Do Households Actually Generate Rational Expectations? "Invisible Hand" for Steady State

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Abstract

The rational expectations hypothesis has been criticized for imposing substantial demands on economic agents, and this problem has not been sufficiently solved by introducing a learning mechanism. I present a new approach to this problem by assuming that households behave on the basis of not the rate of time preference but the capital-output (income) ratio. I show that households can equivalently reach and stay at a steady state without doing anything equivalent to computing a complex macro-econometric model. Although households are not required to implement anything difficult, they look to be behaving fully rationally, led by an "invisible hand."

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Keywords: Capital-output ratio; Rational expectation; Steady state; Sustainable heterogeneity; Time preference

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1 INTRODUCTION

The rational expectations hypothesis has been predominant in economics since it was popularized by Lucas (1972) and Sargent et al. (1973), both of whose papers were based on that of Muth (1961), and rational expectations are currently assumed in the vast majority of economic studies. In general, a rational expectation is understood to be a model-consistent expectation. An economic agent behaves consistently with an economic model while fully utilizing all available information.

However, the rational expectations hypothesis has been criticized for imposing substantial demands on economic agents. It first assumes that there can be an objectively correct and true economic model with corresponding correct and true parameter values. The hypothesis then requires that the models and parameter values that economic agents subjectively hold in their minds and act upon are, on average, equal to the objectively correct and true model and parameter values. Because of this property, economic agents are assumed to make no systematic errors in their expectations on average. However, can economic agents actually generate rational expectations by clearing this significantly high hurdle?

To generate rational expectations, households generally have to do something equivalent to computing complex large-scale non-linear dynamic macro-econometric models. Can a household routinely do such a thing in its daily life? Evans and Honkapohja (2001) argued that this problem can be solved by introducing a learning mechanism (see also, e.g., Marcet and Sargent, 1989; Ellison and Pearlman, 2011), but this solution is not necessarily regarded as being sufficiently successful because arbitrary learning rules have to be assumed.

In this paper, I present an alternative solution. In Ramsey-type growth models, the rate of time preference (RTP) is positively proportionate to the capital-output (income) ratio (COR) and the capital-wage ratio (CWR) at steady state. These relations suggest the possibility that households behave on the basis of COR or CWR and not RTP as required in the standard models. COR and CWR have a clear advantage over RTP in that COR and CWR can be directly and easily observed, whereas RTP cannot. As is well known, we can compute a value of RTP only indirectly on the basis of some assumed models: that is, we cannot know whether the computed value is correct, true, and intrinsic. Given this clear advantage of COR and CWR over RTP, I present an alternative procedure for households to reach a steady state, in which households behave on the basis of CWR.

The alternative procedure is very simple. A household assesses whether the combination of its earned (labor) income and wealth (capital) feels comfortable or not. They are not required to do anything equivalent to computing a complex macro-econometric model. Furthermore, they are not even required to be aware of any sort of economic model. In this new model, households reach a steady state without difficulty, and I show that we can interpret this steady state as the same one reached by the conventionally assumed procedure that relies on generating rational expectations based on RTP. In addition, the essential nature is unchanged, regardless of whether households are homogeneous or heterogeneous. Households appear as if they are unconsciously and unintentionally led to a steady state by a mysterious force or an "invisible hand." Because the alternative procedure is far easier for households to use than the conventionally assumed procedure and leads to the same steady state, it is much more likely that households actually behave as proposed by the alternative procedure rather than the traditional one.

In this case, the invisible hand does not require an objectively correct and true RTP, so the steady state derived by it is not necessarily firmly anchored to some ideal state. Hence, the invisible hand is vulnerable in that the achieved steady state is susceptible to various shocks and therefore the economy will occasionally fluctuate. In particular, heterogeneity among households is an important source of vulnerability, because households have to estimate the values of a few key variables.

2 GUIDE FOR HOUSEHOLD BEHAVIOR

2.1 RTP, COR, and CWR

Suppose a Ramsey-type growth model in which the representative household maximizes its expected utility

$$E\int_0^\infty \exp(-\theta t)u(c_t)dt$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - c_t \ ,$$

where y_t, k_t, and c_t are production, capital, and consumption per capita, respectively, in period t; A is technology; θ (> 0) is RTP; u is the utility function; $y_t = f(A, k_t)$ is the production function; and E is the expectation operator. The production function is assumed to be Harrod neutral such that $y_t = A^\alpha k_t^{1-\alpha}$, where α ($0 < \alpha < 1$) is a constant.

At steady state,

$$\theta = \frac{\partial y_t}{\partial k_t}$$

holds. By the production function,

$$\frac{\partial y_t}{\partial k_t} = (1-\alpha)A^{\alpha}k_t^{-\alpha} = (1-\alpha)\frac{A^{\alpha}k_t^{1-\alpha}}{k_t} = (1-\alpha)\frac{y_t}{k_t} \ , \tag{1}$$

so

$$\theta = (1-\alpha)\frac{y_t}{k_t} \tag{2}$$

at steady state; that is, RTP (θ) is equivalent to COR ($\frac{y_t}{k_t}$) times $(1-\alpha)$. Because the production y_t is distributed by

$$y_t = w_t + \frac{\partial y_t}{\partial k_t}k_t \ , \tag{3}$$

y_t also indicates the sum of the labor income and capital income evaluated by $\frac{\partial y_t}{\partial k_t}$, where w_t is the wage (labor income). By equations (1) and (3),

$$\frac{y_t}{k_t} = \alpha^{-1}\frac{w_t}{k_t} \ . \tag{4}$$

That is, CWR ($\frac{w_t}{k_t}$) is positively proportionate to COR ($\frac{y_t}{k_t}$). Most simply, k_t can be interpreted as wealth. Hence, COR and CWR can be simply interpreted as the wealth-income ratio and the wealth-wage ratio, respectively. Problems concerning this interpretation are discussed in Section 6.1.3.

An important point is that RTP and COR are substitutable at steady state by equation (2); furthermore, RTP and CWR are also substitutable at steady state because, by equations (2) and (4),

$$\theta = \left(\frac{1-\alpha}{\alpha}\right)\frac{w_t}{k_t} \ . \tag{5}$$

This substitutability means that a household may use CWR (or COR) at steady state and not RTP as a guide for its behaviors.

2.2 *Constancy of COR*

Kaldor (1957) noted six remarkable historical constancies, which are known as Kaldor's facts. One of these is the fact that the capital-output ratio (i.e., COR) is roughly constant over long periods of time. Recently, Pickety (2013) showed that COR does not appear to have changed very much since the late nineteenth century.

The constancy of COR has been regarded as an essential element, not only in Ramsey-type growth models such as that shown in Section 2.1, but also in many other types of economic growth models (e.g., Solow, 1956; Romer, 1986). In this sense, the constancy of COR is generally accepted not only as an empirical fact but also as a nature that economic theories require.

Although the constancy of COR is predicted by equation (2), the equation also indicates that, if RTP changes, COR and CWR also change. The effect of a possible change in RTP, COR, and CWR is examined in Section 6.2.

3 WHAT HOUSEHODS SHOULD DO

In this section, I present a procedure according to which households behave considering CWR. For simplicity, it is first assumed that all households are identical, but this assumption is removed in Section 4. The values of α and A are also assumed to be exogenously given and constant.

3.1 *The procedure*

3.1.1 "Comfortability" of CWR

A household should first subjectively evaluate the value of $\frac{\breve{w}_t}{\breve{k}_t}$ where $\breve{w}_t$ and $\breve{k}_t$ are w_t and k_t of the household respectively, that is, how much labor income it earns and how much capital (wealth) it possesses. Let Γ be the subjective valuation of $\frac{\breve{w}_t}{\breve{k}_t}$ by a household and Γ_i be the value of $\frac{\breve{w}_t}{\breve{k}_t}$ of household i ($i = 1, 2, 3, \ldots, M$).

The household should next assess whether it feels comfortable with its current Γ, that is, its combination of income and capital. "Comfortable" in this context means "at ease," "not anxious," and other related feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC), and let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ therefore indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household i's Γ_i when it is at $\tilde{s}_i$.

3.1.2 Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.
Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

With this procedure, a household is not required to do anything equivalent to computing a complex large-scale non-linear dynamic macro-econometric model. It has only to subjectively value its self-assessed combination of labor income and capital, and adjust consumption to the point at which it feels most comfortable.

3.2 Reaching steady state

Let S_t be the state of the entire economy in period t, and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. The production function is assumed to be the same as that assumed in Section 2.1 (i.e., $y_t = A^{\alpha} k_t^{1-\alpha}$).

Lemma 1: If households behave according to Rules 1-1 and 1-2, then one and only one $\tilde{S}_{MDC}$ exists.
Proof: Because all households are identical, the values of Γ of all households are identical and also identical to the value of $\Gamma(S_t)$ in any period. Hence, all households that behave according to Rules 1-1 and 1-2 commonly adjust their consumption so as to reach the value of $\Gamma(\tilde{s}_i)$ that is common for any i, and therefore $\Gamma(S_t)$ approaches the common value of $\Gamma(\tilde{s}_i)$. As a result, $\Gamma(S_t) = \Gamma(\tilde{s}_i)$ is achieved, and when it is achieved, $\Gamma(\tilde{S}_{MDC}) = \Gamma(S_t) = \Gamma(\tilde{s}_i)$.

On the other hand, because

$$\Gamma(S_t) = \frac{y_t}{k_t} = \left(\frac{A}{k_t}\right)^{\alpha}$$

for the entire economy by the production function, then $\Gamma(S_t)$ corresponds to the level of k_t on a one-to-one basis and is a monotonically continuous function of k_t. Hence, there is only one level of k_t that corresponds to $\Gamma(\tilde{S}_{MDC}) = \Gamma(S_t) = \Gamma(\tilde{s}_i)$ because the value of $\Gamma(\tilde{s}_i)$ is unique and common for any i. Therefore, one and only one $\tilde{S}_{MDC}$ exists. ■

Note that even if idiosyncratic shocks occur on individual households, the economy stays at $\tilde{S}_{MDC}$ on average, because if a household's Γ deviates from its $\Gamma(\tilde{s})$, the household no longer will feel most comfortable. Therefore, according to Rule 1-2, it will begin to eliminate this deviation from $\Gamma(\tilde{s})$ by independently adjusting its level of consumption.

3.3 Substitutability

Next, I examine whether $\tilde{S}_{MDC}$ is consistent with households' rational expectations in the sense that $\tilde{S}_{MDC}$ can be interpreted as the same steady state as that reached as a result of households' behaviors based on rational expectations. Let $\tilde{S}_{RTP}$ be the steady state in the Ramsey-type growth model discussed in Section 2, and $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$. As assumed above, the values of α and A are exogenously given and constant.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ in the Ramsey-type growth model, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: By Lemma 1, one and only one $\tilde{S}_{MDC}$ exists. By equation (2) and the production function,

$$\theta = (1-\alpha)A^{\alpha}k_t^{-\alpha} . \qquad (6)$$

Therefore, in the Ramsey-type growth model, there is one and only one $\tilde{S}_{RTP}$ for a given value of θ, the values of per capita capital (k_t) at $\tilde{S}_{RTP}$ are subjected to a one-to-one correspondence to the values of θ, and k_t at $\tilde{S}_{RTP}$ is a monotonically continuous function of θ. Hence, because all households are identical, if the value of θ that is calculated from the value of k_t at $\tilde{S}_{MDC}$ by equation (6) is given as the value of θ in the Ramsey-type growth model, the value of k_t at $\tilde{S}_{RTP}$ is identical to that at $\tilde{S}_{MDC}$. Because the production function is common for both $\tilde{S}_{MDC}$ and $\tilde{S}_{RTP}$ and thereby y_t is subjected to a one-to-one correspondence to k_t equally for both of them, $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$. ■

Proposition 1 indicates that we can interpret that $\tilde{S}_{MDC}$ is equivalent to $\tilde{S}_{RTP}$, and it is therefore consistent with rational expectations. This means that the two procedures (RTP-based and MDC-based) can function equivalently and that MDC is substitutable for RTP as a guide for household behavior.

An important point is that we cannot know whether the achieved $\tilde{S}_{MDC}$ is equal to the objectively correct and true steady state. We know only that it is a steady state at which all households feel most comfortable, and we can interpret that it is equivalent to $\tilde{S}_{RTP}$.

3.4 Responding to technological progresses

Usually, an economy grows steadily thanks to technological progress. Under the MDC-based procedure, however, households behave only according to Rules 1-1 and 1-2, neither of which directly refers to technology. How do households perceive technological progress and respond to it? In this model, there are at least two easy and practical ways for households to respond to technological progress:

(a) If a new version (variety) of a product is introduced into markets with higher performance but the same price as the old version (variety), a household will buy the new version (variety) instead of the old one without changing its $\Gamma(\tilde{s})$.
(b) If a household feels that its income have unexpectedly and permanently increased and that its current Γ is deviating from (and particularly, is higher than) $\Gamma(\tilde{s})$, it will begin to adjust its consumption such that its Γ returns to $\Gamma(\tilde{s})$ according to Rule 1-2. Because of the permanent increase in income, the household will accumulate more capital to make Γ return to its $\Gamma(\tilde{s})$.

Channel (a) is related to the creation of new technology and Channel (b) is related to an increase in productivity, and both are commonly related to technological progress. Hence, both channels describe a household's responses when it faces technological progress. Channels (a) and (b) do not require any change in Rules 1-1 and 1-2; this means that households need do nothing special to respond to technological progress because they already respond to it through these two channels.

Furthermore, Channels (a) and (b) and Rules 1-1 and 1-2 indicate that households are not required to even know the level of technology (A_t). Indeed, it seems highly unlikely that households actually know and use a numerical A_t when they choose their behaviors, even though they look as if they are behaving rationally (i.e., consistent with the model).

Note that, in the above examinations, technological progress is implicitly assumed to be exogenous, and therefore the degree of risk aversion (DRA) is ignored. However, in the framework of endogenous technological progress, DRA should be considered. How sensitively a household responds to new versions (varieties) in Channel (a) and to increases in income in Channel (b) will differ depending on its DRA, and DRA will eventually influence firms' plans to invest in technology.

4 WHAT HETEROGENEOUS HOUSEHODS SHOULD DO

4.1 Consequences of heterogeneous preference

In actuality, households are not identical—they are heterogeneous—and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker, 1980; Harashima, 2010, 2012, 2017). Here, "unilaterally" means that a household behaves without considering the other households' optimality conditions. In particular, it supposes that all other households should behave in the same manner as it does (i.e., supposing that all households, including itself, are identical). For simplicity, it is assumed that only the MDCs of households under the MDC-based procedure and only the RTPs of households under the RTP-based procedure are heterogeneous; that is, households are identical except for either MDC or RTP.

If all households are identical, the value of $\frac{\partial y_t}{\partial k_t}$ is identical among households, but if they are heterogeneous, each household's perceived value of $\frac{\partial y_t}{\partial k_t}$ is not necessarily guaranteed to be identical among households. On the other hand, the real interest rate in markets (r_t) in period t is uniquely determined to be equal to the value of $\frac{\partial y_t}{\partial k_t}$ of the entire economy (i.e., the average value of $\frac{\partial y_t}{\partial k_t}$) and is common knowledge for all heterogeneous households. However, a household does not necessarily use the current value of r_t as its perceived value of $\frac{\partial y_t}{\partial k_t}$, because the current value of r_t is not the value of r_t at steady state. In addition, the stream of r_t is estimated differently by households because they are heterogeneous.

Under an environment in which the estimated streams of r_t are heterogeneous among households, how does a household estimates the value of $\frac{\partial y_t}{\partial k_t}$? If a household behaves unilaterally, it will estimate this value on the basis of its own value of $\Gamma(\tilde{s})$ because it behaves by supposing that the other households should behave in the same manner as it behaves. Particularly, a unilaterally behaving household will estimate that the value of $\frac{\partial y_t}{\partial k_t}$ is equal to its Γ because the household will basically maintain $\Gamma = \Gamma(\tilde{s})$ by behaving according to Rules 1-1 and 1-2, and it will assume that other households should behave in the same manner as it does.

Note that a household here merely estimates its own personal value of $\frac{\partial y_t}{\partial k_t}$; it is not required to use rational expectations (i.e., act model-consistently) to obtain this value. Furthermore, the estimated value need not be the objectively correct and true one.

Because the values of $\frac{\partial y_t}{\partial k_t}$ that are estimated by households are heterogeneous, the capital of each heterogeneous household accumulates differently. As a household's value of $\Gamma(\tilde{s})$ becomes relatively low, it accumulates relatively large amounts of capital, and its $\frac{\breve{y}_t}{\breve{k}_t}$ (Γ) becomes relatively low. Therefore, by the production function and equation (1), the household estimates a relatively low value of $\frac{\partial y_t}{\partial k_t}$ and vice versa. That is, because households hold heterogeneous values of $\frac{\partial y_t}{\partial k_t}$, any differences in capital accumulation are amplified.

Because capital accumulates differently among households, there is no steady state other than corner solutions if households' MDCs (i.e.,$\Gamma(\tilde{s})$) are heterogeneous and the households behave unilaterally, as shown in Lemma 2.

Lemma 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 1-1 and 1-2, $\tilde{S}_{MDC}$ does not exist.

Proof: r_t is determined to be equal to the value of $\frac{\partial y_t}{\partial k_t}$ of the entire economy (i.e., $\Gamma(S_t)$), and the capital of the entire economy accumulates according to the value of r_t. However, each heterogeneous household behaves unilaterally and accumulates capital on the basis of its own estimated value of $\frac{\partial y_t}{\partial k_t}$. A household with a relatively low value of $\Gamma(\tilde{s})$ accumulates a relatively large amount of capital, because a relatively low value of $\Gamma(\tilde{s})$ requires a relatively large amount of capital. Therefore, this household's valuation of $\frac{\partial y_t}{\partial k_t}$ is relatively low by equation (1), and vice versa.

A household whose personal valuation of $\frac{\partial y_t}{\partial k_t}$ is higher than $\Gamma(S_t)$ (i.e., the value of $\frac{\partial y_t}{\partial k_t}$ of the entire economy) accumulates less capital than it estimated, because

r_t is lower than its estimated value of $\frac{\partial y_t}{\partial k_t}$. Therefore, by Rule 1-2, the household decreases its consumption to approach its $\Gamma(\tilde{s})$. However, even after this adjustment, it still accumulates less capital than it estimated by the same reasoning. Hence, by Rule 1-2, it further decreases its consumption, and this process continues until it can no longer decrease its consumption. Once it reaches this point, it has to decrease its capital to sustain its minimum level of consumption and will eventually lose all its capital.

The capital of households whose estimated values of $\frac{\partial y_t}{\partial k_t}$ are higher than $\Gamma(S_t)$ decreases. Therefore, the ratio of capital owned by the households whose values of $\Gamma(\tilde{s})$ are lower than $\Gamma(S_t)$ to all capital in the economy increases, and thereby $\Gamma(S_t)$ and r_t decrease. Because of the decreases in $\Gamma(S_t)$ and r_t, the estimated values of $\frac{\partial y_t}{\partial k_t}$ of more households become higher than $\Gamma(S_t)$, and these households also eventually lose all capital. This process continues until all capital is owned by the lowest $\Gamma(\tilde{s})$ household. Therefore, if the values of $\Gamma(\tilde{s})$ are heterogeneous, $\tilde{S}_{MDC}$ does not exist. ■

The state at which all capital is owned by the household with the lowest $\Gamma(\tilde{s})$ (i.e., the lowest MDC) corresponds to the state at which all capital is owned by the lowest RTP household in Becker (1980). The problem arising from heterogeneity in households is therefore common for both the RTP-based and MDC-based procedures.

Lemma 2 indicates that Proposition 1 is meaningless from the start because it is highly likely that households are heterogeneous and Proposition 1 assumes homogeneous households. However, Harashima (2010, 2012, 2017) showed that a sustainable heterogeneity (SH) at which all optimality conditions of all heterogeneous households are simultaneously satisfied exists under the RTP-based procedure. In the next sections, I will show that SH also exists under the MDC-based procedure.

4.2 SH under the RTP-based procedure

First, I briefly explain the nature of SH under the RTP-based procedure based on the work of Harashima (2010, 2012, 2017).

4.2.1 The model of SH

Suppose for simplicity that there are only two economies—Economy 1 and Economy 2—that are identical except for RTP. Each economy consists of its own identical households respectively. Let θ_1 and θ_2 be RTPs of households in Economies 1 and 2, respectively, and $\theta_1 < \theta_2$. The population growth rate is zero in both economies. The two economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Because the economies are fully open, they are integrated through trade and form a combined economy. The combined economy can be interpreted as the world economy (the international interpretation) or the national economy (the national interpretation). Usually, the concept of the balance of payments is used only for international transactions, but because both national and international interpretations are possible, this concept and terminology are also used for the national economy model in this paper.

Because a balanced growth path requires Harrod-neutral technological progress, the production function of Economy i is assumed to be

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha}$$

for $i = 1$ or 2, where $y_{i,t}$ and $k_{i,t}$ are the per capita production and capital, respectively, of Economy i in period t; A_t is the technology in period t; and α ($0 < \alpha < 1$) is a constant. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies, and the economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ are returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \quad \text{and} \quad \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies, such that

$$\tau_t = \kappa(k_{1,t}, k_{2,t}) .$$

The government (or an international supranational organization under the international interpretation) can intervene in the activities of Economies 1 and 2 by transferring money between the two economies. The amount of transfer from Economy 1 to Economy 2 in period t is g_t, and it is assumed that g_t depends on capital, such that

$$g_t = \bar{g}_t k_{1,t} .$$

$\bar{g}_t$ is an exogenous variable for households and firms and is appropriately adjusted by the government (or an international supranational organization) in every period so as to achieve SH. Because $k_{1,t} = k_{2,t}$ and $\dot{k}_{1,t} = \dot{k}_{2,t}$,

$$g_t = \bar{g}_t k_{1,t} = \bar{g}_t k_{2,t} .$$

Each household in Economy 1 maximizes its expected utility

$$E \int_0^\infty u_1(c_{1,t}) \exp(-\theta_1 t) dt$$

subject to

$$\frac{dk_{1,t}}{dt} = A^\alpha k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha) A^\alpha k_{1,t}^{-\alpha} \left(\int_0^t \tau_s \, ds + z_0 \right) - \tau_t - \bar{g}_t k_{1,t} \quad ,$$

and each household in Economy 2 maximizes its expected utility

$$E \int_0^\infty u_2(c_{2,t}) \exp(-\theta_2 t) dt$$

subject to

$$\frac{dk_{2,t}}{dt} = A^\alpha k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha) A^\alpha k_{2,t}^{-\alpha} \left(\int_0^t \tau_s \, ds + z_0 \right) + \tau_t + \bar{g}_t k_{2,t}$$

where $c_{i,t}$ is the per capita consumption of Economy i in period t, u_i is the utility function of Economy i, and E is the expectation operator.

4.2.2 SH

Harashima (2010, 2017) showed in the framework of endogenous growth that, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ = constant, all the optimality conditions of both economies are satisfied (i.e., SH is achieved). Even if the government does not intervene (i.e., $\bar{g}_t = 0$), if the economies behave multilaterally in the sense that each economy behaves fully considering the optimality conditions of the other economy, SH is achieved. On the other hand, if the economies behave unilaterally, SH is not achieved unless a government appropriately intervenes. The reason why SH can be achieved in the cases of multilateral behaviors and appropriate government intervention is that the capital accumulation of the more advantaged Economy 1 is restrained because of multilateral behaviors and appropriate government interventions. If SH is achieved, the growth rates of consumption in both economies are equally

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \frac{\theta_1+\theta_2}{2}\right], \tag{7}$$

where m, v, and ϖ are positive constants, and $\varepsilon = -\frac{c_{1,t}u_1''}{u_1'} = -\frac{c_{2,t}u_2''}{u_2'}$ is the degree of relative risk aversion and is constant.

Harashima (2010, 2017) indicates that, in the framework of exogenous growth (e.g., Ramsey-type growth models) with a heterogeneous population, SH also exists. The capital accumulation of the more advantaged Economy 1 is also restrained at SH in exogenous growth models. Hence, the capital (wealth) that a household in Economy 1 owns at SH is not k_1 but $k_1 + \Psi$, where Ψ is a negative constant, and the capital a household in Economy 2 owns at SH is not k_2 but $k_2 - \Psi$, where $k_{i,t}$ is identical for any i through market arbitration (i.e., $k_{i,t} = k_t$ for any i).

Note that Harashima (2010, 2017) showed that the two-economy model can be easily extended to multi-economy models, and the results in multi-economy models are basically the same as those in the two-economy model.

4.2.3 Government intervention

Harashima (2012) showed that if a government intervenes such that

$$\lim_{t \to \infty} \bar{g}_t = \frac{\theta_2 - \theta_1}{2} ,$$

then SH is achieved even if Economy 1 behaves unilaterally, and equation (7) is satisfied. When sustainable heterogeneity is achieved, Economies 1 and 2 consist of a combined economy (Economy 1+2) with twice the population and RTP of $\frac{\theta_1 + \theta_2}{2}$. Suppose that there is a third economy with RTP of θ_3, and it is identical to Economies 1 and 2 except for its RTP. Because Economy 1+2 has twice the population of Economy 3, if a government intervenes such that the amount of transfer from Economy 1+2 to Economy 3 in period t is $g_t = \bar{g}_t k_{3,t} = \bar{g}_t k_t$, SH is achieved where $k_{3,t}$ is capital in Economy 3 in period t, and

$$\lim_{t \to \infty} \bar{g}_t = \frac{\theta_3 - \frac{\theta_1 + \theta_2}{2}}{2} .$$

Remember that $k_{i,t}$ is identical for any i through arbitration as shown in Section 4.2.2. By iterating similar procedures, if a government's transfers from Economy 1+2+ · · · + (H – 1) to Economy H is made on the basis of

$$\lim_{t \to \infty} \bar{g}_t = \frac{\theta_H - \frac{\sum_{q=1}^{H-1} \theta_q}{H - 1}}{H} , \tag{8}$$

then SH is achieved.

4.3 SH under the MDC-based procedure

If SH also exists under the MDC-based procedure, it means that $\tilde{S}_{MDC}$ also exists in a heterogeneous population. SH indicates that all heterogeneous households are linked in the sense that a household's behavior must be set so as to be consistent with the behaviors of all the other households, unlike in the case of a homogeneous population where $\tilde{S}_{MDC}$ is achieved even if the households are isolated from each other and behave independently.

The links among households in SH may be voluntarily established by households themselves, or the government may force them to be established. In any case, if the links are established, each household has to behave consistently with the links. A household's behavior is fundamentally affected by how it is linked with other households, and it therefore must obtain information about the links before it makes decisions on its economic activities.

4.3.1 Estimating information about household links

It would seem to be difficult for each household to correctly know the links needed for SH. As will be shown in Section 4.3.4, however, households do not need this information. Instead, a household has only to "estimate" the values of a few variables related to the links. Furthermore, a household does not actually need to know the correct links to achieve SH under the MDC-based procedure. Below, I discuss a few of the variables that households need to estimate.

4.3.1.1 $\boldsymbol{\Gamma(S_t)}$ at SH

$\Gamma(S_t)$ (i.e., CWR of the entire economy) at SH is a particularly important piece of information because the values of w_t and r_t depend on the value of $\Gamma(S_t)$. By equation (3), the value of w_t is determined by $w_t = y_t - r_t k_t$ in period t, and because r_t is equal to the value of $\frac{\partial y_t}{\partial k_t}$ of the entire economy, the value of w_t depends on $\Gamma(S_t)$ by equation (1). Therefore, each household must estimate a value of $\Gamma(S_t)$ to behave consistently with SH. Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by all households when households' MDCs are heterogeneous (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$.

A household will estimate the values of $\Gamma(\tilde{S}_{MDC,SH})$ in various ways. An important piece of information will be a household's recognition of how much it feels either richer or poorer than other households. If a household feels poorer than the average household, it will estimate that $\Gamma(\tilde{S}_{MDC,SH})$ is lower than its Γ because it believes that the average household has a larger amount of wealth than it actually has. Therefore, the poorer a household feels, the lower its perceived value of $\Gamma(\tilde{S}_{MDC,SH})$ than its Γ, and the richer it feels, the higher the value.

4.3.1.2 Net transfers from government

A household must also estimate the amount of government transfers (g_t) for it to behave consistently with SH. These transfers can take various forms. In some cases, a household will receive benefits from the government, but in other cases, it will pay taxes and bear other burdens. Let "net transfer" be the benefits from the government minus the burdens imposed by it with regard to SH in every period, and let T be the net transfer that a household receives. Specifically, let T_i be the net transfer that household i receives (i = 1,2,3, … , M).

Equation (2) implies that the RTP (θ) of a household is positively proportionate to its $\Gamma(\tilde{s})$. Equation (8) implies that a household whose value of $\Gamma(\tilde{s})$ is larger than the average $\Gamma(\tilde{s})$ will basically receive positive net transfers, and one whose value of $\Gamma(\tilde{s})$ is smaller than the average will basically receive negative net transfers. In addition, the net transfer increases (decreases) as the value of $\Gamma(\tilde{s})$ increases (decreases).

Because the production function is $\frac{y_t}{k_t} = A^{\alpha} k_t^{-\alpha}$, when a household's value of $\Gamma(\tilde{s})$ is relatively high, its values of k_t and y_t will be relatively low at SH, which implies that a household with a relatively high value of $\Gamma(\tilde{s})$ is relatively poor and vice versa. Because the amount of net transfer will be correlated with the value of $\Gamma(\tilde{s})$, as shown above, how much a household feels poorer or richer than other households will therefore also provide an important piece of information as they try to estimate *T*. If a household feels poorer than the average household, it will estimate that it will receive a positive net transfer, and as it feels poorer, it will estimate that it will receive a larger net transfer.

In addition, the amount of net transfer that a household currently receives will be directly observable to a considerable extent, and such direct observations will provide important information for households when estimating *T*.

4.3.1.3 Numerical adjustment to the raw (unadjusted) value of Γ

Suppose that a household's $\Gamma(\tilde{s})$ is not affected by its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$; that is, it is constant regardless of the household's estimated value of $\Gamma(\tilde{S}_{MDC,SH})$. On the other hand, a household's raw (unadjusted) value of Γ will take various values depending on its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$, because w_t, r_t, T and Ψ differ depending on $\Gamma(\tilde{S}_{MDC,SH})$. In addition, the capital (wealth) that a household owns consist of not only k_t but Ψ under SH as shown in Section 4.2.2. Therefore, it is possible that even if a household's current state is MDC, its raw (unadjusted) value of Γ is not equal to its constant value of $\Gamma(\tilde{s})$, and the household may therefore wrongly begin to deviate from its MDC state. That is, it is meaningless to simply compare the raw (unadjusted) value of Γ with $\Gamma(\tilde{s})$. A household has to make numerical adjustments to its raw (unadjusted) value of Γ on the basis of the information about its estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, w_t, r_t, T and Ψ, so as to make the comparison with $\Gamma(\tilde{s})$ meaningful.

Let Γ_R be a household's numerically adjusted value of Γ based on the information it has about its estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, w_t, r_t, T and Ψ. Specifically, let $\Gamma_{R,i}$ be Γ_R of household i.

Note that households may actually make numerical adjustments to $\Gamma(\tilde{s})$ rather than Γ, but it is assumed that households make numerical adjustments to Γ, not $\Gamma(\tilde{s})$ in this paper.

4.3.2 SH and the level of inequality

Before a government can intervene, it also has to know the necessary links among households for SH to be achieved. As with households, however, it will be very difficult for a government to know each individual household's links with other households because it cannot know the correct and true RTP of each household. Hence, not only households but also a government has to estimate the links.

How then does a government estimate the links? An important aspect of SH is that, even though households are heterogeneous, the level of inequality neither increases nor decreases at SH because SH indicates a steady state. If SH is not achieved, however, the level of economic inequality will continue to increase or decrease. As Harashima (2010, 2017) showed, if it continues to increase, less-advantaged households will resist the government in various ways to urge it to act to achieve SH (e.g., they will vote against the incumbent government in elections). On the other hand, if a government takes measures to decrease the level of economic inequality, more-advantaged households will oppose the measures. Therefore, the way in which households vote in response to measures to increase or decrease the level of economic inequality represents an important piece of information about the household SH links. Conversely, whether the level of inequality increases, decreases, or is unchanged can also be used as a necessary (but not sufficient) criterion to judge whether SH has been achieved.

According to the median voter theorem (e.g., Downs 1957), a government will intervene up to the point at which the number of votes cast in response to increases in the level of economic inequality is equivalent to that in response to decreases in elections. At this level of intervention, the level of economic inequality will be kept nearly constant, so a necessary condition for SH is satisfied at this state even though it is unknown whether this state is actually SH. A government may quit imposing measures to institute change because it interprets that SH is already achieved at this state.

4.3.3 Revised and additional rules

Sections 4.3.1 and 4.3.2 indicate that the rules of households in a heterogeneous population cannot be the same as those in a homogeneous population (Rules 1-1 and 1-2). A new government rule must also be introduced.

4.3.3.1 Rules for households

Considering the requirement for households to achieve SH shown in Section 4.3.1, the household rules (Rules 1-1 and 1-2) have to be revised in a heterogeneous population as follows:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.

Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

The essential natures of Rules 1-1 and 1-2 are preserved in Rules 2-1 and 2-1: a household should feel comfortable and adjust consumption until it feels most comfortable. It still does not need to do anything equivalent to computing a complex large-scale non-linear dynamic macro-econometric model. However, in a heterogeneous population, a household is also required to estimate the values of $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R.

4.3.3.2 Rule for the government

According to the argument presented in Section 4.3.2, a government can roughly achieve SH by making the number of votes cast in response to increases in the level of economic inequality equivalent to that in response to decreases in elections. Considering this nature, the following rule for a government is introduced.

Rule 3: The government adjusts T_i for some i if necessary so as to make the number of votes cast in response to increases in the level of economic inequality equivalent to that in response to decreases in elections.

4.3.4 Reaching SH

According to Rules 2-1 and 2-2, each household behaves so as to reach its $\tilde{s}$, fully considering its estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R, while the government behaves according to Rule 3.

4.3.4.1 No guarantee of $\tilde{S}_{MDC,SH}$

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. Let $\tilde{S}_{Bel}$ be the state at which all households feel that their MDCs are achieved; that is, they feel that the current state ($\tilde{S}_{Bel}$) is identical to their severally estimated states of $\tilde{S}_{MDC,SH}$. Let $\Gamma(\tilde{S}_{Bel})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{Bel}$. By households behaving according to Rules 2-1 and 2-2, $\tilde{S}_{Bel}$ can be realized at least temporarily. However, once realized, $\tilde{S}_{Bel}$ will not necessarily be stable because there is no guarantee that each household's estimated $\Gamma(\tilde{S}_{MDC,SH})$ is identical to $\Gamma(\tilde{S}_{Bel})$. Rather, $\Gamma(\tilde{S}_{Bel})$ is usually different from most households' estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ because these estimated values are probably heterogeneous. Hence, the capital of the entire economy will not accumulate and the wage will not be determined as most households estimated. As a result, households' values of Γ_R will begin to deviate from the current values even though each of them feels that its current Γ_R is equal to its $\Gamma(\tilde{s})$. Therefore, the current $\tilde{S}_{Bel}$ cannot stay unchanged, and $\tilde{S}_{MDC,SH}$ cannot necessarily always exist.

In addition, even if a government behaves according to Rule 3, it cannot achieve SH precisely because (1) Rule 3 satisfies only a necessary condition for SH as argued in Section 4.3.2 and (2) there are some technical problems such that the amount of net transfer to each household cannot be precisely equal to what is needed to achieve SH. For example, transfers are usually adjusted stepwise and cannot be fine-tuned on a household basis. Hence, it is not necessarily guaranteed that the amounts of net transfers for most households are exactly equal to what is needed to achieve SH.

4.3.5.2 Approximate SH

Although $\tilde{S}_{Bel}$ is not stable and will change, it will eventually reach a state of about $\tilde{S}_{MDC,SH}$ because the government adjusts T_i for some i if necessary according to Rule 3. In response to the government's adjustment of T_i, households will revise their estimated values of T_i and $\Gamma_{R,i}$, and, furthermore, of $\Gamma(\tilde{S}_{MDC,SH})$. Moreover, the government may also revise its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ if the number of votes cast in response to increases in the level of economic inequality is not equivalent to that in response to decreases in elections. As a result, $\tilde{S}_{MDC,SH}$ that the government estimates and $\tilde{S}_{Bel}$ will gradually converge at a unique state at least approximately. Because of the behavior of the government according to Rule 3, the number of votes cast in response to increases in the level of economic inequality will be eventually kept equivalent to that in response to decreases, and therefore the $\tilde{S}_{MDC,SH}$ that the government estimates and $\tilde{S}_{Bel}$ will eventually be approximately fixed. That is, $\tilde{S}_{Bel}$ will eventually become approximately identical to $\tilde{S}_{MDC,SH}$.

As shown in Section 4.3.4.1, government transfers are usually adjusted stepwise. Households are put into several categories according to their income and wealth, and the amount of net transfer to a household differs depending on the category to which it belongs, but all households in the same category receive the same amount. Here, suppose that the net transfer that a household receives is smaller than its estimated T, indicating that the household's actual income is smaller than it estimated, and therefore its capital (wealth) begins to decrease. This means that the household becomes poorer. Hence, the government revises the category to which it belongs to the one similarly poor households belong to. Thereby, the household can receive a larger amount of net transfers than it did previously. The resulting amount of net transfer may exceed its estimated T. Consequently, the household's income and capital will begin to increase. This kind of cycle will be continuously repeated, and the household will go back and forth around $\tilde{S}_{MDC,SH}$. Most households will similarly move around $\tilde{S}_{MDC,SH}$, which means that the economy will approximately remain at $\tilde{S}_{MDC,SH}$.

On the other hand, households may occasionally revise their estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R simply because they guess that they wrongly estimated $\Gamma(\tilde{S}_{MDC,SH})$. As a consequence of such occasional revisions (i.e., trial and error), the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ of most households may roughly converge at a unique value. The degree of instability of $\tilde{S}_{Bel}$ will be mitigated to some extent by these occasional revisions.

Therefore, thanks to the government's intervention and other factors, SH will be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved, as described above. Let also $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. The term "on average" is added because households go back and forth around $\tilde{S}_{MDC,SH}$. Because $\Gamma(\tilde{S}_{MDC,SH,ap})$ is the averaged value, a value of $\Gamma(\tilde{S}_{MDC,SH,ap})$ is uniquely determined for any $\tilde{S}_{MDC,SH,ap}$. Similarly, let $\Gamma_{i,ap}$ be Γ_i at $\tilde{S}_{MDC,SH,ap}$ on average for any i. A value of $\Gamma_{i,ap}$ is also uniquely determined for $\tilde{S}_{MDC,SH,ap}$. In addition, for the previously discussed reasons, the net transfer to a household will fluctuate at $\tilde{S}_{MDC,SH,ap}$, but it will be constant on average. Let $T_{i,ap}$ be the net transfer that household i receives on average from the government at $\tilde{S}_{MDC,SH,ap}$. A value of $T_{i,ap}$ is also uniquely determined for $\tilde{S}_{MDC,SH,ap}$.

Lemma 3: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, and if the government behaves according to Rule 3, then $\tilde{S}_{MDC,SH,ap}$ exists.

Proof: Because households behave according to Rules 2-1 and 2-2, the economy will reach $\tilde{S}_{Bel}$, although it may not be stable. Because of the government's adjustment of T_i for some i according to Rule 3 and the consequent revisions of households' severally estimated values of T_i, $\Gamma_{R,i}$, and $\Gamma(\tilde{S}_{MDC,SH})$ and the government's estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, $\tilde{S}_{Bel}$ and $\tilde{S}_{MDC,SH}$ will approximately converge and will be maintained approximately in the state at which the number of votes cast in response to increases in the level of economic inequality is equivalent to that in response to decreases in elections, indicating that $\tilde{S}_{MDC,SH,ap}$ exists. ■

An important point is that $\tilde{S}_{MDC,SH,ap}$ exists no matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are severally estimated by households. These estimated values need not be objectively correct and true.

4.3.5 Substitutability

4.3.5.1 SH under the RTP-based procedure

Under the RTP-based procedure in a heterogeneous population, both households and the government have to generate rational expectations in order to reach SH. Let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure when households are identical except for their RTPs, and let $\Gamma\left(\tilde{S}_{RTP,SH}\right)$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

4.3.5.2 Substitutability between MDC- and RTP-based procedures

As was the case for $\tilde{S}_{MDC}$ and $\tilde{S}_{RTP}$ in Proposition 1, we can interpret $\tilde{S}_{MDC,SH,ap}$ as $\tilde{S}_{RTP,SH}$ as Proposition 2 shows. Let θ_i be RTP of household i (i = 1, 2, 3, … , M). Remember that, as shown in Section 4.2.2, $k_{i,t}$ is identical for any i through arbitration in markets even in a heterogeneous population, and therefore, $k_{i,t}$ is equal to k_t of the economy. That is, $k_{i,t}$ does not simply indicate the capital owned by household i because Ψ exists.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in which households are identical except for their RTPs, then $\Gamma\left(\tilde{S}_{MDC,SH,ap}\right) = \Gamma\left(\tilde{S}_{RTP,SH}\right)$.

Proof: By Lemma 3, $\tilde{S}_{MDC,SH,ap}$ exists. On the other hand, as shown in Section 4.2, $\tilde{S}_{RTP,SH}$ exists for a given set of θ_i, and T_i and k_t are uniquely determined at $\tilde{S}_{RTP,SH}$. By equation (8), it is possible to calculate back θ_i for any i from the values of $T_{1,ap}$, $T_{2,ap}$, … , $T_{M,ap}$ and k_t at $\tilde{S}_{MDC,SH,ap}$. If the value of θ_i that is calculated back from these values is given as the value of θ_i under the RTP-based procedure for any i, then k_t and T_i at $\tilde{S}_{RTP,SH}$ are identical to k_t and $T_{i,ap}$ at $\tilde{S}_{MDC,SH,ap}$ for any i. Because the production function is common for both MDC- and RTP-based procedures and thereby y_t is subject to a one-to-one correspondence to k_t equally for both, Γ_i at $\tilde{S}_{RTP,SH}$ is identical to $\Gamma_{i,ap}$ at $\tilde{S}_{MDC,SH,ap}$ for any i. Therefore, $\Gamma\left(\tilde{S}_{MDC,SH,ap}\right) = \Gamma\left(\tilde{S}_{RTP,SH}\right)$. ■

Proposition 2 indicates that we can interpret that $\tilde{S}_{MDC,SH,ap}$ is equivalent to $\tilde{S}_{RTP,SH}$, although it is unknown whether the back-calculated value of θ_i is the objectively correct, true, and intrinsic value of RTP of household i. Nevertheless, whether or not it is the objectively correct, true, and intrinsic value does not matter if households use the MDC-based procedure. In addition, Proposition 2 indicates that because $\Gamma\left(\tilde{S}_{MDC,SH,ap}\right)$ exists no matter what values of $\Gamma\left(\tilde{S}_{MDC,SH}\right)$ and T are severally estimated by households as indicated in Lemma 3, $\Gamma(\tilde{S}_{RTP,SH}) = \Gamma(\tilde{S}_{MDC,SH,ap})$ holds for any estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ and T. That is, no matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are severally estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be regarded as the objectively correct and true steady state.

In addition, Proposition 2 means that a government need not necessarily provide the objectively correct $T_{i,ap}$ for SH. It only needs to make the number of votes cast in response to increases in the level of economic inequality equivalent to that in response to decreases in elections. Even if $T_{i,ap}$ is not objectively correct, $\Gamma(\tilde{S}_{RTP,SH}) = \Gamma(\tilde{S}_{MDC,SH,ap})$ is achieved as Proposition 2 shows, and a government is therefore not required to do any more.

5 INVISIBLE HAND

5.1 MDC- and RTP-based procedures: Which is actually used?

Households can use both the MDC- and RTP-based procedures to reach a steady state, but which procedure more likely to be used by them in daily life? It is much more likely that households would choose the more practical and easily usable procedure.

The MDC-based procedure has an important advantage over the RTP-based procedure in that CWR can be directly and easily observed by households, but RTP cannot. Although we can use models to compute the numerical values of households' RTPs indirectly, we cannot know whether the computed numerical values are correct and true. Although it is hard to know the correct and true numerical RTP values, under the RTP-based procedure, the initial consumption level that a household sets can be determined only after it calculates the saddle path to steady state on the basis of the correct and true RTP values that all households possess in their minds; that is, knowing the correct and true RTP values *ex ante* is indispensable under the RTP-based procedure. Clearly, a household is required to engage in more complicated behavior under the RTP-based procedure than under the MDC-based procedure.

Nevertheless, even under the MDC-based procedure, households need to estimate the values of a few variables, but there is an important difference between this process and that of rational expectations. The estimated values under the MDC-based procedure need not be objectively correct and true, but rational expectations under the RTP-based procedure need to be correct, at least on average— otherwise the economy cannot reach the steady state.

Even if the correct and true numerical values of RTP could be known, however, another and more difficult problem must still be solved in the RTP-based procedure. A household must generate rational expectations—that is, it must calculate the saddle path to steady state on the basis of the values of all households' RTPs to reach steady state. A household's initial consumption level can be set only after it has accomplished something equivalent to computing a complex large-scale non-linear dynamic macro-econometric model and thereby eventually finding the optimal consumption path. On the other hand, under the MDC-based procedure, all a household needs to do is simply estimate a few variable values and then adjust its behavior on the basis of its level of comfort with these estimated values.

In addition, there is indirect evidence against the RTP-based procedure. Even before modern economics emerged, households looked as if they behaved rationally—that is, model-consistently. The fact that people behaved without any economic model in mind indicates that the RTP-based procedure could not have been used by households until recently. Moreover, in the early period when modern economics emerged, the economic models were constructed on the basis of observations and data for the periods before these models were constructed. Hence, if these models well describe rational (model-consistent) household behaviors, it means that households actually could behave rationally (model-consistently) without any economic model in mind (i.e., before modern economics emerged). Of course, it is highly likely that even today, the vast majority of people are completely unaware of the models.

The RTP-based procedure imposes extraordinarily difficult (almost impossible) requirements on households. Clearly, the MDC-based procedure is far easier for households to use than the RTP-based procedure, so it is hard to imagine that they choose the RTP-based procedure. It is therefore highly likely that households actually behave according the MDC-based procedure, not the RTP-based one.

5.2 *Invisible hand*

Unlike the case under the RTP-based procedure, a household need not have any economic model in mind under the MDC-based procedure

and furthermore it can behave without even being conscious that it is acting under the MDC-based procedure and still reach $\tilde{S}_{MDC,SH,ap}$. Households look as if they are unconsciously and unintentionally led to $\tilde{S}_{MDC,SH,ap}$ by a mysterious force, or an "invisible hand."

Of course, the meaning of the invisible hand mentioned above is not the same as that used by Adam Smith (1776). Nevertheless, because $\tilde{S}_{MDC,SH,ap}$ can be reached without government interventions in a homogeneous population, the term as used both by Adam Smith and in this paper describes self-interested behaviors that result in a socially desirable state. However, in a heterogeneous population, $\tilde{S}_{MDC,SH,ap}$ will usually be achieved only with the help of government interventions, and therefore using the term "invisible hand" may be somewhat misleading.

6 VULNERABILITY OF THE INVISIBLE HAND

6.1 Difficulties in estimating $\Gamma(\tilde{S}_{MDC,SH})$, *T*, and Γ_R

An important problem with the invisible hand discussed in the previous section is that $\tilde{S}_{MDC,SH,ap}$ crucially depends on the estimated values of a few variables (i.e., $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R). Because these values will generally be estimated with incomplete information, $\tilde{S}_{MDC,SH,ap}$ will be vulnerable to various shocks and could occasionally fluctuate widely.

6.1.1 Limited information

When a household estimates these values, it can access only limited information about various aspects of the economy. A household can know only very limited types and amounts of information through its own direct experiences. Hence, most of the information it uses will be publicly disseminated. Even that, however, will not be comprehensive and, more importantly, it may not necessarily be correct and may even be purposefully distributed misleading or incorrect information. Therefore, the estimated values may be vulnerable to various shocks imposed by different kinds of newly obtained information.

6.1.2 Permanent capital and income

The CWR used under the MDC-based procedure conceptually should be the ratio of permanent labor income to permanent capital. Hence, the value of CWR should be modified by removing any temporal elements, but this modification may not be easy.

6.1.3 Capital or wealth

Conceptually, CWR should be the ratio of labor income to capital, not wealth. However, a household may not easily know how much capital it possesses, whereas it is much easier to know how much wealth it possesses. Even though the concepts of capital and wealth are technically different, it seems likely that many households would use wealth as a substitute for capital when determining their levels of comfortability.

A problem in this substitution is that the prices of various kinds of wealth fluctuate more widely and frequently than both the general price level and the prices of capital. This means that households will often potentially be confused by fluctuations in prices, and their estimated values of CWR may become biased. This vulnerability can be exploited; for example, Harashima (2015, 2018a) showed the possibility that bluffers can generate a bubble-like phenomenon by exploiting the opportunities this vulnerability generates (e.g., by manipulating the prices of some assets).

6.2 Revision of the estimated $\Gamma(\tilde{S}_{MDC,SH})$ as RTP shock

Because of these vulnerabilities, households will occasionally revise their estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R when new pieces of information arrive or some kinds of shocks are recognized. Nevertheless, idiosyncratic revisions by individual households will not have a huge impact on the entire economy. For a revision to cause a large shock to the economy, the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ of many households need to be simultaneously revised.

An occurrence of simultaneous revisions of the estimated $\Gamma(\tilde{S}_{MDC,SH})$ under the MDC-based procedure corresponds to a shock on the expected RTP of a representative household under the RTP-based procedure. In this sense, a shock on the estimated $\Gamma(\tilde{S}_{MDC,SH})$ is equivalent to that on the RTP of the representative household. Harashima (2014) showed that it is very difficult for a household to generate the expected RTP of the representative household, and therefore it is generated heuristically.

6.2.1 Effects of simultaneous revision of the estimated $\Gamma(\tilde{S}_{MDC,SH})$ under the MDC-based procedure

Suppose that, because of some new information that becomes simultaneously known to all households, the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ of all households are simultaneously revised upward, and all households perceive these simultaneous revisions. As mentioned above, this situation is equivalent to an upward RTP shock under the RTP-based procedure. Because of the increases in the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, households begin to guess that w_t and the net transfers they receive from the government will become smaller in the future than they had previously estimated. In other words, they begin to feel poorer. Because they begin to estimate smaller incomes while capital remains unchanged, the households begin to feel that their current values of Γ_R are smaller than previously estimated. Therefore, they begin to adjust the values of Γ_R upwards to make them equal to their values of $\Gamma(\tilde{s})$ by adjusting consumption according to Rule 2-2.

Adjusting the value of Γ_R upwards means that some of the accumulated capital becomes excessive and must be reduced, but how can it be reduced? One possibility is that a household increases its consumption temporarily to reduce its accumulated capital as Rule 2-2 suggests. However, it is highly unlikely that a household will increase its consumption because it has to behave strategically after the simultaneous revisions of the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ by all households. During the simultaneous revisions, all households may change their actions in different ways. Therefore, the effects of the simultaneous revisions on a household will differ depending on the actions of the other households. Hence, a household must behave strategically, and it will therefore not necessarily simply increase its consumption as it would in the "normal" case. Harashima (2004, 2009, 2018b) showed that, in the framework of the RTP-based procedure, it is highly likely that a household instead decreases its consumption together with the other households because of strategic considerations. The same result may be observed under the MDC-based procedure. Because households decrease consumption, the economy falls into a recession and large amounts of resources are left unutilized. Even if such negative effects are generated, households will continue to decrease consumption until $\Gamma_R = \Gamma(\tilde{s})$ is restored. As a result, excessive capital will eventually be destroyed.

Suppose next the case of a downward revision, which is equivalent to a downward RTP shock. Because of decreases in the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, households begin to guess that w_t and the net transfers they receive will become larger in the future than previously estimated. In other words, they begin to feel richer. Hence, households begin to feel that their current values of Γ_R are larger than previously estimated, and thereby they begin to adjust their values of Γ_R downwards to make them again equal to their values of $\Gamma(\tilde{s})$ by adjusting consumption according to Rule 2-2. Unlike the case with upward revision, it is highly likely that households will immediately begin to increase consumption regardless of what happens as a consequence of the increase. As a result, an economic boom will begin and capital and labor will begin to be overused.

6.2.2 Shock on the estimated $\Gamma(\tilde{S}_{MDC,SH})$ or the expected RTP?

Although a shock on the estimated $\Gamma(\tilde{S}_{MDC,SH})$ is equivalent to that on the expected RTP, the former seems to be intuitively easier to accept than the latter, because a shock on the estimated $\Gamma(\tilde{S}_{MDC,SH})$ means a shock to people's feeling of comfort. It is intuitively easy to accept that people will change their behavior if they begin to feel less comfortable. On the other hand, it is not easy to envision a situation where people technically adjust their numerical values of expected RTP. In this sense, it is much more likely that the actual shocks are on the estimated $\Gamma(\tilde{S}_{MDC,SH})$, and shocks on the expected RTP merely mirror those on the estimated $\Gamma(\tilde{S}_{MDC,SH})$; that is, the shocks on the expected RTP are not "real" phenomena.

6.3 Informational superiority of government

There is a possibility that a government has informational superiority with regard to $\tilde{S}_{MDC,SH,ap}$ because it can access much more information than ordinary people can. If a government truly has substantial informational superiority, it will be able to mitigate the vulnerability of the invisible hand to some extent—for example, by leading households to stay at $\tilde{S}_{MDC,SH,ap}$ over a long period of time and to reach $\tilde{S}_{MDC,SH,ap}$ more quickly and smoothly after a shock.

7 DISCUSSION

7.1 Rationality

A rational expectation has usually been regarded as a model-consistent expectation. However, because households can behave without any economic model in mind under the MDC-based procedure, this kind of understanding or definition of rationality seems to be meaningless in this case. At the least, an alternative understanding or definition of rationality may be needed under the MDC-based procedure.

7.2 Expected utility

An important question arises out of the MDC-based procedure. Are households actually maximizing their expected utilities in the first place? We can interpret that they are doing so under the RTP-based procedure; that is, they behave on the basis of carefully thought-out plans made through thorough calculations related to expected utility. However, because the RTP (discount factor) is not used in the MDC-based procedure, households need not—or rather cannot—calculate their expected utilities. Households behave only on the basis of their feelings of comfortability.

Even if households do not calculate their expected utilities, however, it does not mean that they do not consider the future at all under the MDC-based procedure. Humans are endowed with reason and therefore foresee the future and plan for their future actions. Under the MDC-based procedure, this future plan is reflected implicitly in the degree of comfortability, because capital takes effect in the future and the extent to which the current level of capital leads to a comfortable feeling (i.e., feeling secure for the future) is implicitly related to the future plan. Households do not want to accumulate capitals infinitely under the MDC-based procedure, which implicitly means that they unconsciously discount the utilities from their future consumptions. Therefore, households behave fully considering the future and choosing the best option for the future under the MDC-based procedure.

7.3 The correct and true $\tilde{S}_{RTP,SH}$

Another important question arises. Does the objectively correct and true $\tilde{S}_{RTP,SH}$ exist? We cannot know the correct and true RTP values of households. Hence, we cannot judge which $\tilde{S}_{RTP,SH}$ is the objectively correct and true $\tilde{S}_{RTP,SH}$, although any $\tilde{S}_{MDC,SH,ap}$ can be interpreted to be $\tilde{S}_{RTP,SH}$ as Proposition 2 indicates.

This problem may be generated because the concept of RTP used under the RTP-based procedure is not the same as that used in the fields of psychology and experimental economics. Indeed, the examinations in this paper strongly imply that the concepts of RTP under the RTP-based procedure and the one used in those fields are not the same. The concept of RTP used under the RTP-based procedure has usually ignored many observed anomalies that have been reported in the fields of psychology and experimental economics (e.g., Barro, 1999), but these issues may have been ignored because the concepts are different in the first place, even though it was convenient to use the term RTP for the RTP-based procedure. The RTP used in the RTP-based procedure may be only a shadow of a true deep parameter X. In this case, it follows that no objectively correct and true $\tilde{S}_{RTP,SH}$ exists.

8 CONCLUDING REMARKS

The rational expectation hypothesis has been criticized for imposing substantial demands on economic agents, such that households have to do something equivalent to computing complex large-scale non-linear dynamic macro-econometric models. Can a household truly implement such a thing routinely in its daily life? Evans and Honkapohja (2001) argued that this problem can be solved by introducing a mechanism of learning, but this solution is not necessarily regarded sufficiently successful because arbitrary learning rules have to be assumed.

In this paper, I presented an alternative solution. Considering the clear advantage of COR and CWR over RTP in that COR and CWR can be directly and easily observed but RTP cannot, I presented an alternative procedure for households to reach a steady state (the MDC-based procedure) instead of the conventionally assumed procedure based on rational expectations (the RTP-based procedure). The MDC-based procedure is very simple. A household has only to act on its feelings about whether the combination of its labor income and capital (wealth) is comfortable or not. Households are not required to generate rational expectations and, furthermore, are not even required to have any economic model in their minds. For all that, households can reach a steady state without difficulty. I showed that this steady state can be interpreted to be equivalent to the steady state reached under the RTP-based procedure. Furthermore, the essential nature is unchanged, regardless of whether households are homogeneous or heterogeneous. Households look as if they are unconsciously and unintentionally led to a steady state by a mysterious force or "invisible hand." Because the MDC-based procedure is far easier for households to use than the RTP-based procedure but leads to the same steady state as that under RTP-based procedure, it is highly likely that households actually behave under the MDC-based procedure.

The MDC-based procedure does not require an objectively correct and true model and RTP, and therefore no systemic error can exist. In this case, the lack of systemic error means that households can be interpreted to be naturally able to reach the same conclusions as if they generate rational expectations in the framework of the RTP-based procedure. On the other hand, because the steady state is not firmly anchored to some ideal state under the MDC-based procedure, the invisible hand has a vulnerability: the achieved steady state is vulnerable to various shocks and therefore the economy occasionally fluctuates. In particular, heterogeneity among households plays an important role in generating this vulnerability. In addition, the examinations in this paper imply the possibilities that households are not actually calculating the expected utilities and that the concept of RTP used under the RTP-based procedure is not the same as that used in the fields of psychology and experimental economics; this suggests that no objectively correct and true $\tilde{S}_{RTP,SH}$ exists.

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