x'(\phi) \text{ for all } \phi > \phi_x^*, \tag{32}$$

the value-matching condition

$$v_x(\phi_x^*) = 0, \tag{33}$$

and the smooth-pasting condition

$$v_x'(\phi_x^*) = 0. \tag{34}$$

According to Eq. (29), the boundary conditions (30) and (31), and the domestic profit (15), we obtain

$$v_d(\phi) = \frac{fe^{(\sigma-1)\phi}}{r+(\sigma-1)g} - \frac{f}{r} + \left(\frac{f}{r}\right)\left[\frac{(\sigma-1)g}{r+(\sigma-1)g}\right]e^{-rT(\phi)}, \tag{35}$$

where $T(\phi) \equiv \phi/g$ represents the waiting time until the firm with current productivity ϕ exits the domestic market. $\phi = (\phi - 0)$ of the numerator of $T(\phi)$ is the distance from current productivity ϕ to exit cutoff $\phi = 0$. On the contrary, g in the denominator of $T(\phi)$ is the speed to run the given distance $(\phi - 0)$. The sum of the first and second terms on the right-hand side of Eq. (35) represents the discounted value from the domestic market, in which firms continue to operate forever despite losses. Then, the third term is the additional value of the option to exit in the future. The option value decreases in ϕ and converges to zero as $\phi \to \infty$. This is because the exit option is invoked in the remote future when ϕ becomes very large, as captured by $T(\phi)$.

Similarly, according to Eq. (32), the boundary conditions (33) and (34), and the export profit (16), we obtain

$$v_x(\phi) = \left(\frac{1}{\tau}\right)^{\sigma-1}\left[\frac{fe^{(\sigma-1)\phi}}{r+(\sigma-1)g}\right] - \left(\frac{1}{\tau}\right)^{\sigma-1}e^{(\sigma-1)\phi_x^*}\left(\frac{f}{r}\right) + \left(\frac{1}{\tau}\right)^{\sigma-1}e^{(\sigma-1)\phi_x^*}\left(\frac{f}{r}\right)\left[\frac{(\sigma-1)g}{r+(\sigma-1)g}\right]e^{-rT_x(\phi)}, \tag{36}$$

where $T_x(\phi) \equiv (\phi - \phi_x^*)/g$ represents the waiting time until the firm with current productivity ϕ exits the export market. $(\phi - \phi_x^*)$ of the numerator of $T_x(\phi)$ is the distance from current productivity ϕ to export cutoff ϕ_x^*. On the contrary, g in the denominator of $T_x(\phi)$ is the speed to run the given distance $(\phi - \phi_x^*)$. Similarly, the sum of the first and second terms on the right-hand side of Eq. (36) represents the discounted value from the foreign market, in which firms continue to export forever despite losses. Then, the third term is the additional value of the option to exit the export market in the future. The option value decreases in ϕ and converges to zero as $\phi \to \infty$. This is because the exit option is invoked in the remote future when ϕ becomes very large, as captured by $T_x(\phi)$.

We now derive the free-entry condition. Recall that each new entrant can produce one unit of a new variety of the good by using the fixed amount of labor $f_e > 0$. New firms adopt frontier technology $\bar{\phi}$ with exogenous probability $p \in (0, 1)$, while firms adopt the productivity drawn from the endogenous productivity distribution $\mu(\phi)$ with probability $1 - p$. Then, if new firms enter the market, the following free-entry condition must hold:

$$f_e = v_e, \tag{37}$$

where

$$v_e \equiv (1-p)\int_0^{\bar{\phi}} v(\phi)\mu(\phi)d\phi + pv(\bar{\phi}). \tag{38}$$

v_e represents the expected benefit from entry. The first term on right-hand side of Eq. (38) represents the expected value from innovation á la Lucas and Moll (2014) and Perla and Tonetti (2014). The second term represents the expected value from innovation á la Aghion and Howitt (1998, Ch. 3).

2.7 Labor and asset markets

Define $\bar{M}$ as the measure of Ω, which is the set of the available varieties of goods in a typical country, M as the measure of the set of the varieties of domestic goods in a typical country, and M_x as the measure of the set of the varieties of exported goods in a typical country. Then, the following equation holds because of the presence of symmetric countries:

$$\bar{M} = M + NM_x. \tag{39}$$

That is, the total number $\bar{M}$ of varieties used in a country is the sum of the number M of domestic varieties and the number NM_x of imported varieties.

The labor market-clearing condition is

$$L = M\int_0^{\bar{\phi}} l(\phi)\mu(\phi)d\phi + M_e f_e, \tag{40}$$

where M_e represents the number of new entrants per unit of time. Then, according to Eqs. (39) and (40), given the lack of population growth, the following equation must hold in a balanced-growth equilibrium:

$$0 = \frac{\dot{\bar{M}}}{\bar{M}} = \frac{\dot{M}}{M} = \frac{\dot{M}_x}{M_x} = \frac{\dot{M}_e}{M_e}. \tag{41}$$

Next, consider the asset market-clearing condition. Suppose no financial flows across countries. Then, the asset market-clearing condition requires the aggregate assets owned by the representative household to be equal to the aggregate equity value of all domestic firms:

$$aL = M \int_0^{\bar{\phi}} v(\phi)\mu(\phi)d\phi. \tag{42}$$

Supposing either a closed or an open financial market is irrelevant because of the presence of symmetric countries.

3 Productivity distribution

In this section, we derive the endogenous productivity distribution of active firms. We then provide the economic intuitions behind the distribution.

3.1 Stationary productivity distribution

We derive the stationary productivity distribution $\mu(\phi)$ for the log of relative productivity ϕ. We divide time into short intervals of duration $\Delta t > 0$, and the ϕ space into short segments, each of length $\Delta h \equiv g\Delta t$. Productivity ϕ falls by $\dot{\phi}\Delta t = -g\Delta t \equiv -\Delta h$ during time interval Δt. Define the entry rate of new entrants per unit of time as

$$\epsilon \equiv \frac{M_e}{M}, \tag{43}$$

which is constant over time in a balanced-growth equilibrium.

Now, consider the segment centered on $\phi \in (0, \bar{\phi})$, which starts with the number $M\mu(\phi, t)\Delta h$ of varieties at time t. In the next unit time period Δt, all these varieties move to the left segment $\phi - \Delta h$ because of the obsolescence of (reduction in) the log of relative productivity ϕ. New entrants as well as varieties from the right segment $\phi + \Delta h$ arrive to take their places ϕ. Therefore, the evolution of the productivity distribution is

$$M\mu(\phi, t)\Delta h = M\mu(\phi + \Delta h, t - \Delta t)\Delta h + (1 - p)M_e\Delta t\mu(\phi, t - \Delta t)\Delta h, \tag{44}$$

for all $\phi \in (0, \bar{\phi})$, where $\mu(\phi, t)$ represents the probability density function of the log of relative productivity ϕ at time t. The left-hand side of Eq. (44) represents the total number of varieties (located in segment ϕ at time t), the first term on the right-hand side is the inflow of varieties into segment ϕ at time t because of the obsolescence of the log of relative productivity, and the second term on the right-hand side is the inflow of varieties into segment ϕ at time t because of innovations. By canceling the common factor Δh in Eq. (44), dividing both sides by M, and noting that Eq. (43) holds, we yield

$$\mu(\phi, t) = \mu(\phi + \Delta h, t - \Delta t) + (1 - p)\epsilon\Delta t\mu(\phi, t - \Delta t). \tag{45}$$

Expanding $\mu(\phi + \Delta h, t - \Delta t)$ around $\mu(\phi, t)$ following Taylor's theorem yields

$$\mu(\phi + \Delta h, t - \Delta t) = \mu(\phi, t) + \frac{\partial\mu(\phi, t)}{\partial\phi}\Delta h - \frac{\partial\mu(\phi, t)}{\partial t}\Delta t. \tag{46}$$

Higher-order terms of order such as $(\Delta t)^2$ and $(\Delta t)^3$ approach zero faster than Δt, and thus these terms are omitted from Eq. (46). Substituting Eq. (46) into Eq. (45), dividing both sides by Δt, imposing $\Delta t \to 0$, and simplifying yields the Kolmogorov forward equation (KFE):[7]

$$\frac{\partial \mu(\phi, t)}{\partial t} = g\frac{\partial \mu(\phi, t)}{\partial \phi} + (1-p)\epsilon\mu(\phi, t) \text{ for all } \phi \in (0, \bar{\phi}). \tag{47}$$

In the stationary productivity distribution, $\partial\mu(\phi, t)/\partial t = 0$ must hold. That is, the productivity distribution must be independent of time t. Then, the KFE (47) has the following stationary form:

$$0 = g\mu'(\phi) + (1-p)\epsilon\mu(\phi) \text{ for all } \phi \in (0, \bar{\phi}). \tag{48}$$

We now derive the boundary conditions for the stationary distribution. First, consider the segment at $\bar{\phi}$. The outflow from the varieties during time interval Δt is $M\mu(\bar{\phi})\Delta h$ because of the obsolescence of relative productivity. The inflow of varieties into segment $\bar{\phi}$ during time interval Δt is $pM_e\Delta t + (1-p)M_e\Delta t\mu(\bar{\phi})\Delta h$ because of innovations. In the stationary distribution, the inflow and outflow must be equal:

$$M\mu(\bar{\phi})\Delta h = pM_e\Delta t + (1-p)M_e\Delta t\mu(\bar{\phi})\Delta h. \tag{49}$$

Then, dividing both sides of Eq. (49) by $M\Delta t$, noting that Eq. (43) holds, imposing $\Delta t \to 0$, and simplifying yields

$$\mu(\bar{\phi}) = \frac{p\epsilon}{g}. \tag{50}$$

Next, consider the boundary condition for the segment at exit cutoff $\phi = 0$. Define $\delta\Delta t$ as the share of the varieties that reach exit cutoff $\phi = 0$ during interval Δt. That is, the following equation holds:

$$M\delta\Delta t = M\mu(\Delta h)\Delta h, \tag{51}$$

where $M\delta\Delta t$ is the number of exiting firms, which equals the number $M\mu(0 + \Delta h)\Delta h$ of firms located next to exit cutoff $\phi = 0$. Then, dividing both sides of Eq. (51) by Δt, imposing $\Delta t \to 0$, and simplifying yields

$$\delta = \mu(0)g. \tag{52}$$

Finally, we derive the relationship between entry rate ϵ and exit rate δ in a balanced-growth equilibrium. According to Eq. (41), the number M of varieties is constant over time. Then, the inflow of new entrants and outflow of incumbents must be equal:

$$M_e\Delta t = \delta M\Delta t, \tag{53}$$

where the left-hand side represents the inflow of new entrants during time interval Δt and the right-hand side represents the outflow of incumbents (the number of exiting firms) during time interval Δt. Then,

[7] See, for example, Dixit and Pindyck (1994), Shreve (2004), and Stokey (2009) for the KFE.

by dividing both sides of Eq. (53) by $M\Delta t$, according to Eq. (43), we obtain

$$\epsilon = \delta. \tag{54}$$

That is, entry rate ϵ is equal to exit rate δ in the balanced-growth equilibrium.

By solving the ordinary differential equation (48) by imposing the condition $1 = \int_0^{\bar{\phi}} \mu(\phi)d\phi$ and noting Eqs. (52) and (54), we obtain the following lemma.

Lemma 1 *The following equations describe the stationary distribution of the log of relative productivity ϕ:*

$$\mu(\phi) = \left(\frac{\eta}{1-p}\right) e^{-\eta\phi} \text{ for } \phi \in [0, \bar{\phi}], \tag{55}$$

where

$$\eta \equiv \frac{(1-p)\epsilon}{g} > 0 \text{ and} \tag{56}$$

$$\bar{\phi} = \left(\frac{1}{\eta}\right) \ln\left(\frac{1}{p}\right) > 0. \tag{57}$$

Lemma 1 implies that $\mu(\bar{\phi}) = \lim_{\phi \uparrow \bar{\phi}} \mu(\phi)$. Then, the continuity at $\bar{\phi}$ holds in the stationary distribution, $\mu(\phi)$. That is, we can show that the stationary distribution (55) satisfies the boundary condition (50), even though we do not impose the condition (50) to solve the differential equation (48).

Lemma 1 shows that relative productivity ϕ follows the exponential distribution with the bounded support. This finding implies that absolute productivity φ follows the Pareto (power law) distribution with bounded support $\varphi \in [\varphi^*, \bar{\varphi}]$. The variable η is referred to as the Pareto exponent or shape parameter in the literature.

3.2 Intuitions behind the productivity distribution

The intuition behind Lemma 1 is similar to that of Kishi (2016). This subsection applies the intuitions discussed in Kishi (2016) to the presented model.

As shown in Lemma 1, the slope of the productivity distribution is always negative. We consider the economic intuition behind this result. Since we consider the stationary distribution, $\mu(\phi, t)$ is independent of time, that is, $\mu(\phi)$ holds. Then, by noting Eq. (43), we can rewrite Eq. (44) as

$$\mu(\phi)\Delta h = \mu(\phi + \Delta h)\Delta h + (1-p)\epsilon\Delta t\mu(\phi)\Delta h. \tag{58}$$

The left-hand side of Eq. (58) is the share of firms (varieties) around ϕ. The right-hand side represents the composition of the share of firms around ϕ. The first term $\mu(\phi + \Delta h)\Delta h$ of the right-hand side is the share (inflow) of incumbents, which comes from the right-hand neighboring segment $\phi + \Delta h$ because of the obsolescence of (reduction in) the log of relative productivity ϕ. The second term $(1-p)\epsilon\Delta t\mu(\phi)\Delta h$ is the share (inflow) of new entrants, which comes from innovations. The sum of these firms corresponds to the total share $\mu(\phi)\Delta h$ of firms equipped with around ϕ. At frontier technology $\bar{\phi}$, as shown in Fig. 1, there is no inflow of incumbents, while there is an inflow of new entrants that attain $\bar{\phi}$. After the passage of

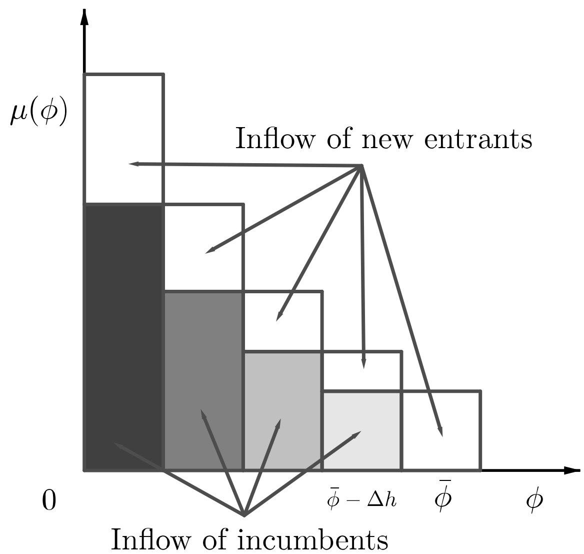

Figure 1: Productivity distribution $\mu(\phi)$ in discrete time.

time interval Δt, the share $\mu(\bar{\phi})\Delta h$ of frontier firms moves to the left-hand neighboring segment $(\bar{\phi}-\Delta h)$ because of the obsolescence of their productivities. Furthermore, the share $(1-p)\epsilon\Delta t\mu(\bar{\phi}-\Delta h)\Delta h$ of new entrants comes into segment $(\bar{\phi}-\Delta h)$ during the time interval because of innovations, which attain productivity $(\bar{\phi}-\Delta h)$. Therefore, the total share of firms in segment $(\bar{\phi}-\Delta h)$ is larger than that in segment $\bar{\phi}$. By maintaining these dynamics of the share of firms for each segment until exit cutoff $\phi=0$, as shown in Fig. 1, productivity distribution $\mu(\phi)$ draws the negative slope for all ϕ.

Next, we consider the economic intuition behind the Pareto exponent η of distribution $\mu(\phi)$. According to Fig. 2, a higher η causes a larger proportion of firms (varieties) around exit cutoff $\phi=0$. According to Eq. (56), an increase in entry rate ϵ raises η. The reasons behind the results are as follows. New entrants tend to develop goods equipped with low productivity since productivity distribution $\mu(\phi)$ has a negative slope because of the repeated summation of the number of new entrants and number of incumbents located in the right-hand neighboring segment. Then, promoting entry rate ϵ leads to a larger inflow of new entrants into the lower productivity region, which results in a higher η. That is, a new entrant tends to meet a low-productive incumbent and then adopt a low-productive technology because of the large number of existing low-productive firms in the economy. This mechanism causes the low-productive entrant effect after the impact of trade liberalization, as shown in Sections 4 and 5.[8]

There is another interpretation of η. According to Eq. (54), a higher entry rate ϵ leads to a higher exit rate, δ, in the balanced-growth equilibrium. Therefore, firms tend to locate around the exit cutoff, $\phi=0$, as shown in Fig. 2, which implies a higher η.

[8]The average productivity of entrants is larger than that of incumbents because entrants adopt the frontier technology with probability $p\in(0,1)$. In this study, the low-productive entrant effect means that *if the entry rate increases*, it causes a larger inflow and thus the accumulation of low-productive firms, as shown in Fig. 2.

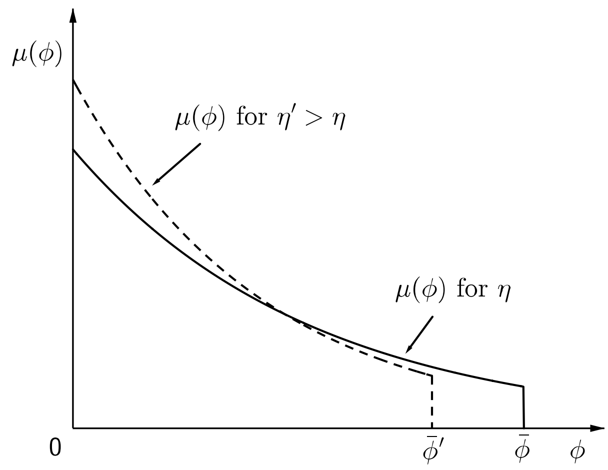

Figure 2: Effects of Pareto exponent η on productivity distribution $\mu(\phi)$.

4 Balanced-growth equilibrium

In this section, we show the existence of the balanced-growth equilibrium and investigate the impact of trade liberalization on the productivity distribution.

4.1 Growth rate

Consider the determination of the economic growth rate. Differentiating Eq. (12) with respect to time t, recalling that $g = \dot{\varphi}^*/\varphi^*$, noting that $\dot{Q}/Q = \dot{c}/c$ from Eq. (10), according to Eq. (5), and simplifying yields

$$\frac{\dot{Q}}{Q} = \frac{\dot{c}}{c} = g. \tag{59}$$

Therefore, the exogenous growth rate g of the world frontier technology coincides with the economic growth rate in the balanced-growth equilibrium; in other words, the model is an exogenous growth model.

If we were to develop additional structures such that the growth g of the frontier technology becomes an endogenous variable, the model would become an endogenous growth model. However, this is beyond the scope of this study. Instead, Appendix C presents the model under $p = 0$, showing that it becomes an endogenous growth model with an exogenous productivity distribution. This result corresponds with those of Lucas and Moll (2014), Perla and Tonetti (2014), Perla et al. (2015), and Sampson (2016). Then, we show that trade liberalization raises the economic growth rate (see Proposition 5 in Appendix C).[9] Further, the model under $p = 0$ does not have a scale effect, suggesting that the growth rate is independent of population size L (see Appendix C).

To satisfy the transversality condition (4), according to Eqs. (5) and (59), the following equation must

[9]Following the seminal international trade models of Eaton and Kortum (2002) and Melitz (2003), researchers have discussed whether trade liberalization enhances growth by extending their models; see, for example, Eaton and Kortum (2001), Dinopoulos and Unel (2011, 2013), Perla et al. (2015), Wu (2015), Ourens (2016), Sampson (2016), and Naito (2017a, 2017b).

hold:

$$-\rho - \theta\frac{\dot{c}}{c} + \frac{\dot{a}}{a} - \frac{\dot{P}}{P} < 0 \iff r = \rho + (\theta - 1)g > 0. \tag{60}$$

For convenience, we restate the above assumption.

Assumption 2 *The equilibrium interest rate r is positive:*

$$r = \rho + (\theta - 1)g > 0. \tag{61}$$

4.2 Equilibrium shape of the productivity distribution

This subsection shows the existence of equilibrium value η and the balanced-growth equilibrium as well as the impact of trade liberalization on the productivity distribution.

The free-entry condition (37) determines the equilibrium Pareto exponent η of the productivity distribution since the expected benefit of entry v_e is a function of η. To show this, define μ_x as

$$\mu_x \equiv \int_{\phi_x^*}^{\bar{\phi}} \mu(\phi) d\phi = \frac{e^{-\eta\phi_x^*} - p}{1 - p}, \tag{62}$$

where μ_x represents both the share of exporting firms in a typical country and the probability of becoming an exporting new entrant conditional on drawing probability $1 - p$. Further, define $\mathbb{E}[\cdot]$ and $\mathbb{E}_x[\cdot]$ as $\mathbb{E}[X(\phi)] \equiv \int_0^{\bar{\phi}} X(\phi)\mu(\phi)d\phi$ and $\mathbb{E}_x[X(\phi)] \equiv \int_{\phi_x^*}^{\bar{\phi}} X(\phi)[\mu(\phi)/\mu_x]d\phi$, respectively. Then, according to Eqs. (28), (35), (36), (38), and (55), we derive the expected benefit v_e of entry:

$$v_e = (1 - p)v_d + (1 - p)\mu_x N v_x + p v_d(\bar{\phi}) + pN v_x(\bar{\phi}), \tag{63}$$

where

$$v_d \equiv \int_0^{\bar{\phi}} v_d(\phi)\mu(\phi)d\phi = \left[\frac{f}{r + (\sigma - 1)g}\right] \mathbb{E}[e^{(\sigma-1)\phi}] - \frac{f}{r} + \left(\frac{f}{r}\right)\left[\frac{(\sigma - 1)g}{r + (\sigma - 1)g}\right] \mathbb{E}[e^{-rT(\phi)}] \tag{64}$$

represents the expected value of the firm from a domestic market and

$$\begin{aligned} v_x &\equiv \int_{\phi_x^*}^{\bar{\phi}} v_x(\phi)\left(\frac{\mu(\phi)}{\mu_x}\right) d\phi \\ &= \left(\frac{1}{\tau}\right)^{\sigma-1}\left[\frac{f}{r + (\sigma - 1)g}\right] \mathbb{E}_x[e^{(\sigma-1)\phi}] - \left(\frac{1}{\tau}\right)^{\sigma-1} e^{(\sigma-1)\phi_x^*}\left(\frac{f}{r}\right) \\ &\quad + \left(\frac{1}{\tau}\right)^{\sigma-1} e^{(\sigma-1)\phi_x^*}\left(\frac{f}{r}\right)\left[\frac{(\sigma - 1)g}{r + (\sigma - 1)g}\right] \mathbb{E}_x[e^{-rT_x(\phi)}], \end{aligned} \tag{65}$$

represents the expected value of the firm from an export market conditional on exporting firms. Since $(\varphi/\varphi^*)^{\sigma-1} = e^{(\sigma-1)\phi}$, $\mathbb{E}[e^{(\sigma-1)\phi}]$ represents average relative productivity across all varieties:

$$\mathbb{E}[e^{(\sigma-1)\phi}] = \int_0^{\bar{\phi}} e^{(\sigma-1)\phi}\mu(\phi)d\phi = \left[\frac{\eta}{\eta - (\sigma - 1)}\right]\left(\frac{1 - pe^{(\sigma-1)\bar{\phi}}}{1 - p}\right). \tag{66}$$

$\mathbb{E}[e^{-rT(\phi)}]$ represents the average discount factor in the event of reaching exit cutoff $\phi = 0$:

$$\mathbb{E}[e^{-rT(\phi)}] = \int_0^{\bar{\phi}} e^{-rT(\phi)} \mu(\phi) d\phi = \left(\frac{\eta}{\eta + \frac{r}{g}}\right)\left(\frac{1 - pe^{-\frac{r}{g}\bar{\phi}}}{1-p}\right). \tag{67}$$

$\mathbb{E}_x[e^{(\sigma-1)\phi}]$ represents average relative productivity conditional on exporting firms:

$$\mathbb{E}_x[e^{(\sigma-1)\phi}] = \int_{\phi_x^*}^{\bar{\phi}} e^{(\sigma-1)\phi}\left(\frac{\mu(\phi)}{\mu_x}\right) d\phi = \left(\frac{1}{\mu_x}\right)\left[\frac{\eta}{\eta - (\sigma - 1)}\right]\left[\frac{e^{-[\eta-(\sigma-1)]\phi_x^*} - pe^{(\sigma-1)\bar{\phi}}}{1-p}\right]. \tag{68}$$

$\mathbb{E}_x[e^{-rT_x(\phi)}]$ represents the average discount factor in the event of reaching export cutoff ϕ_x^* conditional on exporting firms:

$$\mathbb{E}_x[e^{-rT_x(\phi)}] = \int_{\phi_x^*}^{\bar{\phi}} e^{-rT_x(\phi)}\left(\frac{\mu(\phi)}{\mu_x}\right) d\phi = \left(\frac{e^{\frac{r}{g}\phi_x^*}}{\mu_x}\right)\left(\frac{\eta}{\eta + \frac{r}{g}}\right)\left[\frac{e^{-(\eta+\frac{r}{g})\phi_x^*} - pe^{-\frac{r}{g}\bar{\phi}}}{1-p}\right]. \tag{69}$$

From Eq. (57), v_e becomes a function of η. Then, the free-entry condition (37) determines the equilibrium Pareto exponent η of the productivity distribution.

If we can yield an equilibrium Pareto exponent η of the productivity distribution, the other endogenous variables can automatically be determined, as shown in Section 4.3, allowing us to ensure the existence of the balanced-growth equilibrium.

Proposition 1 *Suppose that Assumptions 1 and 2 hold. Then, if entry cost f_e is sufficiently large, there exists a balanced-growth equilibrium.*

Proof. See Appendix A. ■

In the equilibrium, $\bar{\phi} > \phi_x^* \iff \eta < (1/\phi_x^*)\ln(1/p)$ must hold to ensure the existence of exporting firms. A sufficiently large f_e causes a sufficiently small equilibrium η, which ensures that $\bar{\phi} > \phi_x^*$. More precisely, the large fixed cost f_e of entry discourages the entry of new firms, which raises average relative productivity because new entrants tend to draw a lower existing productivity from distribution $\mu(\phi)$. Therefore, the discouragement of new entry caused by a large f_e raises average relative productivity, which implies a lower η, as shown in Fig. 2. Then, in Proposition 1, we require the condition that f_e is sufficiently large to ensure the existence of the balanced-growth equilibrium with $\bar{\phi} > \phi_x^*$. However, when $p \to 0$, $\bar{\phi} > \phi_x^* \iff \eta < (1/\phi_x^*)\ln(1/p)$ is satisfied without the assumption of a sufficiently large f_e. See Appendix B for more details, where Lemma 3 shows the uniqueness of η under the additional assumption $p \to 0$.

Proposition 2 *Suppose that Assumptions 1 and 2 as well as $p \to 0$ hold. Then,*

(i) A decrease in iceberg cost τ increases the equilibrium Pareto exponent η of the productivity distribution;

(ii) A decrease in the fixed cost f_x of the exporting varieties increases the equilibrium Pareto exponent η of the productivity distribution.

Proof. See Appendix B. ■

Proposition 2 implies that trade liberalization via a reduction in τ has a negative effect on average productivity in the economy, as implied in Fig. 2. The following average productivities $\mathbb{E}[e^{(\sigma-1)\phi}]$ and $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ affect welfare, as shown in Section 4.3. Therefore, we now conduct the comparative statics of these average productivities.

Proposition 3 *Suppose that Assumptions 1 and 2 as well as $p \to 0$ hold. Then,*

(i) A decrease in iceberg cost τ decreases average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$ across all varieties;

(ii) A decrease in iceberg cost τ decreases average relative productivity $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ across all exporting (imported) varieties;

(iii) A decrease in the fixed cost f_x of the exporting varieties decreases average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$ across all varieties;

(iv) A decrease in the fixed cost f_x of the exporting varieties decreases average relative productivity $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ across all exporting (imported) varieties.

Proof. See Appendix B. ■

Trade liberalization via a reduction in τ or f_x increases the equilibrium Pareto exponent η of the productivity distribution. This then reduces $\mathbb{E}[e^{(\sigma-1)\phi}]$ and $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ for the following reason. A reduction in τ or f_x increases the expected entry value v_e for a given η since each new entrant expects to earn larger export profits because of lower trade barriers. This then fosters the entry of firms into the market. New entrants tend to have low productivity since productivity distribution $\mu(\phi)$ has a negative slope, as explained in Section 3.2. Therefore, fostering the entry of new firms through trade liberalization reduces $\mathbb{E}[e^{(\sigma-1)\phi}]$ and $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ because of the diffusion of inferior existing technologies. That is, trade liberalization causes the low-productive entrant effect.

4.3 Welfare measure

To prepare for the numerical welfare analysis in Section 5, we consider the determination of welfare and the other variables in the model.

The initial consumption level or real income is the steady-state welfare measure in the model. Define $v \equiv \int_0^{\bar{\phi}} v(\phi)\mu(\phi)d\phi = v_d + N\mu_x v_x$ as the average asset value across all firms. Given that the aggregate asset value is $aL = Mv = M(v_d + N\mu_x v_x)$, according to Eqs. (6) and (42), equilibrium consumption per capita is

$$c = \frac{1 + ra}{P}, \tag{70}$$

where asset value per capita is

$$a = \frac{Mv}{L} = \frac{M(v_d + N\mu_x v_x)}{L}. \tag{71}$$

That is, initial consumption is equal to real income. As we develop the exogenous growth model, we see that initial consumption c, determined from Eq. (70), only affects welfare in the balanced-growth equilibrium. Therefore, the variable c is the welfare measure in the presented model.

Derive the price index as follows:

$$P = (Mp_d^{1-\sigma} + NM_x p_x^{1-\sigma})^{\frac{1}{1-\sigma}}, \tag{72}$$

where

$$p_d^{1-\sigma} \equiv \int_0^{\bar{\phi}} p_d(\phi)^{1-\sigma}\mu(\phi)d\phi = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1} \tag{73}$$

$$p_x^{1-\sigma} \equiv \int_{\phi_x^*}^{\bar{\phi}} p_x(\phi)^{1-\sigma}\left(\frac{\mu(\phi)}{\mu_x}\right)d\phi = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tau^{1-\sigma}\mathbb{E}_x[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}. \tag{74}$$

The variables p_d and p_x represent the weighted averages of domestic prices across all domestic varieties and of import prices across all imported varieties, respectively. Both p_d and p_x are functions of η. Note that $\mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}$ represents average absolute productivity $\varphi^{\sigma-1}$ since $\varphi^{\sigma-1} = (\varphi/\varphi^*)^{\sigma-1}(\varphi^*)^{\sigma-1} = e^{(\sigma-1)\phi}(\varphi^*)^{\sigma-1}$. Then, noting Eq. (57) and $\bar{\phi} = \ln(\bar{\varphi}/\varphi^*)$, the average value of $\varphi^{\sigma-1}$ is $\mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1} = \mathbb{E}[e^{(\sigma-1)\phi}]e^{-(\sigma-1)\bar{\phi}}(\bar{\varphi})^{\sigma-1}$ for a given exogenous state $\bar{\varphi}$. Similarly, $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ represents average absolute productivity $\varphi^{\sigma-1}$ conditional on imported varieties, as given by $\mathbb{E}_x[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1} = \mathbb{E}_x[e^{(\sigma-1)\phi}]e^{-(\sigma-1)\bar{\phi}}(\bar{\varphi})^{\sigma-1}$ for a given exogenous state $\bar{\varphi}$.

Next, consider the labor market equilibrium, which determines the number M of domestic varieties and the number M_x of exporting varieties. Derive average labor demand for production as follows:

$$l \equiv \int_0^{\bar{\phi}} l(\phi)\mu(\phi)d\phi = l_d + \mu_x N l_x, \tag{75}$$

where

$$l_d \equiv \int_0^{\bar{\phi}} l_d(\phi)\mu(\phi)d\phi = f(\sigma-1)\mathbb{E}[e^{(\sigma-1)\phi}] + f \tag{76}$$

represents average labor demand for a domestic market and

$$l_x \equiv \int_{\phi_x^*}^{\bar{\phi}} l_x(\phi)\left(\frac{\mu(\phi)}{\mu_x}\right)d\phi = \left(\frac{1}{\tau}\right)^{\sigma-1} f[(\sigma-1)\mathbb{E}_x[e^{(\sigma-1)\phi}] + e^{(\sigma-1)\phi_x^*}]. \tag{77}$$

represents average labor demand for an export market conditional on exporting firms. From the labor market-clearing condition (40), the number of domestic varieties is

$$M = \frac{L}{l + \epsilon f_e}. \tag{78}$$

Noting $\epsilon = g\eta/(1-p)$ from Eq. (56), the right-hand side of Eq. (78) becomes a function of η. Then, if we yield an equilibrium η, Eq. (78) determines the equilibrium number M of domestic varieties. $M_x = \mu_x M$ determines the number M_x of exporting varieties in a typical country.

Following Arkolakis et al. (2012), Perla et al. (2015), and Sampson (2016), we can rewrite initial consumption c by using the domestic trade share. Define λ as the domestic trade share (i.e., the proportion of domestic revenues in total revenues):

$$\lambda \equiv \frac{M \int_0^{\bar{\phi}} R_d(\phi)\mu(\phi)d\phi}{M \int_0^{\bar{\phi}} R(\phi)\mu(\phi)d\phi}, \tag{79}$$

where

$$R(\phi) \equiv R_d(\phi) + NR_x(\phi) \tag{80}$$

represents the total revenue of a firm whose productivity is ϕ.

$$R_d(\phi) \equiv p_d(\phi)q_d(\phi) = \sigma e^{(\sigma-1)\phi} f \tag{81}$$

represents the domestic revenue of a firm whose productivity is ϕ and $q_d(\phi)$ is the amount of the intermediate good under domestic price $p_d(\phi)$.

$$R_x(\phi) \equiv p_x(\phi)q_x(\phi) = \sigma \left(\frac{1}{\tau}\right)^{\sigma-1} e^{(\sigma-1)\phi} f \tag{82}$$

represents the exporting revenue of a firm whose productivity is ϕ and $q_x(\phi)$ is the amount of the intermediate good under exporting price $p_x(\phi)$. Then, Eq. (79) becomes

$$\lambda = \frac{\mathbb{E}[e^{(\sigma-1)\phi}]}{\mathbb{E}[e^{(\sigma-1)\phi}] + N\left(\frac{1}{\tau}\right)^{\sigma-1} \mu_x \mathbb{E}_x[e^{(\sigma-1)\phi}]}. \tag{83}$$

By using the domestic trade share (83), the price index (72) becomes

$$P = \frac{\left(\frac{\sigma}{\sigma-1}\right)\lambda^{\frac{1}{\sigma-1}}}{M^{\frac{1}{\sigma-1}} \left\{\mathbb{E}[e^{(\sigma-1)\phi}]\right\}^{\frac{1}{\sigma-1}} \varphi^*}. \tag{84}$$

Therefore, initial consumption is

$$c = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{1}{\lambda}\right)^{\frac{1}{\sigma-1}} \overbrace{M^{\frac{1}{\sigma-1}} \underbrace{\left\{\mathbb{E}[e^{(\sigma-1)\phi}]\right\}^{\frac{1}{\sigma-1}} \varphi^*}_{\text{Average absolute productivity}}}^{\text{Aggregate absolute productivity}} (1 + ra). \tag{85}$$

Table 1: Calibration

Parameter	Value	Source or Target
Number of trading partners, N	1	—
Initial frontier technology, $\bar{\varphi}(0)$	1	Normalization
Population size, L	1	Normalization
Fixed production cost, f	1	Normalization
Elasticity of substitution across varieties, σ	3.8	Bernard et al. (2003)
Growth rate of the frontier technology, g	0.02	2 percent economic growth rate
Interest rate, r	0.07	9 percent real rate of return on stocks
Probability of adopting the frontier technology, p	0.0113	Relative frontier firm size= $e^{4.18}$
Iceberg trade cost, τ	1.58	85 percent domestic trade share
Fixed export cost, f_x	1.17	21 percent of firms' exports
Fixed entry cost, f_e	30.6	Zipf's law of the firm size distribution

Then, the following three variables determine initial consumption and welfare: domestic trade share λ, aggregate absolute productivity $M^{\frac{1}{\sigma-1}} \left\{\mathbb{E}[e^{(\sigma-1)\phi}]\right\}^{\frac{1}{\sigma-1}} \varphi^*$, and per capita asset level a.

5 Numerical analysis

In this section, we numerically show the effect of trade liberalization via a reduction in τ on average and aggregate productivities and welfare in the balanced-growth equilibrium. Since the effect of a reduction in f_x is similar to that in τ, we report the case of f_x in Appendix D.

5.1 Calibration

To quantify the productivity and welfare gains or losses from trade liberalization, we calibrate the model. The top panel of Table 1 reports the normalizations and preselected parameters. We set the number of trading partners N to 1.[10] That is, we investigate the counterfactual impact of the bilateral trade agreements between symmetric countries. The initial frontier technology $\bar{\varphi}(0)$ is an initial state variable, which only affects the scale of absolute exit cutoff φ^*. We normalize this to set $\bar{\varphi}(0) = 1$. According to Eq. (78), population size L only proportionally affects the number M of domestic varieties since entry rate ϵ and aggregate labor demand l are independent of L. That is, L determines the scale of M. We normalize this to set $L = 1$. The parameter choice of fixed production cost f has no substantive effect on η, which is a key variable in this paper. Indeed, according to Eq. (17), relative exporting cost $\tau^{\sigma-1} f_x/f$ affects export cutoff ϕ_x^*. Thus, from Eq. (37), the relative exporting cost and relative fixed entry cost f_e/f affect the Pareto exponent η of the productivity distribution. Therefore, we normalize fixed production cost f to unity. The elasticity of substitution across varieties of 3.8 comes from Bernard et al. (2003). This value implies that the gross markup (the ratio of price to marginal cost) is $\sigma/(\sigma-1) = 1.36$, which is in the range of 1.05–1.4 estimated by Norrbin (1993) and Basu (1996).[11] The growth rate g of the frontier

[10]Given $N = 5$, recalibrating the model to match the data in Table 1 yields $p = 0.0113$, $\tau = 2.80$, $f_x = 0.23$, and $f_e = 30.5$. Further, given $N = 10$, recalibrating yields $p = 0.0113$, $\tau = 3.59$, $f_x = 0.12$, and $f_e = 30.6$. Under both sets of calibrated parameters with $N = 5$ and $N = 10$, we confirm that the results described in Sections 5.2 and 5.3 are generally unchanged.

[11]The markup $\sigma/(\sigma-1) \in [1.05, 1.4]$ implies that $\sigma \in [3.5, 21]$. Then, given $\sigma = 3.5$, recalibrating yields $p = 0.0109$, $\tau = 1.67$, $f_x = 1.17$, and $f_e = 32.6$. Further, given $\sigma = 21$, we confirm that no set of parameters exists that satisfies the

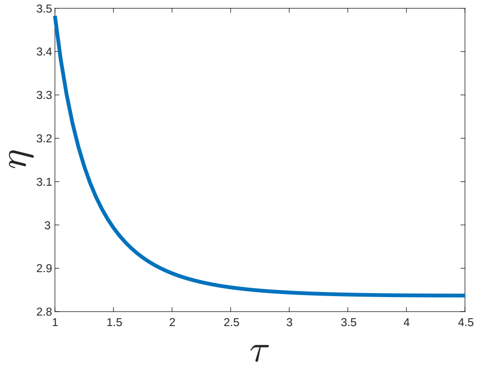

Figure 3: The impact of iceberg trade cost τ on the Pareto exponent η of the productivity distribution.

technology is set to 0.02 to target a 2 percent per capita GDP growth rate in the United States since World War II. The inverse of the intertemporal elasticity of substitution θ and discount rate ρ affect the model only through interest rate r. Then, it is sufficient to specify the value of r. Mehra and Prescott (2003) report a 9 percent average real rate of return on stocks in the United States since World War II. In the model, the real rate of return is $r - \dot{P}/P = r + g$. Given $g = 0.02$, we set $r = 0.07$ to match the historical real rate of return on stocks.[12]

Given the normalizations and preselected parameters, the bottom panel of Table 1 reports the parameters set to target the moments in the data. Benhabib et al. (2017) report that the mean of the ratio of the 90th to 10th percentiles of employment across industries for 1980–2014 in the United States is $e^{4.18}$. The 90th percentile of employment is a proxy for the frontier employment and the 10th percentile is a proxy for the least productive firm's employment. We set $p = 0.0113$ to match $l(\bar{\phi})/l(0) = e^{4.18}$. Benhabib et al. (2017) also report that the mean of the ratio of the 90th to 10th percentiles of revenue is $e^{4.39}$. Under the calibrated parameters in Table 1, our model provides $R(\bar{\phi})/R(0) = e^{4.48}$, which is close to the data. Ramondo et al. (2016) report that the average U.S. domestic trade share in manufacturing over 1996–2001 is 85 percent. To match $\lambda = 0.85$, we set $\tau = 1.58$. We choose $f_x = 1.17$ to match the proportion of U.S. manufacturing plants that exported in 1992, $\mu_x = 0.21$, as reported by Bernard et al. (2003). Luttmer (2007) estimates $\eta/(\sigma - 1) = 1.06$ based on the distributions of firm size (employment). To satisfy $\eta/(\sigma - 1) = 1.06$, we set $f_e = 30.6$.[13]

5.2 Productivity gains from trade liberalization

Given the calibrated parameters in Table 1, we study the impact of trade liberalization via a reduction in τ on the productivity measures. As shown in Fig. 3, the reduction in τ increases the Pareto exponent

data in Table 1. We confirm that $\sigma = 9.65$ is the maximum value that has a solution for the calibration. Given $\sigma = 9.65$, recalibrating yields $p = 0.0139$, $\tau = 1.16$, $f_x = 1.18$, and $f_e = 13.8$. Under both sets of calibrated parameters with $\sigma = 3.5$ and $\sigma = 9.65$, we confirm that the results described in Sections 5.2 and 5.3 are generally unchanged.

[12] Previous studies may set a lower interest rate, r. Then, given $r = 0.01,\ 0.02,\ 0.03,\ 0.04,\ 0.05,$ and 0.06, we recalibrate the model to match the data in Table 1. Under each set of calibrated parameters, we confirm that the results described in Sections 5.2 and 5.3 are generally unchanged.

[13] Zipf's law implies $\eta/(\sigma - 1) = 1$.

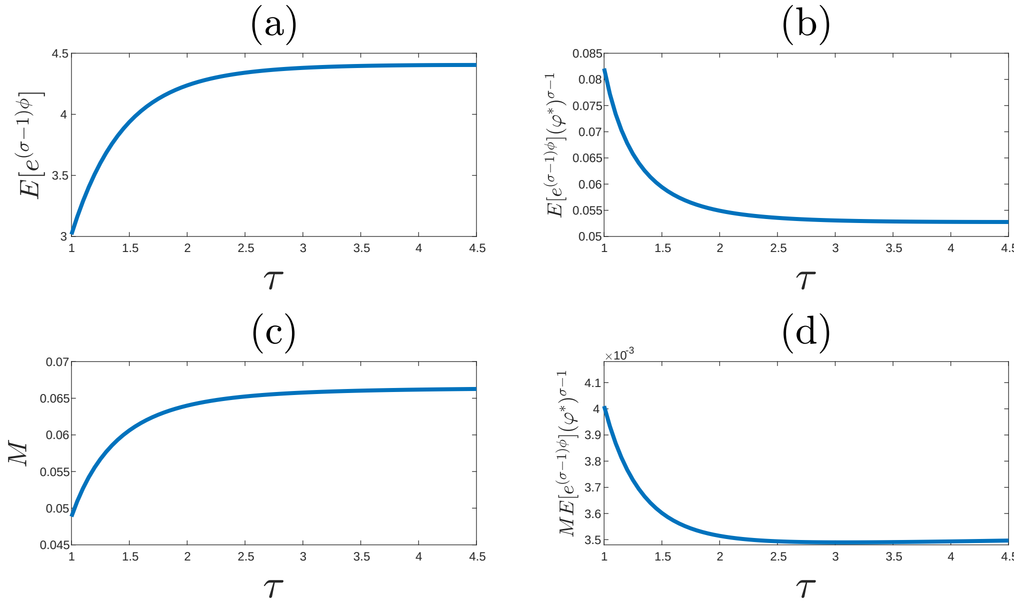

Figure 4: The impact of iceberg trade cost τ on the productivities and the mass of domestic varieties. The vertical axis in Fig. (a) represents average productivity relative to the exit cutoff. The vertical axis in Fig. (b) represents average productivity. The vertical axis in Fig. (c) represents the number of domestic varieties. The vertical axis in Fig. (d) represents aggregate productivity.

η of the productivity distribution. This numerical result is consistent with Proposition 2, which is the analytical result under $p \to 0$. The increase in η implies the low-productive entrant effect, which reduces average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$, as shown in Fig. 4(a). That is, trade liberalization induces low-productive firms to enter the market, which reduces average relative productivity. The numerical result is also consistent with result (i) of Proposition 3 and result (i) of Lemma 2 in Appendix A. As in the Melitz (2003) model, trade liberalization also has the resource reallocation effect caused by market selection. That is, the reduction in τ increases exit cutoff productivity φ^*. This is because trade liberalization reduces price index P, as explained in Section 5.3, which reduces demand for each variety (8) and negatively affects the domestic profit (11). That is, trade liberalization fosters competition between varieties. Thus, according to Eq. (12), trade liberalization has a positive effect on exit cutoff φ^* through price index P. In addition, trade liberalization has a negative effect on exit cutoff φ^* because of the increase in aggregate demand $Q = cL$ for the goods (see Fig. 6(d)). However, the negative effect on φ^* is sufficiently small to ensure that trade liberalization increases exit cutoff φ^*. Consequently, trade liberalization facilitates the exit of low-productive firms, and thus the (labor) resources employed by the exiting low-productive firms can be reallocated toward high-productive firms. This resource reallocation effect contributes to an increase in average absolute productivity in the economy. As shown in Fig. 4(b), the resource reallocation effect dominates the low-productive entrant effect; that is, the reduction in τ increases average absolute productivity $\mathbb{E}[e^{(\sigma-1)\phi}]\,(\varphi^*)^{\sigma-1}$. This numerical result is consistent with analytical result (ii) of Lemma 2 in Appendix A.

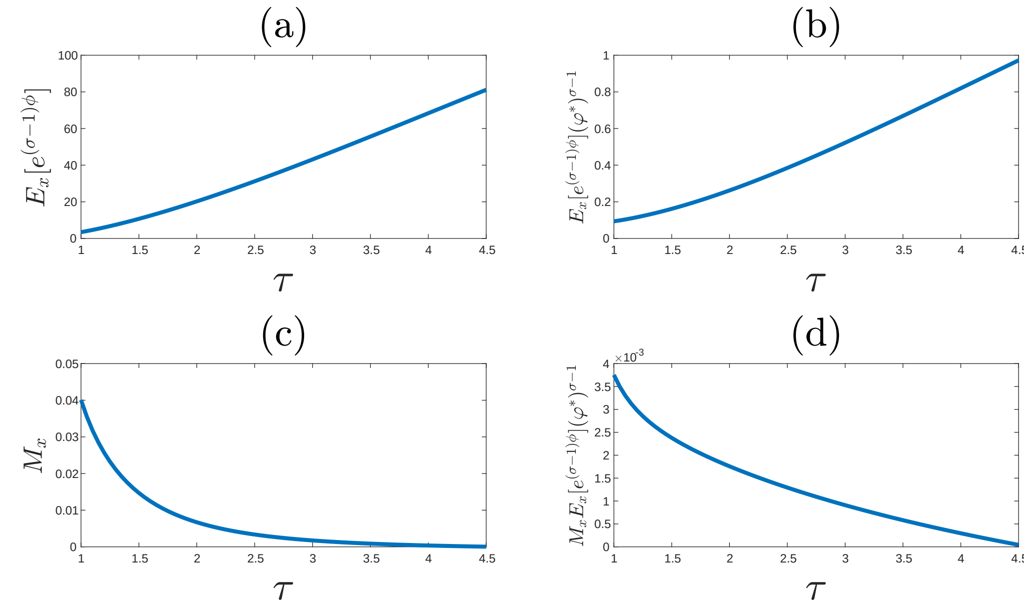

Figure 5: The impact of iceberg trade cost τ on productivities conditional on exporting firms and the mass of exporting varieties. The vertical axis in Fig. (a) represents average productivity relative to the exit cutoff conditional on exporting firms. The vertical axis in Fig. (b) represents average productivity conditional on exporting firms. The vertical axis in Fig. (c) represents the number of exporting varieties. The vertical axis in Fig. (d) represents aggregate productivity conditional on exporting firms.

As in the Melitz (2003) model, according to Fig. 4(c), trade liberalization reduces domestically produced varieties M, which has a negative effect on aggregate absolute productivity in the economy. Fig. 4(d) shows the non-monotonic (U-shaped) effect of trade liberalization on aggregate absolute productivity $M\mathbb{E}[e^{(\sigma-1)\phi}]\,(\varphi^*)^{\sigma-1}$. This result implies that the sum of the negative effect on M and low-productive entrant effect may dominate the resource reallocation effect, and thus trade liberalization may reduce aggregate productivity. Melitz (2003) shows that the resource reallocation effect dominates the negative effect on M, and thus trade liberalization always increases aggregate productivity. On the contrary, by adding the low-productive entrant effect into the Melitz (2003) model, we yield the non-monotonic (U-shaped) relationship between trade liberalization and aggregate productivity.[14] Then, according to Eq. (85), trade liberalization has a non-monotonic effect on initial consumption and welfare via aggregate productivity.

Next, we examine the impact of a reduction in τ on productivities conditional on the exporting (imported) varieties. As shown in Fig. 5(a), trade liberalization via the reduction in τ reduces average relative productivity conditional on the exporting varieties $\mathbb{E}_x[e^{(\sigma-1)\phi}]$, which is a result of the low-productive entrant effect. This numerical result is consistent with result (ii) in Proposition 3. Fig.

[14]This non-monotonic relationship holds under different values of p. We confirm that such non-monotonicity arises under smaller values of p such as $p = 0.001,\ 0.0001,$ and 0.00001, for which we set the other parameters as in Table 1. Similarly, the non-monotonicity arises under larger values of p such as $p = 0.03,\ 0.05,$ and 0.07. However, this non-monotonicity disappears when p is sufficiently large, such as $p = 0.08,\ 0.09,$ and 0.1.

5(b) shows that trade liberalization reduces average absolute productivity $\mathbb{E}_x[e^{(\sigma-1)\phi}]\,(\varphi^*)^{\sigma-1}$ conditional on exporting firms. This finding implies that the low-productive entrant effect dominates the resource reallocation effect when we focus on the average productivity of exporting firms.

According to Fig. 5(c), trade liberalization increases exporting varieties $M_x = \mu_x M$, which has a positive effect on aggregate absolute productivity conditional on exporting firms. Figs. 4(c) and 5(c) imply that trade liberalization increases the share μ_x of exporting varieties. Trade liberalization has both positive and negative effects on share μ_x. According to Eqs. (17) and (62), the reduction in τ has a positive effect on share μ_x because of the reduction in trade barriers. On the contrary, the reduction in τ increases η, which results in the low-productive entrant effect. That is, trade liberalization increases the share of low-productive firms, which reduces the share μ_x of high-productive exporting firms. Consequently, we have a dominant positive effect on μ_x, and thus the reduction in τ increases μ_x. Further, the increase in μ_x is sufficiently strong to raise M_x. Then, Fig. 5(d) shows that the reduction in τ increases aggregate absolute productivity $M_x\mathbb{E}_x[e^{(\sigma-1)\phi}]\,(\varphi^*)^{\sigma-1}$ conditional on exporting firms. This is because the positive effect on M_x and resource reallocation effect dominate the low-productive entrant effect.

We now consider the reason behind the negative effect of trade liberalization on M, as shown in Fig. 4(c). The labor market-clearing condition (78) determines the number of domestic varieties M. According to Eqs. (76) and (77), and Figs. 4(a) and 5(a), the reduction in τ causes the low-productive entrant effect, which has a negative effect on average labor demand l_d for a domestic market and average labor demand l_x for an export market conditional on exporting firms. However, the reduction in τ also has a positive effect on l_x because it contributes to an increase in the exporting profit (16) and thus raises the average labor demand l_x of exporting firms. Furthermore, the reduction in τ raises the share μ_x of exporting firms, according to Eq. (75), which contributes to an increase in average labor demand l. In addition, according to Fig. 3 and $\epsilon = g\eta/(1-p)$, the reduction in τ increases entry rate ϵ because of the reduction in trade barriers. Consequently, trade liberalization must reduce the domestic varieties M to ensure the labor market-clearing condition because it has dominant positive effects on average labor demand l across all firms and labor demand ϵf_e for entering firms.

5.3 Welfare gains from trade liberalization

Given the calibrated parameters in Table 1, we study the impact of trade liberalization via a reduction in τ on welfare. Fig. 6(d) implies that the reduction in τ increases welfare in the balanced-growth equilibrium. We consider the reason behind this result.

Fig. 6(a) reports that the reduction in τ decreases domestic trade share λ. According to Eq. (83) and Fig. 5(a), trade liberalization has a positive effect on λ because of the reduction in average relative productivity $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ conditional on exporting firms. However, Fig. 6(a) implies that this positive effect is relatively small. The following three dominant effects contribute to the reduction in domestic trade share λ as τ decreases: an increase in the share μ_x of exporting firms, the reduction in relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$ due to the low-productive entrant effect, and an increase in the exporting revenue for each firm because of the reduction in marginal costs. As shown in Eqs. (84) and (85), the reduction in domestic trade share λ has a positive effect on initial consumption c and welfare via price index P.

Fig. 6(b) reports that the reduction in τ non-monotonically affects the average value of the firm,

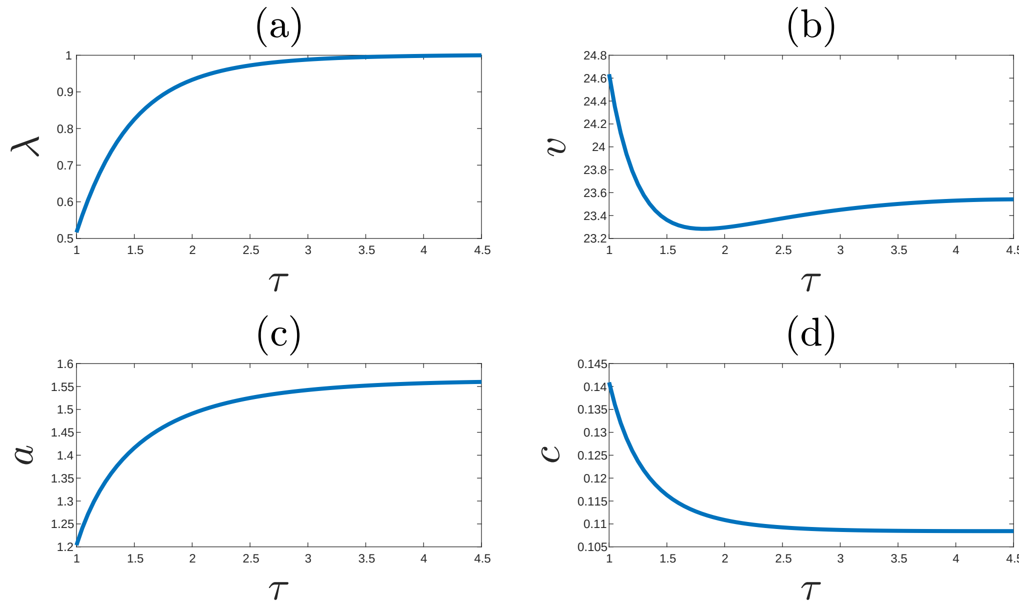

Figure 6: The impact of iceberg trade cost τ on domestic trade share λ, average assets across all firms v, per capita assets a, and initial consumption c.

v. This is because trade liberalization has the following positive or negative effects on v. The low-productive entrant effect reduces v by increasing the share of low-productive firms. On the contrary, trade liberalization increases the share μ_x of exporting firms, and thus it raises v. Further, the reduction in τ increases exporting profits because of the reduction in marginal costs, which increases v. Consequently, as shown in Fig. 6(b), trade liberalization has a non-monotonic effect on the average value of the firm, v. On the contrary, as shown in Fig. 6(c), the reduction in τ monotonically reduces per capita assets a. Noting that $a = Mv/L$, the additional negative effect of M on a contributes to the reduction in a. Then, the reduction in a has a negative effect on initial consumption and welfare because it reduces asset income.

Fig. 6(d) reports that the reduction in τ increases initial consumption, and thus trade liberalization raises welfare. Recall that aggregate productivity, the domestic trade share, and per capita assets determine initial consumption. As shown in Fig. 4(d), trade liberalization non-monotonically affects initial consumption via aggregate productivity. The reduction in per capita assets caused by trade liberalization, as shown in Fig. 6(c), has a negative effect on initial consumption. However, the reduction in the domestic trade share, as shown in Fig. 6(a), is sufficiently strong to increase monotonically initial consumption as τ decreases. In sum, trade liberalization reduces the sum of wage income and asset income $1 + ra$, whereas it sufficiently reduces price index P to increase real income $(1 + ra)/P$. Thus, trade liberalization increases initial consumption and welfare in the balanced-growth equilibrium.

Table 2: Effects of trade liberalization on labor productivity

	(1) OLS	(2) IV	(3) OLS	(4) IV	(5) OLS	(6) IV
Canadian tariffs	1.420***	1.436	-1.237	-4.355	0.249	0.666
	(0.365)	(1.064)	(2.443)	(2.717)	(2.964)	(2.424)
U.S. tariffs	1.113	2.878	-4.205	-11.897	-28.247**	-30.153**
	(0.981)	(2.587)	(11.966)	(13.731)	(12.495)	(11.772)
Business conditions	0.253***	0.258***	0.031	0.070	0.131	0.137*
	(0.040)	(0.039)	(0.102)	(0.095)	(0.080)	(0.072)
U.S. control	0.159*	0.138	0.430**	0.420**	0.106	0.109
	(0.088)	(0.100)	(0.194)	(0.183)	(0.140)	(0.125)
F-value		1.56		5.23		59.18
Hansen test (p-value)		0.83		0.22		0.49
Observations	211	211	31	31	20	20

Notes: The dependent variable is labor productivity. All the estimations include a constant term, although we do not report the results here. The asterisks ***, **, and * indicate the 1%, 5%, and 10% significance levels, respectively. The numbers in parentheses are heteroskedasticity-robust standard errors.

6 Empirical evidence

Fig. 4(d) shows the non-monotonic relationship between trade liberalization and productivity. In this section, we revisit Trefler's (2004) study of the effect of the Canada-U.S. FTA on the Canadian manufacturing sector and confirm the negative impact of trade liberalization in certain circumstances. By using his dataset of four-digit standard industrial classification data (213 industries) in Canada, we estimate the baseline specification:

$$(\Delta y_{i1}-\Delta y_{i0}) = \theta+\beta^{CA}(\Delta\tau_{i1}^{CA}-\Delta\tau_{i0}^{CA})+\beta^{US}(\Delta\tau_{i1}^{US}-\Delta\tau_{i0}^{US})+\delta(\Delta b_{i1}-\Delta b_{i0})+\gamma(\Delta y_{i1}^{US}-\Delta y_{i1}^{US})+v_i. \quad (86)$$

Following the notation presented by Trefler (2004), we let y_{it} be labor productivity in industry i in period t. Δy_{i1} is the average annual log change in labor productivity in the FTA period, specifically $\Delta y_{i1} = (\ln y_{i,1996} - \ln y_{i,1998})/(1996-1988)$. Likewise, Δy_{i0} is the same in the pre-FTA period, specifically $\Delta y_{i0} = (\ln y_{i,1986} - \ln y_{i,1980})/(1986-1980)$. $\Delta\tau_{i1}^{k}$ is the change in the FTA-mandated tariff concessions extended by Canada (the United States) to the United States (Canada) when k indexes CA (US). $\Delta\tau_{i0}^{k}$ is the average change in tariff concessions when industry i is the automotive sector and zero otherwise. Δb_i captures the business conditions and is the proportion of labor productivity driven by movements in GDP and the real exchange rate. Δy_i^{US} is labor productivity in the United States. v_i is an error term. See Trefler (2004) for additional details.

Table 2 shows the estimation results for labor productivity. While Trefler (2004) rescales β^{CA} and β^{US} to pay attention to the most impacted, import-competing group of industries, we multiply these parameters by -1 to ensure that a larger value means more liberalized. Columns (1) and (2) are identical to rows (1) and (12) in Table 2 of Trefler (2004). Column (1) reports the ordinary least squares (OLS) regression of labor productivity. While the U.S. tariff concessions do not have any significant impact on labor productivity, the Canadian tariff concessions significantly increase labor productivity in the Canadian manufacturing sector. Because the Canadian and U.S. tariffs can be endogenous variables, we conduct an instrumental variable (IV) estimation (see column (2)), using the same IVs as in Trefler (2004). Trefler (2004, p. 878) states that his instrument set "consists of 1980 log values for: (1) Canadian hourly wages, which captures protection for low-wage industries [...], (2) the level of employment, which captures protection for large industries [...], (3) Canadian imports from the United States, and (4) U.S. imports from Canada. I also include squares and cross-products as well as any exogenous regressors." Then, the coefficient of the Canadian tariff concessions becomes insignificant. However, this result might suffer from the weak instrument problems since the Kleibergen-Paap F-value is too low.

Trefler (2004) shows that the U.S. tariff concessions do not have any significant impact on labor productivity at the industry level, while they significantly increase labor productivity at the plant level, suggesting that these tariff concessions promote the entry of low-productive and young plants by reducing trade barriers. As a result, this might have a negative impact on labor productivity at the industry level under certain circumstances. To examine non-liberalized industries, columns (3) and (4) show the estimation results when the U.S. tariff concessions are greater than zero. The coefficients of the U.S. tariff concessions are negative but not significant. Columns (5) and (6) report the results when the U.S. tariff concessions are more than 0.03 percent. In this case, the U.S. tariff concessions have a significantly negative impact on labor productivity, suggesting that trade liberalization decreases productivity for non-liberalized industries, consistent with the numerical results in the previous section. Finally, the estimation results are similar to those in Table 2 when using the two alternative measures of labor productivity provided by Trefler (2004). In addition, even if we use other control variables for the business conditions and a U.S. control, the estimation results are generally unchanged.

7 Conclusion

To reconsider the impact of trade liberalization on aggregate (average) productivity, we added the low-productive entrant effect, as implied by Trefler (2004) and Melitz and Polanec (2015), into the seminal international trade model of Melitz (2003). We constructed the low-productive entrant effect by using technology diffusion à la Aghion and Howitt (1998, Ch. 3), Lucas and Moll (2014) and Perla and Tonetti (2014). The new entrant randomly searches for the incumbent and learns its existing technology. We showed that trade liberalization enhances the entry of new firms, whose productivities tend to be low. That is, low-productive technologies tend to diffuse across the economy after the impact of trade liberalization. This low-productive entrant effect contributes to reducing aggregate (average) productivity in the economy. On the contrary, as in the Melitz (2003) model, trade liberalization induces the least productive firms to exit the market because of the intense competition between firms. The resources employed by the exiting firms can then be reallocated toward high-productive firms. In the recent theoretical and empirical literature, this resource reallocation effect is one of the major premises to ensure that trade liberalization improves aggregate (average) productivity in the economy as well as welfare. However, this study showed that some resources can be reallocated toward low-productive entrants because of the low-productive entrant effect, which weakens the resource reallocation effect. Further, as in Melitz (2003), trade liberalization reduces the number of domestic varieties, which also has a negative effect on aggregate productivity. Consequently, trade liberalization causes a non-monotonic effect on aggregate productivity because of the weakened resource reallocation effect and the reduction in number of domestic varieties.

The low-productive entrant effect worsens welfare via the negative effect on aggregate productivity. Further, we showed that trade liberalization reduces asset income, which also has a negative effect on welfare. However, trade liberalization increases welfare in the balanced-growth equilibrium because it sufficiently diminishes the domestic trade share, which reduces the price of the consumption good. Consequently, trade liberalization increases real income and welfare.

As a promising future research topic, researchers could incorporate international technology diffusion into the model. Such diffusion via international trade and foreign direct investment would weaken the low-productive entrant effect because exporting firms and multinationals tend to have high productivity. Thus, trade liberalization diffuses the existing high-productive technologies of those firms. For example, we could consider the following international technology diffusion based on international trade. The entrants search for not only the domestic varieties but also imported varieties. That is, the number of incumbents targeted for the search is $M^* \equiv M + \chi N M_x$, where $\chi \in [0, 1]$ represents the share of imported varieties that can contribute to the technology diffusion. Since the mass of imported varieties NM_x tend to have high-productive technologies, trade liberalization has an additional positive effect on aggregate (average) productivity. The precise analysis remains a topic for future research. If $\chi = 0$, however, the model becomes the baseline model described in the present paper. We speculate that if χ is sufficiently small, we yield similar results to those described in this paper.

Appendix

A Equilibrium without limit $p \to 0$

We first prove Proposition 1. For convenience, we restate the proposition.

Proposition 1 *Suppose that Assumptions 1 and 2 hold. Then, if entry cost f_e is sufficiently large, there exists a balanced-growth equilibrium.*

Proof. First, consider the limit value of v_e when $\eta \to 0$. According to Eq. (57), we yield $\lim_{\eta\to 0} \eta e^{(\sigma-1)\bar{\phi}} = \infty$. Then, according to Eq. (66), we yield $\lim_{\eta\to 0} \mathbb{E}[e^{(\sigma-1)\phi}] = \infty$. Further, according to Eq. (67), $\lim_{\eta\to 0} \mathbb{E}[e^{-rT(\phi)}] = 0$ holds. Therefore, according to Eq. (64), $\lim_{\eta\to 0} v_d = \infty$ holds.

Similarly, noting that $\lim_{\eta\to 0} \mu_x = 1$ from Eq. (62), according to Eqs. (68) and (69), we yield $\lim_{\eta\to 0} \mathbb{E}_x[e^{(\sigma-1)\phi}] = \infty$ and $\lim_{\eta\to 0} \mathbb{E}_x[e^{-rT_x(\phi)}] = 0$. Therefore, according to Eq. (65), $\lim_{\eta\to 0} v_x = \infty$ holds.

Since $\bar{\phi} \to \infty$ as $\eta \to 0$, according to Eqs. (35) and (36), we have $\lim_{\eta\to 0} v_d(\bar{\phi}) = \infty$ and $\lim_{\eta\to 0} v_x(\bar{\phi}) = \infty$.

In sum, according to Eq. (63), we yield the limit value of v_e when $\eta \to 0$: $\lim_{\eta\to 0} v_e = \infty$.

Note that $\eta < (1/\phi_x^*) \ln(1/p) \iff \bar{\phi} > \phi_x^* \iff \mu_x > 0$ must hold in the equilibrium to ensure the existence of exporting firms. Define $\bar{\eta} \equiv (1/\phi_x^*) \ln(1/p)$. Then, according to Eqs. (66) and (67), we yield $\lim_{\eta\uparrow\bar{\eta}} \mathbb{E}[e^{(\sigma-1)\phi}] > 0$ and $\lim_{\eta\uparrow\bar{\eta}} \mathbb{E}[e^{-rT(\phi)}] > 0$. Since $\bar{\phi} \downarrow \phi_x^*$ as $\eta \uparrow \bar{\eta}$, we yield $\lim_{\eta\uparrow\bar{\eta}} v_e = (1-p)\lim_{\eta\uparrow\bar{\eta}} v_d + pv_d(\phi_x^*) > 0$, which is a finite value.

Therefore, according to $\lim_{\eta\to 0} v_e = \infty$ and $\lim_{\eta\uparrow\bar{\eta}} v_e$ has a finite positive value, and the continuity of v_e with respect to η, if f_e is sufficiently large, there exists at least one solution of η that satisfies the free-entry condition $f_e = v_e$. ■

We show the effect of η on $\mathbb{E}[e^{(\sigma-1)\phi}]$ and $\mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}$.

Lemma 2 *Suppose that Assumptions 1 and 2 hold. Then,*

(i) An increase in η reduces average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$ across all varieties;

(ii) An increase in η raises average absolute productivity $\mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}$ across all varieties.

Proof. Proof of part (i). By differentiating Eq. (66) with respect to η and noting Eq. (57), we yield

$$\frac{\partial \mathbb{E}[e^{(\sigma-1)\phi}]}{\partial \eta} \gtreqless 0 \iff y_1(x) \gtreqless y_2(x), \tag{A.1}$$

where

$$y_1(x) \equiv (1-x)\ln\left(\frac{1}{p}\right), \tag{A.2}$$

$$y_2(x) \equiv p^{x-1} - 1, \text{ and} \tag{A.3}$$

$$x \equiv \frac{\sigma-1}{\eta}. \tag{A.4}$$

Since $\partial y_1(x)/\partial x = \partial y_2(x)/\partial x|_{x=1} = \ln p$ and $y_1(1) = y_2(1) = 0$ hold, $y_1(x)$ and $y_2(x)$ can be drawn as in Fig. A.1. Therefore, $y_1(x) \leq y_2(x)$ holds for all $x > 0$. Then, according to Eq. (A.1), we yield the required result:

$$\frac{\partial \mathbb{E}[e^{(\sigma-1)\phi}]}{\partial \eta} \leq 0 \text{ for all } \eta > 0. \tag{A.5}$$

Proof of part (ii). According to Eq. (57) and the definition of $\bar{\phi} = \ln(\bar{\varphi}/\varphi^*)$, absolute exit cutoff productivity is

$$\varphi^* = p^{\frac{1}{\eta}}\bar{\varphi}. \tag{A.6}$$

Frontier technology $\bar{\varphi}$ is an exogenous value. Then, by noting Eqs. (66) and (A.6) and differentiating $\mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}$ with respect to η, we yield

$$\frac{\partial \mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}}{\partial \eta} \gtreqless 0 \iff y_1(x) \gtreqless y_3(x), \tag{A.7}$$

where

$$y_3(x) \equiv 1 - p^{1-x}. \tag{A.8}$$

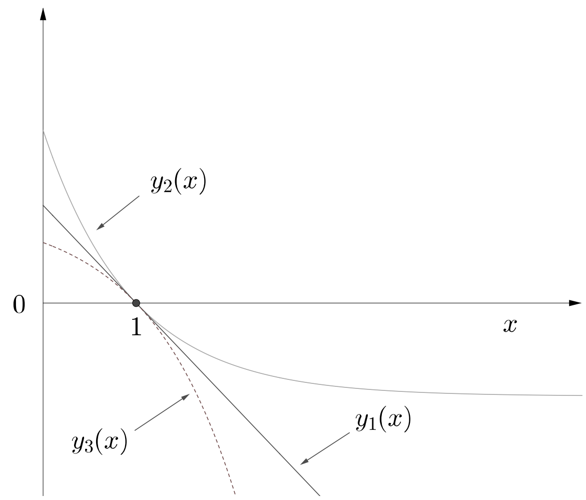

Figure A.1: $y_1(x) \leq y_2(x)$ and $y_1(x) \geq y_3(x)$ hold for all $x > 0$, where the solid black line represents $y_1(x)$, solid gray curve represents $y_2(x)$, and dotted curve represents $y_3(x)$.

Since $\partial y_1(x)/\partial x = \partial y_3(x)/\partial x|_{x=1} = \ln p$ and $y_1(1) = y_3(1) = 0$ hold, $y_1(x)$ and $y_3(x)$ can be drawn as in Fig. A.1. Therefore, $y_1(x) \geq y_3(x)$ holds for all $x > 0$. Then, according to Eq. (A.7), we yield the required result:

$$\frac{\partial \mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}}{\partial \eta} \geq 0 \text{ for all } \eta > 0. \tag{A.9}$$

■

As shown in result (i) of Lemma 2, the higher Pareto exponent η of the productivity distribution causes lower average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$. This is because according to Eq. (56), a higher η implies a higher entry rate ϵ. New entrants tend to have low productivity since the productivity distribution (55) has a negative slope. Then, an increase in the entry rate ϵ implied by a higher η reduces average relative productivity.

However, according to result (ii) of Lemma 2, the higher Pareto exponent η of the productivity distribution causes higher average absolute productivity $\mathbb{E}[e^{(\sigma-1)\phi}](\varphi^*)^{\sigma-1}$. This is because an increase in the entry rate ϵ implied by a higher η causes the market selection effect as in Melitz (2003), which increases absolute exit cutoff productivity. Since the positive effect of market selection on $(\varphi^*)^{\sigma-1}$ outweighs the negative effect on $\mathbb{E}[e^{(\sigma-1)\phi}]$, we obtain result (ii) described in Lemma 2.

B Equilibrium under $p \to 0$

In Proposition 1, we do not show the uniqueness of the equilibrium η. Then, we show the uniqueness of η under the additional assumption $p \to 0$.

Lemma 3 *Suppose that Assumptions 1 and 2 as well as $p \to 0$ hold. Then, there exists a unique equilibrium Pareto exponent η of the productivity distribution.*

Proof. The free-entry condition $f_e = v_e$ determines the equilibrium η. As $p \to 0$, Eq. (63) becomes

$$v_e = v_d + N\mu_x v_x. \tag{B.1}$$

We now show that $\partial v_e/\partial\eta < 0$. Assuming that $\eta > \sigma - 1$ holds in the balanced-growth equilibrium, after some tedious algebra, when $p \to 0$, according to Eqs. (64), (65), (66), (67), (68), and (69), we yield the relationship between v_x and v_d as follows:

$$v_x = \left(\frac{1}{\tau}\right)^{\sigma-1} e^{(\sigma-1)\phi_x^*} v_d. \tag{B.2}$$

The equilibrium condition $\eta > \sigma - 1$ ensures that $\mathbb{E}[e^{(\sigma-1)\phi}]$ and $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ have a finite value when $p \to 0$, and thus v_d and v_x also have a finite value. According to Eq. (62), when $p \to 0$, we yield

$$\mu_x = e^{-\eta\phi_x^*}. \tag{B.3}$$

When $p \to 0$, according to Eqs. (64), (66), and (67), v_d becomes

$$v_d = \left[\frac{f}{r + (\sigma-1)g}\right]\left[\frac{\eta}{\eta - (\sigma-1)}\right] - \frac{f}{r} + \left(\frac{f}{r}\right)\left[\frac{(\sigma-1)g}{r + (\sigma-1)g}\right]\left(\frac{\eta}{\eta + \frac{r}{g}}\right). \tag{B.4}$$

Differentiating Eq. (B.4) with respect to η yields

$$\frac{\partial v_d}{\partial \eta} = \left[\frac{(\sigma-1)f}{r + (\sigma-1)g}\right]\left\{\left(\frac{1}{\eta + \frac{r}{g}}\right)^2 - \left[\frac{1}{\eta - (\sigma-1)}\right]^2\right\} < 0. \tag{B.5}$$

Then, from Eq. (B.2), $\partial v_x/\partial\eta < 0$ holds. Therefore, noting $\partial\mu_x/\partial\eta < 0$ from Eq. (B.3), $\partial(\mu_x v_x)/\partial\eta < 0$ also holds. Then, according to Eq. (B.1), $\partial v_e/\partial\eta < 0$ holds. Further, we can easily confirm that $\lim_{\eta\downarrow\sigma-1} v_e = \infty$ and $\lim_{\eta\to\infty} v_e = 0$. Therefore, there exists a unique equilibrium η as shown in Fig. B.1. ■

We can replicate the result in Luttmer (2012), who shows that the distribution of firm size follows Zipf's law $\eta/(\sigma-1) = 1$ if entry cost f_e is relatively large. In the model under $p \to 0$, if fixed entry cost f_e relative to fixed production cost f is sufficiently large, i.e., $f_e/f \to \infty$, we yield $\eta/(\sigma-1) \to 1$.

Lemma 3 does not imply a unique balanced-growth equilibrium. Under $p \to 0$, according to Eq. (57), we yield $\bar{\phi} \to \infty$. Then, we lose the equation that determines absolute exit cutoff φ^*.[15] Then, there exists a continuum of balanced-growth equilibria, which is parameterized by an absolute exit cutoff φ^*. The different distributions of absolute productivity φ, parameterized by φ^*, induce different price indexes P from Eq. (84), and thus different initial consumptions c from Eq. (70).[16]

[15] Unless $p \to 0$ or $p \to 1$, under the equilibrium value of η, Eq. (57) determines φ^* for a given exogenous state $\bar{\varphi}$.

[16] Benhabib et al. (2017, Sec. 4.2) show that a continuum of equilibria emerges in the case of bounded support of the productivity distribution. On the contrary, by adopting the model in this study, there emerges a continuum of equilibria in

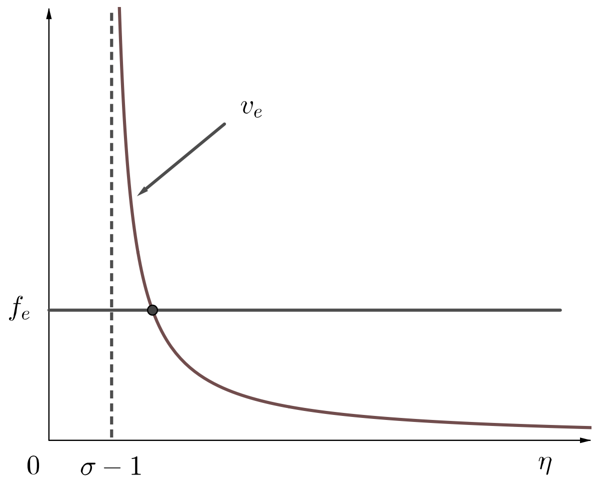

Figure B.1: The existence of a unique Pareto exponent η of the productivity distribution when $p \to 0$.

Next, we prove Proposition 2. For convenience, we restate the proposition.

Proposition 2 *Suppose that Assumptions 1 and 2 as well as $p \to 0$ hold. Then,*

(i) A decrease in iceberg cost τ increases the equilibrium Pareto exponent η of the productivity distribution;

(ii) A decrease in fixed cost f_x for exporting varieties increases the equilibrium Pareto exponent η of the productivity distribution.

Proof. Although τ and f_x change, according to Eq. (B.4), v_d does not change for a given η. According to Eqs. (B.2) and (B.3), the expected value of export $\mu_x v_x$ from entry is

$$\mu_x v_x = \left(\frac{1}{\tau}\right)^{\sigma-1} e^{-[\eta-(\sigma-1)]\phi_x^*} v_d. \tag{B.6}$$

Noting Eq. (17), according to Eq. (B.6), we can easily confirm that $\partial(\mu_x v_x)/\partial\tau < 0$ and $\partial(\mu_x v_x)/\partial f_x < 0$ for a given η. Then, according to Eq. (B.1), we yield $\partial v_e/\partial\tau < 0$ and $\partial v_e/\partial f_x < 0$ for a given η. Therefore, a decrease in τ or f_x shifts v_e upward, and thus the equilibrium η, which is determined from $f_e = v_e$, increases. That is, we yield the required results $d\eta/d\tau < 0$ and $d\eta/df_x < 0$. ■

We prove Proposition 3. For convenience, we restate the proposition.

Proposition 3 *Suppose that Assumptions 1 and 2 as well as $p \to 0$ hold. Then,*

(i) A decrease in iceberg cost τ decreases average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$ across all varieties;

the case of the unbounded support of the productivity distribution. To avoid the continuum of equilibria, Sampson (2016) imposes a certain technical assumption, which exogenously determines the initial value of φ^*.

(ii) A decrease in iceberg cost τ decreases average relative productivity $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ across all exporting (imported) varieties;

(iii) A decrease in fixed cost f_x for exporting varieties decreases average relative productivity $\mathbb{E}[e^{(\sigma-1)\phi}]$ across all varieties;

(iv) A decrease in fixed cost f_x for exporting varieties decreases average relative productivity $\mathbb{E}_x[e^{(\sigma-1)\phi}]$ across all exporting (imported) varieties.

Proof. When $p \to 0$, according to Eqs. (66) and (68), we yield

$$\mathbb{E}[e^{(\sigma-1)\phi}] = \frac{\eta}{\eta - (\sigma - 1)} \tag{B.7}$$

and

$$\mathbb{E}_x[e^{(\sigma-1)\phi}] = e^{(\sigma-1)\phi_x^*}\mathbb{E}[e^{(\sigma-1)\phi}]. \tag{B.8}$$

According to Proposition 2 and Eq. (B.7), we immediately obtain the required results $d\mathbb{E}[e^{(\sigma-1)\phi}]/d\tau > 0$ and $d\mathbb{E}[e^{(\sigma-1)\phi}]/df_x > 0$. By using the results and, according to Eq. (17), $d\phi_x^*/d\tau > 0$, and $d\phi_x^*/df_x > 0$, we yield $d\mathbb{E}_x[e^{(\sigma-1)\phi}]/d\tau > 0$ and $d\mathbb{E}_x[e^{(\sigma-1)\phi}]/df_x > 0$. ■

C Equilibrium under $p = 0$

Developing the model under $p = 0$, this appendix replicates the results of Lucas and Moll (2014), Perla and Tonetti (2014), Perla et al. (2015), and Sampson (2016), who all derive the exogenous Pareto exponent η of the productivity distribution and endogenous economic growth rate g.

Frontier technology $\bar{\varphi}$ is a useless variable in the model when $p = 0$ since no firm can adopt frontier technology $\bar{\varphi}$. For convenience, we may recycle the notations g and η. In this appendix, we define $g \equiv \dot{\varphi}^*/\varphi^*$. In the balanced-growth equilibrium, g coincides with the economic growth rate as in Eq. (59), that is, $g = \dot{c}/c$ holds.

Now, we derive the KFE under $p = 0$, which is identical to the KFE (47) under $p = 0$ except for the support of the distribution. That is,

$$\frac{\partial\mu(\phi,t)}{\partial t} = g\frac{\partial\mu(\phi,t)}{\partial\phi} + \epsilon\mu(\phi,t) \text{ for all } \phi \in (0,\infty). \tag{C.1}$$

The KFE (C.1) is identical to those of Perla et al. (2015).

As in Lucas and Moll (2014), Perla and Tonetti (2014), and Perla et al. (2015), we assume that $\mu(\phi, 0) = \eta e^{-\eta\phi}$ for $\phi \in [0,\infty)$, where η is a Pareto exponent. That is, the initial absolute productivity φ distribution is assumed to be a Pareto. η is an exogenous value because the initial distribution is historically determined. Under the initial distribution, we obtain the productivity distribution in the balanced-growth equilibrium as follows:

$$\mu(\phi,t) = \mu(\phi) = \eta e^{-\eta\phi} \text{ for } \phi \in [0,\infty), \tag{C.2}$$

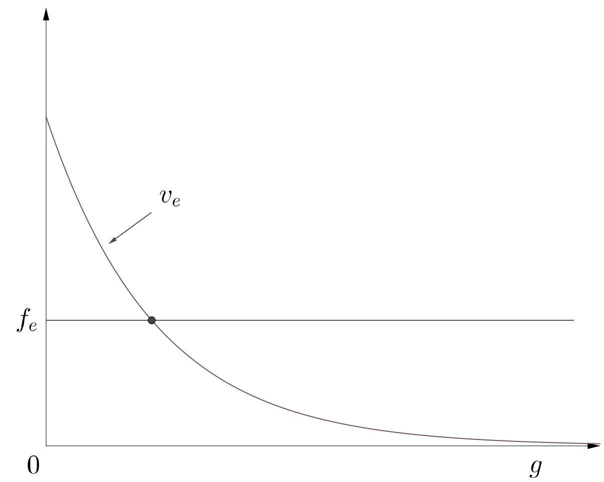

Figure C.1: The existence of a unique economic growth rate g for each initial Pareto exponent η of the productivity distribution.

where $\eta = \epsilon/g$ holds. This is because we can confirm that Eq. (C.2) satisfies the initial condition and KFE (C.1). Therefore, Eq. (C.2) is the particular solution of KFE (C.1). This result is consistent with those of Lucas and Moll (2014), Perla and Tonetti (2014), and Perla et al. (2015). That is, the long-run productivity distribution has an exogenous Pareto exponent η, which is consistent with the Pareto exponent of the initial distribution. This finding implies that there exists a continuum of the long-run productivity distribution for each Pareto exponent η of the initial distribution. In sum, the model is discontinuous at $p = 0$; that is, the model has endogenous η under $p \in (0, 1)$, while the model has exogenous η under $p = 0$. Then, the existence, growth, and adoption of frontier technology $\bar{\varphi}$ are crucial to yielding the endogenous Pareto exponent η of the productivity distribution. See Kishi (2016) for more detailed discussions.

Next, we consider the determinants of economic growth rate g under the exogenous productivity distribution (C.2). The functional form of Eq. (C.2) is identical to that of Eq. (55) under $p \to 0$. The free-entry condition $f_e = v_e$ determines g. Under the assumption of $\eta > \sigma - 1$, the value of entry v_e is given by Eq. (B.1), where v_d, μ_x, and v_x are given by Eqs. (B.4), (B.3), and (B.2), respectively. By assuming $\theta = 1$ (logarithm utility) for simplicity, the next proposition shows the existence of a unique g for each η.

Proposition 4 *Suppose that Assumption 1, $p = 0$, $\eta > \sigma - 1$, and $\theta = 1$ hold. Then, if entry cost f_e is sufficiently small, there exists a unique equilibrium economic growth rate g for each initial Pareto exponent η of the productivity distribution.*

Proof. Under the assumption of $\theta = 1$, according to Eq. (61), $r = \rho$ holds, and thus the transversality condition is always satisfied for any $g > 0$. The assumption $\eta > \sigma - 1$ ensures that v_d and v_x are finite, and thus v_e also has a finite value for a given g.

Differentiating Eq. (B.4) with respect to g yields

$$\begin{aligned}\frac{\partial v_d}{\partial g} = &-\left[\frac{(\sigma-1)f\eta}{\eta-(\sigma-1)}\right]\left(\frac{1}{r+(\sigma-1)}\right)^2 + \left\{\frac{(\sigma-1)f\eta}{[r+(\sigma-1)g]^2}\right\}\left(\frac{\eta}{\eta+\frac{r}{g}}\right) \\ &+\left[\frac{(\sigma-1)f\eta}{r+(\sigma-1)g}\right]\left[\frac{1}{(\eta+\frac{r}{g})^2 g}\right].\end{aligned} \tag{C.3}$$

After some tedious algebra, we yield $\partial v_d/\partial g < 0$ for all $g > 0$.

Further, we yield $\lim_{g\to 0} v_d = (f/r)\{(\sigma-1)/[\eta-(\sigma-1)]\} > 0$ and $\lim_{g\to\infty} v_d = 0$. Thus, according to Eqs. (B.1), (B.2), and (B.3), v_e can be drawn as in Fig. C.1. Therefore, as shown in Fig. C.1, if f_e is sufficiently small, a unique equilibrium g exists that satisfies the free-entry condition $f_e = v_e$. ■

As in the case of $p \to 0$ in Appendix B, we lose Eq. (57), which pins down the initial value of absolute exit cutoff φ^*. Therefore, there exists a continuum of φ^*. In sum, there exists a continuum of balanced-growth equilibria because of a continuum of η and φ^*.

In the next proposition, we show the effect of trade liberalization on growth.

Proposition 5 *Suppose that Assumption 1, $p = 0$, $\eta > \sigma - 1$, and $\theta = 1$ hold. Then, if entry cost f_e is sufficiently small,*

(i) A decrease in iceberg cost τ increases the economic growth rate g;

(ii) A decrease in fixed cost f_x for exporting varieties increases the economic growth rate g.

Proof. The proof is similar to that of Proposition 2 described in Appendix B. Noting Eq. (17), according to Eq. (B.6), we can easily confirm that $\partial(\mu_x v_x)/\partial\tau < 0$ and $\partial(\mu_x v_x)/\partial f_x < 0$ for a given g. Then, according to Eq. (B.1), we yield $\partial v_e/\partial\tau < 0$ and $\partial v_e/\partial f_x < 0$ for a given g. Therefore, a decrease in τ or f_x shifts v_e upward, and thus the equilibrium g, which is determined from $f_e = v_e$, increases. That is, we yield the required results $dg/d\tau < 0$ and $dg/df_x < 0$. ■

The result of Proposition 5 is similar to those of Perla et al. (2015) and Sampson (2016). Trade liberalization via a reduction in τ or f_x encourages the entry of new firms (i.e., the adoption of existing technologies). This increases ϵ, which accelerates growth g from $\eta = \epsilon/g$ since η is exogenous. The expected benefit v_e of entry is independent of population size L, and thus growth g is also independent of L. Therefore, the model is the non-scale effect model of endogenous growth.

D The effect of f_x on productivities and welfare

Given the calibrated parameters in Table 1 in Section 5, we investigate the impact of trade liberalization via the reduction in f_x on average and aggregate productivities as well as welfare. The results are similar to the case of τ described in Section 5. Therefore, we only briefly explain the results.

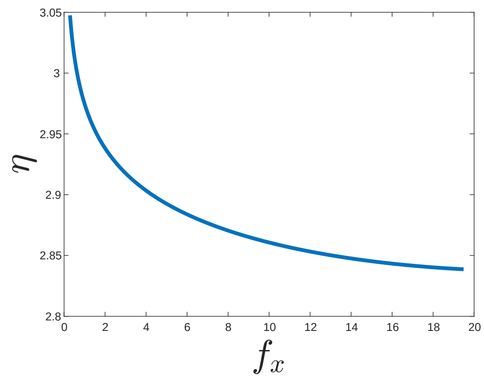

Figure D.1: The impact of fixed exporting cost f_x on the Pareto exponent η of the productivity distribution.

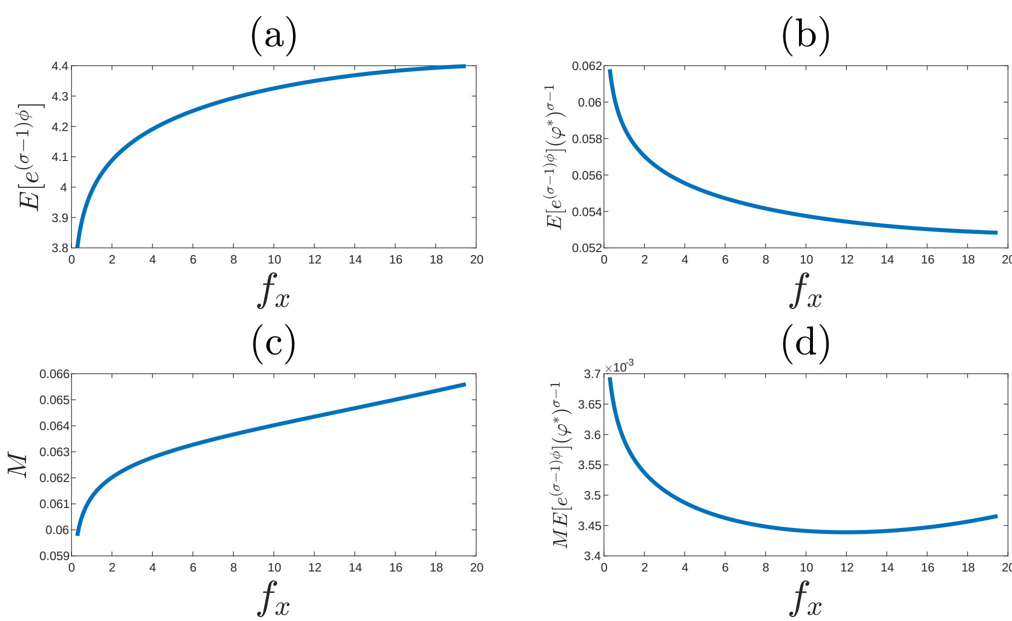

Figure D.2: The impact of fixed exporting cost f_x on productivities and the mass of domestic varieties. The vertical axis in Fig. (a) represents average productivity relative to the exit cutoff. The vertical axis in Fig. (b) represents average productivity. The vertical axis in Fig. (c) represents the number of domestic varieties. The vertical axis in Fig. (d) represents aggregate productivity.

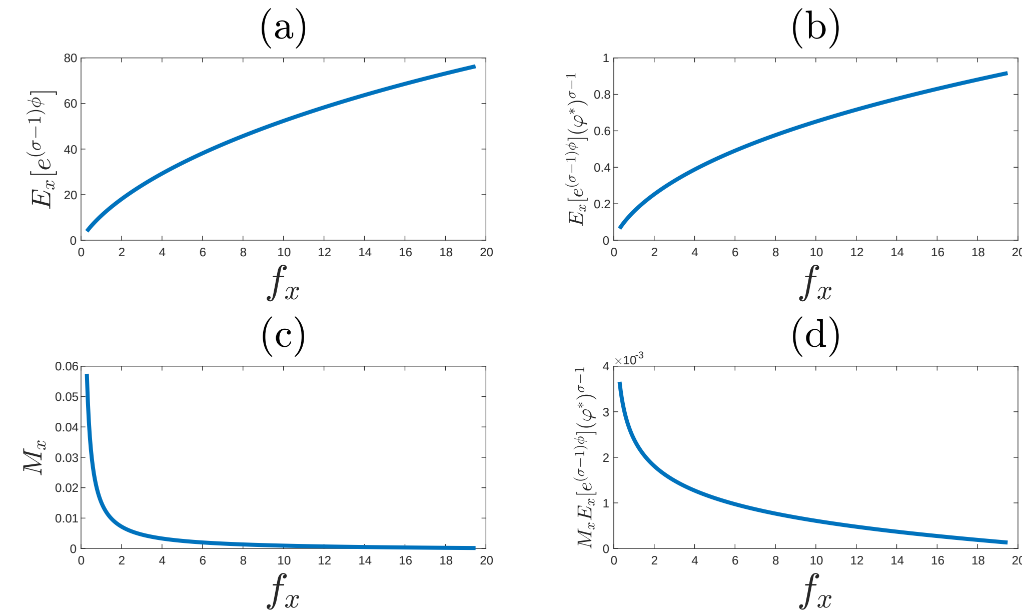

Figure D.3: The impact of fixed exporting cost f_x on productivities conditional on exporting firms and the mass of exporting varieties. The vertical axis in Fig. (a) represents average productivity relative to the exit cutoff conditional on exporting firms. The vertical axis in Fig. (b) represents average productivity conditional on exporting firms. The vertical axis in Fig. (c) represents the number of exporting varieties. The vertical axis in Fig. (d) represents aggregate productivity conditional on exporting firms.

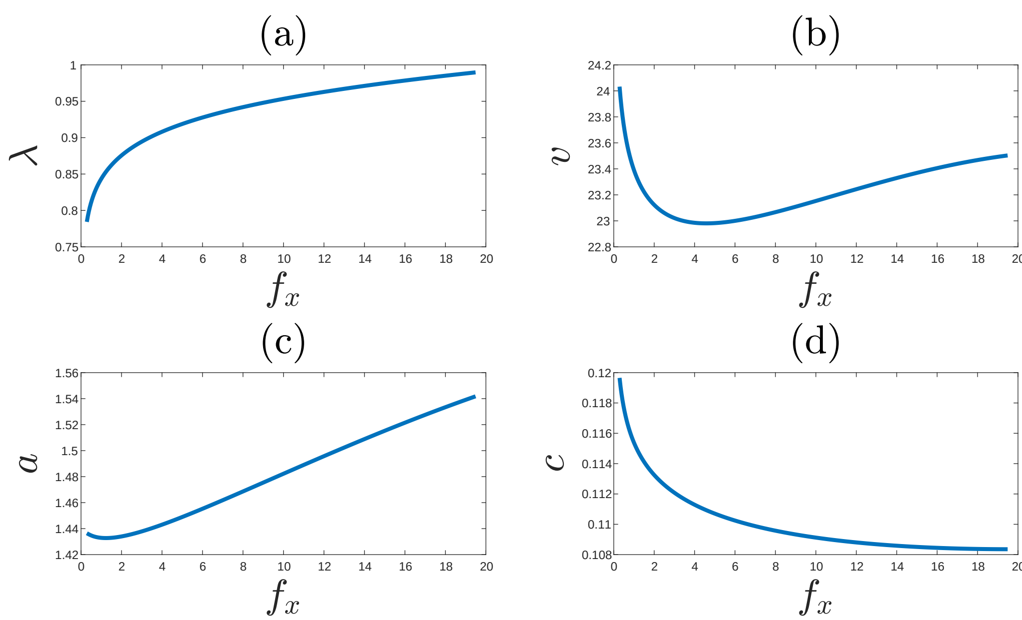

Figure D.4: The impact of fixed exporting cost f_x on domestic trade share λ, average assets across all firms v, per capita assets a, and initial consumption c.

Fig. D.1 reports that the reduction in f_x increases the Pareto exponent η of the productivity distribution. This finding implies that trade liberalization causes the low-productive entrant effect, which reduces the relative productivities of both domestic varieties and exporting varieties, as shown in Figs. D.2(a) and D.3 (a). At the same time, trade liberalization induces the resource reallocation effect caused by market selection (i.e., it increases exit cutoff φ^*). Hence, the resource reallocation effect dominates (is dominated by) the low-productive entrant effect, and thus trade liberalization via a reduction in f_x increases (decreases) average absolute productivity (conditional on exporting firms), as shown in Figs. D.2(b) and D.3(b). On the contrary, according to Fig. D.2(c), the reduction in f_x reduces domestic varieties M. These effects contribute to the non-monotonic relationship between f_x and aggregate absolute productivity, as shown in Fig. D.2(d). According to Fig. D.3(c), the reduction in f_x increases exporting varieties $M_x = \mu_x M$ because of the dominant positive effect of the share μ_x of exporting firms. This effect increases the aggregate absolute productivity of exporting firms, as shown in Fig. D.3(d).

Fig. D.4(d) implies that the reduction in f_x increases welfare. The reduction in f_x decreases the domestic trade share λ, as shown in Fig. D.4(a), which has a negative effect on price index P and a positive effect on real income $(1 + ra)/P$ and welfare. As in the case of τ in Section 5, according to Fig. D.4(b), the reduction in f_x has a non-monotonic effect on the average value of the firm, v. However, in contrast to the case of τ in Fig. 4(c) in Section 5, Fig. D.4(c) reports that the reduction in f_x also has a non-monotonic effect on per capita assets, a. That is, trade liberalization can increase asset income ra, which has a positive effect on welfare. Summing the effects on aggregate productivity, the domestic trade share, and per capita assets, Fig. D.4(d) implies that trade liberalization via the reduction in f_x increases real income $(1+ra)/P$, and thus it raises initial consumption and welfare in the balanced-growth equilibrium.

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