Firm Size, Bank Size, and Financial Development

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Abstract

Financial intermediation facilitates economic development by providing entrepreneurs with external finance. The relative costs of financing depend on the efficiency of the financial sector and the sector using financial intermediation services, the production sector. These costs determine the occupational choices and the set of active establishments in the production and financial sectors. A model of establishment-size distributions in the production and financial sectors results. This model is calibrated to match facts about the U.S. economy, such as the interest-rate spread and the establishment-size distributions in the production and financial sectors. The model is then used to evaluate the importance of the technological progress in the production and financial sectors and the observed decline in the real interest rate for the dynamics of the value added and the average establishment size in the production and financial sectors. The model accounts for the observed positive trend in the share of the value added and the negative trend in the average establishment size in the U.S. and Taiwanese financial sectors during the last three decades.

Keywords: economic development; financial development; technological progress; establishment-size distributions; interest-rate spreads; real interest rate.

JEL Classification Numbers: E13; O11; O16; O41.

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1 Introduction

Financial intermediaries contribute to economic growth, and their role is increasingly important. The share of the value added in the U.S. financial intermediation sector increased by 40 percent in the last three decades, from approximately 3.5 percent in the 1980s to 5 percent at the beginning of the 21st century. Does this increase imply that the financial sector became more efficient relative to the other sectors of the economy? In a competitive economy, the increased efficiency of any sector should lead to greater output given the costs and lower entrepreneurial profits. In particular, the increased efficiency of the financial intermediation (due to technological improvements or changes in regulation) should lead to better allocation of funds, fewer information asymmetries, and increased efficiency of the other sectors. The observed growth in the financial sector share of value added implies that the sector-specific technology growth could have been unbalanced. At the same time, the real interest rate on savings decreased significantly during the last three decades (a phenomenon explained by the emergence of fast-growing economies and their gradual integration in the global financial markets; see Caballero et al., 2008). This implies that the financial intermediaries' cost of capital decreased. The decline in the cost of capital could be another reason behind the growth of the financial sector value added.

This paper analyzes the relative performance of the financial intermediation sector, referred to as the "financial sector," and all other sectors, referred to as the "production sector," and its impact on economic development during the last three decades. For these purposes, I develop an economic model of firm finance with sector-specific occupational choices.

At the heart of the model are two ingredients. First, individuals' occupational choices based on the expected profits of entrepreneurship determine the set of active entrepreneurs in the production and financial sectors. Second, the financial and production sectors' outputs are interdependent, because the entrepreneurs in the production sector rely on the financial intermediaries for the supply of funds. The model characterizes the quantity and quality of entrepreneurs in each sector, sector-specific output, prices, and profits as functions of exogenous sector-specific technological progress and the financial intermediaries' cost of capital. An improvement in the financial sector technology or an increase in the financial intermediaries' cost of capital makes the financial sector relatively more efficient, increases competition within the sector, and pushes the least efficient entrepreneurs out of the financial intermediation activities. At the same time, it decreases the cost of capital faced by production sector entrepreneurs, leading to the entry of less efficient producers. The opposite occurs when the production sector technology improves. The sector-specific average establishment size in terms of employment is an increasing function of the sector's relative efficiency, because more efficient entrepreneurs are able to successfully manage larger-scale projects. As a side product, the model offers a simple formula for evaluating the share of the financial sector value added: It is an increasing function of the interest-rate spread and a decreasing function of the financial intermediaries' cost of capital.

The model is calibrated to the U.S. economy. The country's real gross domestic product (GDP) per capita and the interest-rate spread are then used to trace the path of technological progress in the production and financial sectors of the US and Taiwan, given the financial intermediaries' cost of capital proxied by the country's real interest rate. The model explains a decline in the average establishment size of the financial sector in terms of employment, an increase in the fraction of financial establishments, and an increase in the financial sector value added observed in the US during the last three decades. The results suggest that the U.S. financial sector became less efficient relative to the production sector, and this led to a decline in the probability of successful monitoring of borrowers, defined in model terms, from 0.89 to 0.83 between 1986 and 2005. According to the model, the decline in the relative efficiency of the financial sector and subsequent increase in the financial sector profits and value added are caused by the decrease in the financial intermediaries' cost of capital or the real interest rate on savings.

The model also partially explains the nonlinear trends in the average establishment size of the financial sector in terms of employment and an increase in the financial sector value added observed in Taiwan during the last three decades.

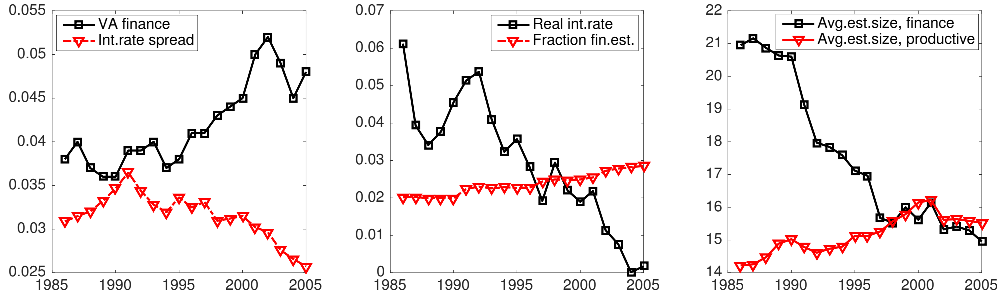
The quantitative analysis suggests that most of the U.S. output growth during the last three decades was due to the growth in the production sector technology and a decline in the financial intermediaries' cost of capital, with improvements in the financial sector technology having a minor impact. For Taiwan, the output growth is mainly explained by the growth of production and financial sector technologies, with the decline in the financial intermediaries' cost of capital having a minor impact.

This paper contributes to the ample literature on the importance of financial development for economic development. Several channels through which the financial development influences economic development have been emphasized: For example, see Khan (2001) and Greenwood et al. (2010, 2013) on the role of information costs; Erosa (2001), Antunes et al. (2008), Amaral and Quintin (2010), and D'Erasmo and Boedo (2012) on the importance of limited enforcement and intermediation costs; and Chiu et al. (2017) on the role of intermediation in efficiency and innovation.

I follow the conventional approach and connect financial development and economic growth by exploiting the consequences of the external finance provision for occupational choices and for the dynamics of the establishment distribution. The importance of financial development for external financing, occupational choice, and firm size has been empirically evaluated by Rajan and Zingales (1998) and Beck et al. (2006), among others, and clarified in the models by Barseghyan and DiCecio (2011); Greenwood et al. (2010, 2013); Arellano et al. (2012); Cooley and Quadrini (2001); Cabral and Mata (2003); Clementi and Hopenhayn (2006); Albuquerque and Hopenhayn (2004); and Buera et al. (2011, 2015), among many others. Most studies model the financial sector as consisting of competitive firms or introduce financial frictions as a borrowing constraint without explicitly considering the problem that the financial intermediary solves when deciding on the allocation of funds. One exception is Laeven et al. (2015) who model economic growth as an outcome of continuous innovations by profit-maximizing entrepreneurs and financiers. However, those authors do not consider the distribution of financial establishments.

In this paper, instead of concentrating on the establishment size in the economy overall, I discuss the dynamics of establishment size in the financial sector and all other sectors, and consider the difference in these dynamics as a signal of the unequal relative efficiency of these sectors. The main distinctive feature of this paper is explicit modeling of the financial intermediaries' profit-maximization problem and the possibility of positive profits from the financial intermediation activities. As a result, this paper sheds some light on how sector-specific technological progress affects the characteristics and dynamics of financial sector establishments and the share of financial sector value

Figure 1: Characteristics of the financial intermediation sector, the US data



Note: The figure presents the US data on the financial sector value added and the interest-rate spread (left graph); the real interest rate on savings and the fraction of financial sector establishments in total establishments (middle graph); the average establishment size, in terms of number of persons engaged, in the financial sector and all other sectors (right graph). Data sources are described in the Appendix.

added.

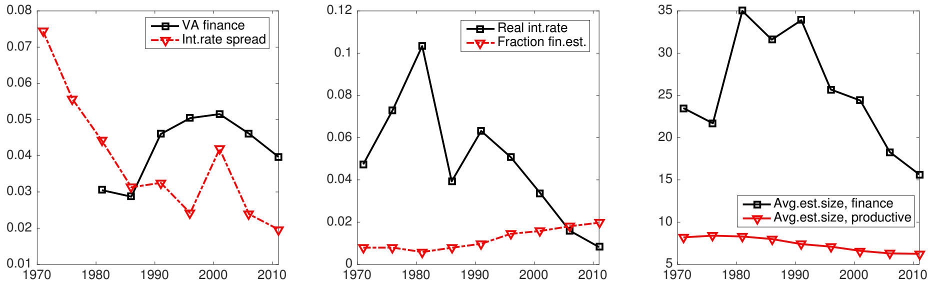
The rest of the paper is organized as follows. Section 2 briefly describes the trends in the financial and production sectors' characteristics using data from the US and Taiwan. Section 3 presents a model that incorporates the profit-maximizing producers and financial intermediaries (bankers) in a general equilibrium framework with exogenous sector-specific technological progress. Section 4 characterizes the properties of key economic indicators derived from the model. Section 5 provides quantitative analysis of the model: calibration to the U.S. economy, analysis of the model performance in replicating the U.S. and Taiwanese data, and a number of counterfactual experiments evaluating the importance of sector-specific technological progress and the financial intermediaries' cost of capital for economic development in the US and Taiwan during the last three decades. Section 6 concludes.

2 The Data and the Modeling Strategy

Figure 1 shows the time series of the key variables of interest: the financial sector share of value added, the interest-rate spread, the average size (in terms of the number of persons engaged) of the financial and production sector establishments, the fraction of financial sector establishments, and the real interest rate on savings, using U.S. data for 1986–2005. All the data and data sources are described in the Appendix.

The model constructed below is intended to explain the trends observed in the first

Figure 2: Characteristics of the financial intermediation sector, Taiwanese data



Note: The figure presents the data for Taiwan on the financial sector value added and the interest-rate spread (left graph); the real interest rate on savings and the fraction of financial sector establishments in total establishments (middle graph); the average establishment size, in terms of number of persons engaged, in the financial sector and all other sectors (right graph). Data sources are described in the Appendix.

five of these variables as an outcome of three possible causes: the unobserved technological progress in the production sector, the unobserved technological progress in the financial sector, and the observed changes in the real interest rate, a proxy of the financial intermediaries' cost of capital. Thus, all key variables of interest (except for the real interest rate) are endogenous variables, and the sector-specific technological progress and the real interest rate are the only exogenous variables in the model.

In this respect, the model does not take into account the potential positive impact of bank branch deregulation on technological progress in the US. The relaxation of intrastate bank branch restrictions that started in the 1970s in the US could have contributed to the financial development and to greater efficiency of the production sector through easier access to loans and better monitoring (see Jayaratne and Strahan, 1996). In addition, it could have led to growth in the fraction of financial sector establishments in the total number of establishments, reported in Figure 1. However, in many U.S. states the bank branch restrictions were lifted by 1986 (the starting year of the analysis in this paper), and even if this deregulation served as a positive shock to financial sector development, the dynamics of the size distribution of the bank branches (financial establishments in the model) and production establishments, the relative prices, and the sector-specific value added can still be characterized as functions of exogenous technological progress.

The other two trends characterizing the U.S. financial sector during the last three decades (not reported in Figure 1) are the increased bank concentration and a significant decrease in the number of banks. The first of these trends is consistent with the outcomes of the model, which predicts that technological progress leads to a larger fraction of total capital being intermediated by the most efficient entrepreneurs. The explanations of the potential drivers behind the second trend are not the purpose of this paper. A number of studies (see, for example, Wheelock and Wilson, 2000; and Berger et al., 1999 for a review of related literature) considered the quality of personnel management and asset management as the potential drivers behind numerous bank failures and the mergers and acquisitions that led to the sharp decline in the number of U.S. banks during the last three decades.

The model considered in this paper is based on interactions between production and financial sector establishments where entrepreneurs in the production sector borrow capital funds from financial sector entrepreneurs. The efficiency of each establishment depends on the ability of the entrepreneur who manages the establishment (corresponding to the bank branch in the financial sector). Thus, the model aims to explain the trends characterizing U.S. bank branches rather than banks.

The US has been characterized by relatively stable economic growth rates during the last three decades. To test the relevance of the model and to evaluate the importance of sector-specific technological progress for economic development in an economy growing at an unbalanced rate, I consider the data for Taiwan. Figure 2 reports the six key variables of interest using the (available years within the period) 1971–2011 data for Taiwan. Although not as transparent as for the US, the trends in the Taiwanese data mostly resemble those for the US, except for the average size of the production sector establishments which decreases for Taiwan.

Next, I describe the model, before taking it to the data and quantifying the impact of financial development, production sector development, and the decline in the real interest rate on the variables shown in Figures 1 and 2.

3 The Model

Consider an economy populated by two types of agents, "potential producers" and "potential bankers," with measure one of individuals of each type. The "potential producers" are able to run a firm in the production sector to produce final goods. The "potential bankers" are able to run a financial intermediary institution to supply loans to entrepreneurs from the production sector and monitor their performance. Thus, the agent's type defines the sector in which the agent can perform entrepreneurial activity. Depending on his ability, the agent can choose whether to become an entrepreneur in his type-specific sector or to be hired as a worker. The distribution of agents' abilities within each type is time-invariant and characterized by cumulative distribution function $F_j(z)$ and probability density function $f_j(z)$, $z \in [\underline{z}_j, \bar{z})$ in the production ($j = e$) and financial ($j = b$) sectors, respectively. The productivity of a worker does not depend on his entrepreneurial type; thus, the labor market is common for the production and financial sectors.

All agents are born with zero assets. At the beginning of their lives, they decide whether to become an entrepreneur of their type or to be hired in the labor market as a worker. Those who decide to become entrepreneurs have a span of control to operate a decreasing returns-to-scale technology and choose the optimal amounts of capital to borrow and labor to hire, given expectations about the output that they can produce. Each entrepreneurial project has a certain probability of failure, and the expected profits depend on the entrepreneur's ability. Each worker supplies one unit of labor in exchange for the expected income given by the market wage. All agents receive their income and decide on consumption and savings allocations at the end of the first period of their lives. They retire and supply their savings to the active financial intermediaries at the beginning of the second period of their lives. Finally, they consume their returned savings and interest at the end of the second period of their lives and then die. Thus, each agent lives for two periods, and in every period, there are two overlapping generations, one working and one retired. The consumption and saving choices of an agent solve the following utility maximization problem:

$$\max_{c_1,h} \; u(c_1) + \beta u(c_2) \tag{1}$$

$$s.t.: c_1 + h = \Pi, \tag{2}$$

$$c_2 = (1 + r_b)h, \tag{3}$$

where $u(c)$ is the utility from consumption, $u'(c) > 0$, $u''(c) < 0$; c_1 and c_2 denote consumption in the first and second periods of the agent's life; Π is the realized income of an agent (realized profits for entrepreneurs and realized wages for workers); $\beta \in (0,1)$ is a discount factor; r_b is the interest rate paid by financial intermediaries (bankers) for savings; and h denotes savings. Assume that $u(0) = \underline{u} \gtrapprox 0$; that is, the agents whose realized income is zero still enjoy some positive consumption (for example, by collecting fruits from publicly available trees). Assume that $\underline{u}$ is sufficiently low so that all agents prefer positive income and therefore, work either as entrepreneurs or as workers.

The problem of the individuals of each type, the role of abilities, and the markets are described in more detail below.

3.1 The problem of a "potential producer"

Each individual of "potential producer" type decides whether to run a firm and produce output in the form of final goods or to be hired as a worker in the labor market. The decision is made based on the expected payoffs of these occupational choices.

The technology that a potential producer can operate has the following form:

$$AS_e(z)^{1-q}(k^a l^{1-a})^q, \tag{4}$$

where k and l are capital and labor hired by the entrepreneur; $a \in (0,1)$ reflects the importance of capital in production; $q \in (0,1)$ is the span of control parameter (as in Lucas, 1978); and $AS_e(z)^{1-q}$ is the productivity level of a firm's production process. The productivity is the product of two components: the aggregate state of technology in the production sector A and an individual-specific productivity $S_e(z)$, which depends on the entrepreneurial ability z, with $S_e'(z) > 0$.

Given that entrepreneurs start life with zero assets, they have to borrow capital to run their firms. The borrowing is complicated by two factors: the ultimate success of the entrepreneur's project is uncertain, and the entrepreneur can hide the final outcome of his production project.

The production sector project is successful if output is produced according to technology (4). The probability of success is $\hat{\pi}$; with probability $1-\hat{\pi}$ the project is unsuccessful and no output is produced.

The producer borrows capital from the financial intermediaries at the risk-adjusted competitive interest rate $r_e/\hat{\pi}$ and hires labor at the competitive expected wage w, before he knows if his project is successful. Once the firm's inputs are employed, a random draw from a uniform distribution on [0,1] determines if the project is successful. The entrepreneur can hide the successful realization of his project from the financial intermediary with probability $1-P$, which depends on the level of financial development in the economy and will be defined below. If the project is successful, the entrepreneur produces final goods according to the technology (4), pays wages $w/\hat{\pi}$ to employed workers, and repays the loan conditional on successful monitoring by the intermediaries. If the project is unsuccessful, the entrepreneur announces bankruptcy and does not repay the loan to the financial intermediaries or the labor costs. For simplicity, the liquidation value of the bankrupt firm is zero.

The maximization problem of the producer is the following:

$$\max_{k,l} \ \hat{\pi} A S_e(z)^{1-q}(k^a l^{1-a})^q - r_e P k - wl. \tag{5}$$

For convenience, re-scale the probability of successful production project as follows: $\hat{\pi} = \pi^{1-q}$. The solution to problem (5) characterizes the optimal capital and labor inputs as functions of producer's ability, given wages, P, and interest rates:

$$l(z) = L_e \pi S_e(z), \tag{6}$$

$$k(z) = h_e l(z), \tag{7}$$

where

$$L_e = \left(\frac{qA(1-a)^{1-aq} a^{aq}}{w^{1-aq}(r^e P)^{aq}} \right)^{\frac{1}{1-q}}, \tag{8}$$

$$h_e = \frac{aw}{(1-a) r^e P}. \tag{9}$$

The expected profits of the potential entrepreneur with ability z can be expressed as follows:

$$E\Pi_e(z) = L_e \pi S_e(z) w \frac{\frac{1}{q} - 1}{1-a}. \tag{10}$$

Each potential producer decides whether to undertake an entrepreneurial project with expected payoff $E\Pi_e(z)$ or to become a worker with expected payoff w. Given that the expected profits are monotone increasing in ability, there is a threshold ability z_e^* such that all potential producers with $z \geq z_e^*$ undertake an entrepreneurial project. This threshold can be found from the following equation:

$$w = E\Pi_e(z_e^*). \tag{11}$$

The potential producers with ability lower than z_e^* choose to become workers, so that the total supply of labor from the group of potential producers is given by $\int_{\underline{z}_e}^{z_e^*} f_e(z)dz$.

Given z_e^*, the total labor, L, and capital, K, demanded by the operating producers are given by:

$$L = L_e \int_{z_e^*}^{\bar{z}} \pi S_e(z) f_e(z) dz. \tag{12}$$

$$K = h_e L. \tag{13}$$

These quantities depend on wages, interest rates, prices and the set of active bankers, which together with the set of active entrepreneurs (captured by z_e^*), are determined in equilibrium.

Note, however, that the labor demand L can be rewritten as a function of $z_e^\star$ only, combining (8), (11), and (12), in particular,

$$L(z_e^*) = \frac{\int_{z_e^*}^{\bar{z}} \pi S_e(z) f_e(z) dz}{S_e(z)} \frac{1-a}{(\frac{1}{q}-1)\pi}. \tag{14}$$

Therefore, the labor demand from the producers depend on wages and prices indirectly, through their impact on the threshold z_e^*.

3.2 The problem of a "potential banker"

Each individual from the group of potential bankers decides whether to run a financial intermediary institution or to be hired as a worker in the labor market. The decision is made based on the expected payoffs of these occupational choices. If the potential banker runs a financial intermediary institution, he can make profits by intermediating the funds from savers to borrowers, the operating producers. The bankers buy deposits

d on the deposits market at a competitive deposit interest rate $\hat{r}_b$ and sell loans to the producers at the competitive expected loan interest rate r_e. The total cost of capital for the financial intermediaries is $r_b = \hat{r}_b + \delta$, where δ is the depreciation rate of capital (similar to Greenwood et al., 2013). Parameter δ is constant throughout the model and r_b will be referred to as deposit interest rate. Each potential banker can operate a common to the financial sector technology, which allows him to monitor borrowers to reduce the probability $1 - P$ of their hiding the successful realizations of projects.

Monitoring requires labor input, therefore the bankers also hire workers in the labor market. The success of the monitoring depends positively on the banker's ability, z, through function $S_b(z)$, with $S_b'(z) > 0$, and labor input, x, and depends negatively on the volume of intermediated funds, d. In particular (similar to Greenwood et al., 2010), the probability of successful monitoring, P, is given by:

$$P = 1 - \frac{1}{(\frac{TS_b(z)^{1-\gamma}x^\gamma}{d})^\psi}, \ \frac{d}{Tx^\gamma} < S_b(z)^{1-\gamma}, \ \psi, \gamma \in (0, 1), \tag{15}$$

where $T > 0$ represents the financial sector's state of technology; γ reflects the importance of labor employed in the financial intermediation activities; and ψ is the span of control in financial intermediation. Specification (15) implies that an increase in the amount of capital intermediated requires more than a proportional increase in the labor employed for the intemediation activities (because $\gamma < 1$), and decreases the probability of successful monitoring (because $\psi < 1$). The inequality implies that the amount of intermediated deposits adjusted for the technology-augmented labor effort must not exceed certain level defined by the individual productivity of the banker to insure a positive probability of successful monitoring. Assume that $a(1 - \gamma) + \gamma/q < 1$ (this assumption is not restrictive for a plausible range of parameters).

The maximization problem of the banker is the following:

$$\max_{d,x} \ \left(1 - \frac{1}{(\frac{TS_b(z)^{1-\gamma}x^\gamma}{d})^\psi}\right) r_e d - r_b d - wx, \tag{16}$$

$$s.t. : \frac{d}{Tx^\gamma} < S_b(z)^{1-\gamma}. \tag{17}$$

The solution to this problem characterizes optimal deposits and labor inputs as functions of the banker's ability, given wages and interest rates:

$$x(z) = L_b S_b(z), \tag{18}$$

$$d(z) = h_b x(z), \tag{19}$$

where

$$L_b = \left(\frac{\gamma \psi T (r^e - r^b)^{\frac{1}{\psi}+1}}{w r^{e^{\frac{1}{\psi}}} (1+\psi)^{\frac{1}{\psi}+1}} \right)^{\frac{1}{1-\gamma}}, \tag{20}$$

$$h_b = \frac{w(1+\psi)}{\gamma \psi (r_e - r_b)}. \tag{21}$$

Substituting the expressions for labor and deposits demand by a financial intermediary, obtain that the probability of success that each operating banker faces in equilibrium depends only on the prices of capital:

$$P = \frac{\psi r_e + r_b}{r_e(1+\psi)}. \tag{22}$$

For positive interest-rate spread, $r_e - r_b$, P is bounded between zero and one.

Therefore, at optimum, the probability of successful monitoring is the same across all active financial intermediaries. The intermediaries with less ability to monitor borrowers will optimally choose to intermediate fewer funds.

The expected profits of the potential banker can be expressed as follows:

$$E\Pi_b(z) = L_b S_b(z) w(\frac{1}{\gamma} - 1). \tag{23}$$

Each potential banker decides whether to run an intermediary institution with expected payoff $E\Pi_b(z)$ or to become a worker with expected payoff w. There is a threshold ability z_b^* such that all potential bankers with $z \geq z_b^*$ run a financial intermediary institution. This threshold can be found from the equation:

$$w = E\Pi_b(z_b^*). \tag{24}$$

The potential bankers with ability lower than z_b^* choose to become workers, so that the total supply of labor in the labor market from the group of potential bankers is given by $\int_{\underline{z}_b}^{z_b^*} f_b(z)dz$.

Given z_b^*, the total labor, X, and capital, D, demanded by the operating bankers are given by:

$$X = L_b \int_{z_b^*}^{\bar{z}} S_b(z) f(z) dz, \tag{25}$$

$$D = h_b X. \tag{26}$$

These quantities depend on wages, interest rates, prices and the set of active entrepreneurs. The labor demand from the financial intermediaries can be rewritten as a function of z_b^* only, combining (20), (24), and (25), in particular,

$$X(z_b^*) = \frac{\int_{z_b^*}^{\bar{z}} S_b(z) f_b(z) dz}{S_b(z)} \frac{1}{(\frac{1}{\gamma} - 1)}. \tag{27}$$

Therefore, the labor demand from the bankers depend on wages and prices indirectly, through their impact on the threshold z_b^*.

Two features of the financial sector are specific to this model and should be highlighted. First, the common probability of successful monitoring makes all financial intermediaries identical from the point of view of both savers and borrowers. The set of active financial intermediaries represents a homogeneous financial system that accepts deposits and issues loans, performing monitoring along the way. Depositors can invest in, and producers can borrow from, several financial intermediaries within a period. In this sense, the model focuses on the determinants of the size of individual financial establishments rather than on the determinants of the size of capital loans issued to particular producers (differently from the related models with incentive compatibility constraints imposed on borrowers, such as Greenwood et al., 2010 and 2013).

Second, the technology of the financial sector T includes the factors that make the financial monitoring more efficient, and as formulated, is incomparable with the Solow residual, commonly reported as an estimate of the sector technology. That is, T represents an unobserved technological progress. This unobserved process can potentially be estimated given the observable variables, such as the interest-rate spread, as explained below.

3.3 The equilibrium

The focus of the analysis is on stationary equilibria. First, the market-clearing conditions are presented. Second, a definition for a stationary equilibrium is given. Third, it is shown that a stationary equilibrium for the model exists.

There are three markets in the model economy: a labor market, a market for deposits, and a market for loans. The labor market is common for both sectors. The entrepreneurs from the production and financial sectors demand labor according to functions L and X, respectively. The agents who choose not to be entrepreneurs supply labor with the total labor supply given by $\int_{z_e^*}^{\bar{z}} f_e(z)d(z) + \int_{z_b^*}^{\bar{z}} f_b(z)dz$. The wage w adjusts to clear the labor market.

The market for loans arises because the borrowers (entrepreneurs from the production sector) can shirk repaying their loans by falsely reporting their production projects as unsuccessful. The financial intermediaries can monitor borrowers' activities and reduce the probability of shirking, but the monitoring process is costly and requires some labor input. Therefore, the interest rate on loans is greater than the deposit interest rate. The supply of loans by the financial intermediaries is given by D, and the demand for loans is given by K. The lending interest rate r_e adjusts to clear the loans market.

The market for deposits is characterized by the demand for deposits from the financial intermediaries, D, and the supply of deposits by the savers. In a closed economy, savers are the retired agents, and the total supply of funds is a fraction of the total profits generated in the previous period. The deposit interest rate r_b adjusts to clear the savings market. In an open economy, capital is supplied at interest rate $\hat{r}_b$ which is taken as given by the savers and the financial intermediaries (who face the capital cost $r_b = \hat{r}_b + \delta$). The data suggests that $\hat{r}_b$ was decreasing during 1986–2005 (see Figure 1), and globalization of financial markets is considered to be the reason behind this decrease (see Caballero et al., 2008). Therefore, further analysis focuses on the open economy model as a more relevant framework.

The market-clearing conditions can be summarized in the following equations:

$$L(z_e^*) + X(z_b^*) = \int_{z_e^*}^{\bar{z}} f_e(z)d(z) + \int_{z_b^*}^{\bar{z}} f_b(z)dz, \tag{28}$$

$$h_e L(z_e^*) = h_b X(z_b^*). \tag{29}$$

These conditions define the prices r_e and w; r_b is given in the open economy. More formally, a competitive stationary equilibrium is defined as follows.

Definition: *A competitive stationary equilibrium given r_b, A, and T is described by the thresholds z_e^*, z_b^*, allocations $\{k(z), l(z)\}_{z_e^*}^{\bar{z}}$, $\{d(z), x(z)\}_{z_b^*}^{\bar{z}}$, wage w, and interest rate r_e, such that:*

i) *given w, r_e, and r_b, all agents maximize their expected income by choosing their occupation, and the thresholds z_e^* and z_b^* are determined, in accordance with (11) and (24);*

ii) *given w, r_e, and r_b, all producers and bankers choose capital and labor inputs to maximize their expected profits; and*

iii) *the wage, w, and the lending interest rate, r_e, are determined so that the markets for labor and loans clear, in accordance with (28)-(29).*

The values of exogenous variables A, T, and r_b determine the equilibrium prices, w and r_e, as well as the set of active producers and bankers.

It is possible to show that given any positive values of these variables, there exists a unique equilibrium for the model economy.

Proposition 1: *For any positive values of A, T, and r_b, there is a unique stationary equilibrium for the model economy.*

(All proofs are in the Appendix.)

The impact of the interest rate r_b and the technological progress, in either the financial or production sector, on the economy can now be characterized.

4 Characterization

The aim is to analyze the impact of sector-specific technological progress and the real interest rate on macroeconomic indicators, such as the economic output, the capital to output ratio, value added by sector, sector-specific distribution of establishments by size in terms of employment, and quantity of establishments by sector. First, the macroeconomic indicators in model terms are defined and their properties discussed.

Second, the pattern of responses of each of these indicators to a change in A, T, or r_b is established. Then, in the next section the model predictions are compared to the patterns in the data, and the model-implied trends of A and T in particular countries are discussed.

Given prices and thresholds z_e^* and z_b^*, the aggregate macroeconomic indicators can be computed as follows.

The interest-rate spread, s, can be derived from the capital market clearing condition:

$$s = r_e - r_b = \frac{(1-a)(1+\psi)X(z_b^*)}{a\gamma\psi L(z_e^*) - (1-a)\psi X(z_b^*)} r_b. \tag{30}$$

The expected total output, Y, is given by the sum of the total expected profits (of entrepreneurs, workers, and savers) or equivalently, by the total expected output in the final goods sector, as follows:

$$Y = \int_{z_e^*}^{\bar{z}} A(\pi S_e(z))^{1-q}(k^a l^{1-a})^q f_e(z)dz = h_e^{aq} L_e^q A \int_{z_e^*}^{\bar{z}} \pi S_e(z) f_e(z)dz. \tag{31}$$

The economy's output is proportional to the production sector technology, A, the quantity of labor hired by the production sector, and the probability of successful production, π, and negatively depends on the producer's costs of labor, w, and capital, reP.

The total capital to output ratio, from (13) and (31), is given by the following expression:

$$\frac{K}{Y} = \frac{aq}{r_e P} = \frac{aq(1+\psi)}{\psi s + (1+\psi)r_b} = \frac{aq(1+\psi)}{\psi r_e + r_b}. \tag{32}$$

The model predicts that the capital-output ratio increases with a decrease in the interest-rate spread, s, or the deposit rate, r_b.

The financial sector value added is given by the total profits of the financial intermediaries plus the labor costs or by the financial sector output minus the financial intermediaries' cost of capital. The financial sector value added as a share of the output is as follows:

$$VA_b = (r_e P - r_b)\frac{K}{Y} = \frac{\psi aqs}{\psi s + (1+\psi)r_b} = \frac{\psi aq(r_e - r_b)}{\psi r_e + r_b}. \tag{33}$$

It is a function of observable variables, the interest-rate spread, s, and the financial intermediaries' cost of capital, r_b. The share of financial sector value added is decreasing in r_b, because it becomes more costly to produce loans as the deposit rate increases and therefore, the supply of loans decreases. The share of financial sector value added is increasing in s and r_e, because it is more profitable to supply loans as their price increases, other things equal.

Similarly, the production sector value added is given by the total profits of the producers plus the labor costs or by the production sector output minus the producers' cost of capital input. The production sector value added as a share of output simplifies as follows:

$$VA_e = \frac{Y - r_e PK}{Y} = 1 - aq. \tag{34}$$

It is constant in the model economy, equal to one minus the share of capital in the production. A fraction aq of value added is generated by the financial intermediation of capital, which contributes VA_b, and by the savers, who invest their capital with financial intermediaries and contribute $r_b K$ to the total value added.

In addition to these standard indicators of economic performance, the measures of establishment size, quantity, and size distribution by sector can be computed in model terms, given the values of A, T, r_b, and parameters. Along with economic output indicators, these measures can be used to evaluate the impact of sector-specific technological progress on the economy.

The quantities of firms operating in the production and financial sectors, Q_e and Q_b, respectively, are given by:

$$Q_e = \int_{z_e^*}^{\bar{z}} f_e(z)dz, \tag{35}$$

$$Q_b = \int_{z_b^*}^{\bar{z}} f_b(z)dz. \tag{36}$$

The sector-specific average size of the establishment can be computed as the ratio of the total sector-specific labor demand to the quantity of establishments in a given sector:

$$LQ_e = \frac{L(z_e^*)}{\int_{z_e^*}^{\bar{z}} f_e(z)dz}, \tag{37}$$

$$LQ_b = \frac{X(z_b^*)}{\int_{z_b^*}^{\bar{z}} f_b(z)dz}, \tag{38}$$

where LQ_e and LQ_b stand for the average size of the establishment in terms of employment in the production and financial sectors, respectively.

The establishment size distribution in a given sector can be characterized by the share of employment in the smallest N percent of establishments in that sector, computed as follows:

$$LQ_{e,N} = \frac{\int_{z_e^*}^{z_e^N} S_e(z)f_e(z)dz}{\int_{z_e^*}^{\bar{z}} S_e(z)f_e(z)dz}, \text{ where } z_e^N \text{ solves: } N = \frac{F_e(z_e^N) - F_e(z_e^*)}{1 - F_e(z_e^*)}; \tag{39}$$

$$LQ_{b,N} = \frac{\int_{z_b^*}^{z_b^N} S_b(z)f_b(z)dz}{\int_{z_b^*}^{\bar{z}} S_b(z)f_b(z)dz}, \text{ where } z_b^N \text{ solves: } N = \frac{F_b(z_b^N) - F_b(z_b^*)}{1 - F_b(z_b^*)}, \tag{40}$$

for different Ns.

The economy growing at an exogenous growth rate determined by the relative growth rates of A and T and given r_b, can now be characterized.

Proposition 2: *Let T grow at rate g, A grow at rate $(1+g)^{1-aq} - 1$, and r_b be constant. There exists a balanced growth path where the wages, output, demand for loans, and demand for deposits all grow at rate g. The thresholds z_e^* and z_b^*, the quantity and the average size of establishments in the production and financial sectors, labor demand and supply, and the lending interest rate remain constant.*

This result is similar to the conclusion of Greenwood et al. (2010) that balanced development of the production and financial sectors does not make the financial sector more efficient. The probability of catching the firm that misrepresents its earnings is constant over time. The quantity of active establishments in both sectors does not change over time.

For a constant r_b, unbalanced growth occurs whenever technology in either sector outpaces the balanced growth of the other sector's technology. Intuitively, faster technological progress in the financial sector makes it more efficient in comparison to the production sector (P increases). The relative cost of monitoring producers drops, leading to higher competition for deposits, lower interest rates for loans, and crowding out of the least efficient financial intermediaries. At the same time, the greater supply and the lower price of funds make borrowing affordable for less efficient producers. Therefore, technological progress in the financial sector leads to worsening of the pool of borrowers from the production sector.

The opposite occurs when the production sector's technology grows relatively faster than the financial sector's technology: The demand for loans increases, increasing their price and crowding out the least efficient producers. High demand for loans makes financial intermediation profitable for less efficient financial intermediaries and reduces the relative efficiency of the financial system.

Proposition 3: *(a) Let A grow at rate $(1+g)^{1-aq}-1$, T grow at rate $g' < (>)\ g$, and r_b be constant. The threshold z_e^*, the financial sector value added, and the interest-rate spread increase (decrease) over time; the threshold z_b^* and the capital-to-output ratio decrease (increase) over time; output grows at a rate lower (greater) than g.*
(b) Let A/T grow at rate $(1+g)^{-aq}-1$, and r_b decrease (increase) over time. The threshold z_e^*, the financial sector value added, and the capital-to-output ratio increase (decrease) over time; the threshold z_b^* and the interest-rate spread decrease (increase) over time; output grows at a rate greater (lower) than g.

A decrease in the exogenously given r_b and a decrease in the financial sector technology T affect the capital-output ratio and the interest-rate spread in opposite directions while having a similar impact on the set of active entrepreneurs in each sector and the financial sector value added. In both cases, the financial sector becomes relatively less efficient. A drop in T implies that the financial sector technology is relatively low, leading to greater producer cost of capital and slower output growth. A drop in r_b implies lower financial intermediaries' cost of capital and leads to the supply of capital for producers at lower prices, accelerating output growth.

Proposition 3 establishes how the quantity of establishments in a particular sector responds to the changes in sector-specific technologies or in the deposit interest rate. An increase in sector-specific technology increases competition in a given sector, pushing the least efficient producers (those with lower z) out of business. As a result, the quantity of establishments decreases. The opposite happens in the sector which experiences a relative decrease in its aggregate sector-specific technology. The impact on the average establishment size, LQ_e and LQ_b, is uncertain in general and depends on the functions S_e and f_e (S_b and f_b). In the quantitative analysis, these functions are selected to replicate the observed patterns of the average establishment size and establishment size distributions in the US.

The next section uses observable data, such as the real interest rates on savings and the interest-rate spread, the total output, the financial sector share of output, and the establishment size distributions to quantitatively characterize the importance of changes in A, T, and r_b for the economy.

5 Quantitative Analysis

The aim of this section is to evaluate the model performance in replicating the data and to quantify the importance of changes in A, T, and r_b for economic development. First, the values of the model parameters are chosen to fit a number of empirical facts from the U.S. economy. Second, the values of macroeconomic indicators predicted by the model are compared to their counterparts in data from the US and Taiwan. Third, the impact of sector-specific technological progress or a decrease in the financial intermediaries' cost of capital on economic output is analyzed.

For quantitative analysis, the functions S_e and S_b, which describe individual-specific productivity in the production and financial sectors, and the abilities distribution functions, f_e and f_b, must be specified. Let the abilities of each type of agent follow Pareto distribution with the following parameters:

$$f_e(z) = v_e \underline{z}_e^{v_e} z^{-v_e-1}, \; z \in [\underline{z}_e, \infty), \; v_e > 1; \tag{41}$$

$$f_b(z) = v_b \underline{z}_b^{v_b} z^{-v_b-1}, \; z \in [\underline{z}_b, \infty), \; v_b > 1. \tag{42}$$

The corresponding cumulative distributions are $F_e(z) = 1 - \underline{z}_e^{v_e} z^{-v_e}$ and $F_b(z) = 1 - \underline{z}_b^{v_b} z^{-v_b}$, for the production and financial sectors, respectively.

The individual-specific productivity functions S_e and S_b must be increasing in ability

z. Consider the following functions:

$$S_e(z) = c_{e,0} + c_{e,1}z^{s_e}, \; c_{e,0}, c_{e,1} > 0, s_e \in (0, v_e); \tag{43}$$

$$S_b(z) = c_{b,0} + c_{b,1}z^{s_b}, \; c_{b,0}, c_{b,1} > 0, s_b \in (0, v_b). \tag{44}$$

The restrictions on the coefficients $c_{e,0}$ and $c_{b,0}$ imply that the average establishment size in the production and financial sectors is increasing in the thresholds z_e^* and z_b^*, respectively. The restrictions on s_e and s_b are necessary to generate finite labor demand.

5.1 Calibration

The model economy is fully characterized by five variables: z_e^*, z_b^*, A, T, and r_b. Only r_b is observable and can be taken directly from the data. The properties of z_e^*, z_b^*, A, and T can be gauged by analyzing the observed characteristics of the variables influenced by these unobservables. This is done as follows.

Parameter δ is assigned the standard value used in the literature, 0.05. Parameter a measures the importance of capital in production and is determined together with parameter q so that the aggregate capital share aq is equal to 0.30. Parameter ψ governs the share of the financial sector value added, VA_b defined by (33). I use $\psi = 0.97$, in line with Greenwood et al. (2013); the values of ψ in the range (0.9–0.99) deliver the model-predicted level of financial sector value added close to that observed in the data. Given the model parameters, PARAM=$\{\{c_{j,0}, c_{j,0}, v_j, s_j, \underline{z}_j\}_{j\in\{e,b\}}, \delta, a, q, \gamma, \psi, \pi\}$, and given the values of the exogenous variables, EXVARS=$\{A, T, r_b\}$, the thresholds z_e^* and z_b^* fully characterize the sector-specific average establishment sizes, LQ_e and LQ_b, and establishment distributions by employment size (which can be described by the share of employment in the smallest N percent of establishments, for different Ns, using (39) and (40)). Therefore, the empirically relevant thresholds z_e^* and z_b^* can be found by choosing the model parameters PARAM$\setminus\{\delta, a, \psi\}$, given the exogenous variables EXVARS, to minimize the distance between the model-predicted and empirical values of the average establishment sizes and establishment size distributions in the production and financial sectors. Following Greenwood et al. (2013), I consider the share of employment in the smallest 60, 75, 87, 95, 98, 99.3, and 99.7 percent of establishments to describe the establishment size distributions in the model and in the data.

The levels of the exogenous sector-specific technologies, A and T, have a direct impact on the production and financial sector output and determine relative prices, including the interest-rate spread. In particular, given r_b and the parameters, the equilibrium of the economy can be considered as providing the mapping between the aggregate level of output, Y, and the interest-rate spread, s, on the one hand, and the state of technology in the production and financial sectors, A and T, on the other. Represent this mapping by (Y,s)=M(PARAM,EXVARS). This mapping can be used to make an inference about (A, T), given an observation on (Y, s), by using the relationship (A,T)=M^{-1}(PARAM,Y,s,r_b), as in Greenwood et al. (2013).

The model is calibrated to the U.S. economy. As a measure of the cost of capital $\hat{r}_b$, I use the estimate of the global real interest rate from Caballero et al. (2008), computed as the weighted average of the interest rates on 3-month Treasury bills adjusted for inflation in the next quarter among the leading economies. This interest rate is highly correlated with the estimates of the U.S. real interest rate and with the estimates of the global real interest rate by King and Low (2014), and it is inconsequential which one is used in the estimation. What matters is the downward trend characterizing the U.S. and the global real interest rates during 1986–2005. The deposit rate r_b is then computed as $\hat{r}_b + \delta$.

Detailed sectoral data on the distribution of establishments by employment size is available on an annual basis starting from 1986 (at the U.S. Census Bureau, County Business Patterns). I consider the data for sectors with SIC codes 6000 and 6100 (NAICS code 522///) which correspond to financial intermediation as a counterpart of the financial sector in the model and data on all other sectors minus financial intermediation as a counterpart of the production sector in the model. During the period 1986–2005, there are no significant changes in the overall establishment size distributions. Therefore, I use the 1986 year data for the calibration targets. At the same time, during the period 1986–2005, the average size of the financial sector establishments decreases, and the average size of the production sector establishments increases. To achieve significant responses of the establishment size to changes in the thresholds z_e^* and z_b^*, PARAM are chosen so that the elasticity of the average production (financial) sector establishment size with respect to threshold z_e^* (z_b^*) is maximized.

To summarize, the calibration procedure is as follows:

(a) given deposit interest rate r_b, the values of parameters δ and ψ, and a guess for the remaining PARAM:

  (a.1) the values of z_e^*, z_b^*, w, and r_e are found from equations (28), (29), (11), and (24) jointly with

  (a.2) the values of A and T such that the model-generated output, Y, and the model-generated interest rate spread, $r_e - r_b$, are equal to the U.S. real GDP per capita (in thousands) and the U.S. interest-rate spread in 1986; and jointly with

  (a.3) the values of parameters γ, q, and a such that the financial and production sectors model-generated average establishment sizes are equal to their counterparts in 1986 U.S. data and the aggregate capital share is 0.30;

(b) step (a) is repeated until the sum of the squares of the differences between the model-generated and the 1986 observed shares of employment in the smallest 60, 75, 87, 95, 98, 99.3, and 99.7 percent of establishments together with the inverses of the model-implied elasticities of the average establishment sizes with respect to thresholds z_e^* and z_b^*, for the production and financial sectors, respectively, is minimized, subject to the following constraints: $s_e < v_e$, $s_b < v_b$, $v_e, v_b > 1$, $c_{e,1}, c_{b,1} > 0$, $0 < \pi < 1$.

Table 1 reports the parameter values. The two parameters that are standard in the literature, the importance of capital in production, $a = 0.365$, and the span-of-control in the production sector, $q = 0.823$, calibrated to match the aggregate share of capital and the production sector average establishment size, are within the range of the values used in the literature (0.28–0.40 and 0.8–1, respectively; see Greenwood et al., 2013 and Guner et al., 2008). The calibrated probability of successful production projects is relatively high, $\pi = 0.873$, suggesting that more than 80 percent of the establishments in the final good sector generate positive output.

Figure 3 presents the cumulative share of employment in the production and financial sector establishments in the 1986 U.S. data and in the model. The model overestimates and underestimates the shares of employment in the lowest and highest tails of the establishment size distributions, respectively, while fitting the middle relatively well. This is because there is a trade-off between the magnitude of the establishment size elasticities, $\frac{LQ_e}{\frac{dLQ_e}{dz_e^*}z_e^*}$ and $\frac{LQ_b}{\frac{dLQ_b}{dz_b^*}z_b^*}$ and a good fit of the establishment size distributions. If the establishment size elasticities are excluded from the minimization function, then the model-generated distributions are closer to their empirical counterparts, but the implied elasticities are zero, which contradicts the data. The experiments suggest that more complex distributions of abilities or individual technologies functions S_e and S_b have little impact on calibration results.

Table 1: Parameter Values

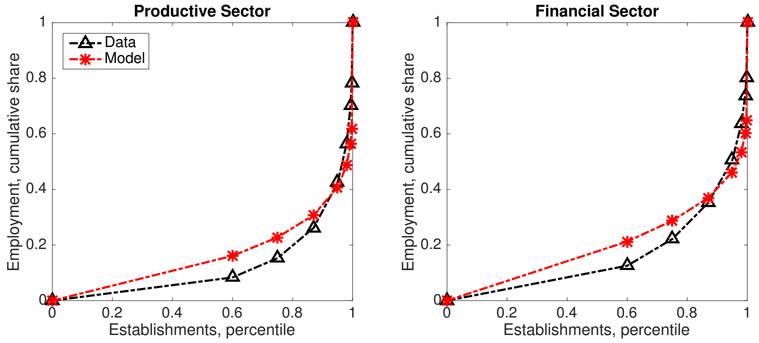
Parameter	Value
Depreciation rate, δ	0.050
Probability of successful production, π	0.873
Importance of capital in production, a	0.365
Importance of labor in finance, γ	0.864
Span of control in production, q	0.823
Span of control in finance, ψ	0.970
Shape parameter, abilities distribution in production, v_e	1.371
Shape parameter, abilities distribution in finance, v_b	1.726
Scale parameter, abilities distribution in production, $\underline{z}_e$	1.036
Scale parameter, abilities distribution in finance, $\underline{z}_b$	0.734
Constant, individual efficiency in production, $c_{e,0}$	5.805
Constant, individual efficiency in finance, $c_{b,0}$	273.322
Scale parameter, individual efficiency in production, $c_{e,1}$	2.264
Scale parameter, individual efficiency in finance, $c_{b,1}$	2.928
Shape parameter, individual efficiency in production, s_e	1.158
Shape parameter, individual efficiency in finance, s_b	1.478

Note: The table reports the values of the parameters obtained from the calibration of the model to the U.S. data.

5.2 Simulations

I simulate the model for the US (on an annual basis, during 1986–2005) and for Taiwan (every fifth year, during 1971–2011) because the necessary data (or plausible proxies) for these countries and time periods are available. For each country, given the values of the parameters reported in Table 1, I simulate the model along the equilibrium growth path at which the model-generated output replicates the annual country's real GDP per capita, the model-generated interest-rate spread replicates the country's annual interest-rate spread, and the interest rate r_b in the model is equal to the country's annual real

Figure 3: Establishment size distribution, 1986 US data and model



Note: The figure presents the cumulative share of employment in the smallest N percent of establishments in the 1986 US data (black line with triangles for data points) and in the model (red line with stars for data points). The left and right graphs report the values for the production and financial sectors, respectively. Data sources are described in the Appendix.

interest rate plus depreciation δ. That is, I repeat steps (a.1) and (a.2) of the calibration procedure for every year from 1986 to 2005 for the US or every fifth year from 1971 to 2011 for Taiwan, using the values of PARAM from Table 1.

The following key model-generated variables are functions of unobservable A and T (or equivalently, of observable Y and $r_e - r_b$) and can be used to evaluate the model performance: the financial sector value added (the production sector value added is constant given by (34)), the sector-specific employment size, and the quantity of establishments in each sector. The performance of the model in replicating each variable for the US and Taiwan is discussed next.

5.2.1 Results for the US

Table 2 summarizes the U.S. data for the first and last years of the time period considered, 1986 and 2005, and the values of the corresponding variables in the model. Figure 4 plots the time series of three key model-generated variables, the financial sector share of value added, the average establishment size in the financial sector, and the fraction of financial sector establishments, and their empirical counterparts for the US. The model captures (with different degrees of quantitative success) the increase in the financial sector value added during 1986–2005 and the trends in the sector-specific establishment characteristics: an increase in the average size of the production sector establishments, a decrease in the average size of the financial sector establishments, and an increase in the fraction of financial establishments in the total number of establishments.

The model-predicted share of financial sector value added, which can be computed independently of any other predictions of the model using the observable values of r_b and s, is very close to its empirical counterpart, and the two variables move together over time (see Figure 4, left panel). The capital-to-output ratio in model terms is linked to the financial sector value added and can be computed from the data on r_b and s and the given parameters a, q, and ψ. The model-generated values are close in magnitude to their empirical counterparts, although the model predicts an increase in the capital-to-output ratio, while in the data the ratio is relatively stable over time.

The model-generated values for the thresholds of abilities characterizing active entrepreneurs, z_e^* and z_b^*, and for the technology levels, A and T, can be used to make inferences about the potential drivers behind the trends observed in Figure 4. According to the model, given the trends in the U.S. output and the interest-rate spread, the minimum (and average) entrepreneurial efficiency in the production sector increased, while the minimum (and average) entrepreneurial efficiency in the financial sector decreased during 1986–2005. Thus, the average financial sector entrepreneur became less efficient over time, and the average production sector entrepreneur became more efficient over time; as a consequence, the fraction of the financial sector establishments increased. This could happen for either or both of the following reasons: (1) because the financial sector technology grew at a rate that was lower than the rate necessary to maintain the constant relative efficiency of the production and financial sector and (2) because the financial intermediaries' cost of capital, the interest rate on savings, decreased. The growth rates of the production and financial technologies over 1986–2005 are 0.0050 and 0.0107, respectively (these values can be obtained from Table 2 using formula $g_Z = (Z_{2005}/Z_{1986})^{1/(2005-1986)} - 1$, where $Z = T, A$). From Proposition 3, for a balanced growth path along which T grows at rate $g = 0.0107$, A should grow at rate $(1+g)^{1-aq} - 1 = 0.0075$. Thus, the financial sector technology grew at a rate greater than the rate required for the balanced growth path. According to the model, this should lead to an increase in the relative efficiency of the financial sector and to the growth of the total output at a rate greater than g.

However, the real interest rate decreased during 1986–2005. The fact that the relative efficiency of the financial sector declined (as reflected in the decline in the average

Table 2: Results: the US data and the model

	1986		*2005*	
	Data	Model	Data	Model
Output, Y	33.400	33.400	49.583	49.583
Int. rate spread, $s = r_e - r_b$	0.031	0.031	0.026	0.026
Finance value added, VA_b	0.038	0.036	0.048	0.059
Capital-to-output, K/Y	3.213	2.373	3.017	4.658
Avg. est. size production, LQ_e	14.223	14.223	15.526	14.346
Avg. est. size finance, LQ_b	20.949	20.949	14.957	15.837
Fraction of finance est., $\frac{1-F_b(z_b^*)}{2-F_b(z_e^*)-F_b(z_b^*)}$	0.020	0.039	0.029	0.081
Min. entrepr.efficiency prod., $S_e(z_e^*)$		19.552		20.072
Min. entrepr.efficiency fin., $S_b(z_b^*)$		445.103		364.742
Production technology, A		4.423		4.860
Finance technology, T		4102.9		5019.9

Note: The table reports the values of the variables observed in the US in 1986 and 2005, in columns "Data," and the corresponding model-generated values, in columns "Model." The model-generated values are obtained from the simulations along the equilibrium growth path at which the model-generated output replicates the annual U.S. real GDP per capita, the model-generated interest-rate spread replicates the U.S. interest-rate spread, and the interest rate r_b in the model is equal to global real interest rate plus depreciation δ. The last four rows report the model-generated variables which do not have observable counterparts in the data.

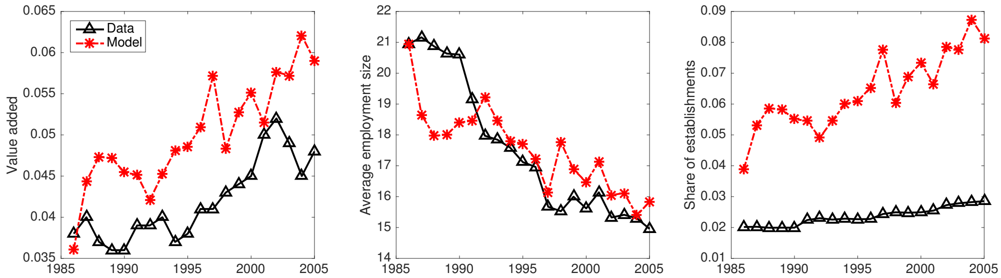
financial sector entrepreneur's efficiency and an increase in the financial sector share of value added) implies that the impact of the relative improvement in the financial sector technology was overturned by the decrease in the real interest rate. This led to a decline in the probability of successful monitoring of borrowers, P, defined in model terms, from 0.89 to 0.83 between 1986 and 2005.

At the same time, according to the model, the decline in the real interest rate contributes to the total output growth. Therefore, the financial sector technology growth rate 0.0107 combined with the observed decline in the real interest rate is consistent with the output growth rate 0.021, as observed in the US during 1986–2005.

5.3 Results for Taiwan

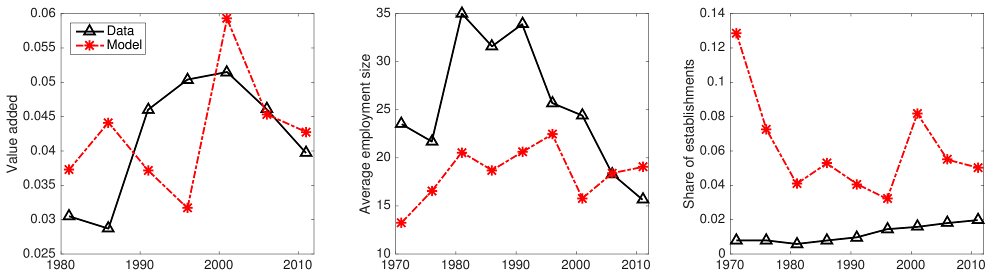
Taiwan is one of the few countries for which some data on the establishment size distribution by sector is available (all data sources are in the Appendix). In addition, Taiwan experienced rapid economic growth in the last three decades and a sharp decline in the interest-rate spread, from 0.0744 in 1971 to 0.0196 in 2011. Therefore, I check the model performance in explaining the patterns of the key variables using Taiwanese data (similar to Greenwood et al., 2013). Data on establishment size is available for every

Figure 4: Characteristics of the financial sector, the U.S. data and the model



Note: The figure presents the U.S. data (black lines with triangles for available data points) and the model-generated data (red lines with stars) for financial intermediation sector. The left graph reports the share of value added; the middle graph reports the average establishment size measured by the number of persons engaged; the right panel presents the share of financial establishments in total establishments. Data sources are described in the Appendix.

Figure 5: Characteristics of the financial sector, Taiwanese data and the model



Note: The figure presents the Taiwanese data (black lines with triangles for available data points) and the model-generated data (red lines with stars) for financial sector. The left graph reports the share of value added; the middle graph reports the average establishment size measured by the number of persons engaged; the right panel presents the share of financial establishments in total establishments. Data sources are described in the Appendix.

fifth year starting from 1971 until 2011. The data on financial sector value added is available from 1981. For consistency with the analysis for the US, I consider 2006 the final year. As an estimate of the real interest rate on savings, I consider the Taiwanese annual deposit interest rate adjusted for inflation. That is because although there is no significant difference between the U.S. real interest rate and the global real interest rate, the Taiwanese real interest rate is significantly higher than the global real interest rate.

Table 3 reports the data and model values for 1981 and 2006. Figure 5 shows the time series for three key variables in the data and corresponding variables generated by the model: the financial sector share of value added, the average establishment size in the financial sector, and the fraction of financial sector establishments.

Table 3: Results: Taiwanese data and the model

	1981		*2006*	
	Data	Model	Data	Model
Output, Y	10.132	10.132	35.958	35.958
Int. rate spread, $s = r_e - r_b$	0.044	0.044	0.024	0.024
Finance value added, VA_b	0.031	0.037	0.046	0.045
Capital-to-output, K/Y	1.436	1.713	2.479	3.859
Avg. est. size production, LQ_e	8.300	14.230	6.300	14.273
Avg. est. size finance, LQ_b	35.000	20.580	18.300	18.419
Fraction of finance est., $\frac{1-F_b(z_b^*)}{2-F_b(z_e^*)-F_b(z_b^*)}$	0.006	0.041	0.018	0.055
Min. entrepr.efficiency prod., $S_e(z_e^*)$		19.579		19.761
Min. entrepr.efficiency fin., $S_b(z_b^*)$		438.139		401.342
Production technology, A		2.118		4.058
Finance technology, T		847.996		4776.4

Note: The table reports the values of the variables observed in Taiwan in 1981 and 2006, in columns "Data," and the corresponding model-generated values, in columns "Model." The model-generated values are obtained from the simulations along the equilibrium growth path at which the model-generated output replicates the annual Taiwanese real GDP per capita, the model-generated interest-rate spread replicates the Taiwanese interest-rate spread, and the interest rate r_b in the model is equal to Taiwanese real interest rate plus depreciation δ. The last four rows report the model-generated variables which do not have observable counterparts in the data.

The model-specific variables measuring the sector-specific entrepreneur's efficiency suggest that the Taiwanese financial sector became relatively less efficient during the considered time period. The computations similar to those performed for the US suggest that the production sector technology grew at a rate of 0.0263, while the financial sector technology grew at a rate of 0.0716, which is greater than the balanced growth path rate given the growth rate of A. At the same time, similar to the US, the financial intermediaries' cost of capital declined in Taiwan. This decline in the real interest rate overturned the impact of the financial sector technology growth on the relative efficiency of the financial sector.

The observed trends in the Taiwanese sector-specific establishment size characteristics are non-linear (see Figure 5). There was an increase in the average size of the financial sector establishments in 1986 compared with previous years, followed by a decline until 2011. The model replicates some of the dynamics (none of the Taiwanese data was used as the calibration target) on the average employment size and the fraction of the financial establishments. The model also predicts the share of financial sector value added which is, on average, close to its empirical counterpart.

Table 4: Counterfactuals for the US and Taiwan

	Benchmark	r_b *fixed*	*T fixed*	r_b, T *fixed*	*A fixed*
		The US			
Output	49.583	38.340	48.722	37.941	43.812
Int. rate spread	0.026	0.030	0.029	0.033	0.024
Finance value added	0.059	0.035	0.065	0.039	0.055
Capital-to-output	4.658	2.383	4.547	2.350	4.728
Avg. est. size, production	14.346	14.217	14.377	14.236	14.327
Avg. est. size, finance	15.837	21.327	15.031	20.202	16.414
Fraction of finance est.	0.081	0.037	0.093	0.043	0.074
		Taiwan			
Output	35.958	31.403	21.795	20.800	7.029
Int. rate spread	0.024	0.026	0.214	0.218	0.009
Finance value added	0.045	0.035	0.185	0.157	0.019
Capital-to-output	3.859	2.726	1.750	1.465	4.252
Avg. est. size, production	14.273	14.215	14.933	14.818	14.132
Avg. est. size, finance	18.419	21.425	9.693	10.162	27.984
Fraction of finance est.	0.055	0.037	0.320	0.275	0.016

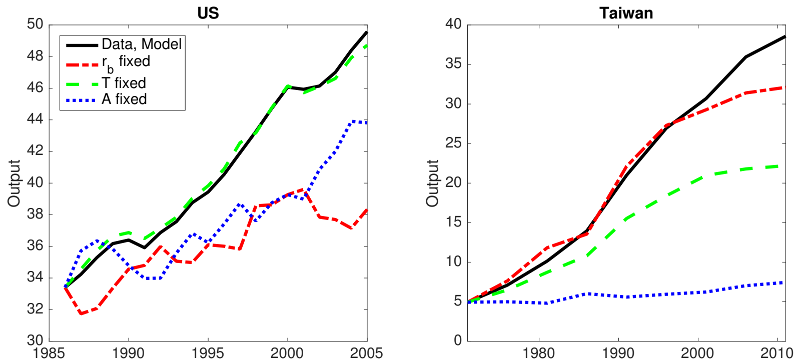
Note: The table presents the values of the variables predicted by the benchmark model for the final period (2005 for the US and 2006 for Taiwan) and the counterfactual model-generated outcomes if one of the following variables is fixed at its initial period value: the cost of capital, the financial sector technology, the cost of capital together with the financial sector technology, and the production sector technology, for the US (top panel) and for Taiwan (bottom panel).

5.4 The Importance of Sector-Specific Technological Progress for Economic Development

Given that the model captures the main trends in the data, it can be used to evaluate the importance of sector-specific technological progress and the observed decline in the real interest rate on economic development measured by output growth. The top and bottom panels of Table 4 report the results of counterfactual experiments for the US and Taiwan, respectively. Table 4 reports the values of variables that, according to the model, would characterize the economy at the final period (2005 for the US and 2006 for Taiwan) if r_b, T, r_b and T, or A were held constant at their initial period value.

First, consider the US. If the production and financial sector technologies grew as predicted by the model (and reported in Table 2) but the real interest rate on savings remained fixed at the 1986 level, then the 2005 output (real GDP per capita) in the US would be 23 percent lower that the observed 2005 output (38.340 thousands versus 49.583 thousands). If the financial sector technology remained fixed at its 1986 level, but the real interest rate changed as in the data and the production sector technology grew

Figure 6: The impact of sector-specific technological progress and the real interest rate on output



Note: The figure presents the data and model-generated economic output (black solid line) and the counterfactual model-generated output for the US (left graph) and for Taiwan (right graph) if one of the following variable is fixed at its initial period value: the cost of capital (red dashed-dotted line), the production sector technology (blue dotted line), and the financial sector technology (green dashed line).

as predicted by the model, then the 2005 output in the US would not be very different from the observed 2005 output. Finally, if the production sector technology A remained fixed at its 1986 level, then the 2005 output in the US would be 12 percent lower than the observed 2005 output. According to these numbers, the economic development in the US during the last three decades was mostly driven by the decline in the real interest rate, while the financial sector technology growth had a minor role.

The implications for the relative efficiency of the production and financial sectors can be easily accessed from Table 4. For example, if the financial sector technology, T, remained fixed at the 1986 level, then the financial sector would become more inefficient compared to the benchmark model where T grows consistently with the observed output growth. The 2005 financial sector value added would be 0.065 (versus 0.059 in the benchmark), and the fraction of the financial sector establishments would be larger (0.093 versus 0.081).

For Taiwan, the picture is different. If the production and financial sector technologies in Taiwan grew as predicted by the model (and reported in Table 3) but the real interest rate on savings remained fixed at the 1971 level, then the 2006 output (real GDP per capita) in Taiwan would be only 13 percent lower that the observed 2006 output (31.403 thousands versus 35.958 thousands). If the financial sector technology remained fixed at its 1971 period level, but the real interest rate changed as in the data and the production sector technology grew as predicted by the model, then the 2006 output in Taiwan would be 39 percent lower than the observed 2006 output. Finally, if the production sector technology A remained fixed at the 1971 level, then the 2006 output in Taiwan would be 80 percent lower than the observed 2006 output. Thus, according to the model, the growth of the production and financial sector technologies were the main drivers of Taiwanese output growth during the last three decades. Figure 6 offers graphical representation of the observed output series for the US and Taiwan, and the levels of output that would occur if one of the exogenous variables remained fixed at its initial period value.

The patterns of output dependence on sector-specific technological progress are consistent with different stages of development in the US and Taiwan during the last three decades. The US, a developed country with relatively high levels of sector-specific technologies, is considered to offer "safe" assets (see, for example, Gourinchas and Rey, 2016; and Caballero et al., 2017) and therefore, enjoyed low interest rates (the financial intermediaries' cost of capital) during the last three decades, which allowed the U.S. output to grow above the balanced growth rate, according to the model. Most of the economic development in Taiwan happened during the last three decades, and the growth in the production and financial sector technologies was at the heart of this development, according to the model.

6 Conclusions

This paper analyzes the impact of unobserved technological progress in the production and financial sectors and the observed decline in the real interest rates for these sectors relative efficiency and the implications for the characteristics of the sector-specific establishments. I construct a model that describes how the occupational choices in the production and financial sectors are affected by the costs of inputs and the relative technological efficiency of these sectors and how these occupational choices define the average size and the quantity of establishments in these sectors. In addition, the model proposes a formula for computing the financial sector share of value added as a function of the interest-rate spread and the financial intermediaries' cost of capital, and the implied values are very close to their empirical counterparts in the US and Taiwan.

The quantitative analysis suggests that the technology growth of the financial sector outpaced the growth of technology in the production sector in the US and Taiwan during the last three decades. However, the decline in the real interest rates on savings, which reflects the financial intermediaries' cost of capital, overturned the positive impact of the financial sector technology, making the financial sector less efficient compared to the production sector. According to the model, the decline in the real interest rate had a significant positive impact on output growth in the US, while for Taiwan, the growth in the financial and production sector technologies was the main driving force behind the output growth observed in the last three decades.

The model captures the trends in all of the key variables for the US except for the capital to output ratio, which increases in the model but is relatively stable, with insignificant negative trend, in the data. The model focuses on the interactions between the production and financial sector establishments and assumes that the intermediated assets can be transformed into physical capital without any cost. It would be interesting to consider an extension where physical capital accumulation process is modeled explicitly and the relative price of investment goods is a function of sector-specific technological progress. In addition, the observed capital to output ratios can, at least partially, be driven by trade frictions in international trade in capital goods (Mutreja et al., 2018) or the presence of private information frictions (Khan and Ravikumar, 2001). A financial intermediation framework with a clear separation between the physical capital and financial assets (as suggested by Gomme et al., 2011) is a promising avenue for future research.

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Appendix

Data Sources

Financial Industry Share of Value Added: For the US, this variable is computed using the industry output data on Financial Intermediation value added from the U.S. Bureau of Economic Analysis website. For Taiwan, this variable is computed using the industry output data on Financial Intermediation and Insurance Services (because more desegregated data is not available) from the National Statistics of Republic of China (Taiwan).

Real GDP per capita: I use the real GDP divided by population, both series taken from the Penn World Tables (Feenstra et al., 2015).

Interest-rate spread: I compute the interest-rate spread for the U.S. as suggested by Mehra et al. (2009) and used by Greenwood et al. (2013). I strictly follow the rules described by Mehra et al. (2009) and use the data sources suggested by these authors. The resulting interest-rate spread path is slightly different from the path reported by Greenwood et al. (2013) which might be due to the adjustment in data measurements that occurred over time (I use the NIPA Tables last revised on August 3, 2017 and the Financial Accounts of the United States issued in 2017). For Taiwan, the interest-rate spread is computed as in Lu (2013), using the data from the Central Bank of the Republic of China (Taiwan). In particular, the spread is computed as the average over monthly differences between the Base Lending Rate and Month Deposit Rate.

Financial Intermediaries' Cost of Capital, the Real Interest Rate on Savings: For the US, the data on the global short term real interest rate, computed as weighted (by the share of the economy in the world GDP) average of the short term interest rates (adjusted for inflation) on treasury bills in major economies (US, UK, Belgium, France, Germany, Netherlands, Sweden, Canada, Japan), is taken from Caballero et al. (2008), available at https://www.aeaweb.org/articles?id=10.1257/aer.98.1.358. It is highly correlated with the real interest rate characterizing the US and with the long-term real interest rates (available from the same source).

For Taiwan, I use a proxy of the real interest rate on savings computed as the annual interest rate on month deposits adjusted for inflation, using the data from the Central

Bank and the National Statistics of Republic of China (Taiwan).

Establishment-size distribution: I look at the distribution of establishment by size measured as the number of person engaged by the establishment. For the U.S., I rely on the data from County Business Patterns from the U.S. Census Bureau. This data with detailed decomposition by sector is available for years 1986–2014 at: https://www.census.gov/data. I consider the data on sectors with SIC codes 6000 and 6100 (NAICS code 522///) which correspond to Financial Intermediation, a counterpart of the financial sector in the model, and the data on all other sectors minus the Financial Intermediation as a counterpart of the production sector in the model (I check the data for the sectors including Financial Intermediation and Insurance services, the trends are very similar to those observed in the sectors including the Financial Intermediation only).

For Taiwan, I use Industry and Service Census data from the National Statistics of Republic of China (Taiwan), available at: http://eng.stat.gov.tw. The summary data is available for years 1971, 1976, 1981, 1986, 1991, 1996, 2001, 2006, and 2011. I consider Financial Intermediation and Insurance Services as a counterpart of the financial sector in the model, because the information on the Financial Intermediation only is not available for Taiwan.

Capital to Output ratio: I use the ratio of capital to GDP, both series taken from the Penn World Tables (Feenstra et al., 2015).

Proofs

Proof of Proposition 1. First, I will summarize all the equations in a function of z_b^*. Second, I will show that this function is monotone decreasing and achieves zero at the value proportional to $A/(r_bT)$. Denote $\int_{z_e^*}^{\bar{z}} S_e(z)f_e(z)dz = I_e(z_e^*)$ and $\int_{z_b^*}^{\bar{z}} S_b(z)f_b(z)dz = I_b(z_b^*)$.

The demand functions (14) and (27) are decreasing: $X'(z_b^*) = (-S_b(z_b^*)f_b(z_b^*)S(z_b^*) - S'(z_b^*)I_b(z_b^*))/S(z_b^*)^2 \frac{1}{(\frac{1}{\gamma}-1)} < 0$, because $S_b(z_b^*), f_b(z_b^*) > 0$ and $S'(z_b^*) > 0$; similarly, $L'(z_e^*) < 0$.

Consider the labor market clearing condition:

$$L(z_e^*) + X(z_b^*) = F_e(\bar{z}) + F_b(\bar{z}) - F_e(z_e^*) - F_b(z_b^*). \tag{45}$$

This equation implicitly defines z_e^* as a function of z_b^*. An increase in z_b^* leads to a decrease in $X(z_b^*)$ and, given that $F_b'(z_b^*) < 0$ and $F_e'(z_e^*) < 0$, z_e^* must be decreasing in z_b^*.

Next, use the equation defining capital market clearing to find interest rate and interest-rate spread:

$$K = D, \tag{46}$$

$$h_e L = h_b X, \tag{47}$$

$$\frac{aw}{(1-a)(\psi r_e + r_b)} L = \frac{w(1+\psi)}{\psi\gamma(r_e - r_b)} X, \tag{48}$$

$$r_e = \frac{(1-a)X + a\gamma\psi L}{a\gamma\psi L - (1-a)\psi X} r_b. \tag{49}$$

$$r_e - r_b = \frac{(1-a)(1+\psi)X}{a\gamma\psi L - (1-a)\psi X} r_b. \tag{50}$$

$$\psi r_e + r_b = \frac{a\gamma\psi(1+\psi)L}{a\gamma\psi L - (1-a)\psi X}. \tag{51}$$

Finally, use the equations defining L_e and L_b in terms of $z_e^\star$ and $z_b^\star$:

$$L_e = \left(\frac{qA(1-a)^{1-aq}a^{aq}}{w^{1-aq}(r^e P)^{aq}}\right)^{\frac{1}{1-q}} = \frac{(1-a)}{(\frac{1}{q}-1)}(\pi S(z))^{-1}, \tag{52}$$

$$L_b = \left(\frac{\gamma\psi T(r^e - r^b)^{\frac{1}{\psi}+1}}{wr^{e^{\frac{1}{\psi}}}(1+\psi)^{\frac{1}{\psi}+1}}\right)^{\frac{1}{1-\gamma}} = \frac{S_b(z)^{-1}}{\frac{1}{\gamma}-1}, \tag{53}$$

$$\tag{54}$$

Consider the ratio of these equations taken to the powers $(1-q)$ and $(1-aq)(1-\gamma)$. Substituting expressions for interest rate and simplifying, obtain:

$$\frac{\left(\frac{L}{X}\right)^{aq}}{\left(1+\frac{a}{1-a}\gamma\psi\frac{L}{X}\right)^{\frac{1}{\psi}(1-aq)}\left(\frac{a}{1-a}\gamma\frac{L}{X}-1\right)}\frac{S_b^{(1-aq)(1-\gamma)}}{S_e^{1-q}} = \frac{\bar{C}A\pi^{1-q}}{r_b T^{1-aq}}, \tag{55}$$

where $\bar{C} = \frac{q(1-a)^q(1/q-1)^{1-q}}{\gamma(1/\gamma-1)^{(1-aq)(1-\gamma)}}$.

Given that $\frac{L}{X} = \frac{S_b I_e}{S_e I_b}$, we can express $S_b = \frac{L}{X}\frac{S_e I_b}{I_e}$, so that the left hand side (LHS) becomes:

$$\frac{\left(\frac{L}{X}\right)^{1-\gamma+aq\gamma}}{\left(1+\frac{a}{1-a}\gamma\psi\frac{L}{X}\right)^{\frac{1}{\psi}(1-aq)}\left(\frac{a}{1-a}\gamma\frac{L}{X}-1\right)}\frac{I_b^{(1-aq)(1-\gamma)}}{I_e^{(1-aq)(1-\gamma)}S_e^{1-q-(1-aq)(1-\gamma)}}. \tag{56}$$

The LHS of (55) tends to infinity as z_b^* tends to some value $\hat{z}$ such that $\frac{L}{X} \to \frac{1-a}{a\gamma}$. The LHS tends to zero as $z_b^* \to \bar{z}$ (to see this, note that $\lim\limits_{z_b^* \to \bar{z}} \frac{L}{X} = \infty$, $\lim\limits_{z_b^* \to \bar{z}} I_b = 0$ and divide the nominator and the denominator by $\frac{L}{X}^{1-\gamma+aq\gamma}$ to conclude that the nominator tends to zero while the denominator tends to infinity). Moreover, given assumption that $a(1-\gamma)+\gamma/q<1$, the LHS is monotone decreasing in $z_b^\star$: the derivative is

$$G_1(z_b^\star)\left(\frac{dL}{dz_b^\star} - \frac{dX}{dz_b^\star}\frac{L}{X}\right)\left[1-\gamma+aq\gamma - \frac{\frac{a}{1-a}\frac{L}{X}\gamma}{\frac{a}{1-a}\gamma\frac{L}{X}-1} - \frac{(1-aq)\gamma\frac{a}{1-a}\frac{L}{X}}{1+\frac{a}{1-a}\gamma\psi\frac{L}{X}}\right] +$$
$$G_2(z_b^\star)\left[\frac{(1-aq)(1-\gamma)}{I_b}\frac{dI_b}{dz_b^\star} - \frac{(1-aq)(1-\gamma)}{I_e}\frac{dI_e}{dz_b^\star} - \frac{1-q-(1-aq)(1-\gamma)}{S_e}\frac{dS_e}{dz_b^\star}\right],$$

where G_1 and G_2 are positive values given $z_b^\star$ and terms in square brackets are negative (when $z_b^\star$ increases, I_b decreases, S_e decreases, I_e and L increases and X decreases). Given that the range of function determined by the LHS is $(0;\infty)$ on the domain $(\hat{z},\bar{z})$, the LHS equals the right hand side (RHS) for positive A, T, and r_b at some $z_b^* \in (\hat{z},\bar{z})$, therefore, equilibrium exists. Given the LHS is monotone function, the equilibrium is unique.■

Proof of Proposition 2. Let A grow at rate $(1+g)^{1-aq}-1$, and T grow at rate g. If there exist a solution, there exist w and r_e that clear the markets, given r_b. Conjecture that along a balanced growth path wages, w, grow at rate g, and interest rate r_e is constant. Then L_e and L_b are constant. From (11) and (24) it means that the thresholds z_e^* and z_b^* are constant. Therefore, given (21) and (20) $d(z)$ grows at rate g. This implies that probability $P(z)$ is constant.

Given that L_e, L_b, z_e^* and z_b^* are constant over time, labor demand functions $l(z)$, $x(z)$ are constant over time. From (9), (13), (21), and (26), the capital loans demand is growing at rate g, same rate as the supply of loans. Finally, output given by (31) is proportional to wages and grows at rate g. Therefore, the conjectured solution for the rates of growth of w and r_e was correct. ■

Proof of Proposition 3. Follows from proofs of Propositions 1 and 2, given equation (55) and the fact that the LHS of this equation is decreasing in z_b^*. Notice that $\frac{ds}{dr_b} = \frac{d(r_e-r_b)}{dr_b} > 0$ from (30) and from the fact that $\frac{d(L/X)}{dr_b} < 0$.■