Simulation of Spar Type Floating Offshore Wind Turbine Subjected to Misaligned Wind-Wave Loading Using Conservation of Momentum Method

Chan, Kemin and Hong, Yu

Shanghai Vocational College of Science Technology

1 September 2018

Online at https://mpra.ub.uni-muenchen.de/88777/
MPRA Paper No. 88777, posted 02 Sep 2018 19:33 UTC
Simulation of Spar Type Floating Offshore Wind Turbine Subjected to Misaligned Wind-Wave Loading Using Conservation of Momentum Method

Kemin Chan, Yu Hong
Shanghai Vocational College of Science & Technology

Abstract

Floating wind turbines are subjected to stochastic wind and wave loadings. Wind and wave loadings are not essentially aligned. The misalignment between wind and wave loadings affects the dynamical response of the floating wind turbines which needs to be studied. For this purpose, the nonlinear equations of motion of the spar type floating wind turbine is derived using the Newton’s second law and conservation of angular momentum theory. The aerodynamic, hydrodynamic, mooring and buoyancy forces are determined and coupled with the system. The dynamic responses of the system are calculated and compared for different wind-wave misalignment angles. The simulation results demonstrate the importance of consideration of wind-wave misalignment angle on the dynamic response for the floating offshore wind turbine.

Keywords: Spar floating wind turbine; conservation of angular momentum; wind-wave misalignment

1. Introduction

By increasing the energy demand and the limitations of fossil fuels, renewable energy sources experience a remarkable growth in the industry. In this regard, different studies and majors have focused on renewable energies [1]-[22].

Higher and steadier wind speed, less visual impacts and noise pollution have led the offshore wind industry to focus on floating platforms. Due to lower center mass, small water area and deep-draft, spar type floating wind turbines are among the most interesting types of floating wind turbines. Floating wind turbines which are subjected to combined wind and wave loadings should withstand in different environmental conditions including misaligned wind-wave loadings. It is crucial simulate the dynamic response of floating wind turbines and make sure these structures withstand in different environmental conditions.

Different studies have focused on modelling improvement, such as: increasing the number of DOFs [23], reducing the computational time [24], improving the modeling of mooring cables [25], considering the flexibility of components [26] to make the system simulation more accurate. Different methods are used to model and simulate the floating wind turbines and its external loadings. Fluid-structure interaction method is implemented to study the dynamic response of the floating wind turbines [27].

Reducing the motion and vibrations of the offshore wind turbines improves the stability of these systems under various environmental conditions. In this regard, different researches have been conducted to reduce the vibrations of offshore wind turbines including fixed and floating wind
turbines. Passive tuned mass dampers have been installed in the nacelle, tower or in the blades to reduce the vibrations of nacelle, tower or blades [28]-[30]. Active tuned mass dampers were shown to be more effective than the passive counterparts [31],[32]. Due to the advantages of semi-active tuned mass dampers over the passive and active tuned mass dampers, researchers have focused on vibration reduction of offshore wind turbines by semi-active tuned mass dampers [33]-[35]. These studies have been limited to environmental conditions where wind and wave loadings are considered to be aligned.

The probability of occurrence for wind and wave loading to be aligned is higher than any other misalignment. However, it is still crucial to study the dynamic responses of offshore wind turbines under misaligned wind and wave loading. Stewart and Lackner [36] used two linear tuned mass dampers to reduce the vibrations of barge type floating wind turbine in the presence of wind-wave misalignment. A simplified structural model was used to evaluate the performance of the controlling method. Sun and Jahangiri [37]-[38] used a pendulum tuned mass damper to mitigate the vibrations of a fixed monopile offshore wind turbine. It should be noted that, the misalignment effect is much more important in floating wind turbines than the fixed counterparts.

Based on the literature review, the present paper models the floating wind turbine using the conservation of angular momentum theory and Newton’s second law. Totally, the floating wind turbine has six unknown degrees of freedom including three translational (surge, sway and heave) and three rotational (pitch, roll and yaw) motions. Also, the system includes two known degrees of freedom, nacelle yaw and spin of the blades. The motion of the floating wind turbine is determined using the Euler angles with exact nonlinear coupling between the degrees of freedom. The aerodynamic, hydrodynamic, mooring and hydrostatic forces are derived and applied to the system’s equations of motion. Also, four representative misalignment angles have been used to study the effects of the misalignment angle on the dynamic response of the floating wind turbine. MATLAB codes have been developed and the system is simulated and solved in MATLAB.

2. Analytical model

In this section, the equations of motion for the floating wind turbine are derived by using conservation of angular momentum and Newton’s second law. It is essential to introduce different coordinate systems and to determine the system’s equations of motion with respect to the mentioned coordinates. Also, the external forces and moments, which are resulted from buoyancy, hydrodynamic, wind, and mooring cables are determined.

The floating wind turbine is considered as two rigid bodies, the tower and the rotor and nacelle assembly. \((X, Y, Z)\) is earth-fixed global coordinate system located at system’s center of mass. \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) are body fixed and are located at first and second body’s center of mass respectively. \((x, y, z)\) is the coordinate system for the entire system, which is assumed to be parallel to \((x_1, y_1, z_1)\) and is located on the system’s center of mass.

Six unknown degrees of freedom is related to the first body which includes three translational, and 3 rotational degrees of freedom. Additionally, two known degrees of freedom for rotor spinning, \(\varphi\), and nacelle yaw, \(\beta\) which describes the relative motion of the second body to the first body.
The schematic image of the floating wind turbine and the coordinates are illustrated in Fig. 1. The misalignment angle between wind and wave loading is denoted by $\theta$.

![Schematic image of the spar type FWT](image)

Fig. 1. Schematic image of the spar type FWT

Newton's second law is applied to the system to determine the translational equations:

$$\sum F^C = (m_1 + m_2) a^C$$  \hspace{1cm} (1)

where $\sum F^C$ is the summation of all external forces. Also, $m_1$ and $m_2$ denote the mass of first and second body respectively. Additionally, the linear acceleration of the system which is shown as $a^C$ is equal to $[\dot{a}_1, \dot{a}_2, \dot{a}_3]$.

To determine the rotational equations of motion, the conservation of angular momentum theory is used as follows:

$$\sum r \times M = I^C_{\hat{\omega}} + \dot{r}^C \times H_{\hat{\omega}}$$  \hspace{1cm} (2)

where $\sum M$ is the external moment resulted from external loadings. Also, $H_{\hat{\omega}}$ denotes the angular momentum of entire system in $C_i$ coordinate system. $I^C_{\hat{\omega}}$ is the change of momentum amplitude for the entire system and $\dot{r}^C$ describes the absolute angular velocity. The total angular momentum is equal to the summation of angular momentum of each body.
The angular momentum of the system can be determined as follows:

\[ H_{G_r} = I_s \dot{\omega}_s + H' \]  \hspace{1cm} (3)

where \( I_s \) is the inertia tensor and \( H' = (-I_{Br} \sin \beta, I_{Br} \cos \beta, I_C \omega_{aw}) \). The absolute time derivative of angular momentum of the system can be determined as:

\[ \ddot{H}_{G_r} = \dot{\omega}_s + I_s \ddot{\omega}_s + \dot{H}' \]  \hspace{1cm} (4)

By substituting the angular momentum of the system and its derivative into the newton’s second law and conservation of angular momentum theory, the equations of the motion can be written in a matrix form as:

\[ \begin{bmatrix} M \mathbf{\dot{x}} + C \mathbf{\dot{x}} + K \mathbf{x} \end{bmatrix} = \mathbf{Q}^{\text{wave}} + \mathbf{Q}^{\text{wind}} + \mathbf{Q}^{\text{buoy}} + \mathbf{Q}^{\text{mooring}} + \mathbf{Q}^{\text{gravity}} \]  \hspace{1cm} (5)

It should be noted that, \( M, C \) and \( K \) are the mass, damping and stiffness matrixes which are time variant. Variables \( \mathbf{Q}^{\text{wave}}, \mathbf{Q}^{\text{wind}}, \mathbf{Q}^{\text{buoy}}, \mathbf{Q}^{\text{mooring}}, \mathbf{Q}^{\text{gravity}} \) are the force and moment vectors resulted from the external loadings.

3. Loading

The mooring forces, buoyancy force, hydrodynamic and wind loads are derived in this section

3.1. Wind Load

An approximate wind thrust force is calculated for the swept area of the blades according to the method, which was introduced in [39]:

\[ F_{\text{wind}} = \frac{1}{2} C_T \rho_{\text{air}} A_{\text{blade}} V_{rb}^2 \]  \hspace{1cm} (6)

where \( C_T \), \( \rho_{\text{air}} \) and \( A_{\text{blade}} \) are the thrust coefficient, air density and swept area of the blades respectively. The amplitude of the wind velocity relative to the nacelle is shown as \( V_{rb} \). To determine the relative velocity of the wind and the blades it is important to model the wind field profile and determine the velocity of the blades.

In this study, wind velocity is represented by summation of a constant velocity with a turbulent component. The logarithmic wind profile is used to calculate the mean wind velocity and the turbulent component is calculated using the IEC Kaimal spectrum. A three dimensional wind field profile is generated using the TurbSim program which was proposed by [40].

The velocity of the center of blade area can be calculated as:

\[ \frac{V_{G_r}}{I_s} = V_{G_r} + T_{s \rightarrow G_s} \left( \omega_s \times \frac{r}{I_{G_r}} \right) \]  \hspace{1cm} (7)

where \( V_{G_r} \) is the linear velocity of the system which can be defined as: \( \mathbf{V}_{G_r} = (X_r, Y_r, Z_r) \).
The wind force in the $C_i$ coordinate system and the moment in the $C_s$ coordinate system is determined as follows:

$$F_{\text{wind}}^{C_i} = T_{2 \rightarrow s} F_{\text{wind}}^2$$
$$M_{\text{wind}}^{C_i} = r_{G_i \rightarrow G_s} \times T_{1 \rightarrow s} F_{\text{wind}}$$  \hspace{1cm} (8)

### 3.2. Wave load

The wave force acting on circular cylindrical structural members can be estimated using Morison’s equation as follows:

$$F_M = \frac{1}{2} \rho C_d D \frac{r}{g} \left| \vec{V}_n \right| - C_a \rho \frac{\pi}{4} D^2 \vec{V}_s + C_m \rho \frac{\pi}{4} D^2 \vec{V}_m$$  \hspace{1cm} (9)

where $C_d$, $C_a$ and $C_m$ are the drag, added mass and inertia coefficient respectively. Also, $\rho$ is the sea water density and $D$ is the diameter of the hull. $\vec{V}_n$, $\vec{V}_s$ and $\vec{V}_m$ are the normal component of wave acceleration, structural acceleration and velocity of the water particle relative to the tower. The normal component of water particle acceleration is determined by: $\vec{V}_n = \vec{r}_i \times (\vec{V} \times \dot{\vec{u}}_i)$. Parameter $\vec{r}_i$ represents the wave acceleration vector in the $C_i$ coordinate system and $\dot{\vec{u}}_i$ is the unit vector along the central axis of the tower in the $C_i$ coordinate system, which is equal to: $\dot{\vec{u}}_i = T_{i \rightarrow s}(0,0,1)$.

The structural velocity and acceleration are calculated by the following equations:

$$\dot{\vec{V}} = (\dot{X}_1, \dot{X}_2, \dot{X}_3) + T_{s \rightarrow i} (\dot{\vec{r}}_{G_i} \times \vec{r}_{G_i})$$  \hspace{1cm} (10)

$$\ddot{\vec{V}} = (\ddot{X}_1, \ddot{X}_2, \ddot{X}_3) + T_{s \rightarrow i} (\ddot{\vec{r}}_{G_i} \times \vec{r}_{G_i} + \dot{\vec{r}}_{G_i} \times (\dot{\vec{r}}_{G_i} \times \vec{r}_{G_i}))$$  \hspace{1cm} (11)

Where $\vec{r}_{G_i}$ is the vector radius from CM of the system to the segment with unit length. The normal velocity of the water particle relative to the tower is expressed as $\vec{V}_n = \vec{r}_i \times (\vec{V}_n \times \dot{\vec{u}}_i)$. The relative velocity of the wave to the segment of the submerged tower is shown as $\vec{V}_{rel} = \vec{V} - \dot{\vec{V}}$ in which $\vec{V}$ is the wave kinematic velocity.

To define the velocity and acceleration of the water particles as it was developed by [41], [42], it is considered that the sea waves have a regular sinusoidal shape. According to this theory the surface elevation, velocity and acceleration of particles are computed from Eqs. (12)-(14):

$$\eta(y,t) = \frac{H}{2} \cos (ky - \omega t)$$  \hspace{1cm} (12)

$$V = (0, u, v)$$

$$\begin{align*}
  u &= \frac{\pi H}{T} \times \frac{\cosh k(x + d)}{\sinh kd} \cos (ky - \omega t) \\
  v &= \frac{\pi H}{T} \times \frac{\cosh k(x + d)}{\sinh kd} \sin (ky - \omega t)
\end{align*}$$  \hspace{1cm} (13)
where $H$, $T$, $k$ and $\omega'$ are wave height, time period of wave, wave number and angular wave frequency respectively. The wave force on the platform of the system and the wave moment in the system is computed by numerically integrating over the submerged length of the tower as follows:

$$
\begin{align*}
\mathbf{F}_{\text{wave}} &= \int_{-\infty}^{\infty} \mathbf{F}_{\text{wave}}(t) dt \\
\mathbf{M}_{\text{wave}} &= \int_{-\infty}^{\infty} \mathbf{M}_{\text{wave}}(t) dt
\end{align*}
$$

The wave loading along surge and sway by considering the wind-wave misalignment angle, $\beta$, can be expressed as:

$$
\begin{align*}
F_{\text{wave,surge}} &= F_{\text{wave}} \cos \beta \\
F_{\text{wave,sway}} &= F_{\text{wave}} \sin \beta
\end{align*}
$$

### 3.3. Mooring forces

A simplified mooring system is assumed to consist of four radial taut lines in which the tension vector can be expressed as:

$$
\begin{align*}
\mathbf{F}_i &= (T_0 + \frac{ES}{L}(r_g/A - L)) \frac{F_{E/A}}{r_{E/A}} \\
\mathbf{F}_{i,\text{force}} &= (0, 0, \rho g \pi r^2 h_i)
\end{align*}
$$

The pretension of each mooring line is shown as $T_0$, also $E$, $S$ and $L$ are the Young's Modulus, line cross sectional area and the initial length of the line respectively. In addition, the resulting external moment is calculated from $\mathbf{M}_i = \mathbf{r}_{B/G} \times \mathbf{F}_i$.

### 3.4. Buoyancy Load

The instantaneous buoyancy of the floating system can be expressed as follows:

$$
\mathbf{F}_b = (0, 0, \rho g \pi r^2 h_i)
$$

where $\rho$ is the water density, $r$ is the radius of the cylinder and $h_i$ is the instant submerged length of the cylinder.

To determine the moments as introduced by [43], the center of buoyancy should be calculated, if the distance between center of buoyancy and CM of the system is assumed as $\mathbf{F}_{B/G} = (x_B, y_B, z_B)$, the distance can be expressed as:
The resulting external moment is defined as 

\[
\begin{align*}
\xi_B &= -\frac{n_{33} r^2}{4 t_{31} h_i} \\
\eta_B &= -\frac{n_{33} r^2}{4 t_{31} h_i} \\
\zeta_B &= -h_G + \frac{h_i}{2} + \frac{r^2 (n_{11}^2 + n_{22}^2)}{8 t_{33}^2 h_i}
\end{align*}
\]  

(19)

The resulting external moment is defined as 

\[
M'_B = F_{B/G} \times T_{c_1 \rightarrow c_2} F'_B
\]

4. Simulation and Results

The NREL 5MW spar type wind turbine model is used as a case study. The wind speed is considered to be 12 m/s with a turbulence intensity of 10%. The significant wave height is 2 m with a 10 s period. The wind time history for the mentioned case is demonstrated in Fig. 2. It should be noted that the wind time history is generated in three different axes and the wind turbine is facing the wind speed with higher values.

Fig. 2. Three dimensional wind time history (v = 12 m/s and turbulence intensity of 10%)

Fig. 3 shows the translational motion of the platform under four different misalignment angles. It can be clearly seen that, by changing the misalignment angle the dynamic response changes especially in surge and sway directions.
Fig. 3. Translational motion of the floating wind turbine under different misalignment angles

Fig. 4 shows the rotational motion of the platform under four different misalignment angles. One can easily notice that the rotational motion of the floating wind turbine is dependent to the misalignment angle. This approach is related to the equipment resilience, which is an important characteristic in the system [44], [45].

Fig. 4. Translational motion of the floating wind turbine under different misalignment angles
5. Conclusion

In this paper, Spar type floating wind turbine is modeled using conservation of angular momentum theory and Newton’s second law with considering the misalignment effects between wind and wave loadings. The proposed model includes six unknowns and two known degrees of freedom. The nonlinearity coupling between motions in all degrees of freedom is considered in the model. The hydrodynamic, wind, mooring and buoyancy forces are calculated for the system. At first, the nonlinear equations of motion are determined and are solved in MATLAB. Then, the system is simulated for four different misalignment angles. Finally, the responses of the system are defined. The results demonstrate the importance of considering the misalignment angle between wind and wave loadings in every degree of freedom. The system’s dynamic response alters with the change of misalignment angle.

References


