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Abstract

The Infant Mortality Rates in 34 sub-Saharan countries are examined in this paper by means of focusing on the degree of persistence and non-linearities. The results indicate that half of the countries examined display non-linearities and the orders of integration are extremely large in all cases, being around 2 in the majority of them. Looking at the growth rate series, we observe significant negative trends in three countries: Chad, Equatorial Guinea and Mozambique, and evidence of mean reversion, and thus, transitory shocks, in the cases of Lesotho, Rwanda, Botswana and Mozambique. As expected, time dynamics of IMR and its growth rates are expected to be persistent in order to ascertain the decline in mortality rates. Serious government interventions are therefore required in health management of infants in those listed countries.

Keywords: Infant Mortality Rates; fractional integration; long range dependence; non-linearity; Africa

JEL Classification: C22, C40, D60

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1. Introduction

Infant mortality rate (IMR) is defined as the probability of dying between birth and the first birthday, and this is measured as deaths per 1000 live births. In advanced countries such as the Organization of Economic Cooperation and Development (OECD) group, government interventions in health technology, better access to health care and disease prevention for infants and children have assisted considerably in reducing the rates of mortality of infants and children. Meanwhile, as a result of limited facilities available in many African countries, the rates of mortality are still alarmingly high, compared to those in non-African countries despite health management interventions. Furthermore, with the overall decline in infant mortality rates in Africa, mortality remains at unacceptably high levels, and about half of all deaths of infants in 2012 are concentrated in Sub-Saharan Africa regions (UNICEF, 2013). Between 1990 and 2011, Liberia, Rwanda, Malawi and Madagascar were among the top ten countries with the greatest decline in infant mortality rates.

Infant mortality rates have implications for life expectancy. Based on World Bank dataset, the average life expectancy in the sub-Saharan African region was 40 years in 1960, and this increased to 60 years in 2015. The entire time series from 1960 formed an increasing non-linear curve as it approached 2016, with life expectancy at a constant age of 50 between 1990 and 2000 (World Bank Group). The understanding of time and trend properties of IMR is of high priority to gauge the socio-economic progress and investment in public health, thus, it is important to study the time trend dynamic properties of mortality rates of infants via correlations and non-linearities in observations.

In this paper, we investigate dependencies in mortality rates of infants in Africa by using models based on long range dependence, and using both linear and non-linear specifications. The non-linear models are based on the approach developed recently in Cuestas and Gil-Alana (2016), which is robust to increasing or decreasing trended time observations.

The remainder of the paper is structured as follows: Section 2 presents the statistical methodology, from linear to non-linear methods, based on fractional integration. Section 3 presents the data and the main empirical results, while Section 4 renders the concluding remarks.

2. Statistical Methodology

In modelling trending time series, the standard statistical model is a linear time function,

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ..., \qquad (1)$$

where in the case of the present paper, y_t represents the logarithmic transformed infant mortality rate, and x_t is the detrended error term. The parameter β_1 is the average yearly reduction in the mortality rate, which is expected to be significantly negative. However, in order to make valid statistical inference about β_1 in (1), it is very important to correctly determine the structure of the error term x_t. Thus, x_t must be an integrated process of order 0 (i.e., $x_t \approx I(0)$), which means that there is no dependence across the observations, or, if there is, it is of a weak form such as the one formed by the Autoregressive of order 1 (AR(1)) process,

$$x_t = \varphi x_{t-1} + \varepsilon_t, \qquad t = 1, 2, ..., \qquad (2)$$

where $| \varphi | < 1$ and ε_t is a white noise process. In this context, Park and Mitchell (1980) and Woodward and Gray (1993) noted that significant size distortions may occur in the test statistic when testing $\beta_1 = 0$ (against $\beta_1 < 0$) if the AR coefficient φ in (2) approaches unity. Thus, as a way out, one needs to set φ equal to 1, and the process is then said to be integrated of order 1, that is x_t ≈ I(1)). Then, the statistical inference is based on the first differences $x_t - x_{t-1}$, which are then stationary. By combining equations (1) and (2) with φ = 1, the model becomes,

$$(1\text{-}B)y_t = \beta_1 + x_t; \qquad t = 1, 2, ..., \tag{3}$$

where B is the backward shift operator ($Bx_t = x_{t-1}$).[1]

A time series {x_t, t = 0, ±1, … } is said to be integrated of order d, and denoted as $I(d)$ if:

$$(1-B)^d x_t = u_t, \quad t = 1, 2, ... \tag{4}$$

with u_t ≈ I(0)), (see Granger and Joyeux, 1980; Hosking, 1981). The left-handed side of (4) can be expanded using the Binomial expansion,

$$(1-B)^d = \sum_{j=0}^{d} \binom{d}{j} (-1)^j B^j = 1 - dB + \frac{d(d-1)}{2} - ... \tag{5}$$

such that the higher the value of d is, the higher the degree of association is between observations distant in time. The parameter d actually plays a very crucial role in determining the degree of persistence of the series. If $u_t = x_t = I(0)$ as when $d = 0$, the process is short memory, and it may still be weakly AR autocorrelated. If $0 < d < 0.5$, x_t is mean reverting and covariance stationary with autocorrelations takes a longer time to disappear than in the case of an $I(0)$ process. For 0.5 ≤ d < 1, the process is no longer covariance stationary but it is still mean reverting in the sense that shocks will disappear in the long run. Finally, if d ≥ 1, x_t is non-mean-reverting and the effects of the shocks will be permanent, persisting forever, and requiring strong measures from the authorities if one wants to recover the original trends.

[1] Vogelsang (1998) constructed a t-statistic based on (3) in the presence of serial correlation.

Apart from the assumption of linearity imposed in (1), the literature have noted evidence of non-linear dynamics in the mortality rates (Hill et al., 1999; Booth et al., 2002; etc.). Cuestas and Gil-Alana (2016) proposed a non-linear deterministic trend of the form,

$$y_t = f(\theta; t) + x_t, \qquad t = 1, 2, \ldots \tag{6}$$

where $f(.)$ is a non-linear function which depends on the unknown parameter vector θ of dimension m. The trend function that we propose in this paper is actually based on Chebyshev polynomials in time, which accommodates very well in fractionally integrated frameworks. Thus, equation (6) is re-written as:

$$y_t = \sum_{i=0}^{m} \beta_i \mathrm{P}_{i,T}(t) + x_t, \quad t = 1, 2, \ldots \tag{7}$$

where T is the sample size; and m indicates the order of the Chebyshev polynomial, i.e.,

$$P_{0,T}(t) = 1, \tag{8}$$

and

$$P_{i,T}(t) = \sqrt{2}\cos\left(i\pi(t-0.5)/T\right), \qquad t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots \tag{9}$$

as detailed in Cuestas and Gil-Alana (2016); the strength of non-linearity increases as m increases from 1. Thus, if m = 0, the model contains only an intercept; if m = 1, it contains an intercept and a linear trend; and if m > 1, it becomes nonlinear, where the higher the value of m is, the higher is the nonlinear structure. In the empirical applications carried out in the following section, to allow for some degree of nonlinearity, we set m = 3, and thus, θ_2 and θ_3 refer to the nonlinear coefficients.

In the linear case, the estimation of d based on equations (1) and (4) will be carried out using the Whittle function in the frequency domain, using a testing Lagrange Multiplier (LM) procedure of Robinson (1994). This procedure tests the null hypothesis,

$$H_0 : d = d_0 \qquad (10)$$

in (1) and (4) for any range of values for *d*. The test statistic is given by,

$$R = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a} \qquad (11)$$

where

$$\hat{a} = -\frac{2\pi}{T} \sum_j^* \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I_{\hat{u}}(\lambda_j), \qquad (12)$$

$$\hat{A} = \frac{2}{T} \left\{ \sum_j^* \psi(\lambda_j) \psi(\lambda_j)' - \sum_j^* \psi(\lambda_j) \hat{u}(\lambda_j)' \left[\sum_j^* \hat{u}(\lambda_j) \hat{u}(\lambda_j)' \right] \sum_j^* \hat{u}(\lambda_j) \psi(\lambda_j)' \right\}, \qquad (13)$$

with

$$\psi(\lambda_j) = \mathrm{Re}\left[\frac{\partial}{\partial d} \log \rho\left(e^{i\lambda_j}; d\right) \right], \qquad (14)$$

and

$$\hat{u}(\lambda_j) \;=\; \log \left| \sin \frac{\lambda_j}{2} \right|, \qquad (15)$$

and the sum over * above refers to all bounded discrete frequencies in the spectrum. For the particular case of white noise u_t, $Var(u_t) = \sigma^2$, and the spectral density function of u_t is then $\frac{\sigma^2}{2\pi}$ and $g(.)$ in (12) becomes 1, and also $\hat{u}(\lambda_j) = 0$. Under very mild regularity condition, i.e. up to second order moments, Robinson (1994) showed that,

$$\hat{R} \rightarrow_d \;\; \chi_1^2 \qquad (16)$$

as $T \rightarrow \infty$.

Cuestas and Gil-Alana (2016) extended the above method to the non-linear case, replacing equation (1) by (7). Note that combining (4) and (7), the model becomes:

$$y_t^* = \sum_{i=0}^{m} \beta_i \mathrm{P}^*_{i,T}(t) + u_t, \quad (1 - B)^d x_t = u_t, \quad t = 1, 2, ... \tag{17}$$

where $y_t^* = (1 - B)^d y_t$; and $P_{i,Tt}^*(t) = (1 - B)^d P_{iT}(t)$, thus, noting that u_t in (17) is I(0) by construction, the estimates of θ in (17) (or in (7)) can be obtained based on least square methods. (See Cuestas and Gil-Alana, 2016).

3. Data and Empirical Results

Infant mortality rate data used in this paper were obtained from the Federal Reserve Bank of St Louis Economic Research Division database website at www.stlouisfed.org. These are mortality rates for infants less than 1 year old in 34 African countries: Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Congo, Cote d'Ivoire, Egypt, Equatorial Guinea, Ghana, Guinea, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mozambique, Niger, Nigeria, Rwanda, Senegal, Somalia, South Africa, Sudan, Togo, Tunisia, Uganda, Zambia and Zimbabwe.

[Insert Table 1 about here]

We report in Table 1 the sample period for each country, along with the mortality rates in two years that are far apart, namely 1960 and 2016, in order to analyse the reduction in the rates over time. By looking at the IMR in 1960, the lowest value in the mortality rate was observed in the case of South Africa (0.0881). Next to this is Madagascar (0.0902) and Zimbabwe (0.0925). The highest mortality rate was observed in Mali (0.2134). At the end of the sample (2016), there has been major economic growth in Africa which has improved the health management of infants. Tunisia indicated the lowest value in IMR (0.0117), followed by Egypt (0.0194) and Algeria (0.0216). In terms of percentage reduction in IMR from 1960 to 2016, Somalia indicated the lowest percentage reduction in IMR (29.1%), followed by Chad (40.8%). Tunisia indicated the highest overall percentage reduction in IMR (93.5%) making her a country with the best infant mortality reduction strategy among the sampled African countries.

Next, we carry out empirical analysis with the non-linear specification. Table 2 reports the estimated coefficients in (7) and (4). The second column in that table refers to the estimated values of *d* in (4) and their corresponding 95% confidence intervals, and the remaining columns display the estimated coefficients in (7), the last two referring to the non-linear terms.

We first look at the estimated values of d, and we observe that they are very high in all cases, being higher than 2 in a number of cases. In fact, the highest estimate corresponds to Burundi (d = 2.74), followed closely by Algeria and Niger (2.70). Along with these three countries, there is a group of thirteen countries where the estimated values of d are significantly higher than 2. These countries are Benin (2.58), Burkina Faso (2.59), Cameroun (2.50), Egypt (2.35), Ghana (2.62), Liberia (2.60), Malawi (2.53), Mali (2.61), Nigeria (2.38), Senegal (2.67), Sudan (2.32), Uganda (2.56) and Zimbabwe (2.25). The I(2) hypothesis cannot be rejected for twelve countries (Angola, Central Africa Republic, Congo, Equatorial Guinea, Guinea, Kenya, Madagascar, Somalia, South Africa, Togo, Tunisia and Zambia), and the lowest degrees of dependence are observed in six countries where the estimates of d are constrained between 1 and 2. These countries are: Botswana, Chad, Cote d'Ivorie, Lesotho, Mozambique and Rwanda. Therefore, these results indicate extremely large degrees of persistence, with shocks having permanent effects; the good thing about this is that in the event of a shock drastically reducing IMRs, its effect will remain forever. However, on the other hand, if the shock increases the level of IMR (i.e.

due to civil conflict, wars, etc.), strong measures must be adopted to recover the original levels (or trends) since the series do not recover by themselves in the long run.

[Insert Table 2 about here]

Concerning the non-linearity of IMR, we observe some evidence of this in a number of cases, noting that β_2 and/or β_3 are statistically significant in many cases. In fact, both coefficients (β_2 and β_3) are statistically significant in the cases of Angola, Botswana, Central Africa Republic, Cote d'Ivorie, Mozambique and Zambia, and β_2 or β_3 are significant in another group of ten countries: Chad, Guinea, Kenya, Lesotho, Liberia, Madagascar, Nigeria, Somalia, South Africa and Tunisia. On the other hand, in half of the countries (that is 17), we do not observe evidence of nonlinearities.

Based on the large degrees of persistence observed in the results reported across Table 2, next we take first differences on the logged series to analyse the growth rates, and consider now the linear model given by equations (1) and (4), i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \qquad (18)$$

assuming that u_t is a white noise process. The results are displayed across Table 3. We consider the three standard cases examined in the literature and corresponding to i) no deterministic terms, ii) an intercept, and iii) an intercept with a linear time trend. We marked in bold in the table the selection made according to the t-values on these deterministic terms. We observe only three countries with significant (negative) trends and they correspond to Chad, Equatorial Guinea and Mozambique. For the rest of the countries, no deterministic terms, or an intercept only are sufficient to describe these deterministic components. Focusing now on the estimated values of d, the largest values are obtained in the cases of Liberia (d = 2.05) and Burundi (2.04), while the lowest values correspond to Lesotho (0.61) and Rwanda (0.66). Apart from these two countries, statistical evidence supporting mean reversion (i.e., significant values below 1) is found in the cases of Botswana (d = 0.73) and Mozambique (0.83), and though d is found to be smaller than 1 in other countries (Cote d'Ivorie, Equatorial Guinea and South Africa), the confidence intervals are so wide that we cannot reject the unit root null hypothesis. (See Table 4 for a summary of the results based on persistence). Thus, special attention should be taken in these four countries where mean reversion takes place (Lesotho, Rwanda, Botswana and Mozambique) noting that significant positive shocks that may reduce significantly IMRs may only have transitory effects, requiring in these cases special measures, to maintain the values at low levels.

[Insert Tables 3 and 4 about here]

4. Concluding Remarks

In this article, we have examined the Infant Mortality Rates (IMR) in a group of 34 African countries by looking at two important features of these data, namely their degree of persistence and the non-linearities. We use a set-up that is based on a fractionally integrated or I(d) model, which is more general than the standard methods based on integer degrees of differentiation (i.e., 0 in case of I(0) processes and 1 for the I(1) case).

We start with a non-linear specification and we choose here the Chebyshev polynomials in time (see Cuestas and Gil-Alana, 2016) that accommodates very well in the context of fractional integration. Our results show evidence of non-linearities in half (17) of the countries investigated, and the orders of integration associated to the series are extremely large in all cases ranging from 1.41 (Lesotho) and 1.50 (Mozambique) to 2.70 (Algeria), that is, decline in infant mortality rate is expected to continue permanently given the current health management strategy in those African countries.

Taking first differences on the logged series and thus looking at the growth rates, we only observe three countries with significant (negative) trends, namely, Chad, Equatorial Guinea and Mozambique, and the orders of integration widely range now between 0.61 (Lesotho) and 2.05 (Liberia). Since mean reversion is only observed in the growth rates of IMR for Lesotho, Rwanda, Botswana and Mozambique, special attention should be paid in these countries, noting that the effects of the (positive) shocks in the growth rates will disappear by themselves in the long run unless policy actions are taken.

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Table 1: Sample period and reduction in IMR by country

Country	Time period	Starting IMR	Ending IMR	% Reduction
Algeria	1960-2016	0.1475 (19)	0.0216 (3)	85.4 (32)
Angola	1980-2016	0.1397 (15)	0.0546 (23)	60.9 (10)
Benin	1960-2016	0.1897 (27)	0.0631 (25)	66.7 (17)
Botswana	1960-2016	0.1152 (5)	0.0326 (5)	71.7 (26)
Burkina Faso	1960-2016	0.1560 (21)	0.0527 (20)	66.2 (16)
Burundi	1964-2016	0.1420 (16)	0.0484 (16)	65.9 (15)
Cameroon	1960-2016	0.1640 (23)	0.0528 (21)	67.8 (19)
C. Afr. Rep.	1960-2016	0.1646 (24)	0.0885 (34)	46.2 (3)
Chad	1972-2016	0.1270 (10)	0.0752 (32)	40.8 (2)
Congo	1969-2016	0.1509 (20)	0.0720 (30)	52.3 (5)
Cote d'Ivoire	1960-2016	0.2121 (33)	0.0660 (26)	68.9 (22)
Egypt	1960-2016	0.2099 (29)	0.0194 (2)	90.8 (33)
Equit. Guinea	1983-2016	0.1466 (18)	0.0662 (27)	54.8 (6)
Ghana	1960-2016	0.1242 (9)	0.0412 (13)	66.8 (18)
Guinea	1960-2016	0.2108 (30)	0.0583 (24)	72.3 (27)
Kenya	1960-2016	0.1182 (7)	0.0356 (9)	69.9 (23)
Lesotho	1960-2016	0.1432 (17)	0.0724 (31)	49.4 (4)
Liberia	1960-2016	0.2115 (31)	0.0512 (19)	75.8 (29)
Madagascar	1968-2016	0.0902 (2)	0.0340 (7)	62.3 (12)
Malawi	1964-2016	0.2117 (32)	0.0389 (11)	81.6 (31)
Mali	1963-2016	0.2134 (34)	0.0680 (29)	68.1 (20)
Mozambique	1964-2016	0.1769 (25)	0.0531 (22)	70.0 (24)
Niger	1967-2016	0.1273 (11)	0.0509 (18)	60.0 (9)
Nigeria	1964-2016	0.1957 (28)	0.0669 (28)	65.8 (14)
Rwanda	1960-2016	0.1310 (13)	0.0292 (4)	77.7 (30)
Senegal	1960-2016	0.1296 (12)	0.0336 (6)	74.1 (28)
Somalia	1983-2016	0.1165 (6)	0.0826 (33)	29.1 (1)
South Africa	1974-2016	0.0881 (1)	0.0342 (8)	61.2 (11)
Sudan	1960-2016	0.1067 (4)	0.0448 (15)	58.0 (8)
Togo	1960-2016	0.1612 (22)	0.0507 (17)	68.5 (21)
Tunisia	1962-2016	0.1788 (26)	0.0117 (1)	93.5 (34)
Uganda	1960-2016	0.1323 (14)	0.0377 (10)	71.5 (25)
Zambia	1960-2016	0.1223 (8)	0.0438 (14)	64.2 (13)
Zimbabwe	1960-2016	0.0925 (3)	0.0400 (12)	56.8 (7)

Note, in parentheses the ranking based on the reduction in infant mortality rates.

Table 2: Estimated coefficients in a nonlinear I(d) model

Series	d	β_0	β_1	β_2	β_3
Algeria	2.70 (2.59, 2.96)	4.5386 (2.34)	0.3150 (0.21)	0.0733 (0.27)	-0.0707 (-0.72)
Angola	2.21 (1.90, 2.73)	4.9906 (47.12)	0.0292 (0.40)	**-0.0918** **(-5.50)**	**0.0309** **(4.45)**
Benin	2.58 (2.40, 2.76)	5.0726 (8.22)	0.0769 (0.17)	0.0377 (0.43)	0.0101 (0.31)
Burkina Faso	2.59 (2.44, 2.89)	5.0887 (8.32)	-0.0625 (-0.14)	0.0653 (0.07)	0.0345 (1.08)
Bostwana	1.58 (1.43, 1.81)	4.3775 (11.51)	0.0243 (0.09)	**0.1730** **(2.32)**	**0.0710** **(1.81)**
Burundi	2.74 (2.63, 2.91)	4.9126 (4.10)	-0.0039 (-0.004)	-0.0546 (-0.03)	0.0301 (0.51)
Central Africa R.	1.94 (1.78, 2.18)	5.2285 (37.27)	-0.1806 (-1.89)	**0.0665** **(2.73)**	**0.0200** **(1.80)**
Cameroun	2.50 (2.27, 2.62)	5.3053 (4.27)	-0.1879 (-0.21)	-0.0238 (-0.13)	0.0731 (1.06)
Chad	1.80 (1.73, 1.98)	4.8088 (128.17)	0.0199 (0.79)	-0.0003 (-0.05)	**0.0066** **(2.03)**
Congo	2.04 (1.84, 2.40)	5.0279 (63.35)	-0.0274 (-0.51)	0.0033 (0.25)	**0.0198** **(3.38)**
Cote d' Ivoire	1.67 (1.52, 1.91)	5.0958 (29.70)	0.0565 (0.49)	**0.0770** **(2.38)**	**0.0659** **(4.02)**
Egypt	2.35 (2.10, 2.63)	4.9657 (7.55)	0.32003 (0.69)	-0.0125 (-0.12)	-0.0270 (-0.68)
Equat. Guinea	2.33 (1.92, 2.79)	4.9464 (37.81)	0.0281 (0.31)	0.0026 (0.13)	0.0048 (0.60)
Ghana	2.62 (2.53, 2.70)	4.9139 (3.55)	-0.0197 (-0.02)	-0.0357 (-0.18)	-0.0171 (-0.23)
Guinea	2.19 (1.98, 2.44)	5.2266 (42.06)	0.1178 (1.37)	**-0.0389** **(-1.96)**	0.0118 (1.43)
Kenya	1.91 (1.73, 2.16)	4.7042 (11.99)	-0.0297 (-0.11)	0.0168 (0.24)	**0.0734** **(2.32)**
Lesotho	1.41 (1.28, 1.58)	4.5405 (26.22)	0.1642 (1.45)	**0.1288** **(3.49)**	0.0065 (0.31)
Liberia	2.60 (2.50, 2.90)	5.2210 (8.05)	0.0747 (0.15)	-0.0913 (-1.00)	**0.1066** **(3.14)**
Madagascar	2.25 (1.94, 2.59)	4.4753 (18.92)	0.1358 (0.82)	**-0.1025** **(-2.78)**	-0.0174 (-1.15)
Malawi	2.53 (2.16, 2.72)	5.2683 (3.12)	0.0573 (0.04)	-0.0589 (-0.24)	0.0655 (0.72)
Mali	2.61 (2.45, 2.80)	5.3786 (9.03)	-0.0144 (-0.03)	-0.0244 (-0.29)	0.0310 (1.01)

Mozambique	1.50 (1.36, 1.67)	4.9706 (45.42)	0.2373 (3.29)	**-0.1457** **(-6.57)**	**0.0515** **(4.26)**
Niger	2.70 (2.52, 2.88)	4.6916 (7.11)	0.1659 (0.33)	-0.0833 (-0.91)	0.0240 (0.73)
Nigeria	2.38 (2.08, 2.59)	5.3931 (14.92)	-0.1427 (-0.56)	0.0639 (0.11)	**0.0616** **(2.89)**
Rwanda	1.63 (1.46, 1.86)	4.8292 (5.34)	0.0542 (0.08)	-0.1244 (-0.71)	0.1081 (1.21)
Senegal	2.67 (2.60, 2.74)	4.9505 (4.82)	-0.0910 (-0.12)	-0.0134 (-0.09)	0.0346 (0.76)
Somalia	2.02 (1.64, 2.96)	4.8612 (50.74)	-0.0947 (-1.45)	0.0061 (0.38)	**0.0194** **(2.71)**
South Africa	1.53 (1.14, 2.16)	4.1535 (15.77)	0.0779 (0.44)	0.0705 (1.34)	**0.0939** **(3.32)**
Sudan	2.32 (2.07, 2.56)	4.7959 (18.84)	-0.0597 (-0.33)	-0.0325 (-0.84)	0.0086 (0.55)
Togo	2.19 (1.92, 2.52)	5.0166 (241.53)	0.0007 (0.05)	0.0328 (1.00)	0.0194 (1.42)
Tunisia	2.07 (1.84, 2.40)	4.5478 (11.50)	0.3946 (1.46)	0.0096 (0.14)	**0.0619** **(2.17)**
Uganda	2.56 (2.32, 2.77)	5.0333 (2.97)	-0.0300 (-0.02)	-0.1306 (-0.54)	0.0617 (0.68)
Zambia	1.73 (1.42, 2.09)	4.7435 (13.74)	0.0485 (0.20)	**-0.1083** **(-1.69)**	**0.1101** **(3.48)**
Zimbabwe	2.25 (2.20, 2.48)	4.8015 (5.02)	-0.3077 (-0.46)	0.1211 (0.81)	0.0017 (0.03)

Note, in parentheses in the second column are the confidence bands for *d*. In column 3-6, the parentheses are t-statistics for estimated parameters for nonlinear time trend. In bold are significant estimates at 5% levels.

Table 3: Estimates of d on the growth rate series

	No det. terms	An intercept	A linear time trend
Algeria	**1.97 (1.72, 2.26)**	1.97 (1.72, 2.26)	1.97 (1.71, 2.26)
Angola	1.45 (1.23, 1.71)	**1.74 (1.60, 1.97)**	1.74 (1.66, 1.98)
Benin	1.36 (1.14, 1.62)	**1.72 (1.45, 2.01)**	1.73 (1.45, 2.04)
Burkina Faso	1.59 (1.35, 1.90)	**1.89 (1.67, 2.17)**	1.90 (1.67, 2.18)
Botswana	0.76 (0.64, 0.95)	**0.73 (0.59, 0.94)**	0.73 (0.59, 0.94)
Burundi	1.92 (1.72, 2.16)	**2.04 (1.88, 2.36)**	2.04 (1.87, 2.36)
Central Africa R.	1.06 (0.96, 1.20)	**1.06 (0.95, 1.17)**	1.06 (0.95, 1.17)
Cameroun	1.51 (1.31, 1.78)	**1.63 (1.46, 1.90)**	1.64 (1.46, 1.93)
Chad	0.83 (0.69, 1.08)	0.87 (0.76, 1.05)	**0.83 (0.67, 1.05)**
Congo	1.05 (0.91, 1.27)	**1.20 (1.09, 1.36)**	1.21 (1.09, 1.37)
Cote de Ivore	0.92 (0.80, 1.09)	**0.89 (0.77, 1.06)**	0.89 (0.77, 1.06)
Egypt	1.13 (0.94, 1.38)	**1.39 (1.22, 1.61)**	1.39 (1.22, 1.62)
Equat. Guinea	0.83 (0.40, 1.26)	0.94 (0.81, 1.15)	**0.93 (0.76, 1.15)**
Ghana	1.57 (1.28, 1.89)	**2.01 (1.75, 2.30)**	1.98 (1.73, 2.27)
Guinea	1.12 (0.98, 1.30)	**1.39 (1.26, 1.55)**	1.39 (1.20, 1.56)
Kenya	0.90 (0.76, 1.08)	**1.07 (0.91, 1.31)**	1.07 (0.91, 1.30)
Lesotho	**0.61 (0.50, 0.78)**	0.59 (0.47, 0.76)	0.60 (0.47, 0.76)
Liberia	**2.05 (1.88, 2.28)**	2.05 (1.88, 2.28)	2.05 (1.88, 2.27)
Madagascar	1.41 (1.27, 1.61)	**1.49 (1.32, 1.72)**	1.49 (1.32, 1.72)
Malawi	**1.54 (1.18, 1.92)**	1.56 (1.22, 1.94)	1.56 (1.22, 1.94)
Mali	1.35 (1.14, 1.61)	**1.85 (1.67, 2.09)**	1.85 (1.67, 2.10)
Mozambique	0.86 (0.74, 1.02)	0.84 (0.74, 1.00)	**0.83 (0.70, 1.00)**
Niger	1.65 (1.46, 1.90)	**1.91 (1.68, 2.19)**	1.91 (1.68, 2.19)
Nigeria	1.24 (1.09, 1.48)	**1.58 (1.45, 1.75)**	1.58 (1.45, 1.75)
Rwanda	**0.66 (0.51, 0.88)**	0.67 (0.53, 0.90)	0.66 (0.48, 0.90)
Senegal	**1.94 (1.74, 2.17)**	1.96 (1.76, 2.18)	1.96 (1.76, 2.18)
Somalia	1.21 (0.99, 1.61)	**1.46 (1.18, 1.90)**	1.46 (1.18, 1.95)
South Africa	0.86 (0.64, 1.27)	**0.75 (0.56, 1.09)**	0.75 (0.55, 1.09)
Sudan	1.04 (0.87, 1.31)	**1.20 (1.10, 1.33)**	1.20 (1.10, 1.33)
Togo	1.02 (0.87, 1.26)	**1.02 (0.89, 1.18)**	1.02 (0.89, 1.18)
Tunisia	0.93 (0.76, 1.18)	**1.03 (0.89, 1.20)**	1.03 (0.89, 1.20)
Uganda	1.52 (1.28, 1.84)	**1.61 (1.38, 1.89)**	1.61 (1.38, 1.90)
Zambia	0.92 (0.76, 1.17)	**1.01 (0.82, 1.32)**	1.01 (0.82, 1.32)
Zimbabwe	1.23 (1.08, 1.43)	**1.31 (1.16, 1.51)**	1.32 (1.16, 1.51)

In bold, the selected models according to the deterministic terms. In parenthesis, 95% confidence bands.

Table 4: How persistent are the IMR growth rates in Africa?

Mean reversion (d < 1)	Unit roots (d = 1)	I(1 < d < 2)	I(2) behaviour
Lesotho (0.61)	South Africa (0.75)	Congo (1.20)	Benin (1.72)
Rwanda (0.66)	Chad (0.83)	Sudan (1.20)	Mali (1.85)
Botswana (0.73)	Cote de Ivorie (0.89)	Zimbabwe (1.31)	Burkina Faso (1.89)
Mozambique (0.83)	Eq. Guinea (0,93)	Egypt (1.39)	Niger (1.91)
	Zambia (1.01)	Guinea (1.39)	Senegal (1.94)
	Togo (1.02)	Somalia (1.46)	Algeria (1.97)
	Tunisia (1.03)	Madagascar (1.49)	Ghana (2.01)
	Central African Rep (1.06)	Malawi (1.54)	Burundi (2.04)
	Kenya (1.07)	Nigeria (1.58)	Liberia (2.05)
		Uganda (1.61)	
		Cameroun (1.63)	
		Angola (1.74)	

In parenthesis, the degree of integration for each series.